2-5: Free-Fall Acceleration

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1 Gravity

Acceleration by gravity is represented by g. The magnitude of acceleration by gravity is:

$$g = 9.8 \, \text{m/s}^2$$

If an object is falling its Acceleration would be equal to $-g$:

$$a = -g = -9.8 \, \text{m/s}^2$$

Note: Do not substitue $-9.81 \, \text{m/s}^2$ for g.

Example   Suppose we tossed a tomato up to the air. The velocity start off positive, but since acceleration is negative, the magnitude of velocity (speed) will decrease until it reaches 0, at which the tomato reached its **maximum height**. Now, the velocity of the tomato becomes increasingly negative as the tomato falls back to the ground.
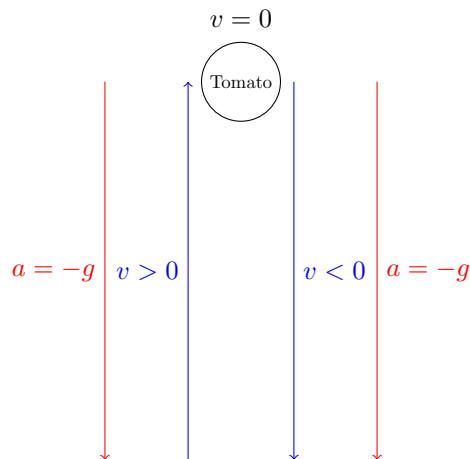
1.1 Problem

In figure 2-13, a pitcher tosses a baseball up along a y axis, with an initial speed of 12 m/s.

(a) How long does the ball take to reach its maximum height?

(b) What is the ball's maximum height above its release point?

(c) How long does the ball take to reach a point 5.0 m above its release point?

1.1.1 Part (a)

To find out how long the ball takes to reach its maximum height, we need to determine the time it takes for $v_1 = 12$ m/s to reach $v_2 = 0$ m/s. Since we know acceleration a is equal to $-g = 9.8$ m/s, we can integrate to get a function for velocity:

$$\int_{t_1}^{t_2} \frac{dv}{dt} = \int_{t_1}^{t_2} a \, dt = \int_{t_1}^{t_2} -9.8 \, dt \tag{1.1}$$

$$v(t)\Big|_{t_1}^{t_2} = a \cdot t \Big|_{t_1}^{t_2} = -9.8 \cdot t \Big|_{t_1}^{t_2} \tag{1.2}$$

$$v(t)\Big|_{t_1}^{t_2} = -9.8 \cdot t \Big|_{t_1}^{t_2} \tag{1.3}$$

We can set $t_1 = 0$s and $t_2 = t$ where t is how long the ball takes to reach its maximum height. Since the ball was launched with an initial speed of 12 m/s, the time at this moment would be 0s, which is equal to t_1. Thus, we can say that $v(t_1) = 12$m/s, leaving us with the only unknown being $v(t_2)$, or simply $v(t)$.

$$v(t_2) - v(t_1) = -9.8 \cdot t_2 - -9.8 \cdot t_1 \tag{1.4}$$

$$v(t_2) - v(0) = -9.8 \cdot t_2 - -9.8 \cdot 0 \tag{1.5}$$

$$v(t_2) - 12 = -9.8 \cdot t_2 \tag{1.6}$$

$$v(t) - 12 = -9.8 \cdot t \tag{1.7}$$

$$\Rightarrow v(t) = -9.8 \cdot t + 12 \tag{1.8}$$

Now, since we want to find time t when velocity $v(t) = 0$, we can set $v(t) = 0$ and solve for t.

$$0 = -9.8 \cdot t + 12 \tag{1.9}$$

$$\Rightarrow -12 = -9.8 \cdot t \tag{1.10}$$

$$\Rightarrow \frac{-12}{-9.8} = t \tag{1.11}$$

$$t = \frac{12}{9.8} \, \text{s} = 1.2 \, \text{s} \tag{1.12}$$

Alternatively, we could utilize the fact that we are looking at two endpoints for velocity, thus we can use the equation for average acceleration:

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \tag{2.1}$$

$$-9.8 = \frac{0 - 12}{t_2 - 0} \tag{2.2}$$

$$-9.8 = \frac{-12}{t_2} \tag{2.3}$$

$$\Rightarrow -9.8 \cdot t_2 = -12 \tag{2.4}$$

$$\Rightarrow t_2 = \frac{-12}{-9.8} \tag{2.5}$$

$$t_2 = \frac{12}{9.8} = 1.2 \tag{2.6}$$

$$t = 1.2\text{s} \tag{2.7}$$

This approach is more optimal in this case, however we wouldn't be able to calculate the velocity at any other point in time t.

1.1.2 Part (b)

Now we want to find the maximum height of the ball. Given that we have an equation for velocity, we can integrate $v(t)$ to get displacement $x(t)$.

$$\text{x}(t)\Big|_{t_1}^{t_2} = \int_{t_1}^{t_2} \text{v}(t)\, dt \tag{3.1}$$

$$\text{x}(t)\Big|_{t_1}^{t_2} = \int_{t_1}^{t_2} -9.8 \cdot t + 12\, dt \tag{3.2}$$

$$\text{x}(t)\Big|_{t_1}^{t_2} = \int_{t_1}^{t_2} -9.8 \cdot t + \int_{t_1}^{t_2} 12\, dt \tag{3.3}$$

$$\text{x}(t)\Big|_{t_1}^{t_2} = -9.8 \cdot \frac{t^2}{2}\Big|_{t_1}^{t_2} + 12t\Big|_{t_1}^{t_2} \tag{3.4}$$

$$\text{x}(t_2) - \text{x}(t_1) = [-9.8 \cdot \frac{(t_2)^2}{2} - -9.8 \cdot \frac{(t_1)^2}{2}] + (12t_2 - 12t_1) \tag{3.5}$$

$$\text{x}(t) - \text{x}(0) = [-9.8 \cdot \frac{t^2}{2} - -9.8 \cdot \frac{(0)^2}{2}] + (12t - 12(0)) \tag{3.6}$$

$$\text{x}(t) - \text{x}(0) = -9.8 \cdot \frac{t^2}{2} + 12t \tag{3.7}$$

$$\tag{3.8}$$

Since the ball is tossed at $t_1 = 0$, we can say that $x(t_1)$ which equals $x(0)$ is equals 0. We also know that, from *part (a)*, the ball reaches its maximum height at time $t = 1.2\,\text{s}$, thus all we need to do is

substitue.

$$x(1.2) - (0) = -9.8 \cdot \frac{(1.2)^2}{2} + 12(1.2) \tag{3.9}$$

$$x(1.2) = -9.8 \cdot 0.72 + 14.4 \tag{3.10}$$

$$x(1.2) = -9.8 \cdot 0.72 + 14.4 \tag{3.11}$$

$$x(1.2) = 7.3 \, \text{m} \tag{3.12}$$

1.1.3 Part (c)

Now we want to find the time it takes the ball to reach $5.0\,\text{m}$ above its release point. Since we have an equation for displacement, we can simply solve for t by setting $x(t) = 5$.

$$x(t) = -9.8 \cdot \frac{t^2}{2} + 12t \tag{4.1}$$

$$x(t) = 5 \tag{4.2}$$

$$5 = -9.8 \cdot \frac{t^2}{2} + 12t \tag{4.3}$$

$$5 = -4.9 \cdot t^2 + 12t \tag{4.4}$$

$$\Rightarrow 0 = -4.9t^2 + 12t - 5 \tag{4.5}$$

Then we use the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = -4.9 \tag{4.6}$$

$$b = 12 \tag{4.7}$$

$$c = -5 \tag{4.8}$$

$$t = \frac{-(12) \pm \sqrt{12^2 - 4(-4.9)(-5)}}{2(-4.9)} \tag{4.9}$$

$$t = \frac{-(12) \pm \sqrt{46}}{2(-4.9)} \tag{4.10}$$

$$t = \frac{-(12) \pm 6.78}{-9.8} \tag{4.11}$$

$$t = \{\frac{-12 + 6.78}{-9.8}, \frac{-12 - 6.78}{-9.8}\} \tag{4.12}$$

$$t = \{0.53\,\text{s}, 1.9\,\text{s}\} \tag{4.13}$$

It turns out that the ball reaches $5.0\,\text{m}$ at two times, $t_1 = 0.53\,\text{s}$ and $t_2 = 1.9\,\text{s}$. This makes sense since the ball will reach $5.0\,\text{m}$ when traveling upwards to its maximum height of $7.3\,\text{m}$ and then once again as it travels downards to the ground.

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