Report 2: Force Tables

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Phys 207 Lab CD4
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Contents

1 Introduction

Objective This lab is meant to develop a deeper understanding of what vectors are and what it means to add vectors. For example, vectors of the same magnitude that are opposite to each other will cancel out, however when modeled in the real world with pulleys and weights, this fact holds more significance. Another example would be when two vectors of equal magnitude, with an angle in-between them, point to the left while a vector pointing to the right, aligned with the midpoint between the two other vectors. With a specific magnitude for the third vector, equilibiurm will be achieved, but on paper, this does not hold as much meaning. When modeled in the real world however, the canceling out of forces due to equilibiurm and the representation of those forces using vectors is clear.

2 Procedure

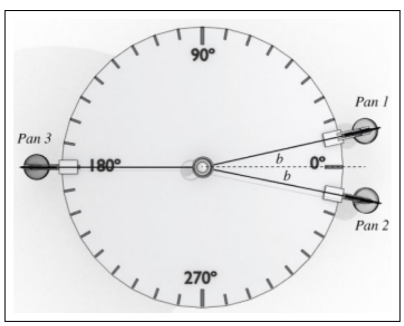
2.1 Experiment 1: Sensitivity of the Instrument

1. Arrange two pulley systems, with one pan at 0° and another pan at 180°.
2. Place 50 grams on both pans.
3. On one of the pans, add 1 gram and check for equilibiurm.
4. Find the maximum mass you can add to this pan before equilibiurm is lost.
5. Record this mass in grams.

Determine how much mass you should need to balance the forces and compare this value to the mass you used. Record the difference, in grams, between your experimental mass and the simulated mass.

2.2 Experiment 3: Find a Function

Set up the force table like the below diagram, where $b = 5°$ and the first two pans (1 and 2) have 50 grams placed onto them:



1. Experiment to find the mass that should be placed on pan 3 such that equilibiurm is achieved.
2. Repeat this experiment by increasing angle b by 5° until $b = 80°$ or equilibiurm cannot be achieved.
3. Record your results in the following table:

Angle	Mass of Pan 3 system
5°	P_3 mass
10°	P_3 mass
⋮	⋮
80°	P_3 mass

Determine a function to predict the mass of pan 3 with respect to angle b, where the summed masses of pan 1 and 2 the constant $C = 200$g. This equation should look like:

$$P_3(b) = C\cos(b)$$

We will then use Microsoft Excel to plot the recorded masses of pan 3 with respect to the cosine of the angle b and plot a linear trendline. The slope of the trendline will be our constant C.

2.3 Return to the Force Table

Use the provided number generator to produce 3 pairs of masses and directions. Each pair will be the mass and direction of a vector. Then, use vector algebra to calculate a fourth vector $\vec{\mathbf{D}}$ such that:

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{D}} = 0$$

Once vector $\vec{\mathbf{D}}$ is calculated, record the values for $|D_x|$ and $|D_y|$. Convert $\vec{\mathbf{D}}$ back to a mass and angle using trigonometric functions. Set up the force table using the masses and directions for all four vectors to see if the calculated prediction is correct, in which equilibiurm occurs.

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3 Data and Calculations

3.1 Data

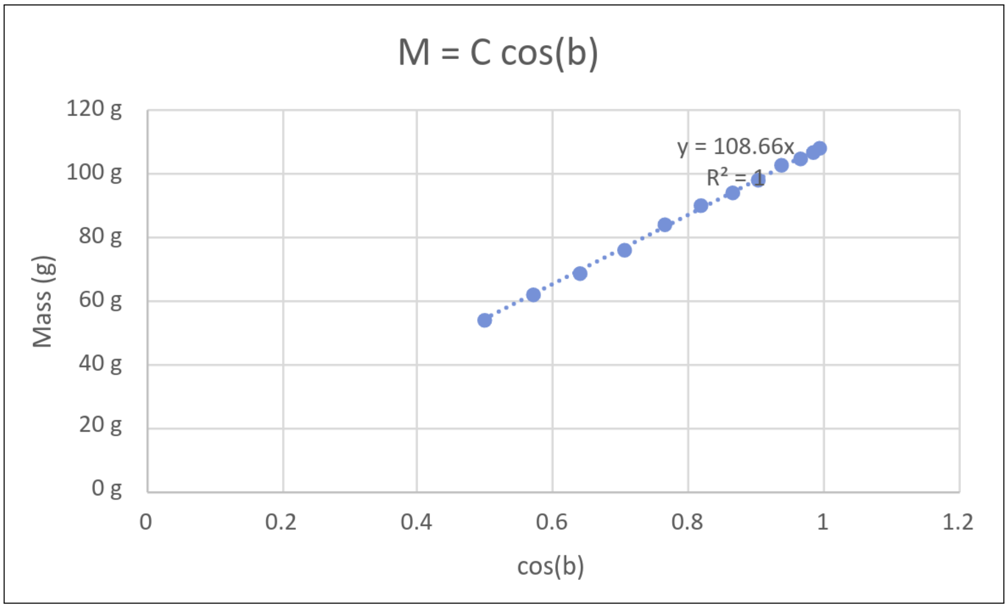
Table 1: Mass of Pan 3 and Angle b

Mass (grams)	b (°)
108 g	5°
107 g	10°
105 g	15°
103 g	20°
98 g	25°
94 g	30°
90 g	35°
84 g	40°
76 g	45°
69 g	50°
62 g	55°
54 g	60°

Table 2: Mass of Pan 3 and cos(b)

Mass (grams)	cos(b)
108 g	0.9961947
107 g	0.98480775
105 g	0.96592583
103 g	0.93969262
98 g	0.90630779
94 g	0.8660254
90 g	0.81915204
84 g	0.76604444
76 g	0.70710678
69 g	0.64278761
62 g	0.57357644
54 g	0.5

Figure 1: Mass of Pan 3 as a Function of $cos(b)$



3.2 Calculations

3.2.1 Calculating $\vec{\mathbf{D}}$

Since we have the equation:

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{D}} = 0$$

this implies that:

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = -\vec{\mathbf{D}}$$

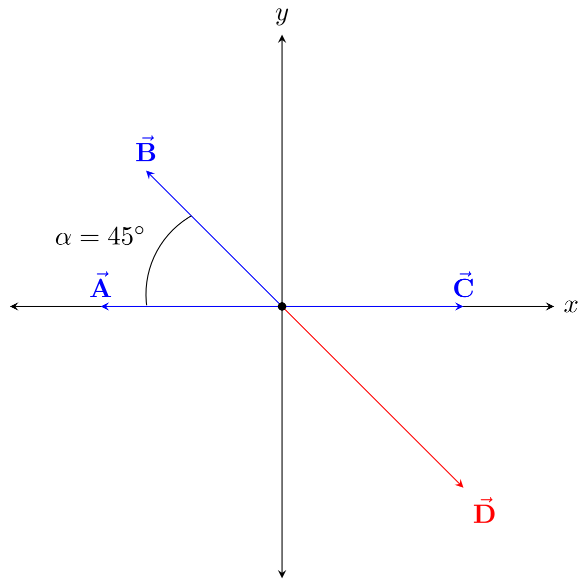
thus:

$$D_x = -(A_x + B_x + C_x) \qquad \text{and} \qquad D_y = -(A_y + B_y + C_y)$$

Since we are looking for the magnitudes of D_x and D_y, this equation becomes:

$$|D_x| = |A_x + B_x + C_x| \qquad \text{and} \qquad |D_y| = |A_y + B_y + C_y|$$

We can draw this as a force diagram, where the origin is the center of the force table:



We can see that $\vec{\mathbf{A}}$ and $\vec{\mathbf{C}}$ do not have y-components, but all three vectors have x-components. We can calculate the x-component of $\vec{\mathbf{D}}$ by taking the magnitudes of $\vec{\mathbf{A}}$, $\vec{\mathbf{C}}$, and B_x. The magnitude of B_x will equal:

$$B_x = -\left|\vec{\mathbf{B}}\right| \cos(\alpha) = -70g\cos(45°) \quad \vec{\mathbf{B}} \text{ exerts a negative force along } x$$

where g is acceleration by gravity (9.8 $\mathrm{m/s^2}$) and 70 is the mass of the object applying force $\vec{\mathbf{B}}$ to the center of the force table. The magnitudes of $\left|\vec{\mathbf{A}}\right|$ and $\left|\vec{\mathbf{C}}\right|$ are:

$$\left|\vec{\mathbf{A}}\right| = -M_A g = -70g \quad \vec{\mathbf{A}} \text{ exerts a negative force along } x.$$
$$\left|\vec{\mathbf{C}}\right| = M_C g = 80g$$

Thus:

$$|D_x| = |-70g + 80g - 70g\cos(45)| \approx 0.387\,\mathrm{N}$$

Since $\vec{\mathbf{A}}$ and $\vec{\mathbf{C}}$ do not have y-components, the only value needed to calculate $|D_y|$ is B_y. B_y will equal:

$$B_y = \left|\vec{\mathbf{B}}\right| \sin(\alpha) = 70g\sin(45)$$

thus:

$$|D_y| = |70g\sin(45)| \approx 0.485\,\mathrm{N}$$

We can calculate $\left|\vec{\mathbf{D}}\right|$ to be:

$$\left|\vec{\mathbf{D}}\right| = \sqrt{D_x^2 + D_y^2} \approx 0.385\,\mathrm{N}$$

since $\mathrm{N} = \mathrm{kg}\cdot\mathrm{m/s^2}$, we can multiply by 1000 and divide by gravity in order to get the analytical mass required to achieve equilibiurm:

$$0.385\,\cancelto{\mathrm{g}}{\mathrm{kg}} \cdot \cancel{\mathrm{m/s^2}} \cdot 1000 \cdot \frac{1}{9.8\,\mathrm{m/s^2}} \approx 39.29\,\mathrm{g}$$

4 Questions

(1) What factors could contribute to this sensitivity?

Regarding the sensitivity of the force table, the friction between the rope and the pulley wheel could contribute to this sensitivity. More friction would require more weight to lose equilibiurm, thus making the force table less sensitive. Less friction would have the opposite effect, making the force table more sensitive.

(2) Report the difference between what you've experimentally measured and what the simulation predicted. Are they within the expected sensitivity of the instrument?

The difference between my experimental mass and the simulated mass was 5.46 ± 0.5g since the experimental mass of the resultant was 79.9 ± 0.5g and the simulated mass was 85.6 ± 0.5g. This is within the expected sensitivity of the instrument since we experimentally determined the sensitivity to be 5.0 ± 0.5g.

(3) On one graph plot the experimental data from your table along with the analytical prediction of the function you found. Do they follow the same trend? Then, find the slope of your linear data and compare it to what the slope should be from your analytical equation. Does it differ from the analytically derived slope by less than the uncertainty?

The experimental data and the analytical prediction follow the same trend, in which both are linear functions. The slope of the linear data was 108.66 grams, and the analytical equation had a slope of 200 grams. However, considering that both pans had 50 grams placed onto them, those grams could cancel out, and the slope could be 100. Since the sensitivity of our force table of two and our slope should theoretically equal the sum of the masses of the two pans:

$$P_3(b) = C\cos(b) \quad \text{where} \quad C = P_1 + P_2$$

the uncertainty in this case will be the sensitivity of our force table multiplied by two to account for sensitivity of the masses two pans we are using:

$$\delta P = 5.0 \pm 0.5\,\text{g} \cdot 2 = 10. \pm 1.0\,\text{g}$$

Thus, experimentally derived slope could differ from the analytically derived slope by less than the uncertainty if we consider the analytical slope to be 100 grams. However, since the analytical slope was originally 200 grams, then the difference between the experimentally derived slope and analytical slope is greater than our uncertainty.

(4) Give the details of this calculation and compare your analytical results with the the experimental results. Draw a vector diagram that shows the table arrangement.

Analytically, we found that a mass of 39.29 grams for the fourth pan would be required to reach equilibiurm. Experimentally, we found that adding 35 grams to the fourth pan would reach equilibiurm. With a difference of 4.29 grams, the two values differ by less than the uncertainty of the force table, which was found to be 5.0 ± 0.5g.

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5 Conclusion

Error The room for error in this lab could be due the force table being slightly off from the provided setup. For example, when calculating D_x and D_y in section 3.2.1, our provided direction for vector $\vec{\mathbf{B}}$ was 55°, but when setting up the pulley system at that angle, we we might have set up a direction of 54° of 56°. The same could apply to weight, where not placing the weights exactly on top of each other, or centered perfectly, could alter the center of mass for the pans, thus the vectors we set up would have different magnitudes than the analytical vectors. We also have to take into the account the sensitivity of the force table, which could present significant gaps between the experimental values and analytical values. With this potential error occuring in four vectors, our experimental observation would not match exactly with the analytical value.

There is also possibility of not accounting for the inherent masses of the pans, which are 50 grams. With a significant mass, not accounting for it by mistake or misunderstanding would produce major errors in experimental and analytical results.

Findings A greater angle between two vectors decreases the magnitude of the resultant. This helps to explain why when the angle between vectors is 180°, in which the vectors are opposite to each other, there is no resultant, or that the magnitude of the resultant is 0 N.

Revisions Perhaps using pans with insignficant masses, such as masses around 1 gram, would make the experiment more clear and cut down on potential errors. Although the masses should theoretically cancel out in a state of equilibiurm, mistakenly not accounting for them would produce large errors, such as when experimentally deriving the slope for experiment 3.