Gama Marius Catalin Gr. 913 08/05/2020

$solve\left(\left\{x - 2 \cdot x \cdot y, \dfrac{x^2}{2} - y\right\}\right);$

$$\{x = 0, y = 0\}, \left\{x = 1, y = \dfrac{1}{2}\right\}, \left\{x = -1, y = \dfrac{1}{2}\right\} \tag{1}$$

$with(linalg) : with(DEtools) : with(VectorCalculus) :$

$Jm := Jacobian\left(\left[x - 2 \cdot x \cdot y, \dfrac{x^2}{2} - y\right], [x, y]\right);$

$$Jm := \begin{bmatrix} -2\,y + 1 & -2\,x \\ x & -1 \end{bmatrix} \tag{2}$$

$A1 := subs([x = 0, y = 0], Jm);$

$$A1 := \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{3}$$

$eigenvalues(A1);$

$$1, -1 \tag{4}$$

$A2 := subs\left(\left[x = 1, y = \dfrac{1}{2}\right], Jm\right);$

$$A2 := \begin{bmatrix} 0 & -2 \\ 1 & -1 \end{bmatrix} \tag{5}$$

$eigenvalues(A2);$

$$-\dfrac{1}{2} + \dfrac{I\sqrt{7}}{2}, \; -\dfrac{1}{2} - \dfrac{I\sqrt{7}}{2} \tag{6}$$
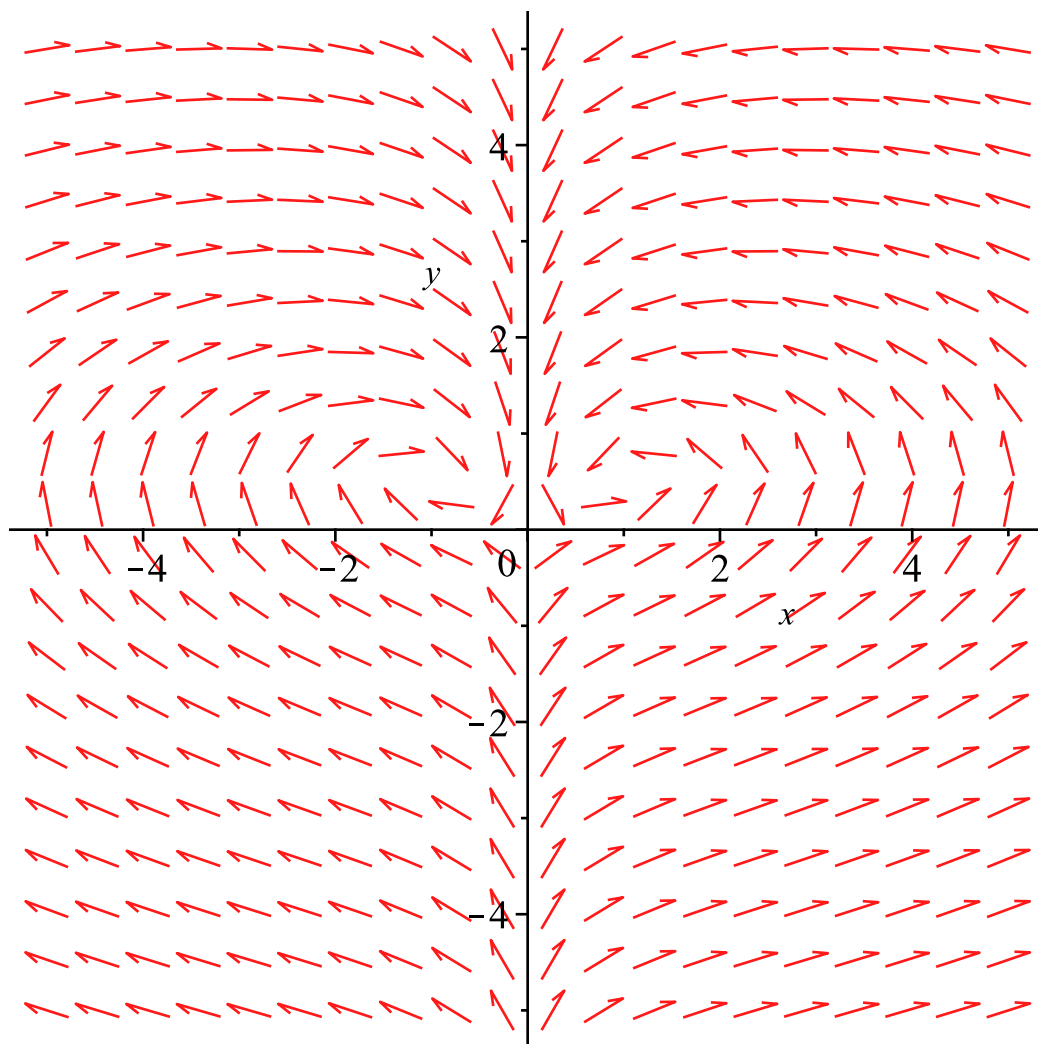
$A3 := subs\left(\left[x = -1, y = \dfrac{1}{2}\right], Jm\right);$

$$A3 := \begin{bmatrix} 0 & 2 \\ -1 & -1 \end{bmatrix} \tag{7}$$

$eigenvalues(A3);$
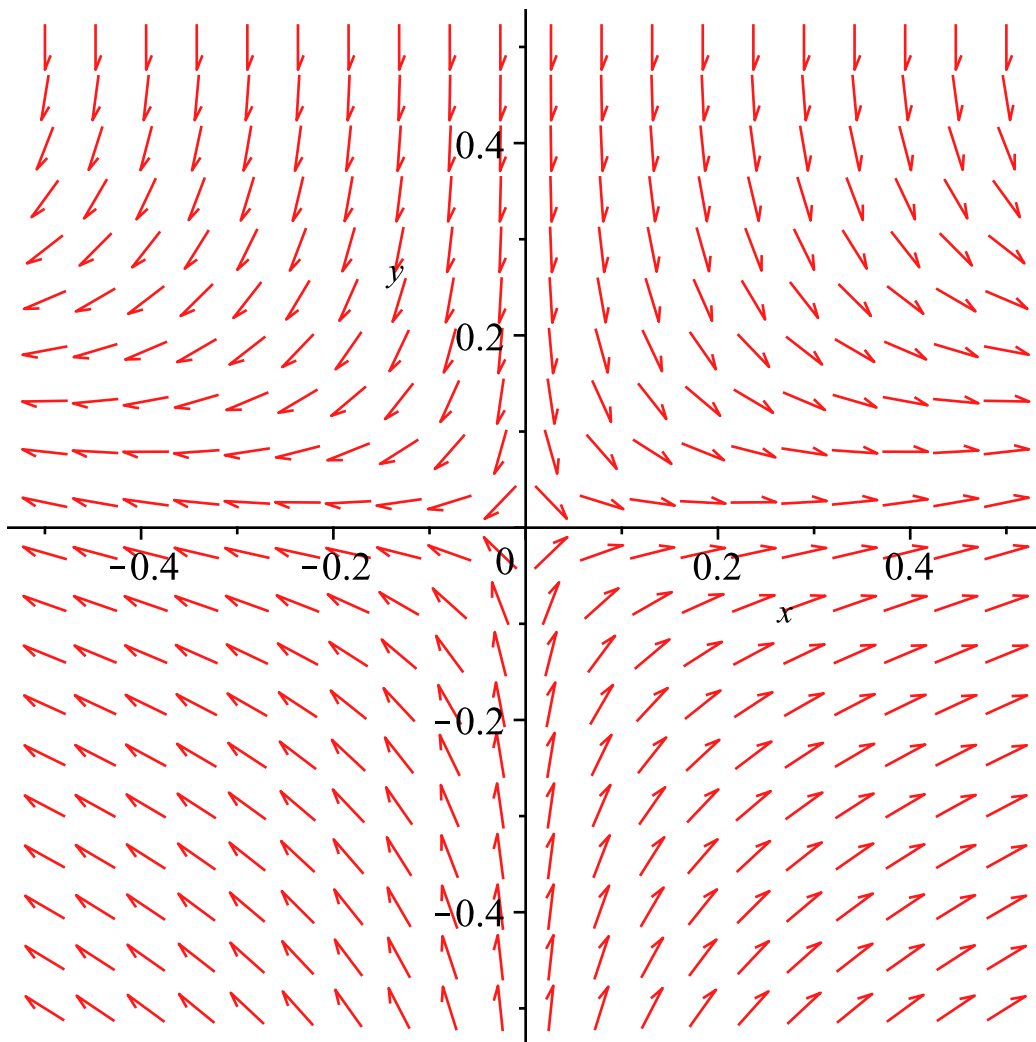
$$-\dfrac{1}{2} + \dfrac{I\sqrt{7}}{2}, \; -\dfrac{1}{2} - \dfrac{I\sqrt{7}}{2} \tag{8}$$

$dfieldplot\left(\left[diff(x(t), t) = x(t) - 2 \cdot x(t) \cdot y(t), \; diff(y(t), t) = \dfrac{x(t)^2}{2} - y(t)\right], [x(t), y(t)], t = 0..1, x = -5..5, y = -5..5\right);$
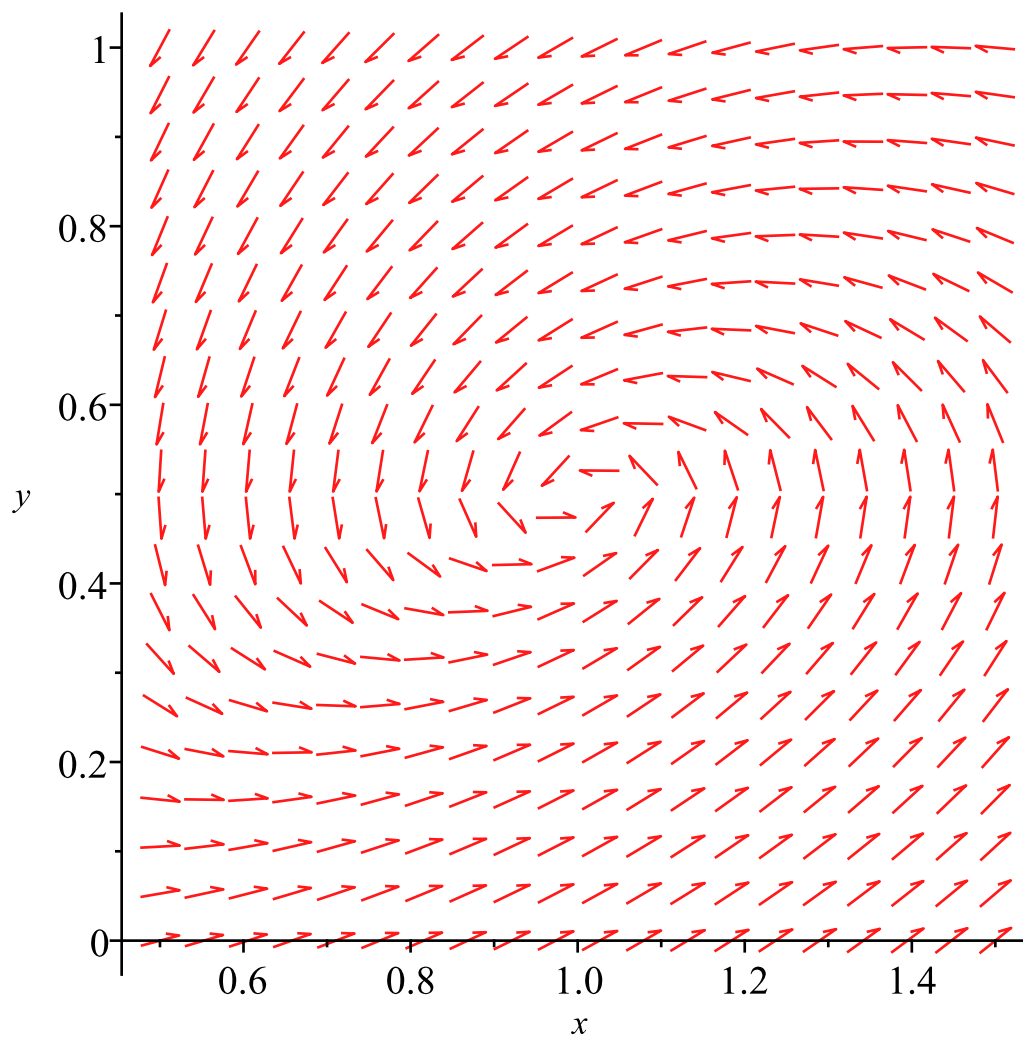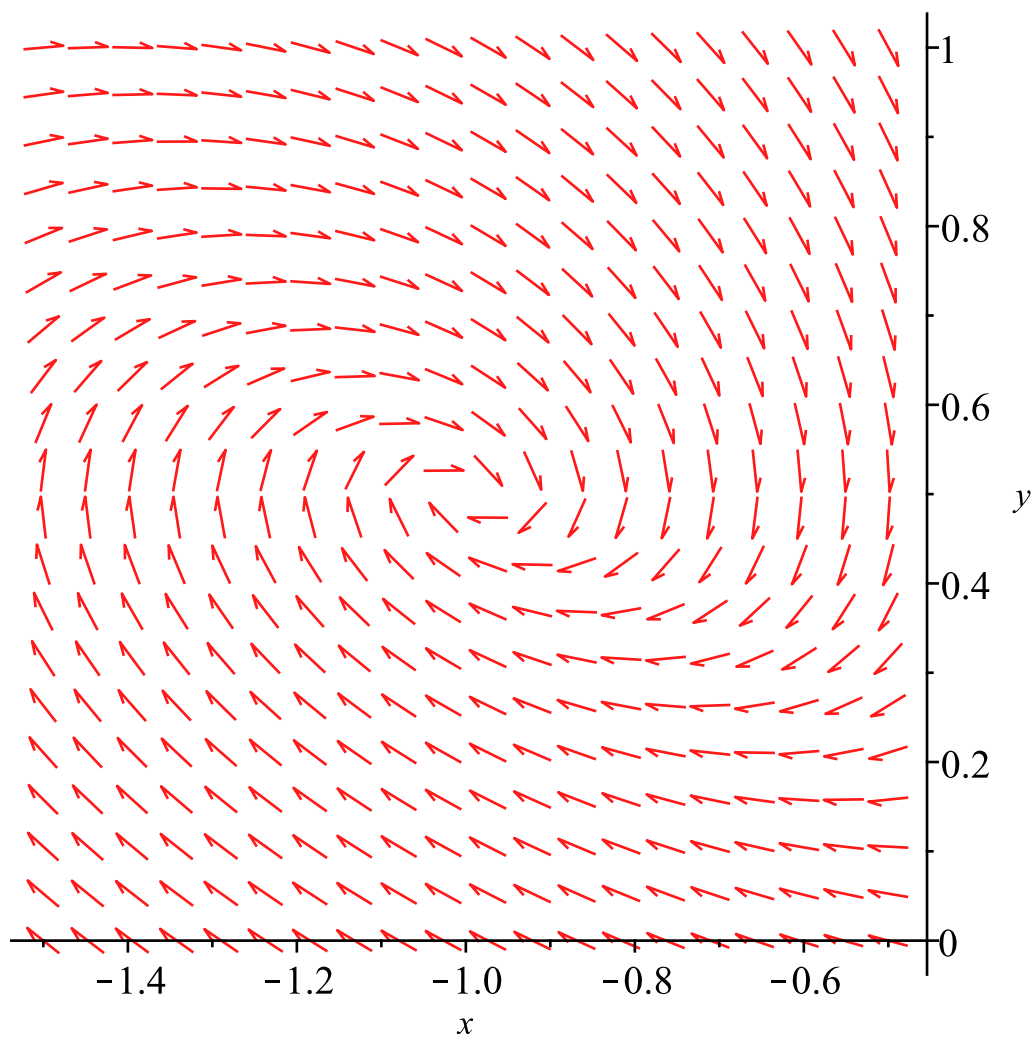
$$dfieldplot\left(\left[diff(x(t), t) = x(t) - 2 \cdot x(t) \cdot y(t), diff(y(t), t) = \frac{x(t)^2}{2} - y(t)\right], [x(t), y(t)], t = 0..1, x = \right.$$

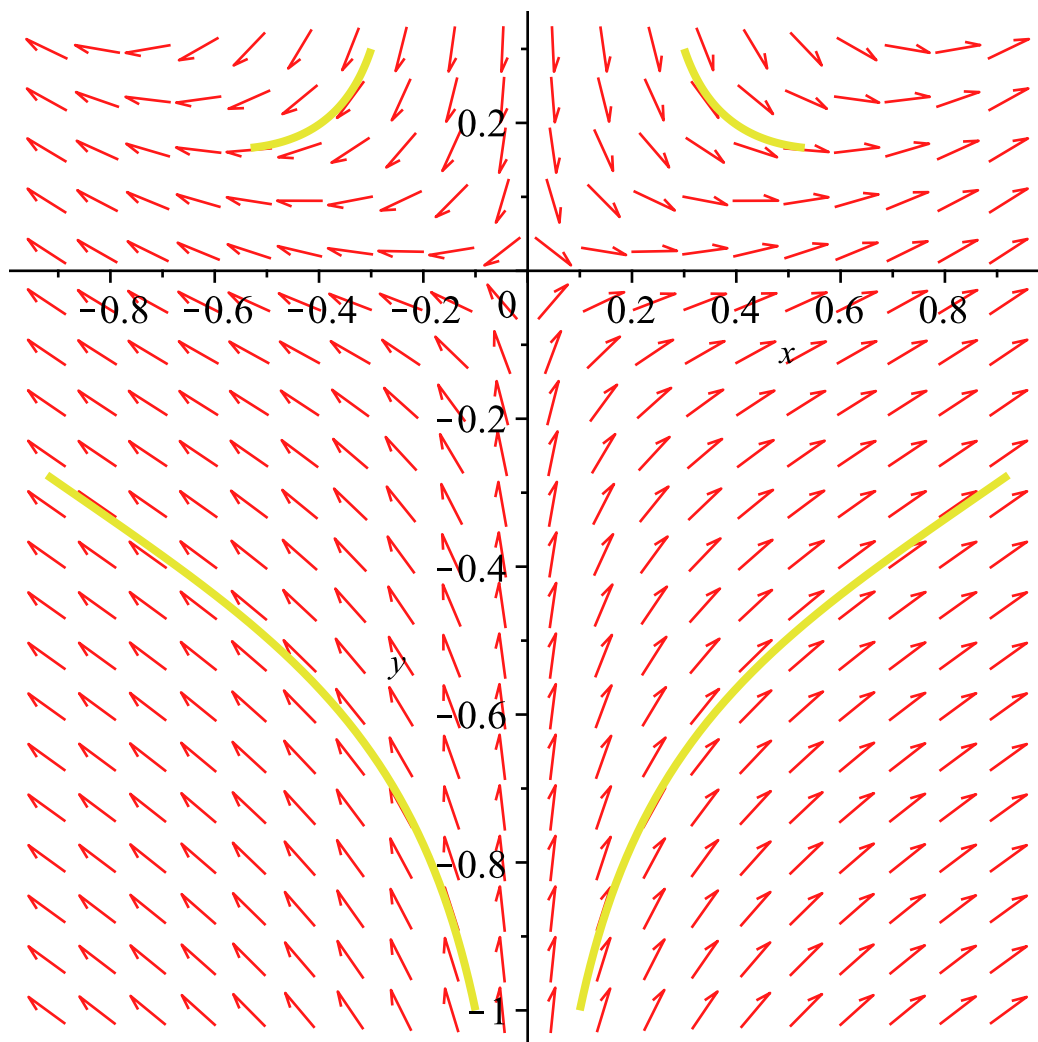$$\left. -0.5..0.5, y = -0.5..0.5\right);$$

$$dfieldplot\left(\left[diff\left(x(t), t\right) = x(t) - 2 \cdot x(t) \cdot y(t), diff\left(y(t), t\right) = \frac{x(t)^2}{2} - y(t)\right], [x(t), y(t)], t = 0\,..1, x\right.$$
$$\left. = 0.5\,..1.5, y = 0\,..1\right);$$

$dfieldplot\left(\left[diff\left(x(t),t\right)=x(t)-2\cdot x(t)\cdot y(t), diff\left(y(t),t\right)=\dfrac{x(t)^2}{2}-y(t)\right], [x(t),y(t)], t=0..1, x=\right.$

$\left.-1.5..-0.5, y=0..1\right);$

$$DEplot\left(\left[diff\left(x(t),t\right)=x(t)-2\cdot x(t)\cdot y(t), diff\left(y(t),t\right)=\frac{x(t)^2}{2}-y(t)\right], [x(t),y(t)], t=0..1,\right.$$
$$\left[[x(0)=0.1, y(0)=1], [x(0)=-0.1, y(0)=1], [x(0)=0.3, y(0)=0.3], [x(0)=-0.3, y(0)\right.$$
$$=0.3]]);$$

GAMA MARIUS CATALIN   GR. 913   08/05/2020

I

$$\begin{cases} \dot{x} = x - 2xy \\ \dot{y} = \dfrac{x^2}{2} - y \end{cases}$$

a) $\begin{cases} x - 2xy = 0 \\ \dfrac{x^2}{2} - y = 0 \end{cases}$ $(\Rightarrow)$ $\dfrac{x^2}{2} = y \Leftrightarrow x^2 = 2y$ $\Big| \Rightarrow x - x^3 = 0 \Rightarrow$

$$\Rightarrow x(x^2 - 1) = 0 \Rightarrow$$

$$\Rightarrow x_1 = 0 \quad x_{2,3} = \pm 1$$

~~x = 0 ⇒ x² = 2y~~

$\begin{aligned} x = 0 \\ x^2 = 2y \end{aligned} \Big| \Rightarrow y = 0 \Rightarrow eq_1 \cdot (0,0)$

$\begin{aligned} x = 1 \\ x^2 = 2y \end{aligned} \Big| \Rightarrow y = \dfrac{1}{2} \Rightarrow eq_2 \left(1, \dfrac{1}{2}\right)$

$\begin{aligned} x = -1 \\ x^2 = 2y \end{aligned} \Big| \Rightarrow y = \dfrac{1}{2} \Rightarrow eq_3 \left(-1, \dfrac{1}{2}\right)$

b)

$$J_f(x,y) = \begin{pmatrix} \frac{df_1}{dx} & \frac{df_1}{dy} \\ \frac{df_2}{dx} & \frac{df_2}{dy} \end{pmatrix} = \begin{pmatrix} 2y+1 & -2x \\ x & -1 \end{pmatrix}$$

$$A_1 = J_f(0,0) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Rightarrow \lambda_1 = 1, \lambda_2 = -1$$

$$A_2 = J_f(1,\tfrac{1}{2}) = \begin{pmatrix} 0 & -2 \\ 1 & -1 \end{pmatrix} \Rightarrow \lambda_1 = -\frac{1}{2} + \frac{i\sqrt{7}}{2}, \lambda_2 = -\frac{1}{2} - \frac{i\sqrt{7}}{2}$$

$$A_3 = J_f(-1,\tfrac{1}{2}) = \begin{pmatrix} 0 & 2 \\ -1 & -1 \end{pmatrix} \Rightarrow \lambda_1 = -\frac{1}{2} + \frac{i\sqrt{7}}{2}, \lambda_2 = -\frac{1}{2} - \frac{i\sqrt{7}}{2}$$

c)