Compiler

Exercises for Chapter 2

Exercises for Section 2.2

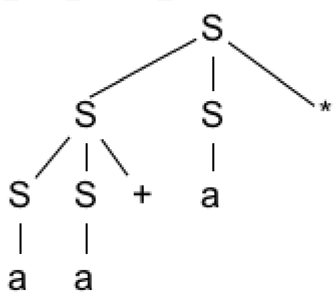
2.2.a

Consider the context-free grammar:

S -> S S + | S S * | a

1. Show how the string `aa+a*` can be generated by this grammar.
2. Construct a parse tree for this string.
3. What language does this grammar generate? Justify your answer.

Answer

1. `S` -> `S` S -> `S` *S* + *S* -> a `S` + S -> *a a* + `S` -> a a + a *
2. 

3. L = {Postfix expression consisting of digits, plus and multiple signs}

2.2.b

What language is generated by the following grammars? In each case justify your answer.

1. S -> 0 S 1 | 0 1
2. S -> + S S | - S S | a
3. S -> S (S) S | ε
4. S -> a S b S | b S a S | ε
5. S -> a | S + S | S S | S * | (S)

Answer

1. L = {0^n1^n | n>=1}
2. L = {Prefix expression consisting of plus and minus signs}
3. L = {Matched brackets of arbitrary arrangement and nesting, includes ε}

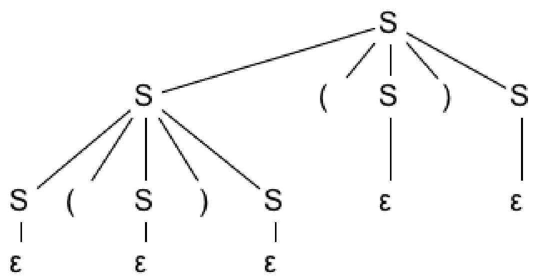
4. L = {String has the same amount of a and b, includes ε}
5. L = {Regular expressions used to describe regular languages}

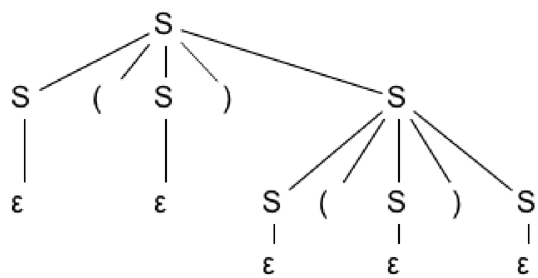
2.2.c

Which of the grammars in Exercise 2.2.2 are ambiguous?

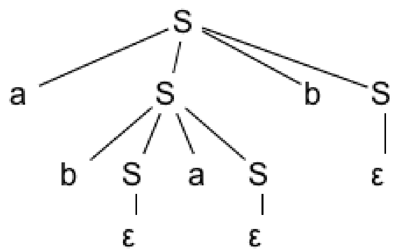
Answer

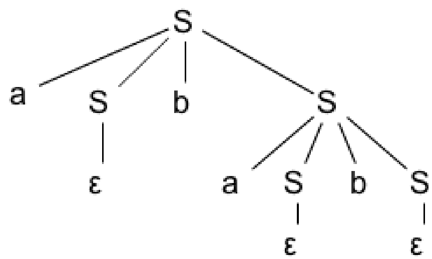
1. No
2. No
3. Yes



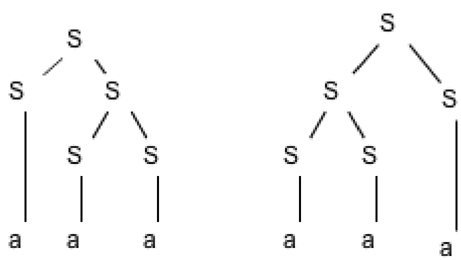


4. Yes





5. Yes



2.2.d

Construct unambiguous context-free grammars for each of the following languages.
In each case show that your grammar is correct.

1. Arithmetic expressions in postfix notation.
2. Left-associative lists of identifiers separated by commas.
3. Right-associative lists of identifiers separated by commas.
4. Arithmetic expressions of integers and identifiers with the four binary operators +,
   -, *, /.
5. Add unary plus and minus to the arithmetic operators of 4.

Answer

```
1.    E -> E E op | num

2.    list -> list , id | id
3.    list -> id , list | id

4.    expr -> expr + term | expr - term | term
term -> term * factor | term / factor | factor
   factor -> id | num | (expr)

5.    expr -> expr + term | expr - term | term
term -> term * unary | term / unary | unary
unary -> + factor | - factor | factor    factor -
> id | num | (expr)
```

2.2.e

1. Show that all binary strings generated by the following grammar have values divisible by 3. Hint. Use induction on the number of nodes in a parse tree.

   num -> 11 | 1001 | num 0 | num num

2. Does the grammar generate all binary strings with values divisible by 3?

Answer

1. Proof

   Any string derived from the grammar can be considered to be a sequence consisting of 11 and 1001, where each sequence element is possibly suffixed with a 0.

   Let `n` be the set of positions where `11` is placed. `11` is said to be at position `i` if the first `1` in `11` is at position `i`, where `i` starts at 0 and grows from least significant to most significant bit.
   Let `m` be the equivalent set for `1001`.
   The sum of any string produced by the grammar is:

   sum

   $= \Sigma_n (2^1 + 2^0) 2^n + \Sigma_m (2^3 + 2^0) 2^m$

   $= \Sigma_n 3\, 2^n + \Sigma_m 9\, 2^m$

   This is clearly divisible by 3.

1. No. Consider the string "10101", which is divisible by 3, but cannot be derived from the grammar.

Readers seeking a more formal proof can read about it below:

Proof:

Every number divisible by 3 can be written in the form `3k`. We will consider `k > 0` (though it would be valid to consider `k` to be an arbitrary integer).
Note that every part of num(11, 1001 and 0) is divisible by 3, if the grammar could generate all the numbers divisible by 3, we can get a production for binary

```
 3k = num   -> 11 | 1001 | num 0 | num num
k = num/3 -> 01 | 0011 | k 0   | k k   k
-> 01 | 0011 | k 0   | k k
```

It is obvious that any value of that has more than 2 consecutive bits set to 1 can k from num's production:

never be produced. This can be confirmed by the example given in the beginning:
10101 is 3*7, hence, k = 7 = 111 in binary. Because 111 has more than 2 consecutive 1's in binary, the grammar will never produce 21.

2.2.6

Construct a context-free grammar for roman numerals.
Note: we just consider a subset of roman numerals which is less than 4k.

Answer

wikipedia: Roman_numerals

- via wikipedia, we can categorize the single roman numerals into 4 groups:

```
  I, II, III | I V | V, V I, V II, V III | I X
```

then get the production:

```
  digit -> smallDigit | I V | V smallDigit | I X
smallDigit -> I | II | III | ε
```

- and we can find a simple way to map roman to arabic numerals. For example:
    - XII => X, II => 10 + 2 => 12
    - CXCIX => C, XC, IX => 100 + 90 + 9 => 199
    - MDCCCLXXX => M, DCCC, LXXX => 1000 + 800 + 80 => 1880
  - via the upper two rules, we can derive the production:

romanNum -> thousand hundred ten digit thousand

-> M | MM | MMM | ε

hundred -> smallHundred | C D | D smallHundred | C

M smallHundred -> C | CC | CCC | ε ten -> smallTen |

X L | L smallTen | X C smallTen -> X | XX | XXX | ε

digit -> smallDigit | I V | V smallDigit | I X smallDigit -> I

| II | III | ε

Exercises for Section 2.3

2.3.a

Construct a syntax-directed translation scheme that translates arithmetic expressions from infix notation into prefix notation in which an operator appears before its operands; e.g. , -xy is the prefix notation for x - y. Give annotated parse trees for the inputs 9-5+2 and 9-5*2.

Answer

productions:

```
expr -> expr + term
| expr - term       |
term term -> term *
factor       | term /
factor       | factor
factor -> digit | (expr)
```

translation schemes:

```
expr -> {print("+")} expr + term
| {print("-")} expr - term
      | term
term -> {print("*")} term * factor
| {print("/")} term / factor
      | factor
factor -> digit {print(digit)}
        | (expr)
```

2.3.b

Construct a syntax-directed translation scheme that translates arithmetic expressions from postfix notation into infix notation. Give annotated parse trees for the inputs 952 *and 952-*.

Answer

productions:

```
expr -> expr expr +
| expr expr -
      | expr expr *
      | expr expr /
      | digit
```

translation schemes:

```
expr -> expr {print("+")} expr +
| expr {print("-")} expr -
      | {print("(")} expr {print(")*(")} expr {print(")")} *
      | {print("(")} expr {print(")/(")} expr {print(")")} /
| digit {print(digit)}
```

Another reference answer

```
E -> {print("(")} E {print(op)} E {print(")"}} op | digit {print(digit)}
```

2.3.c

Construct a syntax-directed translation scheme that translates integers into roman numerals.

Answer

assistant function:

```
repeat(sign, times) // repeat('a',2) = 'aa'
```

translation schemes:

```
num -> thousand hundred ten digit
       { num.roman = thousand.roman || hundred.roman || ten.roman || digit.roman;
print(num.roman)}
thousand -> low {thousand.roman = repeat('M', low.v)} hundred ->
low {hundred.roman = repeat('C', low.v)}
         | 4 {hundred.roman = 'CD'}
         | high {hundred.roman = 'D' || repeat('X', high.v - 5)}
         | 9 {hundred.roman = 'CM'}
ten -> low {ten.roman = repeat('X', low.v)}
     | 4 {ten.roman = 'XL'}
     | high {ten.roman = 'L' || repeat('X', high.v - 5)}
     | 9 {ten.roman = 'XC'}
digit -> low {digit.roman = repeat('I', low.v)}
       | 4 {digit.roman = 'IV'}
       | high {digit.roman = 'V' || repeat('I', high.v - 5)}
       | 9 {digit.roman = 'IX'}
low -> 0 {low.v = 0}      | 1
{low.v = 1}
     | 2 {low.v = 2}
| 3 {low.v = 3} high -> 5
```

```
{high.v = 5}       | 6
{high.v = 6}
      | 7 {high.v = 7}
      | 8 {high.v = 8}
```

2.3.d

Construct a syntax-directed translation scheme that trans lates roman numerals into integers.

Answer

productions:

```
romanNum -> thousand hundred ten digit thousand
-> M | MM | MMM | ε
hundred -> smallHundred | C D | D smallHundred | C M
smallHundred -> C | CC | CCC | ε ten -> smallTen | X
L | L smallTen | X C
smallTen -> X | XX | XXX  | ε
digit -> smallDigit | I V | V smallDigit | I X
smallDigit -> I | II | III | ε
```

translation schemes:

```
romanNum -> thousand hundred ten digit {romanNum.v = thousand.v || hundred.v || ten.v
|| digit.v; print(romanNun.v)}
thousand -> M {thousand.v = 1}
| MM {thousand.v = 2}
          | MMM {thousand.v = 3}
| ε {thousand.v = 0}
hundred -> smallHundred {hundred.v = smallHundred.v}
         | C D {hundred.v = smallHundred.v}
         | D smallHundred {hundred.v = 5 + smallHundred.v}
         | C M {hundred.v = 9}
smallHundred -> C {smallHundred.v = 1}
| CC {smallHundred.v = 2}
              | CCC {smallHundred.v = 3}
| ε {hundred.v = 0} ten -> smallTen
{ten.v = smallTen.v}
     | X L  {ten.v = 4}
     | L smallTen  {ten.v = 5 + smallTen.v}
     | X C  {ten.v = 9} smallTen
-> X {smallTen.v = 1}           |
XX {smallTen.v = 2}
          | XXX {smallTen.v = 3}
| ε {smallTen.v = 0}
digit -> smallDigit {digit.v = smallDigit.v}
       | I V  {digit.v = 4}
       | V smallDigit  {digit.v = 5 + smallDigit.v}
       | I X  {digit.v = 9}
smallDigit -> I {smallDigit.v = 1}
| II {smallDigit.v = 2}
            | III {smallDigit.v = 3}
```

```
                    | ε {smallDigit.v = 0}
```

2.3.e

Construct a syntax-directed translation scheme that translates postfix arithmetic expressions into equivalent prefix arithmetic expressions.

Answer

production:

```
expr -> expr expr op | digit
```

translation scheme:

```
expr -> {print(op)} expr expr op | digit {print(digit)}
```

Exercises for Section 2.4

2.4.a

Construct recursive-descent parsers, starting with the following grammars:

1. S -> + S S | - S S | a
2. S -> S (S) S | ε
3. S -> 0 S 1 | 0 1

Answer

See 2.4.1.1.c, 2.4.1.2.c, and 2.4.1.3.c for real implementations in C.

1） S -> + S S | - S S | a

```
    lookahead = nextTerminal();
  }else{
    throw new SyntaxException()
  }
}
```

```
void S(){
  switch(lookahead){
case "+":
match("+"); S(); S();
break;     case "-":
match("-"); S(); S();
break;     case "a":
match("a");       break;
default:
      throw new SyntaxException();
  } } void
match(Terminal t){
if(lookahead = t){
```

2） S -> S (S) S | ε

```
void S(){
  if(lookahead == "("){
    match("("); S(); match(")"); S();
  }
}
```

3） S -> 0 S 1 | 0 1

```
void S(){
  switch(lookahead){     case "0":
match("0"); S(); match("1");
break;     case "1":
      // match(epsilon);
break;     default:       throw
new SyntaxException();
  }
}
```