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Reputation and signaling in asset sales☆

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ABSTRACT

Static adverse selection models of security issuance show that informed issuers can perfectly reveal their private information by maintaining a costly stake in the securities they issue. This paper shows that allowing an issuer to both signal current security quality via retention and build a reputation for honesty leads that issuer to misreport quality even when owning a positive stake, that is, the equilibrium is neither separating nor pooling. An issuer retains less as reputation improves and prices are more sensitive to retention when the issuer has a worse reputation.

1. Introduction

On April 16, 2010, the US Securities and Exchange Commission (SEC) filed a complaint against Goldman Sachs (GS) alleging that the investment bank had misled investors in the Abacus 2007–AC1 collateralized debt obligation (CDO). In response, GS raised at least two important points that seem to be in line with financial economic theory. First, during GS's first quarter earnings conference call in 2010, Gregory Palm, general counsel for GS, stated that "a significant point missing from the SEC's complaint was the fact that Goldman Sachs retained a significant residual long position in the transaction.... We certainly had no incentive to structure a transaction that was designed to lose money" (Palm, 2010). In other words, GS maintained some skin in the game in the Abacus deal. Second, in a supplemental submission to the SEC, attorneys for GS asserted: "Nor is there any basis to suggest that Goldman Sachs would have intentionally jeopardized its own reputation and relationship with established customers and counterparties.... Goldman Sachs had no reason to mislead anyone" (Klapper et al., 2010). That is, GS would not intentionally mislead investors because doing so could damage its reputation.

The skin-in-the-game defense could follow from the notion that informed issuers reveal their information through costly signaling by maintaining a stake in their is-

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sue and that such a stake should be a sufficient statistic for all private information (Leland and Pyle, 1977; Nachman and Noe, 1994; DeMarzo and Duffie, 1999; DeMarzo, 2005). The reputation defense could follow from the idea that concern for future business can lead issuers to be truthful when making public statements about their issues, such as in marketing materials or a deal prospectus. Considered separately, these defenses make some economic sense, although the interactive effects of signaling and reputation need further exploration. This paper fills that gap and shows that the interaction between reputation and signaling can cause issuers to have a greater incentive to mislead investors. These findings reveal that reputation can improve allocative efficiency, as well as provide for incentives to misrepresent asset quality.

In this paper, I consider an infinitely repeated asset sales game. In each period, a risk-neutral issuer is endowed with an asset to sell. Nature chooses the quality of the asset, which can be either high or low. Asset quality denotes expected cash flow at the end of the period. Patient risk-neutral investors compete to buy the fraction of the asset that is sold. The issuer can perfectly observe the type of the asset, but this information is not available to investors. Thus, a classic lemons problem arises as in Akerlof (1970). The issuer publicly reports the type of asset in a prospectus, but this report need not be truthful. In addition, the issuer can signal asset quality by retaining a fraction of it. Such a signal is credible because the issuer finds it more costly to engage in fractional asset retention when selling a low-quality asset.

Reputation concerns for the issuer arise due to asymmetric information over issuer preferences for honesty. The issuer could be one of two types. The honest-type issuer is committed to truthfully reporting the asset's quality in the prospectus. The opportunistic-type issuer chooses a reporting strategy that maximizes payoffs. Both types optimally choose a fraction of the asset to retain. The issuer's reputation is the probability that the investors place on the issuer being the honest type, as in Kreps and Wilson (1982), Milgrom and Roberts (1982), and Mathis et al. (2009). By mimicking an honest issuer (i.e., truthfully reporting asset quality in the prospectus), an opportunistic-type issuer's reputation can be improved, thereby reducing the lemons discount on the fraction of the asset sold to investors.

A central focus of this paper is the degree to which the issuer's concern for her reputation can improve allocative efficiency. In a static version of my model, under some parameter restrictions, the only equilibrium is the classic least cost separating equilibrium (LCSE). In this equilibrium, the issuer retains just enough of every high-quality asset she sells to signal its quality. This retention leads to forgone gains from trade. Reputation effects impact this result in three ways. First, if gains from trade are high enough, the concern for future reputation leads the opportunistic-type issuer to truthfully report, making retention as a signal of asset quality unnecessary. Second, when the issuer has a higher reputation, investors perceive that a greater chance exists that the issuer is mechanically honest. This raises the price of an asset following a high-quality report even if investors anticipate that the opportunistic-type issuer will pool her low-quality assets with high-quality assets. In effect, a higher reputation means that there is a greater chance that an asset is high quality given a report of high quality. This allows for pooling equilibria with greater allocative efficiency even when they would not exist in the static game. Third, when reputation is too low to sustain a full pooling equilibrium, concern for reputation can leave the opportunistic-type issuer indifferent between truthfully reporting and misrepresentation. In this case, a partial-pooling equilibrium obtains in which the issuer retains some of the asset when she reports that it is high qualty, but not as much as she would if she had zero reputation.

The fact that reputation concerns can lead to a pooling equilibrium gives rise to a criticism of the claim that reputation is the core self-disciplining mechanism for markets, such as that for asset-backed securities (ABS). If one ignores the possibility that issuers can signal asset quality through costly retention, reputation certainly provides some incentives for issuers of ABS to truthfully reveal their private information. However, in a pure costly signaling setting, the issuer would perfectly reveal her information via retention. When reputation and costly signaling interact, the issuer can engage in retention yet misreport at the same time. This feature of the model is appealing given the alleged prevalence of opportunistic behavior in many ABS markets, even though issuers often maintain a position in the assets they sell. For example, Piskorski et al. (2015) show that mortgage-backed securities (MBS) issuers misrepresent important information about the quality of mortgages underlying the securities they sell. Although this type of misrepresentation does not negatively affect the allocative efficiency for the agents in my model, it can have implications for real outcomes outside of the model. For example, misrepresentation in the market for MBS could have led to excessive lending in the primary mortgage market. As such, it is important to understand MBS issuer incentives to misreport.

My theoretical results give rise to new empirical predictions that apply to any financial market with asymmetric information between issuers and buyers, frequent issuances, and observable issuer retention. A good example is the ABS market, although the model can also be applied to other markets such as the repurchase agreement (repo) or asset-backed commercial paper markets. The first empirical prediction is that when discount rates are low or gains from trade are high, there would be no issuer retention and a large dispersion in prices. When discount rates are high or gains from trade are low, the issuer retains a fraction of the asset only when her reputation is low. Retention will decrease as issuer reputation improves because costly signaling and reputation act as substitutes. As such, issuer retention should decrease with good past performance and increase with bad past performance. In the ABS market, this translates to the issuer retaining a larger equity tranche when more of her past tranches have been downgraded. In the repo market, this could translate to larger haircuts when past collateral turns out to be of lower quality than expected. This relation implies that an improvement in issuer reputation will initially decrease the investor's beliefs about the probability that the issuer will truthfully reveal asset quality when reputation is low and increase that probability when reputation is high. In equilibrium, the issuer's actual strategies must be consistent with this belief. Thus, prices should be more sensitive to retention for low-reputation issuers than for high-reputation issuers. The differential sensitivity of prices to issuer retention can help explain why some researchers, for example, Garmaise and Moskowitz (2004), find that price is insensitive to issuer retention.

To close the paper, I demonstrate that reputation can lead to a pooling equilibrium even when imposing a standard refinement criterion for signaling games. In a static setting, pooling equilibria are typically eliminated by the D1 refinement criterion of Banks and Sobel (1987) and Cho and Kreps (1987). When reputation effects are considered, the issuer's preferences no longer satisfy a single crossing property. As a result, a pooling equilibrium with reputation concerns can survive elimination by D1. The intuition is that opportunistic-type issuers selling low-quality assets can be more willing to sacrifice reputation and, hence, investors must ascribe off-equilibrium actions to those types of issuers.

This paper relates to the literature on adverse selection in financial markets. In his seminal paper, Akerlof (1970) shows that private information can cause important distortions in markets. Since Leland and Pyle (1977) and Myers and Majluf (1984), this idea has also been applied to the markets for financial securities. In these markets, informed agents often can take actions that reveal their private information (Spence, 1973). Leland and Pyle (1977) show that informed issuers of financial securities can retain a fraction of a security to signal quality. DeMarzo and Duffie (1999) and DeMarzo (2005) build on this idea to show that when such sellers can signal via retention, issuing a debt like claim backed by assets can be optimal. The signaling mechanism I consider in this paper is similar to that of Leland and Pyle (1977), DeMarzo and Duffie (1999), and DeMarzo (2005).

This paper also relates to the literature on reputation effects in repeated games. Intuitively, agents involved in repeated games could try to acquire a reputation for a certain characteristic in the early stages of the game if that characteristic improves payoffs in later stages. Kreps and Wilson (1982) and Milgrom and Roberts (1982) introduce the notion that imperfect or asymmetric information about player preferences can provide such a mechanism. By observing a given player's previous actions, other agents update their beliefs about that player's type. These beliefs then serve as a form of reputation. In two seminal papers, Diamond (1989) and Diamond (1991) show that reputation can act as an important incentive mechanism in firm financing and investment decisions. Recently, Mathis et al. (2009), Josephson and Shapiro (2012), and Griffin et al. (2013) show that a high reputation can lead informed sellers or intermediaries to misrepresent asset quality. Winton and Yerramilli (2015) consider issuer moral hazard in a repeated setting and find that issuers retain less as their reputation improves. In their model, issuer retention solves an ex ante moral hazard problem. In my model, retention serves as a signaling device to overcome an adverse selection problem. This difference means that, in their model, good behavior monotonically increases with reputation; in my model, the effect is non-monotonic. Chari et al. (2014) consider a reputation model of loan sales with a persistent adverse selection problem. They find that reputation effects can lead to partial pooling equilibria similar to those I consider.

The remainder of the paper proceeds as follows. Section 2 lays out the setup of the model. Section 3 provides an equilibrium analysis of the model. Section 4 introduces novel empirical predictions that follow from the equilibrium analysis. Section 5 examines the application of D1 to the framework of the model. Section 6 concludes. All proofs appear in Appendix C.

2. The model

In this section, I introduce the economic environment of the model and define the equilibrium concept and refinement criteria I use to analyze the model.

2.1. The setup

The economy is populated by an issuer and a unit measure of competitive investors. Trade between the issuer and investors takes place over an infinite succession of periods. At the start of each period, the issuer is endowed with a single asset that produces a cash flow X at the end of the period. Both the issuer and the investors have the same inter-period discount factor, δ. The issuer has an intra-period discount factor of $\gamma < 1$, and the investors have an intra-period discount factor of 1. The inter-period discount factor represents the issuer's preferences for future versus present profits and determines the value of issuer reputation. The difference in intra-period discounting between the issuer and investors represents the relative preference of the issuer for cash over assets and creates the gains from trade in the model. The difference in preferences between the issuer and the investors could arise for a number of reasons, including capital requirements and access to additional investment opportunities, and is a somewhat standard motivation for trade in adverse selection models in finance (DeMarzo and Duffie, 1999; DeMarzo, 2005).

In any given period, the asset's quality is drawn from a binary type distribution and is independent and identically distributed across periods. The asset is of high quality with probability λ and of low quality otherwise. High-quality assets produce a cash flow $(1-\delta)$, and low-quality assets produce a cash flow $(1-\delta)\ell$ with $\ell \in (0, 1)$. Because I focus on the interactive effects of costly signaling and reputation, I consider binary asset cash flows. These cash flows imply perfect observability and allow for explicit characterizations of strategies and value functions. I also abstract from security design issues by limiting the analysis to equity claims on cash flows. For an analysis of security design under a rich set of cash flow distributions in a static setting, see Nachman and Noe (1994) or DeMarzo and Duffie (1999). High- and low-quality assets are denoted h and ℓ, respectively.

At the start of each period, the issuer perfectly observes the quality of the asset, at which point she can sell a fraction $q \in [0, 1]$ of it to investors. In addition to choosing the fraction of the asset offered to investors, the issuer produces a report indicating the quality of the asset in $\{h, \ell\}$. While investors observe both the fraction of the asset on offer and the issuer's report of its quality, they do not observe the asset's true quality. I focus on model parameters that imply a relatively severe adverse selection problem by imposing Assumption 1.

Assumption 1. The unconditional value of the asset to investors is lower than the issuer's private value for a high-quality asset

$$\lambda + (1-\lambda)\ell < \gamma. \tag{1}$$

Following Kreps and Wilson (1982) and Milgrom and Roberts (1982), reputation concerns arise due to incomplete information about issuer preferences. The issuer can be of two types: honest or opportunistic. The honest-type issuer always provides a truthful report and chooses a quantity of the asset to issue that maximizes expected proceeds from securitization and retained assets. The opportunistic-type issuer chooses both a report and a quantity to maximize expected proceeds from securitization and retained assets in the current period, plus the discounted value of her expected future cash flows. The unconditional probability that the issuer is the honest type, denoted ϕ_0, is common knowledge. A convenient way to describe the issuer's type in a given period is as an element from a two-dimensional type space. Along one dimension is the issuer's permanent type given by her preference for honesty and along the other is the quality of the asset. Thus, I call an issuer the honest-high type when she is honest and currently has a high-quality asset to sell, with a corresponding meaning for honest-low, opportunistic-high, and opportunistic-low types.

2.2. Strategies, beliefs, and equilibrium

Given the history of the game and the asset quality she has for sale, the issuer's reporting strategy is described by a function $\pi : \{\ell, h\} \times \{\mathcal{H}_t\} \to [0, 1]$, which maps the quality of the asset the issuer currently has to sell and the history of the game to the probability that she will truthfully report. I restrict attention to strategies that are pure in the quantity dimension conditional on a report. In other words, once the issuer has chosen a report, she does not mix over the quantity she sells. Given this restriction, the issuer's quantity strategy is a function $Q : \{\ell, h\} \times \{\mathcal{H}_t\} \times \{\ell, h\} \to [0, 1]$, which maps the quality of the asset she currently has to sell, the history of the game, and her report of asset quality to the quantity she offers to the market.

Given the observable history of the game, the current report of asset quality, and the quantity of the asset on offer, the investors form beliefs about the issuer's type and reporting strategy. The investors' belief that the issuer is the honest type is called the issuer's reputation and is denoted by ϕ. The investors update this belief via a function $\phi' : \{\mathcal{H}_t\} \to [0, 1]$, which maps the history of the game to a probability that the issuer is the honest type. The investors' common belief that the asset on offer is high quality is given by a function $\mu : \{\ell, h\} \times \{\mathcal{H}_t\} \times [0, 1] \to [0, 1]$ mapping the reported asset quality, the history of the game, and the quantity on offer to a probability that the asset is high quality. Finally, the market price for the asset is a function $P : \{\ell, h\} \times \{\mathcal{H}_t\} \times [0, 1] \to [0, 1]$, which maps the reported asset quality, the history of the game, and the quantity on offer to a per unit price the issuer receives for the asset. For convenience, I assume that the market price is normalized by the cash flow of the high-quality asset, $1-\delta$, so that the proceeds from the sale of q units of an asset are $(1-\delta)qP$.

I use a subscript $i \in \{\ell, h\}$ on strategies, beliefs, or prices that depend on the quality of the asset the issuer currently has to sell or on her report of asset type. For strategies that depend on both, I use a double subscript. The honest- and opportunistic-type issuers can form different strategies, so I indicate their respective strategies with a superscript H or O where appropriate. For example, Q^O_{ij} denotes the quantity that the opportunistic-type issuer sells given that she has an i-quality asset and reports that it is j-quality. The honest-type issuer never gives a false report, i.e., $\pi^H_h = \pi^H_\ell = 1$, so I use π only to refer to the reporting strategy of the opportunistic-type issuer and omit the superscript.

In any given period, the issuer maximizes current stage game payoff plus the discounted expected payoffs of all future stage games given market prices and investors' beliefs. Formally, given a price p as a function of the issuer's report, the quantity on offer, and an action (q, π), an opportunistic-type issuer with a low-quality asset has a stage game payoff of

$$\begin{aligned} U^O_\ell(\pi, q_\ell, q_h; p) = {} & (1-\delta)[\pi_\ell(q_\ell p_\ell(q_\ell) + \gamma(1-q_\ell)\ell) \\ & + (1-\pi_\ell)(q_h p_h(q_h) + \gamma(1-q_h)\ell)]. \end{aligned} \tag{2}$$

If she has a high-quality asset, she has a stage game payoff of

$$\begin{aligned} U^O_h(\pi, q_\ell, q_h; p) = {} & (1-\delta)[\pi_h(q_h p_h(q_h) + \gamma(1-q_h)) \\ & + (1-\pi_h)(q_\ell p_\ell(q_\ell) + \gamma(1-q_\ell))]. \end{aligned} \tag{3}$$

Facing the same price function and given an action q, an honest-type issuer with a low-quality asset has a stage game payoff of

$$U^H_\ell(q; p) = (1-\delta)[q p_\ell(q) + \gamma(1-q)\ell]. \tag{4}$$

If she has a high-quality asset, she has a stage game payoff of

$$U^H_\ell(q; p) = (1-\delta)[q p_h(q) + \gamma(1-q)]. \tag{5}$$

I use the following definition of equilibrium for the majority of the paper.

Definition 1. A set of issuer strategies (π, Q), investor beliefs (μ, ϕ), and market prices P is a Markov perfect equilibrium if

1. All agents' strategies and beliefs depend on the history of past play $\mathcal{H}_t$ only through reputation $\phi_t = \phi'(\mathcal{H}_t)$.
2. The opportunistic-type issuer's strategy is optimal given the strategies of the honest type and the beliefs of the investors.

3. The honest-type issuer's strategy is optimal given the strategies of the opportunistic type and the beliefs of the investors.
4. The investors' beliefs are consistent with Bayes' rule and the issuer's strategies where possible.
5. Market prices are such that investors earn zero profits given their beliefs.

An equilibrium is separating if prices fully reflect the true quality of the asset.

Condition 1 of Definition 1 states that ϕ must be a Markov state variable for the game. That is, given that investors' beliefs depend on $\mathcal{H}_t$ only through ϕ_t, it must be optimal for the issuer to choose a strategy that also conditions on $\mathcal{H}_t$ only through ϕ_t, and vice versa. I consider equilibria that satisfy Conditions 2–5, but not Condition 1, in the Online Appendix. To simplify notation, I use ϕ_t as an argument in place of $\mathcal{H}_t$ wherever possible. Conditions 2–4 are the standard conditions of a perfect Bayesian equilibrium. Given Condition 1, Conditions 2 and 3 are formally stated, respectively as

$$(\pi, Q^O) \in \arg\max_{\tilde{\pi},\tilde{q}} E\left[\sum_{t=0}^{\infty} \delta^t U^O(\tilde{\pi}, \tilde{q}; P)\right] \tag{6}$$

and

$$Q^H \in \arg\max_{\tilde{q}} E\left[\sum_{t=0}^{\infty} \delta^t U^H(\tilde{q}; P)\right]. \tag{7}$$

The expectation operator is taken over the probability distribution of paths of asset quality and the resulting dynamics of reputation induced by the issuer's strategies. Also, reputation ϕ_t affects these payoffs through the price function P. Condition 4 requires that investors' beliefs about asset quality be consistent with the issuer's strategy and Bayes' rule:

$$\mu_\ell(\phi, Q(\phi)) = \frac{\lambda(1-\pi_h(\phi))(1-\phi)}{\lambda(1-\pi_h(\phi)) + (1-\lambda)\pi_\ell(\phi)} \tag{8}$$

and

$$\mu_h(\phi, Q(\phi)) = \frac{\lambda(\phi + (1-\phi)\pi_h(\phi))}{\lambda(\phi + (1-\phi)\pi_h(\phi)) + (1-\lambda)(1-\phi)(1-\pi_\ell(\phi))}. \tag{9}$$

Condition 5 is a proxy for a more fundamental definition of asset market equilibrium and is due to competition among investors. Formally, Condition 5 requires

$$P_i(\phi, q) = (1-\delta)(\mu_i(\phi, q) + (1-\mu_i(\phi, q))\ell) \tag{10}$$

for $i \in \{h, \ell\}$.

2.3. Equilibrium refinement

As is common in signaling games, potential exists for multiplicity of equilibria. To reduce this set, I use a simplified version of the undefeated equilibrium (UE) criterion introduced by Mailath et al. (1993). To that end, consider a proposed equilibrium and a quantity action $\tilde{q}$ not played in that equilibrium. Also, suppose there is an equilibrium of the static version of the stage game without quality reporting such that there is an issuer type who plays the action $\tilde{q}$ and would prefer to play this equilibrium for one period and then return to the repeated game and continue to play the original proposed equilibrium. The proposed equilibrium is defeated if investor beliefs following the action $\tilde{q}$ (regardless of the reporting action) are not consistent with this type. A proposed equilibrium is undefeated if no such equilibrium exists of the static version of the stage game. For a formal equilibrium definition for the static version of the stage game without reporting, see Appendix A. This definition is just a natural restriction of Definition 1 to a one-period game in which the issuer cannot report asset quality. I use the following definition of an undefeated equilibrium.

Definition 2. An equilibrium $(\tilde{Q}, \tilde{\mu}, \tilde{P})$ of the stage game without reporting defeats an equilibrium (π, Q, μ, ϕ, P) if there exists a reputation ϕ and a quantity $\tilde{Q}^*$ such that

1. $Q_i^T \neq \tilde{Q}^*$ for all $(T, i) \in \{O, H\} \times \{h, \ell\}$ and the set K of types that play $\tilde{q}$ in $(\tilde{Q}, \tilde{\mu}, \tilde{P})$ is not empty:
$$K = \{(T, i) \in \{O, H\} \times \{h, \ell\} : \tilde{Q}_i^T = \tilde{Q}^*\} \neq \emptyset; \tag{11}$$
2. For any $(T, i) \in K$,
$$U_i^T(\pi_i, Q_i^T; P) \leq U_i^T(\tilde{Q}^*; \tilde{P}), \tag{12}$$
and there exists $(T, i) \in K$ such that
$$U_i^T(\pi_i, Q_i^T; P) < U_i^T(\tilde{Q}^*; \tilde{P}); \tag{13}$$
and
3. Investor beliefs are
$$\mu(\phi, \tilde{Q}^*) \neq \lambda\rho_h + (1-\lambda)\rho_\ell. \tag{14}$$
where ρ: $\{h, \ell\} \to [0, 1]$ is a function such that $(T, i) \in K$ and $U_i^T(\pi_i, Q_i^T; P) < U_i^T(q; \tilde{P})$ implies $\rho_i = 1$.

An equilibrium (π, Q, μ, ϕ, P) satisfies undefeated equilibrium refinement if there does not exist such an equilibrium $(\tilde{Q}, \tilde{\mu}, \tilde{P})$ of the stage game without reporting.

Intuitively, one can think of the UE refinement as follows. Suppose, in addition to the market for assets described in Section 2.1, the issuer can sell her asset in an anonymous market in any given period. The refinement states that investor beliefs off the equilibrium path must be consistent with the fact that the issuer could go to the anonymous market for a single period and then return to selling assets in the market described in Section 2.1 without any effect on her reputation.

In addition to applying the UE refinement to reduce the set of possible equilibria, I restrict attention to equilibria that satisfy two reasonable criteria. The first restriction is to equilibria in which the investors always believe a report that the asset is low quality. In principle, an equilibrium could exist in which the opportunistic-type issuer chooses to misreport a high-quality asset as a type of code to the investors. In such an equilibrium, investors would place positive probability on the asset being high quality even after receiving a report that it is low quality. However, such equilibria are somewhat pathological in nature in that the issuer's report is used as something other than a representation of quality to investors. The second restriction is to equilibria in which the quantities on offer do not influence investors' beliefs that the issuer is the honest type in subsequent periods. This restriction is roughly equivalent to assuming that investors are short lived and only get to observe the history of past performance and reports of quality. These two restrictions are formally stated as follows.

Restriction 1. In equilibrium, investors always believe the asset is low quality following a report that the asset is low quality:

$$\mu_\ell = 0. \tag{15}$$

Restriction 2. Investors update the issuer's reputation based on the past performance of asset and the issuer reports of asset quality, not based on past quantities:

$$\phi'(\mathcal{H}_t) = \mathbb{P}(\text{issuer is honest}|\{X_s, i_s\}_{s\leq t}). \tag{16}$$

To be clear,equilibria that violate Restrictions 1 and 2 could exist. However, these restrictions are somewhat intuitive and greatly simplify the solution to the model. Also, Restrictions 1 and 2 do not automatically mean that equilibria with those properties exist. It must be shown that when investor beliefs satisfy these restriction, the issuer takes actions consistent with such beliefs.

While focusing on equilibria that satisfies UE together with Restrictions 1 and 2 proves to be tractable, it is certainly not the only way in which to refine the set of equilibria of the game. A more common selection criterion for signaling games is the D1 belief-based refinement of Cho and Kreps (1987). Applying D1 to the game does not allow as sharp results as UE. For the sake of robustness, in Section 5, I dispense with the UE refinement and Restrictions 1 and 2, and I show that some of the main results extend to equilibria that survive D1 refinement.

3. Equilibrium analysis

In this section, I characterize equilibria of the game as well as analyze their properties.

3.1. Equilibrium when the issuer is known to be opportunistic

Before characterizing equilibria for an arbitrary initial reputation, it is useful to consider the simple case when the issuer is known to be the opportunistic type from the start of the game. In this case, the restriction to Markov perfect equilibrium greatly simplifies the set of possible equilibria.

Proposition 1. Suppose $\phi_0 = 0$. There exists a separating equilibrium given by the issuer strategies

$$\pi_h = \pi_\ell = 1, \quad Q^O_{\ell,\ell} = Q^H_{\ell,\ell} = 1, \quad Q^O_{\ell h} = Q^O_{h,h} = Q^H_{h,h} = \hat{q}, \tag{17}$$

investor beliefs

$$\mu_h(q) = \begin{cases} 1 & \text{if } q \leq \hat{q} \\ 0 & \text{otherwise} \end{cases},$$

where

$$\hat{q} = \frac{(1-\gamma)\ell}{1-\gamma\ell}, \quad \mu_\ell(q) = 0 \tag{18}$$

and prices

$$P_\ell(\phi, q) = \ell, \quad P_h(\phi, q) = \ell(1-\mu_h(q)) + \mu_h(q). \tag{19}$$

Moreoever, this equilibrium is the unique equilibrium satisfying UE.

The equilibrium of Proposition 1 as the well-known least cost separating equilibrium. The intuition behind this equilibrium is essentially the same as that of the classic signaling results of Spence (1973) or Leland and Pyle (1977). The quantity $\hat{q}$ is defined so that even when the market responds with a price per share of one for the quantity $\hat{q}$, the issuer with a low-quality asset is better off selling the entire asset for the price ℓ. At the same time, the issuer with a high-quality asset strictly prefers selling the quantity $\hat{q}$ at a price per unit of one to retaining the entire asset. Such a quantity $\hat{q}$ exists because the relative intra-period impatience of the issuer implies that retaining a fraction of the asset is more costly for the issuer when she has a low-quality asset than when she has high-quality asset. This equilibrium is called least cost because it entails the smallest amount of asset retention and, hence, the lowest signaling cost of any separating equilibrium.

If I had not imposed Assumption 1, there would also be pooling equilibria in which the issuer sells the entire asset regardless of its type. However, such pooling equilibria would lead to payoffs equal to those under full information. Moreover, UE refinement does select a pooling equilibrium for some parameter values. For example, if little probability exists that the asset is low quality, then separation can be very costly, and all players would be better off under the pooling equilibrium. Under such values, the adverse selection problem is not sufficiently severe to provide for a role for quality reporting or retention. To focus on cases where quality reporting and retention can impact the game, I maintain Assumption 1.

Proposition 1 also has strong implications for possible equilibria in the game including issuer reputation. If the issuer ever attains a reputation of zero, the only possible equilibrium in continuation play is to repeat the LCSE for all time. In the game with reputation, the issuer trades off one-period profits against a loss in reputation. In Proposition 1, I pin down consequences of such a loss in reputation.

3.2. Reputation dynamics and the issuer's problem

With a fixed equilibrium for the repeated game once the issuer has lost her reputation, I can solve for the dynamics of reputation in equilibrium. I start with the following results in Lemmas 1 and 2, which are key in deriving the equilibrium.

Lemma 1. In any equilibrium that satisfies Restriction 1, the opportunistic-type issuer never misreports the quality of a high-quality asset,

$$\pi_h(\phi) = 1. \tag{20}$$

Lemma 2. In any equilibrium that satisfies Restrictions 1 and 2,

$$Q^O_{\ell\ell} = Q^H_{\ell\ell} = 1 \tag{21}$$

and

$$Q^O_{\ell h} = Q^O_{hh} = Q^H_{hh}. \tag{22}$$

That is, the opportunistic- and honest-type issuers sell the entire asset conditional on reporting that is low quality and always sell the same quantity of the asset conditional on reporting that is high quality.

The intuition behind Lemma 2 is as follows. The opportunistic-type issuer and the honest-type issuer both value instantaneous payoffs from the securitization of an asset identically conditional on quality. At the same time, the opportunistic-type issuer values a higher reputation weakly more than the honest type, because she always has the option to cash in on her reputation while the honest issuer does not. Moreover, an improvement in reputation can only make this option weakly more valuable. Therefore, any quantity strategy that improves reputation is at least as attractive to the opportunistic type as it is to the honest type. This implies that the quantity issued is not a credible signal of issuer type and, on the equilibrium path, cannot contain any new information about whether the issuer is honest. In addition, in Restriction 2, I limit attention to equilibria in which investors do not update reputation by observing off-equilibrium path quantities. Thus, investors update the issuer's reputation only after observing the current period's asset cash flow, not upon observing the current quantity offered. Finally, it cannot be an equilibrium for the issuer to misreport a low-quality asset but then offer a quantity that differs from what she would offer if the asset were truly high quality. In this case, Condition 4 of Definition 1 would require that the investors believe that the asset is low quality and the issuer is opportunistic, in which case the issuer would be better off simply truthfully reporting the low-quality asset.

Given Lemmas 1 and 2, deriving the dynamics of reputation in terms of the reported quality and ex post performance of the asset is straightforward. After the realization of the issuer's actions and asset performance in the previous period, Condition 4 of Definition 1 and Restriction 2 pin down what the investors' beliefs about the issuer's type must be at the start of the next period. Let $\phi'(\mathcal{H}_t) = f(\phi, i, j)$ denote the issuer's start of period reputation, given that in the previous period she had reputation ϕ, took an action consistent with an equilibrium strategy, and reported that the asset was j-quality and the asset cash flow was i-quality. Applying Bayes' rule where possible gives

$$f(\phi, \ell, \ell) = \frac{(1-\lambda)\mu_h}{\mu_h - \lambda}\phi, \tag{23}$$

$$f(\phi, \ell, h) = 0, \tag{24}$$

$$f(\phi, h, \ell) = 0, \tag{25}$$

and

$$f(\phi, h, h) = \phi. \tag{26}$$

Eq. (23) from Bayes' rule and the fact the investors' beliefs π^e_ℓ about the issuer's reporting strategy solve

$$\mu_h = \frac{\lambda}{\lambda + (1-\lambda)(1-\phi)(1-\pi^e_\ell)}. \tag{27}$$

Several aspects of the reputation updating function f are noteworthy. First, if the investors observe that the asset turned out to be of a quality different from what the issuer reported, the issuer's reputation falls to zero. Second, a high-quality cash flow following a high-quality report does not change the issuer's reputation. This is due to the fact that I have restricted attention to equilibria in which $\mu_\ell = 0$. Finally, if the issuer ever obtains a reputation of zero, her reputation stays at zero for the remainder of the game.

I can now simplify the optimization problem facing the issuer given the reputation updating function and the fact that the opportunistic- and honest-type issuers always choose the same retention strategy. Let $V^O(\phi)$ denote the value function of the opportunistic-type issuer fixing the market price P and investor beliefs μ, and let $V^O_\ell(\phi)$ and $V^O_h(\phi)$ denote the value functions when the opportunistic-type issuer is endowed with a low- or high-quality asset, respectively. Then,

$$V^O(\phi) = \lambda V^O_h(\phi) + (1-\lambda)V^O_\ell(\phi), \tag{28}$$

where

$$V^O_h(\phi) = \max_{q_h}\Big\{(1-\delta)(q_h P_h(q;\phi) + \gamma(1-Q)) + \delta V(f(\phi, h, h))\Big\} \tag{29}$$

and

$$V^O_\ell(\phi) = \max_{(\pi, q_h)}\Big\{\pi_\ell((1-\delta)(\ell + \delta V(f(\phi, \ell, \ell))) + (1-\pi_\ell)((1-\delta)(q_h P_h(q;\phi) + \gamma(1-q_h)\ell) + \delta V(0)\Big\}. \tag{30}$$

In solving the above problem in Eqs. (28)–(30), the issuer takes as given the investors' beliefs and, hence, the reputation updating function.

3.3. Separating equilibria

In the full game, separating equilibria can occur in two ways. First, as in the case without reputation, when the issuer has a high-quality asset, she can retain enough of the asset to convince investors that the quality of the asset is high quality. Second, the threat of a loss in reputation from being exposed as the opportunistic type could cause the opportunistic type to always be truthful in her report. I refer to equilibria of this kind as truth-telling. An equilibrium is truth-telling if $Q^O_{hh}(\phi) > \hat{q}$ for some $\phi > 0$ and $\pi_\ell(\phi) = \pi_h(\phi) = 1$ for all $\phi > 0$. In Proposition 2, I give conditions for the existence of the truth-telling equilibrium and the repeated LCSE.

Proposition 2. The repeated version of the LCSE is an equilibrium of the game for all ϕ and all parameterizations of the model. Moreover, the following three statements are equivalent.

1. *The parameters of the model satisfy*

$$d\delta \geq \frac{1-\gamma\ell}{1-\gamma\ell+\lambda(1-\gamma)}. \tag{31}$$

2. *There exists a truth-telling equilibrium.*
3. *There exists a truth-telling equilibrium with $Q^O_{hh} = Q^H_{hh} = 1$.*

Finally, both the truth-telling equilibrium and the LCSE satisfy UE.

The restriction on the parameters that is required for the existence of a truth-telling equilibrium depends on the instantaneous gains to the issuer from misreporting a low-quality asset, as well as the loss in continuation value from being identified as the opportunistic type. To see this, if the investors always believe the report of the issuer, the opportunistic-low type issuer can report high, receive a price of $1-\delta$ for a low-quality asset, and be known as the opportunistic type and revert to the LCSE thereafter. Such a deviation is profitable if and only if

$$\underbrace{(1-\ell)(1-\delta)}_{\text{Gain in proceeds}} \leq \underbrace{\delta(\lambda+(1-\lambda)\ell)}_{\substack{\text{Continuation value}\\ \text{from truth-telling}}} - \underbrace{\delta(\lambda(\hat{q}+\gamma(1-\hat{q}))+(1-\lambda)\ell)}_{\text{Continuation value of LCSE}}, \tag{32}$$

which simplifies to the condition given by the inequality in Eq. (31). An interpretation of this inequality is that the issuer must be sufficiently patient, i.e., have a high inter-period discount factor, so as to make a loss in continuation value severe enough to provide incentives to always accurately report a low-quality asset. Alternatively, the inequality in Eq. (31) is also satisfied when the gains from trade (i.e., the difference between the investors' and issuer's intra-period discount factors $1-\gamma$) are sufficiently large. When these gains are large, maintaining reputation is more valuable to the issuer than the one-period gains from misreporting. Under these conditions, the truth-telling equilibrium exists. If the inter-period discount factor or the gains from trade are small, then no truth-telling equilibrium exists and the most allocatively efficient separating equilibrium is the LCSE.

An important statement in Proposition 2 is the equivalence of Points 2 and 3. That is, if a truth-telling equilibrium exists, then there is a perfectly allocatively efficient truth-telling equilibrium. Thus, if the parameters of the model satisfy inequality in Eq. (31), then perfect allocative efficiency is achievable in equilibrium.

3.4. Partial pooling equilibria

So far I have characterized two types of separating equilibrium. The repeated LCSE always exists, but when it prevails there is a loss of allocative efficiency. To signal asset quality, issuers must retain a fraction of the high-quality assets they have to sell. The truth-telling equilibrium allows for perfect allocative efficiency. However, the truth-telling equilibrium exists only if the issuer is sufficiently patient. This suggests the following question: Are there equilibria that are more allocatively efficient than the repeated LCSE when the issuer is too impatient to support the truth-telling equilibrium? In this subsection, I describe equilibria that achieve better allocative efficiency than the repeated LCSE and call for the opportunistic-low-type issuer to (partially) pool with issuers of high-quality assets some of the time. For the remainder of the paper, I restrict attention to parameters of the model that do not support the truth-telling equilibrium by imposing Assumption 2.

Assumption 2. The parameters of the model do not support the truth-telling equilibrium: $\delta < \frac{1-\gamma\ell}{1-\gamma\ell+\lambda(1-\gamma)}$.

In the static version of the game, the partial pooling equilibrium exists only when the proportion of low-quality assets in the pool is small enough. Otherwise, an issuer would be better off retaining a high-quality asset than pooling with issuers selling low-quality assets. When the issuer has a good reputation, investors perceive that there is a large probability the issuer is the honest type. Thus, even if the opportunistic-type issuer always misreports low-quality assets, the proportion of such assets in the pool following a report of high quality is small. As a result, the pooled price is high and the high-quality asset issuers are better off joining the pool than either retaining the entire asset or attempting to separate via partial retention. In this way, one can think of a high reputation as lowering the proportion of low-quality assets, thus facilitating pooling.

When issuer reputation is not high enough to lead to full pooling, a partial pooling equilibrium can obtain. Consider a candidate equilibrium with $Q^O_{\ell h} = Q^O_{hh} = Q^H_{hh} = \tilde{Q}$. Here, the potential equilibrium quantity $\tilde{Q}$ as well as the investors' beliefs about the opportunistic-low type's reporting strategy determines the viability of a partial pooling equilibrium. The opportunistic- and honest-high types have an incentive to pool (instead of attempting to separate by selling a smaller quantity) only if the price for a reportedly high-quality asset at the pooled quantity is high enough. This price depends on both the reputation of the issuer and the investors' beliefs that the opportunistic-low type truthfully reports. Fixing μ_h and ϕ, the greater is $\tilde{Q}$, the greater is the opportunistic-low type's incentive to misreport. In equilibrium, investor beliefs must reflect this incentive, so that when $\tilde{Q}$ is larger, the equilibrium price is lower. This effect determines how pooling quantities depend on reputation.

When issuer reputation is low, the price investors are willing to pay for reportedly high-quality assets at the quantity that allows for pooling is very sensitive to their beliefs about the opportunistic-type issuer's reporting strategy. When the issuer has a low reputation, the investors believe a large chance exists that the issuer is opportunistic. Thus, to guarantee that the investors' beliefs μ_h correspond to the opportunistic-low type's strategy π_ℓ, the pooling quantity $\tilde{Q}$ must be smaller when reputation is lower. As reputation improves, the effect of the investors' beliefs about the opportunistic-type issuer's reporting strategy on price decreases and equilibrium can be supported with larger $\tilde{Q}$, i.e., a more allocatively efficient equilibrium. In Proposition 3, I formalize this intuition by demonstrating the existence and properties of a particular partial pooling equilibrium.

Proposition 3. There exists an equilibrium satisfying UE with the following properties.

1. *If the issuer's initial reputation is below a threshold $\underline{\phi}$, the strategies and investor beliefs are consistent with the LSCE and reputation never improves.*
2. *If the issuer's initial reputation is above $\underline{\phi}$ but below a threshold $\bar{\phi}$, then $Q^H_{hh} = Q^O_{hh} = \tilde{Q}(\phi) > \hat{q}$ where $\tilde{Q}(\phi)$ increases in reputation and $\tilde{\pi}_\ell(\phi)$ decreases in reputation. Moreover, $\tilde{Q}(\phi) = 1$ for all $\phi \geq \bar{\phi}$.*

Moreover, if

$$\delta \leq \frac{\gamma(1-\ell)}{1-\gamma\ell}, \tag{33}$$

then $\bar{\phi}$ is the lowest possible reputation at which full allocative efficiency can be achieved.

In the proof of Proposition 3, I construct a partial pooling equilibrium using the following approach. First, I show $\bar{\phi}$ exists such that there is an equilibrium with $Q^0_{\ell h}(\phi) = Q^0_{hh}(\phi) = Q^H_{hh}(\phi) = 1$ and $\pi_\ell(\phi) = 0$ for all $\phi \geq \bar{\phi}$. Next, I solve for equilibrium reporting strategies and investor beliefs by working downwards in reputation, applying the single deviation principal and the restriction on off-equilibrium path payoffs given by the UE refinement. If the equilibrium strategies are known for all ϕ greater than some $\tilde{\phi}$, then one can solve for $\pi(\tilde{\phi})$, $\mu_h(\tilde{Q}(\tilde{\phi}), \phi')$, and $\tilde{Q}(\tilde{\phi}) = Q^0_{\ell h}(\phi) = Q^0_{hh}(\phi)$ by comparing the issuer's benefit in current period payoffs from misreporting with her loss in reputation from doing so and checking that investors' beliefs are consistent with the issuer's reporting strategy and respect UE refinement. Formally, I define the issuer's payoff gain from misreporting as

$$\Gamma(\mu,\phi,q) = (\mu + (1-\mu)\ell)q + \gamma(1-q)\ell - \ell - \frac{\delta}{1-\delta}\left(V^0\left(\frac{(1-\lambda)}{\mu-\lambda}\phi\right) - V^0(0)\right). \tag{34}$$

Note that $\Gamma(\mu, \phi, q)$ is increasing in μ. Moreover, the lowest possible investor belief that the asset is high quality is

$$\underline{\mu}(\phi) = \frac{\lambda}{\lambda + (1-\lambda)(1-\phi)}, \tag{35}$$

while the highest possible belief is $\mu = 1$. Finally, the definition of the UE refinement requires that the issuer's payoff when selling a high-type asset be at least what she could obtain in an anonymous market, so that

$$P_h(\phi, \tilde{Q}(\phi))\tilde{Q}(\phi) + \gamma(1-\tilde{Q}(\phi)) \geq \hat{q} + \gamma(1-\hat{q}). \tag{36}$$

I can now identify three regions depending on the value of Γ and the restriction imposed by UE.

Region 1: Full pooling. In Region 1,

$$\Gamma(\underline{\mu}(\phi), \phi, 1) \geq 0 \tag{37}$$

and

$$\underline{\mu} + (1-\underline{\mu})\ell \geq \hat{q} + \gamma(1-\hat{q}). \tag{38}$$

Eq. (37) requires that the payoff gain to misreporting for the opportunistic-low type is always positive regardless of investor beliefs. Eq. (38) guarantees that the equilibrium satisfies UE and requires that an issuer selling a high-quality asset is willing to pool with the opportunistic-low types even if investors believe the opportunistic-low types always misreport. Investor beliefs in Region 1 are

$$\mu_h(\phi,q) = \begin{cases} 1 & \text{if } q \leq \hat{q}, \\ \underline{\mu}(\phi) & \text{otherwise.} \end{cases} \tag{39}$$

The issuer reporting strategy $\pi_\ell(\phi) = 0$ and the quantity strategy $\tilde{Q} = 1$ then satisfy Conditions 2 and 4 of Definition 1, as well as UE.

Region 2: Partial pooling. In Region 2, there exists $\underline{\mu} < \tilde{\mu}(\phi) < 1$ and $\tilde{Q}(\phi)$ such that

$$\Gamma(\underline{\mu}(\phi), \phi, \tilde{Q}(\phi)) = 0 \tag{40}$$

and

$$(\tilde{\mu}(\phi) + (1-\tilde{\mu}(\phi))\ell)\tilde{Q}(\phi) + \gamma(1-\tilde{Q}(\phi)) = \hat{q} + \gamma(1-\hat{q}). \tag{41}$$

Eq. (40) states that the payoff gain to misreporting for the opportunistic-low type given the investor beliefs $\mu_h(\phi, \tilde{Q}(\phi)) = \tilde{\mu}(\phi)$ is exactly zero so that the opportunistic-low type is indifferent between all possible reporting strategies. Eq. (41) guarantees that the candidate strategies satisfy UE and states that an issuer selling a high-quality asset is indifferent between pooling with the opportunistic-low types and separating when investor beliefs about asset quality at the pooled quantity are $\tilde{\mu}(\phi)$. Equilibria could exist for which this condition does not hold with equality in this region. However, they largely resemble the equilibrium studied here. The investor beliefs in Region 2 are

$$\mu_h(\phi,q) = \begin{cases} 1 & \text{if } q \leq \hat{q}, \\ \tilde{\mu}(\phi) & \text{if } \hat{q} < q < \tilde{Q}(\phi), \\ 0 & \text{otherwise.} \end{cases} \tag{42}$$

The issuer reporting strategy

$$\pi_\ell(\phi) = 1 - \left(\frac{1}{1-\phi}\right)\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{1-\tilde{\mu}(\phi)}{\tilde{\mu}(\phi)}\right) \tag{43}$$

and the quantity strategy $\tilde{Q} = 1$ then satisfy Conditions 2 and 4 of Definition 1, as well as UE.

Region 3: Separation. In Region 3, issuer strategies and investor beliefs revert to those of the separating equilibrium.

The key intuition behind the nature of the equilibrium given in Proposition 3 centers on the interaction between the opportunistic-low type's incentives to misreport and the high type's incentive to remain in the pool. Holding quantities and investors' beliefs μ fixed, the opportunistic low type issuer's incentives to misreport are decreasing in reputation. Thus, one could expect that, for lower (higher) levels of reputation, the investors believe that opportunistic-low type accurately reports with lower (greater) probability, i.e., π^e_ℓ is increasing in ϕ. However, investor's beliefs about asset quality, and hence the market price for the asset, following a report of high quality must always be high enough to entice the high type to maintain the pool. In equilibrium, the quantity on offer is set so as to make the opportunistic-low type's incentives to pool equate her optimal reporting strategy with investor beliefs. As a result, the quantity on offer increases in reputation. To illustrate this mechanism in more detail, consider what happens for different levels of the issuer's initial reputation.

If the issuer's initial reputation is low, π_ℓ^e has a large effect on price. At the same time, the price conditional on a report of high quality must be large enough to incentivize the issuers with high-quality assets to pool instead of separate. As a result, to sustain a partial pooling equilibrium, π_ℓ^e must be large. This implies that the increase in reputation following an accurate report of a low-quality asset is small and that the loss in reputation due to misreporting, and hence the incentive to accurately report, is also small. Equilibrium requires that the issuer have incentives to choose a reporting strategy consistent with the investors' beliefs. This in turn requires that the quantity sold be small enough that the payoff gain from misreporting, Γ, for the opportunistic-low type is zero.

If the issuer's initial reputation is somewhat higher, π_ℓ^e has a smaller effect on price. In this case, the price can be high enough to incentivize the issuers selling high-quality assets to pool when π_ℓ^e is low. Thus, the loss in reputation from misreporting is large and provides a large incentive to the issuer to truthfully report. As a result, the quantity sold can increase and the opportunistic-low type is still indifferent between truthfully reporting and not.

If the opportunistic-type issuer begins with a sufficiently high reputation, that is, if initial reputation allows the issuer to sell a larger fraction of reportedly high-quality assets, her reputation gradually improves each time she truthly reports a low-quality asset. This means that she also has a larger incentive to misreport the next low-quality asset. Eventually, the opportunistic-type issuer either has misreported, and hence investors will know she is the opportunistic type, or has accumulated such a good reputation that she cashes in with probability one.

The intuition given above highlights the main reason that the reputation model differs from a repeated games model. In the reputation model, opportunistic behavior has a decreasing impact on equilibrium prices as reputation improves. In a repeated games model without reputation, as discussed in the Online Appendix, these beliefs must always have the same effect on prices regardless of past play.

One important implication of Proposition 3 is that the addition of reputation effects to a standard signaling game recovers pooling equilibrium when they otherwise would not exist. By imposing Assumption 1, I rule out the existence of pooling equilibrium in the static game. The equilibrium presented in Proposition 3 respects Assumption 1 and features pooling. In Section 5, I show that reputation effects also allow pooling equilibria to survive D1 refinement.

Regarding the long-run behavior of the partial pooling equilibrium, when the issuer's initial reputation is low, that is, when $\phi_0 \leq \underline{\phi}$, equilibrium calls for the issuer to repeatedly play the LCSE strategies. In this case, the issuer never has an opportunity to improve her reputation, as investors expect all issuer types to be truthful. When the issuer's initial reputation is somewhat higher, that is, $\phi_0 > \underline{\phi}$, then the issuer has a chance to improve upon her reputation. In the construction of equilibrium contained in the proof of Proposition 3, I show that with probability one the issuer either obtains a reputation greater than $\phi \geq \bar{\phi}$ or misreports at some date. Once the issuer has obtained such a high reputation, she either truthfully reports a sufficient number of low-quality assets to prove that she is the honest type or she misreports a low-quality asset to reveal that she is the opportunist type. One could then speculate that two markets would emerge: a market in which opportunistic issuers transact and a market in which honest issuers transact. This discussion must be taken with a grain of salt, however, because an issuer's true preference for honesty is likely subject to exogenous shocks in reality, so that the equilibrium never fully reveals honest or opportunistic types. In this case, a steady state equilibrium is likely to emerge with similar characteristics to the partial pooling equilibrium.

3.5. Comparative statics for the partial pooling equilibrium

I next examine how the allocative efficiency of the equilibrium in Proposition 3 depends on the model's parameters by studying the threshold $\bar{\phi}$. When δ is low enough, $\bar{\phi}$ is the lowest possible reputation at which it is possible to achieve perfect allocative efficiency. In this case, the solution for $\bar{\phi}$ is

$$\bar{\phi} = \frac{\gamma(1-\ell(1-\lambda))-\lambda}{\gamma(1-\ell)(1-\lambda)}. \tag{44}$$

This solution allows for the comparative statics in Proposition 4.

Proposition 4. When $\delta \leq \frac{\gamma(1-\ell)}{1-\gamma\ell}$, allocative efficiency increases in γ and decreases in ℓ and λ.

The comparative statics in Proposition 4 all follow from the effect that a change in parameters has on the value of reputation versus the one-period gains from misreporting. The comparative statics with respect to γ show that as the gains from trade, $1-\gamma$, increase, allocative efficiency improves. When gains from trade are larger, reputation is more valuable. As a result, the opportunistic-type issuer has a lower incentive to misreport low-quality assets. Thus, the quantity the issuer can sell when reporting she has a high-quality asset increases in that the minimum reputation, $\bar{\phi}$, required for the issuer to sell the entire asset decreases. The comparative statics $\bar{\phi}$ with respect to ℓ and λ show that as the lemons problem becomes less severe, allocative efficiency again improves. When the lemons problem is less severe, the one-period gains from misreporting are lower, and reputation is more valuable.

4. Empirical implications

As is common in signaling games, my model admits multiple equilibrium. To make empirical predictions in a consistent way, I select an equilibrium based on allocative efficiency. I apply the following equilibrium selection in Case I and Case II based on parameters:

Case I: $\delta \geq \frac{1-\gamma\ell}{1-\gamma\ell+\lambda(1-\gamma)}$. The truth-telling equilibrium is selected as it is perfectly allocatively efficient.

Case II: $\delta < \frac{1-\gamma\ell}{1-\gamma\ell+\lambda(1-\gamma)}$. The partial pooling equilibrium of Proposition 3 is selected as it features greater allocative efficiency than the repeated LCSE.

The first set of theoretical results with empirical relevance I explore concerns the qualitatively different implications for allocative efficiency between Case I and Case II. Prediction 1 states that markets are perfectly allocatively efficient if the model's parameters satisfy Case 1. Otherwise, the adverse selection problem will lead to some loss in allocative inefficiency in that the issuer will retain some fraction of the asset for sale.

Prediction 1. Issuer retention is less likely in markets with low discount rates or high gains from trade.

One aspect of Prediction 1 is that it does not condition on reputation and can therefore be tested without a direct measure of it. Consider the mortgage-backed securities market. Suppose one could identify two time periods: one in which gains from trade are low and one in which gains from trade are high. These periods could correspond to cold and hot mortgage markets as measured by volume of new issuance. A hot market could have a large amount of issuance precisely because the gains from trade are high. As a result, issuer retention in the MBS market should be on average higher in cold mortgage markets.

The next implication that follows from the difference between Case I and Case II is that the issuer's equilibrium reporting strategy, and thus the informativeness of prices, can be highly sensitive to the parameters of the model. Consider the equilibrium behavior for high-reputation issuers just above and below the cut-off for the existence of the truth-telling equilibrium. When δ satisfies Case I, the issuer always truthfully reveals asset quality regardless of her type and market prices always reflect an asset's true quality. When δ satisfies Case II, the high-reputation opportunistic-type issuer always misreports low-quality assets. A similar situation applies to the gains from trade, $1-\gamma$. Thus, a small change in the parameters can lead to a discontinuous change in the informativeness of prices. Moreover, prices in markets that function on reputation alone (Case I) are more informative than markets that rely on both reputation and signaling to overcome asymmetric information problems. I summarize these implications in Prediction 2.

Prediction 2. Issuers are more likely to misrepresent asset quality when gains from trade are high or discount rates are low. The prices for high-quality assets and the probability of misrepresentation can be discontinuous in discount rates.

This prediction is consistent with Piskorski et al. (2015), who find that MBS intermediaries misrepresented asset quality during the US housing price boom of the early 2000s. During the housing boom, the expected growth of the economy was high, leading to high discount rates.

Empirical predictions also follow from the characteristics of the partial pooling equilibrium. Prediction 3 relates asset retention and issuer reputation.

Prediction 3. When markets believe an issuer is more likely to be honest, i.e., when an issuer has a higher reputation, that issuer retains less of her assets for sale.

To see how Prediction 3 follows from the analysis above, recall that, in the partial pooling equilibrium in Proposition 3, the quantity the issuer offers for sale (retains) upon reporting that the asset is high-quality increases (decreases) in her reputation. Note that this does not mean that honest- and opportunistic-type issuers retain different amounts of the asset. Rather when the market believes there is a greater chance the issuer is honest (opportunitistic), i.e., when the issuer has a higher reputation, the issuer retains less when reporting that the asset is high quality. This leads to lower asset retention on average for issuers with higher reputations.

Prediction 3 is borne out in the data in the syndicated lending market. Lin and Paravisini (2013) find that lead syndicators that were exposed to the Enron and WorldCom fraud contribute a greater portion of capital, i.e., retain more, of future syndications. Similar evidence is documented by Gopalan et al. (2011). In MBS markets, one way to test the prediction is to examine the correlation between the size of equity tranches and issuer reputation as proxied for by the performance of past deals. If MBS issuers retain equity tranches, then equity tranches should be larger for issuers with worse past performance. A similar prediction would apply to repo markets: borrowers with poor past performance on their repo agreements should face larger haircuts on future repos.

Prediction 4 relates reputation and the sensitivity of prices to retention.

Prediction 4. When discount rates are high, or gains from trade are low, prices are more sensitive to issuer retention for issuers with low reputation than for issuers high reputation.

Prediction 4 could help explain why evidence of signaling in markets with asymmetric information is rarely found. Garmaise and Moskowitz (2004) find no evidence that sellers of commercial real estate signal their private information via seller take-back financing (a form of retention). Even if these sellers are using retention as a signal of asset quality, they could be doing so only when they have low reputations and, hence, such signaling can be difficult to detect without conditioning on reputation. For example, suppose one ran the following regression of prices on retention, reputation, and their interaction:

$$\begin{aligned} Price_i = {} & \beta_0 + \beta_1 Issuer\ Retention_i + \beta_2 Issuer\ Reputation_i \\ & + \beta_3 Issuer\ Retention_i \times Issuer\ Reputation_i + \epsilon_i. \end{aligned} \tag{45}$$

According to standard signaling theory, $\beta_1 > 0$. However, the prediction following from the reputation model, Prediction 4, is that $\beta_3 > 0$. Moreover, this finding can be present even when $\beta_1 = 0$.

One key challenge of applying Predictions 3 and 4 to the data is that they require a measure of issuer reputation. I propose one potential measure for issuer reputation in the MBS markets that is motivated by the model. The issuer's initial reputation does not depend on time. Thus, given a panel of data of MBS deals across time and issuers, one can control for an issuer's initial reputation by using issuer fixed effects. On can measure changes in reputation by observing that reputation decreases when an asset performs worse than the issuer stated it would in the prospectus. For an MBS, such an event would be reflected in a downgrade of the senior tranche. So, a potential measure for negative changes in MBS issuer reputation would be downgrades on the senior tranches of previously issued MBS.

5. D1 refinement and pooling equilibria

In static signaling games, pooling equilibria are typically eliminated by applying the D1 refinement criterion. D1 requires that investor beliefs in response to a given off-equilibrium issuer action must reflect those types who would gain under the largest possible set of beliefs in response to that action. It typically eliminates pooling because higher types usually stand to benefit from breaking a pool for a broader set of belief responses. In the model I present above, this result can be overturned because the issuer's type is given over two dimensions: her preferences for honesty and the quality of the asset she has for sale. An opportunistic-low-type issuer could stand to benefit under the largest set of off equilibrium path beliefs because she values reputation along the equilibrium path the least of all issuer types. To focus on the effect that reputation has on the application of D1 refinement, for the remainder of this section I drop the restrictions to undefeated equilibria and to equilibria in which off-equilibrium path actions do not effect reputation. I provide a detailed definition of D1 refinement in Appendix B.

The key difference between the application of D1 in a static signaling game and in the reputation setting of this paper is that because issuer type is defined over both quality and preferences for honesty, issuer payoffs do not always satisfy a single-crossing property. In the static version of the game, for a given off-equilibrium quantity lower than in equilibrium, the minimum price that makes the issuer better off than in equilibrium is always lower when the issuer has a high-quality asset. This is because asset retention is less costly for the issuer when she has a high-quality asset. D1 then requires that investors believe that deviations to lower quantities than in equilibrium could only mean that the issuer has a high-quality asset. This reasoning typically selects the LCSE as the equilibrium of interest. However, when continuation play contingent on reputation is added to the static game, this reasoning breaks down.

The essential intuition is that although high-type issuers are willing to accept a lower price when deviating than low-type issuers, the opportunistic-low-type issuer is the most willing to sacrifice her reputation. This is because the opportunistic-low-type issuer already anticipates losing her reputation in equilibrium. In other words, she has the least to lose in the continuation game. Now consider a full pooling equilibrium in which the opportunistic type issuer always sells the entire asset and reports that it is high quality. D1 refinement then requires that investors believe the issuer is the opportunistic-low type if the issuer deviates by retaining a fraction of the asset. In this way, a pooling equilibrium can survive D1 refinement. For a more formal development of this intuition, see Appendix B. This intuition leads to Proposition 5.

Proposition 5. Suppose

$$\frac{\gamma(1-\gamma\ell)}{\gamma(1-\gamma\ell)+\lambda(1-\gamma)} < \delta < \frac{1-\gamma\ell}{(1-\gamma\ell)+\lambda(1-\gamma)}, \quad (46)$$

then there exists an equilibrium of the form

$$\pi_h = 1, \quad \pi_\ell = \begin{cases} 1 & \text{for } \phi < \hat{\phi} \\ 0 & \text{otherwise} \end{cases} \quad (47)$$

and

$$Q^H_{hh} = Q^O_{hh} = \begin{cases} \hat{q} & \text{for } \phi < \hat{\phi} \\ 1 & \text{otherwise} \end{cases}, \quad Q^O_{\ell\ell} = Q^O_{\ell h} = 1, \quad (48)$$

for some $\hat{\phi} \in (0, 1)$. Moreover, this equilibrium satisfies D1.

The equilibrium in Proposition 5 calls for full separation for $\phi < \hat{\phi}$ and for full polling otherwise. The cutoff $\hat{\phi}$ is chosen so that for all $\phi > \hat{\phi}$ the following inequality holds:

$$\frac{\delta}{1-\delta}(V^H(\phi) - V^O(0)) > \gamma(1-\ell). \quad (49)$$

This inequality in turn implies that, for all $q < 1$, the set of prices and continuation payoffs that provide the opportunistic-low type an incentive to break the pool at $q = 1$ contains the same set but for the honest-high type. Thus, if the issuer's reputation is above $\hat{\phi}$, and she reports that the asset is high quality but attempts to retain, investors know that the opportunstic-low type stands to gain for the broadest possible set of response beliefs, and D1 requires that investors believe the issuer is the opportunistic-low type. This in turn implies that no issuer has an incentive to break the pool.

6. Conclusion

In this paper, I present a model of an informed securities issuer that unifies signaling and reputation effects. A lemons problem arises due to asymmetric information about the quality of assets. Partial asset retention by the issuer is a credible signal of asset quality, as the issuer is impatient relative to investors causing such retention to be costly. Imperfect information over issuer preferences induces a market reputation for the issuer. A high reputation can increase payoffs for the issuer by reducing issuer retention in equilibrium while decreasing the lemons discount relative to an identical security offered by an issuer with a low reputation. Reputation effects do not imply that an issuer is more likely to perfectly reveal information.

The implications of this model call into question the benefits of issuer reputation as a substitute for regulation and oversight to impose market discipline. For example, although conceptually appealing, the assertion that issuers of ABS behave in the best interests of the wider markets as a consequence of protecting their reputations overlooks an important point. The benefit of having a high reputation can be due to the ability to cash in on this status in the future. In my model, an opportunistic-type issuer cashes in on a high reputation by misreporting low-quality assets. In contrast, signaling in the absence of reputation can force an issuer to reveal the true full information value of any assets underlying ABS at the cost of reducing equilibrium payoffs.

One of the key applications of the theory of costly signaling has been to understand the capital structure of ABS. Future work is to empirically examine the implications of this paper for MBS markets. Some questions that could be answered are: Is the size of the first loss tranche, i.e., the equity tranche often retained by the issuer, related to the reputation of the issuer of the MBS? Do the prices of the senior and mezzanine tranches depend on the size of the first loss tranche? Is that relation altered by issuer reputation? One challenge of formulating empricial questions using the results of this paper is that the simple asset cash flows I model do not allow for a rich security design problem, another topic for future work.

While I mostly discuss the model by referring to a securities issuer, the theory developed in this paper applies equally well to other important financing problems. The key features of the model are that the issuer has valuable private information, a means by which to signal that information in a single period, and a means to generate a reputation for accurate reports. For example, the model could refer to a venture capitalist raising funds from limited partners. Maintaining a larger stake as general partner could be necessary if a good track record of matching investment projects with stated fund goals has not yet been established. Similarly, a private equity firm could need to put up a larger amount of equity capital to implement a leveraged buyout if that firm does not have a long history of accurate analysis of target firms. Finally, both a private equity firm and a venture capitalist could at some point find it advantageous to exploit a good reputation for one-period gains.

Appendix A. Stage game equilibria

In this Appendix, I introduce the definition of stage game equilibrium used in the undefeated equilibrium refinement given in Definition 2. Formally, consider the setup of the game, restricted to a single period and in which the issuer cannot send a public report about asset quality. Under this restriction, an issuer strategy is a function Q: $\{h, \ell\} \to [0, 1]$, which maps the type of asset the issuer has to sell to a quantity that she offers to the market. As in the model with quality reporting, I restrict attention to pure strategies over quantities. Investor beliefs are given by a function μ: $[0, 1] \to [0, 1]$, which maps the quantity on offer to a probability that the asset is high quality. A market price is a function P: $[0, 1] \to [0, 1]$, which maps the quantity on offer to a price for the asset. An equilibrium of this game satisfies Definition 3.

Definition 3. A triple (Q, μ, P) is a perfect Bayesian equilibrium if

1. The issuer's strategy Q is optimal given market prices
$$Q_\ell \in \arg\max_q \{(1-\delta)(P(q)q + \gamma(1-q)\ell)\} \tag{50}$$
and
$$Q_h \in \arg\max_q \{(1-\delta)(P(q)q + \gamma(1-q))\}. \tag{51}$$
2. Investor beliefs are consistent with the issuer's strategies and Bayes' rule, where possible; and
3. Market prices are such that investors earn zero profits given their beliefs,
$$P(q) = \mu(q) + (1-\mu(q))\ell. \tag{52}$$

This is a version of the classic signaling game of Spence (1973). As stated in Proposition 6, this game has two types of equilibria: separating and pooling.

Proposition 6. There exist two types of equilibrium satisfying Definition 3

1. *Separating equilibria with*
$$Q_\ell = 1 \quad P(Q_\ell) = \ell, \tag{53}$$
$$\frac{\gamma-\ell}{1-\ell} \le Q_h \le \frac{(1-\gamma)\ell}{1-\gamma\ell} \quad P(Q_h) = 1. \tag{54}$$
2. *If $\lambda + (1-\lambda)\ell \ge \gamma$, pooling equilibria with*
$$Q_\ell = Q_h \qquad P(Q_h) = P(Q_\ell) = \lambda + (1-\lambda)\ell. \tag{55}$$

Moreover, there are no other equilibria.

Proof. First, consider separating equilibria. Note that $P(q) \ge \ell$ for any equilibrium, so that in any separating equilibrium, it must be the case that $Q_\ell = 1$. Next, to support a separating equilibrium, consider the following beliefs:
$$\mu(q) = \begin{cases} 1 & \text{if } q \le \tilde{q} \\ 0 & \text{otherwise,} \end{cases} \tag{56}$$
for some $\tilde{q}$ such that
$$\frac{\gamma-\ell}{1-\ell} \le \tilde{q} \le \frac{(1-\gamma)\ell}{1-\gamma\ell}. \tag{57}$$
By Condition 3 of Definition 3, these beliefs give rise to the price function
$$P(q) = \begin{cases} 1 & \text{if } q \le \tilde{q} \\ \ell & \text{otherwise.} \end{cases} \tag{58}$$
Given these prices, the issuer's payoff is increasing on $[0, \tilde{q}]$ and again on $(\tilde{q}, 1]$. Thus, it is always optimal for the issuer to offer the quantity $\tilde{q}$ or 1 to the market. When the issuer has a high-quality asset to sell, it is optimal for her to offer $Q_h = \tilde{q}$ because
$$\tilde{q} + \gamma(1-\tilde{q}) \ge \ell, \tag{59}$$
by the definition of $\tilde{q}$. When the issuer has a low-quality asset to sell, it is optimal for her to offer $Q_\ell = 1$ because
$$\tilde{q} + \gamma(1-\tilde{q})\ell \le \ell, \tag{60}$$
again by the definition of $\tilde{q}$. Thus, the proposed strategies satisfy Condition 1 of Definition 3. Moreover, the beliefs

and price function given above satisfy Conditions 2 and 3 by construction. There are no other separating equilibria. If an issuer selling a high-quality asset could separate with a quantity below $Q_h < \frac{\gamma-\ell}{1-\ell}$, she would be better off offering a quantity of one and receiving a price of ℓ. If such an issuer could separate with a quantity $Q_h > \frac{(1-\gamma)\ell}{1-\gamma\ell}$, an issuer with a low-quality asset to sell would better off offering the quantity $Q_\ell = Q_h$ and receiving the price 1.

To support a pooling equilibrium, consider the beliefs

$$\mu(q) = \begin{cases} \lambda & \text{if } q \le \tilde{q} \\ 0 & \text{otherwise.} \end{cases} \tag{61}$$

By Condition 3 of Definition 3, these beliefs give rise to the price function

$$P(q) = \begin{cases} \lambda + (1-\lambda)\ell & \text{if } q \le \tilde{q} \\ \ell & \text{otherwise.} \end{cases} \tag{62}$$

Given these prices, it is optimal for issuers of both asset types to offer $Q_\ell = Q_h = \tilde{q}$, if $\lambda + (1-\lambda)\ell \ge \gamma$. If $\lambda + (1-\lambda)\ell < \gamma$, then an issuer selling a high-quality asset would be better off retaining the entire asset than pooling with the low-quality assets. □

Appendix B. Definition of D1 refinement

I first introduce a definition of D1 refinement as it applies to the game I describe in Section 2. To that end, fix an equilibrium (π, Q, μ, f, P) and start-of-period reputation ϕ. For any off-equilibrium report and quantity pair (j, q) at reputation ϕ for an issuer of type $T \in \{H, O\}$ with an asset of quality $i \in \{h, \ell\}$, let $D(T, i, j, q) \subseteq [\ell, 1] \times [V^T_{\min}, V^T_{\max}]$ be the set of prices in the current period together with payoffs in the continuation game that make a type (T, i) issuer better off than in equilibrium, where $V^T_{\min}$ and $V^T_{\max}$ are the lowest and highest payoffs that can be delivered to the type T issuer, respectively. Formally, the set $D(T, i, j, q)$ is defined by

$$D(T, i, j, q) = \Big\{(p, V) | u(T, i) \le (1-\delta)(pq + \gamma(1-q)i) + \delta V\Big\}, \tag{63}$$

where

$$u(O, i) = U^O(\pi, i; P) + \delta(\pi_i(\phi) V^O(f(\phi, i, i)) + (1 - \pi_i(\phi)) V^O(0)) \tag{64}$$

and

$$u(H, i) = U^H(i; P) + \delta V^H(f(\phi, i, i)) \tag{65}$$

are the equilibrium payoffs to the opportunistic-i- and honest-i- type issuer, respectively. I define the set $D(T, i, j, q)$ in terms of price and continuation payoffs, not in terms of beliefs, for convenience. An equivalent approach is to define this set in terms of beliefs explicitly and then calculate price and continuation payoffs for a given set of beliefs. Such a set $D(T, i, j, q)$ is maximal if it is not strictly contained in another such set $D(T', i', j, q)$. The following definition of D1 is then equivalent to that given by Cho and Kreps (1987).

Definition 4. An equilibrium satisfies D1 if, at any reputation ϕ for any off-equilibrium report j and quantity issued q, investor beliefs are supported on those (T, i) for which $D(T, i, j, q)$ is maximal.

In a standard signaling game, D1 eliminates pooling equilibria because higher types typically stand the most to gain from breaking a pool. Thus, when an off-equilibrium message is sent, the receiver (in my case, the investors) must believe that it was sent by a higher type, which in turn gives the higher types a greater incentive to break the pool. To see why this reasoning does not extend to my model, suppose that there is a reputation ϕ and an equilibrium such that there is full pooling at ϕ so that

$$\pi_h(\phi) = 1, \qquad \pi_\ell(\phi) = 0, \tag{66}$$

and

$$Q^H_{\ell\ell}(\phi) = Q^H_{hh}(\phi) = Q^O_{\ell h}(\phi) = Q^O_{hh}(\phi) = 1. \tag{67}$$

Then the equilibrium payoffs to the issuer depending on her type are given by

$$\begin{aligned} u(O, \ell) &= (1-\delta)\overline{p}(\phi) + \delta V^O(0) \\ u(H, \ell) &= (1-\delta)\overline{p} + \delta V^H(1) \end{aligned} \tag{68}$$

$$\begin{aligned} u(O, h) &= (1-\delta)\overline{p}(\phi) + \delta V^O(\phi) \\ u(H, h) &= (1-\delta)\overline{p} + \delta V^H(\phi) \end{aligned} \tag{69}$$

where

$$\overline{p}(\phi) = \ell + \frac{\lambda}{\lambda + (1-\lambda)(1-\phi)}(1-\ell) \tag{70}$$

is the pooled price.

Now consider a quantity $q < 1$. The set $D(O, \ell, h, q)$ is the set of solutions (p, V) to

$$(1-\delta)(pq + \gamma(1-q)\ell) + \delta V \ge (1-\delta)\overline{p}(\phi) + \delta V^O(0), \tag{71}$$

while the same set for the honest-high-type issuer is given by the set of solutions (p, V) to

$$(1-\delta)(pq + \gamma(1-q)) + \delta V \ge (1-\delta)\overline{p}(\phi) + \delta V^H(\phi). \tag{72}$$

If $V^O(0)$ is sufficiently less than $V^H(\phi)$, then, for a fixed price p, the opportunistic-low type is willing to accept a lower continuation payoff than the honest-high type to deviate to the quantity q. Thus, $D(H, h, h, q) \subset D(O, \ell, h, q)$. A similar argument shows that $D(O, h, h, q) \subset D(H, h, h, q)$. Thus, $D(O, \ell, h, q)$ is maximal and the D1 criterion requires that the off-equilibrium path investor beliefs following the action (h, q) must be $\mu = 0$ and $\phi = 0$.

Appendix C. Proofs

Proof of Proposition 1. Because $\phi_0 = 0$, Bayes' rule implies that $\phi_t = 0$ for all t. First, I verify that each of the conditions of Definition 1 are satisfied.

First, consider Condition 1. The issuer's strategies and investors' beliefs are Markov in reputation by construction. It remains to show that the issuer's strategies are optimal and the investors' beliefs are consistent with Bayes' rule.

Next, consider Condition 2. Because the investors' beliefs are Markov in reputation and that reputation does not change in equilibrium, the opportunistic type's actions in a given period do not affect her payoff in subsequent periods. Consider the opportunistic type's problem when she sells a low-quality asset. She seeks to maximize

$$\begin{aligned} &U_\ell^O(\pi, q_\ell, q_h; P) \\ &= (1-\delta)(\pi(P_\ell(0,q_\ell)q_\ell + \gamma(1-q_\ell)\ell) \\ &\quad + (1-\pi)(P_h(0,q_h)q_h + \gamma(1-q_h)\ell)) \\ &\le (1-\delta)(\pi\ell + (1-\pi)(\hat{q} + \gamma(1-\hat{q})\ell)) \\ &= (1-\delta)\ell \end{aligned} \tag{73}$$

by the definition of P_ℓ, P_h, and $\hat{q}$. The strategy $\pi_\ell = 1$ and $Q_{\ell\ell}^O = 1$ achieves this upper bound. So, the proposed strategy when she sells a low-quality asset is optimal. Now consider the opportunistic type's problem when she sells a high-quality asset. She seeks to maximize

$$\begin{aligned} &U_h^O(\pi, q_\ell, q_h; P) \\ &= (1-\delta)(\pi(P_h(0,q_h)q_h + \gamma(1-q_h)) \\ &\quad + (1-\pi)(P_\ell(0,q_\ell)q_\ell + \gamma(1-q_\ell)\ell)) \\ &\le (1-\delta)(\pi(\hat{q} + \gamma(1-\hat{q})) + (1-\pi)\ell) \\ &\le (1-\delta)(\hat{q} + \gamma(1-\hat{q})) \end{aligned} \tag{74}$$

again by the definition of P_ℓ, P_h, and $\hat{q}$. The strategy $\pi_h = 1$ and $Q_{hh}^H = \hat{q}$ achieves this upper bound. So, the proposed strategy when she sells a high-quality asset is optimal. It is never strictly optimal for the opportunistic-type issuer to condition her strategy on past play, as doing so cannot deliver higher stage game payoffs than the upper bounds given above.

Next, consider Condition 3. Because $\phi_t = 0$ for all $t \ge 0$, Proposition 1 does not specify the strategies of the honest-type issuer. However, when selling a low-quality asset, the honest type seeks to maximize

$$U_\ell^H(q_\ell; P) = (1-\delta)(P_\ell(0,q_\ell)q_\ell + \gamma(1-q_\ell)\ell) \tag{75}$$

$$\le (1-\delta)\ell, \tag{76}$$

by the definition of P_ℓ. The strategy $Q_{\ell\ell}^H = 1$ achieves this upper bound. When selling a high-quality asset, the honest type seeks to maximize

$$U_h^H(q_h; P) = (1-\delta)(P_h(0,q_h)q_h + \gamma(1-q_h)) \tag{77}$$

$$\le (1-\delta)(\hat{q} + \gamma(1-\hat{q})), \tag{78}$$

by the definition of P_h. The strategy $Q_{hh}^H = \hat{q}$ achieves this upper bound. As in the case of the opportunistic issuer, it is never strictly optimal for the honest-type issuer to condition her strategy on past play. Again, doing so cannot deliver higher stage game payoffs than the upper bounds given above.

Next, consider Condition 4. Applying Bayes' rule gives

$$\mu_h(\hat{q}) = \frac{\lambda\pi_h}{\lambda\pi_h + (1-\lambda)(1-\pi_\ell)} = 1 \tag{79}$$

and

$$\mu_\ell(1) = \frac{\lambda(1-\pi_h)}{\lambda(1-\pi_h) + (1-\lambda)\pi_\ell} = 0. \tag{80}$$

Eqs. (79) and (80) also imply that the proposed equilibrium is a separating equilibrium.

Finally, consider Condition 5. This is by construction of P_ℓ and P_h.

To see that the proposed equilibrium is the unique undefeated equilibrium, note that a pure pooling equilibrium does not exist and that any alternative equilibrium (that satisfies Restriction 1) is defeated by the LCSE of the static game given in Appendix A. In any equilibrium in which $0 < \pi_\ell < 1$ and $Q_{\ell h}^O = Q_{hh}^O$, the opportunistic-low type's optimality condition requires that

$$P_h(0, Q_{h\ell}^O)Q_{h\ell}^O + \gamma(1-Q_{h\ell}^O)\ell = \ell. \tag{81}$$

This implies that $Q_{h\ell}^O \ge \hat{q}$ and the payoff to the opportunistic-high type is

$$P_h(0, Q_{hh}^O)Q_{hh}^O + \gamma(1-Q_{hh}^O) = \ell + \gamma(1-Q_{hh}^O)(1-\ell) \tag{82}$$

$$\le \ell + \gamma(1-\hat{q})(1-\ell), \tag{83}$$

i.e., less than the payoff to the high-type in the static LSCE. □

Proof of Lemma 1. Lemma 1 is a direct consequence of Condition 4 of Definition 1 and Restriction 1. Condition 4 of Definition 1 states that investor beliefs must be consistent with the issuer's actions and Bayes' rule, while Restriction 1 states that $\mu_\ell = 0$ so that, in equilibrium, it must be the case that

$$0 = \mu_\ell = \frac{\lambda(1-\pi_h)}{\lambda(1-\pi_h) + (1-\lambda)\pi_\ell}, \tag{84}$$

which implies that $\pi_h = 1$. □

Proof of Lemma 2. For convenience, let $V^H(\phi)$ and $V^O(\phi)$ be the value that honest- and opportunistic-type issuers place on a reputation ϕ, respectively.

First, I show $Q_{\ell\ell}^O = Q_{\ell\ell}^H = 1$. For any equilibrium that satisfies Restriction 1, the price following a report of low quality is ℓ. Conditional on truthfully reporting a low-quality asset, the opportunistic-type issuer then chooses $Q_{\ell\ell}^O(\phi)$ to solve

$$\max_q \left\{ \ell q + \gamma(1-q)\ell + \frac{\delta}{1-\delta}V^O(\phi'(\mathcal{H}_{t+1})) \right\}. \tag{85}$$

By Restriction 2, $\phi'(\mathcal{H}_{t+1})$ does not depend on q, so the solution to Eq. (85) is $Q_{\ell\ell}^O = 1$. The problem for the honest-type issuer is essentially identical. She chooses $Q_{\ell\ell}^H$ to solve

$$\max_q \left\{ \ell q + \gamma(1-q)\ell + \frac{\delta}{1-\delta}V^H(\phi'(\mathcal{H}_{t+1})) \right\}. \tag{86}$$

Again, this problem is solved by $Q_{\ell\ell}^H = 1$.

The proof that $Q_{hh}^O = Q_{hh}^H$ follows a similar argument. When selling a high-quality asset, the opportunistic-type issuer solves

$$\max_q \left\{ P_h(\phi, q)q + \gamma(1-q) + \frac{\delta}{1-\delta}V^O(\phi'(\mathcal{H}_{t+1})) \right\}, \tag{87}$$

and the honest-type issuer solves

$$\max_q \left\{ P_h(\phi, q)q + \gamma(1-q) + \frac{\delta}{1-\delta}V^H(\phi'(\mathcal{H}_{t+1})) \right\}. \tag{88}$$

Because by Restriction 2 $f(\mathcal{H}_{t+1}, h, q)$ does not depend on quantity, these two problems are both equivalent to

$$\max_q \{P_h(\phi, q)q + \gamma(1 - q)\}. \tag{89}$$

While this problem can have multiple solutions, it is without loss of generality to assume that the issuer chooses the largest solution, as a different solution would affect only the stage game payoff and not continuation play.

Finally, to see that $Q^O_{\ell h} = Q^H_{hh}$, note that if the opportunistic-type issuer ever misreports a low-quality asset, her reputation must drop to zero, as such a sequence of events could never happen if the issuer were honest. Now suppose that $Q^O_{\ell h}(\phi) \neq Q^H_{hh}(\phi)$ for some ϕ. Conditions 4 and 5 of Definition 1 then require that $\mu_h(\phi, Q^O_{\ell h}(\phi)) = 0$, so that $P_h(\phi, Q^O_{\ell h}(\phi)) = \ell$, in which case the opportunistic-type issuer would have a profitable deviation truthfully reporting the quality of the asset, because doing so would result in a weakly greater stage game payoff and no loss of reputation. □

Proof of Proposition 2. The proof that the LSCE is an equilibrium for all ϕ is identical to the proof of Proposition 1 once it is observed that investor beliefs about asset quality, and hence prices, do not depend on the issuer reputation. Thus, an issuer with a positive reputation has identical payoff to one with a reputation of zero in the LCSE.

Now suppose there exists a truth-telling equilibrium. In such an equilibrium, Condition 4 of Definition 1 implies that $\mu_h(\phi) = 1$ for all $\phi > 0$, which implies that $f(\phi, \ell, \ell) = \phi$. Because the opportunistic-low-type issuer must prefer to truthfully report in a truth-telling equilibrium, the one-shot deviation principal implies that

$$Q^O_{hh}(\phi) + \gamma(1 - Q^O_{hh}(\phi))\ell - \ell \leq \frac{\delta}{1-\delta}(V^O(\phi) - V^O(0)). \tag{90}$$

Eq. (28) gives

$$V^O(0) = \lambda(\hat{q} + \gamma(1 - \hat{q})) + (1 - \lambda)\ell \tag{91}$$

and

$$V^O(\phi) = \lambda(Q^O_{hh}(\phi) + \gamma(1 - Q^O_{hh}(\phi)) + (1 - \lambda)\ell. \tag{92}$$

Substituting Eqs. (91), (92), and $\ell = \hat{q} + \gamma(1 - \hat{q})\ell$ into Eq. (90) gives

$$(1 - \gamma\ell)(Q^O_{hh}(\phi) - \hat{q}) \leq \frac{\delta\lambda(1-\gamma)(Q^O_{hh}(\phi) - \hat{q})}{1-\delta}. \tag{93}$$

Isolating δ gives

$$\delta \geq \frac{1 - \gamma\ell}{1 - \gamma\ell + \lambda(1-\gamma)}. \tag{94}$$

Now suppose that δ satisfies the inequality given in Eq. (94). I show there exists a truth-telling equilibrium with issuer strategies $Q^O_{\ell\ell} = Q^H_{\ell\ell} = Q^O_{hh}(\phi) = Q^H_{hh}(\phi) = 1$, $\pi_\ell(\phi) = 1$, and $\pi_h(\phi) = 1$ for all $\phi > 1$, together with investor beliefs $\mu_h(\phi, q) = 1$ and $\mu_\ell = 0$ and prices $P_\ell(\phi, q) = \ell$ and $P_h(\phi, q) = 1$. These strategies and beliefs imply

$$V^H(\phi) = V^O(\phi) = \lambda + (1 - \lambda)\ell \tag{95}$$

and $f(\phi, \ell, \ell) = f(\phi, h, h) = \phi$ for all $\phi > 0$. I now check that the proposed strategies satisfy Definition 1.

First, consider Condition 1. In this equilibrium, $\mathbb{I}(\phi > 0)$ is a sufficient statistic for strategies and beliefs. Hence, the Markov property holds. Again, it remains to verify that the proposed strategies are optimal.

Next, consider Condition 2. Consider the opportunistic-type issuer's problem when she sells a low-quality asset. She seeks to maximize

$$\begin{aligned}&(1-\delta)(\pi(\ell q_\ell + \gamma(1 - q_\ell)\ell) + (1-\pi)(q_h + \gamma(1 - q_h)\ell)) \\ &\quad + \delta(\pi V^O(\phi) + (1-\pi)V^O(0)) \\ &\leq \pi((1-\delta)\ell + \delta V^O(\phi)) + (1-\pi)(1 + \delta V^O(0)).\end{aligned} \tag{96}$$

The condition on δ given in Eq. (96) implies $(1 - \delta)\ell + \delta V^O(\phi) \geq 1 + \delta V^O(0)$, so it is optimal for the opportunistic-low type to choose $\pi_\ell = 1$ and $Q^O_{\ell\ell}(\phi) = 1$. Now consider the opportunistic type's problem when she sells a high-quality asset. She seeks to maximize

$$\begin{aligned}&(1-\delta)((1-\pi)\pi(\ell q_\ell + \gamma(1 - q_\ell)) + \pi(q_h + \gamma(1 - q_h))) \\ &\quad + \delta(\pi V^O(\phi) + (1-\pi)V^O(0)) \\ &\leq \pi((1-\delta)\ell + \delta V^O(\phi)) + (1-\pi)(1 + \delta V^O(0)),\end{aligned} \tag{97}$$

so again, it is optimal for the opportunistic-high type to choose $\pi_h = 1$ and $Q^O(\phi)_{hh} = 1$.

Next, consider Condition 3. Consider the honest-type issuer's problem when she sells a low-quality asset. She seeks to maximize

$$(1-\delta)(\ell q_\ell + \gamma(1 - q_\ell)\ell) + \delta V^H(f(\phi, \ell, \ell)), \tag{98}$$

which is clearly maximized by choosing $Q^H_{\ell\ell} = 1$. Now consider the honest type's problem when she sells a high-quality asset. She seeks to maximize

$$(1-\delta)(q_h + \gamma(1 - q_h)) + \delta V^H(\phi), \tag{99}$$

which is clearly maximized by choosing $Q^H_{hh} = 1$.

Next, consider Condition 4. Applying Bayes' rule gives

$$\mu_h(\phi, 1) = \frac{\lambda\pi_h}{\lambda\pi_h + (1-\lambda)(1-\pi_\ell)} = 1 \tag{100}$$

and

$$\mu_\ell(\phi, 1) = \frac{\lambda(1-\pi_h)}{\lambda(1-\pi_h) + (1-\lambda)\pi_\ell} = 0. \tag{101}$$

Belief consistency for the reputation updating function is shown in the text.

Finally, consider Condition 5. This is by construction of P_ℓ and P_h.

Thus, I have shown that Statements 1, 2, and 3 of Proposition 2 are equivalent. Finally, all equilibria mentioned in the proposition deliver the issuer at least as high a payoff when selling a high-quality asset as in the static LCSE and, thus, all equilibria satisfy UE. □

Proof of Proposition 3. The proof proceeds in four steps:

1. Propose conditions that define a candidate equilibrium.
2. Verify this candidate equilibrium satisfies Definition 1.
3. Show that there exists a solution to the conditions that define the candidate equilibrium.
4. Verify that this candidate equilibrium has the properties stated in the proposition. □

Step 1 (Definition of candidate equilibrium). The candidate equilibrium is given by the following investor beliefs and issuer strategies. The investors beliefs are

$$\mu_\ell(\phi, q) = 0 \tag{102}$$

and

$$\mu_h(\phi, q) = \begin{cases} 1 & \text{if } q \le \hat{q} \\ \tilde{\mu}(\phi) & \text{if } \hat{q} \le q \le \tilde{Q}(\phi) \\ \underline{\mu} & \text{if } \tilde{Q}(\phi) < q \end{cases}. \tag{103}$$

Market prices are

$$P_\ell(\phi, q) = \ell \tag{104}$$

and

$$P_h(\phi, q) = \mu_h(q, \phi) + (1 - \mu_h(q, \phi))\ell. \tag{105}$$

The opportunistic-type issuer's reporting strategies are

$$\pi_h(\phi) = 1 \tag{106}$$

and

$$\pi_\ell(\phi) = \begin{cases} 1 - \dfrac{1}{1-\phi}\left(\dfrac{\lambda}{1-\lambda}\right)\left(\dfrac{1-\tilde{\mu}(\phi)}{\tilde{\mu}(\phi)}\right) & \text{if } \phi < 1, \\ 0 & \text{if } \phi = 1 \end{cases}. \tag{107}$$

The issuer's quantity strategies are

$$Q^O_{\ell h}(\phi) = Q^O_{hh}(\phi) = Q^H_{hh}(\phi) = \tilde{Q}(\phi) \tag{108}$$

and

$$Q^O_{\ell\ell}(\phi) = Q^H_{\ell\ell}(\phi) = 1. \tag{109}$$

To define the conditions pinning down $\tilde{\mu}(\phi)$ and $\tilde{Q}(\phi)$, note that, given the strategies and beliefs proposed above, the opportunistic-type issuer's value function is

$$V^O(\phi) = \frac{(1-\delta)(P_h(\tilde{Q}(\phi)), \phi) + \gamma(1-\tilde{Q}(\phi))(\lambda + (1-\lambda)\ell)) + \delta V^O(0)}{1-\delta\lambda}, \tag{110}$$

where

$$V^O(0) = \lambda(\hat{q} + \gamma(1-\hat{q})) + (1-\lambda)\ell. \tag{111}$$

The conditions pinning down $\tilde{\mu}(\phi)$ and $\tilde{Q}(\phi)$ are as follows. First, if the opportunistic-type issuer ever achieves reputation $\phi = 1$, then she sells a quantity 1 regardless of her report. That is,

$$\tilde{Q}(\phi) = 1. \tag{112}$$

Moreover, the investors always believe the issuer's report at $\phi = 1$, so $\tilde{\mu}(1) = 1$. For $\phi < 1$, the conditions are given over three regions in ϕ.

Region 1. If reputation is high enough so that issuers of both low- and high-quality asset are willing to sell the entire asset even when the investors believe the opportunistic-low type always misreports, then the issuer always sells the entire asset regardless of her report and the opportunistic-low type always misreports. That is, if

$$\frac{\lambda}{\lambda + (1-\lambda)(1-\phi)} + \left(1 - \frac{\lambda}{\lambda + (1-\lambda)(1-\phi)}\right)\ell + \frac{\delta}{1-\delta}V^O(0) \ge \ell + \frac{\delta}{1-\delta}V^O(1) \tag{113}$$

and

$$\frac{\lambda}{\lambda + (1-\lambda)(1-\phi)} + \left(1 - \frac{\lambda}{\lambda + (1-\lambda)(1-\phi)}\right)\ell \ge \hat{q} + \gamma(1-\hat{q}), \tag{114}$$

then

$$\tilde{Q}(\phi) = 1, \tag{115}$$

and

$$\tilde{\mu}(\phi) = \frac{\lambda}{\lambda + (1-\lambda)(1-\phi)}. \tag{116}$$

Region 2 If reputation is such that is not possible to satisfy Eqs. (113) and (114), then $\tilde{Q}(\phi)$ and $\tilde{\mu}(\phi)$ solve

$$(\tilde{\mu}(\phi) + (1-\tilde{\mu}(\phi))\ell)\tilde{Q}(\phi) + \gamma(1-\tilde{Q}(\phi))\ell + \frac{\delta}{1-\delta}V^0(0) = \ell + \frac{\delta}{1-\delta}V^O\left(\frac{(1-\lambda)\tilde{\mu}(\phi)}{\tilde{\mu}(\phi)-\lambda}\phi\right) \tag{117}$$

and

$$(\tilde{\mu}(\phi) + (1-\tilde{\mu}(\phi))\ell)\tilde{Q}(\phi) + \gamma(1-\tilde{Q}(\phi)) = \hat{q} + \gamma(1-\hat{q}). \tag{118}$$

Together Eqs. (102)–(118) define the candidate equilibrium. I show there exists a solution to these equations in Step 3.

Step 2 (Candidate equilibrium satisfies Definition 1). In this step, I show that the candidate equilibrium satisfies Definition 1.

First, consider Condition 1. The Markov property holds construction. That is, all beliefs and strategies are functions of ϕ, not the full history of the game. It remains to show that the issuer does not have a profitable deviation by playing a strategy conditional on some aspect of the history of the game not contained in ϕ and that investor beliefs are consistent with the issuer's strategies.

Next, consider Condition 2. The optimality of $Q^O_{\ell\ell}(\phi) = 1$ and $\pi_h(\phi) = 1$ is given by Lemmas 1 and 2. Consider $\phi = 1$. Note that $P_h(q, 1) = 1$, so the opportunistic-high-type issuer's strategy at $\phi = 1$ must solve

$$V^O_h(1) = \max_q \left\{(1-\delta)(q + \gamma(1-q)) + \delta V^O(1)\right\}. \tag{119}$$

This problem is solved by $Q^O_{hh}(1) = 1$ because the objective function in Eq. (119) is increasing in q. Lemma 1 then implies that $Q^O_{\ell h}(1) = 1$. The opportunistic-low type's reporting strategy then solves

$$V^O_\ell(1) = \max_\pi \left\{(1-\pi)((1-\delta) + \delta V^O(0)) + \pi((1-\delta)\ell + V^O(1))\right\}. \tag{120}$$

This problem is solved by $\pi_\ell(1) = 1$ by Assumption 2.

Next, consider $\phi < 1$. When the opportunistic-type issuer has a low-quality asset to sell, the optimality of her strategies is guaranteed by Eqs. (113) and (117) and the single deviation principle. If she were ever to deviate by following a different reporting or quantity strategy, she would always get a weakly lower expected stage game payoff, as well as a weakly greater expected loss in continuation value. In Region 2, the opportunistic-low-type issuer is strictly indifferent between all possible reporting strategies. When the opportunistic-type issuer has a high-quality asset to sell, the optimality of her strategies is guaranteed by Eqs. (114) and (118) and the single deviation principle. The opportunistic-high type cannot gain by reporting that the asset is low quality as doing so given the investor belief $\mu_\ell = 0$ would lead to lower stage game payoffs and a sure loss of reputation.

Next, consider Condition 3. This follows directly from Lemma 2 and the fact that the quantity strategy is optimal for the opportunistic-type issuer.

Next, consider Condition 4. Applying Bayes' rule, investors must believe the asset is low quality if the issuer reports that it is low quality. This is consistent with the beliefs specified in Eq. (102). In equilibrium, investor beliefs about asset quality following a report that the asset is high quality are given by $\tilde{\mu}(\phi)$ defined in Eqs. (103) and (108). If π_ℓ^e and π_h^e are the probabilities the investors place on the event that opportunistic-low and -high types truthfully report, Bayes' rule states that

$$\tilde{\mu}(\phi) = \frac{\lambda(\phi + (1-\phi)\pi_h^e)}{\lambda(\phi + (1-\phi)\pi_h^e) + (1-\phi)(1-\lambda)(1-\pi_\ell^e)}. \tag{121}$$

Solving Eq. (121) for π_ℓ^e, assuming $\pi_h^e = 1$, gives

$$\pi_\ell^e = 1 - \frac{1}{1-\phi}\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{1-\tilde{\mu}(\phi)}{\tilde{\mu}(\phi)}\right). \tag{122}$$

Thus, by Eq. (107), investor beliefs are consistent with the issuer's reporting strategy. Investor beliefs about the issuer's reputation are consistent with Bayes' rule as a direct consequence of the definition of $f(\phi, i, j)$.

Finally, consider Condition 5. Eqs. (104) and (105) are just the expected (normalized by $1-\delta$) cash flow of the asset given investor beliefs.

Step 3 (Existence). Constructing a solution to Eqs. (102)–(118) is complicated algebraically but simple conceptually. The basic approach is to work downwards in reputation, first solving the conditions for $\tilde{\mu}$ and $\tilde{Q}$, assuming the constraint in Eq. (118) does not hold, then working downward from there.

Let $\bar{\phi}$ satisfy

$$\ell + \left(\frac{\lambda}{\lambda + (1-\lambda)(1-\bar{\phi})}\right)(1-\ell) = \max\left\{\hat{q} + \gamma(1-\hat{q}), \ell + \frac{\delta}{1-\delta}(V^O(1) - V^O(0))\right\}. \tag{123}$$

In words, $\bar{\phi}$ is the level of reputation above which both the opportunistic-high and the honest-high types, as well as the opportunistic-low types, prefer to pool. For all $\phi \geq \bar{\phi}$, the conditions given in Eqs. (114) and (113) are satisfied. Thus, $\tilde{Q}(\phi)$ and $\tilde{\mu}(\phi)$ are given by Eqs. (115) and (116).

For $\phi < \bar{\phi}$, the conditions given in Eqs. (113) and (114) do not hold by the definition of $\bar{\phi}$. Thus, the relevant conditions for $\phi < \bar{\phi}$ are given by Eqs. (117) and (118). To solve these equations, I use a technique that closely follows the proof of Proposition 2 in Mathis et al. (2009).

To ease notation, I define two useful functions. I define the function $w(\mu)$ by

$$w(\mu) = (1-\delta)(\mu + (1-\mu)\ell). \tag{124}$$

The function $w(\mu)$ is the stage game payoff to the issuer when she sells the entire asset and the investors believe μ is the probability that the asset is high-quality. I define the function $u(\mu)$ by

$$u(\mu) = (1-\delta)((\mu + (1-\mu)\ell)q(\mu) + \gamma(1-q(\mu))\ell), \tag{125}$$

where $q(\mu)$ solves

$$(\mu + (1-\mu)\ell)q(\mu) + \gamma(1-q(\mu)) = \hat{q} + \gamma(1-\hat{q}). \tag{126}$$

The function $u(\mu)$ is the stage game payoff to the opportunistic-low-type issuer when she reports the asset is high quality, investors believe this report with probability μ, and the quantity she offers is such that high-quality issuers are indifferent between pooling and separating given the quantity $q(\mu)$ and investors beliefs μ.

The function $u(\mu)$ is continuous and strictly decreasing in μ on $[\lambda, 1]$, so that $u^{-1}(\cdot)$ exists and is strictly decreasing on $[u(\lambda), u(1)]$.

Define the sequence μ_n by

$$\mu_1 = \frac{\lambda}{\lambda + (1-\lambda)(1-\bar{\phi})} \tag{127}$$

and

$$\mu_n = u^{-1}(\alpha u(\mu_{n-1}) + \beta), \tag{128}$$

where

$$\alpha = \frac{(1-\lambda)\delta}{1-\delta\lambda} \tag{129}$$

and

$$\beta = (1-\delta)\ell + \delta\left(\frac{\lambda(1-\delta)}{1-\delta\lambda}(\hat{q} + \gamma(1-\hat{q})) - \frac{\delta\lambda(1-\delta)}{1-\delta\lambda}V^O(0)\right). \tag{130}$$

Further define the sequence of functions $\tilde{\mu}_n(\phi)$ by

$$\tilde{\mu}_0(\phi) = \frac{\lambda}{\lambda + (1-\lambda)(1-\phi)}, \tag{131}$$

$$\tilde{\mu}_1(\phi) = u^{-1}\left(\frac{\alpha}{1-\lambda}, w(\tilde{\mu}_0(g(\tilde{\mu}_1(\phi))\phi)) + \eta\right), \tag{132}$$

and

$$\tilde{\mu}_n(\phi) = u^{-1}(\alpha u(\tilde{\mu}_{n-1}(g(\tilde{\mu}_n(\phi))\phi)) + \beta), \tag{133}$$

where

$$g(\mu) = \frac{\mu(1-\lambda)}{\mu - \lambda} \tag{134}$$

and

$$\eta = (1-\delta)\ell - \frac{\delta(1-\delta)}{1-\delta\lambda} V^0(0). \tag{135}$$

Now define the sequence $\phi(n)$ by

$$\phi(1) = \bar{\phi} \tag{136}$$

and

$$\phi(n) = \frac{\mu_n - \lambda}{\mu_n(1-\lambda)\phi(n-1)}. \tag{137}$$

I claim that

$$\tilde{\mu}(\phi) = \tilde{\mu}_n(\phi), \quad \text{such that } \phi \in [\phi(n+1), \phi(n)), \tag{138}$$

and

$$\tilde{Q}(\phi) = \begin{cases} 1 & \text{for } \geq \bar{\phi} \\ \dfrac{\hat{q}(1-\gamma)}{\tilde{\mu}(\phi) + (1-\tilde{\mu}(\phi))\ell - \gamma} & \text{otherwise} \end{cases} \tag{139}$$

solve Eqs. (117) and (118). Eq. (118) holds by the definition of $\tilde{Q}(\phi)$. Also, for $\phi \in (\phi_{n+1}, \phi_n]$,

$$f(\phi, \ell, \ell) = \frac{\tilde{\mu}(\phi)(1-\lambda)}{\tilde{\mu}(\phi) - \lambda}\phi \tag{140}$$

$$= \frac{\tilde{\mu}_n(\phi)(1-\lambda)}{\tilde{\mu}_n(\phi) - \lambda}\phi \in (\phi_n, \phi_{n-1}], \tag{141}$$

by construction.

The rest of the proof of the claim proceeds by induction on n. For $n = 1$, using the strategies defined by Eqs. (138) and (139) and Eq. (28) gives

$$\delta V^0(\phi) = \frac{\delta}{1-\delta\lambda}\Big(w(\tilde{\mu}_0(\phi)) + (1-\lambda)\delta V^0(0)\Big) \tag{142}$$

for $\phi \in [\phi_1, \phi_0] = [\bar{\phi}, 1]$. This implies

$$u(\tilde{\mu}_0(\phi)) + \delta V^0(0) = \frac{\alpha}{1-\lambda} w(\tilde{\mu}_0(f(\phi, \ell, \ell))) + \eta + \delta V^0(0) \tag{143}$$

$$= \frac{\delta}{1-\delta\lambda}\Big(w(\tilde{\mu}_0(f(\phi, \ell, \ell)) + (1-\lambda)\delta V^0(0)\Big) + \ell \tag{144}$$

for $\phi \in [\phi_2, \phi_1)$, by the definition of α and η. Thus, the proposed strategies satisfy Eq. (117) for $\phi \in [\phi_2, \phi_1)$.

For $n = m > 1$, suppose that the proposed strategies satisfy Eq. (117) for all $\phi \in (\phi_m, \phi_{m-1}]$. Then, using the strategies defined by Eqs. (138) and (139) and Eq. (28) gives

$$\delta V^0(\phi) = \frac{\delta}{1-\delta\lambda}\Big(\lambda(1-\delta)(\hat{q} + \gamma(1-\hat{q})) + (1-\lambda)u(\tilde{\mu}_{m-1}(\phi)) + (1-\lambda)\delta V^0(0)\Big) \tag{145}$$

$$= \alpha\tilde{\mu}_{m-1}(\phi) + \beta - (1-\delta)\ell + \delta V^0(0) \tag{146}$$

for $\phi \in (\phi_m, \phi_{m-1}]$. This implies

$$u(\tilde{\mu}_m(\phi)) + \delta V^0(0) = \alpha u(f(\phi, \ell, \ell))) + \beta + \delta V^0(0) \tag{147}$$

$$= (1-\delta)\ell + \delta V^0(f(\phi, \ell, \ell)) \tag{148}$$

for $\phi \in (\phi_{m+1}, \phi_m]$, by the definition of α and β. Thus, the proposed strategies satisfy Eq. (117) for $\phi \in (\phi_{m+1}, \phi_m]$.

Finally, let $\underline{\phi} = \lim_{n\to\infty}\phi_n$ and note that is possible that $\underline{\phi} > 0$, in which case the candidate equilibrium coincides with the least cost separating equilibrium for all $\phi \in [0, \underline{\phi}]$.

Step 4 (Equilibrium properties). To see that $\tilde{Q}(\phi)$ is increasing, consider $\tilde{\phi} < \phi < \bar{\phi}$. Eqs. (117) and (118) must be satisfied at both $\tilde{\phi}$ and ϕ. Substituting Eq. (118) into Eq. (117), and solving for $\tilde{Q}(\phi)$ in terms of $\tilde{Q}(\tilde{\phi})$, gives

$$\tilde{Q}(\phi) = \tilde{Q}(\tilde{\phi}) + \frac{\delta}{\gamma(1-\ell)(1-\delta)} \times \Big(V^0(f(\phi, \ell, \ell)) - V^0(f(\tilde{\phi}, \ell, \ell))\Big). \tag{149}$$

$V^0(\phi)$ must be weakly increasing and $f(\phi, \ell, \ell) \geq f(\tilde{\phi}, \ell, \ell)$. This, together with Eq. (149), implies that $\tilde{Q}(\phi) \geq \tilde{Q}(\tilde{\phi})$, so $\tilde{Q}(\phi)$ is increasing for all $\phi < \bar{\phi}$. $\tilde{Q}(\phi) = 1$ for all $\phi \geq \bar{\phi}$, so that $\tilde{Q}(\tilde{\phi}) \leq \tilde{Q}(\phi)$ for all $\tilde{\phi} < \bar{\phi} < \phi$. Thus, $\tilde{Q}(\phi)$ is increasing for all ϕ.

To see that when δ satisfies the condition given in Eq. (33) the $\bar{\phi}$ is the lowest possible reputation at which full allocative efficiency is achieved, note that when imposing that condition and solving Eq. (123) gives

$$\bar{\phi} = \frac{\gamma(1-\ell(1-\lambda)) - \lambda}{\gamma(1-\ell)(1-\lambda)}. \tag{150}$$

It is straightforward to check that if $\phi < \bar{\phi}$ and $\tilde{Q}(\phi) = 1$, then it is not possible to satisfy either Eqs. (113) and (114) or Eq. (117) and (118) and, thus, an equilibrium with $\tilde{Q} = 1$ cannot exist.

Proof of Proposition 4. Given the definition solution for $\bar{\phi}$ when $\delta \leq \frac{\gamma(1-\ell)}{1-\gamma\ell}$, I have

$$\frac{\partial\bar{\phi}}{\partial\gamma} = \frac{\lambda}{\gamma^2(1-\ell)(1-\lambda)} > 0 \tag{151}$$

and

$$\frac{\partial\bar{\phi}}{\partial\ell} = -\frac{\lambda(1-\gamma)}{\gamma(1-\ell)^2(1-\lambda)} < 0. \tag{152}$$

□

Proof of Proposition 5. To support an equilibrium with the issuer strategies proposed in Proposition 5, consider the investor beliefs

$$\mu_\ell(\phi, q) = 0 \tag{153}$$

and

$$\mu_h(\phi, q) = \begin{cases} 1 & \text{if } \phi \le \hat{\phi} \text{ and } q \le \hat{q} \\ 0 & \text{if } \phi \le \hat{\phi} \text{ and } q > \hat{q} \\ \bar{\mu}(\phi) & \text{if } \phi > \hat{\phi} \text{ and } q = 1 \\ 0 & \text{if } \phi > \hat{\phi} \text{ and } q < 1 \end{cases}, \tag{154}$$

where

$$\bar{\mu}(\phi) = \frac{\lambda}{\lambda + (1-\lambda)(1-\phi)} \tag{155}$$

and

$$f(\phi, \ell, h, q) = 0, \tag{156}$$

$$f(\phi, h, \ell, q) = 0, \tag{157}$$

$$f(\phi, h, h, q) = \begin{cases} \phi & \text{if } \phi \le \hat{\phi} \\ \phi & \text{if } \phi > \hat{\phi} \text{ and } q = 1 \\ 0 & \text{if } \phi > \hat{\phi} \text{ and } q < 1 \end{cases}, \tag{158}$$

and

$$f(\phi, \ell, \ell, q) = \begin{cases} \phi & \text{if } \phi \le \hat{\phi} \\ 1 & \text{if } \phi > \hat{\phi} \text{ and } q = 1 \\ 0 & \text{if } \phi > \hat{\phi} \text{ and } q < 1. \end{cases} \tag{159}$$

To see that these strategies and beliefs satisfy Definition 1, note that for $\phi \le \hat{\phi}$ they coincide with the LSCE. For $\phi > \hat{\phi}$, Conditions 1, 4, and 5 of Definition 1 are satisfied by construction. The verification of Conditions 2 and 3 follow essentially the same argument as for Region 1 in the proof of Proposition 3, given that the restriction on δ given in Proposition 5 implies that does a truth-telling equilibrium does not exist.

The arguments directly preceding Proposition 5 as well as the definition of $\hat{\phi}$ given in Eq. (49) show that the proposed beliefs satisfy D1. Let $\bar{p}(\phi) = \bar{\mu}(\phi) + (1 - \bar{\mu}(\phi))\ell$ be the pooled price at ϕ. To see that Eq. (49) implies that the proposed beliefs satisfy D1, note that

$$D(O, \ell, h, q) = \{(V, p) : \delta V \ge \delta V^O(0) + (1-\delta)(\bar{p}(\phi) - pq - \gamma(1-q)\ell)\}, \tag{160}$$

$$D(H, h, h, q) = \{(V, p) : \delta V \ge \delta V^H(\phi) + (1-\delta)(\bar{p}(\phi) - pq - \gamma(1-q))\}, \tag{161}$$

and

$$D(O, h, h, q) = \{(V, p) : \delta V \ge \delta V^O(\phi) + (1-\delta)(\bar{p}(\phi) - pq - \gamma(1-q))\}. \tag{162}$$

Because $V^O(\phi) \ge V^H(\phi)$, it must be that $D(O, h, h, q) \subset D(H, h, h, q)$. Also, $D(H, h, h, q) \subset D(O, \ell, h, q)$ for all q if and only if

$$\delta V^O(\phi) + (1-\delta)(\bar{p}(\phi) - pq - \gamma(1-q)) > \delta V^O(0) + (1-\delta)(\bar{p}(\phi) - pq - \gamma(1-q)\ell) \tag{163}$$

for all q. Simplifying Eq. (163) gives

$$\frac{\delta}{1-\delta}(V^H(\phi) - V^O(0)) > \gamma(1-\delta)(1-q)(1-\ell) \tag{164}$$

which is clearly implied by the condition given in Eq. (49).

Finally, to see that there exists $\hat{\phi} < 1$ such that it is possible to satisfy Eq. (49), note that given the definition of the strategies above, I have

$$V^H(\phi) = \begin{cases} \lambda(\hat{q} + \gamma(1-\hat{q})) + (1-\lambda)\ell & \text{for } \phi \le \hat{\phi} \\ \bar{p}(\phi) & \text{otherwise} \end{cases} \tag{165}$$

and

$$V^O(0) = \lambda(\hat{q} + \gamma(1-\hat{q})) + (1-\lambda)\ell. \tag{166}$$

So, Eq. (49) reduces to

$$\frac{\delta}{1-\delta}(\bar{p}(\phi) - \lambda(\hat{q} + \gamma(1-\hat{q})) + (1-\lambda)\ell) > \gamma(1-\ell). \tag{167}$$

Now, note that

$$\frac{\delta}{1-\delta}(\bar{p}(1) - \lambda(\hat{q} + \gamma(1-\hat{q})) + (1-\lambda)\ell)$$
$$= \frac{\delta}{1-\delta}(\lambda + (1-\lambda)\ell - \lambda(\hat{q} + \gamma(1-\hat{q})) + (1-\lambda) \tag{168}$$

$$\ge 1 - \ell \tag{169}$$

$$> \gamma(1-\ell), \tag{170}$$

where the first inequality follows from the condition on δ given in the proposition. Thus, it is possible to satisfy the condition given in Eq. (167) and, hence, Eq. (49), at $\phi = 1$. Thus, by the continuity of $\bar{p}$, there exists some $\hat{\phi} < 1$ such that is possible to satisfy the condition given in Eq. (49). □

References

Akerlof, G., 1970. The market for "lemons": quality uncertainty and the market mechanism. Quarterly Journal of Economics 84 (3), 488–500.

Banks, J., Sobel, J., 1987. Equilibrium selection in signaling games. Econometrica 55 (3), 647–661.

Chari, V., Shourideh, A., Zetlin-Jones, A., 2014. Reputation and persistence of adverse selection in secondary loan markets. American Economic Review 104 (12), 4027–4070.

Cho, I., Kreps, D., 1987. Signaling games and stable equilibria. Quarterly Journal of Economics 102 (2), 179–221.

DeMarzo, P., 2005. The pooling and tranching of securities: a model of informed intermediation. Review of Financial Studies 18 (1), 1–35.

DeMarzo, P., Duffie, D., 1999. A liquidity based model of security design. Econometrica 67 (1), 65–99.

Diamond, D.W., 1989. Reputation acquisition in debt markets. Journal of Political Economy 97 (4), 828–862.

Diamond, D.W., 1991. Monitoring and reputation: the choice between bank loans and directly placed debt. Journal of Political Economy 99 (4), 689–721.

Garmaise, M., Moskowitz, T., 2004. Confronting informational asymmetries: evidence from real estate markets. Review of Financial Studies 17 (2), 405–437.

Gopalan, R., Nanda, V., Yerramilli, V., 2011. Does poor performance damage the reputation of financial intermediaries? Evidence from the loan syndication market. Journal of Finance 46 (6), 1335–1365.

Griffin, J., Lowery, R., Saretto, A., 2013. Complex securities and underwriter reputation: do reputable underwriters produce better securities? Review of Financial Studies 27 (10), 2872–2925.

Josephson, J., Shapiro, J., 2012. Credit ratings and security design. Univeristy of Oxford, Oxford, UK. Unpublished working paper

Klapper, R., Tamaino, M., Dunne, C., 2010. In the matter of Abacus CDO: supplemental submission on behalf of Goldman, Sachs & Co., United States of America, before the securities and exchange commission, file no. ho-10911.
Kreps, D.M., Wilson, R., 1982. Reputation and imperfect information. Journal of Economic Theory 27, 253–279.
Leland, H., Pyle, D., 1977. Informational asymmetries, financial structure, and financial intermediation. Journal of Finance 32 (2), 371–387.
Lin, H., Paravisini, D., 2013. Delegated monitoring of fraud: the role of noncontractual incentives. London School of Economics, London, UK. Unpublished working paper
Mailath, G.J., Okuno-Fujiwara, M., Postlewaite, A., 1993. Belief-based refinements in signalling games. Journal of Economic Theory 60 (2), 241–276.
Mathis, J., McAndrews, J., Rochet, J.-C., 2009. Rating the raters: are reputation concerns powerful enough to discipline rating agencies? Journal of Monetary Economics 56 (5), 657–674.
Milgrom, P., Roberts, J., 1982. Predation, reputation, and entry deterrence. Journal of Economic Theory 27 (2), 280–312.
Myers, S., Majluf, N., 1984. Corporate financing and investment decisions when firms have information that investors do not. Journal of Financial Economics 13 (2), 187–221.
Nachman, D., Noe, T., 1994. Optimal design of securities under asymmetric information. Review of Financial Studies 7 (1), 1–44.
Palm, G., 2010. The Goldman Sachs Group, Inc. Q1 2010 earnings call. Seeking Alpha, April 20.
Piskorski, T., Seru, A., Witkin, J., 2015. Asset quality misrepresentation by financial intermediaries: evidence from RMBS market. Journal of Finance 70 (6), 2635–2678. doi:10.1111/jofi.12271.
Spence, M., 1973. Job market signaling. Quarterly Journal of Economics 87 (3), 355–374.
Winton, A., Yerramilli, V., 2015. Lender moral hazard and reputation in originate-to-distribute markets. University of Minnesota, Minneapolis and St. Paul, MN. Unpublished Working Paper