Kinematic Practice Problems

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1 Chapter 2: 1D Motion

1.1 Problem 34: Page 34

A red car and a green car, identical except for the color, move toward each other in adjacent lanes and parallel to an x acis. At time $t = 0$, the red car is at $x_r = 0$ and the green car is at $x_g = 220\,\text{m}$. If the red car has a constant velocity of $20\,\text{km/h}$, the cars pass each other at $x = 44.5\,\text{m}$, and if it has a constant velocity of $40\,\text{km/h}$, they pass each other at $x = 76.6\,\text{m}$. What are:

(a) the initial velocity

(b) the constant acceleration

of the green car?

Figure 2-27 Problems 34 and 35.

1.1.1 Solution

We can define the positions of the red car and green car as functions of time:

$$x_r(t) - x_r = v_r t \tag{R}$$

The acceleration of the red car is 0 because its velocity is constant.

$$x_g(t) - x_g = v_0 t + \frac{1}{2}at^2 \tag{G}$$

We know that the initial position of the red car, x_r, is 0 m and the initial position of the green car, x_g, is 220 m, thus:

$$x_r(t) = v_r t$$
$$x_g(t) - 220 = v_0 t + \frac{1}{2}at^2$$

Since we are given two scenarios, in which the velocity of the red car and the position at which the two cars meet change, but the initial velocity and acceleration of the green car remain the same, we can set up a system of equations to solve for v_0 and subsequently a of the green car:

If the velocity of the red car is $\frac{50}{9}$ m/s, then both cars meet at 44.5 m. We can solve for time t_1:

$$x_r(t_1) = 44.5 \implies 44.5 = \frac{50}{9}t_1 \tag{1A}$$
$$\Rightarrow t_1 = 8.01 \tag{1B}$$

We can do the same if $v_r = \frac{100}{9}$, in which the position is now 76.6 m. We can solve for time t_1:

$$x_r(t_2) = 76.6 \implies 76.6 = \frac{100}{9}t_2 \tag{1C}$$
$$\Rightarrow t_2 = 6.89 \tag{1D}$$

We can plug both times into our function $x_g(t)$ to get a system of two equations:

$$x_g(t_1) - 220 = 44.5 - 220 = -175.5 \implies -175.5 = v_0 t_1 + \frac{1}{2} a t_1^2 \tag{1E}$$

$$x_g(t_2) - 220 = 76.6.5 - 220 = -143.4 \implies -143.4 = v_0 t_2 + \frac{1}{2} a t_2^2 \tag{1F}$$

We can solve for a in equation 1E and plug that value into equation $1F$ to solve for v_0:

$$a = \frac{2}{t_1^2}\left(-175.5 - v_0 t_1\right) \tag{1G}$$

$$\implies -143.4 = v_0 t_2 + \frac{1}{2}\left(\frac{2}{t_1^2}\left(-175.5 - v_0 t_1\right)\right) t_2^2 \tag{1H}$$

$$\Rightarrow -143.4 = v_0 t_2 + \frac{t_2^2}{t_1^2}\left(-175.5 - v_0 t_1\right)$$

$$\Rightarrow -143.4 = t_2\left(v_0 + \frac{t_2}{t_1^2}\left(-175.5 - v_0 t_1\right)\right)$$

$$\Rightarrow -143.4 = t_2\left(v_0 + \frac{t_2}{t_1^2}(-175.5) - v_0 \frac{t_2}{t_1}\right)$$

$$\Rightarrow -143.4 = t_2\left(\left(1 - \frac{t_2}{t_1}\right) v_0 + \frac{t_2}{t_1^2}(-175.5)\right)$$

$$\Rightarrow \frac{-143.4}{t_2} = \left(1 - \frac{t_2}{t_1}\right) v_0 + \frac{t_2}{t_1^2}(-175.5)$$

$$\Rightarrow \frac{-143.4}{t_2} - \frac{t_2}{t_1^2}(-175.5) = \left(1 - \frac{t_2}{t_1}\right) v_0$$

$$\Rightarrow v_0 = \frac{\frac{-143.4}{t_2} - \frac{t_2}{t_1^2}(-175.5)}{1 - \frac{t_2}{t_1}}$$

Substituting in our parameters, we get:

$$v_0 = \frac{\frac{-143.4}{6.89} - \frac{6.89}{8.01^2}(-175.5)}{1 - \frac{6.89}{8.01}} \tag{1I}$$

$$\boxed{v_0 \approx -14.06\,\mathrm{m/s}} \tag{a}$$

Now we can substitute this value into equation 1G to solve for a:

$$a = \frac{2}{8.01^2}[-175.5 + 14.06(8.01)] \approx -1.92\,\mathrm{m/s}^2 \tag{1J}$$

$$\boxed{a \approx -1.92\,\mathrm{m/s}^2} \tag{1K}$$

2 Chapter 3: Vectors

2.1 Problem 31: Page 58

In Fig. 3-30, a vector $\vec{\mathbf{a}}$ with a magnitude of 17.0 m is directed at angle $\theta = 56.0°$ counterclockwise from the $+x$ axis. What are the components (a) a_x and (b) a_y of the vector? A second coordinate system is inclined by angle $\theta' = 18.0°$ with respect to the first. What are the components (c) a'_x and (d) a'_y in this primed coordinate system?

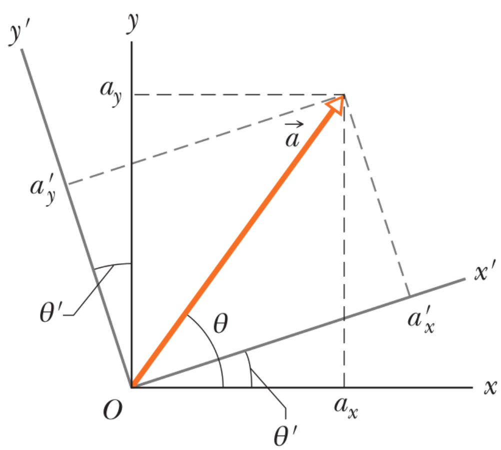


Figure 1: 3-30

2.1.1 Solution

We can draw a triangle using θ to solve for the components using trig functions:

$$\cos(\theta) = \frac{a_x}{\vec{\mathbf{a}}} \implies \boxed{a_x = \vec{\mathbf{a}}\cos(\theta)} \tag{a}$$

$$\sin(\theta) = \frac{a_y}{\vec{\mathbf{a}}} \implies \boxed{a_y = \vec{\mathbf{a}}\sin(\theta)} \tag{b}$$

We can draw another triangle using θ' to solve for the other components:

$$\cos(\theta') = \frac{a'_x}{\vec{\mathbf{a}}} \implies \boxed{a'_x = \vec{\mathbf{a}}\cos(\theta')} \tag{c}$$

$$\sin(\theta') = \frac{a'_y}{\vec{\mathbf{a}}} \implies \boxed{a'_y = \vec{\mathbf{a}}\sin(\theta')} \tag{d}$$

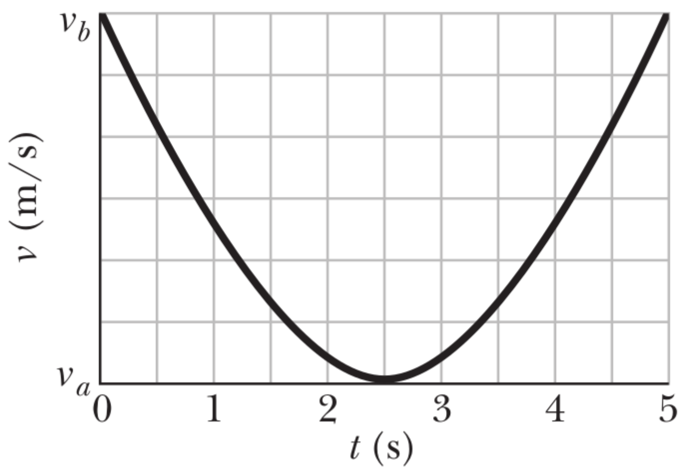
3 Chapter 4: 2D and 3D Motion

3.1 Problem 38: Page 86

A golf ball is struck at ground level. The speed of the golf ball is a function of the time is shown in Fig. 4-36, where $t = 0$ at the instant the ball is struck. The scaling on the vertical axis is set by $v_a = 19\,\text{m/s}$ and $v_b = 31\,\text{m/s}$.

(a) How far does the golf ball travel horizontally before returning to ground level?

(b) What is the maximum height above the ground level attained by the ball?



3.1.1 Solution

Looking at the graph, we can tell that the ball returns to the ground at $t = 5$ since it will travel with the same speed (magnitude of velocity) from 0 to 2.5 s as it does from 2.5 to 5 s, only this time, the ball is traveling downwards instead of upwards. There is vertical acceleration due to gravity, so we need to use the following equation:

$$\Delta r = vt - \frac{1}{2}at^2$$

there will be two different displacements, horizontal and vertical, thus we will have 2 different equations:

$$\Delta r_y = v_y t - \frac{1}{2}gt^2 \qquad \text{(vertical)}$$

$$\Delta r_x = v_x t \qquad \text{(horizontal)}$$

we can assume that horizontal acceleration is 0.

Since the ball reaches its maximum height at $t = 2.5$, the vertical velocity at this moment is 0, thus:

$$\Delta r_y = (0)(2.5) - \frac{1}{2}g(2.5)^2 = \boxed{30.625\,\text{m}} \tag{3A}$$

Since there is no horizontal acceleration, the horizontal velocity is constant, and we can solve for this component of velocity by taking the magnitude of velocity when $v_y = 0$, which as we established, occurs when the ball reaches its maximum height at $t = 2.5$:

$$\vec{\mathbf{v}}(2.5) = \sqrt{v_y(2.5)^2 + v_x^2} \tag{3B}$$

$$\Rightarrow 19 = \sqrt{0 + v_x^2} \implies v_x = 19 \tag{3C}$$

$$\Delta r_x = (19)(5) = \boxed{95\,\text{m}} \tag{3D}$$

3.2 Problem 71: Page 88

A suspicious-looking man runs as fast as he can along a moving sidewalk from one end to the other, taking 2.50 s. Then security agents appear, and the man runs as fast as he can back along the sidewalk to his starting point, taking 10.0 s. What is the ratio of the man's running speed to the sidewalk's speed?

3.2.1 Solution

The first thing we should do is draw the scenario:

We know that the man runs at constant speed since he is always running *as fast as he can.* The moving sidewalk is not said to change speed nor would it make sense for it to change speed, think of it like a flat escalator. Thus, the acceleration in this scenario is 0, meaning it will not have an effect and we can write our equations without it.

We can write a system of equations, one for when the man is running along the sidewalk and the other where he is running against the sidewalk:

We can say that the speed of the man relative to the sidewalk is his speed plus the sidewalk speed in scenario 1 and his speed minus the sidewalk speed in scenario 2. Since the man is able to run against the sidewalk, his speed must be greater than the sidewalk, thus we subtract the speed of the sidewalk from his speed to determine his speed relative to the sidewalk:

$$\frac{\Delta x}{2.5} = \text{man}_v + \text{sidewalk}_v \tag{1}$$

$$\frac{\Delta x}{10} = \text{man}_v - \text{sidewalk}_v \tag{2}$$

We can solve for man_v in equation 1, then substitute that into equation 2 to solve for sidewalk_v, then divide man_v by sidewalk_v to determine the ratio:

$$\text{Equation 1 implies that:} \quad \text{sidewalk}_v = \frac{1}{2.5}\Delta x - \text{man}_v \tag{4A}$$

$$\text{Equation 2 implies that:} \quad \text{man}_v = \frac{1}{10}\Delta x + \text{sidewalk}_v \tag{4B}$$

Solving for man_v:

$$\frac{1}{2.5} = \frac{2}{5}$$

$$\text{man}_v = \frac{1}{10}\Delta x + \left(\frac{2}{5}\Delta x - \text{man}_v\right) \tag{4C}$$

$$\text{man}_v = \frac{1}{10}\Delta x + \frac{4}{10}\Delta x - \text{man}_v \tag{4D}$$

$$2\,\text{man}_v = \left(\frac{1}{10} + \frac{4}{10}\right)\Delta x \tag{4E}$$

$$\text{man}_v == \frac{\left(\frac{5}{10}\right)}{2}\Delta x = \frac{5}{20}\Delta x = \frac{1}{4}\Delta x \tag{4F}$$

Solving for sidewalk_v:

$$\text{sidewalk}_v = \frac{2}{5}\Delta x - \frac{1}{4}\Delta x = \left(\frac{8}{20} - \frac{5}{20}\right)\Delta x \tag{4G}$$

$$\text{sidewalk}_v = \frac{3}{20}\Delta x \tag{4H}$$

Calculating ratio:

$$\frac{\text{man}_\text{v}}{\text{sidewalk}_v} = \frac{\frac{5}{20}\cancel{\Delta x}}{\frac{3}{20}\cancel{\Delta x}} = \frac{20}{3} \cdot \frac{5}{20} = \boxed{\frac{5}{3}} \tag{4I}$$

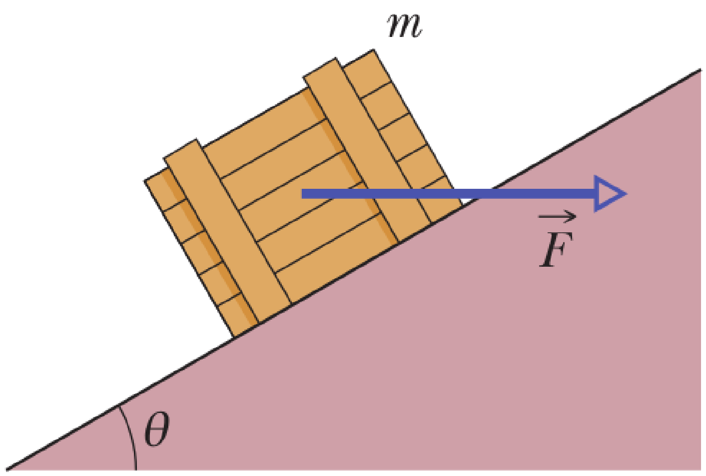
4 Chapter 5: Force and Motion

4.1 Problem 34: Page 119

In Fig. 5-40, a crate of mass $m = 100\,\text{kg}$ is pushed at constant speed up a frictionless ramp ($\theta = 30.0°$) by a horizontal force $\vec{\mathbf{F}}$. What are the magnitudes of:

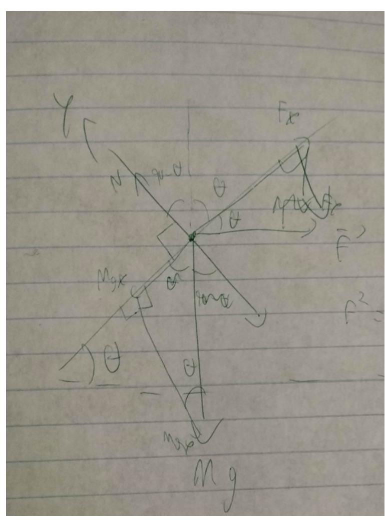
(a) $\vec{\mathbf{F}}$

(b) the force on the crate from the ramp?



4.1.1 Solution

We can draw a freebody diagram, with the center of the box being our origin.



We can use θ to find the components of $\vec{\mathbf{F}}$ and mg along the x-axis since the components will form right triangles. You will see this when you draw the diagram.

Since we have no way of determing the normal force, we should solve for $\vec{\mathbf{F}}$ by determing the net force along the x-axis:

$$\sin(\theta) = \frac{mg_x}{mg} \implies mg_x = mg\sin(\theta) \tag{5A}$$

$$\cos(\theta) = \frac{F_x}{\vec{\mathbf{F}}} \implies F_x = \vec{\mathbf{F}}\cos(\theta) \tag{5B}$$

$$x:\ \vec{\mathbf{F}}\cos(\theta) - mg\sin(\theta) \tag{5C}$$

Since $F = ma$ and the box is moving with constant speed, we can say that acceleration is 0, thus the net force is equal to $m(0) = 0$:

$$\vec{\mathbf{F}}\cos(\theta) - mg\sin(\theta) = 0 \implies \boxed{\vec{\mathbf{F}} = mg\tan(\theta)} \tag{5D}$$

Now that we have $\vec{\mathbf{F}}$, we can calculate normal force $\vec{\mathbf{N}}$ by determing all of components of force along the y-axis. Since the box is not moving along the y-axis, we know the sum of forces along the y-axis will be 0:

$$\cos(\theta) = \frac{mg_y}{mg} \implies mg_y = mg\cos(\theta) \tag{5E}$$

$$\sin(\theta) = \frac{F_y}{\vec{\mathbf{F}}} \implies F_y = \vec{\mathbf{F}}\sin(\theta) \tag{5F}$$

$$y:\ \vec{\mathbf{N}} - mg\tan(\theta)\sin(\theta) - mg\cos(\theta) = 0 \tag{5G}$$

$$\implies \boxed{\vec{\mathbf{N}} = mg\tan(\theta)\sin(\theta) + mg\cos(\theta)} \tag{5H}$$

4.2 Problem 39: Page 119

A sphere of mass 3.0×10^{-4} kg is suspended from a cord. A steady horizontal breeze pushes the sphere so that the cord makes a constant angle of 37° with the vertical. Find:

(a) the push magnitude

(b) the tension in the cord

4.2.1 Solution

First we should draw a force diagram:

We can use the 37° angle, which we will call θ to form a right triangle and solve for the x component of tension, since the push force is the only other horizontal force acting on the sphere.

Since the sphere is suspended, it means it is not moving, thus acceleration is 0. Since $F = ma$, we now know that the net force on the object is 0 Newtons.

$$x: \quad P - T_x = 0 \implies P = T_x \tag{6A}$$

$$\sin(\theta) = \frac{T_x}{\vec{\mathbf{T}}} \implies T_x = \vec{\mathbf{T}}\sin(\theta) P = \vec{\mathbf{T}}\sin(\theta) \tag{6B}$$

We need to solve for the tension $\vec{\mathbf{T}}$ in order to solve for P, we can do so by taking all of the vertical forces acting on the sphere, which are T_y and mg:

$$y: \quad T_y - mg = 0 \implies T_y = mg \tag{6C}$$

$$\cos(\theta) = \frac{T_y}{\vec{\mathbf{T}}} \implies \vec{\mathbf{T}} = \frac{mg}{\cos(\theta)} \tag{6D}$$

Thus we have our answers:

$$\boxed{P = mg\tan(\theta)} \tag{a}$$

$$\boxed{T = mg\sec(\theta)} \tag{b}$$

4.3 Problem 40: Page 119

A dated box of dates, of mass 5.00 kg, is sent sliding up a frictionless ramp at an angle of θ to the horizontal. Figure 5-41 gives,

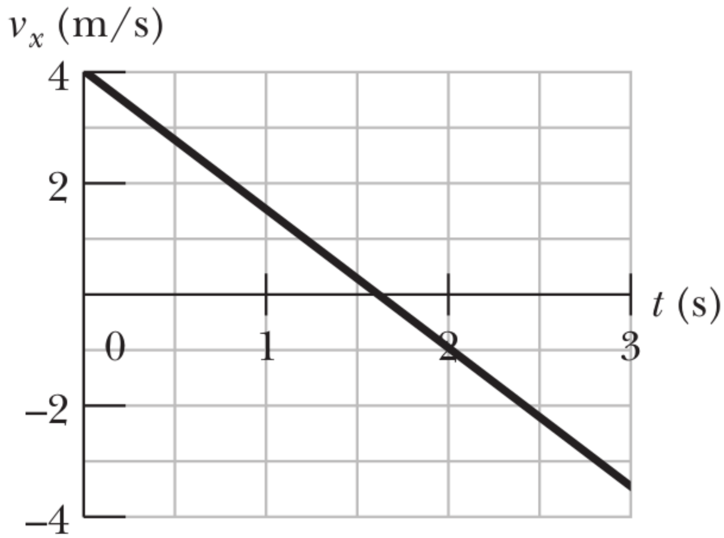


Figure 2: 5-41

as a function of time t, the component v_x of the box's velocity along an x axis that extends directly up the ramp. What is the magnitude of the normal force on the box from the ramp?

4.3.1 Solution

We can draw a free-body diagram of the box sliding up the ramp:

The first thing to do is find acceleration, since we know that there is horizontal acceleration according to the graph of v_x. Taking the slope of the graph using points $t = 0$ to $t = 2$, we find the slope to be:

$$a_x = \text{slope}_{v_x} = \frac{-1-4}{2-0} = \frac{-5}{2}$$

since the object is not moving vertically, we know that $a_y = 0$. We can determine θ.

Since the box only experiences the force of gravity and the ramp on the incline, the equation:

$$a = -g\sin(\theta)$$

holds true. We can use this to solve for θ:

$$\frac{-5}{2} - -g\sin(\theta) \tag{7A}$$

$$\frac{5}{2g} = \sin(\theta) \tag{7B}$$

$$\theta = \arcsin\left(\frac{5}{2g}\right) \tag{7C}$$

Since the box does not move vertically, the sum of forces on the y-axis is 0. Now we can solve for the normal force:

$$y: \quad N - mg_y = 0 \implies N = mg_y \tag{7D}$$

$$\sin(90-\theta) = \frac{mg_y}{mg} \implies mg_y = mg\sin(90-\theta) \tag{7E}$$

$$N = mg\sin(90-\theta) \tag{7F}$$

Thus our answer is:

$$\boxed{N = mg\sin\left(90 - \arcsin\left(\frac{5}{2g}\right)\right)}$$