$$u_k = u_{k+1}p + u_{k-1}q \qquad p+q=1$$

$$u_k(p+q) = u_{k+1}p + u_{k-1}q \; (\cancel{p+q})$$

$$u_k p + u_k q = u_{k+1}p + u_{k-1}q$$

$$u_k p - u_{k+1}p = u_{k-1}q - u_k q$$

$$(u_k - u_{k+1})p = (u_{k-1} - u_k)q$$

$$\left(u_k - u_{k+1} = \frac{q}{p}(u_{k-1} - u_k)\right)(-1)$$

$$k = 1, 2, 3, 4$$

$$u_{k+1} - u_k = \frac{q}{p}(u_k - u_{k-1})$$

$k=1$ $\qquad u_0 = 1$

$$\underline{u_2 - u_1} = \frac{q}{p}(u_1 - 1)$$

$$S^k = 1 + (q/p) + (q/p)^2 + \cdots + (q/p)^k$$

$k=2$ $\qquad S^{k-1} =$

$$u_3 - u_2 = \frac{q}{p}(u_2 - u_1)$$

$$u_3 - u_2 = \frac{q}{p}\left(\frac{q}{p}(u_1 - 1)\right)$$

$$u_3 - u_2 = \boxed{\left(\frac{q}{p}\right)^2 (u_1 - 1)}$$

$$\cancel{u_2} - u_1 = \frac{q}{p}(u_1 - 1)$$

$$\cancel{u_3} - \cancel{u_2} = \left(\frac{q}{p}\right)^2 (u_1 - 1)$$

$$\cancel{u_4} - \cancel{u_3} = \left(\frac{q}{p}\right)^3 (u_1 - 1)$$

$$\vdots$$

$$u_k - \cancel{u_{k-1}} = \left(\frac{q}{p}\right)^{k-1} (u_1 - 1)$$

$$\underline{u_k - u_1 = (S^{k-1} - 1)(u_1 - 1)}$$

$k=3$

$$u_4 - u_3 = \frac{q}{p}(u_3 - u_2)$$

$$u_4 - u_3 = \frac{q}{p}\left(\left(\frac{q}{p}\right)^2 (u_2 - 1)\right)$$

$$u_4 - u_3 = \left(\frac{q}{p}\right)^3 (u_2 - 1)$$

$$\frac{q}{p}(u_1 - 1) + \left(\frac{q}{p}\right)^2 (u_1 - 1) + \cdots + \left(\frac{q}{p}\right)^{k-1} (u_1 - 1)$$

$$(u_1 - 1)(\text{somatória})$$

$u_k - u_1 = (S_{k-1} - 1)(u_1 - 1)$

$k = 0$

$k = a$

$u_k - \cancel{u_1} = S_{k-1} u_1 - S_{k-1} - \cancel{u_1} + 1$

$u_a = 0$

$u_k = S_{k-1} u_1 - S_{k-1} + 1$

$u_k = S_{k-1}(u_1 - 1) + 1 \rightarrow$ $\quad u_k = S_{k-1}\left(\frac{-1}{S_{a-1}}\right) + 1$

$u_a = S_{a-1}(u_1 - 1) + 1$

$0 = S_{a-1}(u_1 - 1) + 1$ $\quad u_k = 1 - \frac{S_{k-1}}{S_{a-1}}$

$-1 = S_{a-1}(u_1 - 1)$

$u_1 - 1 = \frac{-1}{S_{a-1}}$ $\quad u_k = \frac{1}{1} - \frac{k}{a}$ $\quad q = p$

$u_k = \frac{a - k}{a}$

$S_k = 1 + q/p + (q/p)^2 + \dots (q/p)^k$

$S_k = 1 + (1)^2 + (1)^3 + \dots (1)^k$

$S_k = k + 1$

$S_k = 1 + \cancel{r} + \cancel{r^2} + \cancel{r^3} + \dots + \cancel{r^k}$

$rS_k = \cancel{r} + \cancel{r^2} + \cancel{r^3} + r^4 + \dots \cancel{r^k} + r^{k+1}$

$S_k - rS_k = 1 - r^{k+1}$ $\quad q \neq p$

$S_k(1 - r) = 1 - r^{k+1}$

$S_k = \frac{1 - r^{k+1}}{1 - r} = \frac{1 - (q/p)^{k+1}}{1 - (q/p)}$

$S_{k-1} = \frac{1 - (\frac{q}{p})^k}{1 - (q/p)}$ $\quad S_{a-1} = \frac{1 - (q/p)^a}{1 - (q/p)}$

$u_k = 1 - \frac{S_{k-1}}{S_{a-1}}$

$u_k = 1 - \frac{\frac{1 - (q/p)^k}{\cancel{1 - (q/p)}}}{\frac{1 - (q/p)^a}{\cancel{1 - (q/p)}}} = 1 - \frac{1 - (q/p)^k}{1 - (q/p)^a}$

$$u_K = 1 - \frac{1-(q/p)^K}{1-(q/p)^a} = \frac{\cancel{1} - (q/p)^a - \cancel{1} + (q/p)^K}{1-(q/p)^a}$$

$q \neq p$

$$u_K = \frac{(q/p)^K - (q/p)^a}{1-(q/p)^a}$$

$$u_K = \frac{a-K}{a} \qquad p = q = \frac{1}{2}$$