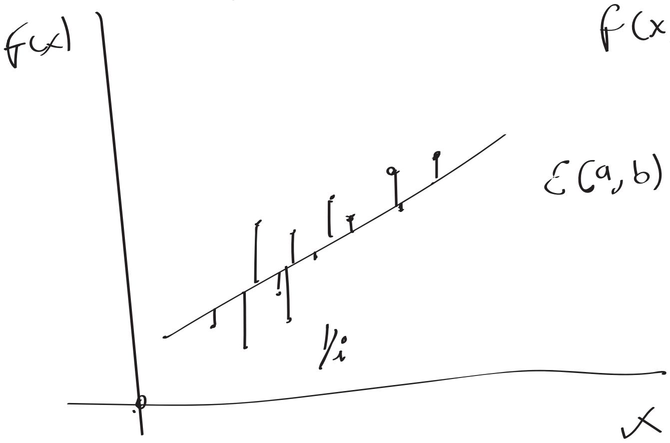
Regresión



$f(x,\theta)$ $\theta(a,b)$

diferencia o el error es menor

menor

$$\mathcal{E}(a,b) = \sum_{i=1}^{N} |ax_i + b - y_i|$$

$l_2 \to$ distancia

$$\mathcal{E}(a,b) = \sum_{i=1}^{N} (ax_i + b - y_1)^2$$

Convexa

Concavas

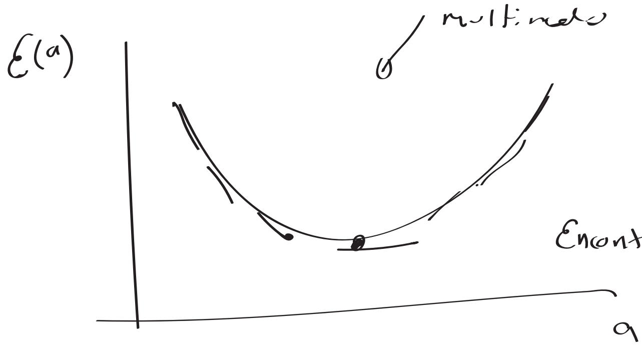
$\theta = \arg\min_{\theta} MSE(\theta)$

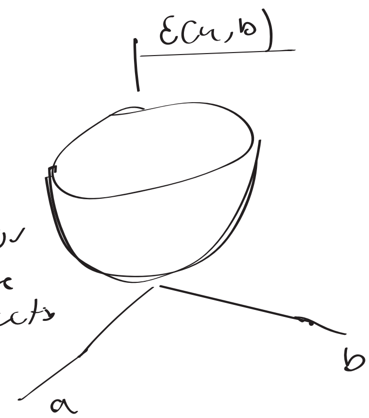
$b \to$ No convexo multimodal

Mejor

Optimizer

Encontrar la mejor solución entre todas rectas





¿Cuál a y b minimizan el error?

$$\frac{\partial \mathcal{E}(a,b)}{\partial a} = 0$$

$$\frac{\partial \mathcal{E}(\theta)}{\partial \theta} = 0$$

$b = 0$ restricción

$$\mathcal{E}(a,b) = \sum_{i=1}^{N} (ax_i + b - y_1)^2$$

$$\mathcal{E}(a) = \sum_{i=1}^{N} (ax_i - y_i)^2$$

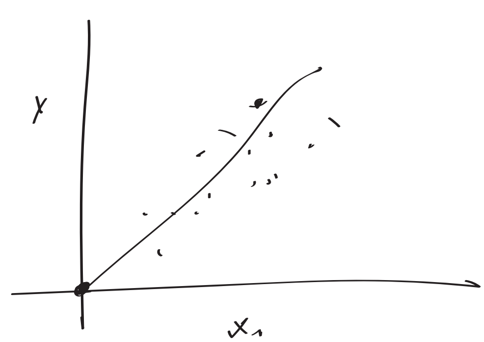
$\arg\min_{a} \mathcal{E}(a)$

$$\frac{\partial \mathcal{E}(a)}{\partial a} = 2\sum_{i=1}^{N} (ax_i - y_i) \cdot x_i$$

$$a \sum x_i^2 - \sum y_i x_i = 0$$

$$a \sum x_i^2 = \sum y_i x_i$$

$$a = \frac{\sum y_i x_i}{\sum x_i^2}$$



$$\mathcal{E}(a,b) = \sum_{i=1}^{N} (a x_i + b - y_i)^2$$

$$2 \sum (a x_i + b - y_i) \cdot 1$$

$$\sum a x_i + \sum b - \sum y_i = 0$$

$$\frac{\partial \mathcal{E}(a,b)}{\partial a} = 0 \qquad \frac{\partial \mathcal{E}(a,b)}{\partial b} = 0$$

$$a \sum x_i^2 + b \sum x_i = \sum y_i x_i \quad \ldots \; 1$$

$$\sum a x_i + \sum_{i=1}^{N} b = \sum y_i \quad \ldots \; 2$$

$$\sum_{i=1}^{N} b = N b$$

$$\left[\begin{array}{cc|c} aN \sum x^2 & + Nb \sum x & N \sum y \cdot x \\ a \sum x & + Nb & \sum y \end{array}\right]$$

mul $1 \times N$ - $2 \sum x$

$$a \sum x + N b = \sum y$$

$$b = \frac{\sum y - a \sum x}{N}$$

$$a N \sum x^2 - a \left(\sum x\right)^2 = N \sum yx - \sum y \sum x$$

$$\boxed{a = \frac{N \sum xy - \sum y \sum x}{N \sum x^2 - (\sum x)^2}}$$

$$b = \frac{\sum y - \left(\frac{N \sum yx - \sum y \sum x}{N \sum x^2 - (\sum x)^2}\right) \sum x}{N}$$

$$b = \frac{N \sum y \sum x^2 - \cancel{\sum y (\sum x)^2} - N \sum yx \sum x + \cancel{\sum y \sum x^2}}{N \left(N \sum x^2 - (\sum x)^2\right)}$$

$$\boxed{b = \frac{\sum y \sum x^2 - \sum yx \sum x}{N \sum x^2 - (\sum x)^2}}$$