

概率统计——习题二参考解答

$$2.1 \quad (1) P = \frac{C_{400}^{90} C_{1100}^{110}}{C_{1500}^{200}}; \quad (2) P = 1 - \frac{C_{1100}^{200}}{C_{1500}^{200}} - \frac{C_{400}^1 C_{1100}^{199}}{C_{1500}^{200}}.$$

$$2.2 \quad (1) P = \frac{3}{5}, (2) P = \frac{1}{10}$$

2.3 设 $B_1 = \{\text{所取的三个字母中不含 } a\}, B_2 = \{\text{所取的三个字母中不含 } b\}.$

$$\text{另见, } A = B_1 B_2, B = B_1 \cup B_2, C = B_1 \bar{B}_2, \text{ 从而 } P(A) = P(B_1 B_2) = \frac{C_6^3}{C_8^3} = \frac{5}{14},$$

$$P(B) = P(B_1 \cup B_2) = P(B_1) + P(B_2) - P(B_1 B_2) = \frac{C_7^3}{C_8^3} + \frac{C_7^3}{C_8^3} - \frac{C_6^3}{C_8^3} = \frac{25}{28},$$

$$P(C) = P(B_1 \bar{B}_2) = \frac{C_1^1 C_6^2}{C_8^3} = \frac{15}{56}.$$

$$2.4 \quad (\text{见学习指南 1.1}) \quad P = 1 - P(\text{无成双}) = 1 - \frac{C_6^4 (C_2^1)^4}{C_{12}^4} = 1 - \frac{C_6^2 2^4}{C_{12}^4} = 1 - \frac{15 \cdot 2^4}{12 \cdot 11 \cdot 10 \cdot 9 / 4!} \\ = 1 - 16/33 = 17/33 \approx 0.515.$$

$$2.5 \quad \text{设 } A_i \text{——第 } i \text{ 人取得红球, 则由乘法公式即得 } P(A_i) = \frac{1}{10}, \quad i = 1, 2, \dots, 10.$$

$$2.6 \quad (1) P(B|A \cup \bar{B}) = \frac{P(BA)}{P(A \cup \bar{B})} = \frac{P(A) - P(\bar{A}B)}{P(A) + P(\bar{B}) - P(\bar{A}B)} \\ = \frac{(1-0.3)-0.4}{(1-0.3)+(1-0.4)-0.4} = 1/3;$$

$$(2) P(A \cup B) = P(A) + P(B) - P(AB) = P(A) + \frac{P(AB)}{P(A|B)} - P(AB) \\ = P(A) + [\frac{1}{P(A|B)} - 1]P(A)P(B|A) = \frac{1}{4} + [\frac{1}{1/2} - 1](\frac{1}{4})(\frac{1}{3}) = \frac{1}{3}.$$

2.7 设 A_1, A_2 ——分别表示取出的零件来自第一、二箱, B_1, B_2 ——分别表示第一、二次取出的零件是一等品, 则

$$(1) P(B_1) = P(A_1)P(B_1|A_1) + P(A_2)P(B_1|A_2) = \frac{1}{2} \frac{C_{10}^1}{C_{50}^1} + \frac{1}{2} \frac{C_{18}^1}{C_{30}^1} = \frac{2}{5};$$

$$(2) P(B_2|B_1) = \frac{P(B_1 B_2)}{P(B_1)} = \frac{\frac{1}{2}(C_{10}^2/C_{50}^2 + C_{18}^2/C_{30}^2)}{2/5} = \frac{230 \times 3}{49 \times 29} \approx 0.4856.$$

2.8 设 H_i ——飞机被击中 i 次, $i=0, 1, 2, 3$, B ——飞机被击落, 则

$$P(B) = \sum_{i=0}^3 P(H_i)P(B|H_i).$$

其中 $P(B|H_0)=0$, $P(B|H_1)=0.2$, $P(B|H_2)=0.6$, $P(B|H_3)=1$;

$$P(H_1) = 0.4(1-0.5)(1-0.7) + (1-0.4)(0.5)(1-0.7) + (1-0.4)(1-0.5)(0.7) = 0.36,$$

$$P(H_2) = 0.4(0.5)(1-0.7) + (0.4)(1-0.5)(0.7) + (1-0.4)(0.5)(0.7) = 0.41,$$

$$P(H_3) = 0.4(0.5)(0.7) = 0.14; \text{ 故}$$

$$P(B) = \sum_{i=0}^3 P(H_i)P(B|H_i) = 0.36(0.2) + 0.41(0.6) + 0.14 = 0.458.$$

2.9 设 A_1 、 A_2 、 A_3 ——分别表示每箱含有 0, 1, 2 只残次品, B 表示顾客买下该箱玻璃杯则

$$P(B|A_0)=1, P(A_0)=0.8$$

$$P(B|A_1) = \frac{C_{19}^4}{C_{20}^4} = 0.8, P(A_1)=0.1,$$

$$P(B|A_2) = \frac{C_{18}^4}{C_{20}^4} = 12/19, P(A_2)=0.1$$

$$P(B) = \sum_{i=0}^2 P(A_i)P(B|A_i) = 0.943, \text{ 由贝叶斯公式, } P(A_0|B) = \frac{P(A_0)P(B|A_0)}{P(B)} = 0.848.$$

2.10 设 A_1 、 A_2 、 A_3 、 A_4 ——分别表示朋友乘火车、轮船、汽车、飞机来, B ——朋友迟到。

$$\text{则由于 } P(B) = \sum_{i=1}^4 P(A_i)P(B|A_i) = 0.3(\frac{1}{4}) + 0.2(\frac{1}{3}) + 0.1(\frac{1}{12}) + 0 = 0.15,$$

$$\text{故 } P(A_1|B) = \frac{0.3(1/4)}{0.15} = 0.5.$$

2.11 (1) 0.092; (2) 互不相容; (3) 相互独立; (4) 相互对立;