VOLUME, SHAPE, AND ROUNDNESS OF QUARTZ PARTICLES

HAKON WADELL
University of Chicago

ABSTRACT

The article deals with methods of measuring the volume, shape, and roundness of sedimentary quartz particles.

INTRODUCTION

The important properties of sedimentary rock fragments are: (1) the mineralogical and chemical composition; (2) the specific gravity; (3) the volume size or the size of the true nominal diameter; (4) the sedimentological shape and sometimes the crystallographic form; and (5) the roundness of the corners and edges. The regional character of a sediment is indicated by the mineralogical composition and the size of the fragments. The presence of rare and "heavy" minerals may give a clue to the source of the particles, or may be used for correlation of sediments within a given region. The volume size, the specific gravity, and the sedimentological shape are the principal factors governing transportation and deposition, so far as the fragments themselves are concerned. The roundness of the corners and edges may indicate the rigor[1] of the last stage of transportation. Increasing rigor increases fracturing and chipping and reduces the roundness of the corners in general. Rounding of sedimentary fragments is a special type of disintegration attributed to attrition and sometimes to solution. High degree of roundness is often an indication of gentle conditions of wear relative to size, hardness, and toughness of the fragment. Rounding frequently takes place in the bed-load material under gentle tractional transportation. Solution processes in deposited sediments and disintegrated rocks sometimes lead to a relatively higher degree of roundness of the smaller particles

[1] C. K. Wentworth introduced "rigor" as a term for the violence of transportation, in "A Field Study of the Shapes of River Pebbles," *U.S. Geol. Surv. Bull.* 730C (1922).

than of the larger ones, which is generally the reverse of what is found in worn sands.[2]

Most of the formulas and terms used in this article have been described and defined in previous papers.[3]

OUTLINE OF THE ANALYSIS

The present research is an attempt to obtain data on the important properties of sedimentary rock fragments, thereby making it possible to group the particles at will on the basis of any specified property. A classification may be made on the basis of the mineralogical composition and the specific gravity of the fragments; each of the obtained classes may be subdivided into volume-classes of proper intervals, and each of the subclasses divided into classes for shape and roundness. By this method, or some variation of it fitted to the purpose of the research, results are obtained which may furnish a clue to the origin of the sediment and to the processes involved in transportation and deposition.

The steps involved in the suggested analysis are illustrated in Table I. The sizes of the class intervals are tentative. The specific gravity classes are suited to the most convenient limits for centrifuge–heavy-liquid separation of quartz from the small amount of non-quartz particles present in St. Peter sandstone.[4] Owing to the

[2] H. C. Sorby, "On the Structure and Origin of Non-calcareous Stratified Rocks," *Quart. Jour. Geol. Soc.*, Vol. XXXVI (1880), Proc., pp. 48–92.

[3] Hakon Wadell, "Volume, Shape and Roundness of Rock Particles," *Jour. Geol.*, Vol. XL (1932), pp. 443–51; "Sphericity and Roundness of Rock Particles," *Ibid.*, Vol. XLI (1933), pp. 310–31; "The Coefficient of Resistance as a Function of Reynolds Number for Solids of Various Shapes," *Jour. Franklin Institute*, Vol. CCXVII (1934), pp. 459–90; "Shape Determinations of Large Sedimental Rock-Fragments," *Pan-American Geologist*, Vol. LXI (1934), pp. 187–220 (Unfortunately several misprints occur in this paper. The integration symbol, $\int$, is used throughout the paper instead of f as symbol for function. Equation (8) on page 203 should read: $\Psi = (d_n/D_s) + 0.1$.); "Some New Sedimentation Formulas," *Physics*, Vol. V (1934), pp. 281–91.

[4] The presence of non-quartz minerals, the specific gravity of which overlaps that of quartz within the class limits given in Table I, does not impair the final result, although the centrifuge–heavy-liquid separation of quartz from the small amount of non-quartz particles of about the same specific gravity may not be satisfactorily accomplished. Only quartz particles were reproduced in the subsequent drawing of the grains under the microscope.

fact that practically nothing is known in respect to the influence of the grain shape on the result of the sifting process, it has been difficult to decide upon the size of the volume-classes. Although an attempt has been made to reduce the "error" due to the particle shape, it is not yet certain how far this correction has been accomplished. The aim has been to obtain purely dimensional size values, without the influence of shape. The volume-classes should not be too large, yet large enough to reduce the influence of the grain shape to a minimum. The number of particles improperly classified (owing to the influence of shape on size determination) has less influence on the structure of the statistical particle distribution when large volume-classes are used, because the number of fragments correctly allotted to a class of given limits increases with increasing size of the class intervals, while the influence of shape on the size determination of the particles is felt only in the immediate vicinity of the class boundaries by relatively few overlapping fragments, compared with the greater number of particles of the entire class. The class limits for sphericity and roundness have been determined by increasing the size of the classes until the column diagram, on the basis of the given number of investigated particles, showed an orderly distribution and "smooth" outline without any excessively irregular structure. In conclusion, Table I illustrates only the general outline of the analysis. It is believed that the size of the class intervals can be further reduced by improved technique in determining the properties, whereby finer variations in the particle distribution may be detected.

INSTRUMENTS AND APPARATUS

The following equipment was used for the present study of sand grains: (1) a Ro-tap testing sieve shaker from W. S. Tyler Co., Cleveland; (2) four testing sieves with openings in millimeters as follows: 0.500, 0.250, 0.125, 0.061, and pan; (3) an E. Leitz's microscope CM and accessories, including an ocular No. 3 and three achromatic objectives, No. 6 (4 mm.), No. 3 (16 mm.), and No. 1 (40 mm.); (4) a mechanical stage; (5) an object-micrometer scale of 5 mm. in tenths; (6) a camera lucida; (7) a polar planimeter from Keuffel and Esser Co.; (8) a circle scale (Fig. 1), consisting of a number of circles drawn on the ground surface of a 20×20-cm. sheet of transparent celluloid of the kind used for photographic cut-film.[5] The innermost circle of 1 mm. radius was drawn in black (India ink); the next five circles of 2, 4, 6, 8, and 10 mm. radii in red (India ink); the following five circles of 12–20 mm. radii in black; and so forth up to the outermost circle of 70 mm. radius. The alternation of colors in

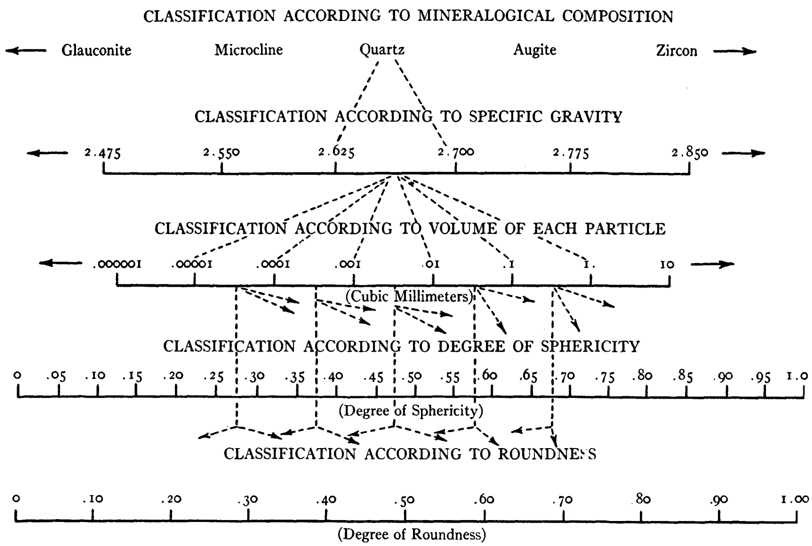
TABLE I

TEST SAMPLE

(116 Grams)

Analysis of 1,000,000 Grains

(Distribution of the Particles according to their Mineralogical Composition and Physical Properties)



sets of five circles (10 mm.) facilitates the reading of the radius. (9) A centrifuge apparatus for mineral separation in heavy liquids; (10) a balance having a sensitiveness of 0.1 mg.; and (11) calculating machines.

PREPARATION OF THE SAMPLE FOR ANALYSIS

For this advance in an almost unknown field, it was deemed desirable to choose a sediment composed entirely, or almost entirely,

[5] Celluloid of this kind may be purchased in any well-stocked shop carrying drawing material. It may also be obtained by boiling a large photographic cut-film in water for a minute or two, thereby removing the gelatinous coat.

of particles of the same specific gravity, thereby avoiding separate analysis of particles of different specific gravities. Through the courtesy of Dr. F. J. Pettijohn a sample of St. Peter sandstone was supplied from the collection of the sedimentological laboratory of

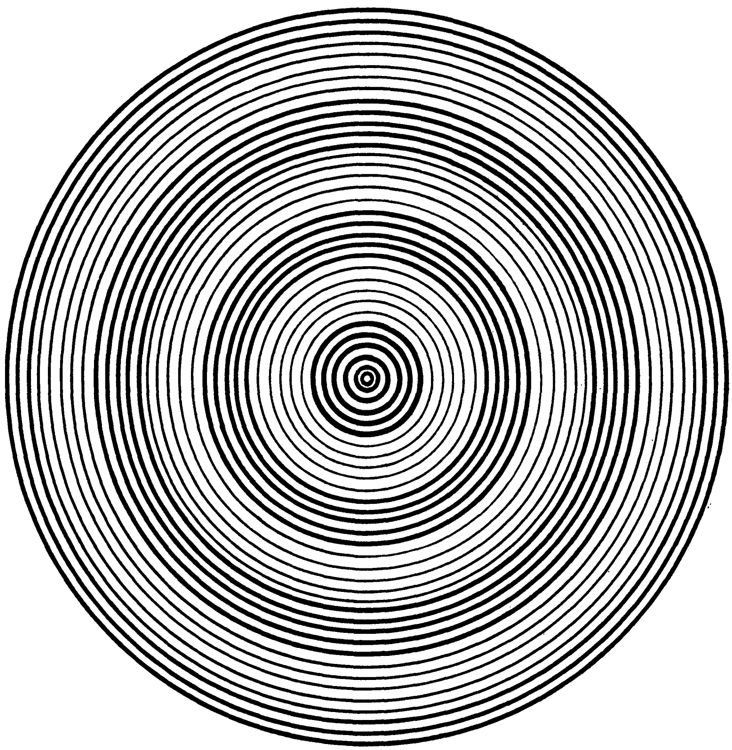


FIG. 1.—Circle-scale of 2 mm. intervals, except for the two innermost circles of 1 and 2 mm. radii. (Reproduced for illustration only, not for accuracy.)

the University of Chicago. It is believed that owing to the generally recognized homogeneity of St. Peter sandstone the sample may be considered to represent the composition of the sandstone in the immediate surroundings of the sample locality. The sample was collected near the base of St. Peter sandstone in Utica Township, T. 33 N., R. 2 E., Sec. 7, where the road crosses Pecumsaugan Creek approximately 3 miles northeast of La Salle, Illinois.

The sample was "cut down" for screen analysis to about 116 grams. The few mineral particles heavier and lighter than quartz were removed in bromoform by centrifuge, and the remaining quartz grains shaken for 15 minutes in a Ro-tap testing sieve shaker. The result of the screen analysis is given in Table II, second and third columns from the left.

The material retained on each sieve was further "cut down" by quartering to a weighable amount small enough to be spread over a

TABLE II

Screen Openings in Mm.	Weight of Material in Mg. on Each Sieve	Number of Grains Present on Each Sieve	Number of Grains per Million	Frequency Value of Each Investigated Grain (F)
0.500	187.2	650	95	3
0.250	61321.4	937,636	136,340	2,726
0.125	49192.8	3,652,027	531,038	10,620
0.061	5446.6	1,876,843	272,909	5,458
Pan	101.9	410,000	59,618	1,192
Total	116,249.9	6,877,156	1,000,000	....

microscopic slide. The grains on each slide (each grade size) were counted under the microscope, using a mechanical stage. Knowing the number of grains present on a slide and their total weight, the number of grains retained on the sieve was computed on the basis of the weight of the sieve material. The results are given in Table II, third column. The fourth column gives the computed number of grains per million.

Each slide was then treated as follows: The sand grains were thoroughly mixed, and one-half to three-fourths of the amount removed after careful quartering. The remaining particles were spread over an area about equal to the size of a cover glass. The slide was given a few gentle taps with a pencil so that all particles came to a stable rest on one of their larger surfaces, more or less parallel to the largest and intermediate diameters.[6] To raise the relief a few drops

[6] The weakest part of an irregular quartz grain is, as a rule, along the direction of its smallest diameter. The results of fracturing, chipping, attrition, and solution of the particles are therefore most noticeable at right angles to the shortest diameter, i.e., in the plane of the longest and intermediate diameters.

of clove oil ($n=1.560$) were added, and a cover glass very gently placed on top. The slide was then ready for reproduction.

REPRODUCTION OF THE SAND GRAINS TO THE STANDARD SIZE

The reader will recall from discussions in previous papers that the measurement of the roundness requires a reproduction of the grains in the "standard size." Large objects, such as boulders, must be reduced, and small ones, like sand grains, magnified to approximately the same size, i.e., the "standard size," on which the roundness measurements are performed. The average diameter of the reproduced grains in the standard size has been fixed at about 7 cm., and quartz particles were given a linear enlargement to about that size by camera lucida.

The standard size was obtained by use of the proper objective for each grade size, and by regulating the distance between the camera lucida mirror and the drawing paper. Thus the particles of the pan and of the 0.061-mm. sieve were given a 600× linear enlargement; those of the 0.125-mm. sieve, a 213× enlargement, and the grains of the 0.250-mm. and 0.500-mm. sieves, a 143× linear enlargement. The magnifications were measured by means of an object-micrometer scale of 5 mm. in tenths, the latter being reproduced in the center of the microscopic field and by means of the camera lucida drawn on the paper. The scale reproduced on the paper was measured with a ruler, and the amount of linear magnification computed. The values for the enlargements thus obtained are sufficiently accurate for the purpose of the investigation. The average maximum diameter of the grain reproductions amounted to about 7 cm., i.e., the standard size. The proper distance between the camera lucida mirror and the drawing paper was determined for a given slide by one of the largest grains present on the slide, in order that no grain, when reproduced to the standard size, would occupy more than one-third of the microscopic field. For reproduction each particle was placed in the center of the field.[7]

[7] No attempt should be made to draw a particle which is not in the center of the field. The distortion in the outer parts of the microscopic field is so great, especially with high-power objectives, that a reasonably accurate outline of the particle is not obtained if the grain occupies more than one-third of the field.

The drawing of the larger grains sometimes required a slight adjustment of the focus for the individual grains, in order to bring out a sharp maximum border line. Whenever the curvature of a corner could be followed inside the maximum outline of the particle, its continuation was traced on the paper, (Fig. 2*a*). All grains showing signs of secondary crystal growth were omitted.

In order to secure a representative result and to avoid error from uneven grain distribution on the slide, the following rule was carried

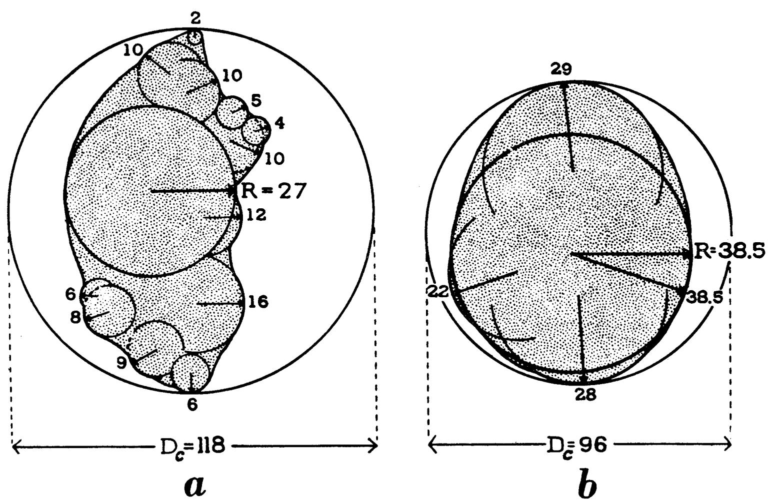


FIG. 2.—Camera lucida drawings, *a*, of a grain retained on the 0.125-mm. sieve, *b*, of a grain retained on the 0.061-mm. sieve. (The figures give the dimensions in millimeters.)

out. Starting with the easternmost grain on the slide, all particles falling on the east-west cross-hair were reproduced as the microscopic view was changed along a straight east-west line by means of the microscopic stage. Then the northernmost grain was used as a starting-point, proceeding in the same way but in a north-south direction, avoiding drawing any grain previously reproduced along the east-west line. If the number of particles thus sketched was not sufficient, the reproduction was continued along some line parallel to those previously used.

Two hundred and twenty-five particles were drawn, 50 of each grade size (of each slide), except for the grains of the 0.500 mm.

sieve, where 25 particles were deemed sufficient to represent the small amount present. The variation in size, shape, and roundness of the grains of each grade size is not likely to be so great that 50 particles will not give a representative picture of the grain properties. It is recommended for the future, however, to choose the number of grains proportionate to the number of particles on the sieve, instead of choosing the same number for all screens as, with one exception, was done in the present study. But not less than 20 particles should be reproduced in any given grade size.

The preceding sections have described the operations leading up to and including the reproduction of sand grains to the standard size. This concludes the mechanical part of the analysis; what follows treats of the methods of computing the sedimentary properties from measurements obtained on the reproductions.

VOLUME, WEIGHT, AND NOMINAL SECTIONAL DIAMETER OF A QUARTZ PARTICLE

The size of a non-spherical particle can be computed with fair accuracy on the basis of the sedimentation velocity within the region of Stokes' law.[8] The use of sieves in mechanical analysis has, however, the advantage of speed. A serious difficulty in connection with screen analysis arises from the fact that the "three diameters" of a sedimentary particle are rarely equal, there usually being a largest, a least, and an intermediate diameter, the last of which ordinarily determines whether the grain will pass the sieve. An accurate size analysis by screening can therefore be achieved only when all particles have the same geometric form. Since this is never the case, there is always an "error" involved, owing to varying non-spherical shapes of the particles. In the present study an attempt has been made to reduce this error somewhat by computing the true nominal diameter from measurements obtained on the reproductions. The diameter and the radius of a sphere of the same volume as a given spherical or non-spherical solid have been previously introduced as the true nominal diameter and the true nominal radius, respectively. "Nominal sectional diameter" is here introduced as a term for the diameter of a circle equal in size to the area of the non-magnified reproduction

[8] Wadell, "Some New Sedimentation Formulas," *loc. cit.*

of a quartz particle in the plane of the largest and intermediate diameters. Our aim is to show that the true nominal diameter can be computed, without too great error, from the nominal sectional diameter obtained and computed from the grain projection in the standard size. The nominal sectional diameter is computed as follows.

Figure 2*a* represents the standard size or the magnified projection of a grain retained on the 0.125-mm. sieve. The area of the original grain reproduction (not of the size given in Fig. 2*a*) as measured by a polar planimeter on the standard size amounted to 5,120 sq. mm., corresponding to a circle area of an 80.7 mm. diameter. Since the grains of the 0.125-mm. sieve had been given a linear magnification of 213×, the value of the nominal sectional diameter, d_{cn}, of the grain amounts to

$$d_{cn}=\frac{80.7}{213}=0.3788 \text{ mm.} \tag{1}$$

The following is intended to show that the nominal sectional diameter, d_{cn}, as computed above, does not deviate excessively from the value of the true nominal diameter, d_n, the latter being a purely dimensional size value by which the volume can be computed.

The nominal sectional diameter of a spherical grain equals the true nominal diameter of the sphere. Thus the volume of a perfect sphere can be computed by the formulas:

$$V_s=\frac{\pi}{6}\cdot d_{cn}^3\,;\; V_s=0.5236\cdot d_{cn}^3\,, \tag{2}$$

where V_s is the volume of the sphere and d_{cn} the nominal sectional diameter.

Assuming that all particles retained on a sieve are spherical, their total volume, V_t, may be obtained approximately by multiplying the arithmetic mean of the volumes of the investigated particles (the volume of each particle computed in the manner demonstrated by Formulas (1) and (2)) by the total number of grains present on the sieve. Thus expressed:

$$V_c=N\cdot\frac{0.5236(\Sigma d_{cn}^3)}{N_i}\,, \tag{3}$$

where V_c is the total volume of the grains as computed by Formula (3), N the total number of particles present on the sieve and computed in a manner previously demonstrated (Table II), N_i the number of investigated particles, reproduced and measured, and Σd_{cn}^3 the sum of the cubes of the nominal diameters, d_{cn}, of the investigated grains.

The total weight, W_c, of the grains is obtained by multiplying their total volume, V_c, by their specific gravity, ρ_s.

$$W_c = V_c \cdot \rho_s \,. \tag{4}$$

It has been found that the computed weight, W_c, generally approaches the value of the actual weight, W_a, obtained by direct weighing of the particles.

When dealing with grains of non-spherical shape, the difference between W_c and W_a increases with departure from spherical shape, and especially with increasing flatness of the particles. This difference computed as percentage of the actual weight, W_a, expresses also the difference between the actual and computed average grain volume. The particles retained on the 0.250-mm. screen are chosen as an example. The values used for the calculation are found in Tables II and III.

$W_a = 61321.4$ mg. = the actual weight of the grains retained on the 0.250-mm. sieve.

$N = 937{,}636$ = the total number of grains retained on the 0.250-mm. sieve.

$N_i = 50$ = the number of investigated grains of 0.250-mm. sieve.

$\rho_s = 2.65$ = the specific gravity of quartz.

$\Sigma d_{cn}^3 = 2.469847$ = the sum of the cubes of the nominal sectional diameters of 50 investigated grains. The numerical values of the nominal sectional diameters (not their cubes) are given in the column marked d_{cn} of Table III.

Applying Formulas (3) and (4) and substituting the numerical values above, the computed weight, W_c, amounts to:

$$W_c = 937{,}636 \cdot \frac{0.5236\ (2.469847)}{50} \cdot 2.65 = 64259.3 \text{ mg.} \tag{5}$$

TABLE III

NUMERICAL VALUES OF ROUNDNESS, SPHERICITY, AND NOMINAL DIAMETER OF FIFTY INVESTIGATED QUARTZ PARTICLES OF THE 0.250-MM. SIEVE

No.	P	ϕ	d_{cn}
1	0.546	0.694	0.3103
2	0.540	0.925	0.4007
3	0.529	0.877	0.5262
4	0.523	0.877	0.3516
5	0.520	0.769	0.1733
6	0.520	0.813	0.3470
7	0.507	0.970	0.3248
8	0.497	0.847	0.2603
9	0.483	0.746	0.3556
10	0.471	0.719	0.4440
11	0.469	0.847	0.3804
12	0.467	0.819	0.2992
13	0.465	0.819	0.2985
14	0.452	0.763	0.3470
15	0.438	0.746	0.2557
16	0.421	0.917	0.3890
17	0.413	0.862	0.2902
18	0.408	0.813	0.3573
19	0.392	0.917	0.3206
20	0.383	0.869	0.3117
21	0.380	0.847	0.2724
22	0.377	0.833	0.3046
23	0.375	0.847	0.5911
24	0.344	0.869	0.4669
25	0.338	0.793	0.2445
26	0.337	0.909	0.4390
27	0.327	0.909	0.3040
28	0.325	0.833	0.3880
29	0.315	0.943	0.3694
30	0.307	0.862	0.3486
31	0.304	0.671	0.4262
32	0.303	0.781	0.3881
33	0.298	0.793	0.3545
34	0.296	0.877	0.3440
35	0.287	0.833	0.3838
36	0.256	0.751	0.3206
37	0.253	0.793	0.3230
38	0.251	0.806	0.3666
39	0.246	0.840	0.3248
40	0.244	0.833	0.5082
41	0.236	0.877	0.3429
42	0.223	0.869	0.3103
43	0.220	0.877	0.4470
44	0.209	0.746	0.2916
45	0.206	0.806	0.3374
46	0.201	0.840	0.3353
47	0.195	0.735	0.2882
48	0.157	0.854	0.3103
49	0.152	0.806	0.3389
50	0.100	0.840	0.3644

The difference between W_c and W_a expressed as percentage of W_a is obtained by the formula:

$$\frac{100\,(W_c - W_a)}{W_a}\,; \tag{6}$$

or substituting the numerical values:

$$\frac{100\,(64259.3 - 61321.4)}{61321.4} = 4.7 \text{ per cent}\,. \tag{7}$$

Since the specific gravity value, 2.65, enters both W_c and W_a, the result, 4.7 per cent, expresses also the difference between the actual and computed average grain volume. Table IV gives the results obtained by similar calculations for the particles of the four sieves and the pan.

TABLE IV

Sieve (Mm.)	$\frac{100(W_c - W_a)}{W_a}$ (Percentage)	Percentage Difference in Respect to Diameter
0.500	8.3	2.7
0.250	4.7	1.5
0.125	37.5	11.2
0.061	10.8	3.4
Pan	51.2	14.7

The high percentage, 51.2, on the pan is probably due to the fact that (1) the very finest grains of a nominal sectional diameter less than 0.04 mm. were omitted in drawing, because their magnification to the standard size would have involved an extra arrangement which was deemed unnecessary, since their total weight amounted to an extremely small fraction of 1 per cent of the whole sample of St. Peter sandstone; and (2) these particles had in general a somewhat more flaky form than the grains of the sieves. The 37.5 per cent obtained for the 0.125-mm. sieve appears abnormally high in view of the low values for the screens above and below. It suggests that the 50 investigated particles are not representative of the entire amount of 3,652,027 particles retained on that sieve, at least not so far as the particle size is concerned. They may, however, be adequate in respect to sphericity and roundness, because these properties are not necessarily subject to the same amount of variation as the particle size. The percentages 8.3, 4.7, and 10.8 for the remaining screens (0.500 mm., 0.250 mm., and 0.061 mm.) can be regarded as reasonably satisfactory. It should be noted that the percentage values express the difference in volume and not in diameter, which is the common basis for size classification. Since the diameter of a sphere equals $\sqrt[3]{\text{Volume}/0.5236}$, the percentage values in respect to the difference in diameters are less conspicuous (Table IV). Even as much as 51.2 per cent difference in volume between two spheres, figured as percentage of the smaller sphere, amounts only to 14.7 per cent when computed on the basis of the diameter values of corresponding spheres. Although the last figure (14.7) is rather high, it compares favorably with many values obtained by various proposed methods and computed from linear measurements of more or less arbitrary diameters in one plane of a non-spherical quartz particle.

In conclusion, the advantages of the nominal sectional diameter are: (1) that its numerical value gives a good average of all diameters in the plane of measurement; (2) that personal opinion in choice of diameters for measurement is avoided; (3) that, whereas many proposed methods, such as the measurement of the "statistical diameter,"[9] give an average size value of a great number of particles, the nominal sectional diameter is a permissible size expression for a single quartz grain; and (4) that the accuracy of the nominal sectional diameter as a size value increases with increasing degree of true sphericity, and, when dealing with perfect spheres, the nominal sectional diameter equals the true nominal diameter, which latter equals the diameter of the sphere. Since quartz has no pronounced cleavage, it is not likely that fracturing and chipping in sedimentary transportation will produce very flat quartz particles; consequently the nominal sectional diameter of a sedimentary quartz grain may generally be used as a size value of fair accuracy.

[9] Geoffrey Martin, "Law Governing the Connection between the Number of Particles and Their Diameters in Grinding Crushed Sand," *Trans. Ceramic Soc.*, Vol. XXIII (1923–24), pp. 61–120.

SPHERICITY AND ROUNDNESS MEASUREMENTS

It has been shown by the coefficient of resistance, as a function of Reynolds number for solids of various shapes, that the sedimentological form, bearing approximately on the terminal uniform settling velocity of solids, can be expressed by the ratio:[10]

$$\frac{s}{S}=\psi\,, \tag{8}$$

where s is the surface area of a sphere of the same volume as the solid, S the actual surface area of the solid, and ψ denotes the degree of true sphericity. The maximum value obtained for ψ is 1, which is the numerical shape value of a sphere.

There are several rather accurate methods for surface determination of small particles in bulk. None of these could, however, be adopted for the present study, because it was desirable for classification in the statistical charts to obtain a sphericity value for each particle rather than bulk values for a great number of granules. Lack of experimental data has made it necessary to adopt the same scheme used in a previous paper dealing with shape determination of larger rock fragments.[11] Table V gives the dimensions of five geometric forms, the same used as prototypes in the earlier paper. Figure 3 illustrates the outline of the same solids as they would appear under the microscope, when they rest on one of their largest faces, more or less parallel with the largest cross-sectional area. It is assumed, for obvious reasons, that the cube rests on one of its square faces, while the other parallelopipeds rest on one of their two larger rectangular faces. It is recalled that the sand grains for reproduction were placed in their most stable position of rest by a few gentle taps on the slide.

A practical formula for computing the shape of a quartz grain must be such that the obtained value approaches as closely as possible the degree of true sphericity. The following formula is suggested:

$$\frac{d_c}{D_c}=\phi\,, \tag{9}$$

[10] Wadell, "The Coefficient of Resistance as a Function of Reynolds Number for Solids of Various Shapes," *loc. cit.;* "Shape Determinations of Large Sedimental Rock-Fragments," *loc. cit.;* "Some New Sedimentation Formulas," *loc. cit.*

[11] Wadell, "Shape Determinations of Large Sedimental Rock-Fragments," *loc. cit.*

where d_c is the diameter of a circle equal in area to the area obtained in the standard size when the grain rests on one of its larger faces, more or less parallel to the plane of the longest and intermediate diameters, and D_c is the diameter of the smallest circle circumscribing the grain reproduction of the standard size. The shape value achieved by this method is denoted by the Greek letter ϕ. It is assumed that the ϕ-value generally approaches the ψ-value, i.e., the value of the degree of true sphericity (Formula (8)).

Table V gives the ϕ- and ψ-values for the five geometric prototypes. The ϕ-value is computed according to Formula (9) and on the basis of d_c being the diameter of a circle area equal to the projection area (Fig. 3) and D_c being the diameter of the smallest circle circumscribing the projection area. An inspection of Table V shows that for spheres $\psi = \phi$. For the other forms the value of ϕ approaches that of ψ, except for parallelopiped No. 4, which is a very flat and rather square-shaped solid. It is especially to be noted that the table is not intended to mask the actual situation. For other geometric forms the difference between the ϕ- and ψ-values may take other proportions. For instance, a circular disc obtains a maximum ϕ-value, i.e., 1, while its actual ψ-value may be very low. Quartz particles, however, do not in general attain very flat shapes, and sedimentary quartz grains of circular disc shape may be considered as extremely rare. Very thin quartz particles of a size larger than 0.1 mm. are readily broken up into several pieces by tractional transportation, each fragment attaining a higher degree of true sphericity (ψ) than the original particle of the same thickness. Irregularly shaped sedimentary quartz may well have rather thin edges, but the thickness

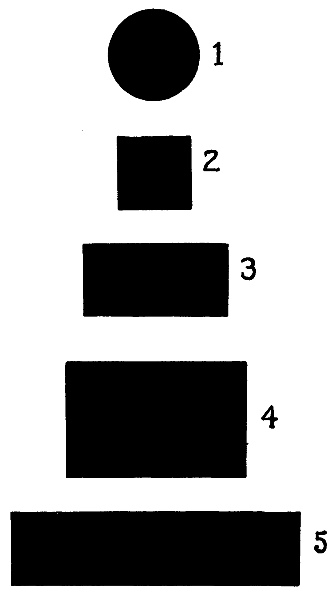


FIG. 3.—Outlines of five geometric prototypes in their most stable position of rest.

generally increases toward the center, thereby increasing the degree of true sphericity. In conclusion, it is believed that the ϕ-value in general approaches the actual ψ-value without excessive deviation. It has also been shown in a previous paper[12] that similar formulas, constructed for larger rock fragments on the basis of the same prototypes presented above, agreed reasonably well with experimental data bearing on the sedimentation velocity and the coefficient of resistance as a function of Reynolds number.

TABLE V

DIFFERENCE BETWEEN THE ψ- AND ϕ-VALUES FOR FIVE GEOMETRIC FORMS

No.	Geometric Form	Dimensions	Volume	ψ	ϕ	$\phi-\psi$
1	Sphere	Diam. 2.48	8	1.00	1.00	0.00
2	Cube	2×2×2	8	0.80	0.79	−0.01
3	Parallelopiped	4×2×1	8	0.69	0.71	0.02
4	Parallelopiped	5×3.2×0.5	8	0.48	0.75	0.27
5	Parallelopiped	8×2×0.5	8	0.46	0.54	0.08

Roundness has been discussed in two previous papers.[13] The arguments presented tended to show that the degree of roundness is essentially a planimetric conception, referring to the smooth curvature of the outline of a plane area, projection area, or cross section. A "plane corner" was defined as every such part of the smooth outline which has a radius of curvature equal to or less than the radius of the maximum inscribed circle in the given area. Thus a plane corner has reached its maximum degree of roundness when the radius of its smooth curvature equals the radius of the maximum inscribed circle. The total roundness of a solid may be obtained by curvature measurements in several cross-sectional planes, but one plane is generally satisfactory when dealing with sedimentary sand grains (see footnote 6).

The roundness of a plane corner is expressed by the ratio r/R, where r is the radius of curvature of the corner, and R is the radius of

[12] Wadell, "Shape Determinations of Large Sedimental Rock-Fragments," *loc. cit.*

[13] Wadell, "Volume, Shape and Roundness of Rock Particles," *loc. cit.;* "Sphericity and Roundness of Rock Particles," *loc. cit.*

the maximum inscribed circle, both values obtained by measurements on the standard size. The maximum value for r/R is 1.00 for a corner (hereafter always understood to be a plane corner if not otherwise specified) of maximum roundness. The total roundness in one plane equals the arithmetic mean of the roundness values of the corners, thus:

$$\frac{\sum\left(\frac{r}{R}\right)}{N}=\mathrm{P}\,, \tag{10}$$

where $\sum\left(\frac{r}{R}\right)$ is the sum of the roundness values of the corners, N the number of corners in the given plane, and P denotes the total degree of roundness. The maximum value obtained by this formula is 1.000 for a particle of maximum roundness in the given cross section or projection plane.

A formula which sometimes gives a slightly different roundness value from that obtained by equation (10) has been used in this study. It reads:

$$\frac{N}{\sum\left(\frac{R}{r}\right)}=\mathrm{P}. \tag{11}$$

The maximum value achieved by this formula is also 1.000 for a solid of maximum roundness in the plane of measurement. Preference is given Formula (11), because it results in a slightly lower value for the roundness of particles having corners of greatly different roundness values, i.e., when the coefficient of variation of the roundness values is high.[14] Relatively well-rounded particles, which, by chipping or fracturing shortly before deposition, have obtained a very low degree of roundness for one or more corners, are generally placed in a lower roundness class by use of Formula (11) instead of Formula

[14] The coefficient of variation as developed by Pearson is the standard deviation as percentage of the arithmetic mean, thus

$$C_v=\frac{\sigma}{M_a}\times 100\,,$$

where C_v is the coefficient of variation, σ the standard deviation, and M_a the arithmetic mean of a given number of variables.

(10). The resulting diagram (by use of Formula (11)) is therefore more influenced by recent events of transportation preceding the deposition.

The roundness of a quartz particle was obtained by the following method. We assume that the grain has been drawn on a paper and that the reproduction represents the standard size of the particle in the plane of the longest and intermediate diameters (Fig. 2). All corners, i.e., curves of the outline of the reproduction having a radius equal to or less than the radius of the maximum inscribed circle, are measured. The radius of curvature of each corner is obtained by placing the transparent celluloid circle-scale (Fig. 1) over the drawing and adjusting its circles to the outline of the reproduction so as to obtain a direct reading of the radius of curvature of a given corner. The circle-scale is adjusted so that one of its circles covers as much as possible of the outline of the given corner. The radius of that circle is then taken as the radius of curvature of the corner. The outline of a corner having a radius of curvature of odd value (5 mm., 11 mm., 13 mm., etc.) appears as a curved line between and parallel with two circles of the scale, the latter having circles with 2 mm. intervals. The radius of curvature for such a corner is, of course, given an odd value. Although the circle-scale allows readings to half a millimeter, it is in general found satisfactory to express the radii values of the corners in whole millimeters, because one may always count on a certain amount of error due to inaccuracy of drawing. Curvatures of a radius less than 1 mm. are beyond accurate measuring, and they are therefore always given a radius value of 0.5 mm. The radius of the maximum circle which can be inscribed within the boundaries of the grain reproduction is readily obtained by placing the circle-scale over the drawing.

The following illustrates the computation of sphericity and roundness on the basis of the values obtained by the method outlined above. Figures 2*a* and *b* represent the standard size of two grains of the 0.125-mm. and 0.061-mm. sieves, respectively. Figure 2*a*, which illustrates the only quartz particle having as many as twelve corners, has been chosen as an example of what may be expected in an extreme case. Figure 2*b* illustrates the normal appearance of a particle of high sphericity and high roundness.

Example: Figure 2a representing the standard size of a quartz grain of grade size 0.125 mm.

The area of the grain reproduction = 5,120 sq. mm.

d_c = 80.7 mm. = the diameter of a circle area of 5,120 sq. mm.

D_c = 118.0 mm. = the diameter of the circumscribing circle.

Applying Formula No. 9 and substituting the numerical values above:

$$\text{Sphericity } (\phi) = \frac{80.7}{118} = 0.68 .$$

R = 27 mm. = the radius of the maximum inscribed circle.

r = the radius of curvature of a corner. The values for r are shown in Figure 2a and in the table below.

N = 12 = the number of corners in Figure 2a.

$\sum\left(\frac{R}{r}\right) = 53.06$ (see table below).

Applying Formula (11) and substituting the numerical values above:

$$\text{Degree of roundness (P)} = \frac{12}{53.06} = 0.22.$$

r	$\frac{R}{r}$
2	13.50
10	2.70
5	5.40
4	6.75
10	2.70
12	2.25
16	1.68
6	4.50
9	3.00
8	3.38
6	4.50
10	2.70
	53.06

About 70 per cent of all particles were found to have six to seven corners, and only one grain as many as twelve (Fig. 2a). The average number of corners suggests an influence of the crystallographic form of quartz.

GRAPHIC PRESENTATION OF THE RESULT

Table II gives the number of particles per million, computed from the number of particles present on each sieve. Since 50 grains of each grade size (except for the 0.500-mm. screen where 25 particles were deemed sufficient) were drawn and measured, each investigated grain represented $\frac{1}{50}$ (and $\frac{1}{25}$) of the number of grains per million present in respective grade sizes. Thus screen 0.250 mm. contained 937,636 particles, corresponding to 136,340 grains per million of the total sample of St. Peter sandstone. Each one of the 50 investigated particles represented consequently $\frac{136,340}{50} = 2,726$ grains per million. The last value, 2,726, is called the frequency value (F) of the investigated particle.

By summarizing the frequency values of grains having the same class characteristics, the total number of particles per million falling in a given class is obtained. Assuming that three particles of the total number of investigated grains attain the sphericity values 0.91, 0.93, and 0.94, they all fall in the same class, 0.90–0.95. For the purpose of illustration assume further, as an extreme case, that the three particles belong to the 0.250-mm., 0.125-mm., and 0.061-mm. sieves, the investigated grains of which have the frequency values 2,726, 10,620, and 5,458, respectively. The total number of grains per million in sphericity class 0.90–0.95 will then be: $2,726+10,620+5,458=18,804$ (example only).

The frequency values have the advantage that they permit manipulation and classification of the particles according to any grain property and scheme suited to the investigation and the purpose of the research.

The dotted lines of Table I show how the analysis of the St. Peter sandstone branches out so as to comprise the essential sedimentary properties of the grains. Figures 4, 5, and 6 illustrate the results of the analysis. The particles have been classified according to their properties, and the number of grains of a given class computed on the basis of the frequency values of the investigated particles. Each diagram shows the distribution of one million grains. Particles having property values falling exactly on a class limit were allotted to the class of lower value, because it has been previously shown that the values obtained by the practical formulas for computing the volume and sphericity (ϕ) were slightly too high.

Figure 4 illustrates the distribution of one million grains classified in five volume-classes. The volume of each investigated particle has been calculated according to Formula (2) and on the basis of the nominal sectional diameter, computed in a manner demonstrated by Formula (1). In practice, however, a simpler method has been followed in distribution of the particles in the proper volume-classes.

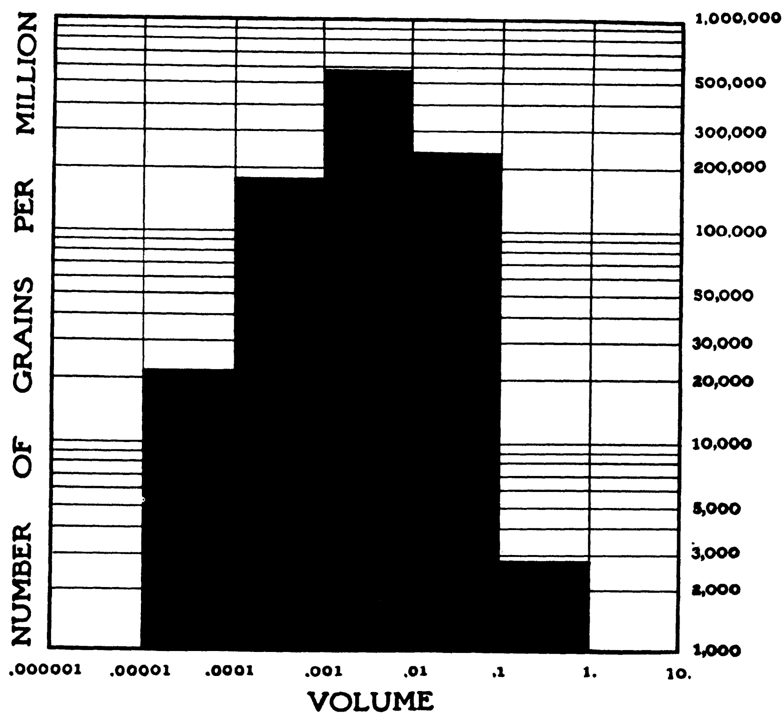


FIG. 4.—Distribution of one million grains classified according to volume (semilogarithmic scale).

The values of the nominal diameters corresponding to the limits of the volume-classes were computed, and the grains were then allotted directly to the proper volume-classes on the basis of their nominal sectional diameters, without the elaborate work of first computing the volume of each investigated particle. The volume-classes 0.00001–0.0001, 0.0001–0.001, 0.001–0.01, 0.01–0.1, 0.1–1.0, cubic mm. have the following limits, expressed as diameters of spheres of respective volumes, 0.02673–0.05758, 0.05758–0.12407, 0.12407–0.2673, 0.2673–0.5758, 0.5758–1.2407, mm.

Figure 5 illustrates the distribution of one million grains over the five volume-classes, the grains of each volume-class being classified according to their ϕ-values. Because of the difficulty in expressing adequately the small number of particles in the highest and lowest volume-classes, italic figures have been inserted to indicate the exact amount in each ϕ-class.

Figure 6 illustrates the distribution of one million grains classified according to volume and P-values. The volume-classes 0.01–0.1 and 0.1–1.0 have been united into one, on account of the few particles present in the uppermost class. The same has been done in respect to the two lowest volume-classes. The structure of the entire diagram becomes thereby more pronounced, and variations in roundness may be better visualized.

Arithmetic scales are used in Figures 5 and 6. In order to avoid an unduly large diagram, Figure 4 has been constructed on the semi-logarithmic scale.

AVERAGE VALUES FOR SPHERICITY AND ROUNDNESS OF PARTICLES ON EACH SIEVE

Lamar[15] has constructed a mechanical device for determining the "roundness" or "angularity"[16] of sand grains in bulk. The *modus operandi* was as follows.

About 60 cc. of sand, carefully screened to a given sieve size and dried at 100°C., were placed in a cylinder. A downward concentration of the sand was produced by a motor-driven plunger, which raised and dropped the cylinder about half an inch at the rate of 100 times a minute. The cylinder struck on a piece of felt, thus reducing the amount of rebound imparted to the sand. The operation was continued until the volume of sand in the cylinder could be reduced

[15] J. E. Lamar, "Geology and Economic Resources of the St. Peter Sandstone of Illinois," *Ill. Geol. Surv. Bull. 53* (1927).

[16] Lamar's terms "roundness" and "angularity" are practically synonymous with the "sedimentological shape" or the "degree of true sphericity" as used in the present paper.

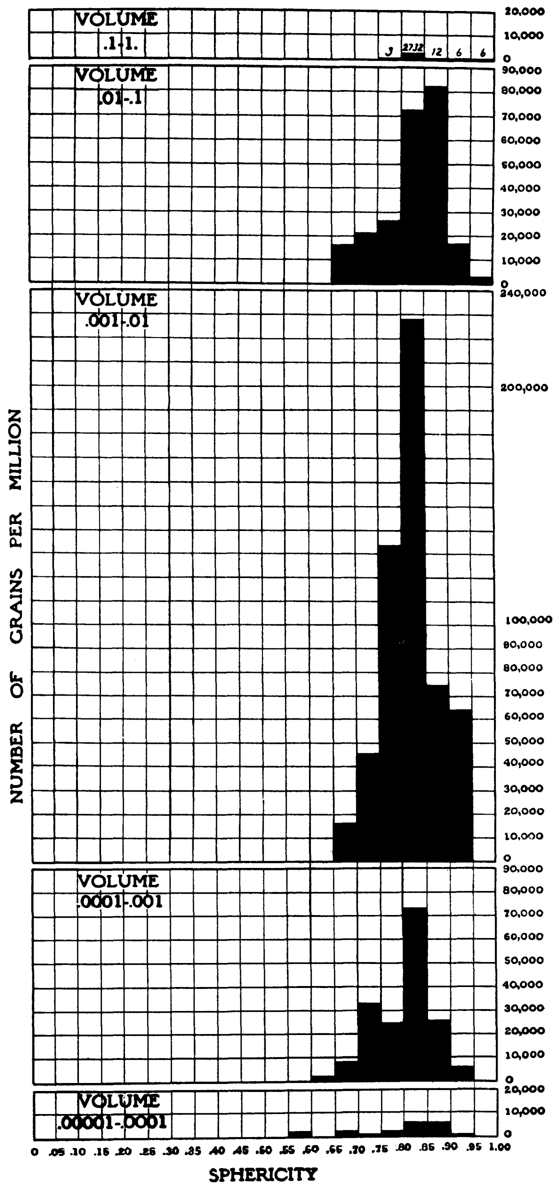


FIG. 5.—Distribution of one million grains classified according to volume and ϕ-value (arithmetic scale).

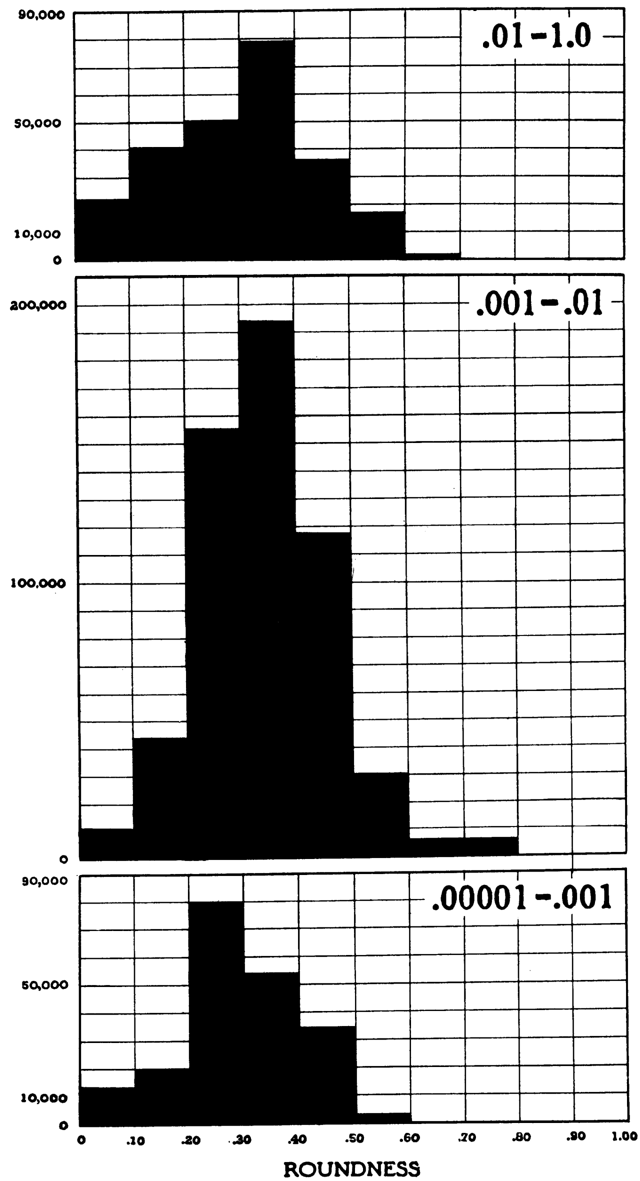


FIG. 6.—Distribution of one million grains classified according to volume and P-value (arithmetic scale).

no further. The percentage of porosity of the compacted sand was determined by the formula:[17]

$$P=\frac{100(C-V)}{C},$$

where, C is the volume of sand and voids, V the actual volume of all sand grains, and P is the percentage of porosity with maximum compaction.

The relative "angularity" of the sand was determined by dividing 25.95, the theoretical minimum porosity for spheres, by the porosity of the compacted sample. Lamar concludes that the nearer the figure is to 1.00, the "rounder" the sand. "Round" must here be considered as synonymous with "spherical."

Lamar's conclusion appears to be justified, provided that: (1) the grains are of approximately the same size so that the pore-space is not filled with smaller particles; (2) the particles do not vary greatly in shape, and (3) the grains are somewhat irregular in shape and do not possess geometric forms which permit complete compaction (cubes). These provisions are reasonably well fulfilled in respect to grade sizes obtained by screening St. Peter sand. Yet, on account of the varying irregular shape of the particles, it is difficult to obtain a clear mathematical analysis of the problem. We may, however, adopt Mitscherlich's viewpoint, namely: (1) the form of the pore-space is governed by the shape of the particles and the mode of packing; and (2) the pore-space becomes more branched with the increasing sum of the particle surfaces.[18]

Since the sum of the surface areas of particles of a given sieve size increases with decreasing degree of true sphericity, it is conceivable, on the basis of the reasoning presented above, that porosity values are to a large extent influenced by the sphericity values of the

[17] Lamar's formula is given as, $P=\frac{C-V\times 100}{C}$; it is obviously a misprint or *lapsus calami.*

[18] Eilh. A. Mitscherlich, *Bodenkunde für Land- und Forstwirte* (1905). P. 84, "Die Gestalt des Hohlraumvolumen des Bodens wird bedingt durch die Gestalt der einzelnen Teilchen und durch die Art der Aneinanderlagerung." P. 86, ". . . . das Hohlraumvolumen um so verzweigten ist, je grösser die Summe der Oberfläche der festen Bodenteilchen ist."

grains. The arithmetic mean of the ϕ-values, obtained for each sieve in the present study, is given in Table VI. Thus, the value 0.829 for the 0.250 mm. sieve is the arithmetic mean of the ϕ-values given in Table III. The corresponding porosity value has been obtained by dividing 25.95, the figure used by Lamar for computing the "angularity" value, by the average ϕ-value. Lamar's porosity and "angularity" values, the latter actually expressing the average grain shape, may serve for comparison. The average values for porosity

TABLE VI

St. Peter Sandstone (Wadell)			Average of 9 Samples of St. Peter Sandstone (Lamar)			Crushed "Cypress Sand" (Lamar)	
On Sieve (Mm.)	Arithmetic Mean of ϕ-Values	Percentage of Porosity	On Sieve (Mm.)	Relative "Angularity"	Percentage of Porosity	Relative "Angularity"	Percentage of Porosity
0.500	0.852	30.4	0.417	0.809	32.1	0.574	45.2
0.250	0.829	31.3	0.295	0.773	33.6	0.649	40.0
0.125	0.824	31.4	0.208	0.735	35.3	0.659	39.4
0.061	0.804	32.2	0.147	0.712	36.2	0.641	40.5

and "angularity" of nine samples of St. Peter sand are given in Table VI, which also includes the values obtained by Lamar for "Cypress sand," to show the difference in result obtained for artificially crushed and very "angular" sand.

Comparing the figures obtained by Lamar and the present writer it should be noted that: (1) the screen openings applied by Lamar are somewhat different from those used in the present study; (2) Lamar's samples were collected in other localities, but, with two exceptions, situated within a few miles' distance from the locality of the sample used in this study; and (3) Table VI gives average values for nine samples. Lamar's maximum and minimum values (not given in Table VI) for "angularity" amount to 0.852 and 0.692 respectively, thus comparing favorably with the maximum and minimum average ϕ-values of Table VI. The relatively higher values, obtained both by Lamar and the present writer for St.

Peter sand, present a notable contrast to the lower values of the artificially crushed "Cypress sand."

Table VII gives the arithmetic mean of the roundness values (P) obtained for each sieve size. For instance, the average roundness value for grains of the 0.250-mm. sieve amounts to 0.350, which is the arithmetic mean of the roundness values (P) listed in Table III. Table VII shows that the average roundness value increases toward

TABLE VII

On Sieve (mm.)	Average P
0.500	0.423
0.250	0.350
0.125	0.332
0.061	0.285
Pan	0.288

the higher grade-sizes, thus in conformity with Sorby's[19] general observations in respect to worn sands.

GENERAL REMARKS

A presentation of new methods is the main purpose of this paper. The diagrams will be interpreted in a forthcoming paper. Only a few points bearing on the accuracy and possibilities of the proposed methods will be taken up here.

Before attempting any interpretation of the diagrams, we must consider how reliably the obtained values express the particle properties. The measurement of the nominal sectional diameter has already been sufficiently discussed and is believed to be superior to any other method proposed for measuring the size of a small quartz particle directly.

The ϕ-values are, perhaps, the most unreliable data presented in this paper. The paucity of available experimental data made it necessary to adopt the five prototypes as a base for reasoning, which, although admittedly weak in many points, nevertheless proved to hold rather well in respect to 62 experimental determinations of

[19] *Op. cit.*

settling velocities of larger fragments (pebbles) of irregular shapes.[20] It was shown that the settling velocities of fragments of the same volume and specific gravity in general decreased with decreasing degree of true sphericity (ψ) and that the same rule can be applied to the ϕ-values with some modification. It was also found that the practical formulas obtained on the basis of the five prototypes were reasonably well in accord with the coefficient of resistance as a function of Reynolds number for the various fragments. It should be especially noted, in defense of any eventual inadequacies in the ϕ-values, that our present interest does not lie so much in the structural details of the resulting diagrams, but rather in their general character, showing the tendency toward high or low sphericity for the majority of particles in each volume-class. For instance, Figure 5 shows a marked increase of the ϕ-values among the particles of the higher volume-classes, 0.01–1.0. The sphericity class 0.80–0.85 is the modal group in the volume-class 0.001–0.01, while the modal sphericity group rises to the ϕ-class 0.85–0.90 in the next higher volume-class, 0.01-0.1. The particles in the highest volume-class, 0.1–1.0, are too few for comparison, but one notes even here a marked general increase in the sphericity, the lowest ϕ-value being represented by 3 particles in the ϕ-class 0.75–0.80, whereas in the volume-class below there are more than 15,000 particles in the ϕ-class 0.65–0.70.

The question arises whether the general increase in sphericity toward the higher volume-classes is in accord with facts or due to inadequacies of the methods. It has been previously pointed out that particles of high ϕ-values may have either a high degree of true sphericity or the shape of a more or less circular disc of very low ψ-value. It was possible, however, to measure the thickness of the particles by well-known microscopic methods, which showed all particles of high ϕ-values to be more or less equidimensional, a fact which suggests that the ϕ-value expresses reasonably well the actual ψ-value.

Exact roundness values can be procured provided that the outlines of the grains are carefully reproduced with steady hand. The

[20] Wadell, "Shape Determinations of Large Sedimental Rock-Fragments," *loc. cit.*

variation in result, due to slight variations in position of a particle in reproduction, does not exceed one class-interval. However, for uniform results, all particles should be reproduced as nearly as possible in the plane of the longest and intermediate diameters.

Both the sphericity and the roundness of a sand grain are visualized by the so-called "image value," which has been described and illustrated in a previous paper.[21] The "image value" has, however, no sedimentological significance.

In an attempt to interpret the structure of the diagrams it is important to bear in mind that the frequencies are built on frequency values of the investigated particles. If the frequency values, as derived from the number of particles on each sieve, vary greatly, the resulting diagrams of volume classes, carrying but a few investigated particles, may present an irregular structure of no real significance. Since the volume-classes are entirely artificial, we cannot expect to find any extreme difference in the sphericity and roundness of particles in two adjacent volume-classes, although in actuality great difference may exist, which could be visualized in a diagram if we could find the proper volume-class limits resulting in maximum differentiation. If, therefore, any excessively irregular structure is encountered in a diagram for a given volume-class, without at least indications of corresponding irregularities in adjacent volume-classes, the irregular structure must in most cases be attributed to the frequency values and an insufficient number of investigated particles. For example, the high frequency in ϕ-class 0.70–0.75 of volume-class 0.0001–0.001 (Fig. 5) is most likely an irregularity of the type described, because there is no indication of pronounced frequency in the same ϕ-classes of the volume-classes next above and below.

If the numbers of investigated particles in one or several volume-classes are too few, these classes are either united or combined with an adjacent volume-class. The diagrammatic structure thereby becomes more pronounced and the tendency of the particle properties is better visualized. Some of the volume-classes in Figure 6 were united and the same could well have been done in respect to corre-

[21] Wadell, "Volume, Shape and Roundness of Rock Particles," *loc. cit.*

sponding volume-classes in Figure 5. It was, however, desirable to retain Figure 5 in its original form in order to illustrate the discussion above.

ACKNOWLEDGMENTS

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