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TABLES 1–4, AUGUST, 1941

MEASUREMENT AND GEOLOGICAL SIGNIFICANCE OF SHAPE AND ROUNDNESS OF SEDIMENTARY PARTICLES

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ABSTRACT

Methods are described for the rapid measurement of shape and roundness of large sedimentary particles. Shape is determined from the long, intermediate, and short diameters of the particles, and roundness is measured by a rapid visual method. The importance of distinguishing between these properties is discussed. Shape and roundness behave differently during abrasion and selective transport, and hence are important in evaluating the effects of these processes during the formation of the deposit.

INTRODUCTION

Two fundamental properties of sedimentary particles which have been much confused are shape and roundness. Some workers apparently consider the distinction a minor one; others recognize its importance, but do do not make the distinction because of the time required. The present paper describes rapid but rigorous methods for the measurement of shape and roundness which reduce the time required per sample to a matter of minutes instead of hours. The paper also discusses the distinction between these attributes on the basis of the behavior of particles during abrasion and transportation to indicate the fundamental importance of the concepts in evaluating the past history of sedimentary deposits.

SHAPE AND ROUNDNESS

Wadell (1932) was the first to show that the commonly used terms "shape" and "roundness" were not synonymous, but really included two geometrically distinct concepts. The shape of a particle is its form, entirely independently of whether the edges or corners are sharp or round. Fundamentally the shape is a measure of the ratio of the surface area of a particle to its volume. For a sphere, this ratio is a minimum, and for all other forms it is larger. Hence the ratio of surface area to volume indicates how closely or remotely the particle approaches a sphere in form. For practical purposes this ratio is difficult to measure, and the actual measurement is expressed in terms of the ratio of the volume of the particle to the volume of its circumscribing sphere. The cube root of this ratio is called the *sphericity* of the particle.

The roundness of a particle is a measure of the curvature of the corners and edges expressed as a ratio to the average curvature of the particle as a whole, independent of its form. For practical purposes the "average curvature" is expressed in terms of the inscribed circle drawn on a projection of the particle in a plane.

One may gain a clear picture of the difference between sphericity and roundness by making several simple tests on actual pebbles. A pebble is held in such a manner that the forefinger and thumb form a circle about the long axis of the pebble. If the pebble occupies only a small part of this "circumscribed sphere," the ratio of its volume to the volume of the sphere is small, and it has a low sphericity. If the pebble essentially fills the "sphere," it has a high sphericity. To visualize roundness, a different technique is used. The pebble is held with its long axis between the extended thumb and forefinger in such a manner that one views the largest projection of the pebble. The largest inscribed circle which can be drawn on the projection is now visualized, and with this in mind, the radius of each corner on the projection is compared with the radius of the inscribed circle. If the corner radii are small in comparison, the roundness is low. If the radii of the corners are of the same magnitude as the inscribed circle the roundness is high. It may be noted, incidentally, that a pebble of high sphericity may have either high or low roundness, as may a pebble of low sphericity—the two concepts are geometrically distinct.

RAPID METHOD FOR SHAPE MEASUREMENT

Wadell's method for determining the sphericity of a pebble requires measurement of the volume of the pebble and of its longest diameter. The diameter of a sphere having the same volume as the pebble is then calculated, and the ratio of this "nominal diameter" to the long diameter is the sphericity of the pebble. In the writer's method only the long, intermediate, and short diameters of the pebble are measured; the sphericity is read directly from a chart by means of two ratios between pairs of the diameters. This "intercept method" reduces the time required to a small fraction of the standard method, and yet the average values of the two methods agree within a few per cent.

The writer's intercept method is based on a triaxial ellipsoid as the reference solid to which the pebble is compared.[a] It is therefore necessary to define the three diameters of the pebble as mutually perpendicular intercepts. This requires some care in choosing the proper diameters to measure. For accurate work a formal routine is followed, which may be even further shortened for approximate field measurements. The formal routine requires at most between one and two hours for 50 pebbles; the field method reduces this to about 30 minutes.

The formal routine is described in detail to establish the principles involved; thereafter the shortened versions may be adjusted to the needs of the individual worker. In measuring the diameters it is convenient to have a sliding rod of the type shown in figure 1. The steps involved are:

1) The pebble is held in position and the longest intercept through it is measured as shown in figure 1.
2) The pebble is then held between the thumb and forefinger by the ends of this longest diameter, and rotated until the largest sec-

[a] The mathematical theory on which the intercept method is based is given in the Appendix at the end of this paper, which also gives comparative figures on the accuracy of the method.

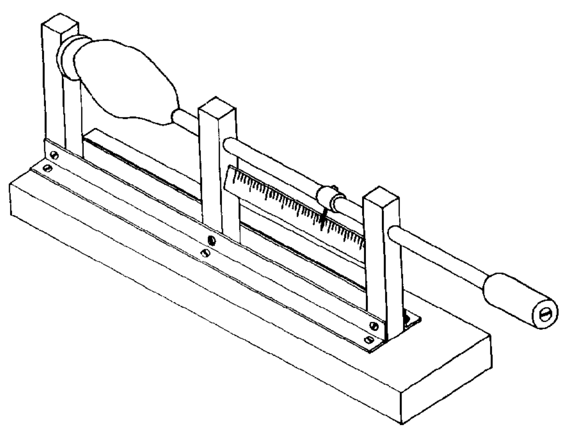

FIG. 1.—Sliding rod caliper for measuring pebble diameters. The rod and uprights are brass, and the device is mounted on a wooden base.

tion is seen by the eye. (This is equivalent to finding the position in which the pebble would cast its largest shadow in a parallel light beam.)

3) The pebble is held with this "maximum projection plane" horizontal, and the widest part is measured in a direction perpendicular to the long axis. (See figure 2, left.)

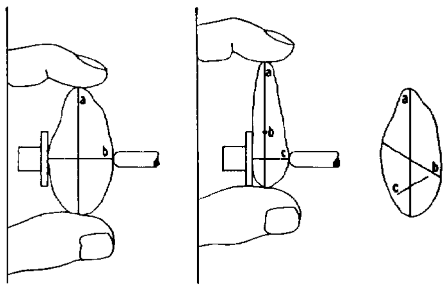

FIG. 2.—Measurement of pebble diameters. *Left*, the b-axis in position; *center*, the c-axis in position. *Right*, the pebble in perspective.

4) With the pebble still held by its long diameter, the maximum projection plane is rotated to a vertical position, which places the short pebble diameter in a horizontal plane.
5) The widest part of the pebble in the horizontal plane is measured, also perpendicular to the long axis. This is the short diameter of the pebble. Figure 2 (center) shows this step, and figure 2 (right) shows the three axes through the pebble. It may be noted that the three diameters do not necessarily intersect at any one point, although they are mutually perpendicular.

The three diameters are designated as a, b, and c for the long, intermediate, and short axes respectively, and are recorded as shown in table 1. Two ratios are then determined with a slide rule, and entered in adjacent columns. The first is the ratio of the intermediate to the long diameter (b/a), and the second is the ratio of the short to the intermediate diameter (c/b). To determine the sphericity, these ratios are laid off on the axes of the chart (figure 3), and where the values intersect

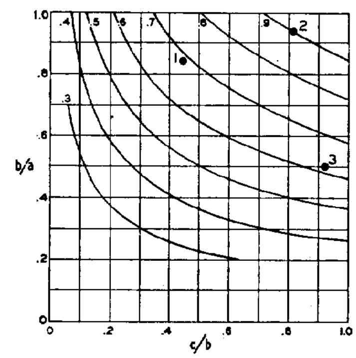

FIG. 3.—Chart for determining sphericity. The curves represent lines of equal sphericity. The numbered circles refer to the examples in table 1.

in the diagram the sphericity may be read off to the nearest two-hundredths. The figure shows the location of the three pebbles in table 1, and the corresponding interpolated sphericities are entered in the last column of the table.

The space required for describing the method is out of proportion to the time actually required, but it insures an added accuracy if the diameters are measured as indicated. The use of a table and the calculation of the ratios may be eliminated by using an "eye-ratio" method which is suitable for rough field measurements. The pebble is held with its long axis lined up with a pencil, and the length of the long axis is indicated by setting

TABLE 1. *Arrangement of data for intercept sphericity*

Pebble	Long Diameter in mm.(a)	Intermediate Diameter (b)	Short Diameter (c)	b/a	c/b	ψ
1	45	38	17	0.84	0.45	0.68
2	51	48	39	.94	.81	.90
3	48	24	22	.50	.92	.62

the thumb nail on the pencil. The pencil is then held parallel to the intermediate diameter, and the ratio estimated by eye to the nearest tenth. This yields the b/a ratio. The thumb nail is then moved to indicate the length of the intermediate diameter, and the pencil held along the short diameter to estimate the c/b ratio. These two ratios are then located on the chart and the sphericity read off.

Regardless of whether the formal routine or the eye-ratio method is used, it should be emphasized that individual pebbles may not agree closely in value with the true sphericity as measured by Wadell's method. However, sets of 25 or more pebbles will afford an average sphericity value very close to Wadell's average value. The average sphericity is found by adding the individual sphericities and dividing by the number of pebbles. The lack of agreement between individual pebbles is due to the fact that whereas single pebbles may not be approximated very closely by a triaxial ellipsoid, a group of pebbles will tend statistically to satisfy that condition.

RELATION BETWEEN SPHERICITY AND ZINGG SHAPE CLASSES

Some years ago Zingg developed a classification of pebble shapes based on the b/a and c/b ratios. The present method of determining sphericity permits a direct reconciliation of Zingg's and Wadell's shape concepts. Zingg set up four classes as shown in figure 4, and defined the classes by the boundaries indicated in table 2. The writer merely

TABLE 2. *Zingg's classification of particle shape*

Class	b/a	c/b	Shape
I	$>2/3$	$<2/3$	Disks
II	$>2/3$	$>2/3$	Spherical
III	$<2/3$	$<2/3$	Blades
IV	$<2/3$	$>2/3$	Rod-like

combined figures 3 and 4 to obtain figure 5, which affords a standard form for the intercept sphericity. Inasmuch as the intercept sphericity is based on any values of b/a and c/b, these ratios may be plotted as points on figure 5, whereupon not only may the sphericity be

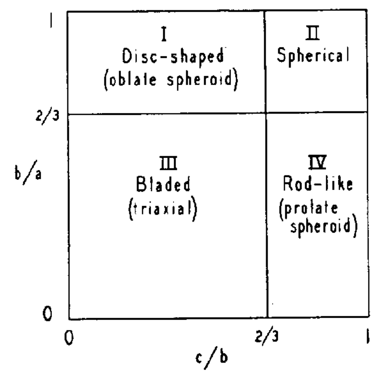

FIG. 4.—Zingg's classification of pebble shapes. (See table 2.)

read, but the pebbles are automatically classified according to Zingg. If the reader will draw the lines of figure 4 on figure 3, he will see that pebble 1 of table 1 is a disk, pebble 2 is spherical, and pebble 3 is cylindrical or rod-shaped.

In practice, the writer uses a separate sheet for each sample of pebbles, with the values indicated by dots, whereupon they may be visualized as a population. The average values of the b/a and c/b ratios may be plotted as a triangle, to indicate the average sphericity of the set. In any study of a set of samples the distribution of the points from sheet to sheet affords data for correlations by sphericity, or may be used to study changes in shape due to selective transportation, etc. Comparisons from sample to sample are most convenient when similar size-ranges are used for both. The writer uses the size range from 16 to 32 mm. for rapid comparisons because of the ease of handling that size.

The present method of shape determination does not lend itself to small particles in which the three axes cannot be separately measured. Sand grains at rest on a microscope slide usually have their short axes nearly vertical, so that the outline seen gives the *a* and *b* axes only.

RAPID METHOD FOR ROUNDNESS ANALYSIS

Wadell's method for measuring the roundness of a pebble requires drawing an image of the pebble at a standard magnification, and determining the radii of the inscribed circle and of the edges and corners of the image. From these data a ratio is calculated which indicates the relative size of the average radius of curvature to the radius of the inscribed circle. This value is the *roundness* of the pebble.

In the writer's method the pebble is compared with standard images of known roundness, and a roundness value assigned to it. The visual roundness method is not a qualitative estimate of roundness such as is commonly used to describe pebbles. The images used for comparison were redrawn from pebbles which were measured by Wadell's method, and statistical studies have shown that the average values agree well

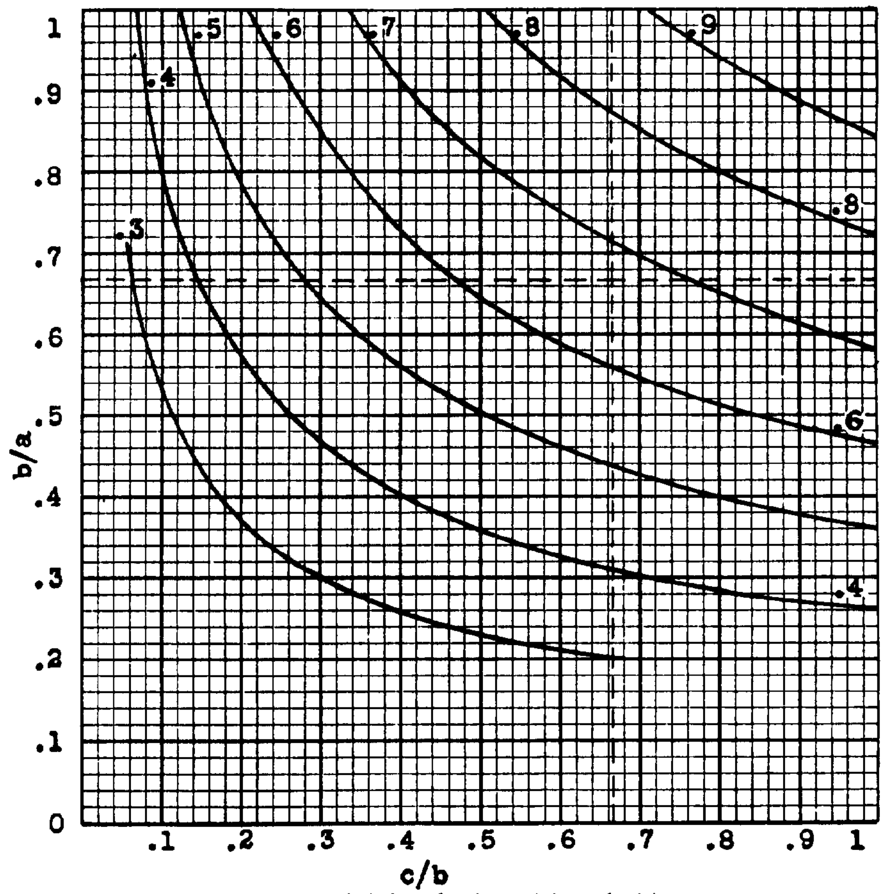

FIG. 5.—Detailed chart for determining sphericity.

with Wadell's values. (See Appendix for details.)

Plate 1 has ten sets of standard images, with long diameters of about 25 mm., for use directly with pebbles in the 16–32 mm. size range. The sets vary in roundness from 0.1 to 0.9. Several broken pebbles are also shown, to indicate the effects of such breakage on roundness. By enlarging or reducing the images photographically, similar sets may be made for any size range. In using the figures, the pebble is held with its long axis between thumb and forefinger in such a manner that one sees the largest projection of the pebble. In this position, the pebble is held against a contrasting background and viewed in silhouette. It is compared with the images, and a value assigned to it. For example, if the pebble is matched by one in the third rectangle from the left, upper row of plate 1, the value 0.3 is assigned to it. A set of pebbles (preferably about 50; but at least 25) is so measured, and the average roundness calculated. The time required is perhaps 30 minutes for 50 pebbles.

In determining the average roundness, a rapid tally method is convenient, as shown in table 3. The first column lists the roundness values; the second column shows the number of pebbles in the sample having these values. The values in the third column are found by multiplying column 1 by column 2. The sum

TABLE 3. *Tally sheet for computing mean roundness*

Roundness, P	Number of particles, f	$P \times f$
0.2	3	0.6
0.3	7	2.1
0.4	19	7.6
0.5	12	6.0
0.6	9	5.4
	50	21.7

of the third column is divided by the number of pebbles to obtain the arithmetic mean roundness. In the example this value is $21.7/50 = 0.43$.

Certain precautions should be followed in determining roundness visually. The pebble should be viewed only in the position specified, because this agrees with the routine originally established by Wadell, and corresponds statistically to the position of rest of pebbles and sand grains when viewed from above. Wadell's roundness is thus expressed in terms of the maximum projection plane of the pebble. There can be no objection to any other orientation if one wishes to use it, except that the values in that case cannot be directly compared with Wadell's values. Similarly, by using arbitrary numbers for each class, a roundness value may be obtained, but again the numbers cannot be directly compared.[2] It is also important to notice all the smaller corners and edges in the silhouetted pebble, because it is these small curvatures which largely control the roundness value. A comparison of the several images in plate 1 will indicate the importance of this point. Rounded pebbles which have been broken require special precautions because the sharp corners on the broken edges reduce the average roundness very markedly. An approximate rule to follow in such cases is to assign a roundness number to the unbroken part of the pebble, and then to halve this number. If the rounded part is matched by a pebble of roundness 0.8, the number to assign to the pebble is 0.4. For odd values, as 0.7, use the smaller half, 0.3. The several examples in the lower right rectangle of plate 1 show actual values.

In any study of roundness, it is advisable to divide the pebbles into size classes and to determine the average roundness for each size class separately. There generally is a marked difference in the roundness of different sized particles in any given sample, so that compari-

[2] Russell and Taylor, for example, used a visual method in which standard sets of grains were numbered arbitrarily, although the original grains had been measured by Wadell's method. The arbitrary numbers varied from 1 up, which departs from the geometrical picture of Wadell's roundness, inasmuch as the latter measures the approach of the curvature of the corners to the curvature of the particle as a whole, and hence varies from 0 to 1. (See R. Dana Russell and R. E. Taylor, "Roundness and shape of Mississippi River Sands," *Jour. Geol.*, vol. 45 (1937), pp. 225–267.

sons from sample to sample are more convenient in terms of a restricted range of sizes.

The roundness figures are suitable for particles of any size, and by using enlargements or drawings of loose grains under the microscope, their roundness may be found as with pebbles. For the binocular microscope a set of photographs from plate 1, reduced to the same scale as the apparent grain size seen by the eye, may be used for rapid estimates.

IMPORTANCE OF SHAPE AND ROUNDNESS IN SEDIMENTARY STUDIES

Geologists are interested in sediments in part for the light they shed on the source of the material, the distance of transport, and the environmental conditions under which the deposit was formed. The basic assumption in these studies is that each sediment is a response to a definite set of environmental conditions, and that the properties of the sediment reflect these conditions to a greater or lesser degree. Reconstruction of the environment, therefore, depends upon an interpretation of the properties of the sediment, supplemented by studies of the fossil content of the rocks and other features.

This point of view is held in common by most workers, but its full implications are sometimes overlooked. Among points often not emphasized is the important one that it may not be possible, *a priori*, to predict which specific properties of the sediment will be most important in interpreting any particular part of its history. Some workers concern themselves with one or two quantitative features of the sediments, such as heavy mineral content or size of particles, and base their interpretations on them. These two are not by any means the only quantitative characteristics of sediments which reflect their history, and it is not even certain that they are the most significant.

The fundamental properties of sedimentary particles are size, shape, roundness, mineralogical composition, surface texture, and orientation. (The latter refers to the orientation in space of the principle axes of the particle while it remains in its parent deposit.) These six attributes control such mass properties as color, average density, porosity, permeability, "fabric," and so on. The complete study of sediments should, therefore, include all the fundamental properties, as well as the mass properties and other characteristics of the rock.

The present paper is concerned mainly with shape and roundness, for which relatively few generalizations have been established, inasmuch as few sedimentary studies have included rigorous investigations of these properties. It seems safe to say, however, that roundness is strongly modified by the abrasion and wear to which particles are subjected, whereas shape may play its most important role in the selective transportation of the particles.

The importance of shape in selective transportation is indicated by the fact that the three properties of particles which control their settling velocities are the size, shape, and density (mineralogical composition). The settling velocities, in turn, are of fundamental importance in the hydrodynamic behavior of the particles. Changes in the dynamical conditions of transport, accordingly, are reflected by changes in the size, shape, and density of the deposits. Further, the ratio of surface area to volume (which is fundamentally what sphericity represents) is important in controlling the response of particles to lifting forces, so that particles of low sphericity may behave differently from particles of high sphericity during transportation. Thus the shape, in conjunction with size and density, should shed light not only on the transportational history of the deposit, but also on the immediate conditions at the site of deposition.

Settling velocity is essentially independent of the roundness of the particles. Hence changes in the conditions of transportation or deposition may not reflect themselves in rounding, except as such changes may affect the rate of wear of the particles. Roundness is extremely sensitive to abrasion, and angular particles change rapidly in their roundness during movement. After the initial stages of rounding the process is much slower. During ordinary abrasion, shape is relatively stable, and the original form is often reflected in the pebble even after considerable wear. Under impact breakage shape may change rapidly. Just what the relative effects are must be determined experimentally, because they depend upon the response of particles to forces of different magnitudes.

Analysis of the dynamics of particle wear and transportation is only in its initial stages, and relatively few rigorous principles have been established, which can be applied directly to sedimentary studies. Some progress has been made, however, and considerably more may be anticipated as studies of shape and roundness become more common. The reader is referred to the excellent articles by Hjulström (1939) and Russell (1939) in *Recent Marine Sediments* for summaries of present knowledge in these subjects. A review of abrasion studies, and an analytical approach to the problem are given in a recent paper by the writer (1941).

APPENDIX

Theory of Intercept Sphericity

Wadell's definition of sphericity (Ψ) may be stated as follows:

$$\Psi = \sqrt[3]{\frac{\text{Volume of the particle}}{\text{Volume of the circumscribed sphere}}} \quad (1)$$

Waddell expressed the volume of the particle in terms of a sphere having the same volume; the diameter of the corresponding sphere is the "nominal diameter" (d) of the particle. On this basis the volume of the particle is $(\pi/6)d^3$. The volume of the circumscribed sphere is in general based on the longest diameter (a) of the particle, so that the volume of this sphere is $(\pi/6)a^3$. By substituting these values in equation (1), Wadell's sphericity reduces to the ratio of the nominal diameter of the particle to its longest diameter:

$$\Psi = \sqrt[3]{\frac{(\pi/6)d^3}{(\pi/6)a^3}} = \sqrt[3]{\frac{d^3}{a^3}} = \frac{d}{a}. \quad (2)$$

For the intercept sphericity, the writer uses the same basic definition (equation 1), but expresses the volume of the particle in terms of a triaxial ellipsoid having the three diameters a, b, and c, where $a > b > c$. The volume of such an ellipsoid is $(\pi/6)abc$. Inasmuch as the volume of the circumscribed sphere is still $(\pi/6)a^3$, these two volumes may be substituted in equation (1), yielding:

$$\Psi = \sqrt[3]{\frac{(\pi/6)abc}{(\pi/6)a^3}} = \sqrt[3]{\frac{bc}{a^2}}. \quad (3)$$

This equation is simplified by cubing both sides, to eliminate the radical sign:

$$\frac{bc}{a^2} = \Psi^3. \quad (4)$$

In order to express the sphericity in terms of the ratios b/a and c/b, equation (4) is manipulated to yield the one ratio on the left hand side and the other on the right. This is done by first dividing through by c, and then multiplying by b, in such manner that the b is placed in the numerator on the left, and in the denominator of a compound fraction on the right:

$$\left(\frac{b}{a}\right)^2 = \frac{\Psi^3}{(c/b)}. \quad (5)$$

Equation (5) is the form on which the hyperbolic curves of figure 5 are drawn. It is an equation of the type $y^2 = k/x$, where $y = b/a$, $k = \Psi^3$, and $x = c/b$. The sphericity was set equal to 0.3, 0.4, $\cdots$, 0.9 successively, and the corresponding curves of figure 5 were calculated. Values of sphericity lower than 0.3 were not included, because of their rarity in nature. If desired, the additional curves may be drawn from values based on equation (5).

It may be seen from this development that the writer's method is based on Wadell's definition of sphericity, so that if actual triaxial ellipsoids were used, the true sphericity could be found from the chart or from equations (4) or (5). Pebbles are not triaxial ellipsoids, but statistically they may be approximated by such a form, so that in practice an excellent agreement is found between the

TABLE 4. *Comparison of standard and rapid methods for measuring shape and roundness*

Sample	Size range (mm.)	Sphericity			Roundness		
		Wadell	Intercept	Relative Error, %	Wadell	Visual	Relative Error, %
Outwash	8–64	0.74	0.75	1.4	0.61	0.60	1.6
Outwash	8–16	.76	.75	1.3	.52	.52	0.0
Outwash	16–32	.75	.76	1.3	.58	.58	0.0
Outwash	32–64	.76	.78	2.6	.61	.61	0.0
Till	32–64	.71	.69	2.8	.50	.50	0.0
Till	8–16	.72	.72	0.0	.48	.49	2.1
Till	16–32	.72	.72	0.0	.52	.50	3.8
Till	16–32	.73	.77	5.5	.48	.54	12.5
Till	16–32	.76	.77	1.3	.51	.59	15.7

average Wadell sphericity and the average intercept sphericity of the same set of pebbles.

Accuracy of Rapid Methods

The writer conducted a number of tests on the intercept sphericity and the visual roundness to determine the accuracy of the methods. It was found that for samples of 25 pebbles the average values obtained by the two methods agreed consistently within 5 per cent. During these studies an opportunity arose for a comprehensive test, based on a number of samples. Through the courtesy of Dr. F. J. Pettijohn of the Department of Geology, the students in a graduate course on sediments measured sets of 100 pebbles for sphericity, and sets of 50 pebbles for roundness, using both standard and rapid methods. The samples were all different, and included pebbles from glacial outwash and glacial till, ranging in size from 8 to 64 mm. diameter. Table 4 summarizes the data. The average relative error of the sphericity measurements (based on 100 pebbles) was 1.8 per cent, and the single large error was 5.5 per cent. In the first seven roundness samples the average error was only 1.1 per cent, but the last two samples had errors of 12.5 and 15.7 per cent. These were entirely out of line with the other samples (all were based on 50 pebbles), and a re-examination showed that the large errors were due to the incorrect evaluation of broken pebbles. It was from this study that the approximate rule for evaluating broken pebbles was developed, in which the roundness of the unbroken half is divided by 2.

The students who conducted these tests had had no previous experience with roundness or sphericity measurements. The results of the roundness study emphasized the need for critical examination of the sharper corners and broken edges of the pebbles. With these precautions taken, the average error lies within about 2 per cent for sets of 50 pebbles.

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