Hyperbolic Functions

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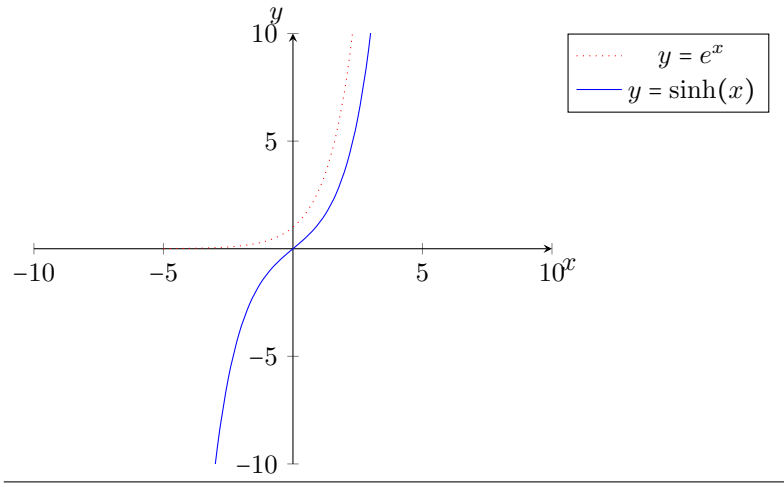
Def: The hyperbolic sine and cosine functions are defined by the equations:

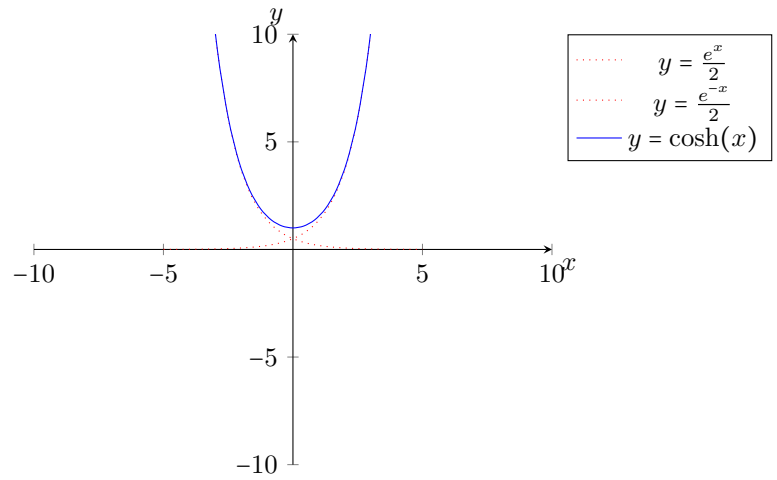
$$\sinh(x) = \frac{e^x - e^{-x}}{2} \tag{1}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \tag{2}$$

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{\sinh(x)}{\cosh(x)} \tag{3}$$

Graph of some hyperbolic function:





$$\coth(x) = \frac{1}{\tanh(x)} = \frac{\cosh(x)}{\sinh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}} \tag{4}$$

$$\operatorname{sech}(x) = \frac{1}{\cosh(x)} = \frac{2}{e^x + e^{-x}} \tag{5}$$

$$\operatorname{csch}(x) = \frac{1}{\sinh(x)} = \frac{2}{e^x - e^{-x}} \tag{6}$$

1. $\cosh^2(x) - \sinh^2(x) = 1$
2. $\cosh^2 + \sin^2(x) = \cosh(2x)$

Proof for identity 2:

$$\cosh(x) = \frac{e^x + e^{-x}}{2}, \sinh(x) = \frac{e^x - e^{-x}}{2} \tag{7}$$

$$\cosh^2(x) + \sinh^2(x) = (\frac{e^x + e^{-x}}{2}) + (\frac{e^x - e^{-x}}{2}) \tag{8}$$

$$= \frac{e^{2x} + 2e^x e^{-x} + e^{-2x}}{4} + \frac{e^{2x} - 2e^x e^{-x} + e^{-2x}}{4} \tag{9}$$

$$= \frac{2e^{2x} + 2e^{-2x}}{4} \tag{10}$$

$$= \frac{e^{2x} + e^{-2x}}{2} = \cosh(2x) \tag{11}$$