Discussion Problems 2

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1 Problem 1

$$\int \cos^2(x)\sin^3(x)\,dx$$

$$\int \cos^2(x)\sin^3(x)\,dx = \int \cos^2(x)\left[1-\cos^2(x)\right]\sin(x)\,dx \tag{1.1}$$

$$u = \cos(x) \quad du = -\sin(x)\,dx \quad dx = \frac{-du}{\sin(x)}$$

$$\Rightarrow \int u^2\left[1-u^2\right]\sin(x)\,\frac{-du}{\sin(x)} \tag{1.2}$$

$$\Rightarrow -\int u^2\left[1-u^2\right]\,du \tag{1.3}$$

$$\Rightarrow -\int u^2 - u^4\,du \tag{1.4}$$

$$\Rightarrow -\left[\int u^2\,du - \int u^4\,du\right] \tag{1.5}$$

$$\Rightarrow -\left[\frac{u^3}{3} - \frac{u^5}{5}\right] + C \tag{1.6}$$

$$\Rightarrow -\left[\frac{5u^3 - 3u^5}{15}\right] + C \tag{1.7}$$

$$\Rightarrow \frac{3u^5 - 5u^3}{15} + C \tag{1.8}$$

$$\Rightarrow \boxed{\frac{3\cos^5(x) - 5\cos^3(x)}{15} + C} \tag{1.9}$$

2 Problem 2

$$\int \frac{[\log_5(x)]^9}{8x}\,dx$$

$$\int \frac{[\log_5(x)]^9}{8x}\,dx = \int \frac{\left[\frac{\ln(x)}{\ln(5)}\right]^9}{8x}\,dx \tag{2.1}$$

$$\Rightarrow \frac{1}{8}\int \frac{\left[\frac{\ln(x)}{\ln(5)}\right]^9}{x}\,dx \tag{2.2}$$

$$\Rightarrow \frac{1}{8}\int \frac{1}{x}\left[\frac{\ln(x)}{\ln(5)}\right]^9\,dx \tag{2.3}$$

$$\Rightarrow \frac{1}{8}\int \frac{1}{x}\left[\frac{\ln^9(x)}{\ln^9(5)}\right]\,dx \tag{2.4}$$

$$\Rightarrow \frac{1}{8\ln^9(5)}\int \frac{1}{x}\ln^9(x)\,dx \tag{2.5}$$

$$u = \ln(x) \quad du = \tfrac{1}{x}dx \quad dx = x\,du$$

$$\Rightarrow \frac{1}{8\ln^9(5)}\int \frac{1}{x}u^9\,x\,du \tag{2.6}$$

$$\Rightarrow \frac{1}{8\ln^9(5)}\int u^9\,du \tag{2.7}$$

$$\Rightarrow \frac{1}{8\ln^9(5)}\left[\frac{u^{10}}{10}\right] + C \tag{2.8}$$

$$\Rightarrow \frac{1}{8\ln^9(5)}\left[\frac{\ln^{10}(x)}{10}\right] + C \tag{2.9}$$

$$\Rightarrow \boxed{\frac{\ln^{10}(x)}{80\ln^9(5)} + C} \tag{2.10}$$

3 Problem 3

$$\int \frac{3x}{e^{x/5}}\,dx$$

$$\int \frac{3x}{e^{x/5}}\,dx = 3\int \frac{x}{e^{x/5}}\,dx \tag{3.1}$$

$$\Rightarrow 3\int x\cdot e^{-x/5}\,dx \tag{3.2}$$

Integrating by parts where:

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int u'(x)v(x)\,dx$$

$$u(x) = x \quad v(x) = \int e^{-x/5} = -5e^{-x/5} \quad u'(x) = 1 \quad v'(x) = e^{-x/5}$$

$$\int x\cdot e^{-x/5} = \left[x\cdot -5e^{-x/5} - \int 1e^{-x/5}\,dx\right]$$

$$\int x\cdot e^{-x/5} = \left[-5xe^{-x/5} - \int e^{-x/5}\,dx\right] \tag{3.3}$$

$$\int x\cdot e^{-x/5} = \left[-5xe^{-x/5} + 5e^{-x/5} + C\right] \tag{3.4}$$

$$\int x\cdot e^{-x/5} = e^{-x/5}\,(5-5x) + C \tag{3.5}$$

Now that we have our integral, we bring back our coefficient of 3:

$$\int \frac{3x}{e^{x/5}} = 3e^{-x/5}\,(5-5x) + C \tag{3.6}$$

$$= \boxed{\frac{3}{e^{x/5}}\,(5-5x) + C} \tag{3.7}$$