Numerical Integration:
Using Area Formulas to Integrate Non-Elementary Functions

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There are some functions that we cannot integrate with the techiques we have used thus far, so we can just use area formulas to approximate the area under the curve. We already touched upon this with **Reimann Sums**:

$$\sum_{i=1}^{n} f(x_i)\Delta x_i \quad \text{where} \quad \Delta x_i = \frac{x_n - x_1}{n}$$

A more precise method of approximation would be to use **Trapezoidal Sums**. Instead of using the area of rectangles, we use the area of trapezoids:

$$\text{Area of a Trapezoid:} \quad \frac{1}{2}(a+b)h$$

As a series, we get:

$$\sum_{i=1}^{n} \frac{1}{2}\Delta x_i (f(x_i) + f(x_{i+1})) \tag{1a}$$

$$\frac{1}{2}\sum_{i=1}^{n} \Delta x_i (f(x_i) + f(x_{i+1})) \tag{1b}$$

Since the next terms will be x_{i+1} and x_{i+2}, in which there will be doubles of x-values between x_i and x_n inclusive, we will get a formula:

$$\boxed{\frac{1}{2}\Delta x_i (f(x_i) + 2f(x_{i+1}) + \cdots + 2f(x_{n-1}) + f(x_n))} \tag{1c}$$