Derivada de la función Sigmoide

$$\sigma(t) = \frac{1}{1+e^{-t}} \qquad \sigma'(t) = \frac{d\left(1+e^{-t}\right)^{-1}}{dt} = -1\,(1+e^{-t})^{-2} \cdot \frac{d(1+e^{-t})}{dt}$$

$$= -(1+e^{-t})^{-2} \cdot -1\, e^{-t} = \frac{e^{-t}}{(1+e^{-t})^2}$$

Haciendo algebra

$$\frac{e^{-t}}{(1+e^{-t})(1+e^{-t})} = \frac{1}{1+e^{-t}} \cdot \frac{e^{-t}}{(1+e^{-t})} = \frac{1}{1+e^{-t}} \cdot \frac{e^{-t}+1-1}{(1+e^{-t})}$$

$$= \left(\frac{1}{1+e^{-t}}\right)\left(\frac{1+e^{-t}}{1+e^{-t}} - \frac{1}{1+e^{-t}}\right) = \frac{1}{1+e^{-t}}\left(1 - \frac{1}{1+e^{-t}}\right)$$

$$\sigma(t) = \frac{1}{1+e^{-t}} \qquad \sigma'(t) = \sigma(t)\left(1-\sigma(t)\right)$$