概率统计——习题十 参考解答

10. 1 $\bar{x}=\frac{1}{n}\sum_{i=1}^{n}x_i=99.625,\quad B_2=\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2=1.7344,\quad S^2=\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2=\frac{n}{n-1}B_2=1.9821$

10.2 (1) $\because E(X)=E(\overline{X})=12,\quad D(\overline{X})=4/5=0.8,\quad \overline{X}^*=\frac{\overline{X}-12}{\sqrt{0.8}}\sim N(0,1),$

$\therefore P\{|\overline{X}-E(X)>1\}=1-P\{|\overline{X}^*|\le 1/\sqrt{0.8}\}=1-[2\Phi(1.118)-1]=2[1-\Phi(1.118)]=0.2628;$

(2) $P\{\min(X_1,\cdots,X_5)<10\}=1-P\{\min(X_1,\cdots X_5)\ge 10\}=1-P\{X_1\ge 10,\cdots,X\ge 10\}$

$$=1-\prod_{i=1}^{5}P\{X_i\ge 10\}=1-[1-\Phi(\frac{10-12}{2})]^5=1-0.8413^5=0.5785.$$

10.3 (1)

$\because \eta=\sum_{i=1}^{10}(\frac{X_i-0}{0.3})^2\sim\chi^2(10),\therefore P\{\sum_{i=1}^{10}X_i^2>1.44\}=P\{\eta>1.44/0.3^2\}=P\{\eta>16\}=0.10;$

(2) $\because E(\overline{X}-\overline{Y})=0,\quad D(\overline{X}-\overline{Y})=\frac{0.3^2}{10}+\frac{0.3^2}{15}=\frac{0.3^2}{6},\therefore(\overline{X}-\overline{Y})^*=\frac{\overline{X}-\overline{Y}}{0.3/\sqrt{6}}\sim N(0,1).$

$$P\{|\overline{X}-\overline{Y}|>0.1\}=1-P\{|(\overline{X}-\overline{Y})^*|\le\frac{0.1}{0.3/\sqrt{6}}\}=1-P\{|(\overline{X}-\overline{Y})^*|\le\sqrt{6}/3\}$$

$$=2[1-\Phi(\sqrt{6}/3)]=2[1-\Phi((0.8165)]=0.4140.$$

10.4 $a=\frac{1}{20},b=\frac{1}{100}$,自由度为2.

10.5 （1）$\frac{\sum_{i=1}^{n}(X_i-\mu)^2}{\sigma^2}\sim\chi^2(n)$ ，$\frac{\sum_{i=1}^{n}(X_i-\overline{X})^2}{\sigma^2}\sim\chi^2(n-1)$ 。

（2）$\frac{X_1+\cdots+X_9}{\sqrt{Y_1^2+\cdots+Y_9^2}}\sim\ t(9)$，参数为 9； $\frac{X_1^2+\cdots+X_9^2}{Y_1^2+\cdots+Y_9^2}\sim F(9,9)$，参数为(9,9)。

10.6 （1）$\because \eta=\frac{(16-1)S^2}{\sigma^2}=\frac{15S^2}{\sigma^2}\sim\chi^2(15),$

$\therefore P\{S^2/\sigma^2\le 2.04\}=P\{\eta\le 15(2.04)\}=P\{\eta\le 3.06\}=1-P\{\eta>30.6\}=1-0.01=0.99.$

（2）$\boldsymbol{D(S^2)=\left(\frac{\sigma^2}{15}\right)^2 D\left(\frac{15S^2}{\sigma^2}\right)=\frac{\sigma^4}{15^2}(2)(15)=\frac{2}{15}\sigma^4.}$

10.7$\because X=\frac{\xi}{\sqrt{\eta/n}}$,其中$\xi\sim N(0,1),\quad \eta\sim\chi^2(n)$， 且ξ，η独立，

$\therefore \xi^2\sim\chi^2(1),\quad X^2=\frac{\xi^2/1}{\eta/n}\sim F(1,n).$

10.8 (1)因为独立正态随机变量的线性组合仍服从正态分布，而

$$E(Z)=\sum_{i=1}^{n}a_iE(X_i)=\mu\sum_{i=1}^{n}a_i,\quad D(Z)=\sum_{i=1}^{n}a_i^2D(X_i)=\sigma^2\sum_{i=1}^{n}a_i^2,$$

$$\text{故}\quad Z\sim N(\mu\sum_{i=1}^{n}a_i,\sigma^2\sum_{i=1}^{n}a_i^2).$$

(2) $\because \dfrac{nB_2}{\sigma^2}=\dfrac{(n-1)S^2}{\sigma^2}\sim\chi^2(n-1),\quad \dfrac{\overline{X}-\mu}{\sigma/\sqrt{n}}\sim N(0,1),$ 且两随机变量独立，

$$\therefore \frac{\overline{X}-\mu}{\sigma/\sqrt{n}}\Bigg/\sqrt{\frac{nB_2}{\sigma^2}\Big/(n-1)}=\frac{\overline{X}-\mu}{\sqrt{B_2/(n-1)}}=T\sim t(n-1).$$

10.9 (1) $(\dfrac{n}{3}-1)\sum_{i=1}^{3}X_i^2/\sum_{i=4}^{n}X_i^2=\dfrac{\sum_{i=1}^{3}\left(\dfrac{X_i}{\sigma}\right)^2\Big/3}{\sum_{i=4}^{n}\left(\dfrac{X_i}{\sigma}\right)^2\Big/(n-3)}\sim F(3,n-3);$

(2) $\because X_{n+1}-\overline{X}\sim N(0,\sigma^2+\dfrac{\sigma^2}{n}),\quad \dfrac{(n-1)S^2}{\sigma^2}\sim\chi^2(n-1)$ 且二者独立，

$$\therefore \frac{X_{n+1}-\overline{X}}{S}\sqrt{\frac{n}{n+1}}=\frac{X_{n+1}-\overline{X}}{\sigma\sqrt{\dfrac{n+1}{n}}}\Bigg/\sqrt{\frac{(n-1)S^2}{\sigma^2}\Big/(n-1)}\sim t(n-1).$$

10.10 (1)略

(2) $C=f_{0.1}(1,1)=39.86$