

# Friction and Circular Motion

Laith Toom

21/2/2023

# 1 Friction

When a object is moving along a surface, there is a force acting against that movement, and that force is friction  $\vec{f}$ .

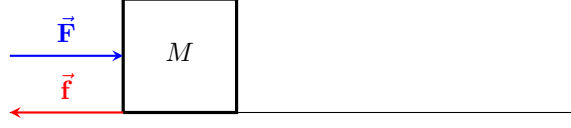
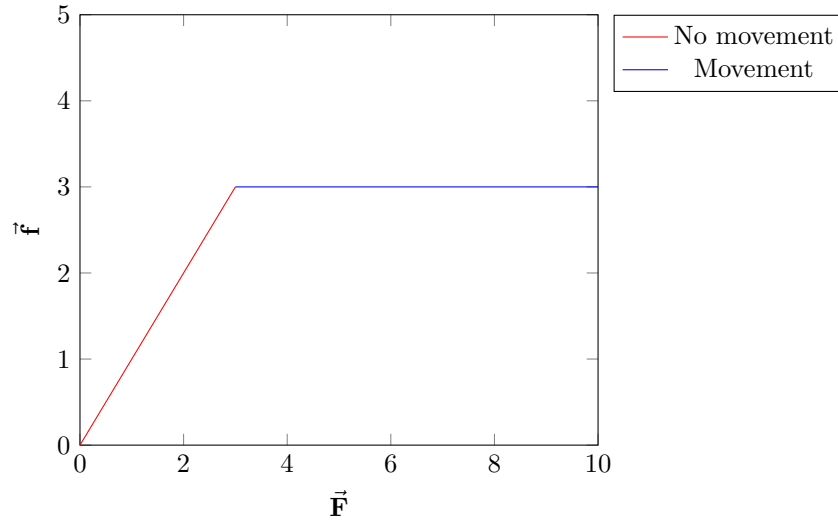


Figure 1: Box moving along ground with friction  $\vec{f}$

The relationship between  $\vec{f}$  and  $\vec{F}$  is linear with a slope of 1, meaning that there is a 1:1 ratio between friction and the force moving the box. As a result, the two forces cancel out and no movement occurs.

However, this only holds true until friction reaches its maximum force:

Figure 2:  $\vec{f}$  with respect to  $\vec{F}$



The maximum force of friction,  $\vec{f}_{\max}$ , is defined as the product of the **coefficient of friction**,  $\mu$ , and the normal force,  $\vec{N}$ , acting on the object:

$$\vec{f}_{\max} = \mu \vec{N}$$

$\mu$  will be a given constant, and it depends on the type of surface where movement is occurring. Slippery surfaces such as ice will have a lower coefficient while rough surfaces such as asphalt or concrete will have a higher coefficient.

## 2 Circular Motion

Does an object in circular motion with a constant speed have a force acting on it?

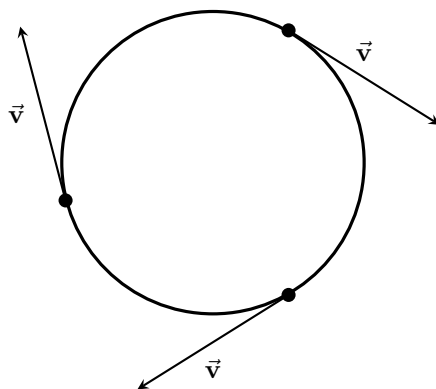


Figure 3: Velocity of object in circular motion

An object in circular motion does have a force acting on it because the direction of velocity changes, which means the components of velocity change, and for there to be change in velocity, there must be acceleration.

However, the magnitude of velocity, **speed**, in this scenario is constant, it is only the components that are changing:

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

$$\frac{d|\vec{v}|}{dt} = 0$$

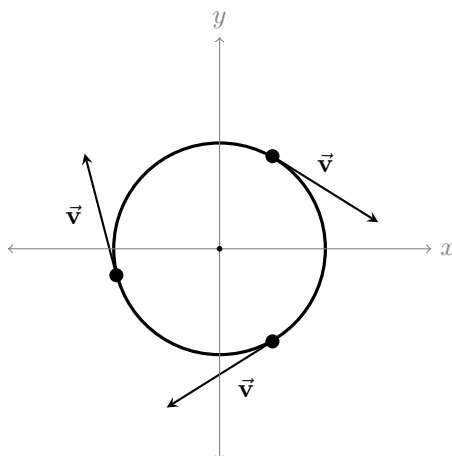
$$\frac{dv_x}{dt} = a_x$$

$$\frac{dv_y}{dt} = a_y$$

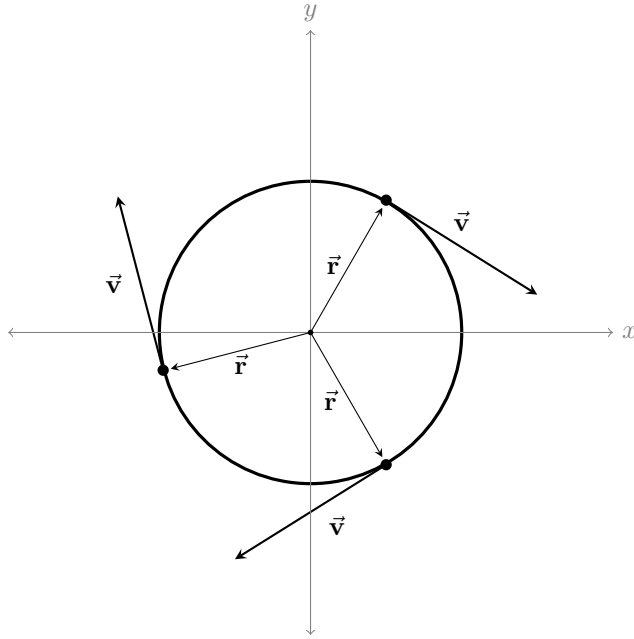
if you forgot, acceleration  $\vec{a}$  is also a vector, with  $x$  and  $y$  components acting on the respective  $x$  and  $y$  components of velocity  $\vec{v}$ .

### 2.1 Determining the Components of Velocity

We can redraw Figure 3 with axes  $x$  and  $y$ :



We can also draw displacement vectors  $\vec{r}$  from the origin to the start of the velocity vectors:



Decomposing the displacement vectors into  $r_x$  and  $r_y$  will form triangles, which will allow us to solve for those components of displacement:

$$r_x = r \cos(\theta)$$

$$r_y = r \sin(\theta)$$

we can then differentiate to get the components of velocity:

$$v_x = \frac{dr_x}{dt} = -r \sin(\theta) \frac{d\theta}{dt}$$

$$v_y = \frac{dr_y}{dt} = r \cos(\theta) \frac{d\theta}{dt}$$

if you can visualize it, as the object moves around the circle, the displacement vector  $\vec{r}$  follows it, and so components of the displacement vector change. Thus, a new triangle is formed, which means a new value for  $\theta$ , thus  $\theta$  does have a rate of change, which comes out during differentiation as seen above.

The rate of change of  $\theta$  will be represented as omega  $\omega$ :

$$\omega = \frac{d\theta}{dt}$$