概率统计——习题五参考解答

5.1 由于 $\boldsymbol{p = P\{X > 10\} = \int_{10}^{\infty} \frac{1}{5} e^{-\frac{1}{5}x} dx = -e^{-\frac{1}{5}x} \Big|_{10}^{+\infty} = e^{-2}}$，则 $\boldsymbol{Y \sim B(5, e^{-2})}$

$$\boldsymbol{P\{Y \geq 1\} = 1 - P\{Y = 0\} = 1 - C_5^0 (e^{-2})^0 (1 - e^{-2})^5 = 1 - (1 - e^{-2})^5}$$

5.2 $P\{X < a\} = \int_0^a 4x^3 dx = a^4$，$P\{X > a\} = \int_a^1 4x^3 dx = 1 - a^4$，所以 $a^4 = 1 - a^4$，故 $a = 1/\sqrt[4]{2}$。

5.3 设三角形 ΔABC 中 AB 边上的高为 h，边 AB 的长度为 a，于是当 $x < 0$ 时，$F_X(x) = P\{X \leq x\} = 0$；当 $x \geq h$ 时，$F_X(x) = P\{X \leq x\} = 1$；

当 $0 \leq x < h$ 时，$F_X(x) = P\{X \leq x\} = \dfrac{2hx - x^2}{h^2}$。

综上有 $$F_X(x) = P\{X \leq x\} = \begin{cases} 0, & x < 0 \\ 1 - \left(\dfrac{h - x}{h}\right)^2 = \dfrac{2hx - x^2}{h^2}, & 0 \leq x < h \\ 1, & x \geq h \end{cases}$$ 。

5.4 1） $$F(x) = \begin{cases} 0 & , \quad x < -1 \\ \dfrac{5}{16}x + \dfrac{7}{16} & , \quad -1 \leq x < 1 \\ 1 & , \quad x \geq 1 \end{cases}$$

2） $\boldsymbol{P\{-1 \leq X < 0\} = P\{X = -1\} + P\{-1 < X < 0\} = F(-1) + F(0) - F(-1) = 7/16}$

5.5 X~U[7:00, 7:30]

（1）P{等车不到 5 分钟}=P{7:10<X<7:15}+P{7:25<X<7:30}=$\frac{1}{3}$;

（2）P{等车超过 10 分钟}=P{7:00<X<7:05}+P{7:15<X<7:20}=$\frac{1}{3}$

5.6 $\because X \sim N(108, 9)$，$\therefore \dfrac{X - 108}{3} \sim N(0, 1)$

(1)

$P\{101.1<X<117.6\} = \boldsymbol{P\{\dfrac{101.1 - 108}{3} < \dfrac{X - 108}{3} < \dfrac{117.6 - 108}{3}\}}$

$$\boldsymbol{= \Phi(\frac{117.6 - 108}{3}) - \Phi(\frac{101.1 - 108}{3})}$$

$= \Phi(3.2) - \Phi(-2.3) = \Phi(3.2) + \Phi(2.3) - 1 = 0.9886$;

(2) $P\{X < a\} = \Phi(\frac{a-108}{3}) = 0.90 = \Phi(1.282)$，$\therefore \frac{a-108}{3} = 1.282$，$a = 111.846$；

(3) $\because P\{|X-a| > a\} = 0.01$，$\therefore 0.99 = P\{|X-a| \le a\} = P\{0 \le X \le 2a\}$

$= \Phi(\frac{2a-108}{3}) - \Phi(\frac{0-108}{3}) \approx \Phi(\frac{2a-108}{3})$，又 $\because 0.99 = \Phi(2.326)$ ，$\therefore \frac{2a-108}{3} = 2.326$ ，

$\therefore a = 57.489$(或者57.50)。

5.7 （1）若有 70 分钟可用，则由于

$$P_1 = P\{0 < X \le 70\} = \Phi(\frac{70-50}{10}) - \Phi(-\frac{50}{10}) \approx \Phi(2) = 0.9772,$$

$$P_2 = P\{0 < X \le 70\} = \Phi(\frac{70-60}{4}) - \Phi(-\frac{60}{4}) \approx \Phi(2.5) = 0.9938 > P_1,$$

故可走第二条路；

（2）若有 65 分钟可用，则由于

$$P_1 = P\{0 < X \le 65\} = \Phi(\frac{65-50}{10}) - \Phi(-\frac{50}{10}) \approx \Phi(1.5) = 0.9332,$$

$$P_2 = P\{0 < X \le 65\} = \Phi(\frac{65-60}{4}) - \Phi(-\frac{60}{4}) \approx \Phi(1.25) = 0.8944 < P_1,$$

故可走第一条路。

5.8 设原件损坏为事件 A，设 X 表示电源电压，$\therefore X \sim N(220, 25^2)$。所以

$\boldsymbol{B_1 = \{X \le 200\}, B_2 = \{200 < X \le 240\}, B_3 = \{X > 240\}}$构成样本空间的一个划分。

（1）由全概率公式有：

$$\begin{aligned}\boldsymbol{\alpha} &= \boldsymbol{P(A) = \sum_{i=1}^{3} P(B_i)P(A \mid B_i)} \\ &= \boldsymbol{P\{X \le 200\} \times 0.1 + P\{200 < X \le 240\} \times 0.001 + P\{X > 240\} \times 0.2 = 0.06415}\end{aligned};$$

（2）由 Bayes 公式有：$\beta = \frac{P\{200 < X \le 240\} \times 0.001}{\alpha} = 0.009$ 。