概率统计——习题十一参考答案

11.1 (1) $\overline{X}, \frac{1}{n}\sum_{i=1}^{n}(X_i-\overline{X})^2$；（2）$1/\overline{X}$.

11.2 由 $E(X)=\overline{X}$，且 $E(X)=mp$，有 $mp=\overline{X}$，得 $p=\overline{X}/m$，

故估计量：$\hat{p}=\frac{1}{m}\overline{X}$，估计值：$\hat{p}=\frac{1}{m}\bar{x}$.

11.3 $\because E(X)=\int_0^1 x(\theta+1)x^{-\theta}dx=(\theta+1)\int_0^1 x^{\theta+1}dx=\frac{\theta+1}{\theta+2}=\overline{X}\quad \therefore \hat{\theta}=\frac{1}{1-\overline{X}}-2.$

11.4 (1) $\overline{X}, \frac{1}{n}\sum_{i=1}^{n}(X_i-\overline{X})^2$；（2）$1/\overline{X}$.

11.5 (1)因为总体 $X\sim P\{X=x\}=\frac{\theta^x}{x!}e^{-\theta},\quad x=0,1,2,\cdots,\quad \theta>0,$

$E(X)=\sum_{x=0}^{\infty}x\frac{\theta^x}{x!}e^{-\theta}=\theta,$ 故 $\hat{\theta}_M=\overline{X}.$

(2) 因为 $L(\theta)=\prod_{i=1}^{n}\frac{\theta^{x_i}}{x_i!}e^{-\theta}=\frac{1}{\prod_{i=1}^{n}x_i!}\theta^{\sum_{i=1}^{n}x_i}e^{-n\theta}, \ln L=-\ln(\prod_{i=1}^{n}x_i)+\sum_{i=1}^{n}x_i\ln\theta-n\theta,$

由 $\frac{d(\ln L)}{d\theta}=\frac{1}{\theta}\sum_{i=1}^{n}x_i-n=0$，得 $\hat{\theta}=\bar{x}$. 故 $\hat{\theta}_L=\overline{X}$.

(3) 因为 $p=P\{X=0\}=e^{-\theta}$,因此，$\hat{p}=e^{-\hat{\theta}}=e^{-\overline{X}}$.

11.6 (1)

$\because E(X)=0\cdot\theta^2+2\theta(1-\theta)+2\times\theta^2+3(1-2\theta)=3-4\theta=\overline{X};$

$\therefore \theta$**的矩估计量为** $\hat{\theta}=\frac{3-\overline{X}}{4}$**,估计值为** $\hat{\theta}=1/4$

$L(x_1,\cdots,x_8;\theta)=\prod_{i=1}^{8}P\{X=x_i;\theta\}=4\theta^6(1-\theta)^2(1-2\theta)^4$

$\ln L=\ln 4+6\ln\theta+2\ln(1-\theta)+4\ln(1-2\theta)\Rightarrow\frac{d\ln L}{d\theta}=\frac{6}{\theta}+\frac{-2}{1-\theta}+\frac{-8}{1-2\theta}=0$

$\Rightarrow 12\theta^2-14\theta+3=0;\Rightarrow\theta_{1,2}=\frac{7\pm\sqrt{13}}{12}$,由于 $\theta=\frac{7+\sqrt{13}}{12}>1/2$ 舍去。

故 θ 的极大似然估计值为：$\hat{\theta}=\frac{7-\sqrt{13}}{12}$

11.7 (1)$\because E(X)=\int_{\mu}^{\infty}\frac{x}{\theta}e^{-\frac{x-\mu}{\theta}}dx \overset{y=\frac{x-\mu}{\theta}}{=} \int_{0}^{\infty}(\theta y+\mu)e^{-y}dy=\theta+\mu \overset{令}{=} \overline{X}$,

$$E(X^2)=\int_{\mu}^{\infty}\frac{x^2}{\theta}e^{-\frac{x-\mu}{\theta}}dx \overset{y=\frac{x-\mu}{\theta}}{=} \int_{0}^{\infty}(\theta y+\mu)^2e^{-y}dy=2\theta^2+2\theta\mu+\mu^2=\theta^2+(\theta+\mu)^2 \overset{令}{=} \frac{1}{n}\sum_{i=1}^{n}X_i^2,$$

故联立上述两方程并解之，得

$$\hat{\theta}_M=\sqrt{\frac{1}{n}\sum_{i=1}^{n}X_i^2-\overline{X}^2}=\sqrt{A_2-A_1^2}=\sqrt{B_2},$$

$$\hat{\mu}_M=\overline{X}-\sqrt{\frac{1}{n}\sum_{i=1}^{n}X_i^2-\overline{X}^2}=A_1-\sqrt{A_2-A_1^2}=A_1-\sqrt{B_2}.$$

(2) $\because L(\theta,\mu)=\prod_{i=1}^{n}\frac{1}{\theta}e^{-\frac{x_i-\mu}{\theta}}=\theta^{-n}e^{-\frac{1}{\theta}\sum_{i=1}^{n}(x_i-\mu)}$, $x_i\ge\mu$, $i=1,2,\cdots,n$

$\ln L=-n\ln\theta-\frac{1}{\theta}\sum_{i=1}^{n}(x_i-\mu)$, 由 $\frac{\partial\ln L}{\partial\theta}=-\frac{n}{\theta}+\frac{1}{\theta^2}\sum_{i=1}^{n}(x_i-\mu)=0$，得 $\theta=\frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)$；

另外，由于 $\mu\le x_{(1)}\le x_i$， μ 越大，$\ln L$ 从而 $L(\theta,\mu)$ 越大，

故估计量：$\hat{\mu}_L=X_{(1)}$， $\hat{\theta}_L=\frac{1}{n}\sum_{i=1}^{n}(X_i-X_{(1)})$.

11.8

$\because E(X)=\int_{1}^{+\infty}x\beta x^{-\beta-1}dx=\frac{\beta}{1-\beta}=\overline{X};$

$\therefore \hat{\beta}_M=\frac{\overline{X}}{\overline{X}-1}$

$\because L=\prod_{i=1}^{n}f(x_i,\beta)=\prod_{i=1}^{n}\beta x_i^{-\beta-1}$， $\ln L=-n\ln\beta-(\beta+1)\sum_{i=1}^{n}\ln x_i$，

$\therefore \frac{d\ln L}{d\beta}=\frac{n}{\beta}-\sum_{i=1}^{n}\ln x_i=0,\Rightarrow \hat{\beta}_L=\frac{n}{\sum_{i=1}^{n}\ln x_i}$

11.9

$\because Y=\ln X\sim N(\mu,\sigma^2)$

$$\therefore X\sim f_X(x)=\begin{cases}\frac{1}{\sqrt{2\pi}\sigma x}e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}, & x>0\\ 0 & x\le 0\end{cases}$$

$$L=\prod_{i=1}^{n} f(x_i;\mu,\sigma^2), \quad \ln L=-\frac{n}{2}\ln(2\pi)-\frac{n}{2}\ln(\sigma^2)-\sum_{i=1}^{n} x_i-\sum_{i=1}^{n}\frac{(\ln x_i-\mu)^2}{2\sigma^2}$$

$$\therefore \frac{d\ln L}{d\mu}=\sum_{i=1}^{n}\frac{(\ln x_i-\mu)^2}{\sigma^2}=0,\Rightarrow \hat{\mu}=\frac{1}{n}\sum_{i=1}^{n}\ln x_i$$

$$\frac{d\ln L}{d\sigma^2}=-\frac{n}{2}\frac{1}{\sigma^2}+\frac{1}{(\sigma^2)^2}\sum_{i=1}^{n}\frac{(\ln x_i-\mu)^2}{2}=0,\Rightarrow \hat{\sigma}^2=\frac{1}{n}\sum_{i=1}^{n}(\ln x_i-\mu)^2$$