



# Impact of idiosyncratic volatility on stock returns: A cross-sectional study



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## ABSTRACT

This paper proposes a new approach to estimate the idiosyncratic volatility premium. In contrast to the popular two-pass regression method, this approach relies on a novel GMM-type estimation procedure that uses only a single cross-section of return observations to obtain consistent estimates. Also, it enables a comparison of idiosyncratic volatility premia estimated using stock returns with different holding periods. The approach is empirically illustrated by applying it to daily, weekly, monthly, quarterly, and annual US stock return data over the course of 2000–2011. The results suggest that the idiosyncratic volatility premium tends to be positive on daily return data, but negative on monthly, quarterly, and annual data. They also indicate the presence of a January effect.

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## 1. Introduction

The finance literature is presently witnessing a debate on idiosyncratic volatility premium. The models of Levy (1978), Merton (1987), Malkiel and Xu (2006), and Epstein and Schneider (2008) predict a positive premium in the capital market equilibrium. However, research based on Kahneman and Tversky's (1979) prospect theory indicates that the premium can be negative (e.g., see Bhootra and Hur, 2011); an additional explanation for why the premium can be negative has been suggested by Peterson and Smedema (2011).<sup>1</sup>

Empirical evidence on the idiosyncratic volatility premium is contradictory. Ang et al. (2006, 2009), Jiang et al. (2009), Guo and Savickas (2010), and Chabi-Yo (2011) document a negative premium. In contrast, Fu (2009) and Huang et al. (2010) find the

premium to be positive. While the existing empirical studies typically apply a version of Fama and MacBeth's (1973) two-pass approach, the divergence in their results can stem from how, exactly, they compute idiosyncratic volatilities of individual stocks in the first pass.<sup>2</sup> For example, Ang et al. employ a realized idiosyncratic volatility measure by using daily stock returns from a previous month. In contrast, Fu uses an expected conditional idiosyncratic volatility measure by estimating an EGARCH model on a time-series of monthly returns (Fu requires having at least 30 observations in the time-series). Peterson and Smedema (2011) indicate that these alternative measures of idiosyncratic volatility can be associated with different effects on returns. Thus, the divergence in the sign of the idiosyncratic volatility premium (estimated in the second pass of the two-pass approach) may have arisen because some researchers use short-term, high-frequency data, whereas other researchers use long-term, low-frequency data—when computing stock-specific idiosyncratic volatilities in the first pass. Hence, specific details of the econometric methodology can play an important role in obtaining empirical conclusions about the idiosyncratic volatility premium (on this point, see also Fink et al., 2012).

The contribution of this paper to the literature is twofold. First, we outline a novel Generalized Method of Moments (GMM)-type econometric procedure that allows us to obtain consistent estimates of parameters of a financial market model (see more

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<sup>1</sup> There is also no consensus in the literature on the issues of the forecasting power and the time-series behavior of idiosyncratic volatility. Goyal and Santa-Clara (2003) document a positive relationship between equal-weighted average stock variance and future market return. However, Bali et al. (2005) show that this predictive relationship does not hold for value-weighted variance. Also, Campbell et al. (2001) report a steady increase in idiosyncratic stock volatility since 1962. However, Brandt et al. (2010) argue that this increase is only an episodic phenomenon. In this paper, we do not investigate the forecasting power and the time-series behavior of idiosyncratic volatility. We focus exclusively on the issue of the idiosyncratic volatility premium.

<sup>2</sup> These computed stock-specific idiosyncratic volatilities are subsequently used to estimate the idiosyncratic volatility premium in the second pass.

on it below), using only a single cross-section of return data. Notably, unlike in the previous studies, having a long historical time-series of returns is not required. This approach could be particularly helpful when a researcher needs to characterize a stock market using only the most current, rather than historical, information. In addition, we empirically illustrate the proposed methodology by estimating the idiosyncratic volatility premium embedded into the US stock returns over the course of 2000–2011. We offer a detailed analysis of the premium using daily, weekly, monthly, quarterly, and annual stock return intervals. This empirical analysis is the second contribution of the paper to the literature.

A parametric financial market model underlying the return data is an important component of the proposed estimation approach. We consider a continuous-time model comprising a well-diversified market portfolio index and a cross-section of individual stocks. The index follows a geometric Brownian motion and is affected by a source of market risk. Individual stocks also follow a geometric Brownian motion and depend on this same source of market risk (i.e., it is a common risk shared by all stocks). In addition, they are affected by stock-specific idiosyncratic risks. We do not take a stance on whether idiosyncratic volatility should command a premium in the capital market equilibrium, but rather we allow for a *potential* effect of a stock's idiosyncratic volatility on the stock's drift term and estimate this effect, if any, from the data.

The estimation approach is empirically illustrated using US stock price data from the Center for Research in Security Prices (CRSP) over the 2000–2011 time period. We estimate the financial market model separately on every return interval in the dataset, and then aggregate the results according to the return data frequency: daily, weekly, monthly, quarterly, and annual. We find that estimates of the idiosyncratic volatility premium computed on daily return data tend to be positive and statistically significant. In comparison, estimates of the premium on weekly return data are, on average, negative but not statistically significant. In turn, premia estimated from monthly, quarterly, and annual return data tend to be negative and statistically significant. Estimates of the idiosyncratic volatility premium for the same time period—but computed at different frequencies—are positively associated. In addition, the calculated values of the idiosyncratic volatility component of the conditional expected return suggest that the impact of the idiosyncratic volatility on the expected return can be economically significant. The results of robustness checks indicate the presence of a January effect. In particular, idiosyncratic volatility premia computed using daily, weekly, and monthly return data over the month of January tend to be higher (and positive, on average) than corresponding estimates from non-January data. Also, in the cases of daily and weekly data, the per annum average estimates of the premium tend to be similar across different calendar years during 2000–2011.

As noted earlier, the existing empirical studies of the idiosyncratic volatility premium typically employ a version of the conventional two-pass regression method of Fama and MacBeth (1973).<sup>3</sup> Despite its intuitive appeal, the two-pass method has several well-known econometric limitations. For example, it delivers consistent estimates only when the time-series length (rather than the number of stocks) grows infinitely large (Shanken, 1992). Also, since the regressors in the second pass (e.g., individual stock-specific idiosyncratic volatilities) are measured with error, the estimator is subject to an errors-in-variables problem (Miller and Scholes, 1972), which may induce an attenuation bias in the estimates (Kim, 1995). The statistical properties of the second-pass estimator are complex. As such, it is not uncommon for these complexities to be ignored in

practice, resulting in biased inference (Shanken, 1992; Jagannathan and Wang, 1998). Moreover, accounting for the time-varying nature of stock betas (Fama and French, 1997; Lewellen and Nagel, 2006; Ang and Chen, 2007) and idiosyncratic volatilities (Fu, 2009) is challenging and requires the imposition of additional assumptions, which further complicate statistical inference.

One of the goals of the estimation approach proposed in this paper is to address these econometric limitations. In particular, the approach delivers consistent estimates as the number of stocks (rather than the time-series data length) grows infinitely large. Thus, it is not affected by the available time-series length of the stock return data. Also, since it does not involve estimating individual stock-specific betas and idiosyncratic volatilities, it is not subject to the errors-in-variables problem arising in the two-pass regression method, and it does not require the imposition of strong assumptions about their time-series behavior. Instead, the approach relies on a parametric model describing a financial market setting, and on a distributional assumption regarding a cross-section of the betas and idiosyncratic volatilities (we model them as random coefficients). While the need to make the distributional assumption might be seen as a potential limitation, practitioners can explore several alternative assumptions to check the robustness of the estimates to a misspecification. Overall, we believe our approach will be an attractive alternative to the two-pass regression method, especially when researchers need to characterize a stock market using only the most current, rather than historical, information.

The remainder of this paper proceeds as follows. Section 2 specifies the financial market model. Section 3 outlines the econometric approach. Section 4 describes the data used in the empirical analysis. Section 5 discusses the results. Section 6 concludes. Selected analytical formulas are derived in the appendix.

## 2. Financial market model

We first specify a financial market model and discuss how it relates to the classical financial framework. We then derive expressions for gross returns and specify distributional assumptions that help implement a GMM-type econometric procedure outlined in Section 3.

### 2.1. Model setup

Financial investors trade in many risky assets in continuous time. One of the assets is a well-diversified stock portfolio bearing only market risk. In what follows, this asset is referred to as “the market index.” Its price at time  $t$  is denoted by  $M_t$ . All other risky assets are individual stocks bearing the market risk and stock-specific idiosyncratic risks. We index the stocks by  $i$ , with  $i = 1, 2, \dots$ , and denote the price of a stock  $i$  at time  $t$  by  $S_t^i$ . In addition to the market index and the stocks, there is a default-free bond that pays interest at a risk-free rate  $r$ .

The price dynamics of the market index follows a geometric Brownian motion and is described by a stochastic differential equation:

$$dM_t/M_t = \mu_m dt + \sigma_m dW_t, \quad (1)$$

with a drift

$$\mu_m = r + \delta \sigma_m, \quad (2)$$

where  $W_t$  is a standard Brownian motion indicating a source of the market risk,  $\sigma_m > 0$  is the market volatility, and  $\delta$  is the market risk premium. As explained in Section 3, the estimation procedure will not allow us to identify  $\delta$ , because this parameter is differenced out when stock returns are conditioned on the market index return.

<sup>3</sup> Black et al. (1972), among others, contributed to the development of the two-pass methodology.

Conditioning on the market index return is a critical step in the econometric procedure that ensures consistency of the estimates.

The price dynamics of a stock  $i$  ( $i = 1, 2, \dots$ ) also follows a geometric Brownian motion and is described by a stochastic differential equation:

$$dS_t^i/S_t^i = \mu_i dt + \beta_i \sigma_m dW_t + \sigma_i dZ_t^i, \quad (3)$$

with a drift

$$\mu_i = r + \delta \beta_i \sigma_m + \gamma \sigma_i, \quad (4)$$

where  $Z_t^i$  is a standard Brownian motion indicating a source of the idiosyncratic risk that only affects stock  $i$ .  $Z_t^i$  is independent of  $W_t$  and every  $Z_t^j$  for  $j \neq i$ . Also,  $\beta_i$  is the stock's beta,<sup>4</sup>  $\sigma_i > 0$  is the stock's idiosyncratic volatility, and  $\gamma$  is the idiosyncratic volatility premium;  $\gamma$  is not sign-restricted and may be zero. Eq. (3) implies that  $W_t$  is a source of common risk among the stocks because an innovation in  $W_t$  results in a shock that is shared by all stocks (i.e., it is a common shock). Sensitivity of individual stock prices to the common shock is allowed to vary. Thus,  $\beta_i$  need not be equal to  $\beta_j$  for  $j \neq i$ . Also, the stocks can have different idiosyncratic volatilities (i.e.,  $\sigma_i$  need not be equal to  $\sigma_j$  for  $j \neq i$ ).

Classical financial models (e.g., Sharpe, 1964; Lintner, 1965) imply that idiosyncratic volatility commands no equilibrium return premium. In that case,  $\gamma = 0$  and the price dynamics of our model would coincide with that of the ICAPM with a constant investment opportunity set (Merton, 1973, Section 6).<sup>5</sup> However, the current finance literature indicates that idiosyncratic volatility can be priced, although there is no consensus among the researchers about the sign of the premium or the underlying pricing mechanism (see a review in the introductory section). To account for a potential premium, we include a term “ $+\gamma\sigma_i$ ” in the specification of the stock's drift in Eq. (4). If idiosyncratic volatility is, indeed, priced, we should expect to estimate a statistically significant  $\gamma$ .

## 2.2. Model solution and distributional assumptions

We focus on a cross-section of returns for a time interval from date  $t = 0$  to date  $t = T$  (a discussion of specific choices of the dates  $t = 0$  and  $t = T$  used in the empirical implementation of the estimation approach is provided in Section 4). More specifically, the data inputs include the gross market index return,  $M_T/M_0$ , and the gross returns on the stocks,  $S_T^1/S_0^1$ ,  $S_T^2/S_0^2$ ,  $\dots$ .

Applying Itô's lemma to Eqs. (1) and (3), we derive expressions for  $M_T/M_0$ :

$$M_T/M_0 = \exp[(\mu_m - 0.5\sigma_m^2)T + \sigma_m W_T], \quad (5)$$

and  $S_T^i/S_0^i$ :

$$S_T^i/S_0^i = \exp[(\mu_i - 0.5\beta_i^2\sigma_m^2 - 0.5\sigma_i^2)T + \beta_i\sigma_m W_T + \sigma_i Z_T^i], \quad (6)$$

where  $W_T$  and  $Z_T^i$  are independent and identically distributed (i.i.d.) as  $N(0, T)$ , and the drifts  $\mu_m$  and  $\mu_i$  are given by Eqs. (2) and (4), respectively.

It is impossible to consistently estimate the individual stock betas  $\beta_1, \beta_2, \dots$  and idiosyncratic volatilities  $\sigma_1, \sigma_2, \dots$  using only a cross-section of returns, because the number of parameters to estimate would grow with the sample size. Therefore, to implement the econometric procedure, we specify the stock betas and idiosyncratic volatilities as random coefficients drawn from a probability distribution (for a survey of the econometric literature on ran-

dom-coefficient models, see Hsiao, 2003, Chapter 6). Parameters of this distribution are estimated simultaneously with other identifiable parameters of the financial market model.

As a practical matter, sensitivity of individual stock returns to the market and idiosyncratic sources of risk should be finite, since infinite stock returns are not observed in the data. Given this consideration, it may be preferable to model the distribution of  $\beta_i$  and  $\sigma_i$  as having a finite support. In addition to being empirically relevant, the finite support requirement helps ensure the existence of theoretical moments of the gross stock return. Also, the support of  $\sigma_i$  must be positive. A negative  $\beta_i$  cannot be ruled out, however; and therefore, the support of  $\beta_i$  should not be sign-restricted. To facilitate an evaluation of the finite-sample properties of the econometric procedure (see details on the Monte Carlo exercise in Section 3), it is also desirable to have a distribution that would let us express moment restrictions analytically. We have explored a number of alternative options and found that the uniform distribution most closely meets these criteria. More specifically, we assume that

$$\beta_i \sim \text{i.i.d. UNI}[\kappa_\beta, \kappa_\beta + \lambda_\beta], \quad (7)$$

where the lower and upper boundaries of the support are parameterized in terms of  $\kappa_\beta$  and  $\lambda_\beta > 0$ , and

$$\sigma_i \sim \text{i.i.d. UNI}[0, \lambda_\sigma], \quad (8)$$

where the upper boundary of the support is  $\lambda_\sigma > 0$ . Here,  $\kappa_\beta, \lambda_\beta$ , and  $\lambda_\sigma$  are the distribution parameters to estimate. Loosely speaking, we are effectively using “non-informative priors” on the random coefficients.

Empirical finance researchers and practitioners could impose a different set of distributional assumptions; for example, they could consider a more flexible case of mutually correlated  $\beta_i$  and  $\sigma_i$ . They could also explore many alternative sets of assumptions, in order to assess the robustness of estimation results to a misspecification. We anticipate that some assumptions may necessitate the use of simulation techniques to approximate theoretical moments, and defer an investigation of this possibility to future research.

## 3. Estimation approach

All individual stocks in a cross-section share the same source of market risk, which makes their returns mutually dependent. This dependence is captured by Eq. (6), which implies that the gross returns  $S_T^1/S_0^1$ ,  $S_T^2/S_0^2$ ,  $\dots$  are all affected by the same random variable  $W_T$ . Hence, dependence between a pair of returns  $S_T^i/S_0^i$  and  $S_T^j/S_0^j$  need not diminish for any  $i \neq j$ . Technically, the gross stock returns  $S_T^1/S_0^1$ ,  $S_T^2/S_0^2$ ,  $\dots$  exhibit strong cross-sectional dependence (for a formal discussion, see Chudik et al., 2011). In this context, conventional laws of large numbers do not apply (these laws presume that data dependence vanishes asymptotically, which is not the case here). Therefore, standard econometric methods (OLS, MLE, etc.) cannot deliver consistent estimates. However, it is possible to achieve consistency using a GMM-type econometric procedure suggested by Andrews (2003, 2005).

Notice that except for  $W_T$ , all other sources of randomness in the expression for  $S_T^i/S_0^i$  in Eq. (6)—namely,  $\beta_i$ ,  $\sigma_i$ , and  $Z_T^i$ —are i.i.d. across the stocks. Also,  $W_T$  is a continuous function of  $M_T/M_0$ , since Eq. (5) implies that  $W_T = \sigma_m^{-1} [\ln(M_T/M_0) - (\mu_m - 0.5\sigma_m^2)T]$ . Therefore, conditional on the market index return  $M_T/M_0$ , the individual gross stock returns  $S_T^1/S_0^1$ ,  $S_T^2/S_0^2$ ,  $\dots$  are i.i.d. This property allows us to modify the standard GMM approach (Hansen, 1982).

More specifically, to estimate the model parameters, we match sample moments of stock returns with corresponding theoretical moments that are conditioned on a realization of the market index return. Conditioning on the market index return is a critical step of

<sup>4</sup> Eqs. (1) and (3) imply that  $\beta_i$  is the ratio of the covariance between the instantaneous returns on the stock and the market index,  $\beta_i \sigma_m^2 dt$ , to the variance of the instantaneous return on the market index,  $\sigma_m^2 dt$ .

<sup>5</sup> We do not consider the case of the ICAPM in which the investment opportunity set is allowed to be time-varying.

the econometric procedure. Intuitively, conditioning here is similar to including a special regressor that “absorbs” data dependence. Once data dependence is dealt with this way, we obtain estimates that are consistent and asymptotically *mixed* normal. The latter property distinguishes them from asymptotically normal estimates in the standard GMM case. However, despite the asymptotic mixed normality, hypothesis testing can be implemented using conventional Wald tests (see Andrews, 2003, 2005).

We collect all identifiable parameters of the financial market model in a vector  $\theta = (\sigma_m, \gamma, \kappa_\beta, \lambda_\beta, \lambda_\sigma)'$ , and define a function

$$g_i(\xi; \theta) = (S_T^i/S_0^i)^\xi - E_\theta \left[ (S_T^i/S_0^i)^\xi | M_T/M_0 \right], \quad (9)$$

where  $\xi$  is a real number and  $E_\theta[\cdot|\cdot]$  denotes a conditional expected value when the model parameters are set equal to  $\theta$ . The function  $g_i(\cdot)$  represents a moment restriction, because  $E_{\theta_0}[g_i(\xi; \theta_0) | M_T/M_0] = 0$ , where  $\theta_0$  is the true parameter vector.

**Proposition 1** in the appendix expresses the conditional expected value  $E_\theta$  analytically. Notably, it does not depend on  $\delta$ . This parameter is differenced out after we condition  $(S_T^i/S_0^i)^\xi$  on  $M_T/M_0$ , but before we apply the distributional assumptions, Eqs. (7) and (8), in the derivation of  $E_\theta$ . Thus, the loss of identification here is not caused by the specific distributional assumptions, but rather is due to conditioning on the market index return. Effectively, once the observation of the market index gross return  $M_T/M_0$  is accounted for in the function  $g_i(\cdot)$  through conditioning, a cross-section of individual stock returns provides no independent information about  $\delta$  that is needed for its estimation.

Now, let  $\xi_1, \dots, \xi_k$  be  $k$  real numbers indicating different moment orders, where  $k$  is at least as large as the number of the model parameters to estimate, that is,  $k \geq 5$ . We define a  $k \times 1$  vector of moment restrictions

$$\mathbf{g}(S_T^i/S_0^i; \theta, M_T/M_0) = (g_i(\xi_1; \theta), \dots, g_i(\xi_k; \theta))',$$

and then write the GMM objective function as:

$$Q_n(\theta; \Sigma) = \left( n^{-1} \sum_{i=1}^n \mathbf{g}(S_T^i/S_0^i; \theta, M_T/M_0) \right)' \times \Sigma^{-1} \left( n^{-1} \sum_{i=1}^n \mathbf{g}(S_T^i/S_0^i; \theta, M_T/M_0) \right),$$

where  $\Sigma$  is a  $k \times k$  positive definite matrix. In this paper, the one-step GMM estimator,  $\hat{\theta}_{1,n}$ , is defined as the global minimizer of the objective function  $Q_n(\cdot)$ , with  $\Sigma$  set equal to an identity matrix  $\mathbf{I}_k$ :

$$\hat{\theta}_{1,n} = \arg \min_{\theta \in \Theta} Q_n(\theta; \mathbf{I}_k).$$

Econometric theory recommends implementing GMM estimation as a two-step procedure. In the first step, a GMM objective function typically utilizes an identity matrix as the weighting matrix. In the second step, the objective function instead incorporates a consistent estimate of an “optimal” weighting matrix. In our case, the second step would amount to replacing  $\Sigma$  with a consistent estimate of the conditional variance of the moment restrictions, matrix  $E_{\theta_0}[\mathbf{g}\mathbf{g}' | M_T/M_0]$ , and then minimizing the objective function to obtain two-step estimates. Asymptotically, a two-step procedure should be more efficient. However, its finite-sample properties are less clear. In fact, researchers find that the one-step GMM estimator tends to outperform the two-step estimator in practice (see Altonji and Segal, 1996 and references therein).

To assess finite-sample properties of the econometric approach and determine whether or not we should apply a two-step (rather than a one-step) procedure in our empirical work, we perform a Monte Carlo simulation exercise. It comprises 1000

estimation rounds on simulated weekly stock return samples of size  $n = 5500$ . The sample size here is set similar to the average number of stocks in actual stock return cross-sections used in the empirical implementation of the estimation approach; in particular, see column “ $n$ ” in Table 2. The returns are simulated by assuming an annual risk-free rate of 1% and using the financial market model solution, Eqs. (5) and (6), and the distributional assumptions, Eqs. (7) and (8). The true parameter vector is set at  $\theta_0 = (0.20, -2.00, 0.50, 3.00, 1.00)'$ , which is similar to our actual estimates for some of the weekly return intervals. Also, we set  $\delta_0 = 0.20$ , which is based on the range of values of the Sharpe ratio of the US stock market, as implied by Mehra and Prescott’s (2003) estimates. In each simulation round, we compute the one- and two-step estimates by utilizing a grid of initial search values and minimizing the objective function numerically with the Nelder-Mead algorithm (a similar approach is applied when computing estimates on actual return data).

The results of the Monte Carlo exercise are summarized in Table 1. We present root mean square errors (RMSEs) of the estimates, absolute values of differences between the medians of the estimates and the true values, and absolute values of the biases. According to these measures (which, ideally, should be as small as possible), the one-step estimates tend to have better finite-sample properties than the two-step estimates. Therefore, we will use the one-step procedure when implementing the estimation approach.

#### 4. Data

Data for the empirical analysis come from the Center for Research in Security Prices (CRSP) and are accessed through Wharton Research Data Services (WRDS). The CRSP provides comprehensive information on all securities listed on the New York Stock Exchange, American Stock Exchange, and NASDAQ. This information is supplemented with interest rate data from the Federal Reserve Bank Reports database, also accessed through WRDS.

We focus on regularly traded stocks of operating companies, and exclude stocks that were delisted during a particular return interval under consideration (see more on the sample time frame and return intervals below). True returns on the excluded stocks are difficult to determine accurately. We also drop shares of closed-end funds, exchange-traded funds, real estate investment trusts, and similar investment vehicles. They pool together many individual assets, and therefore, their price dynamics may not be adequately represented by Eq. (3). In turn, for each included stock, we extract information from the CRSP database on the stock’s daily closing price, adjusted for stock splits, reverse splits, and similar stock events (in what follows, we refer to this price as “adjusted price”), and calculate the gross return on the stock over a particular time interval by taking the ratio of corresponding adjusted prices.

We estimate the idiosyncratic volatility premium using data from the calendar years 2000 through 2011 (inclusive). The year 2000 marked the end of the dot-com bubble and the beginning of a period of dramatic stock market dynamics. For example, the magnitude of the variation in the market valuation observed during 2000–2011 was at its peak in the post-World War II era; see Fig. 1 for an illustration (the figure plots the dynamics of the S&P 500 index). Also, according to Allan Greenspan (2000), the growth of the stock market at the turn of the millennium reached an unprecedented level: “We may conceivably conclude [...] that at the turn of the millennium, the American economy was experiencing a once-in-a-century acceleration of innovation, which propelled forward productivity, output, corporate profits, and stock prices at a pace not seen in generations, if ever.” Thus, the time



**Table 1**

Summary of results of Monte Carlo simulation exercise.

Parameter	RMSE		Median–true value		Mean–true value		True value
	1-step	2-step	1-step	2-step	1-step	2-step	
$\sigma_m$	0.5631	0.7967	0.0916	0.1433	0.2770	0.2643	0.2000
$\gamma$	0.5337	0.7546	0.0014	0.0818	0.0223	0.1013	–2.0000
$\kappa_\beta$	1.3163	1.8470	0.0067	0.0514	0.0209	0.0611	0.5000
$\lambda_\beta$	2.4048	3.3329	0.3979	0.4720	0.0862	0.1034	3.0000
$\lambda_\sigma$	0.0277	0.0394	0.0039	0.0049	0.0052	0.0060	1.0000

This table compares finite-sample properties of the one-step (“1-step”) and two-step (“2-step”) estimates of the financial market model in a Monte Carlo simulation exercise. The number of the simulation rounds is 1000. In each round, we simulate a sample of weekly returns of size 5500. The annual risk-free rate is 1%. The estimates are computed by utilizing a grid of initial search values and minimizing the objective function numerically with the Nelder-Mead algorithm. The vector of moment restrictions is constructed using the moment order vector  $\xi = (-2, -1.5, -1, -0.5, 0.5, 1, 1.5, 2)$ . We present root mean square errors of the estimates across the simulation rounds (column *RMSE*), absolute values of differences between the medians of the estimates and the true parameter values (*|Median–true value|*), and absolute values of the biases (*|Mean–true value|*).



**Fig. 1.** Dynamics of the S&P 500 index, 1950–2011. This figure plots the evolution of the S&P 500 index for the years 1950–2011. The shaded area in the figure indicates the 2000–2011 period, which is studied in detail in this paper.

frame of our empirical analysis—the years 2000–2011—is of a particular interest to investigate in its own right.

The proposed econometric method may be applied to estimate the idiosyncratic volatility premium using a cross-section of returns over a time interval of *any length*. In fact, it may be interesting to compare estimates of the premium computed on return intervals of *different* length. The existing empirical studies of the premium tend to mostly focus on monthly stock holding intervals (e.g., Fu, 2009); these studies do not provide a detailed comparison of premia obtained at different return data frequencies. The article by Ang et al. (2006) is an exception in that they briefly investigate the robustness of premium estimates to a variation in the length of the holding interval; more specifically, they investigate monthly and annual intervals.<sup>6</sup> However, unlike in our case, Ang et al. (2006) compute the premium using portfolio returns, rather than individual stock returns. Moreover, they do not report premia for any other holding interval lengths. In contrast, we estimate, summarize, and compare idiosyncratic volatility premia for daily, weekly, monthly, quarterly, and annual return intervals. To the best of our knowledge, such a comparison has not been provided in any article published to date, and it is one of the two key contributions of our paper to the literature (the other one is the new estimation methodology, as we discussed earlier).

It should be noted that our detailed empirical analysis incurs a substantial computational cost, caused primarily by a large volume

of input data, especially in the case of daily and weekly returns (see the column “Total” in Table 2 for the total number of observations). For example, in the case of daily returns, we estimate the model 3004 times (once for each cross-section of daily returns in the dataset), with each estimation typically lasting 15 hours.<sup>7</sup>

In the case of the daily data frequency, we obtain a cross-section of gross stock returns by using adjusted stock closing prices on adjacent business days (holidays and unscheduled stock exchange closures during the 2000–2011 period are excluded from consideration). Daily return intervals do not overlap. In the case of the weekly frequency, we calculate the returns using Wednesday adjusted closing prices for a particular week and Wednesday prices for the following week (in a small number of instances, we use Thursday prices for the following week, when the Wednesday in question falls on a holiday). Thus, adjacent weekly return intervals do not overlap. We employ Wednesday—rather than Monday or Friday, for example—as a reference day of the week to be in line with Hou and Moskowitz (2005). In the case of the monthly and quarterly data frequencies, returns are calculated between the first Wednesday of a month and a quarter, respectively; and the first

<sup>6</sup> We refer interested readers to Section F, “Robustness to Different Formation and Holding Periods,” in Ang et al. (2006, pp. 292–294).

<sup>7</sup> Much of the time cost is due to our using a large grid of starting points in the numerical optimization search, in order to enhance the accuracy of ultimate estimates. Using the Nelder-Mead routine and the R programming language, each individual optimization round on the grid lasts approximately seven seconds (on a Linux CentOS server with an Intel Xeon CPU at 2.27 GHz). If a researcher is able to narrow down the parameter space, the grid size can be reduced, which may substantially decrease the overall estimation time.

**Table 2**  
Data summary statistics.

Return frequency		Mean	Std.dev.	Min	Max	Median	Skew	Kurt	$M_T/M_0$	$r$	$n$	Total
Daily	Average	1.0006	0.0435	0.5725	1.9106	0.9994	3.7425	132.8706	1.0001	0.0226	5,638	17,016,457
	Std.dev.	0.0131	0.0176	0.1665	0.8864	0.0112	6.1082	334.4249	0.0140	0.0195	753	
Weekly	Average	1.0023	0.0873	0.3743	2.7515	0.9986	3.8345	107.2721	1.0004	0.0227	5,629	3,489,924
	Std.dev.	0.0303	0.0316	0.1565	1.4481	0.0241	5.0752	247.9402	0.0271	0.0195	750	
Monthly	Average	1.0085	0.1732	0.2081	4.1540	0.9984	3.2438	70.8127	1.0015	0.0231	5,609	802,125
	Std.dev.	0.0727	0.0675	0.1029	3.3436	0.0545	4.2824	228.8765	0.0525	0.0195	738	
Quarterly	Average	1.0239	0.3003	0.1182	5.8770	0.9988	3.0422	40.9149	1.0025	0.0237	5,552	260,924
	Std.dev.	0.1361	0.0990	0.0782	2.6541	0.1031	1.8398	39.5717	0.0922	0.0195	710	
Annual	Average	1.1259	0.6889	0.0424	16.0212	1.0242	5.1866	101.5095	1.0233	0.0250	5,197	685,984
	Std.dev.	0.3165	0.3813	0.0367	16.7075	0.2256	4.4985	206.4454	0.2030	0.0192	548	

This table presents descriptive statistics for gross stock returns in the dataset (the years 2000–2011). For each return interval, we compute the mean (Mean), standard deviation (Std.dev.), minimum (Min), maximum (Max), median (Median), skewness (Skew), and kurtosis (Kurt) of the corresponding cross-sectional distribution of gross stock returns. We then calculate and report the average and standard deviation of these statistics over a given return frequency (daily, weekly, etc.). We also calculate and report the average and standard deviation of corresponding gross returns on the market index ( $M_T/M_0$ ), risk-free rates ( $r$ ), and numbers of stocks per return interval ( $n$ ). “Total” is the total number of stock-interval observations for a given return frequency. Return intervals at the annual frequency may partially overlap. Return intervals at the other frequencies do not overlap.

Wednesday of the following month and quarter, respectively. Again, adjacent return intervals at each of these frequencies do not overlap. In contrast, annual return intervals may partially overlap. In particular, we calculate cross-sections of returns between the first Wednesday of January 2000 and the first Wednesday of January 2001, between the first Wednesday of February 2000 and the first Wednesday of February 2001, and so on. The use of partially overlapping annual intervals is motivated by our desire to more fully utilize the available stock data, and therefore, increase the power of statistical inference based on “annual” estimates.<sup>8</sup>

To implement the estimation procedure, in addition to a cross-section of gross stock returns, we must also have an observation of a concurrent return on the market index and a corresponding risk-free interest rate. In line with many empirical studies, we approximate the market index with the value-weighted CRSP market portfolio index. We approximate the risk-free rate with the annualized three-month T-Bill rate.

Table 2 presents selected descriptive statistics for the data. To prepare this table we first compute the mean, standard deviation, minimum, maximum, median, skewness, and kurtosis of the cross-sectional distribution of gross stock returns—for each return interval in the dataset. We then calculate and report the average and standard deviation of these statistics separately for each data frequency. We also report the average and standard deviation of corresponding gross returns on the market index, risk-free rates, and numbers of stocks in a cross-section. As can be seen, the cross-sections used to estimate the model contain about 5200–5600 individual stocks, on average.

## 5. Results

We start with listing estimates of the entire financial market model for selected return intervals. Then, we present the main results for the idiosyncratic volatility premium and discuss our findings. Lastly, we perform an analysis of a “January effect” and show the results of additional robustness checks.

### 5.1. Examples of full model estimates

To illustrate the full outcome of the proposed econometric procedure, Table 3 presents all estimated model parameters using cross-sectional return data for selected examples of weekly

**Table 3**  
Examples of estimated financial market model on weekly return data.

Parameter	January 2008, last week		October 2008, last week	
	Estimate	P-value	Estimate	P-value
<i>Panel A: Moment order vector <math>\xi = (-2, -1.5, -1, -0.5, 0.5, 1, 1.5, 2)</math></i>				
$\sigma_m$	0.0537***	0.0000	0.0672***	0.0000
$\gamma$	-2.1117	0.3368	-1.2936	0.5671
$\kappa_\beta$	0.3417	0.7367	-0.3058	0.7408
$\lambda_\beta$	3.0475***	0.0003	2.8367***	0.0000
$\lambda_\sigma$	1.0580***	0.0000	1.7478***	0.0000
<i>Panel B: Moment order vector <math>\xi = (-1.5, -1, -0.5, 0.5, 1, 1.5)</math></i>				
$\sigma_m$	0.0481***	0.0000	0.0572***	0.0000
$\gamma$	-0.9814	0.7543	-1.7590	0.1515
$\kappa_\beta$	0.0724	0.9518	-0.1863	0.7396
$\lambda_\beta$	2.9699***	0.0000	2.8769***	0.0000
$\lambda_\sigma$	1.0682***	0.0000	1.7148***	0.0000

This table presents examples of estimated financial market model on weekly return data. Parameter estimates and P-values from tests of the statistical significance of the estimates are provided for return data from the last week of January 2008 and the last week of October 2008. To illustrate the robustness of the econometric procedure, the table shows results obtained using two sets of moment restrictions. Panel A reports the results of the estimation using the moment order vector  $\xi = (-2, -1.5, -1, -0.5, 0.5, 1, 1.5, 2)$ . Panel B reports the results for the moment order vector  $\xi = (-1.5, -1, -0.5, 0.5, 1, 1.5)$ . Parameter  $\sigma_m$  is the market volatility,  $\gamma$  is the idiosyncratic volatility premium,  $\kappa_\beta$  is the lower bound of the support of the distribution of the stock betas and  $\lambda_\beta$  is the range of this support, and  $\lambda_\sigma$  is the upper bound of the support of the distribution of the stock-specific idiosyncratic volatilities.

\*\*\* Statistical significance:  $p < 0.01$ .

intervals. We show results for the last week of January 2008 and the last week of October 2008. The year 2008 may be of a particular interest because it was marked with the most serious financial crisis since the Great Depression.<sup>9</sup> To show the robustness of the procedure, we report results for two choices of moment restrictions. Panel A of the table provides estimates under eight restrictions, when the moment order vector  $\xi = (-2, -1.5, -1, -0.5, 0.5, 1, 1.5, 2)$ . Panel B lists estimates under six restrictions, when  $\xi = (-1.5, -1, -0.5, 0.5, 1, 1.5)$ . Values of statistically significant parameter estimates (here,  $\sigma_m$ ,  $\lambda_\beta$ , and  $\lambda_\sigma$ ) are similar between the two moment restriction choices. In the case of not statistically significant parameter estimates (here,  $\gamma$  and  $\kappa_\beta$ ), numerical values differ somewhat, but have the same sign. A similar pattern is observed in additional estimations using other choices of  $\xi$  (results are not reported). Thus,

<sup>8</sup> To account for the possible autocorrelation of estimates when aggregating them, due to the partial return-interval overlap or potential time dependence in the underlying return data, we employ test statistics computed by the Newey and West (1987) method, which are robust to autocorrelation.

<sup>9</sup> To clarify, the purpose of this section is to show what exactly the econometric procedure delivers, using selected return intervals. In addition, the model is estimated for a large number of other intervals between 2000 and 2011. The following Section 5.2 summarizes our main results. Section 5.3 presents robustness checks, including an analysis of the January effect.

our findings tend to be robust to the choice of specific restrictions. In what follows, we focus on estimates obtained using eight restrictions, when  $\xi = (-2, -1.5, -1, -0.5, 0.5, 1, 1.5, 2)$ .

The estimates of the parameter  $\sigma_m$  imply the value of the stock market volatility of 0.0537 (in annual terms) for the last week of January, and 0.0672 for the last week of October 2008. These estimates are lower than the historical average of 0.2, according to Mehra and Prescott (2003), but within the range of values reported in other studies (e.g., Campbell et al., 2001; Xu and Malkiel, 2003). The negative (but not statistically significant) estimates of the idiosyncratic volatility premium,  $\gamma$ , are broadly in line with the findings of Ang et al. (2006) and Jiang et al. (2009). The estimates of the parameters  $\kappa_\beta$  and  $\lambda_\beta$  are broadly consistent with the results reported in the literature. In particular, Eq. (7) implies an average cross-sectional stock beta of  $(\kappa_\beta + \lambda_\beta/2)$ , and a range of betas from  $\kappa_\beta$  to  $(\kappa_\beta + \lambda_\beta)$ . Here, the average stock beta is 1.8655 for January, and 1.1126 for October. In comparison, Fu (2009) reports an average beta of 1.22. The implied range of the betas is from 0.3417 to 3.3892 for January, and from  $-0.3058$  to 2.5309 for October. In comparison, Fama and French (1992) report a range from 0.53 to 1.79 for betas of 100 size-beta stock portfolios. Expectedly, our estimates imply a wider range, since we consider individual stocks. Lastly, the estimates of  $\lambda_\sigma$  are also consistent with the findings in the literature. Eq. (8) implies an average cross-sectional idiosyncratic volatility value of  $\lambda_\sigma/2$ . Thus, annualized average idiosyncratic volatilities are 0.5290 for January, and 0.8739 for October; or 0.1527 and 0.2523 on the monthly basis, respectively. The result for January bears similarity to that of Fu (2009), who computes an average monthly idiosyncratic volatility value of 0.1417, and Ang et al. (2009), who report a value of 0.1472. An increase in the value by two-thirds between the last week of January and the last week of October 2008, in the wake of the financial crisis, is intuitive.

## 5.2. Analysis of main results

We apply the GMM-type procedure outlined in Section 3 to estimate the financial market model. The procedure uses eight moment restrictions, based on a moment order vector  $\xi = (-2, -1.5, -1, -0.5, 0.5, 1, 1.5, 2)$ . The minimization search is based on the Nelder-Mead algorithm, and utilizes a large grid of starting points. The model is estimated separately on every individual return interval in the dataset. In total, we have 3004 daily intervals, 620 weekly intervals, 143 monthly intervals, 45 quarterly intervals, and 132 annual intervals. The intervals at the annual frequency may partially overlap, whereas the intervals at all other frequencies do not overlap, which explains why the number of annual intervals exceeds that of quarterly intervals (for details, see Section 4). In what follows, we focus on the estimates of the idiosyncratic volatility premium,  $\gamma$ . Notably, our paper is the first one to estimate and compare idiosyncratic volatility premia across a range of different return interval lengths.

Table 4 presents aggregate statistics for the estimates of  $\gamma$ . For each given return frequency, we calculate and report a (time-series) average of corresponding cross-sectional estimates of  $\gamma$ , and perform a test of the statistical significance of this average. Due to a potential time dependence in the underlying return data and the partial overlap among annual intervals, the estimates of  $\gamma$  may feature autocorrelation.<sup>10</sup> Therefore, we compute the test statistic for the average by the Newey and West (1987) method, which is robust to autocorrelation.<sup>11</sup> In addition, we report the median of

estimated  $\gamma$ 's, as well as the fractions of such (individual-period) estimates that are positive and statistically significant in a cross-sectional estimation at the 5% significance level, negative and significant at the 5% level, positive but not significant at the 5% level, and negative but not significant at the 5% level.

The average of estimated  $\gamma$ 's at the daily frequency (for brevity, we refer to these estimates as “daily  $\gamma$ 's,” and we use similar terminology for other frequencies) is 5.7668 and highly statistically significant. The median is 3.4752. Also, in individual cross-sectional estimations, a plurality of the daily  $\gamma$ 's are positive and statistically significant. These results suggest that the idiosyncratic volatility premium tends to be positive at the daily frequency.

The average of the weekly  $\gamma$ 's at  $-0.4305$  is not statistically significant.<sup>12</sup> In the case of longer return intervals, the averages of  $\gamma$ 's are negative and statistically significant. In particular, the averages of the monthly and quarterly  $\gamma$ 's are  $-1.3750$  and  $-1.0209$ , respectively, and statistically significant at the 1% level (the medians are  $-1.9883$  and  $-1.5148$ , respectively). Also, the majority of corresponding individual cross-sectional estimates are negative and statistically significant. Similar, although somewhat less pronounced, results are observed for the annual  $\gamma$ 's. Their average is  $-0.3178$  and statistically significant at the 1% level, but smaller in absolute magnitude than at the other frequencies (the median of the annual  $\gamma$ 's is  $-0.4359$ ). Also, fewer individual cross-sectional estimates in this case are statistically significant. Our findings that the idiosyncratic volatility premium tends to be negative at the monthly and annual return frequencies are consistent with Ang et al. (2006) and Jiang et al. (2009), who also estimate a negative premium (by applying a version of the two-pass approach).

To further explore the properties of  $\gamma$  estimates, we perform an additional analysis and ascertain a connection among the estimates at different return frequencies. In principle, the magnitude of  $\gamma$  can vary across the frequencies. Also,  $\gamma$  is likely to vary over time in response to changing stock market circumstances. As such, we expect idiosyncratic volatility premia at “near-by” frequencies to change over time, but to do so (i.e., increase or decrease) in the same direction, which implies that  $\gamma$ 's pertaining to the same period—but estimated at different frequencies—should be *positively correlated*. The finding of a positive association here would help to confirm that our estimates of  $\gamma$  reflect true idiosyncratic volatility premia, as opposed to a collection of unrelated numbers.

Table 5 presents results of ordinary least squares (OLS) regressions, which show positive associations among  $\gamma$ 's computed using daily, weekly, and monthly returns. Panel A reports the regression of a weekly  $\gamma$  on daily  $\gamma$ 's from the *same week*. We group the daily  $\gamma$ 's according to the day of the week on which an underlying daily return interval falls, and denote them as  $\gamma_{\text{Monday}}$ ,  $\gamma_{\text{Tuesday}}$ ,  $\gamma_{\text{Wednesday}}$ ,  $\gamma_{\text{Thursday}}$ , and  $\gamma_{\text{Friday}}$ . A similar approach is implemented in Panel B, which reports a regression of a monthly  $\gamma$  on weekly  $\gamma$ 's from the *same month*; these weekly  $\gamma$ 's are grouped by the week of the month and denoted as  $\gamma_{\text{week}_1}$ ,  $\gamma_{\text{week}_2}$ ,  $\gamma_{\text{week}_3}$ , and  $\gamma_{\text{week}_4}$ . All coefficients on the regressors are positive and statistically significant, indicating the existence of positive associations among the estimates of  $\gamma$ .<sup>13</sup> This result is noteworthy, in view of the fact that we use *different cross-sectional data* to estimate  $\gamma$  across the weeks and comprising them days (Panel A), and across the months and comprising them weeks (Panel B). We run similar OLS regressions using  $\gamma$ 's computed at the monthly, quarterly, and annual frequency, and

<sup>10</sup> Analyses using correlograms suggest the presence of such autocorrelation (results are not reported).

<sup>11</sup> We select the number of lags based on a corresponding correlogram. We always include at least five lags, and add more lags when working with the estimates at the annual frequency.

<sup>12</sup> Section 5.3 and Table 7 show that the average of the weekly  $\gamma$ 's becomes negative and statistically significant (at the 5% level) when only estimates for non-January intervals are considered.

<sup>13</sup> Additional details on the regressions (i.e., the number of observations,  $R^2$ , etc.) can be found in the notes to Table 5.

**Table 4**Aggregate statistics for estimates of idiosyncratic volatility premium ( $\gamma$ ).

Return frequency	Average	Robust <i>t</i> -statistic	Median	Fraction of estimates of $\gamma$ , percent			
				Positive, sign. at 5%	Negative, sign. at 5%	Positive, not sign. at 5%	Negative, not sign. at 5%
Daily	5.7668***	13.7196	3.4752	47.04	24.40	13.91	14.65
Weekly	−0.4305	−1.2438	−1.3827	29.52	43.06	10.00	17.42
Monthly	−1.3750***	−4.9385	−1.9883	12.59	50.35	8.39	28.67
Quarterly	−1.0209***	−5.9491	−1.5148	13.33	55.56	2.22	28.89
Annual	−0.3178***	−2.6344	−0.4359	9.09	23.48	11.36	56.06

This table reports aggregate statistics for the estimates of the idiosyncratic volatility premium,  $\gamma$ , in the financial market model, which are computed using returns in the dataset (the years 2000–2011). For each given return frequency (daily, weekly, etc.), we calculate and report the average of corresponding individual return-interval estimates of  $\gamma$  and perform a test of the statistical significance of this average. The test statistic, presented in the column “Robust *t*-statistic,” is computed by the Newey and West (1987) method and robust to autocorrelation. In addition, we calculate and report the median of the individual return-interval estimates of  $\gamma$ , as well as the fractions (in percent) of such estimates that are positive and statistically significant in a cross-sectional estimation at the 5% significance level, negative and significant at the 5% level, positive but not significant at the 5% level, and negative but not significant at the 5% level. The number of the individual return-interval estimates to calculate the statistics in this table is 3004 for the daily frequency, 620 for the weekly frequency, 143 for the monthly frequency, 45 for the quarterly frequency, and 132 for the annual frequency. Return intervals at the annual frequency may partially overlap. Return intervals at the other frequencies do not overlap.

\*\*\* Statistical significance:  $p < 0.01$ .

**Table 5**Associations among estimates of idiosyncratic volatility premium ( $\gamma$ ) at different return frequencies.

Regressor	Coefficient	<i>t</i> -statistic	Regressor	Coefficient	<i>t</i> -statistic
<i>Panel A: Regression of <math>\gamma</math> at weekly frequency on <math>\gamma</math>'s at daily frequency</i>			<i>Panel B: Regression of <math>\gamma</math> at monthly frequency on <math>\gamma</math>'s at weekly frequency</i>		
$\gamma_{Monday}$	0.1314***	7.1852	$\gamma_{week\_1}$	0.1067***	2.7990
$\gamma_{Tuesday}$	0.0638***	3.2705	$\gamma_{week\_2}$	0.1285***	3.6338
$\gamma_{Wednesday}$	0.0936***	4.8217	$\gamma_{week\_3}$	0.1249***	3.7747
$\gamma_{Thursday}$	0.0474**	2.4808	$\gamma_{week\_4}$	0.0730**	2.1272
$\gamma_{Friday}$	0.0572***	2.8249	Constant	−1.0935***	−5.3262
Constant	−2.9358***	−10.2569			

This table presents ordinary least squares (OLS) regression results illustrating associations among idiosyncratic volatility premia ( $\gamma$ 's) computed using daily, weekly, and monthly returns in the dataset (the years 2000–2011). *Panel A* reports the regression of a  $\gamma$  at the weekly frequency on  $\gamma$ 's obtained on daily returns from the same week. We group the latter  $\gamma$ 's by the day of the week and denote them accordingly. For example,  $\gamma_{Monday}$  refers to the  $\gamma$  obtained on Monday returns. The  $R^2$  in the regression is 0.2514 (adjusted  $R^2$  is 0.2439). The number of observations for the dependent variable is 510; it is less than the total number of the weekly  $\gamma$ 's (620), because some weeks may contain fewer than five business days (e.g., due to holidays or unscheduled stock exchange closures). *Panel B* reports the regression of a  $\gamma$  at the monthly frequency on  $\gamma$ 's obtained on weekly returns from the same month. We group the latter  $\gamma$ 's by the week of the month and denote them accordingly. For example,  $\gamma_{week\_1}$  refers to the  $\gamma$  obtained on the first week's returns. The  $R^2$  in the regression is 0.3446 (adjusted  $R^2$  is 0.3251). The number of observations for the dependent variable is 139; it is less than the total number of the monthly  $\gamma$ 's (143), because some months may contain fewer than four full business weeks (e.g., due to holidays or unscheduled stock exchange closures).

\*\* Statistical significance:  $p < 0.05$ .

\*\*\* Statistical significance:  $p < 0.01$ .

find positive and statistically significant associations among these  $\gamma$ 's as well (results are not reported).<sup>14</sup>

How large is the effect of  $\gamma$  on a cross-section of expected returns? In the following, we show that the effect can be economically significant. To be more specific, Proposition 1 (see the appendix) allows us to calculate a component of the conditional expected gross stock return that depends *only* on the model parameters related to idiosyncratic volatility. We refer to this component as the “idiosyncratic volatility component” and denote it as  $\mathcal{I}$ . In particular, the proposition shows that

$$\mathcal{I} = \frac{\exp(\lambda_\sigma \gamma T) - 1}{\lambda_\sigma \gamma T}, \text{ if } \lambda_\sigma \neq 0 \text{ and } \gamma \neq 0; \text{ and } \mathcal{I} = 1, \text{ otherwise.}$$

Thus,  $\mathcal{I}$  depends on the idiosyncratic volatility premium  $\gamma$ , and on the parameter  $\lambda_\sigma$  of the distribution of  $\sigma_i$ , Eq. (8). Other components of the conditional expected return do not depend on  $\gamma$  and  $\lambda_\sigma$ . Table 6 presents descriptive statistics for the natural logarithm of the idiosyncratic volatility component,  $\log(\mathcal{I})$ . More specifically,

<sup>14</sup> In addition to the regressions, we compute a correlation between an estimate of  $\gamma$  at a given frequency and an average of the estimates of corresponding  $\gamma$ 's at a higher frequency. For example, we compute the correlation between the weekly  $\gamma$  and the average of daily  $\gamma$ 's from the same week. We compute several such correlations by considering different combinations of frequencies. In all instances, the correlation is found to be positive, substantial in magnitude (it typically exceeds 0.5), and highly statistically significant.

**Table 6**

Idiosyncratic volatility component of conditional expected return and prediction discrepancy.

Return frequency		$\log(\mathcal{I})$	$ E - \bar{R} $
Daily	Average	0.01019	0.00069
	Std.dev.	0.02436	0.00868
Weekly	Average	−0.00040	0.00044
	Std.dev.	0.04973	0.00457
Monthly	Average	−0.04328	0.00133
	Std.dev.	0.10648	0.00538
Quarterly	Average	−0.12259	0.00364
	Std.dev.	0.13241	0.00607
Annual	Average	−0.15059	0.02864
	Std.dev.	0.23686	0.05740

This table presents descriptive statistics for the idiosyncratic volatility component ( $\mathcal{I}$ ) of the conditional expected gross return ( $E$ ). It also provides descriptive statistics for the discrepancy between the conditional expected gross return and the corresponding cross-sectional average of actual gross stock returns. This cross-sectional average is denoted here as  $\bar{R}$ ; its model counterpart is  $E$ . The statistics are computed using estimates of the parameters of the financial market model and returns in the dataset (the years 2000–2011). For each return interval, we compute the value of the natural logarithm of the idiosyncratic volatility component,  $\log(\mathcal{I})$ , and the absolute value of the difference between the conditional expected gross return and the cross-sectional average of the actual gross stock returns,  $|E - \bar{R}|$ . We then calculate and report the average and standard deviation (Std.dev.) of these two quantities over a given return frequency (daily, weekly, etc.). The quantity  $\log(\mathcal{I})$  illustrates the contribution of the idiosyncratic volatility component to the *net* conditional expected return.



for each individual return interval in the dataset, we use the parameter estimates to compute the value of  $\log(\mathcal{I})$ , and then calculate and report the average and standard deviation of  $\log(\mathcal{I})$  over each given return frequency. The value of  $\log(\mathcal{I})$  shows the contribution of the idiosyncratic volatility component to the *net* conditional expected return. In line with an earlier finding that daily  $\gamma$ 's are positive on average, the net return tends to increase by approximately 1% on the daily basis, due to the impact of the idiosyncratic volatility component. In comparison, the weekly impact tends to be almost negligible, consistent with the relatively small and not statistically significant average of the weekly  $\gamma$ 's. In turn, due to the impact of the idiosyncratic volatility component, the monthly return decreases by approximately 4% on average, the quarterly return decreases by 12% on average, and the annual return decreases by 15% on average. Overall, the sizeable magnitudes presented here suggest that the impact of the stock-specific idiosyncratic volatility on a cross-section of expected returns can be economically significant.

The estimated positive (on average) impact of the idiosyncratic volatility on the expected return at the daily frequency is consistent with a prediction of Merton's (1987) model. To clarify, Merton assumes that investors cannot fully diversify because of market frictions. In that case, a positive idiosyncratic volatility premium arises in the capital market equilibrium, as investors command higher expected returns for holding stocks with larger idiosyncratic volatilities (all else equal). It is conceivable that many investors may not fully diversify in practice on a daily basis, because of associated trading costs, for example. However, diversification may be easier to implement at lower frequencies. Thus, consistent with our results, we would expect to estimate a positive idiosyncratic volatility premium at the daily frequency, but would be less likely to find a positive premium at the weekly, monthly, etc. frequencies. In comparison, the estimated negative impact of the idiosyncratic volatility on the expected return at the monthly, quarterly, and annual frequencies is more challenging to explain. Peterson and Smedema (2011) and Bhootra and Hur (2011), among others, propose possible explanations for a negative idiosyncratic volatility premium. The gist of their explanations is that stocks with relatively high idiosyncratic volatilities may be overvalued by investors, and thus, such stocks could be associated with relatively low subsequent returns (for an example of this argument, see Peterson and Smedema, 2011, pp. 2556–2557).<sup>15</sup>

Lastly, Table 6 presents descriptive statistics for the values of the absolute difference between the conditional return (denoted here as  $E$ ) and the actual cross-sectional average ( $\bar{R}$ ) of gross stock returns in the sample,  $|E - \bar{R}|$ . The statistics—average and standard deviation—are listed separately for every given data frequency. The discrepancy between  $E$  and  $\bar{R}$  tends to be small in absolute magnitude. To illustrate, for daily intervals, the value of  $|E - \bar{R}|$  is 0.00069 on average. Overall, the relatively small values here suggest that the estimated model is a good fit to the data.

### 5.3. Analysis of January effect and additional robustness checks

Many published papers indicate the presence of a “January effect” (Tinic and West, 1984) in financial data. In particular, Peterson and Smedema (2011), among others, note that the effect may significantly affect inference regarding the idiosyncratic volatility premium. Motivated by these studies, we undertake a closer examination of the estimates of  $\gamma$ , by separating estimates computed for return intervals falling on a calendar month other than January (for brevity, “non-January intervals”) from those computed for intervals falling on January (“January intervals”).

Tables 7 and 8 provide aggregate statistics for estimates of  $\gamma$  for non-January and January intervals, respectively. These tables follow the layout of Table 4 in Section 5.2. As can be seen, the average of the non-January daily  $\gamma$ 's is 5.4798 and statistically significant at the 1% level. In turn, the average of the January daily  $\gamma$ 's is 8.8868 and also statistically significant at the 1% level. We formally test for an equality between these non-January and January averages, and reject the null hypothesis of the equality ( $P$ -value is 0.0075). The averages of the non-January and January weekly  $\gamma$ 's are  $-0.7068$  and  $2.0568$ , respectively; both are significant at the 5% level. Again, a test rejects the null of an equality between these averages ( $P$ -value is 0.0037). In the case of the monthly  $\gamma$ 's, we find that the non-January average is  $-1.6127$  and significant at the 1% level, whereas the January average is  $1.2194$  and not statistically significant. The null of an equality between these averages is rejected ( $P$ -value is 0.0001). Overall, in the cases of the daily, weekly, and monthly  $\gamma$ 's, the January average is higher than the corresponding non-January average.

The averages of the non-January and January quarterly  $\gamma$ 's are  $-1.0792$  (statistically significant at the 1% level) and  $-0.8603$  (significant at the 5% level), respectively. The averages of the non-January and January annual  $\gamma$ 's are  $-0.3326$  (significant at the 1% level) and  $-0.1553$  (not significant). In these instances, the January average tends to exceed the corresponding non-January average, but we are unable to formally reject the null of their equality ( $P$ -values are 0.5833 and 0.3455 for the quarterly and annual cases, respectively).

The results reported here are consistent with the findings in the literature and are indicative of the presence of a January effect. More specifically, this effect tends to increase the idiosyncratic volatility premium in expected returns during the month of January; especially in the cases of daily, weekly, and monthly returns.<sup>16</sup> Still, it should be noted that the results for  $\gamma$ 's estimated on non-January intervals are qualitatively similar to those for the full sample (Table 4), which indicates the robustness of our main findings.

In addition, we investigate whether our results for the idiosyncratic volatility premium at the daily and weekly frequencies may be driven by unusually large or unusually small estimates for any one calendar year in the 2000–2011 period. Table 9 breaks down aggregate statistics for the daily  $\gamma$ 's by calendar year. We find that in every year, the average of the daily  $\gamma$ 's is positive and statistically significant, with the exception of 2002, when the average is still positive but not statistically significant. The qualitative similarity of these year-by-year averages suggests that our finding of a positive average of the daily  $\gamma$ 's in the full sample (the years 2000–2011) is robust and unlikely to be substantially affected by outlier estimates. Table 10 reports a similar analysis using the weekly  $\gamma$ 's. In nearly all instances, the year-by-year averages are seen either to be not statistically significant (six out of twelve cases in total here; see the column “All intervals”), or to be negative and statistically significant (four cases). The two exceptions are the average for 2003, which is positive and marginally statistically significant (at the 10% level); and the average for 2010, which is positive and statistically significant (at the 5% level). These results are consistent with our finding (on the basis of the full sample) that the average of the weekly  $\gamma$ 's is negative and not statistically significant.

The financial crisis of 2008 motivates a closer look at the averages of the daily and weekly  $\gamma$ 's for the calendar years 2007, 2008, and 2009. Although the average of the daily  $\gamma$ 's is positive and statistically significant in 2008, its value at 2.3355 here is smaller than that in 2007 (6.0083) or 2009 (8.7605). As for the weekly  $\gamma$ 's, the 2007 average is  $-2.3823$  and statistically significant at the 1% level.

<sup>15</sup> Peterson and Smedema (2011) link such overvaluation to mispricing. Bhootra and Hur (2011) suggest that it might arise if investors are risk-seeking in the domain of capital losses.

<sup>16</sup> Relatively weaker results for quarterly and annual expected returns here are not surprising. Each of the underlying “January intervals” in these cases is longer than one month. Thus, the interval also covers a time period outside of the month of January, which could “dilute” the “January effect” here.

**Table 7**Aggregate statistics for estimates of idiosyncratic volatility premium ( $\gamma$ ), based on non-January return data.

Return frequency	Average	Robust <i>t</i> -statistic	Median	Fraction of estimates of $\gamma$ , percent			
				Positive, sign. at 5%	Negative, sign. at 5%	Positive, not sign. at 5%	Negative, not sign. at 5%
Daily	5.4798***	12.6585	3.1747	46.27	25.19	13.74	14.79
Weekly	−0.7068**	−1.9925	−1.5978	28.14	44.98	9.32	17.56
Monthly	−1.6127***	−5.8509	−2.3729	10.69	53.44	7.63	28.24
Quarterly	−1.0792***	−4.9395	−1.5262	12.12	51.52	0.00	36.36
Annual	−0.3326***	−2.7812	−0.4381	8.26	23.14	11.57	57.02

This table reports aggregate statistics for the estimates of the idiosyncratic volatility premium,  $\gamma$ , in the financial market model. We restrict the analysis to return intervals from calendar months other than the month of January in the dataset (the years 2000–2011). The table layout is analogous to that of Table 4. The number of the individual return-interval estimates to calculate the statistics in this table is 2751 for the daily frequency, 558 for the weekly frequency, 131 for the monthly frequency, 33 for the quarterly frequency, and 121 for the annual frequency. Return intervals at the annual frequency may partially overlap. Return intervals at the other frequencies do not overlap.

\*\* Statistical significance:  $p < 0.05$ .

\*\*\* Statistical significance:  $p < 0.01$ .

**Table 8**Aggregate statistics for estimates of idiosyncratic volatility premium ( $\gamma$ ), based on January return data.

Return frequency	Average	Robust <i>t</i> -statistic	Median	Fraction of estimates of $\gamma$ , percent			
				Positive, sign. at 5%	Negative, sign. at 5%	Positive, not sign. at 5%	Negative, not sign. at 5%
Daily	8.8868***	7.4139	9.1246	55.34	15.81	15.81	13.04
Weekly	2.0568**	2.3256	2.2826	41.94	25.81	16.13	16.13
Monthly	1.2194	1.7583	0.9932	33.33	16.67	16.67	33.33
Quarterly	−0.8603**	−2.5756	−1.2525	16.67	66.67	8.33	8.33
Annual	−0.1553	−1.0719	−0.4298	18.18	27.27	9.09	45.45

This table reports aggregate statistics for the estimates of the idiosyncratic volatility premium,  $\gamma$ , in the financial market model. We restrict the analysis to return intervals falling on the month of January in the dataset (the years 2000–2011). The table layout is analogous to that of Table 4. The number of the individual return-interval estimates to calculate the statistics in this table is 253 for the daily frequency, 62 for the weekly frequency, 12 for the monthly frequency, 12 for the quarterly frequency, and 11 for the annual frequency. Return intervals at each frequency do not overlap.

\*\* Statistical significance:  $p < 0.05$ .

\*\*\* Statistical significance:  $p < 0.01$ .

**Table 9**Estimates of idiosyncratic volatility premium ( $\gamma$ ) on daily data.

Year	All intervals		Non-January intervals		January intervals	
	Average	Robust <i>t</i> -statistic	Average	Robust <i>t</i> -statistic	Average	Robust <i>t</i> -statistic
2000	3.7280**	2.2076	2.6346*	1.8574	16.2469***	9.0854
2001	3.0725**	2.0638	2.4079**	2.0239	9.6283***	4.4698
2002	0.9954	1.0242	0.7773	0.8421	3.2564	1.1939
2003	8.6333***	7.3155	8.7352***	6.9540	7.5682**	2.3622
2004	6.5235***	4.7984	5.7227***	4.3032	15.3323***	7.0676
2005	7.1393***	8.7940	7.0342***	8.0493	8.2959***	3.0846
2006	7.3010***	7.8196	6.8581***	8.0943	12.1512***	4.8204
2007	6.0083***	6.7171	5.8100***	6.7509	8.1804***	2.9075
2008	2.3355***	2.8295	2.1490**	2.4357	4.2937***	3.2169
2009	8.7605***	7.4556	9.5808***	8.9346	−0.2624	−0.0699
2010	7.9717***	9.9531	7.8145***	9.4720	9.7948**	2.6841
2011	6.5551***	5.6562	5.9734***	5.0541	13.2735***	4.8893

This table reports the average of individual daily return-interval estimates of the idiosyncratic volatility premium,  $\gamma$ , for each calendar year in the dataset. It also provides the test statistic from a test of the statistical significance of this average, computed by the Newey and West (1987) method and robust to autocorrelation (“Robust *t*-statistic”). We present the results for all corresponding daily return intervals from a given year (“All intervals”), and additionally differentiate the results by calendar month: for intervals falling on a month other than January (“Non-January intervals”) vs. intervals falling on January (“January intervals”). The number of the individual return-interval estimates to calculate the statistics in this table varies. For the year 2000, we have a total of 249 estimates (of which 229 are for non-January intervals and 20 are for January intervals); the year 2001: 239 estimates (217 and 22 estimates, respectively); 2002: 250 (228 and 22); 2003: 252 (230 and 22); 2004: 252 (231 and 21); 2005: 252 (231 and 21); 2006: 251 (230 and 21); 2007: 251 (230 and 21); 2008: 253 (231 and 22); 2009: 252 (231 and 21); 2010: 252 (232 and 20); and 2011: 251 (231 and 20).

\* Statistical significance:  $p < 0.10$ .

\*\* Statistical significance:  $p < 0.05$ .

\*\*\* Statistical significance:  $p < 0.01$ .

The 2008 average is also negative (−3.1956) and statistically significant at the 1% level. However, in the post-crisis year 2009, the average changes the sign and loses statistical significance.

In addition, Tables 9 and 10 present year-by-year averages of the daily and weekly  $\gamma$ 's, respectively, by separating the results for non-January return intervals from those for January intervals. The aggregate statistics for January intervals here should be interpreted with some caution because they may be based on relatively small subsamples, especially in the case of weekly data. In nearly

all years, the January averages of daily and weekly  $\gamma$ 's tend to exceed the corresponding non-January averages, with the exception of the years 2003 and 2009 in the case of the daily  $\gamma$ 's, and the years 2002 and 2009 in the case of the weekly  $\gamma$ 's. Overall, these findings provide further support for the presence of the January effect. Also, in line with the existing literature, the results suggest that expected stock returns during January may exceed those during other calendar months, due to a differential idiosyncratic volatility premium effect when all else is equal.

**Table 10**Estimates of idiosyncratic volatility premium ( $\gamma$ ) on weekly data.

Year	All intervals		Non-January intervals		January intervals	
	Average	Robust <i>t</i> -statistic	Average	Robust <i>t</i> -statistic	Average	Robust <i>t</i> -statistic
2000	0.5567	0.5840	−0.0781	−0.0807	6.5230***	5.9916
2001	−1.1268	−1.4825	−1.6024*	−1.9726	2.3608	1.8332
2002	−1.6558**	−2.0694	−1.6216*	−1.8700	−1.9701	−1.0596
2003	1.6186*	1.7342	1.4186	1.4370	3.4984	1.1588
2004	0.2487	0.2674	−0.3663	−0.3895	6.0295**	4.3241
2005	−1.5504*	−1.7771	−1.7258*	−1.7135	0.0980	0.0486
2006	−0.9316	−1.0350	−1.7895**	−2.1321	7.1318**	3.2575
2007	−2.3823***	−3.5326	−2.8660***	−3.6961	1.2459	0.3973
2008	−3.1956***	−6.6551	−3.2295***	−5.2671	−2.9294**	−3.1246
2009	1.5290	1.1864	1.8293	1.4130	−1.2937	−0.4875
2010	1.6406**	2.0582	1.5312*	1.8186	2.6685	0.9967
2011	0.0580	0.0734	−0.1505	−0.1752	2.5085	0.7760

This table reports the average of individual weekly return-interval estimates of the idiosyncratic volatility premium,  $\gamma$ , for each calendar year in the dataset. The layout of the table is analogous to that of Table 9, except that all intervals here are weekly (rather than daily). For the year 2000, we have a total of 52 estimates (of which 47 are for non-January intervals and 5 are for January intervals); the year 2001: 50 estimates (44 and 6 estimates, respectively); 2002: 51 (46 and 5); 2003: 52 (47 and 5); 2004: 52 (47 and 5); 2005: 52 (47 and 5); 2006: 52 (47 and 5); 2007: 51 (45 and 6); 2008: 53 (47 and 6); 2009: 52 (47 and 5); 2010: 52 (47 and 5); and 2011: 51 (47 and 4).

\* Statistical significance:  $p < 0.10$ .

\*\* Statistical significance:  $p < 0.05$ .

\*\*\* Statistical significance:  $p < 0.01$ .

## 6. Conclusion

We propose a new approach to estimate the idiosyncratic volatility premium. Unlike the conventional two-pass method, it allows us to obtain a consistent estimate of the premium by using only a single cross-section of return data; having a long historical financial time-series is not required. The approach is based on a novel GMM-type procedure, which accounts for a strong cross-sectional dependence of stock return observations caused by a common source of market risk. The estimation is performed in the setting of a financial market model comprising a market index and many individual stocks.

We empirically investigate the idiosyncratic volatility premium by applying the proposed approach to daily, weekly, monthly, quarterly, and annual US stock return data over 2000–2011. On average, we find a positive and statistically significant premium for daily return data; a small negative and not statistically significant premium for weekly data; and a negative and statistically significant premium for monthly, quarterly, and annual data. The magnitudes of the estimates suggest that the impact of the idiosyncratic volatility on the expected return should not be ignored. Also, we document that the estimates of the premium using data for daily, weekly, and monthly intervals falling on the month of January tend to be higher than those in the case of the other months.

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## Appendix A

**Proposition 1.** The conditional expected value  $E_\theta \left[ \left( S_T^i / S_0^i \right)^\xi | M_T / M_0 \right]$  can be expressed as

$$E_\theta \left[ \left( S_T^i / S_0^i \right)^\xi | M_T / M_0 \right] = \exp[r\xi T] \times \mathcal{S} \left[ \xi \left( \ln \frac{M_T}{M_0} + \left[ \frac{1}{2} \sigma_m^2 - r \right] T \right), -\frac{1}{2} \xi \sigma_m^2 T \right] \times \mathcal{I} \left[ \xi \gamma T, \frac{1}{2} \xi (\xi - 1) T \right],$$

where the function  $\mathcal{S}[x_S, y_S]$  with  $x_S = \xi \left( \ln \frac{M_T}{M_0} + \left[ \frac{1}{2} \sigma_m^2 - r \right] T \right)$  and  $y_S = -\frac{1}{2} \xi \sigma_m^2 T$  is:

$$\mathcal{S}[x_S, y_S] = \frac{\sqrt{\pi}}{2\lambda_\beta \sqrt{y_S}} \exp \left( -\frac{x_S^2}{4y_S} \right) \left[ \operatorname{erfi} \left( \frac{x_S}{2\sqrt{y_S}} + (\kappa_\beta + \lambda_\beta) \sqrt{y_S} \right) - \operatorname{erfi} \left( \frac{x_S}{2\sqrt{y_S}} + \kappa_\beta \sqrt{y_S} \right) \right] \text{ if } \xi < 0;$$

$$\mathcal{S} = 1 \text{ if } \xi = 0;$$

$$\mathcal{S}[x_S, y_S] = \frac{\sqrt{\pi}}{2\lambda_\beta \sqrt{-y_S}} \exp \left( -\frac{x_S^2}{4y_S} \right) \left[ \operatorname{erf} \left( \frac{x_S}{2\sqrt{-y_S}} - \kappa_\beta \sqrt{-y_S} \right) - \operatorname{erf} \left( \frac{x_S}{2\sqrt{-y_S}} - (\kappa_\beta + \lambda_\beta) \sqrt{-y_S} \right) \right] \text{ if } \xi > 0.$$

In turn, the function  $\mathcal{I}[x_I, y_I]$  with  $x_I = \xi \gamma T$  and  $y_I = \frac{1}{2} \xi (\xi - 1) T$  is:

$$\mathcal{I}[x_I, y_I] = \frac{\sqrt{\pi}}{2\lambda_\sigma \sqrt{y_I}} \exp \left( -\frac{x_I^2}{4y_I} \right) \left[ \operatorname{erfi} \left( \frac{x_I}{2\sqrt{y_I}} + \lambda_\sigma \sqrt{y_I} \right) - \operatorname{erfi} \left( \frac{x_I}{2\sqrt{y_I}} \right) \right] \text{ if } \xi < 0 \text{ or } \xi > 1;$$

$$\mathcal{I}[x_I, y_I] = 1 \text{ if } \xi = 0 \text{ or if } \xi = 1 \text{ and } x_I = 0;$$

$$\mathcal{I}[x_I, y_I] = \frac{\exp(\lambda_\sigma x_I) - 1}{\lambda_\sigma x_I} \text{ if } \xi = 1 \text{ and } x_I \neq 0;$$

$$\mathcal{I}[x_I, y_I] = \frac{\sqrt{\pi}}{2\lambda_\sigma \sqrt{-y_I}} \exp \left( -\frac{x_I^2}{4y_I} \right) \left[ \operatorname{erf} \left( \frac{x_I}{2\sqrt{-y_I}} \right) - \operatorname{erf} \left( \frac{x_I}{2\sqrt{-y_I}} - \lambda_\sigma \sqrt{-y_I} \right) \right] \text{ if } 0 < \xi < 1,$$

where  $\operatorname{erf}(\cdot)$  is the error function,  $\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-t^2) dt$ , and  $\operatorname{erfi}(\cdot)$  is the imaginary error function,  $\operatorname{erfi}(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(t^2) dt$ .

**Proof.** By plugging the expression for  $\left( S_T^i / S_0^i \right)^\xi$  into  $E_\theta \left[ \left( S_T^i / S_0^i \right)^\xi | M_T / M_0 \right]$  and using properties of the moment generating function of a normal random variable  $Z_T^i$ , we obtain  $E_\theta \left[ \left( S_T^i / S_0^i \right)^\xi | M_T / M_0 \right] = E_\theta \left[ \exp \left( \xi (\mu_i - \frac{1}{2} \beta_i^2 \sigma_m^2 - \frac{1}{2} \sigma_i^2) T + \xi \beta_i \sigma_m W_T + \frac{1}{2} \xi^2 \sigma_i^2 T \right) | M_T / M_0 \right]$ . By plugging in the expression for  $W_T$  implied by Eq. (5), we obtain

$$E_{\theta} \left[ \left( S_T^i / S_0^i \right)^{\xi} | M_T / M_0 \right] = \exp(r\xi T) \times E_{\theta} \left[ \exp \left( \xi \left( \ln \frac{M_T}{M_0} + \left[ \frac{1}{2} \sigma_m^2 - r \right] T \right) \cdot \beta_i + \left( -\frac{1}{2} \xi \sigma_m^2 T \right) \cdot \beta_i^2 \right) | M_T / M_0 \right] \\ \times E_{\theta} \left[ \exp \left( \xi \gamma T \cdot \sigma_i + \frac{1}{2} \xi (\xi - 1) T \cdot \sigma_i^2 \right) | M_T / M_0 \right].$$

Importantly, note that this step eliminates  $\delta$ . Next, we take into account distributional assumptions for  $\beta_i$  and  $\sigma_i$  and use Lemma 1 to show that

$$E_{\theta} \left[ \exp \left( \xi \left( \ln \frac{M_T}{M_0} + \left[ \frac{1}{2} \sigma_m^2 - r \right] T \right) \beta_i + \left( -\frac{1}{2} \xi \sigma_m^2 T \right) \beta_i^2 \right) | M_T / M_0 \right] \\ = S \left[ \xi \left( \ln \frac{M_T}{M_0} + \left[ \frac{1}{2} \sigma_m^2 - r \right] T \right), -\frac{1}{2} \xi \sigma_m^2 T \right], \text{ and} \\ E_{\theta} \left[ \exp \left( \xi \gamma T \sigma_i + \frac{1}{2} \xi (\xi - 1) T \sigma_i^2 \right) | M_T / M_0 \right] = \mathcal{I} \left[ \xi \gamma T, \frac{1}{2} \xi (\xi - 1) T \right]. \quad \square$$

**Lemma 1.** Suppose that a random variable  $X$  is uniform,  $X \sim \text{UNI}[\kappa, \kappa + \lambda]$ , where  $\lambda > 0$ . Consider real constants  $a$  and  $b$ . If  $b < 0$ , then  $E[\exp(aX + bX^2)] = \frac{\sqrt{\pi}}{2\lambda\sqrt{-b}} \exp\left(-\frac{a^2}{4b}\right) \times \left[\text{erf}\left(\frac{a}{2\sqrt{-b}} - \sqrt{-b}\kappa\right) - \text{erf}\left(\frac{a}{2\sqrt{-b}} - \sqrt{-b}(\kappa + \lambda)\right)\right]$ . If  $b = 0$ , then  $E[\exp(aX + bX^2)] = \frac{\exp[a(\kappa + \lambda)] - \exp(a\kappa)}{a\lambda}$ . If  $b > 0$ , then  $E[\exp(aX + bX^2)] = \frac{\sqrt{\pi}}{2\lambda\sqrt{b}} \exp\left(-\frac{a^2}{4b}\right) \left[\text{erfi}\left(\frac{a}{2\sqrt{b}} + \sqrt{b}[\kappa + \lambda]\right) - \text{erfi}\left(\frac{a}{2\sqrt{b}} + \sqrt{b}\kappa\right)\right]$ .

**Proof.** The proof is straightforward and available from the authors on request.  $\square$

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