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Operational risk ☆

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Abstract

This paper provides an economic and mathematical characterization of operational risk useful for clarifying the issues related to estimation and the determination of economic capital. The insights for this characterization originate in the corporate finance literature. Operational risk is subdivided into two types, either: (i) the risk of a loss due to the firm's operating technology, or (ii) the risk of a loss due to agency costs. These two types of operational risks generate loss processes with different economic characteristics. We argue that the current methodology for the determination of economic capital for operational risk is overstated. It is biased high because the computation omits the bank's net present value (NPV) generating process. We also show that although it is conceptually possible to estimate the operational risk processes' parameters using only market prices, the non-observability of the firm's value makes this an unlikely possibility, except in rare cases. Instead, we argue that data internal to the firm, in conjunction with standard hazard rate estimation procedures, provides a more fruitful alternative.

JEL classification: G21; G28; G13

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1. Introduction

Risk management concerns the investigation of four significant risks of a loss to a firm or portfolio: market risk, credit risk, liquidity risk, and operational risk (see Jarrow and Turnbull, 2000, p. 587). Market risk includes the risk of a loss due to unanticipated price movements in financial securities or asset values, and it includes price fluctuations due to either equities, interest rates, commodities, or foreign currencies. Credit risk is the risk of a loss due to default, and liquidity risk is the risk of a loss due to the inability to liquidate an asset or financial position at a reasonable price in a reasonable time period. And, according to the revised Basel Committee revised report (Basel Committee on Banking Supervision, 2005) "operational risk is defined as the risk of loss resulting from the inadequate or failed internal processes, people and systems or from external events. This definition includes legal risk". Furthermore, "legal risk includes, but is not limited to, exposure to fines, penalties, or punitive damages resulting from supervisory actions, as well as private settlements".

The existing literature on operational risk almost exclusively focuses on two issues: one, the estimation of operational risk loss processes using either extreme value theory or Cox processes, (see Chavez-Demoulin et al., 2006; Coleman, 2003; de Fontnouvelle et al., 2004; de Fontnouvelle et al., 2005; Ebnother et al., 2001; Embrechts and Puccetti, forthcoming; Jang, 2004; Moscadelli, 2004; Lindskog and McNeil, 2003; Chavez-Demoulin et al., 2006) and two, the application of these estimates to the determination of economic capital, (see de Fontnouvelle et al., 2004; de Fontnouvelle et al., 2005; Moscadelli,

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2004). In the estimation of economic capital for operational risk, the estimates appear to be quite large, in fact, at least as large as that necessary to cover market risk. The magnitude of the necessary capital for operational risk is a surprising result.

As evidenced by these references, the modeling and estimation of operational risk is treated identically to market and credit risk, i.e., a loss process is modeled and estimated. However, this is where the similarity ends. Unlike market and credit risk, which are external to the firm in their origin, operational risk is internal to the firm. Although this asymmetry between external and internal risk generation is well known, the implications of this asymmetry for either: (i) the pricing of financial securities within the firm, or (ii) the determination of economic capital, is not. The purpose of this paper is study these implications.

We argue that the current methodology for the determination of economic capital for operational risk is overstated. It is biased high because the computation omits the bank's net present value (NPV) generating process. In fact, based on standard economic reasoning, we argue that the NPV process itself provides sufficient capital to at least cover the *expected* operational risk losses. Thus, additional economic capital is only needed to cover *unexpected* operational risk losses. The magnitude of the unexpected operational risk losses are potentially significantly less.

To make these arguments, we must step back and revisit the notion of operational risk from a more holistic perspective. In this regard, we generate an economic and mathematical characterization of operational risk, useful for clarifying the issues related to estimation and the determination of economic capital. Our economic characterization is based on insights from the corporate finance literature. The mathematical characterization, as in the existing literature, is analogous to that employed in the credit risk literature. Estimation of the model's parameters is discussed, but its implementation is left for subsequent research.

Our economic characterization partitions operational risk into one of two fundamental types, either: (i) the risk of a loss due to the firm's operating technology/system, including failed internal processes and transactions, or (ii) the risk of a loss due to agency costs, including fraud and mismanagement.[1] These two types of operational risks are generated by different economic considerations. One is based on the production/trading process/system generating revenues, the other is based on managerial incentives. As such, they will have different economic characteristics.

Our mathematical characterization for both of these operational risks is similar to the modeling of default risk in the reduced form credit risk literature. This mathematical characterization leads to the generation of simple valuation formulas for pricing (and hedging) financial securities. These valuation formulas are firm specific, akin to standard net present value calculations often discussed in the selection of investment projects in capital budgeting.

This mathematical characterization also provides insights into the estimation of operational risk losses. We show that although it is conceptually possible to estimate the operational risk processes' parameters using only market prices, the non-observability of the firm's value makes this an unlikely possibility, except in rare cases. Instead, we argue that given data internal to the firm and databases of collections of internal data across many firms, standard hazard rate estimation procedures provide a more fruitful alternative. Only the agency cost component can possibly be estimated using data external to the firm's operating system.

An outline of this paper is as follows: Section 2 provides the economic based definition. Section 3 provides a simple, yet robust mathematical characterization of the operational event risk processes. Section 4 discusses estimation, and Section 5 concludes.

2. The definition

This section provides our economic and mathematical characterization of operational risk. Based on the standard definition, we divide operational risk into two types. Type one corresponds to the risk of a loss due to the firm's operating *system*, i.e., a failure in a transaction or investment, either due to an error in the back office (or production) process or due to legal considerations. And, type two corresponds to the risk of a loss due to *incentives*, including both fraud and mismanagement.[2] The second type of operational risk represents an agency cost, due to the separation of a firm's ownership and management. Agency costs are recognized as a significant force in economics, and they have received significant study in the corporate finance literature as key determinants of the firm's capital structure and dividend policy (see Brealey and Myers, 2004). Both types of operational risk losses occur with repeated regularity, and they can be small or catastrophic. Spectacular catastrophic examples include Orange County, the Barings bank failure, or the bankers trust and Procter and Gamble fiasco (see Risk Books, 2003). Of course, both the system and agency type operational risks can be further subdivided into event type and business line categories as detailed in the revised Basel II report (Basel Committee on Banking Supervision, 2005) and discussed in Section 4. For the moment, however, to keep the logic simple we will confine our discussion to this coarser partitioning.

[1] Of course, each of these two types of operational risk can be divided into subcategories, for example, system risk can be divided into business lines (corporate finance, sales and trading, retail banking, commercial banking, payments and settlement, agency services, asset management, retial brokerage) and agency risk can be divided into fraud and production related factors (see Section 4).

[2] There is some ambiguity with respect to the classification of human error. If the human error is due to misaligned incentives, then it should be included in the agency operational risk category. Otherwise, it is system related risk.

We consider a finite horizon, continuous trading setting with $t \in [0, T]$ on a filtered probability space $(\Omega, F, \{F_t\}, P)$ satisfying the usual conditions (see Protter, 2005) with P the statistical probability measure. Let r_t denote the default free spot rate of interest, and let S_t^i represent the market value of asset $i \in \{1, \ldots, n\}$ at time t. For simplicity, we assume that these assets have no cash flows over their lives.[3] We assume that the market for these assets is arbitrage free, so that there exists an equivalent martingale probability measure Q such that

$$S_t^i = E_t^Q\left\{S_T^i e^{-\int_t^T r_v \, dv}\right\} \quad \text{for all } i,$$

where $E_t^Q\{\cdot\}$ is under the martingale probability. Under this structure, market prices can be computed as the expected discounted value of their future cash flows under the martingale probability. The markets need not be complete, so the martingale probability need not be unique.

We consider a firm operating in this setting, trading financial securities or investing in real assets with prices S_t^i for $i = 1, \ldots, n$, generating a firm value of V_t. As indicated, the firm is conceptualized as a portfolio of financial securities and/or real assets. The firm's value represents the aggregate value of the left side of the firm's balance sheet. The right side of the firm's balance sheet consists of the firm's liabilities and equity, which we also assume trade in an arbitrage free, but possibly incomplete market. Under this assumption, the firm's value V_t trades. For simplicity, we assume that the firm's value has no cash flows. This implies, of course, that all revenues generated within the firm are reinvested and not distributed as dividends. Consequently, its time t value can also be represented as an expected discounted future value using the martingale measure, i.e.

$$V_t = E_t^Q\left\{V_T e^{-\int_t^T r_v \, dv}\right\}.$$

The economic setting can be understood by examining Fig. 1. The firm is represented by an operating technology (the green box) that takes as inputs traded financial securities and real assets, with prices S_t^i, and returns as an output the value of the firm V_t. The operating technology transformation represented within the firm is discussed subsequently. Also, for subsequent usage, we let $\mathbf{X}_t$ denote a vector of state variables, F_t – *measurable*, that characterize the state of the economy at time t. Included in this set of state variables are the spot rate of interest r_t and the market prices S_t^i. We let F_t^X denote the (completed to include all zero probability events) filtration generated by the state variables $\mathbf{X}_t$ up to and including time t.

As is well known, this valuation methodology has embedded the implicit assumption that both the "market" and the "firm" have the "same" information sets, to the extent that their (conditional) probability measures must be equivalent, that is, they must agree on zero probability events. If this is not the case, then the firm's values will differ from the market's. With respect to operational risk events, it is conceivable that the firm might have private information which could violate this condition. Pillar 3 of the revised Basel II accord is designed to eliminate such information asymmetries via the public disclosure requirement (see Basel Committee on Banking Supervision, 2005, see pages 184, 199). To the extent that Pillar 3 is successful, the following methodology applies. We will return to this issue of informational asymmetries when discussing the NPV value process below.

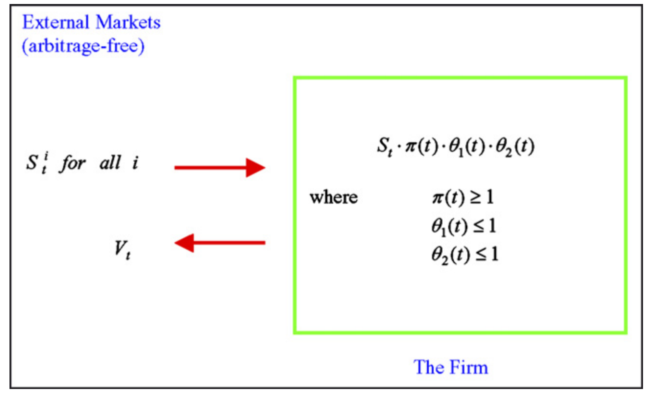

Fig. 1. The economic setting.
S_t^i are prices of traded assets.
S_t represents the aggregate value of the asset portfolio purchased by the firm.
V_t is the firm's value.
$\pi(t)$ is the proportionate change in the value of the firm's asset portfolio due to the firm's operating technology.
$\theta_1(t)$ is the proportionate change in the value of the firm's asset portfolio due to the system operational risk.
$\theta_2(t)$ is the proportionate change in the value of the firm's asset portfolio due to the agency cost operational risk.

2.1. The NPV of the firm's operating technology

In our setting, the existence of an operating technology distinguishes a firm from an individual trading in the market. An operating technology transforms the assets the firm purchases into more valuable objects. As argued in Jarrow and Purnanandam (2004), this is possible if the firm has some special talent, information, or managerial expertise in selecting assets for investment. It is interesting to point out that the NPV process has long been recognized, and even estimated, for credit cards and consumer deposits (see Chatterjea et al., 2003; Janosi et al., 1999). In this context, the NPV process is due to the fact that banks can pay below market rates on demand deposits or charge above market rates on credit card loans. Lastly, note that capital budgeting is the science of computing this NPV process (see Ross et al., 2002).

We let this increase in asset value due to the firm's operating technology be represented by the F_t – *measurable*

[3] Cash flows only complicate the notation, but not the logic of the subsequent analysis, and are therefore omitted for clarity.

stochastic process $\pi(t) \geqslant 1$ for all t.[4] Letting S_t represent the aggregate market value of the firm's asset portfolio, the change in value due to the firm's operating technology at time T is[5]

$$S_T \cdot \pi(T) \geqslant S_T.$$

See Fig. 1. $\pi(t)$ is called the firm's net present value (NPV) process.

2.2. An owner-managed firm

To introduce the characterization of operational risk, we start with the simplest economic setting – that of an owner-managed firm (or portfolio). The owner or manager invests his capital in some assets, either real or financial and she is concerned about the risk of a loss from her investments.

As in the previous section, the time t marked-to-market value of the firm's asset portfolio is denoted S_t. In a traditional risk management model, one would only be concerned with the probability distribution of this price process S_t, and various risk measures could be used to characterize the potential losses on this position due to either market, credit or liquidity risk. To an owner-managed firm, operational risk represents the additional risk due to a failed transaction, perhaps an error in the executive of a trade (or investment), a legal dispute, or possibly an error in judgement.

We represent this additional loss by the multiplication of the previous portfolio's value $S_t \cdot \pi(t)$ by the F_t – *measurable* stochastic process $\theta_1(t) \leqslant 1$ for all t. The quantity $\theta_1(t)$ represents the accumulated time t "recovery" value after all operational risk events of the first type are incurred. The internal value of the firm's assets at time T, after the inclusion of operational risk, is thus

$$V_T = S_T \cdot \pi(T) \cdot \theta_1(T).$$

See Fig. 1. It is important to emphasize that this reduction in asset value due to system operational risk is realized internally within the firm. Given that V_t is a traded asset, the market value of the owner-managed firm's assets is

$$V_t = E_t^Q\left\{S_T \cdot \pi(T) \cdot \theta_1(T) \cdot \mathrm{e}^{-\int_t^T r_v\,\mathrm{d}v}\right\}. \quad (1)$$

The market value of the firm V_t equals the present value of the firm's asset portfolio S_T (as seen by the market at time T), adjusted by the NPV and operational risk processes.

2.3. An agent-managed firm

Next, we consider a firm managed by an agent. Of course, for practical applications, this is the most relevant case. Given an agent-managed firm, the second type of operational risk is also present. We represent this additional loss by the multiplication of the previous firm value $S_t \cdot \pi(t) \cdot \theta_1(t)$ by the F_t – *measurable* stochastic process $\theta_2(t) \leqslant 1$ for all t which represents the accumulated time t "recovery" value after all operational risk events of the second type.

By the same line of reasoning, the internal valuation of the firm's portfolio at time T is

$$V_T = S_T \cdot \pi(T) \cdot \theta_1(T) \cdot \theta_2(T).$$

See Fig. 1, and the time t market value of the firm's assets is

$$V_t = E_t^Q\{S_T \cdot \pi(T) \cdot \theta_1(T) \cdot \theta_2(T) \cdot \mathrm{e}^{-\int_t^T r_v\,\mathrm{d}v}\}. \quad (2)$$

2.4. The firm's NPV process – revisited

It is important to stress the economic importance of including the firm's NPV process when considering operational risk for the purposes of risk measure computations, as with the revised Basel II capital requirements (Basel Committee on Banking Supervision, 2005). As verified by expression (2), operational risk reduces the firm's value, relative to market prices (marking-to-market), because operational risk always has a non-positive impact on firm value ($\theta_1(T) \cdot \theta_2(T) \leqslant 1$). This implies that, without the inclusion of the non-negative NPV process ($\pi(t) \geqslant 1$), there would be no reason for this firm to exist. Indeed, shareholders could generate higher wealth themselves by directly purchasing the firm's assets. This statement follows from that fact that without the NPV process included, $V_t \leqslant S_t$ for all t with probability one.[6] In fact, in well-functioning financial markets, we would expect that for healthy firms, the reverse inequality $V_t \geqslant S_t$ holds for all t with probability one. That is, on average (as measured by expression (2) in market prices), the NPV process dominates (is larger than) the average loss due to operational risk.

This is an important conceptual observation for the determination of a firm's fair economic capital. It implies that the estimates of economic capital for operational risk as contained in de Fontnouvelle et al. (2004), de Fontnouvelle et al. (2005), Moscadelli (2004) are overstated. These estimates include capital for the *expected* operational risk losses. Our model implies that this is unnecessary. The NPV generating process provides enough "capital" to cover expected operational risk losses. And, only the unexpected operational risk losses need to be considered when computing economic capital. Of course, the NPV dominates operational risk only on average, and in the tail of

[4] For simplicity of presentation we have assumed that $\pi(t) \geqslant 1$ for all t with P probability 1. However, this can be substantially weakened. As will be seen below, all we need is for $Q(\pi(T) \geqslant 1) > 0$ and that this probability is large enough so that $V_0 \geqslant S_0$ at the start of the model (see expression (2)).

[5] The multiplicative form of this relationship is without loss of generality (since $\pi(T)$ can itself depend on S_T).

[6] The algebra is that without $\pi(t)$, $\widehat{V}_t = E_t^Q\left\{S_T \cdot \theta_1(T) \cdot \theta_2(T) \cdot \mathrm{e}^{-\int_t^T r_v\,\mathrm{d}v}\right\} \leqslant E_t^Q\left\{S_T \mathrm{e}^{-\int_t^T r_v\,\mathrm{d}v}\right\} = S_t$.

the distribution, operational risk may far exceed the firm's NPV (in fact, it could exceed both the firm's NPV and the market price component). The extent to which this conceptual observation is important empirically depends crucially on whether the expected loss is large relative to the unexpected losses. If this difference is not large, then for all practical purposes, the estimates and conclusions contained in de Fontnouvelle et al. (2004), de Fontnouvelle et al. (2005), Moscadelli (2004) remain unaffected. The relative importance of including expected loss or not remains an open empirical question to be resolved by subsequent research.

It is important to emphasize that this argument depends on an implicit assumption. The assumption is that there are no information asymmetries between the "market" and the "firm", as previously discussed above. If there were information asymmetries, then a firm may attempt to hide operational risk in order to lower its capital requirements. That is, given differential information, the market's valuation might exceed the firm's. And, the expected operational risk losses could be underestimated by the market. However, to the extent that the information dissemination as required by Pillar 3 of the revised Basel II accord is successful, this differential information concern disappears.

3. The NPV and operational risk processes

This section presents a simple, yet robust formulation of the NPV and operational risk event processes. We model these stochastic processes following the jump process formulation often used in the reduced form credit risk literature (see Bielecki and Rutkowski, 2002). Other formulations are possible, and these extensions are left for subsequent research.

3.1. The NPV process

This section presents the stochastic process for the NPV process $\pi(t)$. Let $N_0(t)$ be a doubly stochastic (Cox) counting process, initialized at zero ($N_0(0) = 0$), that counts the number of positive NPV events that occur between time 0 up to and including time t. We assume that this counting process is measurable with respect to the given filtration F_t, and that it has an intensity per unit time given by $\lambda_0(s) = \lambda_0(\mathbf{X}_s) \geqslant 0$ that is F_s^X – *measurable*. Given F_T^X, we assume that $N_0(t)$ is independent of S_t. This is often called the "conditional independence" assumption. Continuing, we let a NPV event at time t cause a percentage increase in firm value equal to $\alpha(t) = \alpha(\mathbf{X}_t) > 0$ that is also F_t^X – *measurable*.

Thus

$$\pi(t) = \prod_{i=0}^{N_0(t)} (1 + \alpha_{T_i}), \tag{3}$$

where $\pi(0) = 1$, T_i for $i = 0, 1, 2, \ldots$ are the jump times of $N_0(t)$ with $T_0 = \alpha_{T_0} = 0$. We assume that the intensity and drifts processes satisfy the technical conditions needed for the existence of the various processes and the subsequent computations (see Bremaud, 1981).

Note that in general, the NPV process $\pi(t)$ is correlated to the portfolio's market value S_t due to their mutual dependence on the state variables $\mathbf{X}_t$. However, after conditioning upon these variables, the randomness generating the counting process $N_0(t)$ in the asset's NPV is idiosyncratic and firm specific, not otherwise related to market prices or the state variables. Even so, this jump NPV process risk could still require a market risk premium if these risks are not diversifiable in a large portfolio (see Jarrow et al., 2005). To accommodate this possibility, under the equivalent martingale measure Q, we let the counting process have the intensity $\lambda_0(s)\mu_0(s)$ where $\mu_0(s) > 0$ is the risk premium associated with the NPV process (see Bremaud, 1981, p. 241).

3.2. System and agency type operational risks

First, let us consider system type operational risk. We want a formulation of the $\theta_1(t)$ process that can be utilized in practice. Conceptually, it is reasonable to believe that the occurrence of an operational risk event is related to the volume of transactions underlying the firm's portfolio. Such a detailed implementation would require decomposing the firm's portfolio into its component parts, and then modeling the trading process of each individual asset, keeping track of the number of transactions and the operational risk events. The occurrence of an operational risk event could then be modeled at the transaction level. This would be a very complex procedure (see Leippold and Vanini, 2005 for one such approach). Although perfectly reasonable to pursue, we follow a "reduced form approach" instead and concentrate on the entire portfolio's value.[7] Refinements in this methodology are delegated to subsequent research.

Formally, let $N_1(t)$ be another doubly stochastic (Cox) counting process, initialized at zero ($N_1(0) = 0$), that counts the number of system operational risk events that occur between time 0 up to and including time t. We assume that this counting process is measurable with respect to the given filtration F_t, and that it has an intensity per unit time given by $\lambda_1(s) = \lambda_1(\mathbf{X}_s) \geqslant 0$ that is F_s^X – *measurable*. We let an operational risk event at time t cause a percentage reduction in firm value equal to $-1 < \delta_1(t) = \delta_1(\mathbf{X}_t) < 0$ that is F_t^X – *measurable*. Hence

$$\theta_1(t) = \prod_{i=0}^{N_1(t)} (1 + \delta_1(T_i)), \tag{4}$$

[7] In fact, this should really be thought of as the portfolios related to the various business lines as specified in the revised Basel II report (Basel Committee on Banking Supervision, 2005), p. 139. The subsequent analysis follows with the appropriate and straightforward aggregation of the business lines into the firm's entire portfolio.

where $\theta_1(0) = 1$, T_i for $i = 0, 1, 2, \ldots$ are the jump times of $N_1(t)$ with $T_0 = \delta_1(T_0) = 0$.

The agency cost operational risk event is modeled in the same way. Using superscripts "2" to indicate a type 2 operational risk event, we have

$$\theta_2(t) = \prod_{i=0}^{N_2(t)} (1 + \delta_2(T_i)), \tag{5}$$

where $\theta_2(0) = 1$, T_i for $i = 0, 1, 2, \ldots$ are the jump times of $N_2(t)$, and $T_0 = \delta_2(T_0) = 0$.

Similar to the firm's NPV process, we assume that conditional upon the path of the state variables $\mathbf{X}_t$ up to time T, $N_2(t)$, $N_1(t)$, $N_0(t)$ and S_t are conditionally independent. Again, this assumption is equivalent to saying that any additional randomness present in this operational risk is idiosyncratic and firm specific. As before, this risk could still require a market risk premium if these risks are not diversifiable in a large portfolio. To accommodate this possibility, under the equivalent martingale measure Q, we let the ith counting process have the intensity $\lambda_i(s)\mu_i(s)$, where $\mu_i(s) > 0$ is the risk premium.

Also, unconditionally, all of the $N_2(t)$, $N_1(t)$, $N_0(t)$ and S_t are correlated processes. They are correlated through the common state variables. This correlation is important, for example, because one might believe that as the value of the firm's asset portfolio increases, the size of the system operational risk losses may decline due to an improved back office systems, or agency cost operational risk losses may increase due to the firm's managers trying to increase their performance and save their jobs. Although modeled using similar stochastic processes, the key distinction between the two types of operational risk is that, most likely, $|\delta_2| \gg |\delta_1|$ and $\lambda_2 \ll \lambda_1$, that is agency cost operational risk results in a larger loss than systems operational risk, but is less likely to occur.

3.3. The firm's internal value generating process

In conjunction, the firm's time T internal value generating process is given by

$$\begin{aligned} &S_T \cdot \pi(T) \cdot \theta_1(T) \cdot \theta_2(T) \\ &\quad = S_T \prod_{i=0}^{N_0(T)} (1 + \alpha_{T_i}) \prod_{j=1}^{2} \left[\prod_{i=0}^{N_j(T)} (1 + \delta_j(T_i)) \right], \end{aligned} \tag{6}$$

where the counting processes $\{N_0(t), N_1(t), N_2(t)\}$ have the intensities $\{\lambda_0(t), \lambda_1(t), \lambda_2(t)\}$ under the statistical probability measure P and $\{\lambda_0(t)\mu_0(t), \lambda_1(t)\mu_1(t), \lambda_2(t)\mu_2(t)\}$ under the martingale probability measure Q. Of course, for the computation of risk measures, like Value at Risk, the statistical measure is the relevant probability, while for valuation and hedging the martingale measure is the appropriate choice.

For practical applications, although not necessary, we assume that within a business line, the NPV and operational risk processes $(\pi(t), \theta_1(t), \theta_2(t))$ do not depend on the traded assets S_t^i (or S_t) as given in Fig. 1. This implies that the firm's internal value process is linear in S_t so that the internal value process applies to the individual traded assets S_t^i as well as to portfolios of traded assets S_t. This observation is used in Example 1 without further comment.

3.4. Asset pricing and risk measures

For pricing and risk measure computation, we have that the market value of the firm's value generating process can be represented as

$$V_t = E_t^Q \left\{ S_T e^{-\int_t^T r_v \, dv} \prod_{i=0}^{N_0(T)} (1 + \alpha_{T_i}) \prod_{j=1}^{2} \left[\prod_{i=0}^{N_j(T)} (1 + \delta_j(T_i)) \right] \right\}. \tag{7}$$

This expression can be simplified to[8]

$$V_t = E_t^Q \left\{ S_T e^{-\int_t^T [r_s - \alpha_s \lambda_0(s)\mu_0(s) - \delta_1(s)\lambda_1(s)\mu_1(s) - \delta_2(s)\lambda_2(s)\mu_2(s)] \, ds} \right\}. \tag{8}$$

We see that the firm value equals the portfolio's market price process S_T discounted by the spot rate after an adjustment for the NPV process and operational risk. The spot rate process is decreased by the expected increase in value due to the NPV process (recall that α_s is positive), but increased to reflect both types of operational risk (recall that both $\delta_1(s)$ and $\delta_2(s)$ are negative). This simplification is important because it demonstrates that pricing in the presence of operational risk can be handled via a simple adjustment to the discount rate (this same adjustment is used in the credit risk literature). Then, the direct application of the mathematics developed for the pricing of interest rate derivatives under default free term structure evolutions can be directly applied to the computation of the relevant quantities given operational risk. For example, if one assumes affine processes for the combined jumps and recovery rate processes, then closed form solutions for these expressions and various options on the firm's cash flows can be obtained (see Shreve, 2004, Chapter 10). To illustrate these computations, we provide the following two-factor Gaussian example.

Example 1. Two-factor affine model

Consider a two-factor Gaussian model as in Shreve (2004), p. 406. The CIR two-factor model follows similarly (see p. 420 instead). Here, the state variables follow diffusion processes given by

$$\begin{aligned} dX_1(t) &= -\phi_1 X_1(t)dt + dW_1(t), \\ dX_2(t) &= -\phi_{21} X_1(t)dt - \phi_2 X_2(t) + dW_2(t), \end{aligned}$$

where $W_1(t)$ and $W_2(t)$ are independent Brownian motions under Q, and $\phi_1 > 0$, $\phi_2 > 0$, ϕ_{21} are constants.

[8] The proof of this expression can be obtained from the author by request.

Let the spot rate follow an affine process in the state variables given by

$$r_t = a_0 + a_1X_1(t) + a_2X_2(t).$$

The value of a default free zero–coupon bond in the market (where $S_T = 1$ with probability one) is

$$S_t = E_t^Q\left\{1 \cdot \mathrm{e}^{-\int_t^T r(u)\,\mathrm{d}u}\right\} = \mathrm{e}^{-X_1(t)C_1(T-t)-X_2(t)C_2(T-t)-A(T-t)},$$

where $C_1(0) = C_2(0) = A(0) = 0$. If $a_1 \neq a_2$

$$C_1(\tau) = \frac{1}{\phi_1}\left(a_1 - \frac{\phi_{21}a_2}{a_1}\right)(1-\mathrm{e}^{-\phi_1\tau}) + \frac{\phi_{21}a_2}{a_2(a_1-a_2)}(\mathrm{e}^{-\phi_2\tau}-\mathrm{e}^{-\phi_1\tau}),$$
$$C_2(\tau) = \frac{a_2}{\phi_2}(1-\mathrm{e}^{-\phi_2\tau}),$$
$$A(\tau) = \int_0^\tau\left(-\frac{1}{2}C_1^2(u) - \frac{1}{2}C_2^2(u) + a_0\right)\mathrm{d}u.$$

Now, let the NPV process and operational risk event intensities, under Q, also satisfy an affine process in the state variables given by

$$\alpha_t\lambda_0(t)\mu_0(t) = b_0 + b_1X_1(t) + b_2X_2(t),$$
$$\delta_1\lambda_1(t)\mu_1(t) = c_0 + c_1X_1(t) + c_2X_2(t),$$
$$\delta_2\lambda_2(t)\mu_2(t) = d_0 + d_1X_1(t) + d_2X_2(t).$$

Then, define new parameters by

$$\psi_0 = a_0 - b_0 - c_0 - d_0,$$
$$\psi_1 = a_1 - b_1 - c_1 - d_1,$$
$$\psi_2 = a_2 - b_2 - c_2 - d_2$$

and an adjusted spot rate process by

$$R(t) = r_t - \alpha_t\lambda_0(t)\mu_0(t) - \delta_1\lambda_1(t)\mu_1(t) - \delta_2\lambda_2(t)\mu_2(t) = \psi_0 + \psi_1X_1(t) + \psi_2X_2(t).$$

The value within the firm for a traded Treasury zero–coupon bond, according to expression (8), is

$$V_t = E_t^Q\left\{1 \cdot \mathrm{e}^{-\int_t^T R(u)\,\mathrm{d}u}\right\} = \mathrm{e}^{-X_1(t)\widetilde{C}_1(T-t)-X_2(t)\widetilde{C}_2(T-t)-\widetilde{A}(T-t)},$$

where $\widetilde{C}_1(0) = \widetilde{C}_2(0) = \widetilde{A}(0) = 0$. If $\psi_1 \neq \psi_2$, then

$$\widetilde{C}_1(\tau) = \frac{1}{\phi_1}\left(\psi_1 - \frac{\phi_{21}\psi_2}{\psi_1}\right)(1-\mathrm{e}^{-\phi_1\tau}) + \frac{\phi_{21}\psi_2}{\psi_2(\psi_1-\psi_2)}(\mathrm{e}^{-\phi_2\tau}-\mathrm{e}^{-\phi_1\tau}),$$
$$\widetilde{C}_2(\tau) = \frac{\psi_2}{\phi_2}(1-\mathrm{e}^{-\phi_2\tau}),$$
$$\widetilde{A}(\tau) = \int_0^\tau\left(-\frac{1}{2}\widetilde{C}_1^2(u) - \frac{1}{2}\widetilde{C}_2^2(u) + \psi_0\right)\mathrm{d}u.$$

These two values for the default free zero–coupon bond differ by the NPV and operational risk processes impact within the firm. In general, we would expect that $V_t \geqslant S_t$.

Expression (6) is also directly relevant for computing various risk management measures. For example, computing the 5% Value at Risk measure over the horizon $[0,T]$ for the firm's asset value requires finding the smallest $\eta > 0$ such that

$$P(V_T \leqslant -\eta) = 0.05.$$

Using expression (6) yields

$$P\left(S_T\prod_{i=0}^{N_0(T)}(1+\alpha_{T_i})\prod_{j=1}^{2}\left[\prod_{i=0}^{N_j(T)}(1+\delta_j(T_i))\right] \leqslant -\eta\right) = 0.05.$$

Given diffusion processes for the state variables $\mathbf{X}_t$, a process for S_T, e.g., geometric Brownian motion, and using the fact that $N_j(t)$ are all mutually independent given F_T^X, this is easily computed using standard Monte-Carlo techniques (see Glasserman, 2004). Of course, the computation of Value at Risk is under the statistical probability measure P using the intensities $\{\lambda_0(t),\ \lambda_1(t),\ \lambda_2(t)\}$ for the relevant counting processes.

To further develop our understanding of expression (8) and its uses, it is instructive to consider the constant parameter case.

Example 2. Constant parameters

Assuming that

$$\alpha_t\lambda_0(t)\mu_0(t),\quad \delta_1(t)\lambda_1(t)\mu_1(t),\quad \delta_2(t)\lambda_2(t)\mu_2(t)$$

are constants, expression (8) simplifies to

$$V_t = S_t(1+\alpha)^{N_0(t)}(1+\delta_1)^{N_1(t)}(1+\delta_2)^{N_2(t)} \times \mathrm{e}^{[\alpha\lambda_0\mu_0+\delta_1\lambda_1\mu_1+\delta_2\lambda_2\mu_2](T-t)}. \tag{9}$$

Here, the firm's value is seen to be equal to the portfolio's market value at time t adjusted to reflect all past NPV and operational risk shocks, plus anticipated changes in these events.

In this constant parameter case, the adjustments to the marked-to market value of the firm's portfolio S_t to reflect operational risk are easy to compute. They amount to a deterministic and proportional change in value as represented by the terms following S_t in expression (9).

As a first pass in implementing operational risk into a firm's risk management procedure, the constant parameter case, expression (9), could prove a very useful tool. Its implementation would require minimal changes to any existing risk management procedure. For example, computing prices and hedges at time 0 amounts to using the following expression:

$$V_0 = S_0\mathrm{e}^{[\alpha\lambda_0\mu_0+\delta_1\lambda_1\mu_1+\delta_2\lambda_2\mu_2]T}. \tag{10}$$

In this expression, the modification is to multiply the market value S_0 by a deterministic proportionality constant which is greater than or equal to 1 under the reasonable assumption that $V_0 \geqslant S_0$. For computing risk measures, like Value at Risk, one only needs to modify the existing procedure for computing the market value of the portfolio S_t by a proportionality factor, obtained by running three independent Poisson processes $\{N_0(t), N_1(t), N_2(t)\}$. In contrast, expression (8) requires the specification of stochastic processes for the same quantities and a more complex adjustment to the computation of the expectation operator (the integral) as illustrated in Example 1.

4. Estimation

This section discusses the estimation of the NPV and operational risk processes' parameters using market prices. It is argued below that it is conceptually possible to estimate the NPV and operational risk factor parameters using only market prices. However, from a practical perspective, except in rare cases, this conceptual possibility cannot be achieved. In contrast, the NPV and operational risk processes can more easily be estimated using both data internal to the firm and collections of internal data aggregated across many firms, using standard hazard rate estimation procedures. The estimation of the various risk premium relevant to operational risk may be estimated using techniques recently employed in the credit risk literature (see Driessen, 2005; Berndt et al., 2005). The estimation of these operational risk premium are not discussed further in this paper.

To see the validity of our assertions regarding the estimation of the NPV and operational risk processes' parameters, let us set up the preliminaries of the argument. First, the market is assumed to observe the prices of the traded assets S_t and V_t. These prices would be recorded in the financial press. We note, for subsequent usage, that V_t represents the total value of the firm's liabilities and equity. Second, the technology's NPV and operational risk factors $\pi(t)$, $\theta_1(t)$, $\theta_2(t)$, being firm specific and internal to the firm are not directly observable to the market. Consequently, the issue is whether one can infer the NPV and operational risk factors via market prices alone. We next argue that this is conceptually possible.

To understand why, consider expression (9), where market prices give us the left side of the following expression:

$$\frac{V_t}{S_t} = (1+\alpha)^{N_0(t)}(1+\delta_1)^{N_1(t)}(1+\delta_2)^{N_2(t)} \times e^{[\alpha\lambda_0\mu_0+\delta_1\lambda_1\mu_1+\delta_2\lambda_2\mu_2](T-t)}.$$

After normalizing the firm value by the market value of the underlying asset portfolio $\left(\frac{V_t}{S_t}\right)$, changes in the left side represent changes in market prices due to the NPV and operational risk processes alone. When $\frac{V_t}{S_t}$ jumps, it is due to one of the counting processes $\{N_0(t), N_1(t), N_2(t)\}$[9] changing, and the percentage change in $\frac{V_t}{S_t}$ is due to the amplitude of the relevant jump process: $\{\alpha, \delta_1, \delta_2\}$. Given a reasonable collection of time series observations of the left side, it should be possible using standard statistical procedures (e.g., maximum likelihood estimation) to estimate the NPV and operational risk processes' parameters.

However, there is practical problem. As noted in the empirical literature estimating the structural approach to credit risk, the firm value process V_t is, except in rare cases, not observable. This is due to the fact that not all of the firm's liabilities and equity trade in liquid markets (e.g., unfunded pension obligations, private bank loans, lines of credit, etc.). Consequently, although conceptually possible, in most cases, market prices alone are not sufficient to estimate the NPV and operational risk processes.

An alternative and perhaps more fruitful approach for estimating the NPV and operational risk processes' parameters is to use data on these processes that are available either internally to the firm or via collections of internal data obtained from many firms. Two such databases of internal data are currently available:[10] (i) the operational riskdata exchange association (ORX) database,[11] and (ii) the OpVantage database, a subsidiary of fitch risk.[12] The OpVantage database, available for a fee, includes more than 12,000 loss events recorded over the past 15 years. Included in OpVantage are the dates of the occurrence of the various operational risk events (categorized via the revised Basel II classification) and the losses that result at each occurrence. For a good discussion of the available databases see Aue and Kalkbrener (2006), p. 9.

To estimate operational risk according to the revised Basel II guidelines (Basel Committee on Banking Supervision, 2005), we need to further decompose our coarse agency and system risks into a finer partitioning as detailed in Table 1. The key determinant of the categorization scheme is whether these event risks would exist in an owner-managed firm. If so, they are system type; otherwise agency.

One needs to estimate a separate point process and loss distribution for each of these (event × business lines) risk processes – the multi-dimensional extension of our methodology. This multi-dimensional extension is straightforward. Standard statistics can then be used to obtain the estimated gain/loss rates, and standard hazard rate estimation procedures can be used to obtain the intensity processes (a good source is Fleming and Harrington, 1991). The feasibility of this estimation is documented by three recent papers, see Aue and Kalkbrener (2006), Dutta and Perry (2006), El-Gamal et al. (2006).[13]

For agency cost type operational risk events, if large enough to be publicly reported, the counting process $N_2(t)$ is observed externally to the firm. Then, observing $\frac{V_t}{S_t}$ enables one to estimate the dollar losses ($\delta_2(s)$) directly

[9] Note that two or more counting processes jumping at the same time occurs with probability zero under our structure.

[10] There is another data set known as the loss data collection exercise (LDCE) collected by US banking regulatory agencies, see Dutta and Perry (2006) for a description of this data.

[11] See www.orx.org for a description of the data and organization.

[12] See www.algorithmics.com/solutions/opvantage for a detailed description of the database.

[13] It should be noted that Aue and Kalkbrener (2006), p. 16 frequency distribution estimation is related to our suggested hazard rate procedure.

Table 1
Decomposition of operational risk event types

Event types	Business lines
Agency	Corporate finance
Internal fraud	Trading and sales
Employment practices and workplace safety	Retail banking
System	Commercial banking
External fraud	Payment and settlement
Clients, products and business practices	Agency services
Damage to physical assets	Asset management
Business disruption and system failures	Retail brokerage
Execution, delivery and process management	

without data internal to the firm. Indeed, one can condition on the time series observations of agency cost operational risk events and apply standard hazard rate estimation techniques (see Jarrow and Chava, 2004) to estimate $\lambda_2(s)$. Given the occurrence of a catastrophic event, one can then measure the dollar losses $\delta_2(s)$ using the changes in $\frac{V_t}{S_t}$. We remark that this approach still has some remaining difficulties: (i) this approach does not include estimates for the NPV and system type operational risk parameters (α, λ_0, δ_1, λ_1), (ii) nor does it include agency cost operational loses not significant enough to be reported in the financial press, and (iii) finally, this procedure still requires an estimate of the change in the firm's value when the agency cost event occurs. These remaining difficulties are easily overcome using data internal to the firm.

5. Conclusion

This paper provides an economic and mathematical characterization of operational risk. This characterization originates in the corporate finance and credit risk literature. Operational risk is of two types, either: (i) the risk of a loss due to the firm's operating technology, or (ii) the risk of a loss due to agency costs. These two types of operational risks generate loss processes with different economic characteristics, both modeled as Cox counting processes. Different parameter values differentiate the operational risk processes. We show that although it is conceptually possible to estimate the operational risk processes' parameters using only market prices, the non-observability of the firm's value makes this an unlikely possibility, except in rare cases. Instead, we argue that data internal to the firm, in conjunction with standard hazard rate estimation procedures, provides a more fruitful alternative. Finally, we show that the inclusion of operational risk into the computation of fair economic capital (as with revised Basel II) without the consideration of a firm's NPV, will provide biased (too large) capital requirements.

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References

Aue, F., Kalkbrener, M., 2006. LDA at work. Working paper, Deutsche Bank AG, Frankfurt, Germany.

Basel Committee on Banking Supervision, November 2005. International convergence of capital measurement and capital standards: A revised framework. Bank for International Settlements, Press and Communications, Basel, Switzerland.

Berndt, A., Douglas, R., Duffie, D., Ferguson, M., Schranz, D., 2005. Measuring default risk premia from default swap rates and EDFs. Working paper, Carnegie Mellon University.

Bielecki, T., Rutkowski, M., 2002. Credit Risk: Modeling, Valuation and Hedging. Springer-Verlag, New York.

Brealey, R., Myers, S., 2004. Principles of Corporate Finance, sixth ed. McGraw Hill, New York.

Bremaud, P., 1981. Point Processes and Queues: Martingale Dynamics. Springer Verlag, New York.

Chatterjea, A., Jarrow, R., Neal, R., 2003. How valuable is credit card lending. The Journal of Derivatives 11 (2), 39–52.

Chavez-Demoulin, V., Embrechts, Pl., Neslehova, J., 2006. Quantitative models for operational risk: Extremes, dependence and aggregation. Journal of Banking and Finance 30, 2635–2658.

Coleman, R., 2003. Op risk modelling for extremes. Operational Risk (January), 6–9.

de Fontnouvelle, P., Rosengren, E., Jordan, J., 2004. Implications of alternative operational risk modelling techniques. Working paper, Federal Reserve Bank of Boston.

de Fontnouvelle, P., Jordan, J., DeJesus-Ureff, V., Rosengren, E., 2005. Capital and risk: New evidence on implications of large operational losses. Working paper, Federal Reserve Bank of Boston.

Driessen, J., 2005. Is default event risk priced in corporate bonds? Review of Financial Studies 18 (1), 165–195.

Dutta, K., Perry, J., 2006. A tale of Tails: An empirical analysis of loss distribution models for estimating operational risk capital. Working paper no. 06-13, Federal Reserve Bank of Boston.

Ebnother, S., Vanini, P., McNeil, A., Antolinez-Fehr, P., 2001. Modelling operational risk. Working paper, ETH Zurich.

El-Gamal, M., Inanoglu, H., Stengel, M., 2006. Multivariate estimation for operational risk with judicious use of extreme value theory. Working paper, Office of the Comptroller of the Currency.

Embrechts, P., Puccetti, G., forthcoming. Aggregating risk capital with an application to operational risk, The Geneva Risk and Insurance Review, forthcoming.

Fleming, T., Harrington, D., 1991. Counting Processes and Survival Analysis. John Wiley and Sons, New York.

Glasserman, P., 2004. Monte Carlo Methods in Financial Engineering. Springer Verlag, New York.

Jang, J., 2004. The Laplace transform of the distribution of the cox process with shot noise intensity and its application to reinsurance and operational risk. Working paper, University of New South Wales.

Janosi, T., Jarrow, R., Zullo, F., 1999. An empirical analysis of the Jarrow van Deventer model for valuing non-maturity demand deposits. The Journal of Derivatives (Fall), 8–31.

Jarrow, R., Chava, S., 2004. Bankruptcy prediction with industry effects. Review of Finance 8 (4), 537–569.

Jarrow, R., Purnanandam, A., 2004. Capital structure and the present value of a firm's investment opportunities: A reduced form credit risk perspective. Working paper, Cornell University.

Jarrow, R., Turnbull, S., 2000. Derivative Securities, second ed. South Western College Pub., Cincinnati, Ohio.

Jarrow, R., Lando, D., Yu, F., 2005. Default risk and diversification: Theory and applications. Mathematical Finance 15 (1), 1–26.

Leippold, M., Vanini, P., 2005. The quantification of operational risk. The Journal of Risk 8 (1), 59–85, Fall.

Lindskog, F., McNeil, A., 2003. Common Poisson shock models: Applications to insurance and credit risk modelling. ASTIN Bulletin 33 (2), 209–238.

Moscadelli, M., 2004. The modelling of operational risk: Experience with the analysis of the data collected by the basel committee. Technical Report 517, Banca d'Italia.
Protter, P., 2005. Stochastic integration and differential equations, second ed. Springer-Verlag, Heidelberg, version 2.1.
Risk Books, 2003. Modern Risk Management: A History, London.
Ross, S., Westerfield, R., Jaffe, J., 2002. Corporate Finance, sixth ed. McGraw-Hill Irwin, New York.
Shreve, S., 2004. Stochastic Calculus for Finance II: Continuous Time Models. Springer Verlag, New York.