Newton's Law

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1 Laws:

First Law:

If an object is not experiencing the effect of any force, then it will either remain stationary **or** keep moving with constant velocity.

Second Law:

If force $\vec{\mathbf{F}}$ is acting on an object with mass m, then the acceleration $\vec{\mathbf{a}}$ is given by:

$$\vec{\mathbf{F}} = M\vec{\mathbf{a}}$$

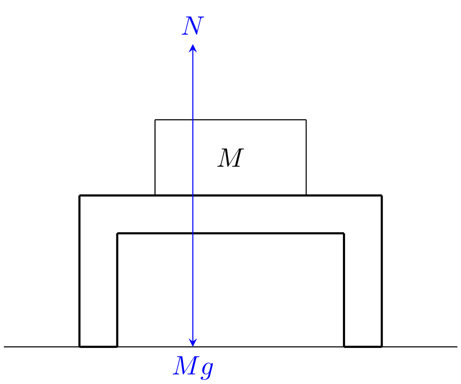


Figure 1: Book on a Table

Since the book is not moving, the net forces (sum of the forces) should equal 0 N. Since the forces are vectors, the forces must act in opposite directions in order for them to cancel out in addition to having the same magnitude. Like you can see in the above figure, the normal force N of the table acts upwards, which is opposite to the downward acting force of gravity Mg.

Mathematically, this would look like:

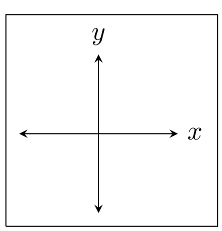
$$\sum F_{net} = N + Mg = 0\,\mathrm{N}$$

Generally speaking, **a net force of 0 N means all components of force are 0**:

$$\vec{\mathbf{F}}_{net} = (0, 0, 0)$$

2 Free-Body Diagrams

We can use free-body diagrams to model forces:



Free-body diagrams can be rotated to match the orientation of the object in question: where with respect to the object, the vertical force vectors form the y-axis and the horizontal force vectors form the x-axis.

3 Types of Forces

1. $\vec{\mathbf{N}}$: Normal force; the force that acts perpendicular to a surface.
2. $\vec{\mathbf{Mg}}$: Gravitational force; the force that acts downwards on an object, calculated by multiplying the mass of an object by acceleration due to gravity.

<u>Remark:</u> acceleration $\vec{\mathbf{a}}$ is not a force, but it is caused by forces.

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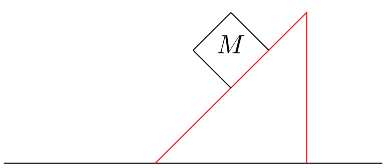
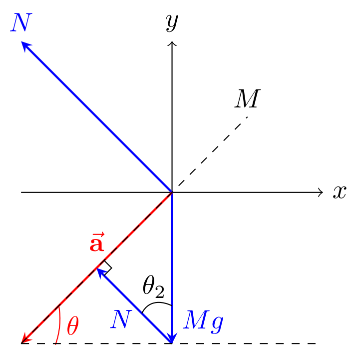
4 Example Problem

Determine the x and y components of the net force on the box:



We can draw a free-body diagram to model the forces acting on the box with mass M:



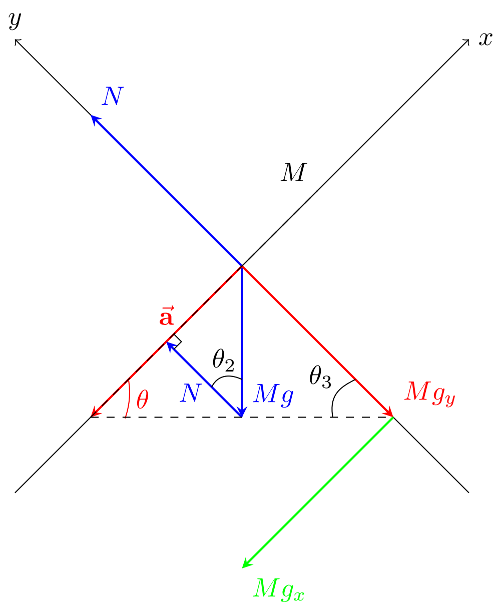
For the x component, we need to look at the angle θ_2 formed between the normal force vector N and gravity vector Mg. We can see that the vectors form a triangle, which allows us to use the trigonometric functions to setup an equation in which we can solve for x. Since the only force acting horizontally on the box is Mg, we can just solve for the height of the triangle (the side aligned with the incline) using sin.

$$\sin(\theta_2) = \frac{x}{Mg}$$
$$\Rightarrow \boxed{x = \sin(\theta_2)Mg}$$

For the y component, there are multiple forces acting in this direction. Thus, we have to determine what these forces are and their y components.

One of these forces is the normal force N. With respect to the box, it is only pushing up on the box. This means that N has a y-component, but no x-component (or an x-component of 0 N). As a result, the y-component of N will be itself (this might not remain true for a 3D space where z-components exist).

The next force is Mg, since in addition to horizontal force, it also has a vertical force acting on the box, in which Mg is bring the box to the ground. We can rotate our free-body diagram to match the orientation of the box in order to more easily break apart Mg into its x and y components:



Now, using the angle θ_3 formed between Mg_y and the ground, we can solve for Mg_y using Mg and sin:

$$\sin(\theta_3) = \frac{Mg}{Mg_y}$$

Since Mg_y is directed downwards, the value of Mg_y is negative, thus we need to multiply by -1.

$$\Rightarrow Mg_y = -\frac{Mg}{\sin(\theta_3)}$$
$$\boxed{Mg_y = -\csc(\theta_3)Mg}$$

Since there are no other forces acting on the box with respect to y, we can sum these two forces to get the y-component of the net force.

$$\begin{aligned}\sum F_{net\,y} &= N + Mg_y \\ &= N + (-\csc(\theta_3)Mg) \\ &= N - \csc(\theta_3)Mg\end{aligned}$$

Thus:

$$\sum F_{net\,y} = N - \sin(\theta_3)Mg$$

Finally, we have the x and y components of F_{net}:

$$\boxed{\sum F_{net} = (\sin(\theta_2)Mg,\, N - \csc(\theta_3)Mg)}$$