Integration by Parts

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1 What is Integration by Parts?

Let us say that we had an expression

$$f(x)g(x)$$

and that expression was differeniated:

$$\frac{\mathrm{d}}{\mathrm{d}x} f(x)g(x) = f'(x)g(x) + f(x)g'(x) \tag{1}$$

in order to get back to $f(x)g(x)$, we would need to integrate (obviously):

$$\begin{aligned} \int \frac{\mathrm{d}}{\mathrm{d}x}\left[f(x)g(x)\right]\,dx &= \int \left[f'(x)g(x) + f(x)g'(x)\right]\,dx \\ f(x)g(x) &= \int f'(x)g(x)\,dx + \int f(x)g'(x)\,dx \end{aligned}$$

We can now solve for the integrals:

$$\int f'(x)g(x)\,dx$$

$$\int f(x)g'(x)\,dx$$

but we will primarily look to solve for the latter integral $\int f(x)g'(x)$. Solving for that integral, we get the formula:

$$\boxed{\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx} \tag{2}$$

1.1 Example

We have an integral:

$$\int x\cos(x)\,dx$$

We will assume this integral is in the form:

$$\int f(x)g'(x)\,dx$$

thus we can use the formula for integration by parts:

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$$

which when plugging in our functions becomes:

$$\int x\cos(x)\,dx = x\int \cos(x)\,dx - \int \frac{\mathrm{d}}{\mathrm{d}x}[x]\cos(x)\,dx$$

evaluting this expression, we get:

$$\begin{aligned} \int x\cos(x)\,dx &= x\int \cos(x)\,dx - \int \frac{\mathrm{d}}{\mathrm{d}x}[x]\cos(x)\,dx && \text{(1a)} \\ &= x\sin(x) - \int (1)\cos(x)\,dx && \text{(1b)} \\ &= x\sin(x) - \int \cos(x)\,dx && \text{(1c)} \\ &= x\sin(x) - \sin(x) + C && \text{(1d)} \end{aligned}$$

thus, we find that:

$$\boxed{\int x\cos(x)\,dx = x\sin(x) - \sin(x) + C}$$

1.1.1 Remarks

In this example:

$$f(x)g(x) = x\sin(x)$$

thus the expression that was differeniated in equation 1 was $x\sin(x)$.