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SPHERICITY AND ROUNDNESS OF ROCK PARTICLES

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The purpose of my article "Volume, Shape, and Roundness of Rock Particles," which appeared in the *Journal of Geology*,[1] was to secure priority and to announce that a more comprehensive work had been prepared for publication.[2] This preliminary article is the subject of criticism by Dr. C. K. Wentworth in this number of the *Journal*. Because of this criticism it seems advisable to go further into the problem and also to consider critically that part of Wentworth's own research work which is the foundation for his opinion.

WENTWORTH'S SHAPE DETERMINATIONS

Wentworth states his opinion that the surface ratio (sphericity), as I have used it, is ". . . . only one of the unique geometrical properties of a sphere and cannot be regarded as the exclusive measure of true sphericity." He also points out that he has listed elsewhere[3] two other properties of a sphere. These unique properties, as he calls them, are ". . . . constant curvature over the entire surface and constant diameters in all directions through a common center." The earlier statement to which Wentworth refers[4] is as follows:

> In studying this change in shape it is necessary to recognize varying degrees of roundness, in other words, to have a numerical answer to the question of how round a given piece is. At least three criteria of roundness readily occur to one in considering the question. These are (1) the ratio of surface area to volume, (2) the average deviation of diameters from a mean diameter, (3) the average deviation of convexities from a mean convexity. Each of these values or coefficients of roundness reaches a minimum in the case of a sphere and has a maximum in the case of a line or plane without volume.

[1] Vol. XL (1932), p. 443–51.

[2] A typewriten copy is now available in the University of Chicago Library (Geological Section).

[3] Chester K. Wentworth, "A Laboratory and Field Study of Cobble Abrasion," *Jour. Geol.*, Vol. XXVII (1919), pp. 507–21.

[4] *Ibid.*

It is possible that the ideas expressed by Wentworth in the present number of the *Journal* may have been the foundation for his statement quoted above, but there is quite a difference between the meanings of the quotations from the two papers. Wentworth's first point, "the ratio of surface area to volume," is only the third type of *specific surface* as suggested by Wo. Ostwald[5] in 1909. A solid can have every possible shape and still have the same ratio of surface area to volume. A cube with a surface area of 6 square centimeters and a volume of 1 cubic centimeter has a specific surface of $\frac{6}{1}=6$, just as the sphere inscribed in the same cube (surface area of sphere $=3.1416$; volume $=0.5236$; specific surface $=6$).

How Wentworth can obtain any reasonable numerical value for shape according to his second and third points is a puzzle which he has never solved. He proposed a "practical solution of the problem." Before discussing its value, it will first be shown that his first and second points are actually expressions for the *mean deviations* of the diameters and the convexities respectively.

There are several types of averages, such as the arithmetic mean, the geometric mean, the harmonic mean, etc. Wentworth failed to

[5] Wo. Ostwald, *Grundriss der Kolloidchemie* (1909). On page 84 the third type of the specific surface is expressed as the ratio:

$$\frac{\text{Absolute Oberfläche eines einzelnen Teilchens der dispersen Phase}}{\text{Volume eines einzelnen Teilchens der dispersen Phase}}.$$

There is a confusion in the literature in respect to the *specific surface*, because of a superimposed meaning given to this term by Fritz Paneth in 1922. According to him, the specific surface is the size of the surface area of one gram of the solid. This meaning of the specific surface is for sedimentological purpose less suitable because the density of the material enters as a factor.

Fritz Paneth, "Über Eine Methode zur Bestimmung der Oberfläche Adsorbierender Pulver," *Zeitschrift für Elektrochemie und Angewandte Physikalische Chemie*, Vol. XXVIII (1922), pp. 113–15.

It should be noted that Jackson, in 1903, proposed the term "surface factor," the numerical value of which is supposed to express the total surface of the particles in a given weight of dry material. In 1905 Purdy changed the definition, the *surface factor* being the relative surface of the particles per unit volume. When dealing with a single particle, I believe that Ostwald's term *specific surface* has been more generally adopted.

W. Jackson, "The Calculation of the Fineness of Ground Materials," *Trans. English Ceramic Soc.*, Vol. III (1903), pp. 16–22.

Ross C. Purdy, "The Calculation of the Comparative Fineness of Ground Materials by Means of a Surface Factor," *Trans. Amer. Ceramic Soc.*, Vol. VII (1905), pp. 441–47.

state which kind of average he had in mind. It is here assumed to be the arithmetic mean, the simplest and the one most commonly used. The first part of his second point, i.e., "the average deviation of diameters" thus becomes synonymous with the *arithmetic mean of the deviations of the diameters;* and the entire second point receives the following form: *The arithmetic mean of the deviations of the diameters from a mean diameter.* This is the expression for the *mean deviation* of the diameters. Accordingly, the second and third of Wentworth's points may be simplified to *the mean deviation of the diameters* and *the mean deviation of the convexities* respectively. Since the numerical value of the mean deviation of a group of items depends upon the numerical values of these items, it follows that the mean deviation cannot be used as an expression for shape. For the sake of simplicity, assume that the three diameters of a solid of a given shape are 1, 2, and 3 centimeters; thus the mean deviation equals 0.666. . . . . For a larger solid of exactly the same shape and of proportionally larger diameters, 10, 20, and 30 centimeters, the mean deviation is 6.666. . . . . The same holds for the mean deviation of the convexities. Thus it is evident that the mean deviation cannot be used, because the numerical value supposed to express the shape is influenced by the size of the solid (0.666 and 6.666). Furthermore, I have already shown that the diameters are out of the question as factors for comparison of solids of different shapes.[6]

Wentworth decided upon a "practical solution of the problem." To that end he used "the ratio of radius of curvature of the most convex part of the surface to half of the longest diameter through that point." An analytical discussion will show that the problem of expressing the shape of a solid is not solved by this method. As will be shown later, Wentworth's error rests actually upon his confusion of roundness and shape. The most convex parts of a geometric cube are its solid corners.[7] Being a geometric solid, these corners are infinitely sharp, and their radius of curvature consequently equals

[6] Wadell, *op. cit.*, pp. 443–44.

[7] In my larger paper I have used the expressions *plane corner* and *solid corner*, the former defined as every such part of the outline (of a plane area) which has a radius of curvature equal to or less than the radius of curvature of the maximum inscribed circle of the same area; a *solid corner* is understood to be the intersection of three or more converging surfaces of a solid.

zero. Thus, according to Wentworth's shape determination, there is to be measured the longest diameter through one of the solid corners of the cube, i.e., the diagonal. If the diagonal of the cube is d, the shape of a geometric cube, according to Wentworth's formula, is $\frac{2\times 0}{d}=0$. It is evident that the same value will be obtained for all shapes of geometric solids, the most convex part of which is a solid corner (radius of curvature equals zero).

That Wentworth's idea of the shape of a solid is not clear is further shown by his later papers, in which he elaborates upon the *flatness ratio*, an invention for the purpose of expressing the shape of beach pebbles.[8] The flatness ratio is expressed by $\frac{D'+D''}{2D'''}$, i.e., the average of the length and breadth divided by the thickness. Accordingly, a sphere has the same flatness ratio as a cube.

It has been shown above that those "unique properties," if used for expressing shape in the way described by Wentworth, are of doubtful value. That the diameters are out of the question as factors in shape determinations has been discussed. It is also impossible to

[8] Chester K. Wentworth, "Shapes of Beach Pebbles," *U.S. Geol. Surv., Prof. Paper 131-C* (1922), pp. 75–83.

Besides papers already mentioned, the following articles deal with suggestions made by geologists for the determination of shape of rock particles. In addition a recent paper by Wo. Ostwald is included.

J. Hirschwald und J. Brix, "Untersuchungen an Kleinschlagdecken behufs Gewinnung einer Grundlage für die Prüfung der naturlichen Gesteine auf ihre Verwendbarkeit als Strassenbaumaterial," *Bautechnische Gesteinsuntersuchungen* (Mitteilungen aus dem Mineralog.-geol. Inst. d. K. Techn. Hochschule Berlin) (1918), p. 39–49; Chester K. Wentworth, "A Method of Measuring and Plotting the Shapes of Pebbles," *U.S. Geol. Surv. Bull. 730-B* (1922), pp. 91–102; "A Field Study of the Shapes of River Pebbles," *ibid., 730-C* (1922), pp. 103–14; A. C. Trowbridge and M. E. Mortimore, "Correlation of Oil Sands by Sedimentary Analysis," *Econ. Geol.*, Vol. XX (1925), pp. 409–23; E. P. Cox, "A Method of Assigning Numerical and Percentage Values to the Degree of Roundness," *Jour. Paleon.*, Vol. I (1927), pp. 179–83; J. E. Lamar, "Geology and Economic Resources of the St. Peter Sandstone of Illinois," *State Geol. Surv. Ill., Bull. 53* (1927), pp. 44–46, 148–51; A. Pentland, "A Method of Measuring the Angularity of Sands," *Proc. and Trans. Roy. Soc. Can.*, Ser. 3, Vol. XXI (1927), App. C, Titles and Abstracts, p. xciii; Allen C. Tester, "The Measurement of the Shapes of Rock Particles," *Jour. Sed. Petr.*, Vol. I (1931), pp. 3–11; F. G. Tickell, *The Examination of Fragmental Rocks* (Stanford University Press, 1931), pp. 6–7; Wo. Ostwald, "Ueber difforme System. I. Stereometri und Systematik difforme Systeme," *Kolloid-Zeitschrift*, Vol. LV, Heft 3 (1931), pp. 257–72.

get a numerical expression for the shape of a solid with reference to "the constant curvature over the entire surface" of a sphere. All geometric solids, including geometric configurations of crystals, which are composed of plane faces, solid corners, and edges, will accordingly have the same shape, because the radii of curvature of the corners and edges equal zero, and the radius of curvature of the plane faces is infinitely large. Finally, none of the "unique properties" suggested by Wentworth is a maximum property. The maximum property (*Maximumeigenschaft*) of a sphere is an isoperimetric property. Further discussion on this subject appears in a later paragraph.

THE DEGREE OF SPHERICITY OF A PARTICLE

Wentworth criticises my term "degree of true sphericity." He expresses his belief ". . . . that most geologists regard *sphericity* as a precise term, indicating exactly the shape of a sphere."

The *Oxford English Dictionary*[9] says that "sphericity" meaning a spherical body is obsolete. The *Century Dictionary* gives "sphericity" as the character of being in the shape of a sphere and as synonymous with the French *sphéricité*. *Nouveau Larousse* gives under the word *sphéricité* an example: "La sphéricité de la terre n'est pas absolue." Thus it is evident that sphericity is a quality or character and as such is expressible in various degrees. The formula which I have suggested for expressing the degree of sphericity leads to a numerical value which expresses the shape-character of a solid relative to that of a sphere of the same volume and specific gravity, and on the basis of the isoperimetric property of the sphere.

The geometric foundation for the concept "degree of true sphericity" rests upon the isoperimetric property of a sphere. The first attempt to prove that among all solids of a given surface area the sphere had the greatest volume was made by Lhuilier.[10] J. Steiner[11]

[9] James A. H. Murray, *A New English Dictionary* (Oxford, 1914–15).

[10] S. A. J. Lhuilier, "De relatione mutua capacitatis et terminorum figurarum, geometrice considerata; seu de maximis et minimis. Varsaviae 1782." Not seen. Reference obtained from: Ernst Steinitz, "Polyeder und Raumeinteilungen," *Encyklopädie der Mathematischen Wissenschaften*, Bd III, 1^2, AB 12 (1914–31), p. 38.

[11] K. Weierstrass, *Jacob Steiner's Gesammelte Werke* (Bd. I [1881]; Bd. II [1882]), Vol. II, p. 304.

simplified the proof and drew up the following thesis: "Unter allen Körpern von gleichem Inhalte hat die Kugel die kleinste Oberfläche; und unter allen Körpern von gleicher Oberfläche hat diesselbe den grossten Inhalt." Further proofs were furnished by H. A. Schwarz[12] and H. Minkowski.[13] Wilhelm Blaschke[14] gave a neat analytical formulation of the isoperimetric property of a sphere. He called this property the *Maximumeigenschaft* ("maximum property") of a sphere.

In suggesting a formula for obtaining a numerical value expressing the shape of a solid, I was aware that the geometricians have made a number of more or less successful attempts to classify solids, especially the convex polyhedron. The geometrician is, however, interested in such a classification for the sake of geometry. He is not concerned with the physico-chemical origin of the solid or the way its properties interact with external forces. As previously pointed out, I am not interested in all kinds of geometric properties, but only in those which are of sedimentological importance. Nevertheless, I have shown in the preceding pages that, basing my sphericity formula on the "maximum property" of a sphere, the concept "degree of true sphericity" has also a strictly geometric foundation. My formulas for sphericity and roundness are also applicable to all kinds of solids, including geometric. All geometric solids composed of plane faces, corners, and edges have a roundness value of zero, thus in accordance with the views expressed in my preliminary paper in respect to unworn solids, exhibiting non-rounded, sharp corners. The roundness is independent of the sphericity, which is expressed by the ratio of the surface area of a sphere of the same volume as the solid to the actual surface area of the solid.

The shape of solids (in physico-chemical meaning) has been discussed in many different fields of science, but nowhere have I found any clue as to the principal factor involved in the term. Both the so-called *volume-weight* and *sediment-volume* are to no small extent

[12] H. A. Schwarz, reference not seen. See Blaschke below.

[13] Hermann Minkowski, "Ueber die Begriffe Länge, Oberfläche und Volumen," *Jahresbericht der Deutschen Mathematiker-Vereinigung*, Vol. IX (1901), pp. 115–21.

[14] Wilhelm Blaschke, "Kreis und Kugel," *Jahresbericht der Deutschen Mathematiker-Vereinigung*, Vol. XXIV (1915), pp. 195–207.

influenced by the shape of particles. The minimum porosity of quartz sand, composed of particles of approximately the same size, is influenced by the shape of the granules.[15] It is generally assumed that the shape of the particles plays an important rôle in ore treatment and in respect to the properties of cement, concrete, pigment powder, and other industrial products. Gilbert in his classic work, "The Transportation of Débris by Running Water,"[16] has given considerable attention to the shape of rock particles in transportation and accumulation of the débris. It is generally assumed that the shape of particles influences the settling velocities. Freundlich[17] has expressed the opinion that, since Stokes' law only holds for spherical particles, its invalidity can often be considered as a proof for the non-spherical form of the particles. The shape of particles influences probably the capillary rise of water in sand.[18] P. Curie[19] said, "Lorsque le corps se déforme, l'énergie en volume est constante et l'énergie totale varie proportionnellement à la variation de surface." An interesting discussion on shape of coal particles in respect to combustion has been given by Rosin and Kayser.[20] Consideration is given to the consumed volume in proportion to the surface area of fire attack. Terzaghi,[21] discussing the elasticity properties of soils with respect to the presence of "gedrungenen Kornes" (apparently high degree of sphericity) and "Schuppen" (lamina-shaped particles), has pointed out that the contact points of the latter are at least a hundred times greater than for the same size of particles of "gedrungenen" form. Eilh. A. Mitscherlich discussed the

[15] J. E. Lamar, *op. cit.*

[16] G. K. Gilbert, "The Transportation of Débris by Running Water," *U.S. Geol. Surv., Prof. Paper 86* (1914).

[17] Herbert Freundlich, *Kapillarchemie* (1932), Bd. II, p. 82.

[18] Alfred Mitscherlich, "Untersuchungen über die physikalischen Bodeneigenschaften," *Landwirtschaftliche Jahrbücher*, Vol. XXX (1901), pp. 361–445.

[19] P. Curie, "Sur la formation des cristaux et sur les constantes capillaires de leurs différents faces," *Bull. de la Société Minéralogique de France*, Vol. VIII (1885), p. 146.

[20] P. Rosin und H.-G. Kayser, "Zur Physik der Verbrennung fester Brennstoffe," *Zeitschrift des Vereins Deutscher Ingenieure*, Vol. LXXV, (1931), pp. 849–57.

[21] Karl v. Terzaghi, "Festigkeitseigenschaften der Schüttungen, Sedimente und Gele," *Handbuch der Physikalischen und Technischen Mechanik* (edited by F. Auerbach und W. Hort), Bd. IV, Part 2 (1931), p. 516.

compactibilty (not compactness) of particles. "Die Gestalt des Hohlraumvolumen des Bodens wird bedingt durch die Gestalt der einzelnen Teilchen und durch die Art der Aneinanderlagerung."[22] ". . . . das Hohlraumvolumen um so verzweigten ist, je grösser die Summe der Oberfläche der festen Bodenteilchen ist."[23] Mitscherlich, however, did not give any definition of shape, though he apparently saw some relationship between shape, volume, and surface. *What, then, is the principal factor involved in the shape of a solid?*

It may be taken for granted that none of the writers on the sedimentological significance of shape have had the geometrician's classification of solids or the crystallographer's definition of form in mind. It seems, on the basis of the quotations in the preceding paragraphs, that there should be some kind of relationship between volume, surface, and the sedimentological shape. These three factors are, together with Steiner's thesis,[24] the basis for the following:

1. Solids of equal surface areas and equal volumes have the same shape.

2. Solids of equal surface areas but of different volumes have different shapes.

3. Solids of equal volumes but of different surface areas have different shapes.

We have, then, two ways of expressing the sedimentological shape, either on the basis of equal surface areas but different volumes or by equal volumes but different surface areas. I have chosen the latter because it is for sedimentological purposes more convenient. In view of the truly unique, isoperimetric property of a sphere and with regard to the extraordinary rôle this shape plays a natural phenomena, it was adopted as a standard shape for comparison. Maintaining the third proposition above as a foundation, the numerical value of the shape-character of a particle was then expressed with reference to a sphere by the ratio of the surface area of a sphere, of the same volume as the particle, to the actual surface area of the particle; thus

$$\frac{s}{S} = \text{Degree of true sphericity,}$$

[22] Eilh. A. Mitscherlich, *Bodenkunde für Land- und Forstwirte* (1905), p. 84.

[23] *Ibid.*, p. 86.

[24] Quoted on p. 315.

where s is the surface area of a sphere of the same volume as the particle, and S is the actual surface area of the particle. The value obtained is independent of the size of the particle, and thus dimensionless.

Variations of the crystallographic habit do not indicate any change in the real crystallographic form of a crystal. In the same way, two solids of the same sedimentological shape, or rather the same degree of true sphericity, may have a different appearance, i.e., they may vary in respect to surface details, roundness of the corners, and non-metric properties such as the number of corners and edges present or in respect to the relative position of the plane surfaces.

Though the word *shape* has been freely used in the preceding pages with a rather definite meaning, it would perhaps be unfortunate at present to attach a definition. The true sphericity, denoted by the Greek letter ψ is rather the term to be used and according to the definition given above.

ROUNDNESS

Wentworth dismisses my strictly mathematical expression and definition of roundness by the statement: "Roundness is a term which may properly be more loosely used, with reference both to solid shapes and plane figures. But when applied to a solid in the phrase 'maximum degree of roundness' most will agree that a perfect sphere is implied." A numerical value expressing the degree of a property must obviously be based on a sharp definition of the property. It is difficult to see the reason why Wentworth at this time advocates loose usage of the term "roundness," after years of endeavor to define roundness and to find a numerical value for the degree of this property. He said: "In studying this change in shape it is necessary to recognize varying degree of roundness, in other words, to have a numerical answer to the question of how round a given piece is."[25] Terms of loose usage should, of course, be avoided in scientific literature. If a term remains non-defined, it is permanently ambiguous.

The *Oxford* dictionary says that *roundness* is the quality of being round and that the meaning of *round* as a spherical or globular body

[25] "A Laboratory and Field Study of Cobble Abrasion," *loc. cit.*, p. 513.

is "somewhat rare." The following quotations give an idea of the general meaning of "round":

"The circumference or outer bounds of some circular objects; the complete circle of something . . . . a circle, ring or coil . . . . a circular part, form or arrangement of natural origin . . . . cylindrical, circular in respect to section; . . . . exhibiting a curvilinear form or outline; curved; forming a segment of a circle, etc."[26]

"Round. Circular, etc. F. rond, OF. roond, L. rotundus, from rota, wheel."[27]

"Rond (ron) Ronde lat. rotundus, de rota, roue ad. Qui a tous les points de la surface egalemente distance d'un point ou d'un axe central, soit sous une forme circulaire."[28]

Round as derived from the Latin *rota* ("wheel") does not mean spherical, and consequently a perfect sphere is not implied by the use of the term "maximum degree of roundness." A sphere is round because its circumference is round. Observing that a solid such as the moon is round, we are, however, not entitled to conclude that the moon also is a sphere. So far as its roundness is concerned, the moon may have any shape which in one section has a circular outline (cylinder, discus, cone, etc.).

According to the quotation from the *Oxford* dictionary, it is evident that *round* refers both to a circle and a segment of a circle. The correctness of this view is further shown by the use of such parasynthetic combinations as *round-backed*, *round-edged*, *round-cornered*, etc.

The formula suggested in my preliminary paper for expressing degree of roundness refers plainly to the roundness of the corners, yet it fits in perfectly with the idea that a circle has a maximum degree of roundness. The ratio of the radius of curvature of a circular outline (a circle) to the radius of curvature of the maximum inscribed circle (obviously the same circle) gives the numerical value 1, which is the value of the maximum degree of roundness. A cylinder terminated at each end by a half-sphere has also a maximum degree of roundness, because the ratio of the radius of curvature of its ends (corners) to the radius of curvature of the maximum inscribed circle in the given section equals the numerical value 1. Thus my formula

[26] *The Oxford English Dictionary.*

[27] Ernest Weekly, *An Etymological Dictionary of Modern English* (1921).

[28] Claude Augé, *Nouveau Larousse Illustré* (Dictionaire Universal Encyclopédique).

is an adequate expression for roundness in view of the common interpretation of the word "round." It fulfils the requirement that a circle has a maximum degree of roundness, and it can at the same time be used for expressing the degree of roundness of corners and edges and is therefore in agreement with the use of the parasynthetic combinations *round-cornered, round-edged,* etc.

Wentworth comments as follows on my statement that a cylinder terminated at each end by a half-sphere cannot become more rounded:

If one accepts Wadell's definition of roundness in strictness, this is true but in the light of a more general concept of roundness such a statement is subject to challenge not only on geometrical grounds but also with reference to natural or experimental shape modifications.

Wentworth's arguments with reference to "natural or experimental shape modifications" will be discussed later. No indication is given of the reason why it is objectional on geometrical grounds, and a reply is therefore not possible. My roundness formula refers strictly to the corners and edges. That a solid has a circular outline in one cross section is, as we shall see later, rather a factor in the shape than in respect to the roundness. It is also to be noted that there is a difference between the roundness and the shape of a corner. As already pointed out in my preliminary article, the roundness is obtained by measurements of the radii of curvature in *one* plane. The total roundness of a solid may be obtained by measurements in three planes at right angles to each other; but one plane is generally satisfactory when dealing with quartz particles, provided that the measurements are undertaken in the largest plane parallel to the longest and intermediate diameters. The weakest part of an irregular quartz grain is as a rule along the direction of its smallest diameter. The results of fracturing, chipping, attrition, or solution of the particle are therefore most noticeable at right angles to the shortest diameter, i.e., in the plane of the longest and intermediate diameters. As we will see later, there are also other reasons for adopting this plane for measurements.

In hydraulics and gas dynamics an orifice is generally classified on the basis of the size and shape of the contracted area with specification in respect to sharp-edged and round-edged. Thus an orifice may be circular and round-edged or sharp-edged.

Stanton[29] found that the mean resultant pressure on dissimilar plates varied considerably for the same speed of a current of air, the mean pressure on a long narrow rectangular plate being nearly 60 per cent greater than that on a circular one. It should be noted that this difference is due to the difference in shape and not due to the difference between the degree of roundness of a rectangular and circular plate.

The shape of a plane figure may be expressed on the basis of the isoperimetric property of a circle[30] by the ratio:

$$\frac{c}{C} = \text{Degree of circularity,}$$

where c is the circumference of a circle of the same area as the plane figure, and C is the actual circumference of the plane figure. The maximum value obtained by this formula is 1.000, which is the numerical value of the shape of a circle.

The difference between the degree of circularity (also the degree of sphericity) and the degree of roundness may now be illustrated. A circle has a maximum degree of circularity and also a maximum degree of roundness. A dodecagon is a closed plane figure composed of twelve straight sides and twelve plane corners, the radii of curvature of the former being infinitely large, and the radii of curvature of the latter equal zero. The degree of circularity of a dodecagon is 0.988, thus approaching very closely that of a circle, yet the degree of roundness of its corners is infinitely small or equals zero.

Plane figures could be constructed with still higher degree of circularity, yet with a roundness value of zero. The limit is reached in a perfect circle with an infinite number of "corners" joined together in a smooth circular outline with a maximum degree of roundness.

[29] T. E. Stanton, "On the Resistance of Plane Surfaces in a Uniform Current of Air," *Minutes of Proc. Inst. Civil Engineers*, Vol. CLVI (1903–4), pp. 78–139.

[30] C. Caratheodory und E. Study, "Zwei Beweise des Satzes, dass der Kreis unter allen Figuren gleichen Umfanges den grössten Inhalt hat," *Mathematische Annalen*, Vol. LXVIII (1910), pp. 133–40.

It follows that one of the essential parts of the roundness is a smooth outline.

In the practical solution of obtaining the roundness of the corners of a plane figure, corresponding to the projection area of a solid, various difficulties occurred. Any solid, no matter how smooth to the naked eye, will with increased magnification show unevenness of surface with corners and edges, and consequently also an uneven outline of the projection area. In order to obtain comparable values of the roundness, a standard size must be adopted. Large objects, such as boulders, must be reduced; and small ones, like sand grains, magnified to approximately the same size, i.e., the standard size, on which the measurements are performed. The average diameter of the standard size used for measurements of rock particles has been fixed at 70 mm., and microscopic particles have been enlarged to about that size by camera lucida or screen projection.

The following is intended to illustrate the practical results which under certain conditions may be obtained by the use of the roundness formula given in my preliminary paper. Assume that we have two cubes of sodium chloride, one with an edge of 70 mm. (approximately the standard size), the other with an edge of 1 mm. The solids are immersed in pure water. After a few minutes the smaller cube assumes a spherical shape, thus possessing a maximum degree of roundness. The corners of the larger cube have, during the same time, obtained approximately the same radius of curvature as the smaller, spherical particle; but the two solids would, of course, not be considered to have the same degree of roundness. According to the roundness formula, the sphere has a maximum degree of roundness, while the roundness value of the larger cube is very small.

If, on the other hand, the two cubes are subject to attrition by rolling without solution, the larger cube is more rapidly rounded than the smaller.[31] Thus, distinctly different results are obtained by the two processes—solution (in pure water) and attrition. Sorby made some observations bearing on these points. He said:

> In the specimens of decomposed granite which I have examined in greatest detail, the larger grains of quartz have a somewhat opaque surface, as if cor-

[31] No experiments have been made on attrition of cubes of sodium chloride. The outcome of such an experiment is inferred from the results obtained by attrition of solids of other composition. Some data are given later in the text.

roded, and the angles are rounded. This rounding is relatively much greater in the case of the smaller grains, which is the reverse of what is met with in worn sand. On the whole the facts seem to indicate that the quartz has been more or less corroded and dissolved by the action of the alkaline silicates set free by the decomposition of the felspar.[32]

The roundness of corners has a definite importance. Rounded corners offer under certain conditions less resistance than sharp ones in a current fluid. Rounding of sedimentary particles is a special type of disintegration attributed to attrition and sometimes to solution. Roundness is destroyed or diminished by fracturing and chipping, and a high degree of roundness is, therefore, often an indication of gentle conditions of wear relative to the size, hardness, and toughness of the particle. High degree of roundness is frequently the result of tractional[33] transportation of the particles, while sand grains carried in suspension are relatively unaffected by attrition.

THE PRACTICAL METHOD OF DETERMINING THE SPHERICITY VALUE OF QUARTZ PARTICLES

Wentworth asks what method I advocate for the measurement of surface areas to obtain valid averages with economy of time. There are several rather accurate methods used in connection with ore crushing and pulverization[34] and in physical chemistry.[35] None of

[32] H. C. Sorby, "On the Structure and Origin of Non-calcareous Stratified Rocks," *Quar. Jour. Geol. Soc.*, Vol. XXXVI (1880), Proc., pp. 48–92.

[33] For the meaning of traction see Gilbert, *op. cit.*

[34] Besides the references already given in this article, the following papers deal with surface determinations and related subjects.

P. R. v. Rittinger, *Aufbereitungskunde* (1867) (not seen); Karl v. Reytt, "Bestimmung des Arbeitsaufwandes zur Zerkleinerung der Aufbereitungsprodukte," *Oesterreichische Zeitschrift für Berg- und Hüttenwesen*, Vol. XXXVI (1888), pp. 229–31, 255–58, 268–70, 283–85; Ernest A. Hersam, "Economy of Power in Crushing Ore," *Mining and Scientific Press*, Vol. XCV, Part 2 (1907), pp. 621–26; Carl Naske, *Zerkleinerungsvorrichtungen und Mahlanlagen* (1921), pp. 1–13; Fritz Paneth und Walter Vorwerk, "Über eine Methode zur Bestimmung der Oberfläche adsorbierender Pulver," *Zeitschrift für Physikalische Chemie*, Vol. CI (1922), pp. 445–79; Hans Wolff, "Bestimmung der Oberffäche von Glaspulver," *Zeitschrift für angewandte Chemie*, Vol. XXXV (1922), pp. 138–40; W. A. Koehler and J. H. Mathews, "The Heat of Wetting of Lead Sulfate," *Jour. Amer. Chem. Soc.*, Vol. XLVI (1924), pp. 1158–83; Geoffrey Martin, "A Method of Accurately Determining Experimentally the Surface of Crushed Sand Par-

[Footnote continued on next page]

[35] E. K. Rideal, *An Introduction to Surface Chemistry* (1930); H. Freundlich, *Kapillarchemie* (1932); F.-V. v. Hahn, *Dispersoidanalyse* (1928).

these methods was adopted, because it was especially desirable for classification in the statistical charts to obtain sphericity values for each particle rather than bulk values for a great number of granules. A complete description of the method used will be given in a forthcoming paper. Only the principles are presented here.

For reasons which space does not permit to discuss here, the maximum projection area was used as a base for sphericity determinations.[36] It should be noted, however, that the most desirable value is that of the degree of true sphericity as obtained by the formula previously given. For quartz particles the value of the degree of true sphericity was found to agree reasonably well with the value obtained by the practical formula given below.

To obtain the shape of quartz grains, the particles were placed under the microscope. The slide was given a few gentle taps with a pencil so that all particles would come to rest on one of their largest surfaces, parallel to the longest and intermediate diameters. This gives approximately the largest projection area. The standard size was obtained by the use of the proper objective for each grade size, and by regulating the distance between the camera lucida mirror and the drawing paper. The area of the reproduced particle

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ticles," *Trans. Ceramic Soc.*, Vol. XXV (1925–26), pp. 51–62; J. Gross, S. R. Zimmerley, S. J. Swainson, "Surface Measurement on Finely Ground Particles and Its Relationship to the Crushing Laws," *Bull. Univ. Utah*, Vol. XVI, No. 4 (1925), pp. 57–76; Herbert F. Kriege, "Relation between the Fineness of Limestone and Its Rate of Solution," *Rock Products*, Vol. XXIV (1926), pp. 65–68 (not seen); W. A. Koehler, "A Method for the Determination of the Relative Surface Areas of Powdered Materials," *Jour. Amer. Ceramic Soc.*, Vol. IX (1926), pp. 437–43; J. Gross and S. R. Zimmerley, "Crushing and Grinding. I. Surface Measurement of Quartz Particles," *Amer. Inst. Min. and Metall. Engineers, Tech. Pub. No. 46* (1928), pp. 1–16; Fr. W. Freise, "Untersuchung von Gesteinen auf Abnutzbarkeit bei Verfrachtung im Wasser," *Mineralogische und Petrographische Mitteilungen*, Bd. XLII (1932), pp. 48–58.

[36] In respect to roundness determinations a reason has already been given on p. 320.

In respect to sphericity determinations it should be noted that, as the shape departs from that of a sphere, i.e., with decreasing degree of sphericity, an increase usually takes place of (1) the longest diameter, (2) the largest cross-sectional area, (3) the surface of the solid, and (4) the sum of the projection areas of the solid in different positions. On the other hand, the degree of circularity of the different cross-sectional areas of the solid decreases with decreasing degree of sphericity. All these factors influence the resistance, exerted by the fluid on the particle in translation in different positions. The coefficient of resistance or drag as a function of *Reynolds number* is therefore influenced by the shape of the particle. The author will return to this question in a special paper.

was measured with a polar planimeter. The following formula was adopted for obtaining the sphericity value:

$$\frac{\delta}{\Delta} = \text{Degree of sphericity,}$$

where δ is the diameter of a circle of an area equal to the area obtained in the standard size by projecting the grain at rest on one of its surfaces parallel to the plane of the longest and intermediate diameters of the particle, and where Δ is the diameter of the smallest circle that will circumscribe the grain projection. The sphericity value achieved by this method is denoted by the Greek small letter ϕ.

Space does not permit here any arguments for this method. Details will be given in a forthcoming paper. Reasons for reproduction and projection of the plane parallel to the longest and intermediate diameters has been given in the preceding chapters both in respect to roundness and sphericity. The method applies only to quartz grains. For other minerals other methods have been suggested. In the course of the work various new ideas have occurred, and further experimental studies are planned.

My diagrams are based not on any averages but on a reasonably accurate determination of the *true* sphericity for each quartz particle. It is probable that the error due to the formula given above in general does not exceed one class-interval. The classes used range from 0–.05, .05–.10, .10–.15, .15–.20, and so forth, up to .95–1.00. The accuracy of the determinations was to some extent checked by taking the average sphericity value for each sieve grade size and comparing it with Lamar's[37] results, which are based on porosity determinations with a device, essentially of the type used for dispersion measurement on the basis of pore space. My average values for grade sizes of St. Peter sand grains were practically identical with those obtained by Lamar by an entirely different method.

MODIFICATION OF SHAPE AND ROUNDNESS OF ROCK PARTICLES BY NON-CHEMICAL FORCES

The shape of rock particles may depend on a variety of factors. One is the difference in cohesive forces in different directions. Owing to the pronounced cleavage of mica, this mineral is never found in

[37] *Op. cit.*

the shape of a sphere. The mode of crushing influences, in all probability, the average shape of particles. It is reasonable to assume that different results are produced by grinding and by direct blows. The shape is influenced by the crystal structure, inequalities of expansion, inclusions, etc. Beckenkamp[38] said:

> Ein Zerreissen eines Körpers tritt dann ein, wenn die (etwa durch das Einpressen einer Messerschneide hervorgerufene) Deformation einen solchen Betrag erreicht, dass die inneren attraktiven Kräfte den inneren repulsiven und den äusseren Kräften nicht mehr das Gleichgewicht halten können. Es tritt demnach Spaltung senkrecht zu solchen Richtungen ein, für welche die eine derartige Deformation erzeugenden äusseren Kräfte einen minimalen Wert haben.

It is also likely that, because of capillary action, different shapes are produced by disintegration in air and water. This tentative conclusion is based upon the method of making thin laminae of gypsum and mica for optical purposes.[39] Much thinner laminae are produced by splitting the mica in water than in air. Should experimental results show that micas by transportation and disintegration in water break up in thinner flakes, thus achieving a lower sphericity, than by disintegration in air, a distinction between eolian and aqueous deposits on this basis will eventually be possible.

In the present issue of the *Journal*, Wentworth expresses his opinion that the result of wear is ". . . . due in part to the decrease in ratio of supporting volume to area of attacked surface." He should not draw any conclusion of this kind from his experimental results. He simply weighed the solids at certain intervals and calculated the distance covered by the revolving drum. The surface areas produced in the course of attrition were never measured.[40] He also

[38] J. Beckenkamp, "Atomanordnung und Spaltbarkeit," *Zeitschrift für Kristallographie*, Vol. LVIII (1923), p. 17.

[39] J. Hirschwald, *Handbuch der bautechnischen Gesteinsprüfung* (1911), p. 275.

[40] On p. 514 of his paper "A Laboratory and Field Study of Cobble Abrasion," Wentworth makes the following statement: "In (1) the measurement of surface area with sufficient accuracy, while not impossible, was too laborious to use with more than one or two specimens." The method used and the result obtained were never published. It is evident that no systematic measurements were undertaken, because on p. 511 of the same paper Wentworth says in respect to wear: "This is probably in part due to the increasing ratio of surface to weight." It is evident that Wentworth has actually no results to fall back on when he criticizes my views as "objectional to natural or experimental shape modifications."

neglected to consider that as the solids are reduced in size their circumferences are also reduced, with the results that the number of revolutions of the solids per mile is increased.

When a solid of cubic shape is reduced in size by attrition in a revolving drum, the volume decrease does not necessarily have any definite proportion to the surface decrease. The change in surface area depends on the change of shape and the decreasing volume. If a cube of 1 cubic decimeter and a surface area of 6 square decimeters (specific surface 6) is reduced in size without change in shape to a cube of 0.5236 cubic decimeters, the surface of the smaller cube amounts to 3.9878 square decimeters and the specific surface is increased to $\frac{3.9878}{0.5236} = 7.44$. If the original cube, however, is reduced in size to 0.5236 cubic decimeters (the volume of the inscribed sphere) and at the same time undergoes a change in shape to a globe, the produced sphere has a surface of 3.1416 square decimeters and a specific surface of 6. Thus the surface-volume ratio is the same as for the original cube. When a cube is reduced in size by attrition in a drum, the produced sphere is, of course, actually smaller than the sphere inscribed in the original cube; but the example shows that the change in shape must be taken into consideration in an experiment of the kind conducted by Wentworth.

My views on the factors involved in attrition of a solid in a revolving drum are different from those of Wentworth. It is perhaps true that the rate of attrition and loss of volume is influenced, as in the case of solution, by an increasing ratio of surface to volume. Thus the shape may eventually influence the rate of attrition. In Wentworth's discussion in the present issue of the *Journal* the problem is, however, to what extent the attrition influences the shape—an entirely different question. The rate of attrition may be influenced by the shape, but the shape is determined primarily by the properties of the particle and secondarily by the rigor of attrition.

Assume that a cube, subject to attrition in a revolving drum, is composed of numerous small spheres arranged according to the rule for maximum pore space or open packing, so that each sphere has six contacts with the surrounding spheres. Assume, also, that the cohesional[41] forces between the spheres are equal in all contacts. The spheres situated in each apex of the solid corners are bound to the mother-cube by three contacts; those in the edges by four contacts; while those in the plane faces are tied to the mother-cube by five contacts. It is therefore to be expected that the solid corners offer the least resistance to attrition, the edges more, and the plane faces most. By removing one sphere from the apex of a solid corner, three other spheres are exposed, each tied to the mother-cube by only three contacts. We may then expect that these three units may be removed with the same ease as the first sphere. Continuing the removal of spheres as they are successively exposed with three contacts, the mother-cube gradually shrinks in a linear direction from the apex of the corners toward the center of the edges, from the center of the edges toward the center of the plane faces as well as toward the center of the cube, and finally also from the center of the plane faces toward the center of the cube. The volumetric loss is greatest in the corners, and the linear centripetal shrinkage is greater than in the edges and plane faces. The final result is a sphere. When this form has been reached, the solid maintains this shape, because the formation of a corner would immediately result in the detachment of the minute sphere so situated.

If, on the other hand, the cohesion between the minute spheres in their contacts with each other is different in different directions, the magnitude of these cohesional forces and the rigor of the attrition will determine whether a sphere or some other shape is produced. If the attrition is feeble, the solid will tend to shape itself according

[41] For the sake of the example the term "cohesion" is here used in a broader sense, meaning the bonds which tend to keep the minute parts together. In actuality we have both attractive and repulsive forces. Beckenkamp (*op. cit.*) said: "Der Widerstand gegen die Trennung der Atome hat einen minimalen Wert, wenn die auf auf die Elektronen zurückzuführenden Kräfte einen minimalen attraktiven oder einen maximalen repulsiven Wert haben."

Huggins gave the following rules governing crystal cleavage: "(1) Cleavage tends to occur so as to leave the two new crystal surfaces electrically neutral. (2) Where some of the bonds in a crystal are weaker than others, cleavage will take place in such a way as to rupture the weaker bonds in preference to the stronger ones. (3) All bonds being equally strong, cleavage will occur between the planes connected by the fewest bonds per unit area (of the cleavage plane)."

Maurice L. Huggins, "Crystal Cleavage and Crystal Structure," *Amer. Jour. Sci.*, 5th ser., Vol. V (1923), pp. 303–13.

to the difference in the cohesional forces within; in other words, those spheres are most readily removed which are so situated that they are bound to the mother-solid by weaker forces. It is evident that some spheres bound to the solid by stronger forces are also removed as they are exposed, according to the views expressed in the preceding paragraph. The total result, however, is that the final shape is determined by the difference in the cohesional forces within the solid.

Wentworth's own experimental work gives an example of the views expressed above. He found that when the marble cubes were rounded and reduced to a weight of 3.29 grams by attrition in the partially water-filled drum, they began to decrease in roundness and apparently also in the degree of sphericity. Wentworth explained this by the durability, as he called it, along various axes. He said that for any given rock there is an ellipsoid of equilibrium for wear by abrasion which depends on the durability along the axes. For a perfectly homogeneous rock this ellipsoid is a sphere, as Wentworth expressed it. For a non-homogeneous rock it is an ellipsoid of greater or less eccentricity. The hypothesis advanced by Wentworth is that the eccentricity is greater for smaller than for larger particles. He said:

> Thus it is conceived that at 3.29 grams the marble cobbles had nearly or quite reached the ellipsoid of durability for that size and from then on were practically following the equilibrium figure in its decreasing values of R as the size grew less.

The hypothesis actually does not explain anything, since the principal question is left open, namely, why the eccentricity is greater for smaller than for larger particles. Wentworth did not consider that he had water in the revolving drum. As long as the solids were sufficiently large and heavy, the slight difference in the cohesional forces within the marble could not be reflected in the shape, because the weight and the momentum of the marble was great enough to eliminate any slight difference in cohesion. In other words, the minute particles constituting the surface were rubbed away by attrition irrespective of a slight difference in cohesion in different directions. As the marble approached the weight of 3.29 grams, the water current produced in and by the revolving drum buoyed up the marble somewhat so as to make the impacts less severe. As the weight was further reduced, the attrition became so feeble that the minute parts of weaker cohesion were more readily detached from the marble than those bound by stronger cohesion. The solid tended to shape itself according to the difference in the cohesional forces within. Thus Wentworth's results find a simple explanation on the basis of the views set forth in this paper.

MODIFICATION OF SHAPE AND ROUNDNESS OF ROCK PARTICLES BY SOLUTION

Wentworth's ideas on shape modification by solution are too generalized, and lack in many cases experimental support. If a process of solution consists of a number of successive stages, the velocity of the whole process depends on that of the slowest component stage. The rate of diffusion is a part of the process of solution, but it is not necessarily the determinant factor in the rate of solution. It seems to me that if the diffusion (without stirring of the solvent) has anything to do with the change of shape of particles subjected to solution, such changes may eventually be due to difference in the density of the diffusion layer in the corners, edges, and plane faces. Even this, however, is rather uncertain in view of experimental results. It is well known that the area of the attacked surface usually has been found to influence the rate of solution, but the influence of the solution process in shaping the solid is a different question. While the volume and the surface area are generally decreasing till the solid is dissolved (the specific surface is increased), the shape usually soon takes a definite character[42] (*Endkörper*) which under constant conditions is maintained to the end of the process.

[42] F. Becke, "Aetzversuche am Fluorit," *Tschermak's Mineralogische und Petrographische Mitteilungen*, Vol. XI (1890), pp. 349–437; G. Wulff, "Zur Frage der Geschwindigkeit des Wachsthums und der Auflösung der Krystallflächen," *Zeitschrift für Krystallographie und Mineralogie*, Vol. XXXIV (1901), pp. 449–530; V. Rosicky, "Über die Symmetrie des Steinsalzes," *Beiträge zur Krystallographie und Mineralogie*, Vol. I (1914–18), pp. 241–56; O. Meyer and S. L. Penfield, "Results Obtained by Etching a Sphere and Crystals of Quartz," *Trans. Conn. Acad.*, Vol. VIII (1889), pp. 158–65; S. L. Penfield and L. V. Pirson, *Contributions to Mineralogy and Petrography*, pp. 160–67; W. Poppe, "Über die Auflösung, von Natriumchlorid- und von Natriumchlorat-Kristallen," *Neues Jahrbuch für Min. Geol. und Paleon.*, Vol. XXXVIII (1915), Beilage-Band, pp. 363–428; H. Bauhansund V. Goldschmidt, "Über Endkörper und Lös-

In the modification of shape and roundness by solution, the result depends mainly upon three factors and their influence on the process. These factors are: (1) the properties of the dissolving particle, (2) the properties of the solvent, and (3) currents and eddies[43] produced in the solvent by the process of solution itself or by stirring of the fluid.

Space limitation does not permit further discussion on the subject. The facts remain that, under certain conditions, spherical or highly spherical and well-rounded solids are produced without stirring of the solvent, while in other cases a decrease in both roundness and sphericity takes place, probably due to the atomic arrangement and the cohesional forces within the solid and a weaker interaction between solute and solvent. If the solid has a fixed position and strong solution currents are present or artificially produced, then the shape modification is influenced by the current velocity and eddies produced about the solid, especially on its lee side. Finally if a number of sand grains in a solvent are stirred around in the bottom of the vessel, their shape is to a large extent a result of attrition. The last is important to have in mind in experimental shape modification of rock particles in solution.

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ungsschwindigkeit von Flussspat," *Beiträge zur Krystallographie und Mineralogie*, Vol. I (1914–18), pp. 219–40; J. J. Galloway, "The Rounding of Grains of Sand by Solution," *Amer. Jour. Sci.*, 4th ser., Vol. XLVII (1919), pp. 270–80; J. J. P. Valeton, "Wachstum und Auflösung der Kristalle," *Zeitschrift für Kristallographie*, Vol. LIX (1924), pp. 135–69, 335–65; Vol. LX, pp. 1–38; G. Aminoff, "Versuche über Verdampfung von Kristallen," *Zeitschrift für Kristallograpie*, Vol. LXI (1925), pp. 373–79.

[43] P. Rosin und H.-G. Kayser, *op. cit.*