

Fórmula general

a{x}^2 + b{x} + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}

Identidades Trigonométricas

csc(\theta) = \frac{1}{\sin(\theta)}

\sec(\theta) = \frac{1}{\cos(\theta)}

\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}

\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)} = \frac{1}{\tan(\theta)}

\sen^2(\theta) + \cos^2(\theta) = 1

\tan^2(\theta) + 1 = \sec^2(\theta)

\cot^2(\theta) + 1 = \csc^2(\theta)

\sen(\alpha \pm \beta) = \sen(\alpha) \cos(\beta) \pm \sen(\beta) \cos(\alpha)

\cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sen(\beta) \sen(\alpha)

\tan(\alpha \pm \beta) = \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha) \tan(\beta)}

\sen(2\theta) = 2 \sen(\theta) \cos(\theta)

\cos(2\theta) = \cos^2(\theta) - \sen^2(\theta) = 1 - 2 \sen^2(\theta) = 2 \cos^2(\theta) - 1

\tan(2\theta) = \frac{2 \tan(\theta)}{1 - \tan^2(\theta)}

\sen^2(\theta) = \frac{1 - \cos(2\theta)}{2}

\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}

\sen(\alpha) \sen(\beta) = \frac{\cos(\alpha - \beta) - \cos(\alpha + \beta)}{2}

\cos(\alpha) \cos(\beta) = \frac{\cos(\alpha - \beta) + \cos(\alpha + \beta)}{2}

\sen(\alpha) \cos(\beta) = \frac{\sen(\alpha - \beta) + \sen(\alpha + \beta)}{2}

\cos(\alpha) - \cos(\beta) = -2 \sen\left(\frac{\alpha + \beta}{2}\right) \sen\left(\frac{\alpha - \beta}{2}\right)

\cos(\alpha) + \cos(\beta) = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)

\sen(\alpha) + \sen(\beta) = 2 \sen\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)

\sen(\alpha) - \sen(\beta) = 2 \sen\left(\frac{\alpha - \beta}{2}\right) \cos\left(\frac{\alpha + \beta}{2}\right)

\alpha	\sen(\alpha)	\cos(\alpha)	\tan(\alpha)
0^\circ	0	1	0
30^\circ	\frac{\pi}{6}	\frac{1}{2}	\frac{\sqrt{3}}{2}
45^\circ	\frac{\pi}{4}	\frac{\sqrt{2}}{2}	\frac{\sqrt{2}}{2}
60^\circ	\frac{\pi}{3}	\frac{\sqrt{3}}{2}	\frac{1}{2}
90^\circ	\frac{\pi}{2}	1	0

Propiedades de los Logaritmos

\log_b a = c \Leftrightarrow b^c = a

\log_a c = \frac{\log_b c}{\log_b a}

\log_b(xy) = \log_b(x) + \log_b(y)

\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)

\log_b(x^y) = y \log_b(x)

\log_b(\sqrt[y]{x}) = \frac{\log_b x}{y}

Funciones hiperbólicas

\senh(x) = \frac{e^x - e^{-x}}{2}

\cosh(x) = \frac{e^x + e^{-x}}{2}

\tanh(x) = \frac{\senh(x)}{\cosh(x)}

\cosh^2(x) - \senh^2(x) = 1

1 - \tanh^2(\theta) = \sech^2(\theta)

\coth^2(x) - 1 = \csch^2(\theta)

\lim_{x \rightarrow 0} \frac{\sen(x)}{x} = 1

\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x} = 0

\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1

\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = \lim_{x \rightarrow 0} (1 + x)^{\frac{1}{x}} = e

Tabla de Derivadas

f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow x} \frac{f(x) - f(h)}{x - h}

D(uv) = u v' + u' v

D\left(\frac{u}{v}\right) = \frac{v u' - u v'}{v^2}

D(au) = a u'

D(u^n) = n u^{n-1} u'

D(e^u) = e^u u'

D(a^u) = \ln(a) a^u u'

D(u^v) = u^{v-1} (u v' \ln(u) + v u')

D(\ln(u)) = \frac{u'}{u}

D(\log_a(u)) = \frac{u'}{u \ln(a)}

D(\sen(u)) = \cos(u) u'

D(\cos(u)) = -\sen(u) u'

D(\tan(u)) = \sec^2(u) u'

D(\sec(u)) = \sec(u) \tan(u) u'

D(\csc(u)) = -\csc(u) \cot(u) u'

D(\cot(u)) = -\csc^2(u) u'

D(\arc \sen(u)) = \frac{u'}{\sqrt{1-u^2}}

D(\arc \cos(u)) = -\frac{u'}{\sqrt{1-u^2}}

D(\arctan(u)) = \frac{u'}{1+u^2}

D(\arcsec(u)) = \frac{u'}{u \sqrt{u^2-1}}

D(\senh(u)) = \cosh(u) u'

D(\cosh(u)) = \senh(u) u'

D(\tanh(u)) = \sech^2(u) u'

D(\sech(u)) = -\sech(u) \tanh(u) u'

D(\csch(u)) = -\csch(u) \coth(u) u'

D(\coth(u)) = -\csch^2(u) u'

D(f^{-1}(x)) = \frac{1}{f'(f^{-1}(x))}

Tabla de Integrales

\int u dv = uv - \int v du

\int \frac{1}{u} du = \ln |u|

\int \frac{u^n}{n+1} du = \frac{u^{n+1}}{n+1} \text{ si } n \neq -1

\int \sen(u) du = -\cos(u)

\int \cos(u) du = \sen(u)

\int \tan(u) du = \ln |\sec(u)| = -\ln |\cos(u)|

\int \sec(u) du = \ln |\sec(u) + \tan(u)|

\int \csc(u) du = \ln |\csc(u) - \cot(u)|

\int \cot(u) du = \ln |\sen(u)|

\int \sec^2(u) du = \tan(u)

\int \csc^2(u) du = -\cot(u)

\int \sec(u) \tan(u) du = \sec(u)

\int \csc(u) \cot(u) du = -\csc(u)

\int \sen^2(u) du = \frac{u}{2} - \frac{\sen(2u)}{4}

\int \cos^2(u) du = \frac{u}{2} + \frac{\sen(2u)}{4}

\int e^u du = e^u

\int a^u du = \frac{a^u}{\ln(a)}

\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan\left(\frac{u}{a}\right)

\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right|

\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right|

\int \sqrt{u^2 \pm a^2} du = \frac{u \sqrt{u^2 \pm a^2}}{2} \pm \frac{a^2 \ln |u + \sqrt{u^2 \pm a^2}|}{2}

\int \sqrt{a^2 - u^2} du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \arcsen\left(\frac{u}{a}\right)

\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsen\left(\frac{u}{a}\right)

\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln \left| u + \sqrt{u^2 \pm a^2} \right|

\int \frac{du}{u \sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arccsc}\left(\frac{u}{a}\right)

\int \senh(u) du = \cosh(u)

\int \cosh(u) du = \senh(u)

\int \sech^2(u) du = \tanh(u)

\int \csch^2(u) du = -\coth(u)

\int \sech(u) \tanh(u) du = -\sech(u)

\int \csc(u) \coth(u) du = -\csch(u)

\int x^m (a + bx^n)^{\frac{p}{n}} dx, (p, q) = 1

Si \frac{m+1}{n} \in \mathbb{Z}, t^q = a + bx^n

Si \frac{m+1}{n} + \frac{p}{q} \in \mathbb{Z}, t^q = ax^{-n} + b

R(\sen(x), \cos(x))

t = \tan\left(\frac{x}{2}\right); dx = \frac{2dt}{1+t^2}

\sen(x) = \frac{2t}{1+t^2}; \cos(x) = \frac{1-t^2}{1+t^2}

R(\sen(x), \cos(x)) = R(-\sen(x), -\cos(x))

t = \tan(x); dx = \frac{dt}{1+t^2}

\sen(x) = \frac{t}{\sqrt{1+t^2}}; \cos(x) = \frac{1}{\sqrt{1+t^2}}

Curvas Paramétricas: r(t) = x(t)i + y(t)j

r' = x' i + y' j \quad v = r'; \quad T = \frac{r'}{|r'|};

a = r''; \quad m = \frac{dy}{dx} = \frac{y'}{x'}; \quad \frac{dy^2}{dx^2} = \frac{m'}{x'}

Volumen del sólido al rotar alrededor del eje:

X: \pi \int f^2(x) dx \quad Y: 2\pi \int x f(x) dx

Longitud de arco: \int \sqrt{x'^2 + y'^2} \quad \int \sqrt{1 + y'^2}

Área de la superficie rotar alrededor del eje:

X: 2\pi \int f \sqrt{1 + f'^2} \quad Y: 2\pi \int x \sqrt{1 + f'^2}

X: 2\pi \int y \sqrt{x'^2 + y'^2} \quad Y: 2\pi \int x \sqrt{x'^2 + y'^2}

Centroide

M = \int f dx; \quad M_y = \int x f dx; \quad M_x = \frac{1}{2} \int f^2 dx

\bar{x} = \frac{M_y}{M} \quad \bar{y} = \frac{M_x}{M}

Coordenadas Polares

x = r \cos(\theta); \quad y = r \sen(\theta); \quad r^2 = x^2 + y^2

\theta = \begin{cases} \arctan\left(\frac{y}{x}\right) & \text{si } x > 0 \\ \arctan\left(\frac{y}{x}\right) + \pi & \text{si } x < 0 \\ \operatorname{sgn}(y) \frac{\pi}{2} & \text{si } x = 0 \end{cases}

Longitud: \int \sqrt{r'^2 + r^2} \quad \text{Área: } \int \frac{1}{2} r^2 d\theta

Pendiente: \frac{r' \cos(\theta) + r' \sen(\theta)}{r' \cos(\theta) - r' \sen(\theta)}

Coordenadas Esféricas

x = p \sen(\phi) \cos(\theta), \quad y = p \sen(\phi) \sen(\theta)

z = p \cos(\phi)

Criterio de derivadas parciales

Si \nabla f(a, b) = \langle 0, 0 \rangle, \quad A = \frac{\partial^2 f}{\partial x^2}(a, b)

B = \frac{\partial^2 f}{\partial x \partial y}(a, b), \quad C = \frac{\partial^2 f}{\partial y^2}(a, b), \quad \Delta = B^2 - AC

\Delta > 0 \Rightarrow (a, b, f(a, b)) \text{ es un punto silla.}

\Delta < 0 \text{ y } A > 0 \Rightarrow f(a, b) \text{ es un mínimo local.}

\Delta < 0 \text{ y } A < 0 \Rightarrow f(a, b) \text{ es un máximo local.}

Series

\sum_{k=1}^n k = \frac{n(n+1)}{2} \quad \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}

\sum_{k=1}^n k^3 = \left(\frac{n(n+1)}{2}\right)^2 \quad \sum_{k=0}^n r^k = \frac{r^{n+1} - 1}{r - 1}

\sum_{k=0}^\infty x^k = \frac{1}{1-x}, \text{ si } |x| < 1 \quad \sum_{k=0}^\infty \frac{x^k}{k!} = e^x

\sum_{k=0}^\infty \frac{(-1)^k x^{2k}}{(2k)!} = \cos(x)

\sum_{k=0}^\infty \frac{(-1)^k x^{2k+1}}{(2k+1)!} = \sen(x)

Ecuación del Plano

Pasa por (x_0, y_0, z_0), perpendicular a (A, B, C)

A(x - x_0) + B(y - y_0) + C(z - z_0) = 0

Producto Punto y Producto Cruz

u \cdot v = |u||v| \cos(\alpha)

\operatorname{proy}_B A = \frac{A \cdot B}{|B|^2} B \quad \operatorname{comp}_B A = \frac{A \cdot B}{|B|}

|A \times B| = |A||B| \sen(\alpha)

(A \times B) \cdot C = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}

Ecuación lineal de primer orden

y' + py = q; y_h = e^{-\int p}; u = \int \frac{q}{y_h}; y = u y_h + k y_h

Ecuación de Bernoulli y' + py = qy^n. Sustituir y = u^{1/(1-n)}.

Ecuación exacta M(x,y)dx + N(x,y)dy = 0 es exacta ⇔ M_y = N_x.

Factor integrante para hacerla exacta:

μ(x) = e^{∫ \frac{My - Nx}{N} dx} o μ(y) = e^{∫ \frac{Nx - My}{M} dy}

Ecuación lineal con coeficientes constantes

α_n y^{(n)} + α_{n-1} y^{(n-1)} + ... + α_0 y = 0

Si r es raíz simple y real: y = C e^{rx}

Si α ± β i son raíces simples y = C_1 e^{αx} cos(βx) + C_2 e^{αx} sen(βx)

Si r es raíz de multiplicidad m y = C_1 e^{rx} + C_2 x e^{rx} + ... + C_m x^{m-1} e^{rx}

Ecuación de Euler

α_n x^n y^{(n)} + α_{n-1} x^{n-1} y^{(n-1)} + ... + α_{n-1} x y' + α_0 y = 0

Si r es raíz simple y real y = C x^r

Si α ± β i son raíces simples: y = C_1 x^α cos(β ln(x)) + C_2 x^α sen(β ln(x))

Si α ± β i son raíces simples: y = C_1 x^r + C_2 x^r ln(x) + ... + C_m x^r ln^{m-1}

Formas de solución para α_n y^{(n)} + α_{n-1} y^{(n-1)} + ... + α_0 y = f(x)

Las raíces del polinomio característico son de multiplicidad r

P, Q, R y S son polinomios de grado m, n o k según se indique. k = máximo {m, n}

f(x)	Raíces	Formas de la solución
P_n	x = 0	x^r Q_n
P_n e^{αx}, α real	α	x^r Q_n e^{αx}
P_m cos(βx) + Q_n sen(βx)	±β	x^r (R_k cos(βx) + S_k sen(βx))
e^{αx} (P_m cos(βx) + Q_n sen(βx))	α + β i	x^r e^{αx} (R_k cos(βx) + S_k sen(βx))

Función Gamma

Γ(x) = ∫_0^∞ t^{x-1} e^{-t} dt

Γ(x + 1) = x Γ(x)

Si n ∈ N, Γ(n + 1) = n!, Γ(0) = 1, Γ(1/2) = √π

Variable Compleja

z = x + yi

z̄ = x - yi

Re(z) = x, Im(z) = y; |z| = √(x^2 + y^2)

arg(x + yi) = 2πk + {arctan(y/x) si x > 0; arctan(y/x) + π si x < 0; sgn(y) π/2 si x = 0 e y ≠ 0}

(rcis(θ))^n = r^n cis(nθ)

w^n = rcis(θ) ⇒ w = √[n]{rcis(θ + 2πk/n)}, k = 0, 1, ..., (n - 1)

e^z = e^x (cos(y) + i sen(y))

log(z) = ln |z| + i arg(z)

sen(z) = sen(x) cosh(y) + i cos(x) senh(y) = \frac{senh(iz)}{i}

cos(z) = cos(x) cosh(y) - i sen(x) senh(y) = cosh(iz)

a_b = e^{b log(a)}

Ecuaciones de Cauchy Riemman

u_x = v_y, u_y = -v_x

Fórmula de la integral de Cauchy

∫_C \frac{f(z)}{z - z_0} dz = 2π i f(z_0), z_0 ∈ Int(C)

∫_C \frac{f(z)}{(z - z_0)^{n+1}} dz = \frac{2π i}{n!} f^{(n)}(z_0), z_0 ∈ Int(C)

Transformada de Laplace

f(t)	ℒ(f) = F(p)
f(t)	∫_0^∞ e^{-pt} f(t) dt
f(t) + kg(t)	F(p) + kG(p)
1	1/p, p > 0
e^{αt}	1/(p - α), p > 0
sen(αt)	p/(p^2 + α^2), p > 0
cos(αt)	p/(p^2 + α^2), p > 0
senh(αt)	α/(p^2 - α^2), p > 0
cosh(αt)	p/(p^2 - α^2), p > 0
t^n, n ∈ ℕ	n!/(p^{n+1}), p > 0
t^r, r > 0	Γ(r+1)/p^{r+1}, p > 0
δ(t - α)	e^{-αp}, α > 0
H(t - α) = { 1 t ≥ α; 0 t < α	e^{-αp}/p, α > 0
f * g = ∫_0^t f(x)g(t-x)dx	F(p)G(p)
f(αt)	1/α F(p/α), α > 0
t^n f(t)	(-1)^n F^{(n)}(p), p > 0
f^{(n)}(t)	p^n F(p) - ∑_{k=1}^n p^{n-k} f^{(k-1)}(0), p > 0
f de periodo T	∫_0^T f(t) e^{-pt} dt / (1 - e^{-pT})
e^{αt} f(t)	F(p - α)
f(t - α)H(t - α)	e^{-αp} F(p)

Transformada Z

x_n	Z{x_n}
x_k	∑_{k=0}^∞ \frac{x_k}{z^k}
{ 1 k = m; 0 k ≠ m	1/z^m
C	Cz/(z-1), z > 1
a^k	z/(z-a), z > a
k	z/(z-1)^2, z > 1
ka^{k-1}	z/(z-a)^2, z > a
cos(αk)	z(z-cos(α))/(z^2-2z cos(α)+1), z > 1
sen(αk)	z sen(α)/(z^2-2z cos(α)+1), z > 1
x_{k+m}	z^m Z{x_k} - ∑_{n=0}^{m-1} x_n z^{m-n}

Residuo de f en un polo z_0 de orden n

Res(f, z_0) = 1/(n-1)! lim_{z -> z_0} d^{n-1}/dz^{n-1} (f(z)(z - z_0)^n)

Serie de Fourier

f(x) definida en [-L/2, L/2]

f(x) ≈ a_0 + ∑_{n=1}^∞ (a_n cos(2π/L nx) + b_n sen(2π/L nx))

a_0 = 1/L ∫_{-L/2}^{L/2} f(x) dx;

a_n = 2/L ∫_{-L/2}^{L/2} f(x) cos(2π/L nx)

b_n = 2/L ∫_{-L/2}^{L/2} f(x) sen(2π/L nx) dx

Serie de Fourier en cosenos

f(x) definida en [0, L]

f(x) ≈ a_0 + ∑_{n=1}^∞ (a_n cos(π/L nx))

a_0 = 1/L ∫_0^L f(x) dx;

a_n = 2/L ∫_0^L f(x) cos(π/L nx)

Serie de Fourier en senos

f(x) definida en [0, L]

f(x) ≈ ∑_{n=1}^∞ b_n sen(π/L nx)

b_n = 2/L ∫_0^L f(x) sen(π/L nx) dx

Forma compleja de Fourier

f(x) definida en [-L, L];

f(x) = ∑_{n=-∞}^{+∞} c_n e^{i 2π n x / L};

c_n = 1/L ∫_{-L/2}^{L/2} f(x) e^{2π n i / L x} dx