Particle Shape Quantities and Measurement Techniques–A Review

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ABSTRACT

It has been shown in the early 20th century that particle shape has an influence on geotechnical properties. Even if this is known, there has been only minor progress in explaining the processes behind its performance and has only partly implemented in practical geotechnical analysis. This literature review covers different methods and techniques used to determine the geometrical shape of the particles. Particle shape could be classifying in three categories; sphericity - the overall particle shape and similitude with a sphere, roundness - the description of the particle's corners and roughness - the surface texture of the particle. The categories are scale dependent and the major scale is to sphericity while the minor belongs to roughness. The overview has shown that there is no agreement on the usage of the descriptors and is not clear which descriptor is the best. One problem has been in a large scale classify shape properties. Image analysis seems according to the review to be a promising tool, it has advantages as low time consumption or repeatability. But the resolution in the processed image needs to be considered since it influences descriptors such as e.g. the perimeter. Shape definitions and its potential role in soil mechanics are discussed.

KEYWORDS: Particle shape, Quantities, Image analysis.

INTRODUCTION

Effects on soil behavior from the constituent grain shape has been suggested since the earliest 1900's when Wadell (1932), Riley (1941), Pentland (1927) and some other authors developed their own techniques to define the form and roundness of particles. Into the engineering field several research works conclude that particle shape influence technical properties of soil material and unbound aggregates (Santamarina and Cho, 2004; Mora and Kwan, 2000). Among documented properties affected by the particle shape are e.g. void ratio (porosity), internal friction angle, and hydraulic conductivity (permeability) (Rousé *et al.*, 2008; Shinohara *et al.*, 2000; Witt and Brauns, 1983). In geotechnical guidelines particle shape is incorporated in e.g. soil classification (Eurocode 7) and in national guidelines e.g. for evaluation of friction angle (Skredkommisionen, 1995). This classification is based on ocular inspection and quantitative judgment made by the individual practicing engineer, thus, it can result in not repeatable data. The lack of possibility to objectively describe the shape hinders the development of incorporating the effect of particle shape in geotechnical analysis.

The interest of particle shape was raised earlier in the field of geology compared to geotechnical engineering. Particle shape is considered to be the result of different agent's transport of the rock from its original place to deposits, since the final pebble form is hardly influenced by these agents (rigor of the transport, exfoliation by temperature changes, moisture changes, etc.) in the diverse stages of their history. Furthermore, there are considerations regarding on the particle genesis itself (rock structure, mineralogy, hardness, etc.) (Wentworth 1922a). The combination of transport and mineralogy factors complicates any attempt to correlate length of transport and roundness due that soft rock result in rounded edges more rapidly than hard rock if both are transported equal distances. According to Barton & Kjaernsli (1981), rockfill materials could be classified based on origin into the following (1) quarried rock; (2) talus; (3) moraine; (4) glaci-fluvial deposits; and (5) fluvial deposits. Each of these sources produces a characteristic roundness and surface texture. Pellegrino (1965) conclude that origin of the rock have strong influence determining the shape.

To define the particle form (morphology), in order to classify and compare grains, many measures has been taken in consideration (axis lengths, perimeter, surface area, volume, etc.). Furthermore, corners also could be angular or rounded (roundness), thus, the authors also focus on develop techniques to describe them. Additionally corners can be rough or smooth (surface texture). Nowadays some authors (Mitchell & Soga, 2005; Arasan *et al.*, 2010) are using these three sub-quantities, one and each describing the shape but a different scale (form, roundness, surface texture).

During the historical development of shape descriptors the terminology has been used differently among the published studies; terms as roundness (because the roundness could be apply in the different scales) or sphericity (how the particle approach to the shape of a sphere) were strong (Wadell, 1933; Wenworth, 1933; Teller, 1976; Barrett 1980; Hawkins, 1993), and it was necessary in order to define a common language on the particle shape field; unfortunately still today there is not agreement on the use of this terminology and sometimes it make difficult to understand the meaning of the authors, that's why it is better to comprehend the author technique in order to misinterpret any word implication.

Several attempts to introduce methodology to measure the particle's shape had been developed over the years. Manual measurement of the particles form is overwhelming, thus, visual charts were developed early to diminish the measuring time (Krumbein, 1941, Krumbein and Sloss, 1963; Ashenbrenner, 1956; Pye and Pye, 1943). Sieving was introduced to determine the flakiness/elongation index but it is confined only for a certain particle size due the practical considerations (Persson, 1988). More recently image analysis on computer base has been applied on sieving research (Andersson, 2010, Mora and Kwan, 2000, Persson, 1998) bringing to the industry new practical methods to determine the particle size with good results (Andersson, 2010). Particle shape with computer assisted methods are of great help reducing dramatically the measuring time (Fernlund, 2005; Kuo and Freeman 1998a; Kuo, *et al.*, 1998b; Bowman, *et al.*, 2001).

In the civil industry e.g. Hot Asphalt mixtures (Kuo and Freeman, 1998a; Pan, et al., 2006), Concrete (Mora *et al.*, 1998; Quiroga and Fowle, 2003) and Ballast (Tutumluer et al., 2006) particle's shape is of interest due the material's performance, thus, standards had been developed (e.g. EN 933-4:2000 Tests for geometrical properties of aggregates; ASTM D 2488-90 (1996) Standard practice for description and identification of soils).

Sieving is probably the most used method to determine the particle size distribution. This traditional method, according to Andersson (2010) is time consuming and expensive. Investigations shows that the traditional sieving has deviations when particle shape is involve; the average volume of the particles retained on any sieve varies considerably with the shape (Lees, 1964b), thus, the passing of the particles depend upon the shape of the particles (Fernlund, 1998). In some industries the Image analysis is taking advantage over the traditional sieving technique regardless of the intrinsic error on image analysis due the overlapping or partial hiding of the rock particles (Andersson, 2010). In this case the weight factor is substitute by pixels (Fernlund *et al.*, 2007). Sieving curve using image analysis is not standardized but after good results in the practice (Andersson, 2010) new methodology and soil descriptions could raise including its effects.

Describing the particle's shape is the main objective, there are 42 different quantities in this document, and it is required to review the information about them to comprehend and interpret the implication of each quantity to determine them usability and practice.

DESCRIPTION OF SHAPE PROPERTIES

Particle shape description can be classified as qualitative or quantitative. Qualitative describe in terms of words the shape of the particle (e.g. elongated, spherical, flaky, etc.); and quantitative that relates the measured dimensions; in the engineering field the quantitative description of the particle is more important due the reproducibility.

Quantitative geometrical measures on particles may be used as basis for qualitative classification. There are few qualitative measures in contrast with several quantitative measures to describe the particle form. Despite the amount of qualitative descriptions none of them had been widely accepted; but there are some standards (e.g., ASTM D5821, EN 933-3 and BS 812) specifying mathematical definitions for industrial purposes.

Shape description of particles is also divided into two:

-3D (3 dimensions): it could be obtained from a 3D scan or in a two orthogonal images and

-2D (2 dimensions) or particle projection, where the particle outline is drawn.

3D and 2D image analysis present challenges itself. 3D analysis requires a sophisticated equipment to scan the particle surface and create the 3D model or the use of orthogonal images and combine them to represent the 3 dimensions. The orthogonal method could present new challenges as the minimum particle size or the placing in orthogonal way of the particles (Fernlund, 2005). 2D image analysis is easy to perform due the non-sophisticated equipment required to take pictures (e.g. regular camera or the use of microscope for smaller particles). In 2D image analysis the particle is assumed to lay over its more stable axis (e.g. longest and intermediate axis lie more or less parallel to the surface while the shortest axis is perpendicular) or random, some authors publish their own preferences about this issue (Wadell, 1935; Riley, 1941; Hawkins, 1993).

SCALE DEPENDENCE

In order to describe the particle shape in detail, there are a number of terms, quantities and definitions used in the literature. Some authors (Mitchell & Soga, 2005; Arasan et al., 2010) are using three sub-quantities; one and each describing the shape but at different scales. The terms are morphology/form, roundness and surface texture. In Figure 1 is shown how the scale terms are defined.

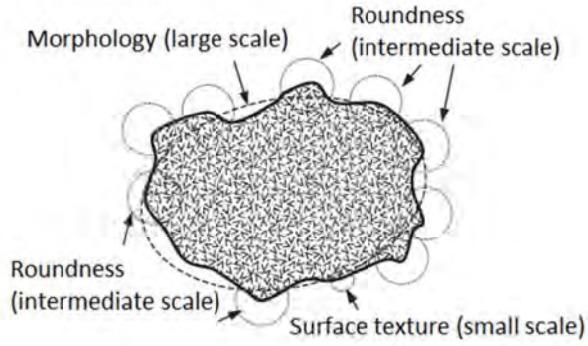

Figure 1: Shape describing sub quantities (Mitchell & Soga, 2005)

At large scale the particle's diameters in different directions are considered. At this scale, describing terms as spherical, platy, elongated etc., are used. An often seen quantity for shape description at large scale is sphericity (antonym: elongation). Graphically the considered type of shape is marked with the dashed line in Figure 1. At intermediate scale it is focused on description of the presence of irregularities. Depending on at what scale an analysis is done; corners and edges of different sizes are identified. By doing analysis inside circles defined along the particle's boundary, deviations are found and valuated. The mentioned circles are shown in Figure 1. A generally accepted quantity for this scale is roundness (antonym: angularity). Regarding the smallest scale, terms like rough or smooth are used. The descriptor is considering the same kind of analysis as the one described above, but is applied within smaller circles, i.e. at a smaller scale. Surface texture is often used to name the actual quantity. The sub-quantities and antonyms are summarized in table 1.

Table 1: Sub-quantities describing the particle's morphology and its antonym

Scale	**Quantity**	**Antonym**
Large scale	Sphericity	Elongation
Intermediate scale	Roundness	Angularity
Small scale	Roughness	Smoothness

FORM (3D)

Wentworth in 1922 (Blott and Pye, 2008), was probably one of the first authors on measure the particle dimensions, this consisted on the obtaining of the length of the tree axes perpendicular among each other (see Figure 2) on the tree dimensions (where a≥b≥c) to obtain the sphericity (Equation 1).

$$\Psi = \frac{a+b}{2c} \tag{1}$$

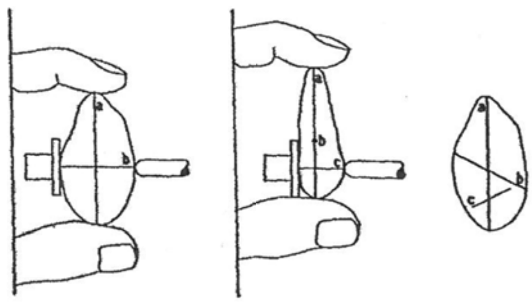

Figure 2: Measurement of the 3 axes perpendicular among each other (Krumbein, 1941)

Krumbein (1941) develop a rapid method for shape measurement to determine the sphericity; this is done by measuring the longest (a), medium (b) and shorter (c) axes diameters of the particle, it can be seen in Figure 2 (Always perpendicular among each other). The radios b/a and c/b are located in the chart developed by his own where it can be found the Intercept sphericity as he called (See Figure 3). This chart is an easy graphical way to relate the dimensions.

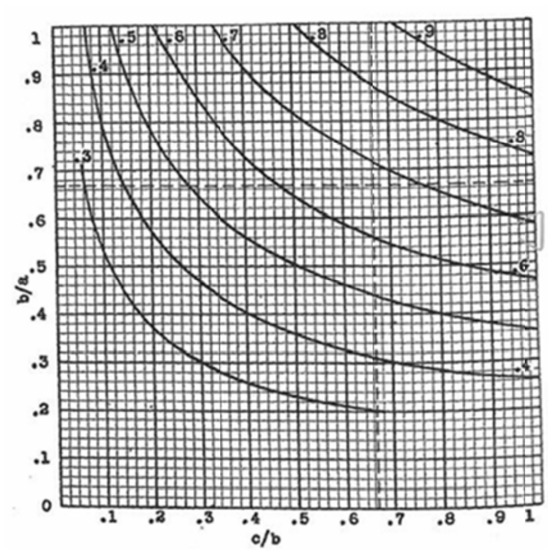

Figure 3: Detailed chart to determining Krumbein intercept sphericity (Krumbein 1941).

Wadell (1932) defined the sphericity as the specific surface ratio (Equation 2). Figure 4 is a schematic representation of the sphere surface and particle surface, both particle and sphere of the same volume.

$$\Psi = \frac{s}{S} \tag{2}$$

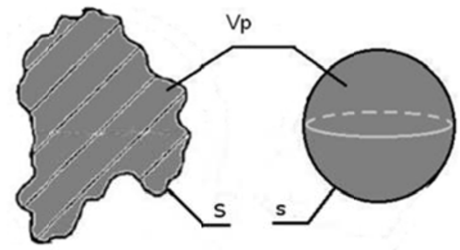

Figure 4: Same volume sphere surface (s) and particle surface (S). (modified after Johansson and Vall, 2011).

This way to obtain the sphericity is almost impossible to achieve, as Hawkins (1993) declares, due the difficulty to get the surface area on irregular solids.

Wadell (1934) also defined the sphericity based upon the particle and sphere volumes, as Equation 3 (see Figure 5):

$$\Psi = \sqrt[3]{\frac{V_P}{V_{CIR}}} \tag{3}$$

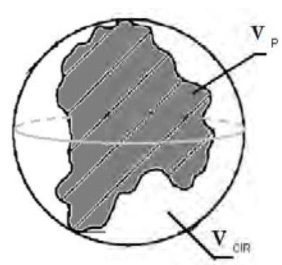

Figure 5: Relation between the volume of the particle and the volume of the circumscribed sphere (Johansson and Vall, 2011).

Wadell (1934) used a new formula simple to manage using the diameters (see Figure 6 and Equation 4).

$$\Psi = \frac{D_{SV}}{D_{CIR}} \tag{4}$$

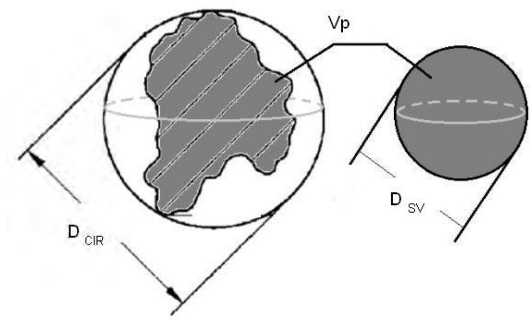

Figure 6: The relation between the diameter of a circumscribed sphere and the diameter of a sphere of the same volume as the particle (Johansson and Vall, 2011).

Zingg (Krumbein, 1941) develop a classification based on the 3 axes relation, in this way it is easy to find out the main form of the particles as a disks, spherical, blades and rod-like; this is summarized on Figure 7. Zingg's classification is related with Krumbein intercept sphericity and the Figure 3.

Class	b/a	c/b	Shape
I	>2/3	<2/3	Disks
II	>2/3	>2/3	Spherical
III	<2/3	<2/3	Blades
IV	<2/3	>2/3	Rod-Like

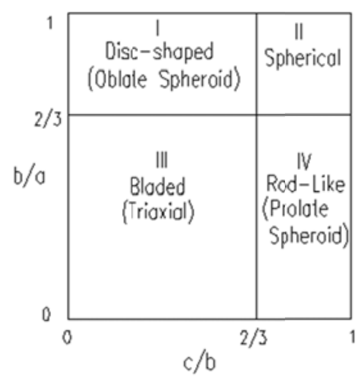

Figure 7: Zingg's classification of pebble shape based on ratios *b/a* and *c/b* (Krumbein 1941).

In Figure 8 the Figures 3 and 7 are combined, the relation in the two classifications can be seen, it is an easy way to understand the morphology regarding on the a, b and c dimensions

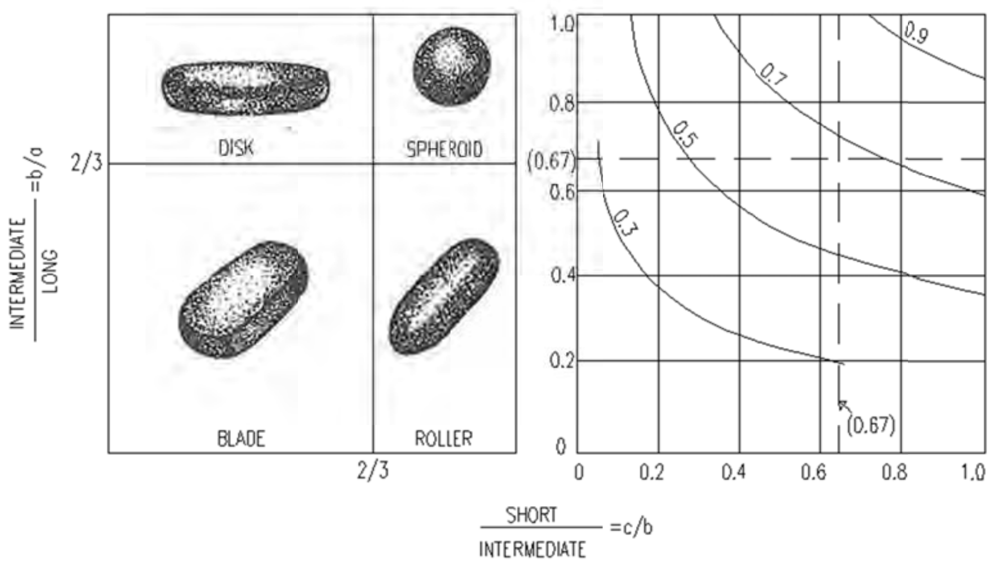

Figure 8: Classification made by Zingg's and chart to determine sphericity (Krumbein and Sloss, 1963)

Pye and Pye (1943), in the article "sphericity determinations of pebbles and sand grains" compare the Wadell's sphericity developed in 1934 (based on the diameter) with "Pebble sphericity" based on an ellipse, this last Equation (number 5) appears two years early published by Krumbein (1941). Axis measurement is done as Figure 1 denotes for Equations 5 trough 12 with exception of Equation 8 where the original document was not possible to obtain. Sneed & Folk in (1958) describes a relation between the tree dimensional axes called "Maximum Projection Sphericity" (Equation 6).

$$\Psi = \sqrt[3]{\frac{b \cdot c}{a^2}} \tag{5}$$

$$\Psi = \sqrt[3]{\frac{c^2}{a \cdot b}} \tag{6}$$

In a similar way Ashenbrenner (1956) showed his Equation (7) at that time named "Working Sphericity"

$$\Psi = \frac{12{,}8 * \sqrt[3]{(c/b)^2 * (b/a)}}{1 + (c/b)(1 + (b/a)) + 6 * \sqrt{1 + (c/b)^2 (1 + (b/a)^2)}} \tag{7}$$

Form or shape factor names are used by authors like Corey (shape factor, eq. 8) in the paper published on 1949, Williams (shape factor, eq. 9) in 1965, Janke (form factor, eq. 10) in 1966 and Dobkins & Folk (oblate-prolate index, eq. 11) in 1970 (Blott and Pye, 2008).

$$\Psi = \frac{c}{\sqrt{ab}} \tag{8}$$

$$\Psi = 1 - \frac{ac}{b^2} \text{ when } b^2 > ac, \frac{b^2}{ac} - 1 \text{ when } b^2 \le ac \tag{9}$$

$$\Psi = \frac{c}{\sqrt{\frac{a^2 + b^2 + c^2}{3}}} \tag{10}$$

$$\Psi = \frac{10\left(\frac{a-b}{a-c} - 0.5\right)}{\frac{c}{a}} \tag{11}$$

Aschenbrenner (1956) developed the shape factor by using the relation of the tree axis but the square of the middle one.

$$\Psi = \frac{ac}{b^2} \tag{12}$$

Table 2: General overview over different particle shape definitions for 3D sphericity has been compiled and arranged chronologically

Aspect	Name	Author	Year	Based on
Sphericity (3D)	Flatness index	Wentworth	1922a	3-axes
	True Sphericity	Wadell	1932	Surface
	Operational sphericity	Wadell	1932	Volume
	Sphericity	Wadell	1934	Sphere diameter
	Zingg's classification	Zingg's[1]	1935	3-axes
	Intercept sphericity chart	Krumbein	1941	3-axes
	Pebble sphericity	Pye and Pye	1943	3-axes
	Corey shape factor	Corey[2]	1949	3-axes
	Working sphericity	Ashenbrenner	1956	3-axes
	shape factor	Ashenbrenner	1956	3-axes
	Maximum projection sphericity	Sneed & Folk	1958	3-axes
	Williams shape factor	Williams[2]	1965	3-axes
	Janke form factor	Janke[2]	1966	3-axes
	Oblate-prolate index	Dobkins & Folk	1970	3-axes

1) Krumbein and Sloss, 1963
2) Blott and Pye, 2008

FORM (2D)

The technique to measure the sphericity is based in three dimensions, it can be found in literature some ways to measure the "two dimensions sphericity" which is simply the perimeter of the particle projection, some authors named "particle outline" or "circularity".

Wadell in 1935 (Hawkins, 1993) adopt a conversion of his 1934 3D sphericity formula (Equation 4) to a 2D outline. He defined an orientation on the particles and they were based on the maximum cross sectional area (outline of the particle projecting the maximum area). The Equations (13) show the relation between diameters of a circle of same area (D_A) and smallest circumscribed circle (D_C). He also used the term "degree of circularity" (Equation 14) as the ratio of the perimeter of a circle of same area (P_C) and the actual particle perimeter (P).

Tickell in 1931 (Hawkins, 1993) used his empirical relation (Equation 15). The particle orientation proposed was a random one. It is described by the ratio between the area outline (A) and the area of smallest circumscribed circle (A_C).

$$C = \frac{D_A}{D_C} \tag{13}$$

$$C = \frac{P_C}{P} \tag{14}$$

$$C = \frac{A}{A_C} \tag{15}$$

Some other authors has been working with the "circularity" concept and had develop them own Equations as Pentland (1927) relating the area (A) outline and area of a circle with diameter equal to longest length outline (A_{C2}), and Cox (Riley, 1941) with the ratio area (A) and perimeter (P) time a constant, Equations 16 and 17 respectively. Both authors did not define any definite orientation of the grains.

$$C = \frac{A}{A_{C2}} \tag{16}$$

$$C = \frac{4\pi A}{P^2} \tag{17}$$

Riley (1941), realize the problems that an area, perimeter and some other measurements proposed by the above authors can carry as the time consuming and tedious work (at that time were not computer, all was made by hand), and that's why he develop this Equation easy to handle called "inscribed circle sphericity". He used the same particle orientation proposed by Wadell and the relation of diameters of inscribed (D_I) and circumscribed (D_C) circles (Equation 18). Horton 1932 (Hawkins, 1993) use the relation of the drainage basing perimeter (P_D) and the perimeter of a circle of the same area as drainage basin (P_{CD}), see Equation 19.

$$C = \sqrt{\frac{D_I}{D_C}} \tag{18}$$

$$C = \frac{P_D}{P_{CD}} \tag{19}$$

Janoo in 1998 (Blott and Pye, 2008) develop his general ratio of perimeter (P) to area (A), Equation 20. Sukumaran and Ashmawy (2001) develop his own shape factor (SF) defined as the deviation of the global particle outline from a circle. Figure 9 can be used as a reference to determine the items used in the Equation 21 (N, referred to the number of samples intervals or radial divisions).

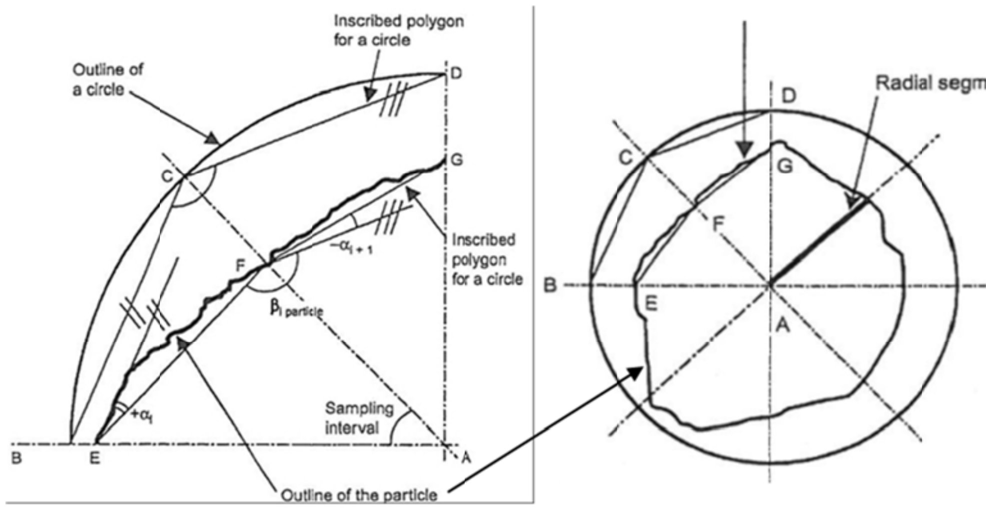

Figure 9: Description of the Sukumaran factors to determine the shape and angularity (Sukumaran and Ashmawy, 2001).

$$C = \frac{P^2}{A} \tag{20}$$

$$SF = \frac{\sum_{i=1}^{N} \left| \alpha_i \text{Particle} \right|}{N \times 45^\circ} \tag{21}$$

ROUNDNESS OR ANGULARITY

Roundness as described previously is the second order shape descriptor. Sphericity lefts beside the corners and how they are, this was notice by most of the authors sited before and they suggested many ways to describe this second order particle property.

Roundness is clearly understandable using the Figure 1. Particle shape or form is the overall configuration and denotes the similarities with a sphere (3D) or a circle (2D). Roundness is concerning about the sharpness or the smoothness of the perimeter (2D). Surface texture (Barret, 1980), is describe as the third order subject (form is the first and roundness the second), and it is superimposed in the corners, and it is also a property of particles surfaces between corners.

Wadell (1935) describes his methodology, calling it total degree or roundness to obtain the roundness of a particle using the average radius of the corners (r) in relation with the inscribed circle diameter (Rmax-in), see Figure 11 and Equation 22. In the same study Wadell (1935) has used Equation 23 (N, is the number of corners). This two last Equation shows slightly differences on the results (Wadell, 1935).

Table 3: General chronological overview of the particle shape definitions for 2D sphericity.

Aspect	Name	Author	Year	Based on
Circularity (2D)	roundness	Pentland	1927	area
	roundness	Cox[1]	1927	area-perimeter
	roundness	Tickell[2]	1931	area
	Circularity	Horton[2]	1932	drainage basin
	outline circularity	Wadell	1935	Circle diameter
	degree of circularity	Wadell	1935	Perimeter
	inscribed circle sphericity	Riley	1941	Circle diameter
	Circularity	Krumbein and Sloss	1963	chart
		Janoo	1998	area-perimeter
	Shape factor	Sukumaran	2001	Segmentation of particle and angles

1) Riley, 1941
2) Hawkins, 1993

$$R = \frac{\sum\left(\frac{r}{R_{max-in}}\right)}{N} \quad (22) \qquad\qquad R = \frac{N}{\sum\left(\frac{R_{max-in}}{r}\right)}$$
)

Powers (1953) also published a graphic scale to illustrate the qualitative measure (Figure 12). It is important to highlight that any comparing chart to describe particle properties has a high degree of subjectivity. Folk (1955) concludes that when charts are used for classification, the risk of getting errors is negligible for sphericity but large for roundness.

Figure 11: Wadell's method to estimate the roundness, corners radius and inscribed circle (Hawkins, 1993).

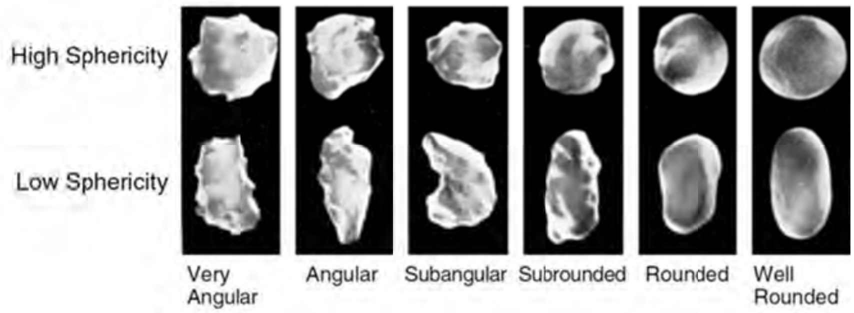

Figure 12: A Roundness qualitative scale (Powers, 1953)

Some authors as Russel & Taylor in 1937, Pettijohn in 1957 and Powers in 1953 developed a classification based on five and six classes (Hawkins, 1993) each one with its own class limits; it is important to denote that the way they measure the roundness is the developed by Wadell (1935). This classification and class limits are showed in the Table 4.

Table 4: Degrees of roundness: Wadell Values. (Hawkins, 1993), N/A = no-applicable

Grade terms	Russell & Taylor (1937)		Pettijohn (1957)		Powers (1953)	
	Class limits (R)	Arithmetic midpoint	Class limits (R)	Arithmetic midpoint	Class limits (R)	Arithmetic midpoint
Very angular	N/A	N/A	N/A	N/A	0.12-0.17	0.14
Angular	0.00-0.15	0.075	0.00-0.15	0.125	0.17-0.25	0.21
Subangular	0.15-0.30	0.225	0.15-0.25	0.200	0.25-0.35	0.30
Subrounded	0.30-0.50	0.400	0.25-0.40	0.315	0.35-0.49	0.41
Rounded	0.50-0.70	0.600	0.40-0.60	0.500	0.49-0.70	0.59
Well rounded	0.70-1.00	0.800	0.60-1.00	0.800	0.70-1.00	0.84

Krumbein and Sloss (1963) published a graphical chart easy to determine the sphericity and roundness parameters using comparison. See Figure 13. (Cho, et al. 2006).

Fischer in 1933 (Hawkins, 1993) used a straightforward method to quantify roundness using a central point in the outline and dividing the outline in angles around this point that were subtended by the straight or non-curved parts of the profile were measured. This is illustrated in Figure 14.

To express the angularity value Fischer used the ratio of angles standing linear parts (ANG_{PLA}) on the outlines and concave (ANG_{CON}) Equations 24 and 25 respectively.

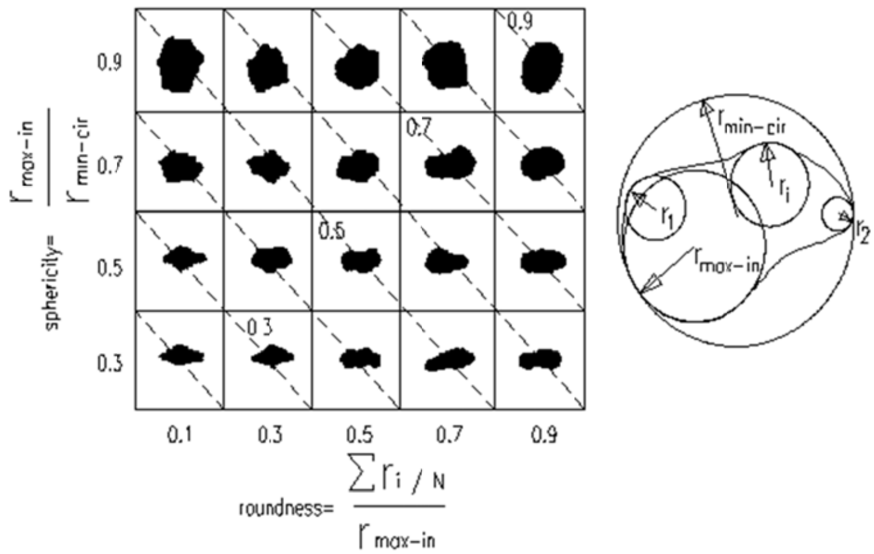

Figure 13: Sphericity and roundness chart. (Cho et, al., 2006). The roundness Equation that appears here in the chart is the wadell’s Equation number 22

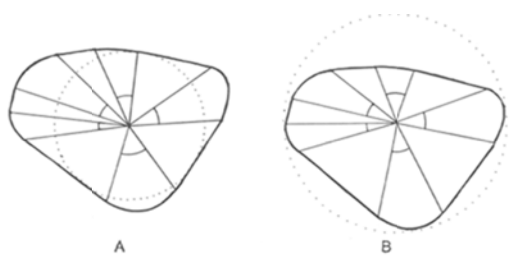

Figure 14: Fischer’s methods of angularity computation (Hawkins, 1993)
A=inscribed circle; B=circumscribed circle

$$R=\frac{\sum ANG_{PLA}}{360°} \qquad R=\frac{\sum ANG_{CON}}{\sum ANG_{PLA}}$$

Figure 14 left (A) and right (B), gives a similar angularity of approximately 0.42 using the above Equations. (Hawkins, 1993).

Wentworth in 1922 (Equation 26) used the maximum projection to define the position of the particle to obtain the outline or contour (Barret, 1980). The Equation reflects the relation of the diameter of a circle fitting the sharpest corner (D_S) and the longest axis (L) plus the shortest axis c in minimum projection (S_M). Wentworth (Hawkins, 1993) also expressed the roundness as the ratio of the radius of curvature of the most convex part (**R**$_{CON}$) and the longest axis (L) plus short axis (B), see Equation 27.

$$R=\frac{D_S}{(L+S_M)/2} \quad) \qquad R=\frac{R_{CON}}{(L+B)/4}$$

Actually these last two Equations are the same, just expressed in different terms, when the particle is in its maximum projection.

Dimensions can be seen on Figure 15, L and B represents the mayor axis a and intermediate axis b. The intention is to make difference between the 2 and 3 dimensions (L and B are for 2D as a, b and c are for 3D).

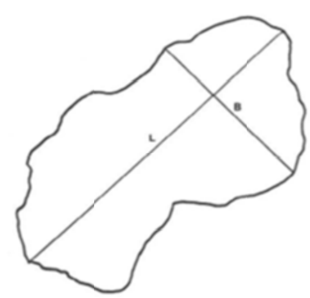

Figure 15: Description of L and B axes (Hawkins, 1993)

Wentworth 1919 has a second way to express the roundness called Shape index (Barrett, 1980) and it relates the diameter of the sharpest corner (D_S) and the diameter of a pebble trough the sharpest corner (D_X).

$$R = \frac{D_s}{D_x}$$

Wentworth (1922b), used define the roundness as the ratio of the sharpest corner (R_{CON}) and the average radius of the pebble (R_{AVG}):

$$R = \frac{R_{CON}}{R_{AVG}} \qquad (29) \qquad 2 * R_{AVG} = D_{AVG} = \sqrt[3]{a * b * c}$$

Cailleux (Barrett, 1980) relates the radius of the most convex part and the longest axis (Equation 31). Kuenen in 1956 show his roundness index (Barrett, 1980) between the sharpest corner (D_S) and the breath axis (B), Equation 32. Dobkins & Folk (1970) used a modified Wentworth roundness with the relation of sharpest corner (D_S) and inscribed circle diameter (D_I), Equation 33.

$$R = \frac{R_{CON}}{L/2} \qquad) \qquad R = \frac{D_s}{B} \qquad (32) \qquad R = \frac{D_s}{D_I}$$

Swan in 1974 shows his Equation (Barrett, 1980) relating the sharpest (or the two sharpest) corner(s) (D_{S1} and D_{S2}) and inscribed circle diameter (D_I), Equation 34. Szadeczsky-Kardoss has his Average roundness of outline (Krumbein and Pettijohn, 1938) relating the concave parts perimeter (P_{CON}) and the actual perimeter (P), Equation 35.

$$R = \frac{(D_{s1} + D_{s2})/2}{D_I} \qquad (34) \qquad R = \frac{P_{CON}}{P} * 100$$

Lees (1964a) developed an opposite definition to roundness, it means that he measures the angularity instead of the roundness, and he calls it Degree of angularity. Figure 16 shows the requirements considered when Equation 36 applies as the angles (α), inscribed circle (R_{max-in}) and the distance (x). See Equation 36.

In order to apply the Equation 36 corners needs to be entered in the formula, and each individual result will add to each other to obtain the final degree of angularity.

$$R = (180 - \alpha)\frac{x}{R_{max-in}} \tag{36}$$

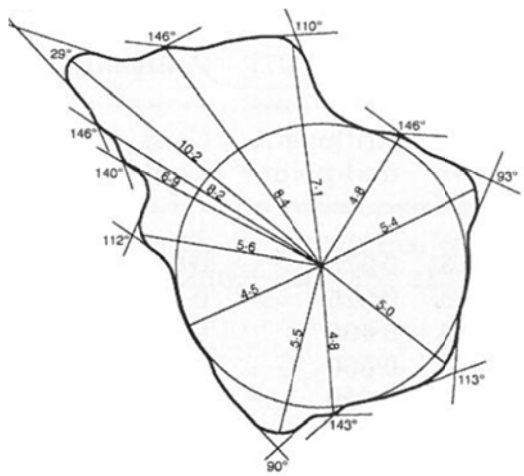

Figure 16: Degree of angularity measurement technique (Blot and Pye, 2008)

A roundness index appears on Janoo (1998), Kuo and Freeman (1998a) and Kuo, et, al. (1998b) it is described as:

$$R = \frac{4\pi A}{P^2}$$

The last Equation appears also as a 2D descriptor because there is not a general agreement on the definition furthermore some authors had used to define the roughness, this is not the only Equation that has been used trying to define different aspects (sphericity, roundness or roughness) but it is a good example of the misuse of the quantities and definitions.

Table 5: General chronological overview of the particle roundness

Aspect	Name	Author	Year	Based on
Roundness	shape index	Wentworth	1919[1]	diameter of sharper corner
	shape index	Wentworth	1922b	sharpest corner and axis
	roundness	Wentworth	1933	convex parts
		Fischer	1933[2]	noncurved parts outline
		Fischer	1933[2]	noncurved-straight parts outline
	Average roundness of outline	Szadeczsky-Kardoss	1933[3]	convex parts-perimeter
	roundness	Wadell	1935	diameter of corners
	roundness	Wadell	1935	diameter of corners
	roundness	Russel & Taylor	1937[2]	class limit table
	roundness	Krumbein	1941	chart
		Cailleux	1947[1]	convex parts
	roundness	Pettijohn	1949[4]	class limit table
	roundness	Powers	1953	chart and class limit table
		Kuenen	1956[1]	axis-convex corner
	roundness	Krumbein and Sloss	1963	chart
	degree of angularity	Lees	1964a	corners angles and inscribed circle
		Dobkins & Folk	1970	diameter of sharper corner
		Swan	1974[1]	diameter of sharper corners
	Angularity factor	Sukumaran and Ashmawy	2001	Segmentation of particles and angles

1) Barret, 1980
2) Hawkins, 1993
3) Krumbein and Pettijohn, 1938
4) Powers, 1953

Sukumaran and Ashmawy (2001) present an angularity factor (AF) calculated from the number of sharpness corners (Equation 37). Angles βi required to obtain the angularity factor are shown in Figure 9:

$$AF = \frac{\sum_{i=1}^{N}(\beta_i \text{Particle} - 180°)^2 - \sum_{i=1}^{N}(\beta_i \text{circle} - 180°)^2}{3 \times (180°)^2 - \sum_{i=1}^{N}(\beta_i \text{circle} - 180°)^2} \quad (37)$$

Sukumaran and Ashmawy (2001) also suggested use not bigger sampling interval of N=40 because it is the cut off between angularity factor and surface roughness. If so this Equation could be used to describe the roughness.

ROUGHNESS OR SURFACE TEXTURE

A third property called texture appears early in the literature with the sphericity and roundness properties, since then, texture property was longed described but it was in accordance with the authors, at that time, not measurable.

Wright in 1955 developed a method to quantify the surface texture or roughness of concrete aggregate using studies done on 19 mm stones. The test aggregates were first embedded in a synthetic resin. The stones were cut in thin sections. The sections projection was magnified 125 times. The unevenness of the surface was traced and the total length of the trace was measured. The length was then compared with an uneven line drawn as a series of chords (see Figure 17). The difference between these two lines was defined as the roughness factor. (Janoo 1998).

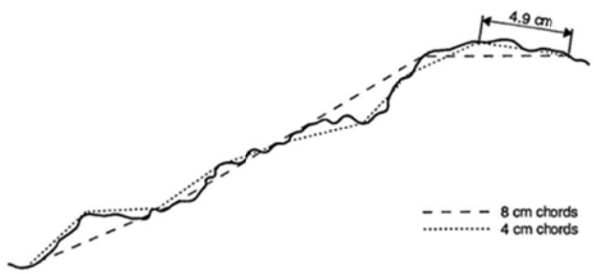

Figure 17: Measurement method for characterizing the surface texture of an aggregate (Janoo, 1998)

However, with the advance of technology it has become easier measure the roughness and here is presented some researcher's ideas how this property should be calculated.

One technique used by Janoo (1988) to define the roughness can be seen in Figure 18a and is defined as the ratio between perimeter (P) and convex perimeter (C_{PER}).

$$R_O = \frac{P}{C_{PER}} \tag{38}$$

The convex perimeter is obtained using the Feret's box (or diameter) tending a line in between the touching points that the Feret's box describes each time it is turn (Figure 18b).

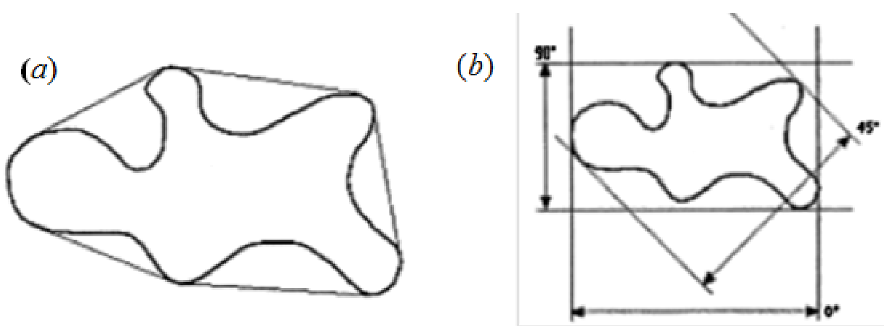

Figure 18: (*a*) Convex perimeter (C_{PER}), (*b*) Feret measurement (modified after Janoo, 1998)

Kuo and Freeman (1998a) and Kuo *et al.*, (1998b) use the roughness (R_O) definition as the ratio perimeter (P) and average diameter (D_{AVG}), Equation 39.

Erosion and dilatation image processing techniques are used to obtain the surface texture. Erosion is a morphological process by which boundary image pixels are removed from an object surface, which leaves the object less dense along the perimeter or outer boundary. Dilatation is the reverse process of erosion and a single dilatation cycle increases the particle shape or image dimension by adding pixels around its boundary (Pan *et al.*, 2006). The "n" erosion and dilatation cycles are not standardized. A represents the original area and A1 is the area after "n" cycles of erosion-dilatation (Equation 40).

$$R_O = \frac{P}{\pi * D_{AVG}}$$)

$$R_O = \frac{A - A1}{A} * 100$$

Mora and Kwan (2000) used the "convexity ratio, CR" (Equation 41) and the "fullness ratio, FR" (Equation 42) in their investigation, they are:

$$R_o = \frac{A}{A_{CON}}$$)

$$R_o = \sqrt{\frac{A}{A_{CON}}}$$

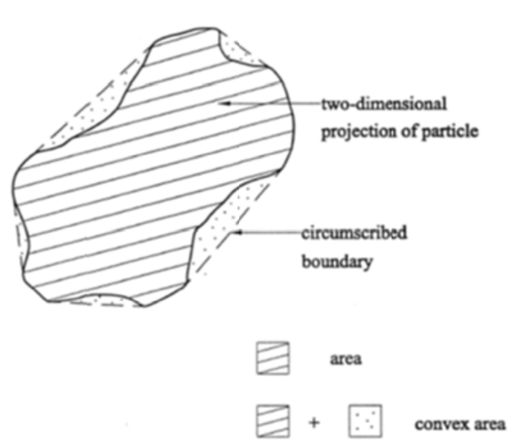

Figure 19: Evaluation of area and convex area (Mora and Kuan, 2000)

The convex area is the area of the minimum convex boundaries circumscribing the particle. This is illustrated in the Figure 19. The convex area is obtained in a similar way as the convex perimeter but in this case the area between the original outline and the convex perimeter is our convex area

TECHNIQUES TO DETERMINE PARTICLE SHAPE

HAND MEASUREMENT

Hand measurement technique was the first used by obvious reasons, in order to improve the accuracy special devices developed as the "sliding rod caliper" used by Krumbein (1941), it works placing the sample on the sliding road calliper as show Figure 20b the length in different positions can be obtain by using the scale provided in the handle; the "convexity gage" that was actually used by opticians to measure the curvature of lenses but easily applicable to the particle shape analysis (Wentworth, 1922b) works measuring the movement of the central pivot as Figure 20a shows (the two adjacent pivots are invariable) as many the central pivot moves more is the curvature; or the "Szadeczky-Kardoss's apparatus" develop in 1933 that traces the profile of the rock fragment, so, the outline traced is then analyzed (Krumbein and Pettijohn, 1938) Figure 20c show equipment.

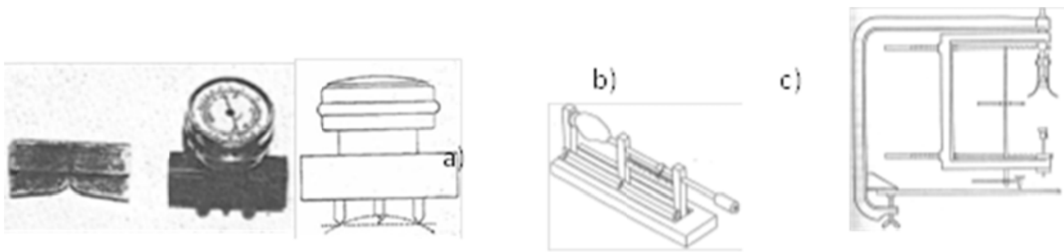

Figure 20: (*a*) convexity gage, used to determine the curvature in particle corners (Wenworth, 1922b), (*b*) sliding rod caliper, device to measure the particle axis length (Krumbein, 1941) and c) Szadeczky-Kardoss (1933) apparatus, it was utilized to obtain the particle outline.

Another helpful tool to determine the particle dimensions was the "camera lucida" to project the particle's contour over a circle scale appearing in Figure 21, thus it is possible to measure the particle's diameter.

Figure 21: Circle scale used by Wadell (1935) to determine particle's diameter and roundness

SIEVE ANALYSIS

Sieving, e.g. according to EN 933-3:1997, can be used to determine simple large scale properties. By combining mesh geometries the obtained results can be used to quantify flakiness and elongation index, ASTM D4791 (Flat and elongated particles are defined as those coarse aggregate particles that have a ratio of length to thickness equal to or greater than a specified value such as 5:1. The index represents the percentage on weight of these particles). The method is not suitable for fine materials. This due to the difficulty to get the fine grains passed through the sieve, and the great amount of particles in relation to the area of the sieve (Persson, 1998) e.g. EN 933-3:1997 related to flakiness index. The test is performed on aggregates with grain size from 4 mm and up to 63 mm. two sieving operations are necessary, the first separates on size fraction and the second use a bar sieve, after the first sieving the average maximum diameter of the particles is obtain and with the second sieving (bar sieving) the shortest axis diameter is found, finally with this two parameters the flakiness index is determined. There are more standards related with the particle shape but, this above presented are probably the most known using sieve analysis to determine particle's geometrical properties. Sieve analysis is facing the computers age and image analysis sieving research is taking place (Andersson, 2010; Mora and Kwan, 2000; Persson, 1998). Industry is also applying the image analysis sieving with decrees on the testing time compare with the traditional sieving method. An inconvenient of image analysis is the error due the overlapping or hiding of the particles during the capture process but the advantages are more compare with disadvantages (Anderson, 2010).

CHART COMPARISON

Charts developed over the necessity of faster results because the long time consuming required when measuring each particle. Krumbein (1941) present a comparison roundness chart for pebbles which were measured by Wadell's method because this property was the most difficult to measure due to the second order scale that roundness represents. (See Figure 22).

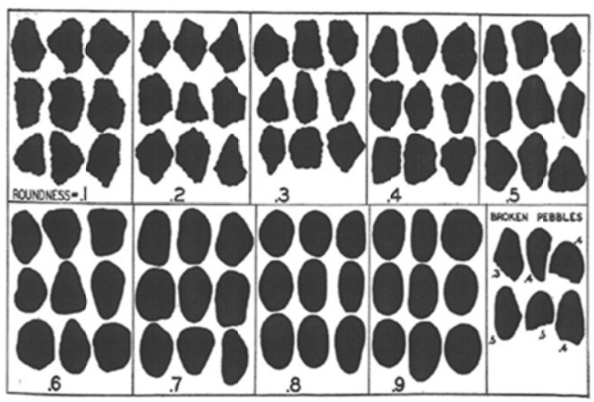

Figure 21: Krumbein (1941) comparison chart for roundness

A qualitative chart by Powers (1953) try to include both (sphericity and roundness) particle's characteristics, it was divided on six roundness ranges (very angular, angular, sub-angular, sub-rounded, rounded and well rounded) and two sphericity series (high and low sphericity). This chart was prepared with photographs to enhance the reader perspective. (See Figure 12)

A new chart including sphericity and roundness appear, this time it was easier to handle the two mean properties of particle's shape, furthermore, there was included the numerical values that eliminated the subjectivity of qualitative description. The chart is based on Wadell's definitions. (Krumbein and Sloss, 1963). (See Figure 13).

Folk (1955) worried about the person's error on the chart's comparison studied the determination of sphericity and angularity (he used the Powers 1953 comparison chart), he found that the sphericity determination by chart comparison has a negligible error while the roundness, he concluded, it was necessary to carry out a more wide research due the high variability show by his study.

IMAGE ANALYSIS

Image analysis is a practical method to use for shape classification since it is fast and can be automated. Different techniques appear to process these images, among them are:

- Feret Diameter: the Feret diameter is the longitude between two parallel lines, this lines can rotate around one particle, or outline, to define dimensions, as it is shown in Figure 22 (left) these method is not a fine descriptor, but as it was say above it is a helpful tool to determine diameters (Janoo, 1988)
- Fourier Technique: It produces mathematical relations that characterize the profile of individual particles (Equation 43). This method favours the analysis of roughness and textural features for granular soils. The problem in the methodology remains in the re-entrant angles in order to complete the revolution (Bowman et al., 2001), see Figure 22 (right).

$$R(\theta) = a_0 + \sum_{n=1}^{N} (a_n \cos n\theta + b_n \sin n\theta)$$

- Fractal Dimension: Irregular line at any level of scrutiny is by definition fractal (Hyslip and Vallejo, 1997), Figure 23 shows fractal analysis by the dividing method. The length of the fractal line can be defined as Equation 44.

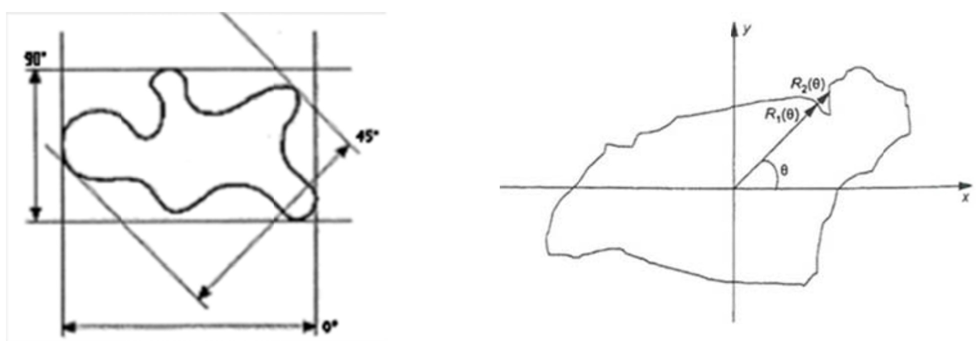

Figure 22: (left) Feret measurement technique is defined by two parallel lines turning around the particle to define the shortest and longest Feret diameter (Janoo, 1988), (right) Fourier technique with two radiuses at one angle (Bowman et al., 2001)

$$P(\lambda) = n\lambda^{1-D_R}$$

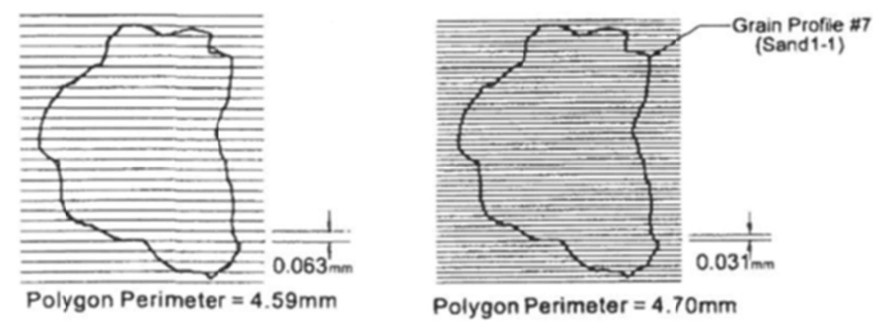

Figure 23: Fractal analysis by the dividing method at different scrutiny scale (Hyslip and Vallejo, 1997)

- Orthogonal image analysis: This technique is basically the use of two images orthogonal between them to acquire the three particle dimensions (Fernlund, 2005), any of the above techniques can be used in this orthogonal way.
- Laser Scanning Technique: this kind of laser scanning 3D is one of the most advanced techniques. In Figures 24a) is showed the laser head scanning the rock particles, the particles have control points in order to keep a reference point when move them to scan the lower part, in Figure 24b) it can be see the laser path followed. (Lanaro and Tolppanen, 2002).
- Laser-Aided Tomography (LAT), in this case a laser sheet is used to obtain the particles surveying (see Figure 25, left)). This technique is different and has special requirements as to use liquid with same refractive index as the particles, particles must let the laser or certain percent of light go through. (Matsushima et al., 2003).

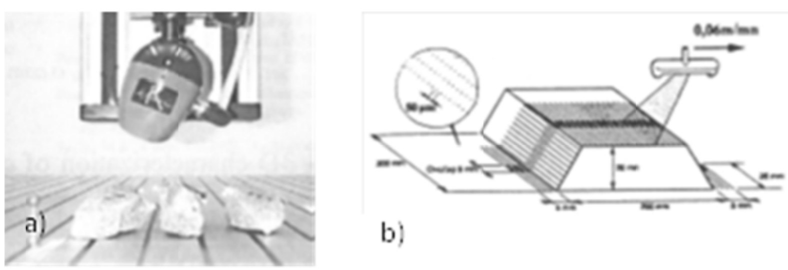

Figure 24: a) Scanning head, b) scanning path (Lanaro and Tolppanen, 2002)

Last two 3D techniques obtain the particle shape that is later used to achieve measures as we can see in Figure 25 (right).

All these previous techniques are easily written in codes or scripts to be interpreted in a digital way obtaining the desired measurement, but there are some interesting points in the image analysis regarding on the errors involve, among them are image resolution and orientation of the particles; orientation is not relevant when it is random and large number of particles are involve; resolution have an influence on the accuracy. (Zeidan et al., 2007)

When resolution is increase more accuracy is obtain and the object representation match better with the real form, in the other hand, more resolution means more spending on memory and time; thus, resolution needs to be according with the goal and precision needed in any work. (Schäfer, 2002).

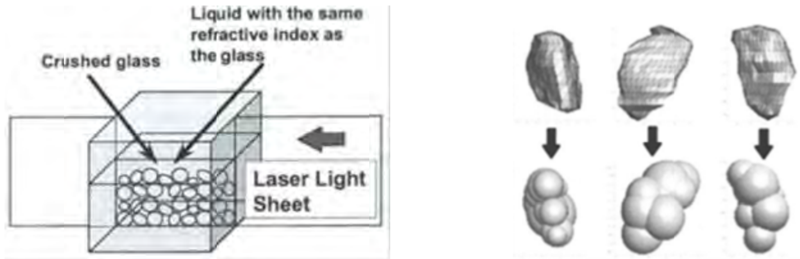

Figure 25: (left) LAT scaning particles (Matsushima et al., 2003), (right) 3D scan completed and mesh generated. (Matsushima et al., 2003)

Schäfer (2002) conclude that attributes like length when measuring digital images present relative high errors. It can be vanish or at least diminish using high resolution just for diameter but not for perimeter that keep the error as big as initially. Johansson and Vall (2011) obtain similar results when 3 different resolutions were used in the same particle obtaining an unstable output for those terms/quantities that involve the perimeter. Thus all quantities relating the perimeter should be treated with care.

DISCUSSION

TERMS, QUANTITIES AND DEFINITIONS

In order to describe the particle shape in detail, there are a number of terms, quantities and definitions (qualitative and quantitative) used in the literature (e.g. Wadell, 1932, 1934; Krumbein, 1941; Sneed & Folk, 1958). All mathematical definitions (quantitatives) are models used to simplify the complexity of shape description. Some authors (Mitchell & Soga, 2005; Arasan et al., 2010) are using three sub-quantities; one and each describing the shape but at different scales. The terms are morphology/form, roundness and surface texture (Figure. 1). The three sub-quantities are probably the best way to classify and describe a particle because not a single definition can interpret the whole morphology. Common language is needed when descriptors are explained, and these three scales represent an option. It is evident in the reviewed literature that many of the shape descriptors are presented with the same name but also that there is not a clear meaning on what this descriptor defines, e.g. when there is no upper limit in the roundness, does it means that the angularity never ends? Could they be more and more angular? Probably they could be on theory but not in reality. Physical meaning of the quantities is difficult to recognize in most of the cases.

IMAGE ANALYSIS

Many image analysis techniques had been used to describe the particle shape, e.g. Fourier analysis, fractal dimension, tomography, etc., (Hyslip and Vallejo, 1997) but there is not agreement on the usage or conclusion to ensure the best particle descriptor for geotechnical or other applications.

There are several shape descriptors and also various techniques to capture the particles profile (3-dimensions, 3-dimension orthogonal and 2-dimensions). Each technique presents advantages and disadvantages. *3-dimensions* is probably the technique that provide more information about the particle shape but the precision also lies in the resolution; the equipment required to perform such capture could be more or less sophisticated (scanning particles laying down in one position and later move to complete the scanning or just falling down particles to scan it in one step). *3-dimensions orthogonal*, this technique use less sophisticated equipment (compare with the previous technique) but its use is limited to particles over 1cm, also, information between the orthogonal pictures is not capture. *2-dimensions* require non sophisticated equipment but at the same time the shape information diminish compare with the previous due the fact that it is possible to determine only the outline; as the particle measurements are performed in 2-dimensions it is presumed that they will lie with its shortest axis perpendicular to the laying surface when they are flat, but when the particle tends to have more or less similar axis the laying could be random.

Advantages on the use of image analysis are clear; there is not subjectivity because it is possible to obtain same result over the same images. Electronic files do not loose resolution and it is important when collaboration among distant work places is done, files can be send with the entire confidence and knowing that file properties has not been changed. Technology evolutions allowed to work with more information and it also applies to the image processing area were the time consumed has been shortened (more images processed in less time).

One important aspect in image analysis is the used resolution in the analysis due the fact that there are measurements dependent and independent on resolution. Thus, those dependent measurements should be avoided due the error included when they are applied, or avoid low resolution to increase the reliability. Among these parameters length is the principal parameter that is influences by resolution (e.g. perimeter, diameter, axis, etc.). Resolution also has another aspect with two faces, quality versus capacity, more resolution (quality) means more storage space, a minimum resolution to obtain reasonable and reliable data must be known but it depend on each particular application.

APPLICATIONS

Quantify changes in particles, in the author’s thought, is one of the future applications due the non-invasive methods of taking photographs in the surface of the dam’s slope, rail road ballast or roads. Sampling of the material and comparing with previous results could show volume (3D analysis) or area (2D analysis) loss of the particles as well as the form, roundness and roughness. This is important when it has been suggested that a soil or rock embankment decrees their stability properties (e.g. internal friction angle) with the loss of sphericity, roundness or roughness.

Seepage, stock piling, groundwater, etc., should try to include the particle shape while modelling; seepage requires grading material to not allow particles move due the water pressure but in angular materials, as it is known, the void ratio is great than the rounded soil, it means the space and the possibilities for the small particles to move are greater; stock piling could be modelled incorporating the particle shape to determine the bin’s capacity when particle shape changes (void ratio changes when particle shape changes) Modelling requires all information available and the understanding of the principles that apply.

Industry is actually using the particle shape to understand the soil behaviour and transform processes into practical and economic, image analysis has been included in the quality control to determine particle shape and size because the advantages it brings, e.g. the acquisition of the sieving curve for pellets using digital images taken from conveyor, this allows to have the information in a short period of time with a similar result, at least enough from the practical point of view, as the traditional sieving.

CONCLUSIONS

• A common language needs to be built up to standardize the meaning on geotechnical field that involve the particle shape.

• Based on this review it is not clear which one is the best descriptor.

• Image analysis tool is objective, make the results repeatable, obtain fast results and work with more amount of information.

• Resolution needs to be taken in consideration when image analysis is been carried out because the effects could be considerable. Resolution must be set according to the necessities. Parameters as perimeter can be affected by resolution.

• There are examples where particle shape has been incorporated in industries related to geotechnical engineering, e.g. in the ballast and asphalt industry for quality control.

FURTHER WORK

Three main issues have been identified in this review that will be further investigated; the limits of shape descriptors (quantities) influence of grading and choice of descriptor for relation to geotechnical properties.

Shape descriptors have low and high limits, frequently the limits are not the same and the ability to describe the particle's shape is relative. The sensitivity of each descriptor should be compare to apply the most suitable descriptor in each situation.

Sieving curve determine the particle size in a granular soil, particle shape could differ in each sieve size. There is the necessity to describe the particle shape on each sieve portion (due to practical issues) and included in the sieve curve. Obtain an average shape in determined sieve size is complicated (due to the possible presence of several shapes) and to obtain the particle shape on the overall particle's size is challenging, how the particle shape should be included?

Since several descriptors have been used to determine the shape of the particles but how is the shape related with the soil properties? It is convenient to determine the descriptor's correlation with the soil properties.

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