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Assignment 6

Q1 $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

$x_1, x_2, x_3 \ldots x_n \rightarrow$ Sample of size n

$L(x_1, x_2, \ldots x_n) = f(x_1).f(x_2) \ldots f(x_n)$

$\Rightarrow \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x_1-\mu)^2}{2\sigma^2}} \times \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x_2-\mu)^2}{2\sigma^2}} \ldots$

Taking ln on both sides

$\ln(L) = \frac{-n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \left(\frac{(x_i-\mu)^2}{2\sigma^2}\right) \quad -①$

Take partial derivative wrt μ of above eqn

$\frac{d \ln(L)}{d\mu} = 0 + \sum_{i=1}^{n} -\left(\frac{2(x_i-\mu)}{2\sigma^2}\right) = 0$

$= \sum_{i=1}^{n} (x_i - \mu) = 0$

$n\bar{x} - n\mu = 0$

Hence $\theta_1 = \bar{x}$ is sample mean

Taking derivative wrt σ^2 (of eq ①)

$\frac{d \ln(L)}{d\sigma^2} = \frac{-n}{2\sigma^2} + \sum_{i=1}^{n} \frac{-(x_i-\mu)^2}{2\sigma^2} = 0$

$\Rightarrow -n + \sum_{i=1}^{n} \frac{-(x_i-\mu)^2}{\sigma^2} = 0$

$\Rightarrow n = \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{\sigma^2} \Rightarrow \sigma^2 = \frac{1}{n}\left(\sum_{i=1}^{n} (x_i-\mu)^2\right)$

$\Rightarrow \theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i-\mu)^2$

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Q2 Binomial distribution $\Rightarrow {}^nC_{x_i}\,\theta^{x_i}(1-\theta)^{n-x}$

$$L = \prod_{i=1}^{n} {}^nC_x\,\theta^{x_i}(1-\theta)^{n-x_i}$$

log on both sides

$$\log L = \sum_{i=1}^{n}\left(\log({}^nC_{x_i}) + \log\theta^{x_i} + \log(1-\theta)^{n-x_i}\right)$$

$$= \sum_{i=1}^{n}\log({}^nC_{x_i}) + \log\theta\sum_{i=1}^{n}x_i + \log(1-\theta)\sum_{i=1}^{n}(n-x_i)$$

Differentiate wrt θ

$$\frac{d\log(L)}{d\theta} = 0$$

$$\frac{1}{\theta}\sum x_i - \frac{1}{1-\theta}\sum(n-x_i) = 0$$

$$\frac{1}{\theta}\sum x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta}\sum x_i = 0$$

$$\frac{1}{\theta(1-\theta)}\sum x_i = \frac{n^2}{1-\theta} \Rightarrow \boxed{\theta = \frac{\sum x_i}{n^2}}$$