$$(p+q)\, d_k = \left(p\, d_{k+1} + q\, d_{k-1} + 1\right) \left(\frac{1}{p+q}\right) \quad k \geq 1$$

$$(p+q) = 1$$

$$d_k p + d_k q = 1 + p d_{k+1} + q d_{k-1}$$

$$\left(p d_{k+1} - p d_k = q d_k - q d_{k-1} - 1\right) \frac{1}{p}$$

$$d_{k+1} - d_k = \frac{q}{p}(d_k - d_{k-1}) - \frac{1}{p}$$

$$\underline{S_k = 1 + \frac{q}{p} + \cdots + \left(\frac{q}{p}\right)^k}$$

Sum $k = 1, 2, 3 \ldots$

$$\underline{d_2 - d_1} = \frac{q}{p}(d_1 - \cancel{d_0}^{\,0}) - \frac{1}{p}$$

$$d_3 - d_2 = \frac{q}{p}(d_2 - d_1) - \frac{1}{p} \qquad d_a = d_0 = 0$$

$$d_3 - d_2 = \frac{q}{p}\left(\frac{q}{p}(d_1 - \cancel{d_0}^{\,0}) - \frac{1}{p}\right) - \frac{1}{p}$$

$$d_3 - d_2 = \left(\frac{q}{p}\right)^2 (d_1 - \cancel{d_0}^{\,0}) - \left(\frac{q}{p}\right)\left(\frac{1}{p}\right) - \frac{1}{p}$$

$$d_3 - d_2 = \left(\frac{q}{p}\right)^2 (d_1) - \frac{q}{p}\left(\frac{1}{p}\right) - \frac{1}{p}$$

$$\underline{d_3 - d_2} = \left(\frac{q}{p}\right)^2 d_1 - \left[\frac{q}{p} + 1\right]\frac{1}{p}$$

$$d_4 - d_3 = \frac{q}{p}\left[d_3 - d_2\right] - \frac{1}{p} = \frac{q}{p}\left[\left(\frac{q}{p}\right)^2 d_1 - \left[\frac{q}{p} + 1\right]\frac{1}{p}\right] - \frac{1}{p}$$

$$d_4 - d_3 = \left(\frac{q}{p}\right)^3 d_1 - \left[\left(\frac{q}{p}\right)^2 + \frac{q}{p}\right]\frac{1}{p} - \frac{1}{p} = \left(\frac{q}{p}\right)^3 d_1 - \left[\left(\frac{q}{p}\right)^2 + \frac{q}{p} + 1\right]\frac{1}{p}$$

$$\cancel{d_2} - d_1 = \left(\frac{q}{p}\right) d_1 - \frac{1}{p} S_0$$

$$\cancel{d_3} - \cancel{d_2} = \left(\frac{q}{p}\right)^2 d_1 - \frac{1}{p} S_1$$

$$\vdots$$

$$d_K - \cancel{d_{K-1}} = \left(\frac{q}{p}\right)^{K-1} d_1 - \frac{1}{p} S_{K-2}$$

$$S_{K-1} = 1 + q/p + \cdots \left(\frac{q}{p}\right)^{K-1}$$

$$S_{K-1} - 1 = q/p + \cdots \left(\frac{q}{p}\right)^{K-1}$$

$$d_K - d_1 = d_1 (S_{K-1} - 1) - \frac{1}{p} \sum_{i=0}^{K-2} S_i$$

$$d_K - \cancel{d_1} = d_1 S_{K-1} - \cancel{d_1} - \frac{1}{p} \sum_{i=0}^{K-2} S_i$$

$$d_K = \underline{d_1} S_{K-1} - \frac{1}{p} \sum_{i=0}^{K-2} S_i \quad \ldots \text{Eq } 1$$

$d_a = 0$

$$0 = d_1 S_{a-1} - \frac{1}{p} \sum_{i=0}^{a-2} S_i$$

$$d_1 = \frac{1}{S_{a-1}} \frac{1}{p} \sum_{i=0}^{a-2} S_i \quad \text{sus. ea 1}$$

$$S_K = 1 + q/p + \cdots + \left(\frac{q}{p}\right)^K$$

$$S_K = \begin{cases} K+1 & p = q \\ \dfrac{1 - (q/p)^{K+1}}{1 - q/p} & p \neq q \end{cases}$$

$$d_K = \frac{S_{K-1}}{S_{a-1}} \frac{1}{p} \sum_{i=0}^{a-2} S_i - \frac{1}{p} \sum_{i=0}^{K-2} S_i$$

$p = q$

$$d_K = \frac{K}{a} \frac{1}{p} \sum_{i=0}^{a-2} S_i - \frac{1}{p} \sum_{i=0}^{K-2} S_i$$

$p = q$

$1 + \left(\frac{p}{q}\right) + \left(\frac{p}{q}\right)^2 + \cdots + \left(\frac{p}{q}\right)^k$

$\underbrace{\underbrace{1 + 1} + 1} + \cdots + 1$

$$\sum_{i=0}^{a-2} S_i = S_0 + S_1 + S_2 + S_3 + \cdots + S_{a-2}$$

$$= 1 + 2 + 3 + a + \cdots + a - 2$$

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$$

$$T = 1 + 2 + 3 + \cdots + (n-2) + (n-1) + n$$

$$T = n + (n-1) + (n-2) + \cdots + 3 + 2 + 1$$

$$2T = (n+1) + (n+1) + (n+1) + \cdots + (n+1) + (n+1) + (n+1)$$

$$2T = n(n+1)$$

$$T = \frac{n(n+1)}{2}$$

$$\sum_{i=0}^{a-2} S_i = \frac{(a-1)(a-\cancel{1}+\cancel{1})}{2} = \frac{a(a-1)}{2}$$

$$\sum_{i=0}^{k-2} S_k = \frac{k(k-1)}{2}$$

$p = q = \frac{1}{2}$

$\frac{1}{p} = \frac{1}{1/2} = 2$

$$d_k = \frac{k}{\cancel{a}} \frac{\cancel{2}(a-1)\cancel{a}}{\cancel{2}} - \cancel{2}\left(\frac{k(k-1)}{\cancel{2}}\right) = k(a-1) - k(k-\cancel{1})$$

$$= k(a - \cancel{1} - k + \cancel{1}) = k(a-k)$$

$$d_k = \frac{S_{k-1}}{S_{a-1}} \frac{1}{p}\sum_{i=0}^{a-2} S_i - \frac{1}{p}\sum_{i=0}^{k-2} S_i \qquad S_k = \begin{cases} k+1 & p=q \\ \dfrac{1-(q/p)^{k+1}}{1-(q/p)} & p\neq q \end{cases}$$

$p \neq q$

$$\sum_{i=0}^{a-2} S_i = S_0 + S_1 + S_2 \cdots + S_{a-2} = \frac{1-(q/p)}{1-q/p} + \frac{1-(q/p)^2}{1-q/p} + \cdots + \frac{1-(q/p)^{a-1}}{1-q/p}$$

$$= \frac{1}{1-q/p}\left[1 - q/p + 1 - (q/p)^2 + \cdots + 1 - (q/p)^{a-1}\right]$$

$$= \frac{1}{1-q/p}\left[a - 1 - q/p - (q/p)^2 - \cdots - (q/p)^{a-1}\right]$$

$$= \frac{1}{1-q/p}\left\{a - \left[1 - q/p + (q/p)^2 + \cdots + (q/p)^{a-1}\right)\right. = \frac{1}{1-q/p}\left[a - S_{a-1}\right]$$

$$\sum_{i=0}^{k-2} S_i = \frac{1}{1-q/p}\left[k - S_{k-1}\right]$$

$$d_k = \frac{S_{k-1}}{S_{a-1}} \frac{1}{p}\left[\frac{1}{1-q/p}\left[a - S_{a-1}\right]\right] - \frac{1}{p}\left[\frac{1}{1-q/p}\left[k - S_{k-1}\right]\right]$$

$$d_k = \frac{1}{p-q}\left[\frac{S_{k-1}}{S_{a-1}}\left[a - S_{a-1}\right] - k + S_{k-1}\right]$$

$$d_k = \frac{1}{p-q}\left[\frac{a S_{k-1}}{S_{a-1}} - \cancel{S_{k-1}} - k + \cancel{S_{k-1}}\right] = \frac{1}{p-q}\left[\frac{a S_{k-1}}{S_{a-1}} - k\right] \quad \left(\frac{-1}{-1}\right)$$

$$d_k = \frac{1}{q-p}\left[k - \frac{a S_{k-1}}{S_{a-1}}\right] = \boxed{\frac{1}{1-2p}\left[k - a\left(\frac{1-(q/p)^k}{1-(q/p)^a}\right)\right]}$$

$$\frac{1}{q-p} \qquad q = 1-p$$

$$\frac{1}{1-p-p} = \frac{1}{1-2p}$$

$$\frac{S_{k-1}}{S_{a-1}} = \frac{\dfrac{1-(q/p)^k}{\cancel{1-q/p}}}{\dfrac{1-(q/p)^a}{\cancel{1-q/p}}}$$