Repaso de Álgebra lineal

Reconocimiento de patrones

Gamaliel Moreno

gamalielmch@uaz.edu.mx
http://pds.uaz.edu.mx/

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Contenido

Escalar

Un escalar es un número real

$$\boldsymbol{s} \in \mathbb{R}$$

Vector

Vector de m dimensiones $\boldsymbol{x} \in \mathbb{R}^m$ (m escalares)

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

Vector fila

Vector de n dimensiones $\boldsymbol{x} \in \mathbb{R}^n$ (n escalares)

$$\boldsymbol{x}^T = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}$$

Matriz

Matriz de $m \times n$ $\boldsymbol{A} \in \mathbb{R}^{m \times n}$

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

- m: files
- n: columnas
- En notación a_{ij} primer subíndice i es la fila y el segundo la columna
- Todas las matrices en $\mathbb{R}^{m \times n}$ conforman un espacio lineal. Esto es $B = \{B_i | B_i \in \mathbb{R}^{m \times n}\}$, entonces $M = \sum_i c_i B_i$

Tensor

Generalización de conceptos anteriores

- Escalar $s \in \mathbb{R}$: tensor 0
- Vector $\boldsymbol{v} \in \mathbb{R}^n$: tensor 1
- Matriz $\boldsymbol{M} \in \mathbb{R}^{m \times n}$: tensor 2
- En general $\mathcal{T} \in \mathbb{R}^{n_1 \times n_2 \times \cdots n_q}$ es un tensor de orden q

Matriz en vector

Matriz $\boldsymbol{A} \in \mathbb{R}^{m \times n}$ se descompone en m vectores fila

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = \begin{bmatrix} \boxed{\boldsymbol{a}_{1,:}{}^{T}} \\ \boxed{\boldsymbol{a}_{2,:}{}^{T}} \\ \vdots \\ \boxed{\boldsymbol{a}_{m,:}{}^{T}} \end{bmatrix}$$

o en vectores columna

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = \begin{bmatrix} \boxed{\boldsymbol{a}_{:,1}} & \boxed{\boldsymbol{a}_{:,2}} & \vdots & \boxed{\boldsymbol{a}_{:,n}} \end{bmatrix}$$

Vectores como matrices

- Observe que todo vector es un tipo particular de matriz
  - Vector fila: matriz de dimensiones $1 \times n$
  - Vector columna: matriz de dimensiones $m \times 1$
- Propiedades de matrices aplicarán a vectores

Matriz transpuesta 1

Si

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

entonces

$$\boldsymbol{A}^T = \begin{bmatrix} a_{11} & a_{21} & \dots & a_{m1} \\ a_{12} & a_{22} & \dots & a_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \dots & a_{mn} \end{bmatrix}$$

- En otras palabras, si $\boldsymbol{B} = \boldsymbol{A}^T$ entonces $b_{ij} = a_{ij}$
- $(\boldsymbol{A}^T)^T = \boldsymbol{A}$
- $(\boldsymbol{AB})^T = \boldsymbol{B}^T\boldsymbol{A}^T$
- $(\boldsymbol{A} + \boldsymbol{B})^T = \boldsymbol{A}^T + \boldsymbol{B}^T$

Blocks

Definition (Greetings)

Hello World

Theorem (Fermat's Last Theorem)

$a^n + b^n = c^n, n \leq 2$

Uh-oh.

By the pricking of my thumbs.

Uh-oh.

Something evil this way comes.

Notation

Definition (Random Variable)

Consider Ω, F, μ, with Ω being the set of events, F the σ-algebra on Ω and some arbitrary measure μ. Further consider an observation space Ω', F', μ'... A random variable is a deterministic function that 'transports/maps' events from Ω to Ω' and effectively induces a new measure μ'. When $\mu'(\Omega') = 1$, it is a probability measure.

Notation

definition

definition