Исследовать ф-цию на условный экстремум

1. $U = 3 - 8x + 6y, \quad x^2 + y^2 = 36$

$L(\lambda_1, x, y) = 3 - 8x + 6y + \lambda_1 (x^2 + y^2 - 36)$

$$\begin{cases} L'_x = -8 + \lambda_1 \cdot 2x = 0 \\ L'_y = 6 + \lambda_1 \cdot 2y = 0 \\ L'_{\lambda_1} = x^2 + y^2 - 36 = 0 \end{cases} \Rightarrow \begin{cases} x = \frac{8}{2\lambda_1} \\ y = -\frac{6}{2\lambda_1} \\ \frac{64}{4\lambda_1^2} + \frac{36}{4\lambda^2} = 36 \end{cases} \Rightarrow$$

$$\begin{cases} x = \frac{8}{2\lambda_1} \\ y = -\frac{6}{2\lambda_1} \\ \lambda_1^2 = \frac{25}{36} \end{cases} \Rightarrow \left(\frac{5}{6}, \frac{24}{5}, -\frac{18}{5}\right), \left(-\frac{5}{6}, -\frac{24}{5}, \frac{18}{5}\right)$$

$L''_{xx} = 2\lambda_1 \quad L''_{yy} = 2\lambda_1 \quad L''_{\lambda_1\lambda_1} = 0$

$L''_{xy} = 0 \quad L''_{x\lambda_1} = 2x \quad L''_{y\lambda_1} = 2y$

$$\begin{vmatrix} 0 & 2x & 2y \\ 2x & 2\lambda_1 & 0 \\ 2y & 0 & 2\lambda_1 \end{vmatrix} = -2x \cdot 2x \cdot 2\lambda_1 - 4y^2 \cdot 2\lambda_1 =$$

$$= -8x^2\lambda_1 - 8y^2\lambda_1 = -8\lambda_1(x^2 + y^2) \Rightarrow$$

$$\Rightarrow \left(\frac{5}{6}, \frac{24}{5}, -\frac{18}{5}\right) - \text{мин}, \left(-\frac{5}{6}, -\frac{24}{5}, \frac{18}{5}\right) - \text{max}$$

2. $2x^2+12xy+32y^2+15$, $x^2+16y^2=64$

в ф-ции U $2x^2+32y^2$ можем заменить на 128:

$$U = 12xy+143$$

$$L(\lambda_1, x, y) = 12xy+143+\lambda_1(x^2+16y^2-64)$$

$$\begin{cases} L'_x = 12y+\lambda_1 \cdot 2x = 0 \\ L'_y = 12x+\lambda_1 \cdot 32y = 0 \\ L'_{\lambda_1} = x^2+16y^2-64=0 \end{cases}$$

① $y = -\dfrac{\lambda_1 \cdot x}{6}$

② $12x - \lambda_1 \cdot 32\dfrac{\lambda_1 x}{6} = 0 \Rightarrow \lambda^2 = \dfrac{9}{4} \Rightarrow \lambda_1 = \pm\dfrac{3}{2}$

Подставим λ_1 в систему (1-е и 2-е уравн. станут равнозначными):

$$\begin{cases} 12y+3x=0 \\ x^2+16y^2=64 \end{cases} \Rightarrow \begin{cases} x=-4y \\ y^2=2 \end{cases}$$

$y_{1,2} = \pm\sqrt{2}$, $x_{1,2} = \mp 4\sqrt{2}$

$$\begin{cases} 12y-3x=0 \\ x^2+16y^2=64 \end{cases} \Rightarrow \begin{cases} x=4y \\ y^2=2 \end{cases}$$

$y_{3,4} = \pm\sqrt{2}$, $x_{3,4} = \pm 4\sqrt{2}$

Точки экстремумов:

$M_0\left(\frac{3}{2}, -4\sqrt{2}, \sqrt{2}\right)$, $M_1\left(\frac{3}{2}, 4\sqrt{2}, -\sqrt{2}\right)$

$M_2\left(-\frac{3}{2}, 4\sqrt{2}, \sqrt{2}\right)$, $M_3\left(-\frac{3}{2}, -4\sqrt{2}, -\sqrt{2}\right)$

$L''_{xx} = 2\lambda_1$, $L''_{yy} = 32\lambda_1$, $L''_{\lambda_1\lambda_1} = 0$

$L''_{xy} = 12$, $L''_{x\lambda_1} = 2x$, $L''_{y\lambda_1} = 32y$

$$\begin{vmatrix} 0 & 2x & 32y \\ 2x & 2\lambda_1 & 12 \\ 32y & 12 & 32\lambda_1 \end{vmatrix} = -2x(2x\cdot 32\lambda_1 - 384y) + 32y(24x - 64\lambda_1 y) =$$

$$= -128x^2\cdot\lambda_1 + 768xy + 768xy - 2048y^2\lambda_1$$

разделим определитель на 128:

$-x^2\lambda_1 + 12xy - 16y^2\lambda_1 = 12xy - 16y^2\lambda_1 - x^2\lambda_1$

$M_0: 12\cdot(-8) - 16\cdot 2\cdot\frac{3}{2} - 32\cdot\frac{3}{2} < 0 - \min$

$M_1: 12(-8) - 16\cdot 2\cdot\frac{3}{2} - 32\cdot\frac{3}{2} < 0 - \min$

$M_2: 12\cdot 8 + 16\cdot 2\cdot\frac{3}{2} + 32\cdot\frac{3}{2} > 0 - \max$

$M_3: 12\cdot 8 + 16\cdot 2\cdot\frac{3}{2} + 32\cdot\frac{3}{2} > 0 - \max$

3. Найти произв. ф-ции по направлению в точку:

$U = x^2 + y^2 + z^2$, $\vec{c}(-9, 8, -12)$, $M(8; -12; 9)$

$|\vec{c}| = \sqrt{81 + 64 + 144} = \sqrt{289} = 17$

$\vec{c_0} = \frac{\vec{c}}{|\vec{c}|} = \left(-\frac{9}{17}; \frac{8}{17}; -\frac{12}{17}\right)$

$U'_x = 2x$, $U'_y = 2y$, $U'_z = 2z$

$\operatorname{grad} U|_{(8;-12;9)} = (16, -24, 18)$

$U'_{\vec{c}}|_{(8;-12;9)} = -\frac{16\cdot 9}{17} - \frac{8\cdot 24}{17} - \frac{18\cdot 12}{17} = -\frac{552}{17}$

4. $U = e^{x^2+y^2+z^2}$, $\vec{d} = (4, -13, -16)$, $L(-16, 4, -13)$

$|\vec{d}| = \sqrt{16 + 169 + 256} = 21$

$U'_x = 2x e^{x^2+y^2+z^2}$, $U'_y = 2y e^{x^2+y^2+z^2}$, $U'_z = 2z e^{x^2+y^2+z^2}$

$\operatorname{grad} U|_{(-16,4,-13)} = (-32e^{441}, 8e^{441}, -26e^{441})$

$U'_{\vec{a}}|_{(-16,4-13)} = \frac{-32\cdot 4e^{441} - 13\cdot 8e^{441} + 16\cdot 26e^{441}}{21} =$

$= \frac{184}{21} e^{441}$