Introduction to Physics

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1 What is Physics?

Physics is the study of how the objects around us change over time. This includes their **position**, **displacement**, **distance**, and **speed**.

1.1 Position

The **position** of an object is its relative distance from a reference point. For example, if we had a red dot and a green dot on a line, then we could determine the position of the red dot to be its distance from the green dot and the position of the green dot to be its distance from the red dot. We can express position as a function x(t).

1.2 Displacement

Displacement is the difference between an objects new position and its previous position. If we had a red dot move from point 1 to point 2, then the displacement of the red dot would be the difference in position of point 2 and point 1. Mathematically, we can represent this as Δx.

$$\Delta x = x_2 - x_1$$

Since position is a function, x_2 and x_1 can be written as $x(t_2)$ and $x(t_1)$, where t_2 is the time at which the object was at position 2 and $t - 1$ is the time at which the object was at position 1. Thus, we can write displacement as:

$$\Delta x = \mathrm{x}(\mathrm{t}_2) - \mathrm{x}(\mathrm{t}_1)$$

1.3 Distance

Distance is the length of the path traveled by an object from one point to another.

1.4 Speed

Speed is how fast an object moved from one point to another. As an extension, **velocity** is just speed, but in specific direction. In other words, speed is a *scalar*, velocity is a *vector*, but both represent how fast an object moves. The rate at which velocity change over time is known as

acceleration. We can represent velocity as a function v(t). We could try to define this function as the ratio between displacement and time elapsed between two points:

$$\overline{\mathrm{v(t)}} = \frac{\Delta x}{\Delta t}$$

The issue with this equation is that velocity can change at any given time, thus in order to get closer to a true representation of velocity, we need to look at instantaneous intervals of velocity. In other words: we differentiate position.

$$\mathrm{v(t)} = \frac{dx}{dt}$$

By differentiating position, we will get the rate at which position changes at a specific time t, which is the velocity at that specific time.

1.5 Acceleration

Acceleration is the rate at which the velocity of an object changes, however with a few exceptions, we will assume acceleration to be constant for the majority of this course. Thus, we will represent acceleration as a variable a, and if acceleration is not constant, a function a(t).

Since acceleration is the rate at velocity changes, we can differentiate velocity to get acceleration:

$$\mathrm{a} = \frac{dv}{dt}$$

This also means that if we want to get velocity, but only have acceleration, we can integrate acceleration to get velocity:

$$\begin{aligned}
\mathrm{v(t)}\Big|_{\mathrm{t_1}}^{\mathrm{t_2}} &= \int_{t_2}^{t_1} a\,dt \\
\mathrm{v(t)}\Big|_{\mathrm{t_1}}^{\mathrm{t_2}} &= a \cdot t\Big|_{t_1}^{t_2} \\
\mathrm{v(t)}\Big|_{\mathrm{t_1}}^{\mathrm{t_2}} &= a \cdot t_2 - a \cdot t_1 = a \cdot (t_2 - t_1) \\
\mathrm{v(t_2)} - \mathrm{v(t_1)} &= a \cdot (t_2 - t_1)
\end{aligned}$$

2 Time

Objects always move with respect to time, as the definition of physics would imply. Thus, we treat time as an independant variable; while position, displacement, distance, speed, velocity, and acceleration are all functions of time.

3 Problems

3.1 The Rabbit

A rabbit is moving along a straigh line. At time $t_1 = 0$ the raddit is distance $x_1 = 1\mathrm{m}$ from your right and is moving towards the right with a velocity of $v_1 = 1\frac{\mathrm{m}}{\mathrm{s}}$. The rabbit has a constant acceleration of $a = 1\frac{\mathrm{m}}{\mathrm{s}^2}$ towards the right. What is the position of the rabbit as time $t_2 = 10s$?

3.1.1 Solution

First we list our known variables followed by our unknowns:

<u>Knowns</u>

$x_1 = 1\text{m}$

$t_1 = 0\text{s}$

$v_1 = 1\dfrac{\text{m}}{\text{s}}$

$t_2 = 10\text{s}$

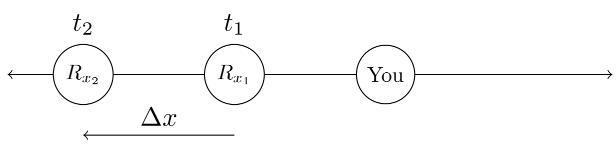
$a = 1\dfrac{\text{m}}{\text{s}^2}$

<u>Unknowns</u>

$x_2 = ?$

$v_2 = ?$

Then we illustrate the scenario: While we do know velocity at time $t = 0\text{s}$, we do not know velocity



at time $t = 10\text{s}$. We don't need to know the value of velocity at this point, it will give us the grounds to define an expression for v(t), which we can differentiate to get a function for position, allowing us to calculate the final position of the rabbit.

Since we know acceleration $a = 1\frac{\text{m}}{\text{s}^2}$, including the fact that it is constant, we can differentiate acceleration to find a function for velocity:

$$a = \frac{dv}{dt} \tag{1.1}$$

$$\int_{t_1}^{t_2} a\,dt = \int_{t_1}^{t_2} \frac{dv}{dt} \tag{1.2}$$

$$a \cdot t\Big|_{t_1}^{t_2} = \mathrm{v}(t)\Big|_{t_1}^{t_2} \tag{1.3}$$

$$\Rightarrow \mathrm{v}(t_2) - \mathrm{v}(t_1) = a \cdot (t_2 - t_1) \tag{1.4}$$

$$\Rightarrow \mathrm{v}(t_2) = a \cdot (t_2 - t_1) + \mathrm{v}(t_1) \tag{1.5}$$

Now that we have a function for velocity, we can utilize the fact that velocity is the derivative of position, allowing us to set our function equal to dx:

$$\frac{dx}{dt} = \mathrm{v}(t_2) = a \cdot (t_2 - t_1) + \mathrm{v}(t_1)$$

We can now omit v(t_2) since we aren't looking for it, giving us a clean equation that we can integrate to get the value of x_2. Naturally, we want to integrate with the upper limit t_2 and the lower limit t_1:

$$\int_{t_1}^{t_2} \frac{dx}{dt}\,dt = \int_{t_1}^{t_2} [a \cdot (t_2 - t_1) + v_1]\,dt \tag{1.6}$$

$$\int_{t_1}^{t_2} \frac{dx}{dt}\,dt = \int_{t_1}^{t_2} a \cdot (t_2 - t_1)\,dt + \int_{t_1}^{t_2} v_1\,dt \tag{1.7}$$

$$\mathrm{x}(t)\Big|_{t_1}^{t_2} = \int_{t_1}^{t_2} (a \cdot \Delta t)\,dt + (v_1 \cdot t)\Big|_{t_1}^{t_2} \tag{1.8}$$

$$\mathrm{x}(t)\Big|_{t_1}^{t_2} = a\frac{\Delta t^2}{2}\Big|_{t_1}^{t_2} + (v_1 \cdot t)\Big|_{t_1}^{t_2} \tag{1.9}$$

Note: $\Delta t = t - t_1$ where t is a parameter.

$$\mathrm{x}(t_2) - \mathrm{x}(t_1) = a(\frac{\Delta t^2(t_2)}{2} - \frac{\Delta t^2(t_1)}{2}) + v_1 \cdot (t_2 - t_1) \tag{1.10}$$

$$\mathrm{x}(t_2) - \mathrm{x}(t_1) = a(\frac{(t_2 - t_1)^2}{2} - \frac{(t_1 - t_1)^2}{2}) + v_1 \cdot (t_2 - t_1) \tag{1.11}$$

$$\mathrm{x}(t_2) = a(\frac{(t_2 - t_1)^2}{2} - \frac{(t_1 - t_1)^2}{2}) + v_1 \cdot (t_2 - t_1) + \mathrm{x}(t_1) \tag{1.12}$$

$$x_2 = a(\frac{(t_2 - t_1)^2}{2} - \frac{(t_1 - t_1)^2}{2}) + v_1 \cdot (t_2 - t_1) + x_1 \tag{1.13}$$

$$\tag{1.14}$$

We now have an equation for x_2. Now all we need to do now is plug in our knowns.

$$x_2 = (1)(\frac{(10 - 0)^2}{2} - \frac{(0 - 0)^2}{2}) + (1) \cdot (10 - 0) + (1) \tag{1.15}$$

$$x_2 = 1(\frac{(10)^2}{2} - \frac{(0)^2}{2}) + 1 \cdot 10 + 1 \tag{1.16}$$

$$x_2 = 1(\frac{100}{2} - \frac{0}{2}) + 10 + 1 \tag{1.17}$$

$$x_2 = 1(50 - 0) + 11 \tag{1.18}$$

$$x_2 = 50 + 11 \tag{1.19}$$

$$x_2 = 61 \tag{1.20}$$

$$\tag{1.21}$$

Our answer: 61m. The final position of the rabbit (the position of the rabit at time $t_2 = 10$s) is 61 meters from our right.

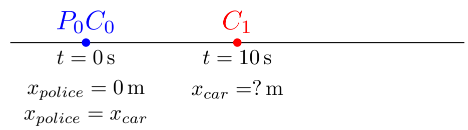
3.2 The Police Car and the Speeding Car

A car is moving with a constant speed v. It passes a police car on the side of the road. The officer realized that the speed of the car was over the limit and starts to chase it at time $t = 10\,\mathrm{s}$ after the car passed by. The police car moves with a constant acceleration of $a = 10\,\mathrm{m/s^2}$ and reaches the car at $t = 30\,\mathrm{s}$. What is the velocity of the speeding car?

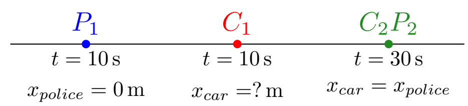
3.2.1 Solution

Lets draw the scenario:

Before the chase:

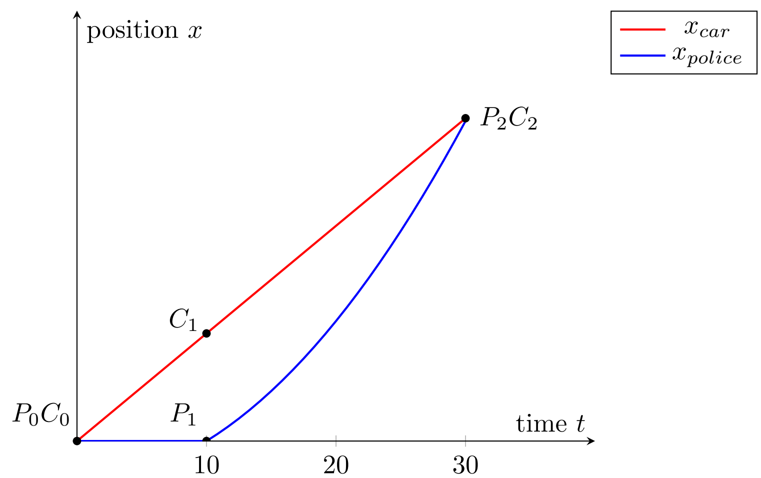


After the chase:



We can even take a closer look by roughly graphing the positions of the car with respect to time:

Rough Model of $\mathrm{x}_{car}(t)$ & $\mathrm{x}_{police}(t)$



From the graph, we can see that the speeding car moves for the entirety of the chase whereas the police car only begins to move 10 seconds into the chase. We can also see that both cars are at the same position at time $t = 0$ and $t = 30$, and so this will be our reference position. As a result, we will set $t_2 = 30$ for both cars, but $t_1 = 0$ for the speeding car and $t_1 = 10$ for the police car.

Since the positions of both cars are equal at t_2, we can say that:

$$\mathrm{x}_{car}(t_2) = \mathrm{x}_{police}(t_2)$$

Now, while we know that $t_2 = 30$, we still need a expression for each function.

The Police Car In order define $\mathrm{x}_{police}(t)$, we need to integrate the velocity of the police car, however we do not have a velocity. Thus, we have to integrate acceleration to get velocity, and we are given a constant acceleration of $a = 10$:

$$\mathrm{v}_{car}(t) = \int_{t_1}^{t} a\,\mathrm{d}t \tag{2.1}$$

$$\mathrm{v}_{car}(t) = \int_{t_1}^{t} 10\,\mathrm{d}t \tag{2.2}$$

$$\mathrm{v}_{car}(t) = [10(t) + C] - [10(t_1) + C] \tag{2.3}$$

$$\mathrm{v}_{car}(t) = 10(t) - 10(10) \tag{2.4}$$

$$\mathrm{v}_{car}(t) = 10t - 100 \tag{2.5}$$

$$\tag{2.6}$$

Now to derive the function for position:

$$\mathrm{x}_{car}(t) = \int_{t_1}^{t} \mathrm{v}_{car}(t)\,\mathrm{d}t \tag{2.7}$$

$$\mathrm{x}_{car}(t) = \int_{t_1}^{t} 10t - 100\,\mathrm{d}t \tag{2.8}$$

$$\mathrm{x}_{car}(t) = [10\frac{(t)^2}{2} - 100t + C] - [10\frac{(t_1)^2}{2} - 100t + C] \tag{2.9}$$

$$\mathrm{x}_{car}(t) = [10\frac{(t)^2}{2} - 100t] - [10\frac{(10)^2}{2} - 100(10)] \tag{2.10}$$

$$\mathrm{x}_{car}(t) = 5t^2 - 100t - (-500) \tag{2.11}$$

$$\mathrm{x}_{car}(t) = 5t^2 - 100t + 500 \tag{2.12}$$

$$\tag{2.13}$$

The Speeding Car We will have to integrate velocity to derive a function for the speeding car's position, however while we do not know the value of its velocity, we can just take the integral of a constant:

$$\mathrm{x}_{car}(t) = \int_{t_1}^{t} v_{car} \tag{2.14}$$

$$\mathrm{x}_{car}(t) = t \cdot v_{car} - t_1 \cdot v_{car} \tag{2.15}$$

$$\mathrm{x}_{car}(t) = t \cdot v_{car} - (0) \cdot v_{car} \tag{2.16}$$

$$\mathrm{x}_{car}(t) = t \cdot v_{car} \tag{2.17}$$

Now to set the two functions equal to each other at $t = t_2 = 30$:

$$\mathrm{x}_{car}(30) = \mathrm{x}_{police}(30) \tag{2.18}$$

$$30 \cdot v_{car} = 5(30)^2 - 100(30) + 500 \tag{2.19}$$

$$30 \cdot v_{car} = 2000 \tag{2.20}$$

$$\Rightarrow v_{car} = \frac{2000}{30} \tag{2.21}$$

$$\boxed{v_{car} = \frac{200}{3}\ \mathrm{m/s}} \tag{2.22}$$

Thus, the velocity of the speeding car was $\frac{200}{3}$ m/s.