Digital Signal Processing Exercise

A rare cosmic energy wave has traveled millions of light-years and finally reaches Earth.

A research station's ultra-sensitive antenna captures this wave, but before it reaches the receiver, the signal gets mixed with Earth's atmospheric noise, satellite interference, and background radiation.

The antenna output is converted into an electrical signal and enter the Signal Processing Module. Captured continuous-time cosmic wave is as follows:

$$x(t) = \sin(16\pi t) + 0.7\sin(100\pi t) + 0.5\sin(200\pi t) + n(t)$$

where $n(t)$ is atmospheric and satellite noise.

1. The receiver samples this signal at sampling frequency $Fs = 500\ Hz$ for a duration of 2 seconds.
   a) Generate and sample the signal $x(t)$ in MATLAB to obtain the discrete-time sequence $x[n]$.
   b) comment whether the chosen sampling frequency satisfies the Nyquist criterion for the given signal.

2. Using FFT, analyze the frequency spectrum of the captured cosmic wave.

   a) Compute the Fast Fourier Transform (FFT) of $x[n]$ and analyze the frequency components by plotting the magnitude spectrum for frequencies from $0\ to\ Fs/2 = 250\ Hz$
   b) Identify which frequencies likely belong to the true cosmic wave.

3. The scientists reveal that the real message exists only within the band 2 Hz to 20Hz which is known as the cosmic pulse band.

   All other components must be suppressed.

   a) Design a band-pass FIR filter using a Hamming window in MATLAB. Use normalized
   b) passband edge frequencies fp = [0.008, 0.08] (normalized values) and a filter order of N = 80.
   c) Plot the magnitude response of your FIR filter (in dB).
   d) Comment on the main-lobe width, the side-lobe level, and why the Hamming window is a reasonable choice for extracting the cosmic band.

4. Design a similar cosmic band-pass filter using the FDA Tool in MATLAB with the following specifications:
   Filter Type: Bandpass
   Design Method: FIR Equiripple
   Filter Order: 80
   Sampling Frequency: 500 Hz
   Passband should cover 2–20 Hz range, with suitable stopbands on ±1Hz
   Magnitude Specifications: W_{stop1}=20, W_{pass}=1, W_{stop2}=20.

   a) Generate and view the following using FDA Tool:
      - Magnitude response
      - Phase response
      - Impulse response and step response
   b) Export the designed filter to the MATLAB workspace as filter coefficients as a (denominator) and b (numerator) using the FDA Tool.
      - Use 'freqz' to plot the magnitude response of the FDA Tool filter.
      - Compare this response with the FIR filter you coded in Part 3 on the same figure.
   c) Briefly discuss:
      - Any differences in transition band sharpness,
      - Passband flatness,
      - Stopband attenuation.
      - Which design (Part 3 vs FDA Tool) seems more suitable for preserving the cosmic message while removing interference?

5. Use the FDA Tool filter (from Part 4) to filter the sampled signal x[n].

   a) Apply the filter using the MATLAB command:
      $$y[n] = filter(b, a, x[n])$$
   b) Plot the following in the time domain:
      - Original noisy signal x[n]
      - Filtered signal y[n]

6) Perform IFFT-based reconstruction as follows:

   a) Compute the FFT of $x[n]$.
   b) Create a binary frequency mask that keeps only components within 2–20 Hz and sets all others to zero.
   c) Apply this mask to the FFT spectrum.
   d) Compute the inverse FFT (IFFT) to obtain a reconstructed time-domain signal $x_{\text{rec}}[n]$
   e) Plot:
      - The reconstructed signal
      - The FDA Tool filtered signal.
      - Comment on the similarity and differences between the time-domain signals obtained by FIR/FDA filtering and Frequency-domain masking & IFFT.

7) Scientists wants to analyze this cosmic wave by using an IIR band-pass filter to isolate the same cosmic frequency band used in previous parts of the assignment.

Design an IIR band-pass filter in MATLAB that meets the following conditions.

Design specifications are as follows:

Sampling frequency	: Fs = 500 Hz
Passband frequencies	: 2 Hz ≤ f ≤ 20 Hz
Stopband frequencies	: 0 Hz≤ f ≤ 1 Hz and 25 Hz ≤ f ≤ 250 Hz
Passband ripple requirement	: A_p=1 dB
Stopband attenuation requirement:	A_s = 40 dB

a) Select a suitable IIR filter from Butterworth, Chebyshev Type-I, Chebyshev Type-II, Elliptic (Cauer) and justify your answer.
b) Compute the normalized passband and stopband edges for the selected IIR filter. Here consider a 4^{th} Oder Filter.
c) Obtain the numerator and denominator coefficient vectors: b_{iir} and a_{iir}
d) Using MATLAB 'freqz' function, plot Magnitude response (in dB) and Phase response. The x-axis must be in Hz over the full range 0–250 Hz.
e) Filter the previously obtained noisy discrete-time signal $x[n]$ to obtain the IIR-filtered signal

$$y_{iir}[n] = filter(b_{iir}, a_{iir}, x[n])$$

Plot the first 1second of the original noisy signal and the IIR-filtered signal on the same figure.

f) Generate a pole-zero plot for the constructed filter and comment on the stability and attenuation of the system.
g) Compier the stability and the attenuation of the filter designed using FDA Tool and the constructed IIR filter.