Group Seven

Number Theory Final Presentation
Spring 2020

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Introduction

Introduction

This project is meant to show how addition and multiplication interact to generate positive integers. For a positive integer n is to be the smallest number of ones required to write the number n using only addition, multiplication, and grouping symbols. Our task as a group was to learn as much about the sequence of numbers C_n

Background, Definitions, and Motivation

Outline

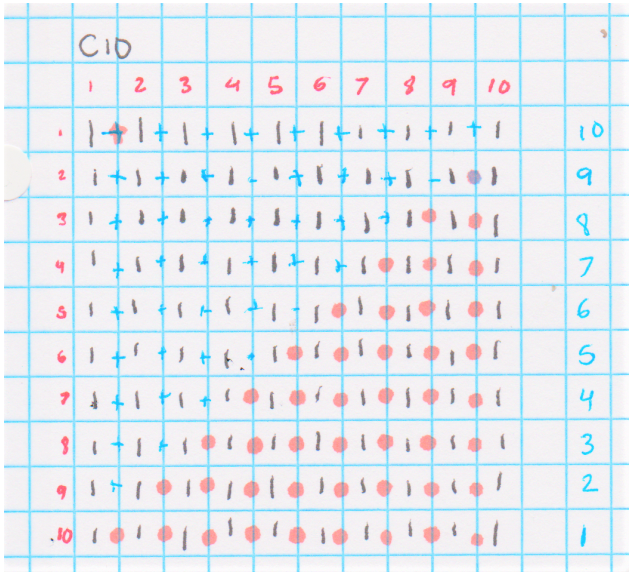
Step 1: We will begin talking about our initial findings of n amount of ones being generated with the minimal information given in project information.

Step 2: Steps about finding our basic formula needed to find the lowest amounts of ones to generate n. And trying to come up an alternative way.

Step 3: Generating code and scatter plot with our basic formula.

Week 1

To start our project, we wanted to see how many values, n amounts of ones can be generated with addition and/or multiplication



Week 1

Conclusion: The sum of n amount of ones, results in n; The product of n amount of ones, will result in one.

This approach did not include grouping, which can drastically alter the equation's value.

Week 2

We discovered that we can drastically lower the amounts of ones needed to generate n by dividing n by 2

The sum of 10 ones is equal to...

$10 = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$

The sum of 5 ones times the sum 2 ones

$10 = (1 + 1 + 1 + 1 + 1)\ \ (1 + 1)$

Week 2

This can only be applied to even numbers only since integers cannot be divided. To overcome this, we just subtract one from n to make it even.

The sum of 11 ones is equal to...

$11 = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$

The sum of 5 ones times the sum 2 ones plus one

$11 = ((1 + 1 + 1 + 1 + 1)\ \ (1 + 1)) + 1$

Week 2

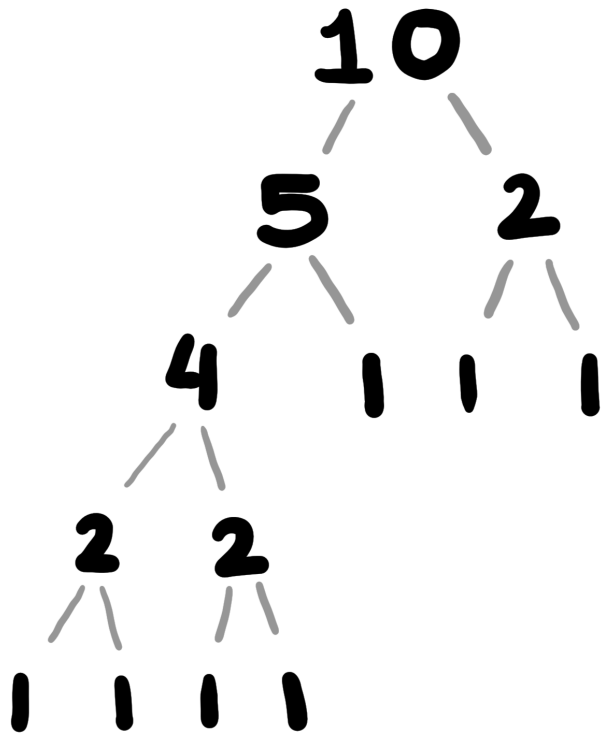
Conclusion: We found our basic formula needed to find the lowest amounts of ones to generate n

If n is even then **n / 2**

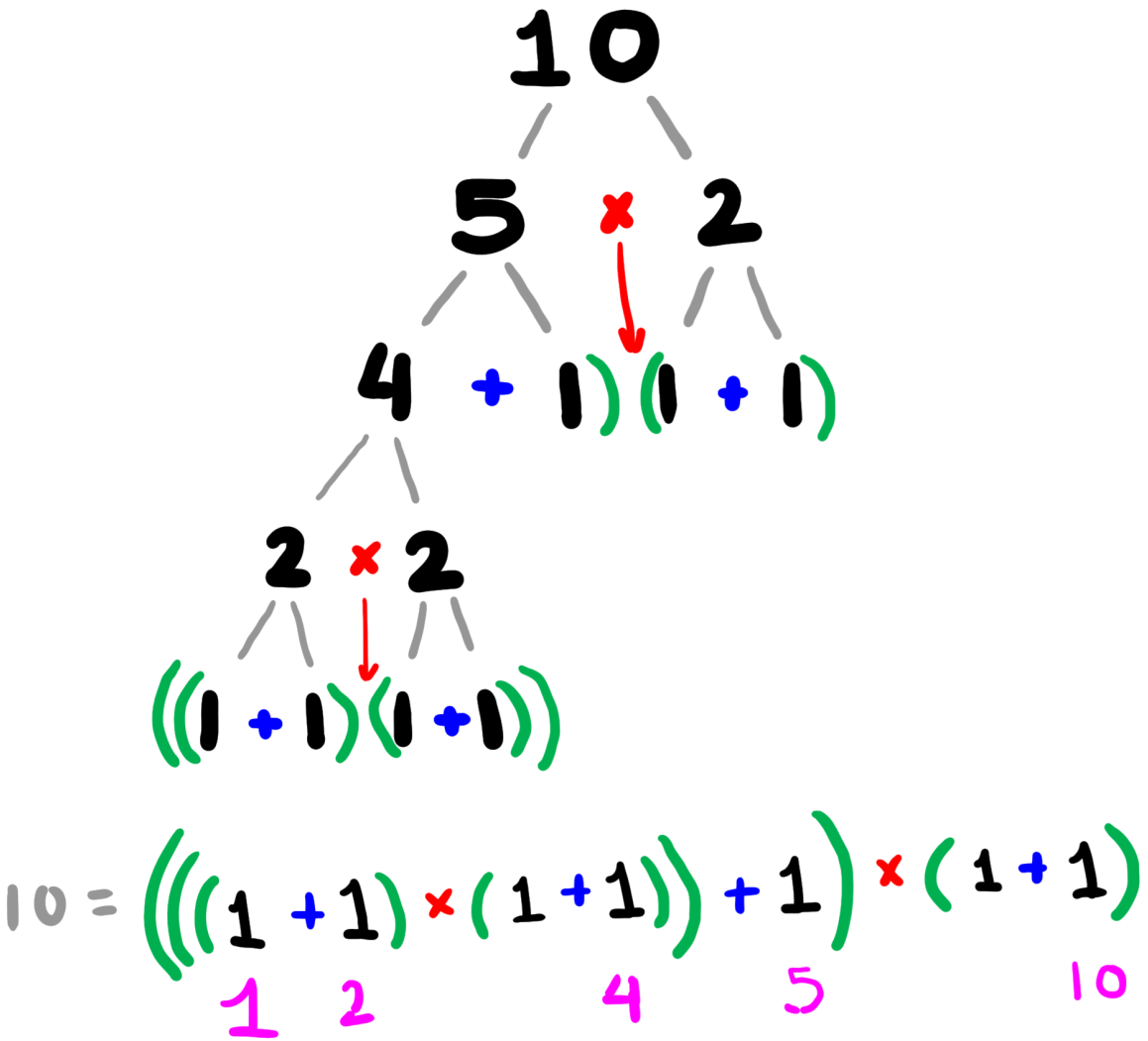
If n is odd then **(n – 1) / 2**

Week 3

We discovered, that we can apply the formula over and over. Using this strategy we created a binary tree (inspired by a factor tree).



Week 3



Week 3

```
def recursionPrinting(n, str):
    if n == 0:  # if no number, then 0
        str = "0"
        return str
    elif n == 1:  # once n has been broken down to 1
        str.append(n)
        return str
    elif (n % 2) == 0:  # is even
        str.append(n)
        n /= 2  # n/2
        return recursionPrinting(n,str)
    elif (n % 2) == 1:  # is odd
        str.append(n)
        n = (n - 1)  # (n - 1) + 2
        return recursionPrinting(n, str)
    else:
        print("Not a valid input")
```

```
>print(recursionPrinting(10,[]))

[10, 5.0, 4.0, 2.0, 1.0]
```

Week 3

```
def eqFormat(n):
    eq = ""
    for i in range(len(n)):
        if n[i] == 1: #starting point
            eq = "1"
        elif n[i] == 2: #the second step, requiered to
    keep things neat
            eq = "( " + eq + " + 1 )"
        elif (n[i] % 2) == 0: #if the number is even,
   then opposite of /2 is ( X 2 )
            eq = "( " + eq + " * ( 1 + 1 ) )"
        elif (n[i] % 2) == 1:# if the number is odd,
   the opposite of - 1 is + 1
            eq = "( " + eq + " + 1 )"
        else: #basic else statement
            print("something went wrong")
    return eq
```

```
> n = [1, 2, 4, 5, 10]
> print(eqFormat(n))

( ( ( ( 1 + 1 ) * ( 1 + 1 ) ) + 1 ) * ( 1 + 1 ) )
```

Week 3

10

```
Enter the number(enter 0 to quit): 10
C 10  <=  7

 10  =   ( ( ( 1 + 1 ) * ( 1 + 1 ) ) + 1 ) * ( 1 + 1 )
```

7

100

```
Enter the number(enter 0 to quit): 100
C 100  <=  14

 100  =   ( ( ( ( ( ( 1 + 1 ) + 1 ) * ( 1 + 1 ) ) * ( 1 + 1 ) ) * ( 1 +
1 ) ) + 1 ) * ( 1 + 1 ) ) * ( 1 + 1 )
```

14

1,000

```
Enter the number(enter 0 to quit): 1000
C 1000  <=  23

 1000  =   ( ( ( ( ( ( ( ( ( ( 1 + 1 ) + 1 ) * ( 1 + 1 ) ) + 1 ) * (
 1 + 1 ) ) + 1 ) * ( 1 + 1 ) ) + 1 ) * ( 1 + 1 ) ) * ( 1 + 1 ) ) + 1 ) * (
 1 + 1 ) ) * ( 1 + 1 ) ) * ( 1 + 1 )
```

23

5,000

```
Enter the number(enter 0 to quit): 5000
C 5000  <=  28

 5000  =   ( ( ( ( ( ( ( ( ( ( ( ( 1 + 1 ) * ( 1 + 1 ) ) * ( 1 + 1 )
 ) + 1 ) * ( 1 + 1 ) ) + 1 ) * ( 1 + 1 ) ) + 1 ) * ( 1 + 1 ) ) * ( 1 + 1 )
 ) * ( 1 + 1 ) ) * ( 1 + 1 ) ) + 1 ) * ( 1 + 1 ) ) * ( 1 + 1 ) ) * ( 1 + 1
 )
```

28

1,000,000

```
Enter the number(enter 0 to quit): 1000000
C 1000000  <=  44

 1000000  =   ( ( ( ( ( ( ( ( ( ( ( ( ( ( ( ( ( ( 1 + 1 ) + 1
) * ( 1 + 1 ) ) + 1 ) * ( 1 + 1 ) ) + 1 ) * ( 1 + 1 ) ) * ( 1 + 1 ) ) + 1
) * ( 1 + 1 ) ) * ( 1 + 1 ) ) * ( 1 + 1 ) ) * ( 1 + 1 ) ) * ( 1 + 1 ) ) +
1 ) * ( 1 + 1 ) ) * ( 1 + 1 ) ) * ( 1 + 1 ) ) + 1 ) * ( 1 + 1 ) ) * ( 1 +
1 ) ) * ( 1 + 1 ) ) * ( 1 + 1 ) ) * ( 1 + 1 ) ) * ( 1 + 1 )
```

44

Week 4

Heading in a different direction with what we've already proved in previous approach. We wanted to find a new approach finding least number of ones and see if it's more efficient. We did so by forming factor families breaking down to ones.

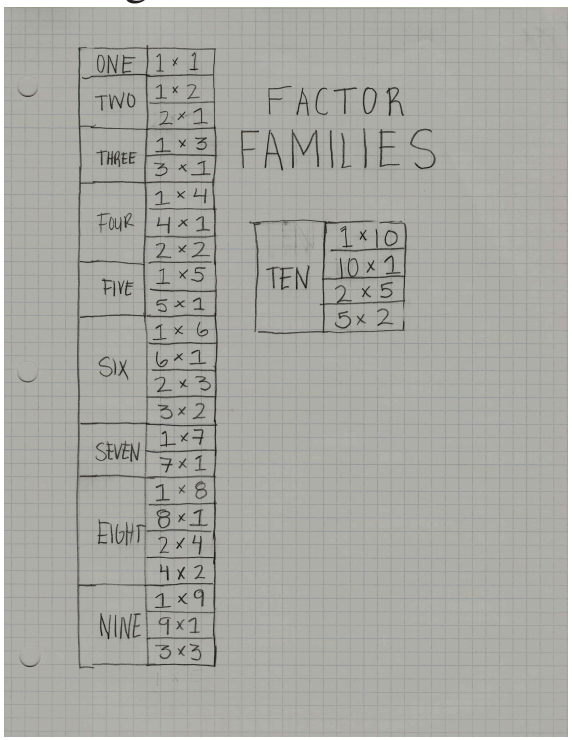


Figure: Factor Families

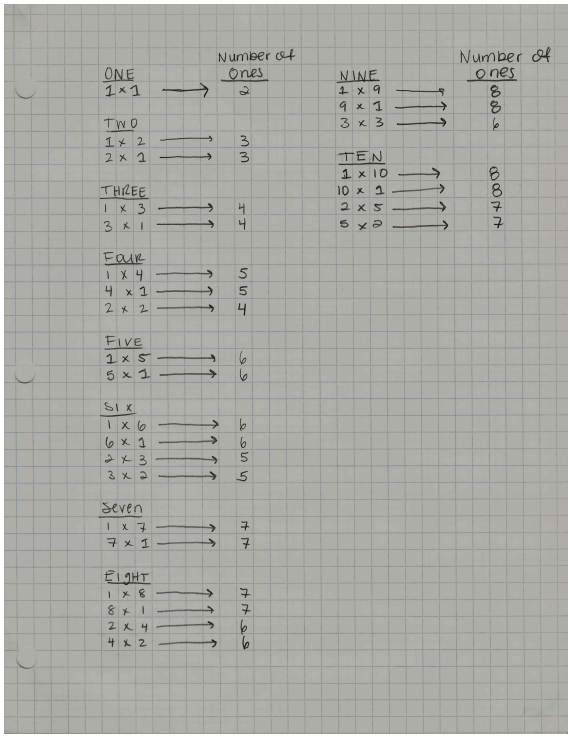


Figure: Number of Ones

Week4

We then came up with eleven ways in which we add numbers that would equal ten.

- 0 + 10
- 10 + 0
- 1 + 9
- 9 + 1
- 2 + 8
- 8 + 2
- 3 + 7
- 7 + 3
- 4 + 6
- 6 + 4
- 5 + 5

Seeing from the amount of ones each equation generated, order at this point did not matter.

Week 4

We then created steps in order to find the least number of ones.

Step 1: Use table 1 to break down the equation, use factor families corresponding to the number with the least number of ones.

Step 2: Avoid choosing the factor family where the number multiplies itself and 1, from what we saw it just generated a lot more ones.(E.g. 1*2,1*3,1*4, etc.)

Step 3: When breaking down the equation and you get 2. Don't use the factor family of (1*2), because this generates 3 ones. Instead just do (1+1), because this generates 2 ones.

Step 4: If the number is odd, use its closest lower even number plus one. For example if you have...

- 3 = 2+1
- 5 = 4+1
- 7 = 6+1
- 9 = 8+1

Week 4

Conclusion: This method was not as efficient because it would take longer for larger n values.

Week 5

With the program created, we've used it to create a scatter plot. The main function recursion() is what generates m (least amount of 1s needed to generate n).

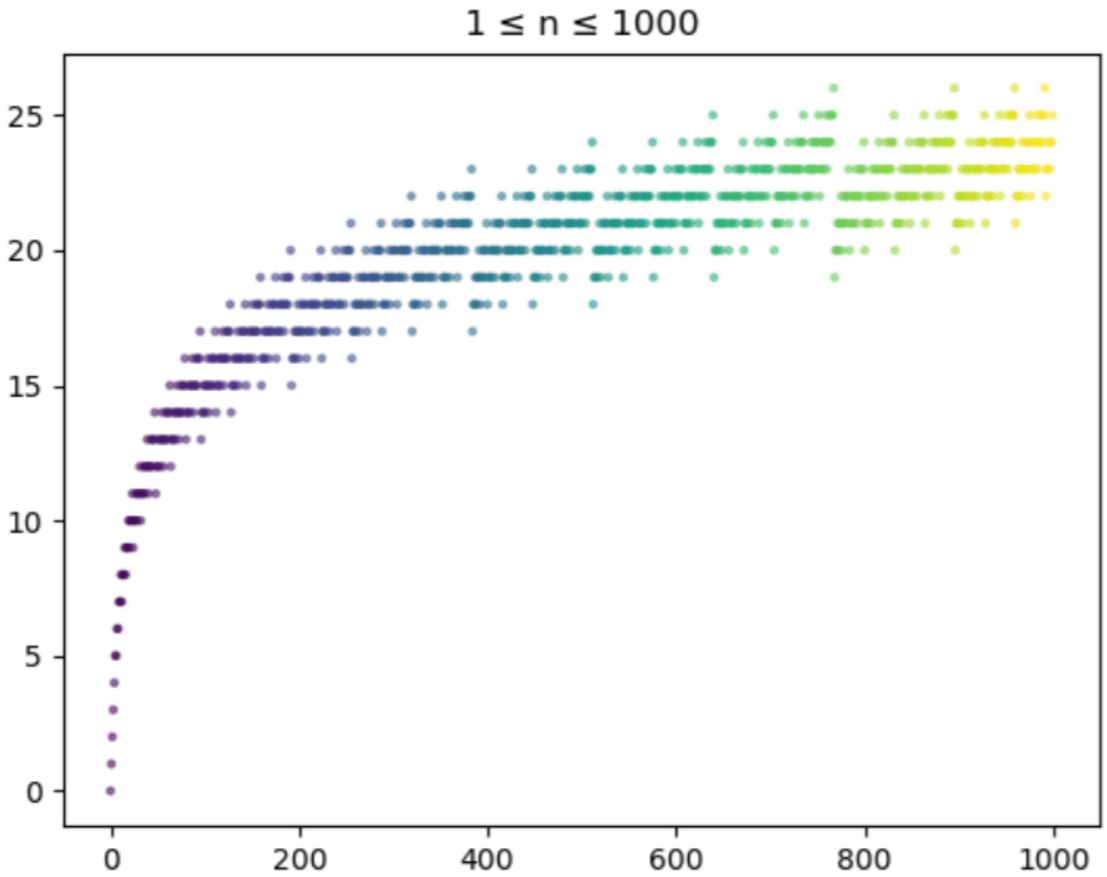
```
def recursion(n, c):
    if n == 0:  # if no number, then 0
        c += 0
        return c
    elif n == 1:  # once n has been broken down to 1
        if n > c:  # this only applies to 1 other wise
    C1 --> 0
            c += 1
        return c
    elif (n % 2) == 0:  # is even
        n /= 2  # n/2
        c += 2
        return recursion(n, c)
    elif (n % 2) == 1:  # is odd
        n = (n - 1) / 2  # (n - 1) + 2
        c += 3  # (-1) and /2 all result to the use of
    3 Ones
        return recursion(n, c)
    else:
        print("Not a valid input")
```

Week 5

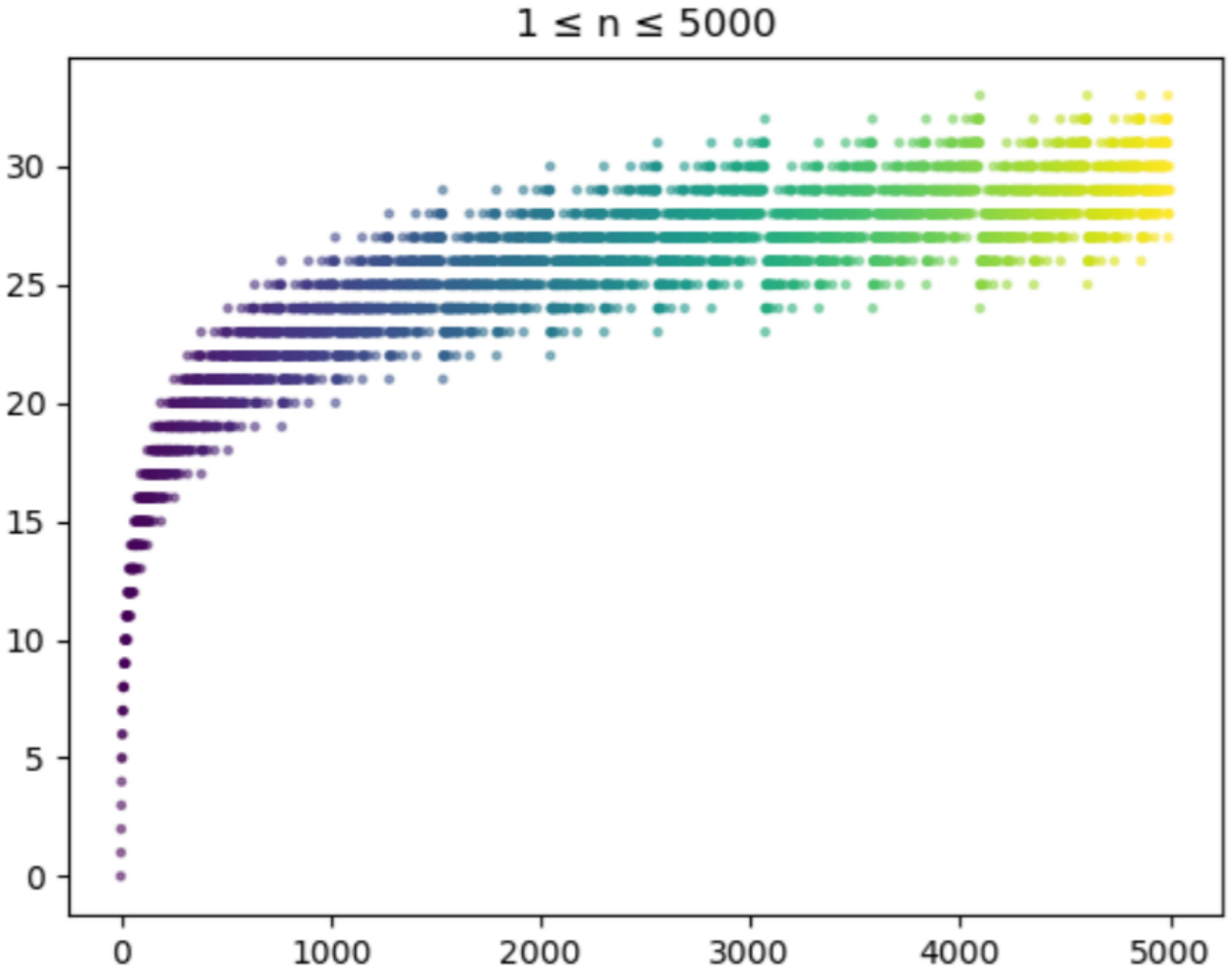
```
x = []
y = []
max = int(input("Enter Number: "))
N = max
colors = []
area = []
diction = {}

for i in range(max):
    x.append(i)
    y.append(recursion(i, 0))
    colors.append(i)
    area.append(i / 10)

title = "1      n      " + str(max)
plt.title(title)
plt.scatter(x, y, s=5, c=colors, alpha=0.5)
plt.show()
```

Week 5

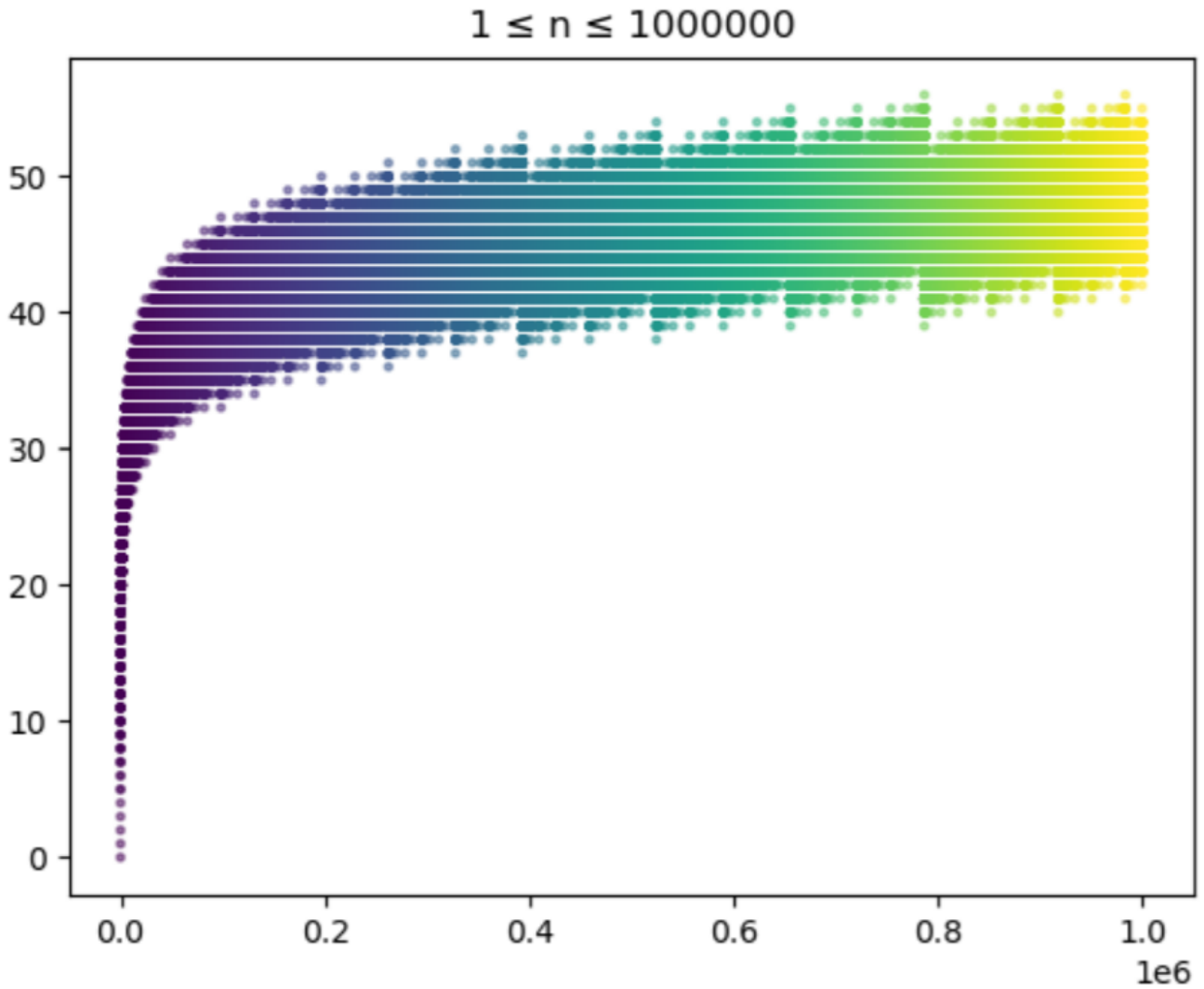
The Results



Week 5



Week 5



Summary

Summary

Week by week we found important information that led us to the end of our project, finding smallest number of ones required to write the number n using only addition, multiplication, and grouping symbols.

Week 1: The sum of n amount of ones, results in n; The product of n amount of ones, will result in one. This approach did not include grouping, which can drastically alter the equation's value.

Week 2: We found our basic formula needed to find the lowest amounts of ones to generate n.

Week 3: We discovered, that we can apply the formula over and over. Using this strategy we created a binary tree. As well as generating code using recursive function to do so.

Week 4: We found an alternative way to find the sum of n amounts of ones. But this method was not as efficient because it would take longer for larger n values.

Week 5: Used code to generate a scatter plot that would help us generate m(least amount of 1s needed to generate n).