Constant Acceleration Textbook Problems

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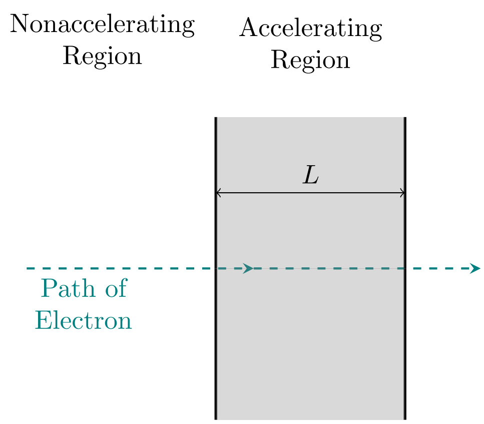
1 ★Problem 23

An electron with an initial velocity $v_0 = 1.50 \times 10^5$ m/s enters a region of length $L = 1.00$ cm where it is electrically accelerated. It emerges with $v = 5.70 \times 10^6$ m/s. What is its acceleration, assumed constant?



1.1 Solution

We have an initial velocity $v_0 = 1.50 \times 10^5$ m/s, final velocity $v = 5.70 \times 10^6$ m/s, and distance $L = 1.00$ cm. We could use the below formula:

$$v^2 = v_0^2 + 2a(x - x_0)$$

Since we already have v and v_0 as well as x and x_0, all we need to do is isolate acceleration:

$$v^2 = v_0^2 + 2a(x - x_0) \tag{1.1}$$

$$\Rightarrow v^2 - v_0^2 = 2a(x - x_0) \tag{1.2}$$

$$\Rightarrow \frac{1}{2}(v^2 - v_0^2) = a(x - x_0) \tag{1.3}$$

$$\Rightarrow \frac{\frac{1}{2}(v^2 - v_0^2)}{x - x_0} = a \tag{1.4}$$

$$a = \frac{(v^2 - v_0^2)}{2(x - x_0)} \tag{1.5}$$

Before we substitue in our values, we need to convert our distance from cm to m in order to keep our units consistent and thus our answer correct:

$$L = 1.00\,\mathrm{cm} \times \frac{1\,\mathrm{m}}{100\,\mathrm{cm}} = 0.01\,\mathrm{m}$$

Now to substitute in our values:

$$a = \frac{((5.7 \times 10^6)^2 - (1.5 \times 10^5)^2)}{2(0.01 - 0)} \tag{1.6}$$

$$a = \frac{((5.7^2 \times 10^{6\times2}) - (1.5^2 \times 10^{5\times2}))}{2(0.01)} \tag{1.7}$$

$$a = \frac{((32.5 \times 10^{12}) - (2.25 \times 10^{10}))}{0.02} \tag{1.8}$$

$$a = \frac{10^{10}(32.5 \times 10^2 - 2.25)}{0.02} \tag{1.9}$$

$$a = \frac{10^{10}(3250 - 2.25)}{0.02} \tag{1.10}$$

$$a = \frac{10^{10}(3247.75)}{0.02} \tag{1.11}$$

$$a = \frac{10^{10}(3.24775 \times 10^3)}{0.02} \tag{1.12}$$

$$a = \frac{(3.24775 \times 10^{13})}{0.02} \tag{1.13}$$

$$a = 162.3875 \times 10^{13} \tag{1.14}$$

$$a = 1.623875 \times 10^{15} \tag{1.15}$$

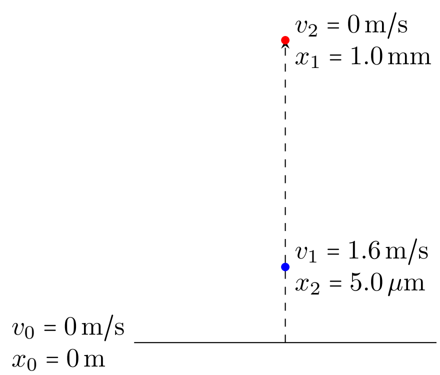
$$a = 1.624 \times 10^{15} \tag{1.16}$$

2 ★ Problem 24

Catapulting mushrooms. Certain mushrooms launch their spores by a catapult mechanism. As water condenses from the air onto a spore that is attached to the mushroom, a drop grows on one side of the spore and a film grows on the other side. The spore is bent over by the drop's weight, but when the film reaches the drop, the drop's water suddenly spreads into the film and the spore springs upward so rapidly that it is slung off into the air. Typically, the spore reaches a speed of 1.6 m/s in a 5.0 μm launch; its speed is then reduced to zero in 1.0 mm by the air. Using those data and assuming constant accelerations, find the acceleration in terms of g during (a) the launch and (b) the speed reduction.

2.1 Solution

We can draw the scenario:



We need to find the acceleration from the ground to the blue point and then the acceleration from the blue point to the red point, both in terms of gravity g. Since we have values for velocity and position, we can use this formula:

$$v^2 = v_0^2 + 2a(x - x_0) \Rightarrow a = \frac{v^2 - v_0^2}{2(x - x_0)}$$

For the first acceleration:

$$\begin{aligned}
v &= 1.6\,\mathrm{m/s} && (2.1.1)\\
v_0 &= 0\,\mathrm{m/s} && (2.1.2)\\
x &= 5.0\,\mu\mathrm{m} = 5.0\cdot 10^{-6}\,\mathrm{m} && (2.1.3)\\
x_0 &= 0\,\mu\mathrm{m} && (2.1.4)
\end{aligned}$$

$$\begin{aligned}
a &= \frac{(1.6)^2-(0^2)}{2(5.0\cdot 10^{-6}-0)} && (2.1.5)\\
&= \frac{1.6^2}{2(5.0\cdot 10^{-6})} && (2.1.6)\\
&= \frac{2.56}{10\cdot 10^{-6}} && (2.1.7)\\
&= \frac{2.56}{10^{-5}} && (2.1.8)\\
a &= 2.56\cdot 10^5\,\mathrm{m/s^2} && (2.1.9)
\end{aligned}$$

Now to write this in terms of gravity, we divide this value by $g = 9.8\,\mathrm{m/s^2}$.

$$\begin{aligned}
a &= 2.56\cdot 10^5\,\mathrm{m/s^2}\cdot\frac{g}{9.8\,\mathrm{m/s^2}} && (2.1.10)\\
a &= \frac{2.56\cdot 10^5}{9.8}g && (2.1.11)\\
a &= 2.61\cdot 10^4 g && (2.1.12)
\end{aligned}$$

For the second acceleration:

$$\begin{aligned}
v &= 0\,\mathrm{m/s} && (2.2.1)\\
v_0 &= 1.6\,\mathrm{m/s} && (2.2.2)\\
x &= 1.0\,\mathrm{mm} = 1.0\cdot 10^{-3}\,\mathrm{m} && (2.2.3)\\
x_0 &= 5.0\cdot 10^{-5}\,\mathrm{m} && (2.2.4)
\end{aligned}$$

$$\begin{aligned}
a &= \frac{(0)^2-(1.6^2)}{2(1.0\cdot 10^{-3}-5.0\cdot 10^{-5})} && (2.2.5)\\
&= \frac{-(1.6^2)}{2\cdot 10^{-3}(1.0-5.0\cdot 10^{-2})} && (2.2.6)\\
&= \frac{-2.56}{2\cdot 10^{-3}(0.95)} && (2.2.7)\\
&= \frac{-2.56}{0.0019} && (2.2.8)\\
a &= -1.347\cdot 10^3\,\mathrm{m/s^2} && (2.2.9)\\
\textbf{In terms of gravity: } a &= \frac{-1.347\cdot 10^3}{9.8}g && (2.2.10)\\
a &= -1.37\cdot 10^2 g && (2.2.11)
\end{aligned}$$