Universität Zürich[UZH]

Institute for Banking and Finance
Plattenstrasse 14
CH-8032 Zürich
www.bf.uzh.ch

Prof. Dr. Marc Paolella
Professor of Empirical Finance
Marc.pao

Research Proposal

for:

Jan Heinrich Schlegel 19-747-096
Dorfstrasse 3, 9479 Malans SG
janheinrich.schlegel@uzh.ch

Title

Portfolio Value at Risk Forecasting with GARCH-Type Models

Hypothesis

Regulators use risk measures such as the Value at Risk (VaR) to define the adequate amount of risk capital that financial institutions have to hold. Thus, it is essential to determine how said risk measure should be predicted so that it neither severely underestimates nor overestimates unexpected losses that might occur in the future. Therefore, I want to investigate how well different approaches forecast the VaR. I hope to find answers to the following questions:

- Which of the models performs the best overall?
- Do the studied multivariate models accomplish better performance than the univariate models?
- Does the COMFORT model outperform the Copula-GARCH models described in Fortin et al. (2022)?
- Do the models that incorporate factor returns yield better forecasts?

Relevance

After the financial crisis of 2008, reducing systemic risk has become paramount. Consequently, regulators enforced more stringent risk management in financial institutions to ensure the stability of the financial system. Hence, companies are obliged to hold risk capital that can be used as a buffer to compensate (potential) unexpected future losses. To determine what amount of risk capital is deemed adequate, risk measures such as the VaR and the ES are used. Even though the VaR is more widely used in regulatory frameworks there are some frameworks such as the Swiss Solvency Test, which applies to Swiss insurances, that use the ES. Another application of risk measures is portfolio optimization (see e.g. Rockafellar and Uryasev, 2000). Further, the Modigliani–Miller theorem states that in an ideal world with complete markets a firm's capital structure is irrelevant for the firm's value. In the real world (with presence of taxes, bankruptcy costs and transaction costs) however, risk management may also be beneficial from a corporate perspective since it can reduce taxes and the likelihood of bankruptcy.

Existing Literature

Risk measures are a very active research area due to their practical relevance. Of special interest are risk measures that are coherent. It is important to note that VaR does not fullfill the conditions of a coherent risk measure introduced by Artzner et al. (1999). The ES however is a coherent risk measure (Acerbi & Tasche, 2002). Further important differences between VaR and ES are demonstrated for example in Embrechts et al. (2001) and Embrechts et al. (2014). In Bachelor courses which treat VaR one often discusses the variance-covariance method to estimate said risk measures that assumes normally distributed returns and constant volatility (homoskedasticity). However, in financial time series data we can often observe heteroskedastic behaviour in the form of volatility clustering, a phenomenon first described by Mandelbrot (1963). To deal with this problem of conditional heteroskedasticity Engle (1982) introduced the ARCH model. This model was later generalized to the GARCH model by Bollerslev (1986). Subsequently, models such as the Exponential GARCH (EGARCH) by Nelson (1991), the Threshold GARCH (TGARCH) by Zakoian (1994) or the Asymmetric Power ARCH (APARCH) by Ding et al. (1993) have been proposed to allow negative and positive returns to have a different impact on the volatility to account for the so-called leverage effect. Consequently, new approaches have been suggested to estimate risk measures that use some variation of a GARCH model to forecast the conditional volatility (see e.g. Angelidis et al. (2004)). Additionally, it is consensus in finance literature that the normal distribution is inappropriate for modeling stock returns. Hence, GARCH models with an innovation distribution that can incorporate skewness and fat tails yield better VaR forecasts than GARCH models with normal innovation terms (Kuester et al., 2006). In addition to the univariate models mentioned above, multivariate GARCH models have been presented to not only model an asset's conditional volatility but also the conditional dependency structure between several assets over time. Examples for such multivariate GARCH models are the Constant Conditional Correlation GARCH (CCC-GARCH) by Bollerslev (1990) or the Dynamic Conditional Correlation GARCH (DCC-GARCH) by Engle (2002). These multivariate GARCH models can be used to directly forecast VaR or as a foundation for more complex models such as the COMFORT model described in Paolella and Polak (2015).

Another frequently seen toolkit in financial econometrics are copulas. Particularly in quantitative risk management it may be useful to model the marginals independently of the copula structure. Several papers such as Palaro and Hotta (2006) and Huang et al. (2009) simulate observations from a copula to forecast VaR. These papers first fit ARMA-GARCH processes to the marginals (i.e. return series of single stocks) and then model the dependence structure using a conditional copula. They found that said models produce better VaR forecasts than traditional methods. In a recent paper Fortin et al. (2022) also used Copula-GARCH models to forecast VaR and ES. Interestingly, they used this Copula-GARCH structure to simulate future factor returns and only in a second step calculated the forecasted stock returns using Carhart's four-factor model. However, they found no significant outperformance of these complex multivariate models of individual asset returns over simpler univariate models for portfolio returns.

An important part of research in this area is comparing different models, especially univariate and multivariate approaches (see e.g. Santos et al., 2013). Thus, it is essential to assess how well a certain model forecasted the VaR. For this the VaR forecasts are backtested. Notable publications in this field are for example Kupiec (1995) and Christoffersen (1998). In addition, the relative performance of models is often of interest. Consequently, tests such as the conditional predictive ability (CPA) test by Giacomini and White (2006) have been developed to compare the performance between different models.

Methodology

I want to compare different univariate and multivariate methods for predicting portfolio VaR. In particular, I want to revisit the paper by Fortin et al. (2022) and compare the VaR forecasts of the models described in this paper to other established models in literature. But contrary to Fortin et al., 2022 I am planning to use daily data instead of weekly data since a lower data frequency can hurt multivariate models. The forecasting will be done using a rolling window approach i.e. using the last 1000 observations to forecast the one-step-ahead 1%-VaR. As a baseline I am planning to use a standard GARCH(1,1) model with normal innovation. Other univariate models that will be included are the GJR-GARCH, the Mix-Normal GARCH as described in Kuester et al. (2006), the EWMA as used in JP Morgan's *RiskMetrics Technical Document* (1996) and a selection of the univariate NGARCH models in Fortin et al. (2022). For the multivariate models I want to include a selection of the Copula-GARCH models stated in Fortin et al. (2022), the COMFORT model by Paolella and Polak (2015) and a DCC-GARCH for reference. Backtesting will be done as in Santos et al. (2013) using tests for independence, conditional and unconditional coverage. Additionally, the relative performance of the models will be tested using the CPA test from Giacomini and White (2006). The portfolio will be equal weighted and consists of the same ten large capitalization companies that were used in Fortin et al. (2022). For this, I require the historical daily returns of said stocks in the time frame January 2nd 2001 to December 30th 2011. The required stock data is freely available on Yahoo Finance. Additionally, this thesis requires the historical daily factor returns of the three Fama-French factors and of the Momentum factor. This is due to the fact that the models in Fortin et al. (2022) require the equity factors from the Carhart four-factor model. This data can be found on Kenneth French's data library. As for software, I am planning to use R.

Schedule

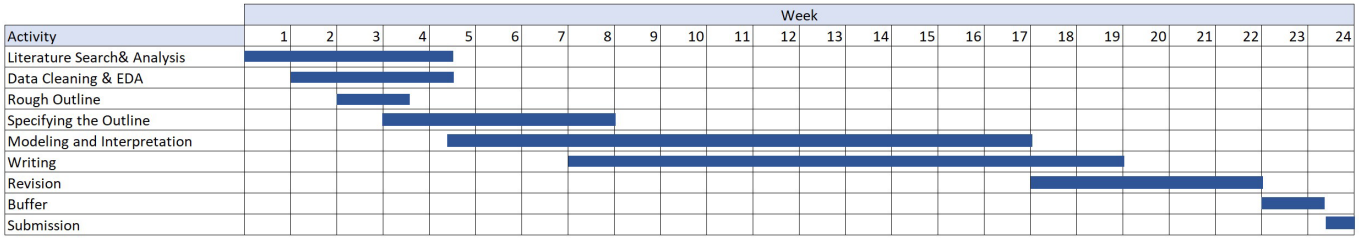

Figure 1: Schedule of my Bachelor Thesis

Outline

1. Introduction
2. Theoretical Framework
   - 2.1. Introduction to Risk Measures
     What is a risk measure? How is the VaR defined?
   - 2.2. Univariate Models
     Presenting the univariate GARCH models
   - 2.3. Multivariate Models with Factor Returns
     Presenting the Copula-GARCH models used in Fortin et al. (2022) following a succinct introduction to copula theory.
   - 2.4. COMFORT Model
     Presenting the COMFORT model as described in Paolella and Polak (2015)
3. Methodology
   - 3.1. Data
     Exploratory data analysis and source of the data
   - 3.2. VaR Forecasts

Describing how the VaR will be forecasted with the different models.

3.3. Backtesting
Describing how backtesting will be conducted

4. Results
Presenting my results and comparing them to the existing literature

5. Conclusion

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For any further questions, please contact Prof. Marc Paolella, Department of Banking and Finance, marc.paolella@bf.uzh.ch.

I wish you good luck and success!

Kind regards

Marc Paolella
Professor of Empirical Finance
Universität Zürich
Institut für Banking und Finance