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The shape of rock particles, a critical review

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ABSTRACT

An attempt was made to distinguish aspects of the shape of rock particles, and to discover by analysis and empirical considerations the most appropriate parameters for describing these aspects. The shape of a rock particle can be expressed in terms of three independent properties: form (overall shape), roundness (large-scale smoothness) and surface texture. These form a three-tiered hierarchy of observational scale, and of response to geological processes. Form can be represented by only two independent measures from the three orthogonal axes normally measured. Of the four pairs of independent measures commonly used for bivariate plots, the two sphericity/shape factor pairs appear to be more efficient discriminators than simple axial ratios. Of the two, the most desirable pair is the maximum projection sphericity and oblate-prolate index for both measures show an arithmetic normal distribution for the range investigated. A measure of form that is independent of the three orthogonal axes, and measures derived from them, is the angularity measure of Lees. Roundness has measures of three types, those estimating average roundness of corners, those based on the sharpest corner, and a measure of convexity in the particle outline. Although each type measures a different aspect, they are not independent of each other. Only roundness from corners is considered in detail. As neither average nor sharpest corner measures are inherently more objective or more quantitative, purpose should determine which is more appropriate. Of the visual comparison charts for average roundness, Krumbein's appears best. The Modified Wentworth roundness is the most satisfactory for estimating roundness from the sharpest corner. The Cailleux Roundness index should not be used because it includes aspects of roundness and form. Shape is a difficult parameter to use for solving sedimentological problems. Even the best of the commonly used procedures are limited by observational subjectivity and a low discriminating power. Unambiguous interpretation of particle shape in terms of source material and processes will always be made difficult by the large number of natural variables and their interactions. For ancient sediments satisfactory results can be expected only from carefully planned studies or rather unusual geological situations.

INTRODUCTION

There have been two main approaches to investigations of shape of rock particles. The experimental approach, using tumbling devices or abrasion mills, allows observed changes to be related to starting materials, processes and time. In the empirical approach, pebbles are measured in sedimentary environments where the processes modifying pebble shape are believed to be known. The problems of measurement have also been examined, notably by Griffiths and his co-workers (summarized in Griffiths, 1967). As a result, there are many parameters for describing the shape of a pebble (Table 1) but none that is universally accepted. Confusion appears to exist over what the various parameters of shape actually measure and how they are related. This paper aims to clarify the relationships between various aspects of shape and to find the most effective parameters to estimate them.

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Table 1. Parameters and features used to describe aspects of shape of rock particles

Property	Parameters or features
Form	Elongation & flatness (Wentworth, 1922a; Zingg, 1935; Luttig, in Sames, 1966; Cailleux, 1947) Angularity (Lees, 1964) Sphericity (Wadell, 1932; Krumbein, 1941; Aschenbrenner, 1956; Sneed & Folk, 1958) Form ratio (Sneed & Folk, 1958) Factor 'F' (Aschenbrenner, 1956; shape factor of Williams, 1965) Use of unranked shape classes (Holmes, 1960)
Roundness	Roundness of sharpest corner (Wentworth, 1919, 1922b; Cailleux, 1947; Kuenen, 1956; Dobkins & Folk, 1970) Average roundness for corners (Wadell, 1932; Russell & Taylor, 1937; Krumbein, 1941; Pettijohn, 1949; Powers, 1953) Average roundness from convexity of outline (Szadecsky-Kardoss, see Krumbein & Pettijohn, 1938)
Surface texture	*Markings due to contact with other rocks (pebble features catalogued by Conybeare & Crook, 1968; quartz grain features catalogued by Krinsley & Doornkamp, 1973) *Surface texture resulting from internal texture, important for small pebbles of crystalline rock

*Numerical parameters have not yet been proposed.

THE MEANING OF 'SHAPE'

Shape is the expression of external morphology, and for some is synonymous with form (Shorter Oxford English Dictionary, 1955; Gary, McAfee & Wolf, 1972). However, Sneed & Folk (1958) used the term *form* for overall particle shape, to be obtained from measurement of the three orthogonal axes, and plotted on a *form triangle*. Used in this way 'form' clearly excludes other aspects of shape, such as roundness. In contrast, Whalley (1972) saw *form* as the appropriate term for external morphology, but regarded *shape* as only one of several properties contributing to it.

Shape may also have different meanings for the same person. For example, Griffiths (1967) has two notions of shape, one being the expression of external morphology (p. 110), and the other 'overall shape' being related to the original form of the particle (p. 111), and excluding roundness and surface texture. Further on (p. 113 *et seq.*) he used sphericity to estimate shape (meaning overall shape presumably), though it is now clear that sphericity contains only part of the information on overall shape.

The two concepts of shape recognized by Griffiths are maintained here, though terminology and usage are clarified. *Shape* is taken to include every aspect of external morphology, that is, overall shape, roundness (=smoothness) and surface texture, *Form* is used, following Sneed & Folk (1958), for the gross or overall shape of a particle, and is independent of roundness and surface texture.

THE RELATIONSHIP BETWEEN FORM, ROUNDNESS AND SURFACE TEXTURE

Form, roundness and surface texture are essentially independent properties of shape because one can vary widely without necessarily affecting the other two properties (Fig. 1). Wadell (1932, 1933) long ago established the independence of sphericity and roundness, but since then sphericity has come to be recognized as only one aspect of form (Aschenbrenner, 1956). Surface texture gives rise to occasional practical difficulties in the measurement of shape, but it is often not considered in discussions of shape. Whalley (1972) stated 'surface texture can not be recognized in the projected outline of a particle . . . ', but this is not necessarily true for crystalline rock particles, for example. Surface texture bears the same relationship to roundness as roundness does to form. These three properties can be distinguished at least partly because of their different scales with respect to particle size, and this feature can also be used to order them (Fig. 2). Form, the first order property, reflects variations in the proportions of the particle; roundness, the second order property, reflects variations at the corners, that is, variations superimposed on form.

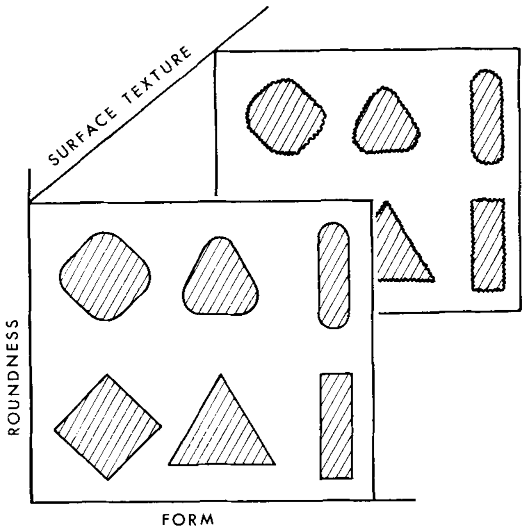

Fig. 1. A simplified representation of form, roundness and surface texture by three linear dimensions to illustrate their independence. However, note that each of these aspects of shape can itself be represented by more than one dimension.

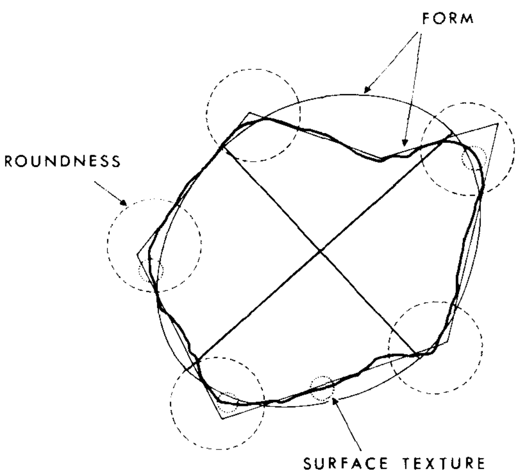

Fig. 2. A particle outline (heavy solid line) with its component elements of form (light solid lines, two approximations shown), roundness (dashed circles) and texture (dotted circles) identified.

Surface texture, the third order effect, is superimposed on the corners, and is also a property of particle surfaces between corners.

This hierarchical view of form, roundness and texture is supported by the geological behaviour of rock particles. Changes in surface texture need not affect roundness. Weathering may enhance the surface roughness of a pebble, though the well rounded corners remain easily discernible. Striae, chatter marks and other features may also be acquired without changing the roundness. This does not preclude the processes producing these textures also changing the roundness over a long period of time. Roundness of rock particles, which normally increases through abrasion, can change greatly without much effect on form. In contrast, a change in form inevitably affects both roundness and surface texture, because fresh surfaces are exposed, and new corners appear, and a change in roundness must affect surface texture, for each change results in a new surface.

PARAMETERS FOR THE ESTIMATION OF SHAPE

It is clear that no one parameter can be devised to characterize the shape of a rock particle, and indeed it is easy to see how several might be needed to describe adequately each property that contributes to shape. The precision or level of description (and hence number of parameters) will depend on the problem being studied. There are, however, at least two properties that the parameters themselves should have. (1) Each should represent an aspect that has some physical meaning, so that they can be related to the processes that determine particle shape. (2) Each should represent a combination of measurements from the same aspect of shape, that is, from the same hierarchical level.

Various parameters that estimate particular aspects of shape are discussed below, taking form and roundness in turn. Surface texture will not be considered further, as numerical parameters are yet to be devised.

Form

Almost all parameters of particle form are based on the longest, shortest and intermediate orthogonal axes (Table 2). Shape parameters should be independent of size, and therefore normally take the form of ratios of the axes. From three axes only two independent ratios can be obtained, and this is the limit for the number of independent parameters of form. Zingg's (1935) diagram, in which I/L is plotted against S/I, is an early and clear expression of this.

The concept of sphericity, as Wadell (1932, 1933)

Table 2. Parameters for estimating aspects of form from three axes L = long axis, I = intermediate axis, S = short axis, $P = I/L$, $Q = S/I$

Author	Formula	Name or description	Range
Indices of flatness			
Wentworth, 1922	$\frac{L+I}{2S}$	Flatness index	$1-\infty$
Cailleux, 1945			
Zingg, 1935	$\frac{I}{L}, \frac{S}{I}$	Ordinate and abscissa for a plot to characterize shape	0–1
Luttig (In Sames, 1966)	$\frac{I \,.\, 100}{L}$	Elongation	0–100
	$\frac{S \,.\, 100}{L}$	Flatness	0–100
Sneed & Folk, 1958	$\frac{S}{L}$	Flatness	0–1
	$\frac{L-I}{L-S}$	Flatness I^r to the long axis	0–1
Indices of sphericity			
Wadell, 1932	$\sqrt[3]{\frac{\text{Vol of particle}}{\text{Vol of circumscribing sphere}}}$		0–1
Krumbein, 1941	$\sqrt[3]{\frac{I \,.\, S}{L^2}}$	Intercept sphericity	0–1
Sneed & Folk, 1958	$\sqrt[3]{\frac{S^2}{L \,.\, I}}$	Maximum projection sphericity	0–1
Aschenbrenner, 1956	$\frac{12{\cdot}8\sqrt{P^2 \,.\, Q}}{1+P(1+Q)+6\sqrt{1+P^2(1+Q^2)}}$	Working sphericity	0–1
Other shape factors			
Dobkins & Folk, 1970	$\frac{10\left(\frac{L-I}{L-S}-0{\cdot}50\right)}{S/L}$	Oblate–prolate index (OP index)	$0-\infty$
Aschenbrenner, 1956	$\frac{L \,.\, S}{I^2}$	Shape factor F	$0-\infty$
Williams, 1965	$I-\frac{L \,.\, S}{I^2}$ if $I^2 > L \,.\, S$	Williams shape factor	0–1
	$\frac{I^2}{L \,.\, S} - +$ if $L^2 < L \,.\, S$		0–(−1)

developed it, represents a different aspect of shape. Wadell argued well for the sphere as a reference form, and considered that deviations were best represented by ratios of particle volume to the volume of the circumscribing sphere (Table 1). Although Wadell is best remembered for his demonstration that sphericity and roundness are separate aspects of shape, his sphericity is sensitive to roundness as well as form. Rounding the edges of a cube changes its Wadell sphericity but not its form. Therefore Wadell's sphericity is not a parameter of form alone, but includes a pinch of roundness, making it a difficult parameter to deal with, conceptually at least.

The differences between the procedures of Zingg and Wadell for describing particle shape were substantially reduced by Krumbein (1941a), who derived an equation for estimating Wadell's sphericity from measurement of the three orthogonal axes of a particle. The principal assumption is that the rock particle approximates an ellipsoid, Krumbein's intercept sphericity being a function of the volume ratio of the ellipsoid defined by the three axes to the circumscribing sphere. Whilst he regarded the intercept sphericity as an approximation to true sphericity Krumbein (1941a, p. 65) had in fact created a conceptually purer parameter than Wadell's sphericity, for intercept sphericity measures form alone. This was the time for the term *equantcy*, proposed recently by Teller (1976) for intercept sphericity, to be introduced.

Krumbein (1941a) recognized that lines of equal intercept sphericity plot as hyperbolic curves on Zingg's diagram (Fig. 3), but it was left to Aschenbrenner (1956) to recognize the need for a parameter to describe variations in form for particles of equal sphericity. His shape factor F (Table 2) had a range from 0 to infinity, but Williams (1965) has provided a transformation to give the shape factor a range from $+1$ to -1 (Fig. 3).

Aschenbrenner's (1956) main purpose, however, was to develop a measure of sphericity that used a reference form closer to real rock debris than an ellipsoid. He wanted a plane-sided figure and chose the tetrakaidekahedron which he thought represented a better aproximation to natural particle shape. Also it was relatively easy to handle mathematically. He took true sphericity to be the ratio of the surface area of the rock particle to the surface area of the reference form, and derived a formula that allowed sphericity to be calculated from the three orthogonal axes, using the tetrakaidekahedron as the reference form. However, he noted that it is not possible to reach a sphericity of 1·0 unless the reference form is an orthotetrakaidekahedron. Aschenbrenner arbitrarily and perhaps regrettably, set the formula for his 'working sphericity' halfway between the two (Table 2). Although he could derive a formula using the orthotetrakaidekahedron capable of yielding a sphericity of 1·0, the reference form would itself have a 'true sphericity' of only 90·1. He appears not to have recognized that the difference in sphericity values results from a difference in roundness of the reference forms.

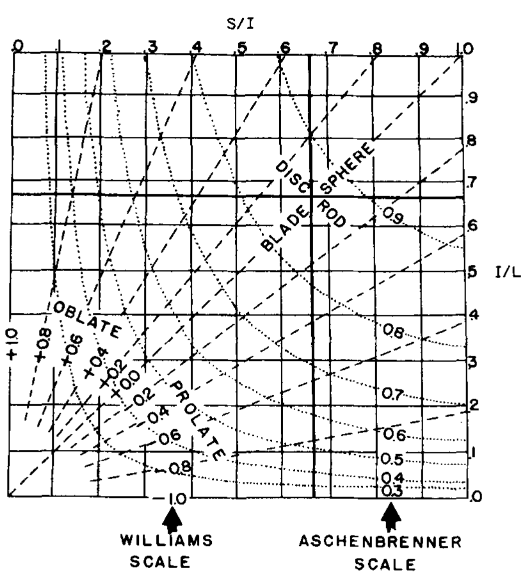

Fig. 3. Zingg's diagram, showing the relationship between the axial ratios I/L and S/I, Aschenbrenner's working sphericity and Williams shape factor (from Drake, 1970).

Sneed & Folk (1958) suggested that the sphericity of a particle should express its behaviour in a fluid. Noting that particles tend to orientate themselves with maximum projection area normal to the flow, they proposed a *maximum projection sphericity* derived from the ratio of a sphere equal to the volume of the particle to a sphere with the same maximum projection area. Sneed and Folk did not compare their measure with other measures of sphericity, but simply presented the results of a major study on river pebbles using the new measure. The widespread acceptance of their measure may reflect as much the usefulness of the results as the power of their argument for the measure. The use of behaviouristic measures can lead to problems in interpretation. A measure may be inappropriate when the behaviour assumed in deriving it may be unimportant or different in the particular situation in which one wants to use the measure. Should a measure appropriate for water-deposited pebbles be used for pebbles deposited from ice? Perhaps the answer can be avoided by noting that the formula for Sneed and Folk's measure is very close to that of intercept sphericity of Krumbein (1941a), which it was designed to replace. The only difference is that maximum projection sphericity uses the short axis as a reference, whereas intercept sphericity uses the long axis (Table 2). Thus the two formulae appear to be equally valid measures of sphericity from a conceptual point of view.

Sneed & Folk also proposed the use of a triangular diagram for plotting pebbles' form, the three poles representing platy, elongated and compact (equant) pebbles (Fig. 4). Unlike most such diagrams where the location of a point is determined by the proportions of the three end members, the location here is determined by the value of the apex end member—compactness, measured by S/L, and a proportion $(L-I)/(L-S)$ measured parallel to the base, which divides pebbles into three classes, platy, bladed and elongated. The diagram emphasizes the fundamental character of these shapes, and the way in which they converge on a single type, compact.

The relationship between the form triangle and maximum projection sphericity is similar to that between Zingg's diagram and intercept sphericity. For each maximum projection sphericity value there is a unique curve on the form triangle. The need for a complementary shape property was not immediately recognized, but in 1970 Dobkins & Folk offered the OP index (oblate–prolate index, Table 2), which was based on the ratio $\frac{L-I}{L-S}$ though it also took into account degree of compactness. The OP index ranges from $-\infty$ to $+\infty$, unlike most shape measures, which range from 0 or -1 to $+1$.

The measures proposed by Folk and his students allow the same pebble data to be plotted in two different ways (Fig. 4): (1) sphericity against OP index on orthogonal axes; (2) S/L against $(L-I)/(L-S)$ on triangular graph paper (the form diagram).

In the equivalent diagram using the procedures of Zingg, Aschenbrenner and Williams (Fig. 3), the same pebble data can be plotted as: (1) I/L against S/I; (2) Aschenbrenner working sphericity against Williams shape factor.

Each of the four plots derives from the same basic data, the lengths of the three principal axes. Therefore a trend in one diagram cannot be legitimately confirmed by a similar trend in another.

The only other common form index that uses the same three axial measurements is the flatness index of Wentworth (1922a) (Table 2). The index was adopted by Cailleux (1945) and now his name is commonly associated with it.

As each pair of measures expresses the same information they are now compared, using two criteria, namely: (1) their effectiveness in discriminating between different shapes, measured by the ratio of range to average standard deviation; and (2) the degree to which each measure follows a normal distribution, extreme deviations making a measure difficult to use for statistical tests.

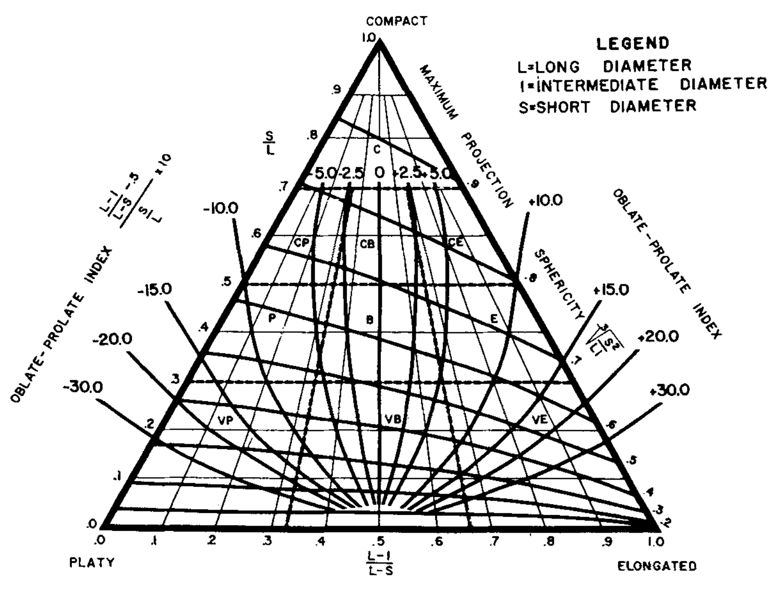

Fig. 4. Folk's form diagram, showing the relationship between the defining parameters S/L and $(L-I)/(L-S)$, and maximum projection sphericity and oblate–prolate index (from Dobkins & Folk, 1970).

The data used for the evaluation are from pebbles in the range 8–64 mm, collected and organized into sets of 20–30 pebbles. Each set represents a particular rock type and sedimentary environment. The pebbles came from two areas, Hooker Glacier in the Southern Alps of New Zealand (20 sets and 597 pebbles), where the rock types distinguished were quartz schist and pelitic schist, and Taylor Valley, Antarctica (29 sets and 706 pebbles), where the rock types sampled were granite, porphyry, vein quartz, dolerite and basalt. The environments sampled in both areas include the subglacial (basal till), superglacial (talus and stream deposits), ice-marginal streams and proglacial streams.

The total range of mean values was found for each measure from both suites of pebbles, and divided by the average standard deviation (Table 3). Whereas most measures show some difference in the average mean between Hooker and Taylor samples, representing differences in average shape, the average standard deviations are all very similar, indicating a similar natural variability in the measures regardless of their mean value. The two pairs of sphericity and oblate–prolate measures have a similar high ratio of range to standard deviation, showing that there is little to choose between them as effective discriminators of form. Both are clearly better than either of the simpler ratios of axes. The value obtained from Wentworth flatness is the highest of all, though the effectiveness of the measure is reduced by not being paired with a complementary form measure.

The shape of the frequency distribution for each measure was examined by comparing the average skewness and kurtosis values for each suite of pebbles (Fig. 5). Only two of the nine measures have a skewness value clearly different from 0. Aschenbrenner's sphericity is strongly negatively skewed, suggesting that the distribution may be constrained by some practical upper limit of sphericity. Wentworth flatness showed, on average, a strong positive skewness. Most measures have a peak that is broad compared with a normal distribution (platykurtic) but the values are not extreme. Again Aschenbrenner sphericity and Wentworth flatness stand out from the rest by showing a pointed (leptokurtic) frequency distribution, and they have a larger variability in kurtosis values than the other measures. In every case for both skewness and kurtosis there is no significant difference in the mean value from each suite of pebbles, suggesting that the features of the frequency distributions described above are of a general nature. Of the measures examined therefore,

Table 3. Mean ($\bar{x}$), standard deviation (s) and the ratio of range in mean to standard deviation for pebbles from Hooker Valley (20 sets) and Taylor Valley (29 sets) to express the effectiveness of each measure in discriminating aspects of form

Measure		Average $\bar{x}$	Range in $\bar{x}$	Total range	Average s	Range in $\bar{x}$ / s
Max. projection sphericity	Hooker	0·62	0·47–0·72	0·31	0·10	3·1
	Taylor	0·69	0·53–0·78		0·10	
Oblate–prolate index	Hooker	−0·54	−6·59–3·05	11·54	5·71	2·5
	Taylor	−0·32			4·82	
S/L	Hooker	0·42	0·28–0·54	0·32	0·11	2·9
	Taylor	0·49	0·35–0·60		0·11	
$(L-I)/(L-S)$	Hooker	0·49	0·37–0·61	0·45	0·21	2·1
	Taylor	0·50	0·33–0·78		0·22	
Aschenbrenner working sphericity	Hooker	0·83	0·70–0·89	0·22	0·07	3·4
	Taylor	0·87	0·77–0·92		0·06	
Williams shape factor	Hooker	0·16	0·01–0·41	0·62	0·28	2·3
	Taylor	0·11	−0·21–0·40		0·25	
I/L	Hooker	0·72	0·64–0·77	0·17	0·13	1·4
	Taylor	0·75	0·66–0·81		0·12	
S/I	Hooker	0·59	0·40–0·71	0·46	0·15	3·1
	Taylor	0·67	0·46–0·86		0·15	
Wentworth flatness index	Hooker	2·31	1·71–3·75	2·26	0·70	3·8
	Taylor	1·91	1·49–2·81		0·49	

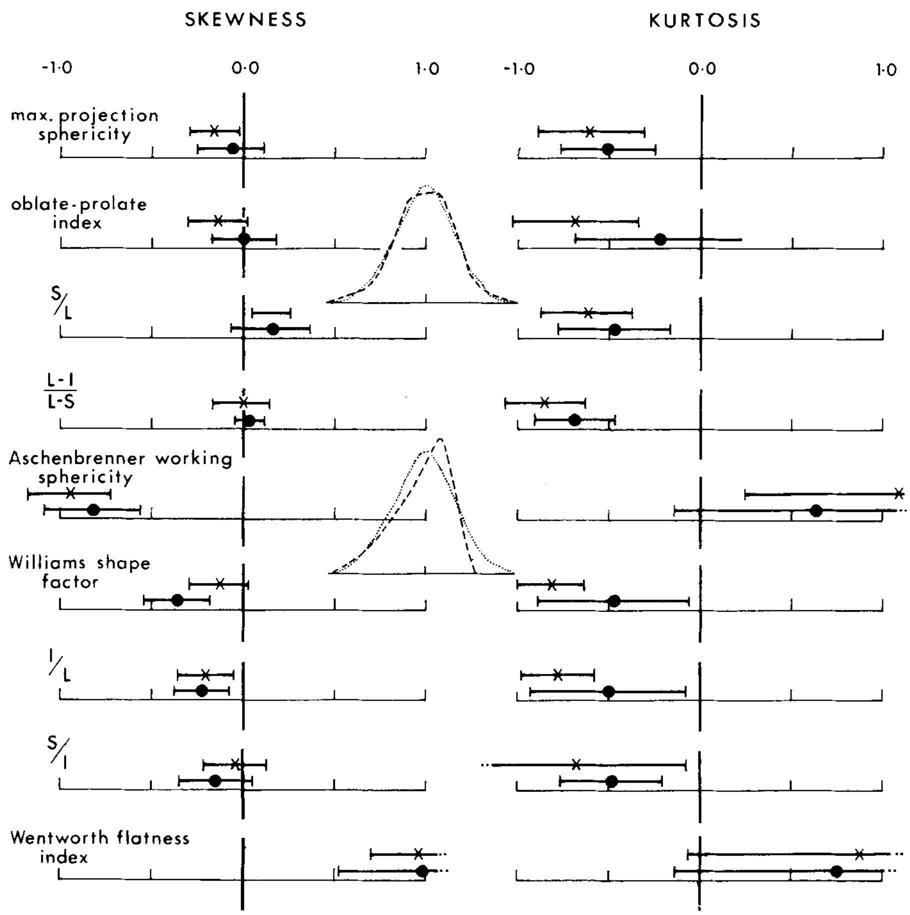

Fig. 5. Average skewness and kurtosis for various form measures (with 95% confidence intervals) for pebbles from the Hooker Valley (●, 20 samples) and Taylor Valley (×, 29 samples). Sketches indicate the way in which the frequency distributions for oblate–prolate index and Aschenbrenner working sphericity differ from a normal distribution (dotted).

maximum projection sphericity and oblate prolate index are the most satisfactory for describing form, in that they both approximate the normal distribution, and are relatively efficient discriminators.

The measures reviewed above are all based on three orthogonal axes, and are not satisfactory discriminators of some forms. In particular they do not separate particles with triangular, rectangular and pentagonal cross-sections. Perhaps this is why Holmes (1960) used verbally defined categories instead of ratio scale measures for describing pebbles from Pleistocene till.

The forms above can clearly be distinguished on the number of sides (or angles). This aspect of shape seems therefore to be an aspect of form, and not roundness, at least when the number of sides is small. Rounding of the corners can vary without affecting the number of sides, though not beyond the point when an entire side is lost. Several practical problems not yet overcome are: (1) being consistent in deciding on the number of (planar) sides for many rock particles with irregular forms; (2) distinguishing the number of sides objectively on particles with a large number of sides and corners; and (3) resolving planar sides on rounded particles.

A measure that takes some account of this aspect of form is that of Lees (1964), who proposed the following measure for the degree of angularity:

$$\sum_{i=1}^{3} \sum_{j=1}^{n} (180^\circ - \alpha_{ij}) \frac{x_{ij}}{r_i}$$

with a range from $0 \rightarrow \infty$ where α_{ij} is the angle of each corner and x_{ij} is the distance of the corner from the centre of the maximum inscribed circle (radius r_i) for each of the three sections through the long, intermediate and short axes of the rock particle.

Defined this way angularity is increased by (1) increase in acuteness of the corners; (2) increase in number of corners; (3) increase in relative distance of corners from the centre of the particle. Although (1) and (2) are clearly aspects of form (the angle of a corner being geometrically independent of its roundness), (3) includes elements of both form and roundness, for particles with rounded corners will have a lower x/r ratio than particles with sharp corners but the same acuteness of angle and number of corners (Fig. 6). For angularity to be a measure of form alone, the distance x', and not x, should be measured.

Roundness

The claim that roundness of rock particles is a property independent of sphericity was first made by Wadell (1932), and was immediately attacked by Wentworth (1933). Wentworth argued that the distinction defied common usage of the terms, on the grounds that roundness is a property best displayed by a sphere. Wadell (1933) responded with points from both the dictionary and common usage, supporting his use of roundness, which incorporated a sense of smoothness or lack of angularity. He observed that whilst roundness is an essential property of a sphere many other forms could be equally well rounded. Wadell's view prevailed.

Measurement of roundness has always posed difficulties quite different from those of form measurement. Although roundness is clearly a three-dimensional (3D) property all methods of measuring roundness to date have begun with 2D

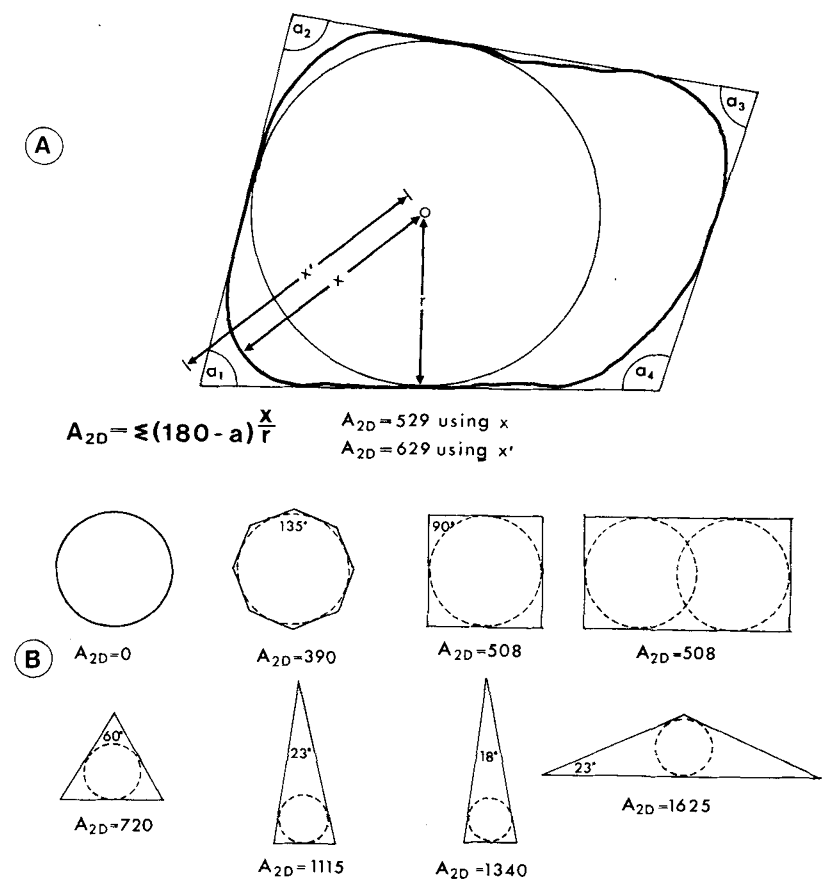

Fig. 6. (a) Construction and formula for determining 2D angularity of a particle by the method of Lees (1964). (b) Values for degree of angularity (A_{2D}) for various regular figures (mainly from Lees, 1964, Fig. 8, with a corrected value for the octagon).

projections of the particle. A truly 3D roundness measure would involve fitting 3D reference surfaces (such as spheres of varying radii) to all corners on the pebble surface. Wadell (1932) believed that measurement of roundness from a 2D projection did not cause serious bias, and everyone since has implicitly agreed, though perhaps largely because of the impracticality of measuring true 3D roundness.

Wadell's procedure, as well as his concept of roundness, has survived with little change. The key to it is the corner, which he defined as 'every such part of the outline of an area (projection area) which has a radius of curvature equal to or less than the radius of curvature of the maximum inscribed circle of the same area'. Each corner of the maximum projection outline is measured with a template of concentric circles of known diameter by finding the largest circle that will fit. Most pebbles have between two and six corners, as defined above; the diameters are averaged and divided by the maximum inscribed circle to provide a measure of average roundness (Table 4). Swan (1974) has suggested that the limitation on the size of corners was a matter of convenience to get a measure that would not exceed 1·0, but Wadell's limitation on what constitutes a corner is essential for separating the corners from the rest of the particle outline. If the limit were a straight part of the outline, all pebbles with convex outlines would consist entirely of corners, some of which would have a radius of curvature of almost infinite size. This would certainly be out of keeping with the common notion of a corner. Because of the time-consuming nature of the procedure, Krumbein (1941a) prepared a set of pebble images of predetermined Wadell roundness from 0·1 in steps of 0·1 to 0·9 for faster roundness determination. The price of faster measurement is the higher subjectivity of the values obtained, because an entire image is being compared, rather than a set of single corners one at a time. Another source of error derives from Krumbein's images

Table 4. Parameters for estimating roundness. L = longest axis, I = intermediate axis, S_m = short axis in maximum projection plane, $D_s(D_{s1}, D_{s2})$ = diameter of circle fitting sharpest corner (two sharpest corners), D_x = diameter of pebble particle through D_s, D_i = diameter of inscribed circle, D_k = diameter of circles fitting corners, n = number of corners, C = circumference of particle

Author	Formula	Name or description	Range
Roundness of sharpest corners			
Wentworth (1919)	D_s/D_x	Shape index	0–1
Wentworth (1922)	$\dfrac{D_s}{(L+S_m)/2}$		0–1
Cailleux (1947)	$\dfrac{D_s \times 1000}{L}$	Cailleux roundness index	0–1000
Kuenen (1956)	$\dfrac{D_s}{I}$	Kuenen roundness index	0–1
Dobkins & Folk (1970)	$\dfrac{D_s}{D_i}$	Modified Wentworth roundness	0–1
Swan (1974)	$\dfrac{(D_{s1}+D_{s2})/2}{D_i}$		0–1
Average roundness of corners			
Wadell (1932)	$\dfrac{(\sum_{k=1}^{n} D_k)/n}{D_i}$	Wadell roundness	0–1
[note also pebble comparison charts in Russell & Taylor (1937), Krumbein (1941) and Powers (1953), all of which are keyed to Wadell roundness]			
Average roundness of outline			
Szadeczky-Kardoss (in Krumbein & Pettijohn, 1938)	$\dfrac{\text{Length of convex parts of } C}{\text{Total length of } C} \times 100$		0–100

having corners all of similar curvature, whereas many natural pebbles have corners with a range of curvatures. This is particularly evident in the three classes of least rounded pebbles. However, as long as the operator compares corners, and not the whole shape, the roundness values obtained should be as accurate as other limitations of the procedure will allow (Krumbein, 1941a, p. 72).

Another pebble comparison chart developed from the idea that particle roundness can best be described as a series of five stages of development, i.e. from angular to very rounded, each stage being characterized by different features (Russell & Taylor, 1937). They transformed the data from ordinal to ratio scale by keying the stage boundaries to Wadell's average roundness values, and since then further modifications have been made. Pettijohn (1949) noted that by moving stage boundaries slightly the intervals represented a geometric progression. Powers (1953) added another class (very angular), and Folk (1955), noting the geometric progression in interval size, proposed a logarithmic transformation to give a roundness scale from 1 to 6 (the rho scale). However, it is difficult to see how distinct natural classes of rounding can develop from continuous or even episodic abrasion. If this point is conceded and it is agreed that roundness forms a continuum, then the most useful comparison chart will be the one with the largest number of classes, as long as adjacent classes can be distinguished. This makes Krumbein's chart the most satisfactory of those available, although the verbal classes of roundness are still useful for purposes of discussion.

The importance of the sharpest point on the outline of a particle was first recognized by Wentworth (1919), who proposed as a measure of shape the ratio of the diameter of curvature of the sharpest corner to the diameter of the particle through that point (not the longest diameter, as stated by Dobkins & Folk, 1970, among others). He later changed the divisor to the average of the long and short diameter of the particle in the plane of projection (Wentworth, 1922b). Cailleux (1947) proposed a similar measure of roundness, namely the diameter of the sharpest corner to the longest diameter of the particle (long axis). This measure has been criticized by Kuenen (1956), Dobkins & Folk (1970), Swan (1974) and Folk (1977), because it confuses both roundness and form in the same measure. It is difficult to understand the continued use of Cailleux's measure (e.g. Briggs, 1977) in the face of this substantial objection. Kuenen modified the Cailleux formula by replacing the long axis with the intermediate axis, whereas Dobkins and Folk suggested using the largest inscribed circle, and Swan proposed a modification of the Dobkins and Folk procedure by averaging the diameters for the two sharpest corners (Table 4).

A concept of roundness independent of the character of corners was proposed by Szadeczky-Kardoss in 1933 (see Krumbein & Pettijohn, 1938; Müller, 1967). The measure, ρ, is the percentage of convex parts along the circumference of a rock particle, and is obtained from an enlarged image using a measuring wheel, or by measuring the total angular distance about the centre of the inscribed circle subtended by convex parts of the circumference. A complementary measure of angularity (sum of angles subtended by plane sides 360°) was proposed independently about this time by Fischer (see Krumbein & Pettijohn, 1938).

In contrast to Wadell's roundness, which estimates degree of curvature, ρ compares the direction or sense of curvature. A visual comparison chart of images with values in the range 10–90% has been prepared by Sames (1966). However, roundness measured in this way is rarely reported in the English-language literature. In practice, it is difficult to locate the boundary between convex, planar and concave segments of the circumference of the particle because they grade into each other. Müller (1967) reported a precision of 3%, but this relates only to measurement and appears not to take account of differences in locating the limits of convex segments.

Each of the three groups of measurements in Table 4 estimates a different aspect of roundness. Roundness estimated from corners has a clearer physical meaning than that based on convexity of the outline, and while the validity of the latter approach is not denied, it is not considered further here. Of the others only Wadell's procedure measures average roundness, and of the various charts available for visual estimation Krumbein's (1941a) is preferred. Of those measures estimating roundness from the sharpest corner, the Modified Wentworth roundness (Dobkins & Folk, 1970) is the most desirable. It uses as denominator a length (diameter of the largest inscribed circle) that can be taken at the same time as that of the sharpest corner from the image of the rock particle, whatever its scale. Swan's (1974) modification of the measure (by averaging the two sharpest points), is unsatisfactory because it yields a value somewhere between the least and the average roundness. If one is going to measure more than the one sharpest corner one might as well measure the total of four or five needed to calculate Wadell sphericity, a long established measure with a clear meaning.

Dobkins and Folk (1970) implied that the Modified Wentworth roundness is more objective and quantitative than any other procedure for measuring roundness, including Wadell's, but in both respects it is no better and no worse. The problem of recognizing corners (one of their objections) is inherent in both methods, and it is in fact most vexing for small angular coarse-grained particles, where it may be difficult to distinguish the sharpest corner from a textural feature. Dobkins & Folk (1970, p. 1170) argued that the sharpest corner provided the best measure of roundness, because it 'best reflects the amount of rounding going on in the latest environment.' This is a good point for river pebbles, but in other situations, for example, beneath glaciers, the level of rounding attained, rather than roundness change since last breakage, may be of most interest.

Wadell roundness using Krumbein's visual comparison chart is now compared with Modified Wentworth roundness, again using the glacial and fluvioglacial pebbles from Hooker Valley and Taylor Valley (p. 297). The discriminating power of each measure is about the same (Table 5). However, whereas the average skewness and kurtosis values of the Wadell–Krumbein measures are approximately normally distributed, values for the Wentworth measures show them to be consistently positively skewed, with kurtosis highly variable. This is not so much a feature of the measure, but of the restricted range in roundness of the particles, close to the lower bound of the measure. The distribution of Modified Wentworth roundness values for beach and river pebbles, which are typically in the range 0·25–0·80, is approximately normal (Folk, 1972). The data presented here suggest that the measure should be used with care where roundness values are low.

CONCLUSIONS

Difficulties in using aspects of shape as geological evidence remain considerable. Even the best of the commonly used procedures have their limitations for both description and hypothesis testing because their discriminating power is low compared with most other sedimentological procedures (e.g. measurement of size, composition, orientation). In addition, each measure depends at some point on subjective assessment, whether it be the location of the three orthogonal axes or the fitting of arcs to particle outlines. Further limitation arises from the observation that shape may be determined by a large number of natural variables interacting in different ways (Krumbein, 1941b; Kuenen, 1956), but can be assessed so far by only three independent measures of form (sphericity, oblate–prolate index, angularity) and one independent measure of roundness (all three types discussed above are related).

The seductive ease with which measurements of form and roundness can be made is in contrast to the difficulties encountered in ascribing a geological meaning to them. Causal relationships between aspects of shape and the natural variables that lead to them can be established only where the variables are controlled, where the operator error can be assessed and where a large number of measurements can be made (to narrow the confidence interval on mean values). Confidence in the inferences made about ancient sediments on the basis of particle shape studies must also depend to a considerable degree on independent knowledge of the natural variables (rock type, particle size, flow direction,

Table 5. Comparison of Wadell roundness using Krumbein's comparison chart and Modified Wentworth roundness for suites of pebbles from the Hooker Valley (20 sets, 597 pebbles) and Taylor Valley (29 sets, 706 pebbles) $\bar{x}$-mean, s-standard deviation, Sk-skewness, K-kurtosis

Measure	Suite	Average $\bar{x}$	Range in $\bar{x}$	Average s	Total range s	Average Sk (=0 for normal distribution)	Average K (=0 for normal distribution)
Wadell by Krumbein	Hooker	0·41	0·16–0·63	0·10	4·8	0·06(±0·34)	0·18(±0·43)
	Taylor	0·33	0·15–0·50	0·10		0·27(±0·23)	−0·31(±0·39)
Modified Wentworth	Hooker	0·14	0·02–0·35	0·08	4·7	0·96(±0·40)	1·25(±1·22)
	Taylor	0·09	0·02–0·19	0·06		1·10(±0·21)	0·78(±0·63)

character of flow). Therefore unambiguous interpretations from shape studies of ancient sediments can be expected only from carefully planned studies of rather unusual geological situations.

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