



A Survey of Learning-Based Intelligent Optimization Algorithms

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Abstract

A large number of intelligent algorithms based on social intelligent behavior have been extensively researched in the past few decades, through the study of natural creatures, and applied to various optimization fields. The learning-based intelligent optimization algorithm (LIOA) refers to an intelligent optimization algorithm with a certain learning ability. This is how the traditional intelligent optimization algorithm combines learning operators or specific learning mechanisms to give itself some learning ability, thereby achieving better optimization behavior. We conduct a comprehensive survey of LIOAs in this paper. The research includes the following sections: Statistical analysis about LIOAs, classification of LIOA learning method, application of LIOAs in complex optimization scenarios, and LIOAs in engineering applications. The future insights and development direction of LIOAs are also discussed.

1 Introduction

An intelligent optimization algorithm, also known as a modern heuristic algorithm, is an algorithm with global optimization performance and strong versatility that is suitable for parallel processing. This type of algorithm can find the optimal solution or approximate optimal solution within a certain period of time.

There are many intelligent optimization algorithms. For example, Holland [1] proposed a genetic algorithm (GA) that simulates the biological evolution of the natural selection and genetic mechanism of Darwin's theory of biological evolution. Inspired by the behavior of ants to find paths in the process of finding food, Dorigo [2] proposed the ant colony optimization (ACO) algorithm. Storn and Price [3] proposed the differential evolution (DE) algorithm. Eberhart

and Kennedy [4] simulated the flock foraging behavior and proposed the particle swarm optimization (PSO). Based on the behavior of *E. coli* (*Escherichia coli*) to devour food in human intestines, Passino [5] proposed the bacterial foraging optimization algorithm (BOA). Eusuff and Lanse [6] proposed the shuffled frog-leaping algorithm (SFLA) by simulating frog behavior.

Karaboga [7], inspired by bee behavior, proposed artificial bee colony (ABC). Krish-nanand and Ghose [8], inspired by the firefly flashing behavior, proposed the firefly algorithm (FA). Mehrabian et al. [9] proposed a random search algorithm based on the principle of weed evolution in nature, called invasive weed optimization (IWO). Yang and Deb [10] proposed the cuckoo search (CS) algorithm by simulating the parasitic brooding behavior of certain species of cuckoo. Yang [11] proposed the bat algorithm (BA), which simulates the behavior of bats in nature to use sonar to detect prey and avoid obstacles.

Mirjalili et al. [12], inspired by the gray wolf's predator activity, proposed the grey wolf optimizer (GWO) algorithm. Inspired by the behavior of marine predators, Faramarzi et al. [13] proposed the marine predator algorithm (MPA). Mirjalili et al. [14] proposed the salp swarm algorithm (SSA) based on the movement and foraging behavior of salps in the ocean. Mirjalili [15] proposed the dragonfly algorithm (DA) based on the theoretical basis of dragonfly hunting and avoidance of natural enemies. Mirjalili [16] proposed the ant lion optimizer (ALO) algorithm. Gandomi [17] proposed the internal search algorithm (ISA) inspired by

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interior design and decoration. Wang et al. [18] the proposed monarch butterfly optimization (MBO) algorithm by simplifying and idealizing monarch butterfly migration. Wang et al. [19–21] proposed the elephant herding optimization (EHO) algorithm by simulating elephant herds' behavior. Gandomi and Alavi [22] proposed the krill herd (KH) algorithm based on simulations of individual shrimp. Since then, many researchers have improved the KH algorithm [23–26]. There are also many researchers who apply the intelligent optimization algorithms to machine learning [27–31].

Inspired by nature, these intelligent optimization algorithms are applied to solve NP-hard problems such as scheduling [32–37], image [38–40], feature selection [41–43], detection [44, 45], path planning [46, 47], cyber-physical social system [48, 49], texture discrimination [50], factor evaluation [51], saliency detection [52], classification [53, 54], object extraction [55], gesture segmentation [56], economic load dispatch [37], shape design [57], big data and large-scale optimization [58–62], signal processing [63], multi-objective optimization [48, 57, 62, 64], unit commitment [65], vehicle routing [66], knapsack problem [67–69], and fault diagnosis [70–72].

Based on these intelligent optimization algorithms, by combining learning operators or adopting a special learning mechanism, the algorithm itself has a certain learning ability, called learning-based intelligent optimization algorithm (LIOAs). Compared with the traditional intelligent optimization algorithms, LIOAs have a certain learning ability that makes individuals communicate more frequently between populations and facilitates the exchange of information between individuals, so as to accelerate the convergence speed of the algorithm and better find the optimal solution.

Many researchers have conducted a great deal of research on LIOAs. Feng [73] proposed an evolutionary PSO learning algorithm for self-generating, radial basis function neural networks to solve nonlinear problems. The goal of learning is to choose a suitable radial basis function and used the least radial basis function. Liang et al. [74] proposed a comprehensive learning particle swarm optimizer that updates the speed of particles by learning the best historical information of all other particles. This learning strategy preserves the diversity of the population while preventing premature convergence. Wang et al. [75] proposed an enhanced PSO based on generalized opposition-based learning and Cauchy mutation.

Xia et al. [76] proposed a multi-strategy particle swarm optimization algorithm, including Tabu detection, reduction, and local learning strategies. The learning strategy increases the accuracy of the solution by learning the difference between two elite particles. Lim et al. [77] proposed a teaching and peer-learning PSO based on a teaching-learning-based optimization algorithm. The algorithm updates particles through two phases of teaching and peer-learning

to improve the algorithm's performance. Lim et al. [78] also proposed a bidirectional teaching and peer-learning PSO. The two stages of teaching and peer learning are used to update individuals.

Hu et al. [79] proposed a memetic algorithm, based on comprehensive learning particle swarm optimization, which can improve both feature selection and parameter optimization. The algorithm can perform personal learning for local search of specific problems, thereby enhancing the use of comprehensive learning particle swarm optimization. Chen et al. [80] proposed a variation immunological system algorithm with radial basis function neural network learning for function approximation and industrial applications.

Chen et al. [81] proposed a new algorithm for a learning backtracking search algorithm that updates the current individual's position by learning from the best individual, the worst individual, and another individual. Gong et al. [82] developed a new framework for organic hybridization of PSO and genetic algorithms into a genetic learning PSO. Genetic operators are used to generate examples from which particles learn, and in turn, historical search information for particles also provides guidance for the development of examples. Chen et al. [83] presented a precision-based learning fuzzy classifier system for explicit processing of continuous state input and continuous action output in a multi-step information learning process.

Song et al. [84] proposed a simple brainstorm optimization algorithm with a periodic quantum learning strategy. Introducing a quantum behavior mechanism into the algorithm through periodic learning strategies provides individuals with new motivation and enables them to escape local optimization. Mei et al. [85] proposed an improved, brain-inspired emotional learning algorithm for fast classification. This algorithm simulates the high speed of the emotion learning mechanism in the mammalian brain. Yu et al. [86] proposed an adaptive particle speed update strategy for swarm comprehensive learning particle swarm optimization to improve search efficiency. According to the elite state in each dimension, each particle adaptively selects the learning target to update the dimension.

We conducted a comprehensive survey of LIOAs in this paper. The rest of the paper is arranged as follows. Section 2 introduces the statistical analysis of 154 papers about LIOAs. Section 3 reviews the classification of LIOAs learning methods. The applications of LIOAs in complex optimization scenarios are introduced in Sect. 4. The applications of LIOAs in engineering optimization are presents in Sect. 5. Finally, Sect. 6 draws the conclusions and the expectation for the future.

2 Statistical Analysis of LIOAs

We searched through Google Scholar with the keyword “learning” and found papers related to intelligent optimization algorithms. From the various papers collected for this study, we selected and used 154 representative papers from 1 January 2006 to 20 March 2020 for our survey, mainly from the following 4 aspects, as shown in Figs. 1–5.

Fig. 1 The number of publications regarding BLAIOs in the last decades

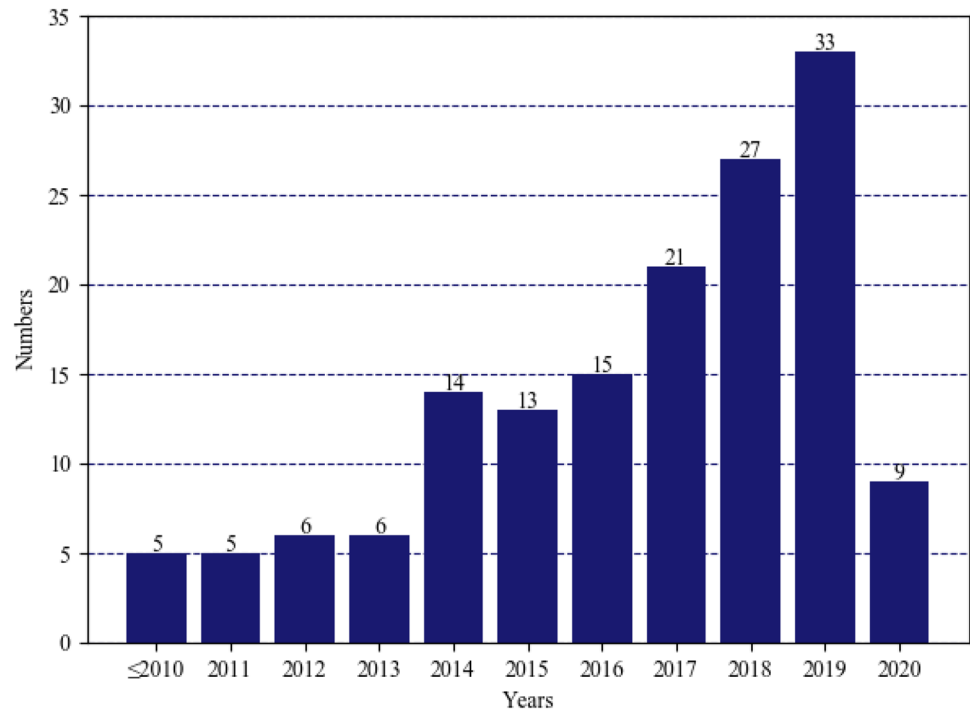


Fig. 2 The classification proportions of learning methods used in LIOAs

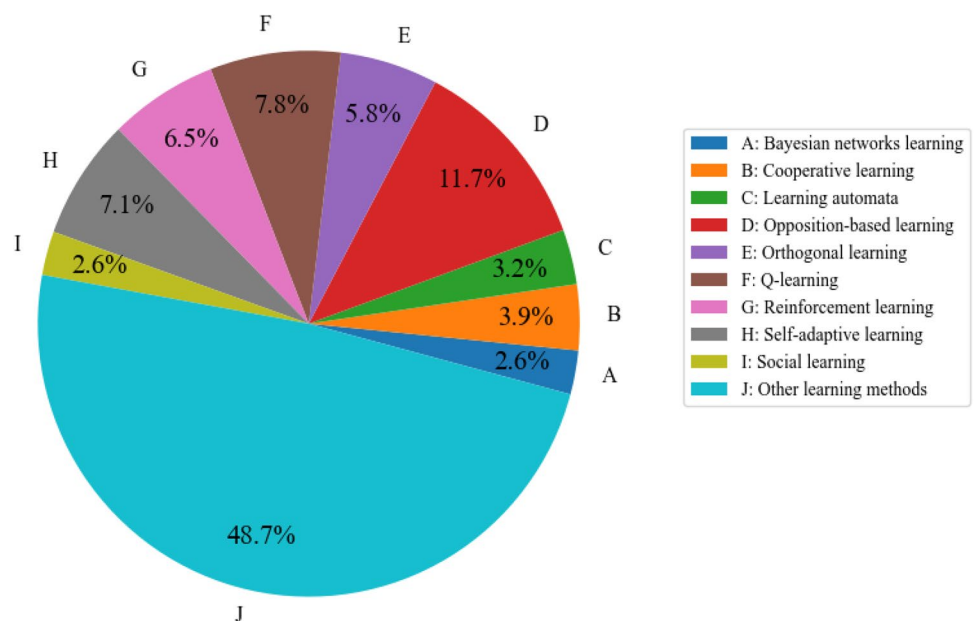


Figure 1 shows the number of publications regarding LAIOs in the last decades. From Fig. 1, we can see the number of papers published each year. It is easy to see that the number of papers is on the rise as a whole. The number of papers published each year has been over 10 since 2104. The number of papers published each year has exceeded 20 and shows a trend of rapid growth trend, especially in the past three years.

Figure 2 presents the classification proportions of learning methods used in LIOAs. Figure 2 shows that this paper

divides the learning methods used in LIOAs into 10 categories. Four methods used sparingly (less than 4%) are Bayesian networks learning, cooperative learning, learning automata, and social learning. The methods of orthogonal learning, Q-learning, reinforcement learning, and self-adaptive learning ratio are used less than 10%. Furthermore, opposition-based learning is used relatively frequently, with its proportion exceeding 10%.

Figure 3 shows 24 other, most-used learning methods. These other learning methods are active learning, algorithm-based learning, associative learning, cellular learning, comprehensive learning, covariance matrix learning, dimension Learning, dual preferred learning mutation, dynamic learning, elitist learning, example-based learning, hierarchical learning, information learning, interactive learning, knowledge learning, learning a value function, learning

classification, learning interval, multiple learning, neighbor-based learning, parameter learning, query-based learning, self-paced learning, and others.

Figure 4 shows the application proportions of LIOAs in complex optimization scenarios. We divide the applications of LIOAs in complex scenarios into five parts. Among them, the most-used applications are combinatorial optimization and multi-objective optimization, accounting for 28%. Next, constrained optimization accounted for 18%. Dynamic optimization accounts for 14%. Relatively few applications are continuous optimization, accounting for 12%. It should be noted that the complex optimization scenario mentioned in this paper only includes the just-mentioned five parts and does not include the evaluation of the benchmark functions. Figure 4 also shows the statistics of 50 papers that belong to those 5 parts from 154 papers, not the statistics of all papers.

Fig. 3 The classification proportions of other learning methods

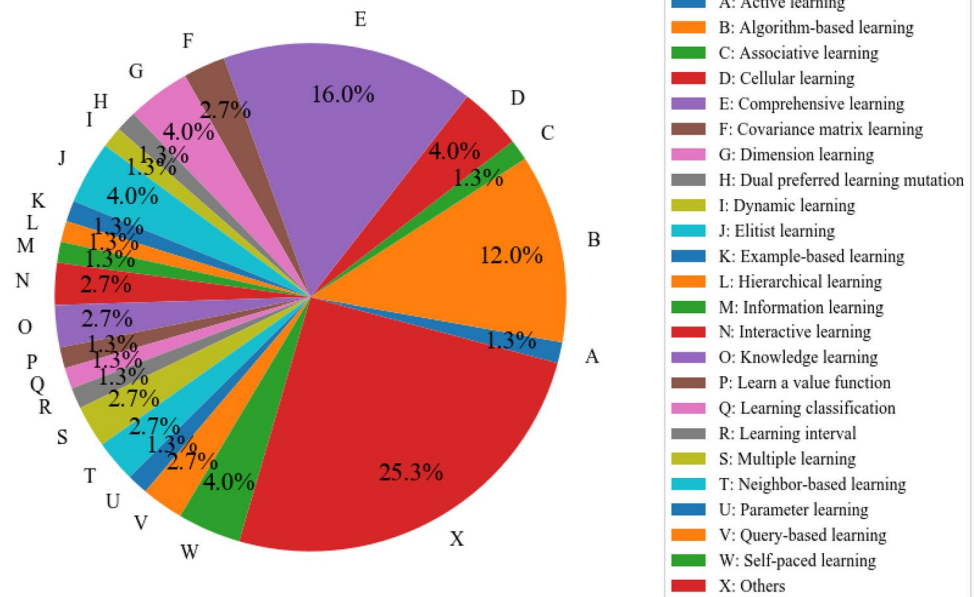


Fig. 4 The application proportions of LIOAs in complex optimization scenarios

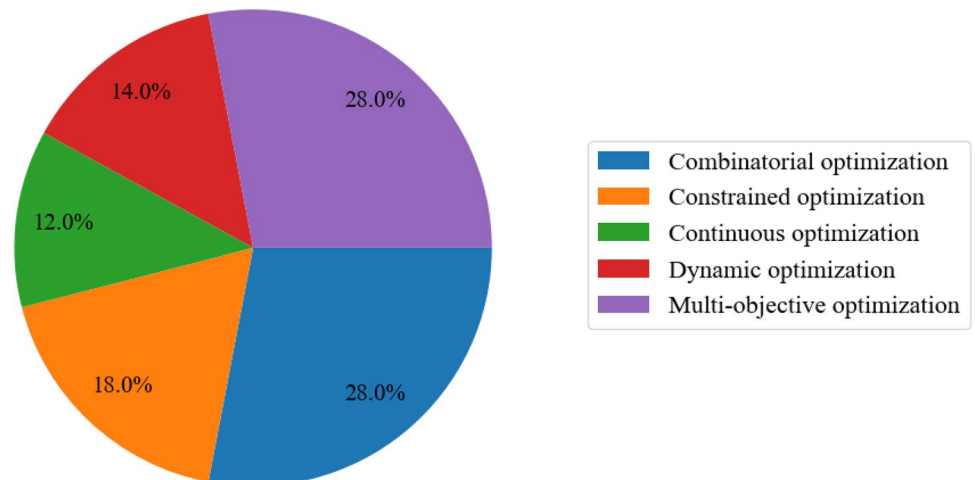


Figure 5 shows the proportion of the type of problems solved by LIOAs. We divided LIOAs into 4 categories according to the different ways to verify LIOAs and application scenarios. Among those categories, LIOAs solve the most benchmark functions, accounting for 37.7%. The second way is to apply LIOAs to Fig. 4 complex optimization scenarios, accounting for 31.4%. Furthermore, the proportion of LIOAs used to solve Engineering application problems is the smallest, only 10.1%. Finally, using LIOAs to solve some other problems accounted for 20.8%.

3 Classification of Learning Methods Used in LIOAs

We divide LIOAs into 10 categories in this section according to different learning methods. The categories' alphabetical order is Bayesian networks learning, cooperative learning, learning automata, opposition-based learning, orthogonal learning, Q-learning, reinforcement learning, self-adaptive learning, social learning, and other learning methods. Figure 2 and Table 1 show their specific contents.

3.1 Bayesian Network Structure Learning

Bayesian network structure learning is learning the Bayesian network structure from a given data set, that is, the

dependencies between nodes. Only after the structure is determined the network parameters, it can learn, that is to say, the conditional probability of dependence between nodes. The learning of network structure is not only the basis of the entire learning process, but it is also an NP problem, so it has attracted the attention of a large number of researchers. Gheisari et al. [88] proposed a Bayesian network structure learning algorithm based on particle swarm optimization, that is, a Bayesian network construction algorithm using particle swarm optimization (PSO). The algorithm uses edge processing to make the particles obtain the best solution. At the same time, it uses a loop delete program to prevent the generation of invalid solutions. Li et al. [87] proposed an immune binary particle swarm optimization method based on a maximum likelihood tree to learn the structure of probabilistic relational models from relational data. The results show that the proposed method can obtain better results by learning the probability structure.

3.2 Cooperative Learning

This concept of collaborative learning can occur without interaction between learning agents, that is, ensemble learning, or interaction can occur during the learning phase, that is, collaborative learning [164]. Alexandridis et al. [108] proposed a new evolutionary cooperative learning scheme based on the collaborative particle swarm optimization

Fig. 5 The proportion of the type of problem solved with LIOAs

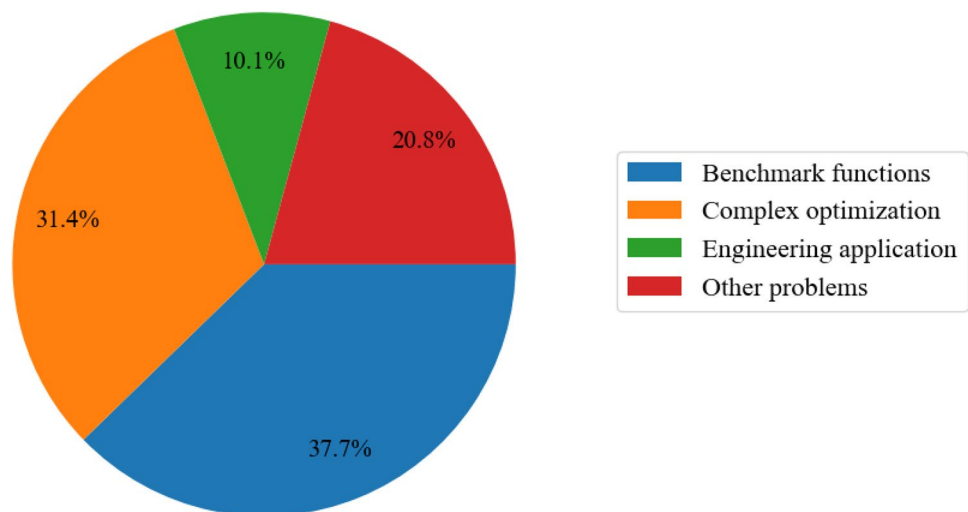


Table 1 The classification of learning methods used in LIOAs

Category	Literatures	Category	Literatures
Bayesian networks learning	[87–90]	Q-learning	[91–102]
Cooperative learning	[103–108]	Reinforcement learning	[83, 109–117]
learning automata	[118–122]	Self-adaptive learning	[123–133]
Opposition-based learning	[75, 134–150]	Social learning	[151–154]
Orthogonal learning	[155–163]	Other learning methods	Table 2

framework. This scheme can solve function approximation and classification problems with higher accuracy and generalization ability. Boryczka et al. [107] merged methods developed to better construct ant decision trees. This method is tested in a cooperative learning mechanism, which means that agents-ants can interact through pheromone values in the building decision tree. This cooperation is an opportunity for better results. Xie et al. [103] proposed a distributed cooperative learning algorithm for a radial basis function network. Ma et al. [105] developed a multi-swarm artificial bee colony algorithm with cooperative learning for data aggregator units in a smart grid to minimize utility companies' costs.

3.3 Learning Automata

Learning automata adjust themselves through constant interaction with a random environment, that is, they gain experience through continuous communication with the environment to improve their behavior so they can choose the environment in a selectable action [165]. The optimal action is the action with the highest probability of getting the environmental reward in the current environment. Anari et al. [122] proposed a new clustering method, that is, a learning automata-based clustering algorithm that uses learning automata and ant swarm intelligence. This method solves the problem of finding the optimal position of the grid for deleting data items by using a learning automaton. Hashemi et al. [119] studied the ability of learning automata to adapt to PSO parameter selection. The authors introduced two kinds of algorithms based on learning automata for adaptive selection of inertial weight and acceleration coefficient. A swarm of particles uses the same parameter values adjusted by the learning automata in the first kind.

3.4 Opposition-Based Learning

Opposition-based learning is a new concept in machine learning inspired by the opposite relationship between entities [140]. It is also a new concept in computational intelligence that has proven to be an effective concept for enhancing various optimization methods.

Liu et al. [142] proposed a modification strategy based on opposition-based learning and integrated it into the bare-bones particle swarm optimization algorithm to help individuals jump out of local optimal values by changing their flight direction. Park et al. [138] combined opposition-based learning with the cuckoo search to make the convergence rate of the cuckoo search faster without reducing the algorithm's search ability. Guo et al. [145] proposed an improved global harmony search based on the generalized opposition-based learning strategy. The probability of finding the global optimal value is increased by introducing the learning strategy

based on generalized opposition. Wei et al. [134] proposed a new, constrained differential evolution framework that uses generalized opposition-based learning. The basic structure of the shuffled frog-leaping algorithm divides, frogs are divided into memplex according to their adaptability value of foraging in food.

Sharma et al. [137] embedded opposition-based learning concepts into memplex before the frogs started foraging. Sun et al. [135] developed a novel monarch butterfly optimization algorithm based on opposition-based learning and random local perturbation. The opposition-based learning method produces opposition-based population from the primitive population. Better individuals can be selected and passed on to the next generation by comparing the opposition-based population with the original population. Oliva et al. [139] proposed an improved brainstorm optimization algorithm using the chaotic opposite-based learning method with a disruption operator for global optimization and feature selection. Mahdavi et al. [140] published a review on opposition-based learning in 2017 that introduced the opposition-based learning method in more detail.

3.5 Orthogonal Learning

Orthogonal learning is a strategy that predicts the best combination of two solution vectors and performs a deep search in the solution space based on limited experiments instead of exhaustive experiments [163]. Zhan et al. [155] proposed a PSO algorithm based on orthogonal learning. Orthogonal learning strategies can guide particles to construct an effective paradigm so that PSO can find more useful information. Li et al. [160] developed a cuckoo search algorithm based on orthogonal learning. A new search strategy based on the orthogonal learning strategy is used to enhance the exploitation ability of the basic cuckoo search algorithm. The algorithm used an orthogonal array design and the orthogonal learning strategy to update the search mechanism.

Liu et al. [158] proposed an orthogonal learning particle swarm optimization algorithm based on quadratic interpolation, which adopted a personal history best experience construction strategy based on quadratic interpolation. The authors also introduced opposition-based learning and Gaussian mutation to increase the diversity of the population and prevent premature convergence. Peng et al. [157] proposed a hybrid learning clonal selection algorithm that combined two learning mechanisms, Baldwin learning and orthogonal learning, and introduced the clonal selection algorithm to guide the immune response process. Li et al. [159] used the orthogonal learning cuckoo search algorithm to estimate the parameters of chaotic systems. This algorithm can combine the random exploration of the cuckoo search with the exploitation ability of the orthogonal learning strategy.

3.6 Q-learning

Q-Learning is a value-based algorithm in reinforcement learning algorithms [102]. Q is $Q(s, a)$, which is the expectation that the action a can obtain benefits under the state s at a certain moment. Thus, the main idea of the algorithm is to construct state and action into a Q -table to store the value Q and then select the action that can obtain the maximum benefit according to the value Q . Sadhu et al. [93] proposed an improved firefly algorithm that involved its own Q -learning framework. Q -learning was used during the learning phase to learn each firefly in the population through the Q -learning strategy and then apply it during execution. Li et al. [96] extended the cuckoo search algorithm through Q -learning and genetic operators and proposed a dynamic, step-size cuckoo search algorithm. Ding et al. [99] proposed a Q -learning-based, task-scheduling framework for energy-efficient cloud computing. Liu et al. [95] proposed a Flying Ad Hoc Networks multi-objective optimized routing protocol based on Q -learning. Protocols can be proposed through Q -learning parameters to adapt to the high dynamics of Flying Ad Hoc Networks.

3.7 Reinforcement Learning

Reinforcement learning, also known as evaluation learning, is one of the paradigms and methodologies of machine learning [166]. It is used to describe and solve agents' learning through interaction with the environment to achieve the maximizing returns. Reinforcement learning is a type of algorithm that allows computers to realize nothing from the beginning. They learn from their mistakes through continuous trial and error and finally find the rules.

Abed-alguni [117] proposed a new action selection method for reinforcement learning based on the cuckoo search algorithm, which is called the cuckoo action selection method. The knowledge about rewards in reinforcement learning is explicitly expressed as a probability distribution in this method. Chen et al. [112] introduced the integration of differential evolution and collaborative co-evolution methods based on reinforcement learning in a compensatory neuro-fuzzy controller. This method used the fitness function of the related reinforcement learning method to enhance the signal to determine the controller control problem that can be used to solve the problem. Hou et al. [110] proposed an evolutionary transfer reinforcement learning framework for developing intelligent agents that can adapt to the dynamic environment of multi-agent systems.

3.8 Self-adaptive Learning

Self-adaptive learning refers to someone or something that can learn or guide itself in an adaptive way during the

evolution process. Li et al. [130] proposed a new algorithm for global optimization called self-learning particle swarm optimization. Each particle in this algorithm has four strategies for dealing with different situations in the search space. Sun et al. [129] proposed a self-adaptive, multi-objective, evolutionary algorithm for multi-objective optimization problems. Based on the regularity of the optimal set, this algorithm adopts the clustering method to adaptively learn the manifold structure of the optimal set. Li et al. [125] proposed a new hybrid model based on self-adaptive learning particle swarm optimization and support vector regression for ore grade estimation. The hybrid model used the particle swarm optimization algorithm based on self-adaptive learning to search for the best parameters of support vector regression; then, it used the best parameters to construct the support vector regression model. Birjali et al. [132] proposed a self-adaptive e-learning model that provides the most appropriate learning content for each learner. Their model is based on two levels of self-adaptive e-learning. Gu et al. [131] improved the iterative learning control algorithm through particle swarm optimization and proposed a particle swarm optimization algorithm for adaptive optimal iterative learning control. Experimental results show that this algorithm can both improve the convergence speed and solve the model's uncertainty.

3.9 Social Learning

Social learning plays an important role in the behavioral learning of social animals. The advantage of social learning, in contrast to individual learning, is that it allows individuals to learn behaviors from others without incurring the cost of individual trial and error [153]. Inspired by the phenomenon of social learning in animals, Cai et al. [154] proposed an adaptive social learning strategy for differential evolution to extract the neighborhood relationship information of individuals in the current population. Zhang et al. [151] proposed an improved social learning PSO algorithm, namely the differential mutation and novel social learning PSO. Dynamic inertial weight is also introduced to replace the random inertial weight of the social learning PSO, and a method and an example learning method are proposed to replace the imitation component and the social influence component of the social learning PSO, respectively. Cheng et al. [153] introduced the social learning mechanism into PSO. Particles in classical PSO are updated based on historical information, including the best solution found by the entire population and the best solution found by each particle. Each particle in this method can learn from any better particle in the current group.

3.9.1 Other Learning Methods

Other learning methods are divided into 24 categories: active learning, algorithm-based learning, associative learning, cellular learning, comprehensive learning, covariance matrix learning, dimension learning, dual preferred learning mutation, dynamic learning, elitist learning, example-based learning, hierarchical learning, information learning, interactive learning, knowledge learning, learning a value function, learning classification, learning interval, multiple learning, neighbor-based learning, parameter learning, query-based learning, self-paced learning, and others. Figure 3 and Table 2 show their specific contents.

(1) Active learning

Cao et al. [167] proposed an active learning brain-storm optimization algorithm based on the dynamic cluster cycle. The algorithm's performance was verified by a set of test functions and two real-world problems.

(2) Algorithm-based learning

Chen et al. [174] proposed a variation of the teaching–learning-based optimization algorithm with multiple classes of cooperative and simulated annealing operators. Deb et al. [170] developed an improved version of the chicken swarm optimization algorithm with an improved rooster update equation and a novel constraint-handling mechanism. They also proposed the synergy between the improved chicken swarm optimization version and the teaching–learning-based optimization algorithm.

(3) Associative learning

Heidari et al. [179] proposed the whale optimization algorithm with the associative learning method

and combined a recent mountain climbing local search to enhance its exploitation ability.

(4) Cellular learning

Vafashoar et al. [182] proposed a hybrid model based on cellular learning automata and a differential evolution algorithm for global numerical optimization. One of the main ideas of this method is to use cellular learning automata to learn the most promising areas of the search space.

(5) Comprehensive learning

Yu et al. [188] proposed two enhancements to the comprehensive learning particle swarm optimization. One is to add the perturbation term to the velocity update process for each particle. The other is to determine the learning probability of the particle adaptively. Lynn et al. [190] proposed a comprehensive learning particle swarm optimization algorithm with enhanced exploration and exploitation, called grouped comprehensive learning particle swarm optimization. The algorithm divided the population into two sub-populations, each of which is assigned to focus solely on exploration or exploitation. It uses the comprehensive learning strategy to generate samples of two subgroups.

(6) Covariance matrix learning

Wang et al. [196] proposed a method for establishing an appropriate coordinate system for the crossover operator based on covariance matrix learning to balance the exploration and exploitation abilities of differential evolution (DE), alleviate the dependence of DE on coordinate systems, and enhance the ability of DE to solve high-variable related problems. The crossover operator developed a new DE variant by introducing covariance matrix learning and bimodal distribution parameter settings into DE.

Table 2 The classification of other learning methods

Category	Literatures	Category	Literatures
Active learning	[167]	Information learning	[168]
Algorithm-based learning	[77, 78, 170–177]	Interactive learning	[177, 178]
Associative learning	[179]	Knowledge learning	[180, 181]
Cellular learning	[182–184]	Learning a value function	[185]
Comprehensive learning	[74, 79, 86, 186–194]	Learning classification	[195]
Covariance matrix learning	[196, 197]	Learning interval	[198]
Dimension learning	[199–201]	Multiple learning	[202, 203]
Dual preferred learning mutation	[204]	Neighbor-based learning	[205, 206]
Dynamic learning	[207]	Parameter learning	[208]
Elitist learning	[209–211]	Query-based learning	[212, 213]
Example-based learning	[214]	Self-paced learning	[215–217]
Hierarchical learning	[218]	Others	[73, 76, 80–82, 84, 85, 219–230]

- (7) Dimension learning
Xiao et al. [200] proposed an improved artificial bee colony algorithm based on elite strategy and dimension learning. Dimension learning greatly improves the difference between two random dimensions in this algorithm.
- (8) Dual preferred learning mutation
Duan et al. [204] proposed a differential evolution algorithm with their dual preferred learning mutation. The algorithm can learn from both individuals with better fitness and individuals with better diversity through the dual preferred learning mutation.
- (9) Dynamic learning
Ye et al. [207] proposed a new multi-swarm PSO algorithm with dynamic learning strategy to improve the performance of PSO.
- (10) Elitist learning
Lim et al. [211] proposed an adaptive two-layer PSO algorithm with elite learning strategies whose search capability was superior to the classical PSO.
- (11) Example-based learning
Huang et al. [214] proposed an example-based learning PSO algorithm to overcome the disadvantages of PSO by maintaining a balance between population diversity and convergence speed.
- (12) Hierarchical learning
Chen et al. [218] introduced the hierarchical learning method and proposed the hierarchical learning water cycle algorithm to improve the algorithm's global search ability effectively.
- (13) Information learning
Inspired by the important role of division and cooperation in human history, Gao et al. [168] developed a new artificial bee colony algorithm based on information learning.
- (14) Interactive learning
Inspired by the phenomenon of human society, it is believed that interactive learning behavior can occur between different groups. Qin et al. [177] proposed an improved PSO algorithm with an inter-group interactive learning strategy that overcame the shortcomings of standard PSO.
- (15) Knowledge learning
Li et al. [180] proposed a cuckoo search extension algorithm based on self-adaptive knowledge learning. They introduced the learning model with personal history knowledge and population knowledge into the cuckoo search algorithm.
- (16) Learning a value function
Branke et al. [185] proposed an interactive, multi-objective, evolutionary algorithm that obtained the real preference of users by learning a value function.
- (17) Learning classification
Al-Obeidat et al. [195] used the differential evolution algorithm to learn and optimize the output of the classification method PROAFTN.
- (18) Learning interval
Almaraashi et al. [198] used simulated annealing for learning the best configurations interval and general type-2 fuzzy logic systems to maximize their modeling capabilities.
- (19) Multiple learning
Xu et al. [203] proposed an improved PSO algorithm based on the multi-objective sorting learning strategy. The multi-objective sorting learning strategy can guide particles to fly in better directions by building guidance examples with better fitness value and diversity.
- (20) Neighbor-based learning
Zhang et al. [205] proposed a gravitational search algorithm based on the dynamic neighborhood learning strategy.
- (21) Parameter learning.
Cai et al. [208] proposed an improved grey wolf optimization algorithm based on the parameter learning strategy.
- (22) Query-based learning
Chang et al. [212] proposed a PSO algorithm with query-based learning to improve the exploration and exploitation ability of PSO and to solve the multi-objective power contract problems.
- (23) Self-paced learning
Li et al. [216] proposed a Pareto self-paced learning algorithm based on differential evolution. Wang et al. [215] proposed an imbalanced sampling approach via self-paced learning to effectively solve the imbalanced cancer data pre-diagnosis.
- (24) Others
Zhang et al. [220] proposed a whale optimization algorithm based on Lamarckian learning to solve high-dimensional function optimization problems. Peng et al. [222] proposed a learning-based memetic algorithm to solve the multiple vehicle pickup and delivery problem; the problem is a generalization of the traveling salesman problem. Li et al. [224] proposed an improved learning-based, multi-strategy elephant herding optimization algorithm to solve numerical optimization problems.

4 Applications of LIOAs in Complex Optimization Scenarios

Figure 4 shows how we divide the applications of learning-based intelligent optimization algorithms (LIOAs) in complex scenarios into five types. The five types are combinatorial optimization, constrained optimization, continuous optimization, dynamic optimization and multi-objective optimization. Table 3 presents a summary of the applications of LIOAs in complex optimization scenarios.

4.1 Combinatorial Optimization

Combinatorial optimization is finding the optimal method to deal with problems such as optimal arrangement, grouping, order or screening of discrete events. Combinatorial optimization problems are a type of optimization problem that finds the optimal solution from a limited set of feasible solutions [231]. Many researchers have applied LIOAs to combinatorial optimization.

Wu et al. [118] proposed the function optimization by learning automata to solve complex function optimization problems. The proposed method was applied to solve the optimal power flow problem of a power system; the results showed that the method can reduce the fuel cost and improve the fuel cells' voltage stability. Chen et al. [229] proposed a particle swarm optimization (PSO) algorithm with learning effect to minimize the later work complexity of the

flow-shop system. Cheng et al. [228] introduced a position-weighted learning effect model based on sum-of-logarithm-processing-times to solve flow-shop scheduling problems.

Nitisiri et al. [223] proposed a parallel, multi-objective, evolutionary algorithm based on a hybrid sampling strategy and learning-based mutation to solve the railway train scheduling problem. Better solutions can be found by combining learning techniques with multi-objective genetic algorithms. Ahmadi et al. [102] proposed a genetic algorithm based on dynamic Q-learning to solve the job sequencing and tool switching problems through Q-learning, the method learned from the experience of selecting mutation and crossover operator order in each generation of a genetic algorithm.

Zhong et al. [186] proposed a discrete, comprehensive learning PSO algorithm based on the acceptance criteria of the simulated annealing algorithm for the traveling salesman problem. Marandi et al. [94] developed a learning-oriented simulated annealing algorithm based on Q-learning to solve the network configuration, multi-factory scheduling with batch delivery problem. Arin et al. [101] integrated the estimation of distribution algorithm and Q-learning method into a metaheuristic for randomized priority search to solve the 0–1 multidimensional knapsack problems. Aiming at combinatorial optimization problems, Jiang et al. [97] developed a novel method based on Q-learning and applied it to the optimization scheme of coordinated urban rail transit lines.

Cao et al. [114] proposed a cuckoo search algorithm with reinforcement learning and the surrogate model to solve semiconductor final test scheduling problems with

Table 3 A summary of the LIOAs applications in complex optimization scenarios

Category	Problem	References
Combinatorial optimization	Optimal power flow problem	Wu et al. [118], Bai et al. [163]
	Flow shop scheduling problem	Chen et al. [229], Cheng et al. [228]
	railway train scheduling problem	Nitisiri et al. [223]
	Job sequencing and tool switching problem	Ahmadi et al. [102]
	Network configuration multi-factory scheduling problem	Marandi et al. [94]
	Semiconductor test facility scheduling problem	Cao et al. [114]
	QoS-aware cloud service composition problem	Zuo et al. [123], Liu et al. [152]
	Traveling salesman problem	Peng et al. [222], Zhong et al. [186]
	0–1 knapsack problem	Arin et al. [101]
	Urban rail transit lines problem	Jiang et al. [97]
Constrained optimization	Network structure search problem	Alonso et al. [90], Contaldi et al. [89]
	Economic dispatch problem	Xiong et al. [156], Hsieh et al. [98], Cardoso Bora et al. [113], Wang et al. [126], Bahmani-Firouzi et al. [133]
	Constrained engineering design problem	Samma et al. [92]
	Constrained portfolio optimization	Almahdi et al. [116]
Continuous optimization	Clustering problem	Anari et al. [122], Cao and Wang [167]
	Channel equalization problem	Balusu et al. [121]
	Training artificial neural network	Dora et al. [226], Darwish et al. [162], Emary et al. [111]

multi-resource constraints. Zuo et al. [123] proposed a PSO-based, self-adaptive, learning constraint task scheduling method to solve the hybrid infrastructure as a service (Issa) cloud problem. Inspired by the evolution process of human intelligence and the theory of social learning, Liu et al. [152] proposed a new social learning optimization algorithm to solve the problem of QoS-aware cloud service composition.

4.2 Constrained Optimization

Constrained optimization, that is, the constraint optimization problems, is a branch of the optimization problems. It is finding a set of parameter values under a series of constraints to optimize the objective value of a function or a set of functions [232]. Many researchers have applied LIOAs to constrained optimization. Alonso et al. [90] proposed an improvement to Bayesian network structure learning based on a greedy equivalence search using a metaheuristic method of local search. Contaldi et al. [89] proposed an improved hybrid learning strategy characterized by the use of the parametric genetic algorithm to learn the Bayesian network structure under a group of data samples.

Hsieh et al. [98] regarded the optimization problem as a kind of reinforcement learning problem and proposed a swarm optimization algorithm based on Q-learning to solve the economic dispatch problem of power systems. Almahdi et al. [116] proposed a constrained portfolio trading system for asset allocation and constraint optimization by combining recurrent reinforcement learning and particle swarm optimization with the Calmar ratio. Bora et al. [113] proposed an improved non-dominated sorting genetic algorithm (NSGA-II) method that combined with a reinforcement learning technique to achieve parameter-free self-tuning, thus achieving multi-objective optimization of environmental/economic dispatch problems.

4.3 Continuous Optimization

Continuous optimization is also a type of optimization problem. Continuous optimization problems generally exist in the research fields and application practice of mathematics, computer science, finance, engineering, and so on. Many researchers have applied LIOAs to solve continuous optimization problems. Balusu et al. [121] adopted the adaptive decision strategy of learning automata and the powerful searchability of genetic algorithm to solve the multicast channel allocation problems in multi-radio multi-channel and reduce the interference in the network. Dora et al. [226] proposed a new learning algorithm to maximize the inter-class margin, a three-layer spiking neural network for pattern classification problems. The algorithm's performance is illustrated by a set of test functions and two practical problems.

Cao et al. [167] proposed an active learning brainstorm optimization algorithm based on the dynamic cluster cycle. The algorithm's performance was verified by a set of test functions and two real-world problems. Darwish et al. [162] developed two integrated models of pre-trained convolutional neural networks by classifying images of healthy and unhealthy leaves and proposed a particle swarm optimization algorithm based on convolutional neural networks and orthogonal learning to diagnose plant diseases. Emary et al. [111] proposed a variant of the grey wolf optimization algorithm that used reinforcement learning principles and neural networks to improve the performance of the algorithm. Its purpose is to overcome the need to set the correct parameters for the algorithm through reinforcement learning.

4.4 Dynamic optimization

Compared with static problems such as extremum function problems, many real-world optimization problems are dynamically changing. This type of problem is called a dynamic optimization problem or a dynamic environment optimization problem. The objective function, constraints, pareto front, etc., may be changing all the time in such problems [233]. Many researchers have applied lioas to solve dynamic optimization problems.

Vafashoar et al. [183] proposed a multi-population differential evolution algorithm to solve the dynamic optimization problems. The cellular learning automaton adjusted the behavior of each subpopulation by adaptive control of its update scheme in the proposed method. Zhu et al. [219] proposed a learning enhanced differential evolution algorithm for processing optimal power flow problems. The results show the efficiency of this algorithm by experimenting on the dynamic ieee 30-bus system and ieee 118-bus system. The proposed method can be learned from the personal best knowledge of each particle and the characteristics of the problem at hand.

Cao et al. [206] proposed a novel particle swarm optimization algorithm to solve the dynamic optimization problems. The proposed algorithm combined the neighbor-based learning strategy into the speed update of pso to enhance the particle exploration and exploitation capability. Chang et al. [213] used the query-based learning dynamic particle swarm optimization algorithm to solve dynamic optimization problems. The proposed query-based learning mechanism included two learning strategies that integrated the concepts of diversity and memorability into pso.

Feng et al. [147] proposed a generalized opposition-based learning monarch butterfly optimization algorithm with gaussian perturbation in which an opposition-based learning strategy was applied to half of the individuals in the population at the later stage of evolution, and gaussian perturbation was applied to some individuals with poor

fitness. Shahrabi et al. [109] used the reinforcement learning method with q factor to solve the parameter estimation problems of dynamic job-shop scheduling. Das et al. [100] proposed a new hybrid method through introducing q-learning based on four basic principles and pso. The method determined the path optimization trajectory velocity algorithm of multiple robots in the debris environment by modifying the parameters and differential perturbation to improve the convergence.

4.5 Multi-objective Optimization

Multi-objective optimization refers to the simultaneous optimization of multiple objectives in a given region. The solution of multi-objective optimization is usually a set of equilibrium solutions, that is, a set of optimal solutions composed of many Pareto optimal solutions. Each element in the set is called a Pareto optimal solution or a non-inferior optimal solution. Many researchers have applied LIOAs to solve multi-objective optimization problems.

Dai et al. [120] proposed an orthogonal evolutionary algorithm based on learning automata to solve the multi-objective optimization problem of continuous variables. The experiments showed that the algorithm can effectively obtain the accurate pareto optimal set and the broad pareto optimal frontier. Antonelli et al. [230] proposed a method based on multi-objective evolutionary algorithms to simultaneously learn the rules and databases of fuzzy rule-based classifiers. Lv et al. [210] proposed an effective multi-objective firefly algorithm based on the compensation factor and elite learning. Cheng et al. [172] proposed a hybrid multi-objective algorithm based on teaching learning and PSO with cyclic crowding sort for solving multi-objective problems.

Yu et al. [187] proposed a multi-swarm comprehensive learning particle swarm optimization for multi-objective optimization. The method involved multiple groups: Each group was associated with a single original objective, and the individual optimal position of each particle was determined only according to the corresponding single objective. Jiang et al. [197] proposed a multi-objective differential evolution algorithm based on dynamic covariance matrix learning and based on all or part of the population information distribution to establish an appropriate coordinate system for the binomial crossover operator by eigen decomposition. Al-Obeidat et al. [195] used a differential evolution algorithm to learn and optimize the output of the classification method PROAFTN.

Chang et al. [212] proposed a PSO algorithm with query-based learning to improve the exploration and exploitation ability of PSO and to solve the multi-objective power contract problems. Gong et al. [217] proposed a multi-objective, self-paced learning method to optimize both the loss function and the self-paced regularize. Ma et al. [141] integrated

opposition-based learning into the multi-objective evolutionary algorithm based on the decomposition framework to speed up its convergence. Shen et al. [91] proposed a multi-objective, two-archive memetic algorithm based on Q-learning to solve software project scheduling problems in a proactive rescheduling way. Wang et al. [127] developed an evolutionary multi-objective optimization algorithm based on adaptive population structure learning.

5 Applications of LIOAs in Engineering Optimization

Figure 5 shows that the proportion of engineering applications is relatively low compared with other issues, but a large number of researchers have also applied learning-based intelligent optimization algorithms (LIOAs) to engineering applications.

Bai et al. [163] proposed an improved artificial bee colony algorithm based on orthogonal learning. The algorithm was validated by the IEEE-30 and 118 bus test systems and applied to the optimal power flow problem. Xiong et al. [156] proposed an orthogonal learning competitive swarm optimizer to solve the economic dispatch problems. The orthogonal learning strategy provided a systematic search engine. Samma et al. [92] developed a simulated annealing algorithm based on Q-learning to solve constrained engineering design problems. Wang et al. [126] proposed a PSO algorithm based on self-adaptive learning for the economic load dispatch problems of power systems.

Bahmani-Firouzi et al. [133] developed a scenario-based fuzzy, self-adaptive learning, particle swarm optimization (PSO) method to consider the dynamic economic emission dispatch problems in combination with wind power plants. Das et al. [171] proposed a hybrid algorithm that combined a support vector machine with teaching-based learning optimization for commodity futures index forecasting. Mahadevan et al. [189] proposed a comprehensive learning PSO algorithm to solve the reactive power dispatch problems. Three different test cases are being studied by using standard IEEE-30 bus and 118 bus test systems. Ding et al. [106] developed a fruit fly optimization algorithm based on hybrid adaptive cooperative learning to solve complex image processing problems.

Maitra et al. [104] proposed a particle swarm optimization algorithm based on cooperative-comprehensive learning to solve the multi-threshold image segmentation problem. Bhandari et al. [175] proposed a 3D Otsu algorithm based on a context-sensitive energy threshold, which used human learning optimization to process multilevel color image segmentation. Ewees et al. [150] proposed an improved grasshopper optimization algorithm based on opposition-based learning strategies to solve optimization functions and

engineering problems, including welded beam design problem, tension/compression spring design problem, three-bar truss design problem, and pressure vessel design problem.

Gao et al. [146] developed a harmony search algorithm with opposition-based learning to solve the optimal wind generator design case. Bora et al. [115] proposed an improved NSGA-II based on reinforcement learning to solve the satellite coverage problems. Xue et al. [124] proposed a discrete differential evolution algorithm based on self-adaptive learning to solve the cooperative jamming weapon-target assignment problems. Li et al. [125] proposed a new hybrid model based on self-adaptive learning particle swarm optimization and support vector regression for ore grade estimation. Hu et al. [79] proposed a memetic algorithm based on comprehensive learning particle swarm optimization for short-term load forecasting.

6 Conclusions

This paper systematically summarized and studied learning-based intelligent optimization algorithms (LIOAs). We searched through Google Scholar with the keyword “learning” and found papers related to intelligent optimization algorithms. From the various papers collected for this study, we selected and used 154 representative papers from 1 January 2006 to 20 March 2020 for our survey. Through the summary analysis of these papers from different perspectives, such as learning methods, optimization scenarios, and application examples, we can see the development trend and space of LIOAs. Researchers used a large number of learning mechanisms to improve the intelligent optimization algorithms to improve the optimization performance of these algorithms and successfully applied them to various optimization fields. LIOAs have achieved good results in many optimization problems and engineering applications, but we still consider that some problems are worth studying during the next few years.

- (1) Sections 4.1 and 4.3 pointed out that LIOAs have achieved some notable accomplishments in solving discrete and continuous optimization problems despite different learning mechanisms. Therefore, expanding the application scope of LIOAs and designing suitable discrete operators and continuous operators based on the learning mechanism will be a challenge in future research.
- (2) Section 4.5 pointed out that LIOAs have been used by many researchers to solve multi-objective problems and have achieved some significant results. Multi-objective problems usually refer to the problems with 2–3 objectives. However, a lot of many-objective problems in the real world are problems with more than four objective

functions. Therefore, extending LIOAs to solve many-objective problems is also a future challenge.

- (3) We can see in Fig. 5 that a large proportion of LIOAs is used for benchmark evaluation (accounting for 37.7%) and has not been applied to specific optimization problems or application engineering. For example, some researchers evaluated related algorithms on CEC 2013 [128, 161, 184, 209], CEC 2014 [144, 181, 199], CEC 2015 [148, 149], CEC 2017 [136, 173, 191–193], and other benchmarks [169, 178, 194]. Some LIOAs are also used for other problems (accounting for 20.8%), such as [143, 198, 201, 202, 221, 225, 227]. Therefore, it will be of far-reaching significance to expand the application scope of LIOAs and apply more LIOAs to the solution of more real problems.
- (4) Figure 5 shows that LIOAs have a lower proportion of engineering applications than the others. This is undoubtedly a shortcoming of LIOAs. Therefore, expanding LIOAs for more engineering applications is also an important challenge in future research work.
- (5) The current LIOAs have achieved significant optimization effects in various aspects, yet most researchers have only focused on the optimization effects of LIOAs, and there is not enough explanation for theoretical analysis. Therefore, strengthening the theoretical analysis of LIOAs and proper mathematical interpretation of the LIOA model is also a severe challenge in future research work.

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