

# Santorini

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## Goal

This semester, I worked with Matthew Yu on the game Santorini. Our goal was to define a tier-based system for the game, then calculate an upper bound on positions in each tier.

## Tier System

One aspect that makes Santorini unique is that the initial location of player pieces is not deterministic – the first player must first decide on what squares to place his two pieces, then the second player decides; only then does the moving/building phase start.

Thus, we define tier 0 to be the empty board, tiers 1 and 2 as the first player's piece locations, and tiers 3 and 4 as the second player's piece locations. To advance to tier 5 and beyond, a player moves and places a building block (e.g. tier 6 contains game states with two blocks on the board).

## Counting

To count the number of positions in each tier (5 and above), we use the following steps:

- 1) Iterate over all possible numbers of layer 0 blocks.
- 2) Calculate how many ways the layer 0 blocks can be rearranged.
- 3) Iterate over all possible numbers of layer 1 blocks.
- 4) Calculate how many ways the layer 1 blocks can be rearranged, on top of the layer 0 blocks.
- 5) Iterate over all possible numbers of layer 2 blocks.
- 6) Calculate how many ways the layer 2 blocks can be rearranged, on top of the layer 1 blocks.
- 7) The values from (1), (3), and (5) as well as the current tier number should determine the number of layer 4 blocks (blue caps).
- 8) Calculate how many ways the layer 4 blocks can be rearranged.
- 9) Calculate how many ways the four player pieces can be rearranged, among squares that do not contain layer 4 blocks.

To compute these values efficiently, I've written a Python script that takes in multiple parameters (width/height, max number of pieces per layer, number of player pieces) and computes an upper bound for each tier.

## Results

Here is sample output from the script, using default parameters – 5x5 board, 2 pieces per player, and max 22-18-14-18 blocks per layer:

```
Tier 0: 1
Tier 1: 25
Tier 2: 300
Tier 3: 6900
Tier 4: 75900
Tier 5: 1897500
Tier 6: 24667500
Tier 7: 222007500
Tier 8: 1553748900
Tier 9: 9004320600
Tier 10: 44929005000
Tier 11: 198178695000
Tier 12: 787461057900
Tier 13: 2858571530100
Tier 14: 9583395475500
Tier 15: 29926491457500
Tier 16: 87650854773900
Tier 17: 242148403998600
Tier 18: 633990041223000
Tier 19: 1579385205816000
Tier 20: 3756410701965900
Tier 21: 8554733484155100
Tier 22: 18702057208840500
Tier 23: 39335432332498500
Tier 24: 79751009696853000
Tier 25: 156132837123473100
Tier 26: 295612483893655500
Tier 27: 542019362784718500
Tier 28: 963610389600490500
Tier 29: 1662876493296770400
Tier 30: 2788176734089236600
Tier 31: 4546431280991826600
Tier 32: 7215430229918252100
Tier 33: 11153554688900220300
Tier 34: 16803987348004814100
Tier 35: 24689994305177001900
Tier 36: 35397803058665668500
Tier 37: 49544188847351068200
Tier 38: 67727252780426197200
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Tier 39: 90461044761157092000
Tier 40: 118097494100606994600
Tier 41: 150742214849746852200
Tier 42: 188173645244388751200
Tier 43: 229777069824875727000
Tier 44: 274505790438737655600
Tier 45: 320880629702717442600
Tier 46: 367035870196848382200
Tier 47: 410814733321294599000
Tier 48: 449910947867982090600
Tier 49: 482045535216779516400
Tier 50: 505160773785154436400
Tier 51: 517608110303833296000
Tier 52: 518305682905804932600
Tier 53: 506845736838117883800
Tier 54: 483542082098386891800
Tier 55: 449419175936464080600
Tier 56: 406151649061734655200
Tier 57: 355962322830100265400
Tier 58: 301480791472407511800
Tier 59: 245563569014396443800
Tier 60: 191089570882088003400
Tier 61: 140761080321428120400
Tier 62: 96924238712882182800
Tier 63: 61361286085715470800
Tier 64: 34988024688901706400
Tier 65: 17536695021883184400
Tier 66: 7514810239745160000
Tier 67: 2667000541409226000
Tier 68: 754679541754314000
Tier 69: 162080142386580000
Tier 70: 24583633004700000
Tier 71: 2324147197680000
Tier 72: 101936280600000
Tier 73: 0
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Total: 8149911102734199893126

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Here, the maximum tier possible is tier 72, since there can only be up to 68 blocks on the board ( $22 + 18 + 14 + 14$ ).

The total number of positions is roughly  $8 \cdot 10^{21}$ , which is much too big to solve.

One possible variant that brings the game into solving territory is making the board 3x3, which results in an upper bound of  $604800298 \approx 6 \cdot 10^8$  positions (over 37 tiers).