Introduction to Physics

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1 What is Physics?

Physics is the study of how the objects around us change over time. This includes their **position**, **displacement**, **distance**, and **speed**.

1.1 Position

The **position** of an object is its relative distance from a reference point. For example, if we had a red dot and a green dot on a line, then we could determine the position of the red dot to be its distance from the green dot and the position of the green dot to be its distance from the red dot. We can express position as a function x(t).

1.2 Displacement

Displacement is the difference between an objects new position and its previous position. If we had a red dot move from point 1 to point 2, then the displacement of the red dot would be the difference in position of point 2 and point 1. Mathematically, we can represent this as Δx.

$$\Delta x = x_2 - x_1$$

Since position is a function, x_2 and x_1 can be written as $x(t_2)$ and $x(t_1)$, where t_2 is the time at which the object was at position 2 and $t - 1$ is the time at which the object was at position 1. Thus, we can write displacement as:

$$\Delta x = \mathrm{x(t_2) - x(t_1)}$$

1.3 Distance

Distance is the length of the path traveled by an object from one point to another.

1.4 Speed

Speed is how fast an object moved from one point to another. As an extension, **velocity** is just speed, but in specific direction. In other words, speed is a *scalar*, velocity is a *vector*, but both represent how fast an object moves. The rate at which velocity change over time is known as **acceleration**. We can represent velocity as a function v(t). We could try to define this function as the ratio between displacement and time elapsed between two points:

$$\overline{\mathrm{v(t)}} = \frac{\Delta x}{\Delta t}$$

The issue with this equation is that velocity can change at any given time, thus in order to get closer to a true representation of velocity, we need to look at instantaneous intervals of velocity. In other words: we differentiate position.

$$\mathrm{v(t)} = \frac{dx}{dt}$$

By differentiating position, we will get the rate at which position changes at a specific time t, which is the velocity at that specific time.

1.5 Acceleration

Acceleration is the rate at which the velocity of an object changes, however with a few exceptions, we will assume acceleration to be constant for the majority of this course. Thus, we will represent acceleration as a variable a, and if acceleration is not constant, a function a(t).

Since acceleration is the rate at velocity changes, we can differentiate velocity to get acceleration:

$$\mathrm{a} = \frac{dv}{dt}$$

This also means that if we want to get velocity, but only have acceleration, we can integrate acceleration to get velocity:

$$\begin{aligned}
\mathrm{v(t)}\Big|_{\mathrm{t_1}}^{\mathrm{t_2}} &= \int_{t_2}^{t_1} a\,dt \\
\mathrm{v(t)}\Big|_{\mathrm{t_1}}^{\mathrm{t_2}} &= a \cdot t\Big|_{t_1}^{t_2} \\
\mathrm{v(t)}\Big|_{\mathrm{t_1}}^{\mathrm{t_2}} &= a \cdot t_2 - a \cdot t_1 = a \cdot (t_2 - t_1) \\
\mathrm{v(t_2)} - \mathrm{v(t_1)} &= a \cdot (t_2 - t_1)
\end{aligned}$$

2 Time

Objects always move with respect to time, as the definition of physics would imply. Thus, we treat time as an independant variable; while position, displacement, distance, speed, velocity, and acceleration are all functions of time.

3 Problems

3.1 The Rabbit

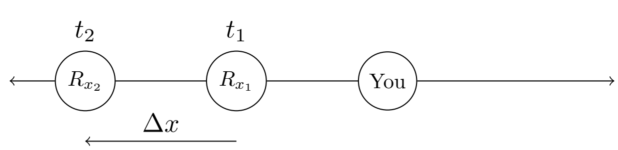
A rabbit is moving along a straigh line. At time $t_1 = 0$ the raddit is distance $x_1 = 1\text{m}$ from your right and is moving towards the right with a velocity of $v_1 = 1\frac{\text{m}}{\text{s}}$. The rabbit has a constant acceleration of $a = 1\frac{\text{m}}{\text{s}^2}$ towards the right. What is the position of the rabbit as time $t_2 = 10s$?

3.1.1 Solution

First we list our known variables followed by our unknowns:

$$
\begin{aligned}
&\underline{\text{Knowns}}\\
&x_1 = 1\text{m}\\
&t_1 = 0\text{s}\\
&v_1 = 1\frac{\text{m}}{\text{s}}\\
&t_2 = 10\text{s}\\
&a = 1\frac{\text{m}}{\text{s}^2}\\
&\underline{\text{Unknowns}}\\
&x_2 = ?\\
&v_2 = ?
\end{aligned}
$$

Then we illustrate the scenario: While we do know velocity at time $t = 0\text{s}$, we do



not know velocity at time $t = 10\text{s}$. We don't need to know the value of velocity at this point, it will give us the grounds to define an expression for v(t), which we can differentiate to get a function for position, allowing us to calculate the final position of the rabbit.

Since we know acceleration $a = 1\frac{\text{m}}{\text{s}^2}$, including the fact that it is constant, we can differentiate acceleration to find a function for velocity:

$$a = \frac{dv}{dt} \tag{1.1}$$

$$\int_{t_2}^{t_1} a\,dt = \int_{t_2}^{t_1} \frac{dv}{dt} \tag{1.2}$$

$$a \cdot t\Big|_{t_1}^{t_2} = \mathrm{v}(t)\Big|_{t_1}^{t_2} \tag{1.3}$$

$$\Rightarrow \mathrm{v}(t_2) - \mathrm{v}(t_1) = a \cdot (t_2 - t_1) \tag{1.4}$$

$$\Rightarrow \mathrm{v}(t_2) = a \cdot (t_2 - t_1) + \mathrm{v}(t_1) \tag{1.5}$$

Now that we have a function for velocity, we can utilize the fact that velocity is the derivative of position, allowing us to set our function equal to dx:

$$\frac{dx}{dt} = \mathrm{v}(t_2) = a \cdot (t_2 - t_1) + \mathrm{v}(t_1)$$

We can now omit $\mathrm{v}(t_2)$ since we aren't looking for it, giving us a clean equation that we can integrate to get the value of x_2. Naturally, we want to integrate with the upper limit t_2 and the lower limit t_1:

$$\int_{t_1}^{t_2} \frac{dx}{dt}\,dt = \int_{t_1}^{t_2} [a \cdot (t_2 - t_1) + v_1]\,dt \tag{1.6}$$

$$\int_{t_1}^{t_2} \frac{dx}{dt}\,dt = \int_{t_1}^{t_2} a \cdot (t_2 - t_1)\,dt + \int_{t_1}^{t_2} v_1\,dt \tag{1.7}$$

$$\mathrm{x}(t)\Big|_{t_1}^{t_2} = \int_{t_1}^{t_2} (a \cdot \Delta t)\,dt + (v_1 \cdot t)\Big|_{t_1}^{t_2} \tag{1.8}$$

$$\mathrm{x}(t)\Big|_{t_1}^{t_2} = a\frac{\Delta t^2}{2}\Big|_{t_1}^{t_2} + (v_1 \cdot t)\Big|_{t_1}^{t_2} \tag{1.9}$$

Note: $\Delta t = t - t_1$ where t is a parameter.

$$\mathrm{x}(t_2) - \mathrm{x}(t_1) = a(\frac{\Delta t^2(t_2)}{2} - \frac{\Delta t^2(t_1)}{2}) + v_1 \cdot (t_2 - t_1) \tag{1.10}$$

$$\mathrm{x}(t_2) - \mathrm{x}(t_1) = a(\frac{(t_2 - t_1)^2}{2} - \frac{(t_1 - t_1)^2}{2}) + v_1 \cdot (t_2 - t_1) \tag{1.11}$$

$$\mathrm{x}(t_2) = a(\frac{(t_2 - t_1)^2}{2} - \frac{(t_1 - t_1)^2}{2}) + v_1 \cdot (t_2 - t_1) + \mathrm{x}(t_1) \tag{1.12}$$

$$x_2 = a(\frac{(t_2 - t_1)^2}{2} - \frac{(t_1 - t_1)^2}{2}) + v_1 \cdot (t_2 - t_1) + x_1 \tag{1.13}$$

$$\tag{1.14}$$

We now have an equation for x_2. Now all we need to do now is plug in our knowns.

$$x_2 = (1)(\frac{(10-0)^2}{2} - \frac{(0-0)^2}{2}) + (1) \cdot (10-0) + (1) \tag{1.15}$$

$$x_2 = 1(\frac{(10)^2}{2} - \frac{(0)^2}{2}) + 1 \cdot 10 + 1 \tag{1.16}$$

$$x_2 = 1(\frac{100}{2} - \frac{0}{2}) + 10 + 1 \tag{1.17}$$

$$x_2 = 1(50-0) + 11 \tag{1.18}$$

$$x_2 = 50 + 11 \tag{1.19}$$

$$x_2 = 61 \tag{1.20}$$

$$\tag{1.21}$$

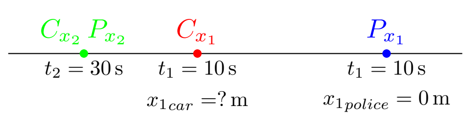
Our answer: 61m. The final position of the rabbit (the position of the rabit at time $t_2 = 10$s) is 61 meters from our right.

3.2 The Police Car and the Speeding Car

A car is moving with a constant speed v. It passes a police car on the side of the road. The officer realized that the speed of the car was over the limit and starts to chase it at time $t_1 = 10\,\mathrm{s}$ after the car passed by. The police car moves with a constant acceleration of $a = 10\,\mathrm{m/s^2}$ and reaches the car at $t_2 = 30\,\mathrm{s}$. What is the velocity of the speeding car?

3.2.1 Solution

Lets draw the scenario:



Lets list our known and unknown variables:

Knowns
$t_1 = 10\,\mathrm{s}$
$t_2 = 30\,\mathrm{s}$
$v_{1police} = 0\,\mathrm{m/s}$
$a_{police} = 10\,\mathrm{m/s^2}$
$x_{1police} = 0\,\mathrm{m}$
$a_{car} = 0\,\mathrm{m/s^2}$
Unknowns
$x_{1car} =?$
$x_2 =?$
$v_{2police} =?$
$v_{car} =?$

We will make the initial position of the police car our reference position, thus it will be equal to $0\,\mathrm{m}$. Since the speeding car has a constant speed, its acceleration equals $0\,\mathrm{m/s^2}$ because the speed is not changing.

Since the police car and the speeding car reached the same position at $t = \,\mathrm{s}$, by finding the equations of position for both cars, we can then solve for the velocity of the speeding car.

Since the police car started to move at $t_1 = 10\,\mathrm{s}$ and reached the position of the speeding car at $t_2 = 30\,\mathrm{s}$, we can say that the police car took $20\,\mathrm{s}$ ($t_2 - t_1 = 20\,\mathrm{s}$) to reach the car. We can also say that the velocity of the police car at $t_1 = 10\,\mathrm{s}$ was $0\,\mathrm{m/s}$ since it only began to move at that instant. With an acceleration of $10\,\mathrm{m/s^2}$, we can integrate acceleration to get a function for velocity:

$$\mathrm{v}_{police}(t) = \int a\,dt \tag{2.1}$$
$$\mathrm{v}_{police}(t) = a \cdot t \tag{2.2}$$
$$\tag{2.3}$$

Then we can integrate again to get position:

$$\mathrm{x}_{police}(t) = \int \mathrm{v}_{police}(t)\,dt \tag{2.4}$$
$$\mathrm{x}_{police}(t) = \int a \cdot t\,dt \tag{2.5}$$
$$\mathrm{x}_{police}(t) = a \cdot \frac{t^2}{2} \tag{2.6}$$
$$\mathrm{x}_{police}(t) = 10 \cdot \frac{t^2}{2} \tag{2.7}$$

Now for the speeding car, while we do not know its velocity, we can still integrate its acceleration of $0\,\mathrm{m/s^2}$ to get a function for velocity:

$$\mathrm{v}_{car}(t) = \int 0\,dt \tag{2.8}$$

$$\mathrm{v}_{car}(t) = C \tag{2.9}$$

$$\tag{2.10}$$

Then for position:

$$\mathrm{x}_{car}(t) = \int \mathrm{v}_{car}(t)\,dt \tag{2.11}$$

$$\mathrm{x}_{car}(t) = \int C\,dt \tag{2.12}$$

$$\mathrm{x}_{car}(t) = C \cdot t \tag{2.13}$$

Now we set these two equations equal to each other, in which we know t to be $30\,\mathrm{s}$:

$$\mathrm{x}_{car}(t) = x_{police}(t) \tag{2.14}$$

$$\mathrm{x}_{car}(30) = x_{police}(30) \tag{2.15}$$

$$C \cdot 30 = 10 \cdot \frac{30^2}{2} \tag{2.16}$$

$$C \cdot 30 = 5 \cdot 30^2 \tag{2.17}$$

$$\Rightarrow C = \frac{5 \cdot 30^2}{30} \tag{2.18}$$

$$C = 5 \cdot 30 \tag{2.19}$$

$$C = 150\,\mathrm{m/s} \tag{2.20}$$

$$\tag{2.21}$$

We find the velocity of the speeding car to be $150\,\mathrm{m/s}$.