Value–at–Risk Models

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Abstract In this chapter, we build first a univariate and then a multivariate filtered historical simulation (FHS) model for financial risk management. Both the univariate and multivariate methods simulate future returns from a model using historical return innovations. While the former relies on portfolio returns filtered by a dynamic variance model, the latter uses individual or base asset return innovations from dynamic variance and correlation models. The univariate model is suitable for passive risk management or risk measurement whereas the multivariate model is useful for active risk management such as optimal portfolio allocation. Both models are constructed in such a way as to capture the stylized facts in daily asset returns and to be simple to estimate. The FHS approach enables the risk manager to easily compute Value-at-Risk and other risk measures including Expected Shortfall for various investment horizons that are conditional on current market conditions. The chapter also lists various alternatives to the suggested FHS approach.

1 Introduction and Stylized Facts

In this chapter, we apply some of the tools from previous chapters to develop a tractable dynamic model for computing the Value-at-Risk (VaR) and other risk measures of a portfolio of traded assets.

The VaR is defined as a number such that there is a probability p of exhibiting a worse return over the next K days, and where p and K must

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be predetermined by the risk manager. The VaR is thus simply a quantile of the return distribution. Clearly, the quantile of a distribution does not tell us everything about risk. Importantly, it does not tell us how large the likely magnitude of losses is on those days when the return is worse than the VaR. Expected Shortfall (ES), which is defined as the expected return conditional on the return being worse than the VaR, has been suggested as an alternative to VaR and will also be discussed in this chapter. But, VaR remains by far the most common risk metric used in practice.

The so-called historical simulation (HistSim) method has emerged as the industry standard for computing VaR. It computes VaR in two simple steps. First, a series of hypothetical historical portfolio returns are constructed, using today's portfolio weights and historical asset returns. Second, the quantile of the hypothetical historical portfolio returns is computed. Advocates of the HistSim approach highlight its "model-free" nature. However it is clearly not "assumption-free". HistSim essentially assumes that asset returns are i.i.d. which is unfortunately not the case empirically.

The objective in this chapter is therefore to design a dynamic alternative to the static HistSim approach. Specifically, we wish to build a risk model with the following characteristics:

- The model is a fully specified data-generating process which can be estimated on daily returns
- The model can be estimated and implemented for portfolios with a large number of assets
- VaR can be easily computed for any prespecified level of confidence, p, and for any horizon of interest, K
- VaR is conditional on the current market conditions
- Risk measures other than the VaR can be calculated easily

To deliver accurate risk predictions, the model should reflect the following stylized facts of daily asset returns

- Daily returns have little or no exploitable conditional mean predictability
- The variance of daily returns greatly exceeds the mean
- The variance of daily returns is predictable
- Daily returns are not normally distributed
- Even after standardizing daily returns by a dynamic variance model, the standardized daily returns are not normally distributed
- Positive and negative returns of the same magnitude may have different impact on the variance
- Correlations between assets appear to be time-varying
- As the investment horizon increases, the return data distribution approaches the normal distribution

Again, the objective is to build a dynamic market risk management model that captures these salient features of daily asset returns, that contains only few parameters to be estimated, and that is easily implemented on a large set of assets.

In Section 2, we will consider a univariate approach and, in Section 3, a multivariate approach to dynamic risk modeling. The univariate model simulates historical portfolio return shocks from a dynamic variance model, and the multivariate model simulates historical asset return shocks by means of both dynamic variance and correlation models. The univariate model is suitable for passive risk management or risk measurement, whereas the multivariate model is useful for active risk management such as optimal portfolio allocation. The end of each section will discuss alternatives to the approach taken here. Section 4 concludes.

2 A Univariate Portfolio Risk Model

In this section, we will consider a simple univariate approach to modeling the dynamic risk of a portfolio. Just as in the HistSim approach mentioned above, we consider a time series of T hypothetical historical portfolio returns computed using today's portfolio weights, and historical returns on n assets

$$\{r_t\}_{t=1}^{T} \equiv \left\{ \sum_{j=1}^{n} w_{T,j} r_{t,j} \right\}_{t=1}^{T},$$

where $r_{t,j}$ denotes the log return on asset j from the market close on day $t-1$ to market close on day t, that is, $r_{t,j} = \ln(S_{t,j}/S_{t-1,j})$, and where $w_{T,j}$ denotes today's weight of asset j in the portfolio.

The univariate risk model proceeds by simply modeling the properties of the univariate portfolio return, r_t. One great advantage of this approach is that the correlations and other interdependencies between the n assets do not need to be modeled. The downside however of the approach is that it is conditional on the portfolio weights. When these weights change, then so should the estimated risk model. This portfolio level approach is sometimes referred to as a *passive* risk model as it does not directly allow for studying the effects of actively managing the risk of the portfolio by changing the portfolio weights.

We proceed by making some key assumptions on the daily portfolio return process. We will assume that

$$r_t = \sigma_t z_t \qquad z_t \overset{i.i.d}{\sim} G(0,1), \tag{1}$$

where the dynamic return volatility σ_t is known at the end of day $t-1$, and where the independent and identically distributed (i.i.d.) return shock, z_t, is from a potentially nonnormal distribution, $G(0,1)$, with zero mean and unit variance. Note that we set the return mean to zero, allowed for time-varying volatility, and for conditional nonnormality. These assumptions are all in line with the stylized facts outlined in Section 1. Note also, however, that we have ruled out time-varying conditional skewness and kurtosis which is sometimes found to be relevant in asset return modeling. See for example Hansen (1994) and Harvey and Siddique (1999).

We proceed by first modeling and estimating σ_t, and then, subsequently, moving on to the specification of $G(0,1)$.

2.1 The dynamic conditional variance model

In order to capture the time-varying volatility found in the daily returns, we rely on the NGARCH(1,1) model. In this model, the variance for date t can be computed based on the return and the variance for date $t-1$ as follows:

$$\sigma_t^2 = \omega + \alpha \left(r_{t-1} - \theta\sigma_{t-1}\right)^2 + \beta\sigma_{t-1}^2,$$

where a positive θ captures the fact that a negative return will increase variance by more than a positive return of the same magnitude. This asymmetry effect is one of the stylized facts listed in Section 1.

The unconditional –or long-run– variance in this model can be derived as

$$\sigma^2 = E\left[\sigma_t^2\right] = \frac{\omega}{1-\alpha\left(1+\theta^2\right)-\beta} \equiv \frac{\omega}{\kappa},$$

where $\kappa \equiv 1-\alpha\left(1+\theta^2\right)-\beta$ is interpretable as the speed of mean reversion in variance.

Setting $\omega = \sigma^2\kappa$ and substituting it into the dynamic variance equation yields

$$\begin{aligned}\sigma_t^2 &= \sigma^2\kappa + \alpha\left(r_{t-1} - \theta\sigma_{t-1}\right)^2 + \beta\sigma_{t-1}^2 \\ &= \sigma_{t-1}^2 + \kappa\left(\sigma^2 - \sigma_{t-1}^2\right) + \alpha\left(r_{t-1}^2 - \sigma_{t-1}^2 - 2\theta r_{t-1}\sigma_{t-1}\right),\end{aligned} \tag{2}$$

where, in the second line, we have simply expanded the square and applied the definition of κ.

The advantage of writing the model in this form is two-fold. First, we can easily impose the long-run variance, σ^2, to be the sample variance, before estimating the other parameters. This is referred to as variance targeting. Second, we can easily impose variance stationarity on the model, by ensuring that $\kappa > 0$ when estimating the remaining parameters. Finally, we guarantee variance positivity by forcing $\alpha > 0$ when estimating the parameters.

The parameters $\{\kappa, \alpha, \theta\}$ that determine the volatility dynamics are easily estimated by numerically optimizing the quasi maximum likelihood criterion of the estimation sample

$$QMLE\,(\kappa,\alpha,\theta) = -\frac{1}{2}\sum_{t=1}^{T}\left(\ln\left(\sigma_t^2\right) + r_t^2/\sigma_t^2\right). \tag{3}$$

Typically, κ is found to be close to zero reflecting slow mean reversion and thus high predictability in the daily variance. The autocorrelation function of the absolute shock, $|z_t| = |r_t/\sigma_t|$, provides a useful diagnostic of the volatility model.

2.2 Univariate filtered historical simulation

We now turn to the specification of the distribution $G(0,1)$ of the return shock, z_t. The easiest way to proceed would be to assume that the shocks follow the standard normal distribution. As the standard normal distribution has no parameters, the specification of the model would then be complete and the model ready for risk forecasting. From the list of stylized facts in Section 1, we know however that the assumption of a normal distribution is not appropriate for most speculative assets at the daily frequency.

The question which alternative distribution to choose then arises? Rather than forcing such a choice, we here rely on a simple resampling scheme, which, in financial risk management, is sometimes referred to as filtered historical simulation (FHS). The term "filtered" refers to the fact that we are not simulating from the set of raw returns, but from the set of shocks, z_t, which are returns filtered by the GARCH model.

It is simple to construct a one-day VaR from FHS. We calculate the percentile of the set of historical shocks, $\{z_t\}_{t=1}^T$, where $z_t = r_t/\sigma_t$, and multiply that onto the one-day ahead volatility

$$VaR_{T,1}^p = \sigma_{T+1} Percentile\left\{\{z_t\}_{t=1}^T, 100p\right\}. \tag{4}$$

where the *Percentile* function returns a number, z_p, such that $100p$ percent of the numbers in the set $\{z_t\}_{t=1}^T$ are smaller than z_p. Note that, by construction of the GARCH model, the one-day-ahead volatility is known at the end of the previous day, so that $\sigma_{T+1|T} = \sigma_{T+1}$ and we simply use the latter simpler notation. The Expected Shortfall for the one-day horizon can be calculated as

$$ES_{T,1}^p = \sigma_{T+1}\frac{1}{p*T} * \sum_{t=1}^{T} z_t * \mathbf{1}\left(z_t < VaR_{T,1}^p/\sigma_{T+1}\right),$$

where $\mathbf{1}(*)$ denotes the indicator function returning a 1 if the argument is true, and zero otherwise.

When computing a multi-day ahead VaR, the GARCH variance process must be simulated forward using random draws, $z_{i,k}$, from the historical shocks, $\{z_t\}_{t=1}^T$. The random drawing can be operationalized by generating a discrete uniform random variable which is distributed from 1 to T. Each draw from the discrete distribution then tells us which shock to select. We build up a distribution of hypothetical future returns as

$$\begin{array}{ccccccc} & & z_{1,1} & \rightarrow & r_{1,T+1} & \rightarrow & \sigma^2_{1,T+2} & \cdots \\ & \nearrow & & & \cdots & & & \cdots \\ \sigma^2_{T+1} & \longrightarrow & z_{i,1} & \rightarrow & r_{i,T+1} & \rightarrow & \sigma^2_{i,T+2} & \cdots \\ & \searrow & & & \cdots & & & \cdots \\ & & z_{M,1} & \rightarrow & r_{M,T+1} & \rightarrow & \sigma^2_{M,T+2} & \cdots \end{array}$$

$$\begin{array}{ccccccccc} z_{1,k} & \rightarrow & r_{1,T+k} & \rightarrow & \sigma^2_{1,T+k+1} & \cdots & z_{1,K} & \rightarrow & r_{1,T+K} \\ & & \cdots & & & \cdots & & & \cdots \\ z_{i,k} & \rightarrow & r_{i,T+k} & \rightarrow & \sigma^2_{i,T+k+1} & \cdots & z_{i,K} & \rightarrow & r_{i,T+K} \\ & & \cdots & & & \cdots & & & \cdots \\ z_{M,k} & \rightarrow & r_{M,T+k} & \rightarrow & \sigma^2_{M,T+k+1} & \cdots & z_{M,K} & \rightarrow & r_{M,T+K} \end{array}$$

where $r_{i,T+k}$ is the return for day $T+k$ on simulation path i, M is the number of times we draw with replacement from the T standardized returns on each future date, and K is the horizon of interest. At each time step, the GARCH model in (2) is used to update the conditional variance and the return model in (1) is used to construct returns from shocks.

We end up with M sequences of hypothetical daily returns for day $T+1$ through day $T+K$. From these hypothetical daily returns, we calculate the hypothetical $K-$day returns as

$$r_{i,T:K} = \sum_{k=1}^{K} r_{i,T+k}, \text{ for } i = 1, 2, ..., M.$$

If we collect the M hypothetical K-day returns in a set $\{r_{i,T:K}\}_{i=1}^M$, then we can calculate the $K-$day Value at Risk simply by calculating the $100p$ percentile as in

$$VaR^p_{T,K} = Percentile\left\{\{r_{i,T:K}\}_{i=1}^M, 100p\right\}.$$

At this point it is natural to ask how many simulations, M, are needed? Ideally, M should of course be as large as possible in order to approximate closely the true but unknown distribution of returns and thus the VaR. On modern computers, taking $M = 100,000$ is usually not a problem and would yield on average $1,000$ tail observations when computing a VaR for $p = 0.01$. It is important to note that the smaller the p the larger an M is needed in order to get a sufficient number of extreme tail observations.

The ES measure can be calculated from the simulated returns by taking the average of all the $r_{i,T:K}$ that fall below the $VaR^p_{T,K}$ number, that is

$$ES^p_{T,K} = \frac{1}{p * M} * \sum_{i=1}^{M} r_{i,T:K} * \mathbf{1}\left(r_{i,T:K} < VaR^p_{T,K}\right).$$

The advantages of the FHS approach are threefold. First, it captures current market conditions by means of the volatility dynamics. Second, no assumptions need to be made on the distribution of the return shocks. Third, the method allows for the computation of any risk measure for any investment horizon of interest.

2.3 Univariate extensions and alternatives

The GARCH model that we used in (2) is taken from Engle and Ng (1993). Andersen, Bollerslev, Christoffersen and Diebold (2006a) survey the range of viable volatility forecasting approaches. The filtered historical simulation approach in (4) was suggested by Barone-Adesi, Bourgoin, and Giannopoulos (1998), Diebold, Schuermann, and Stroughair (1998), and Hull and White (1998).

The general univariate model in (1) and (2) contains a number of standard risk models as special cases:

- The i.i.d. Normal model where $G(0,1) = N(0,1)$ and $\kappa = \alpha = \theta = 0$
- The RiskMetrics model where $G(0,1) = N(0,1)$ and $\kappa = 0$ and $\theta = 0$
- The GARCH-Normal where $G(0,1) = N(0,1)$
- The GARCH-CF where $G^{-1}(0,1)$ is approximated using the Cornish-Fisher approach
- The GARCH-EVT model where the tail of $G(0,1)$ is specified using extreme value theory
- The GARCH-t(d) where $G(0,1)$ is a standardized Student's t distribution

As discussed in Christoffersen (2003), these models can be estimated relatively easily using a variant of the likelihood function in (3) or by matching moments of z_t with model moments. However, they all contain certain drawbacks that either violate one or more of the stylized facts listed in Section 1, or that fail to meet one or more of the objectives listed in Section 1 as well: The i.i.d. Normal model does not allow for variance dynamics. The RiskMetrics model (JP Morgan, 1996) does not aggregate over time to normality nor does it capture the leverage effect. The GARCH-Normal does not allow for conditional nonnormality, and the GARCH-CF and GARCH-EVT (McNeill and Frey, 2000) models are not fully specified data-generating processes. The GARCH-t(d) (Bollerslev, 1987) comes closest to meeting our objectives but needs to be modified to allow for conditional skewness. See, for example, Hansen (1994).

Some quite different approaches to VaR estimation have been suggested. The Weighted Historical Simulation approach in Bodoukh, Richardson and Whitelaw (1998) puts higher probability on recent observations when computing the HistSim VaR. However, see Pritsker (2001) for a critique. The CaViaR approach in Engle and Manganelli (2004) and the dynamic quantile approach in Gourieroux and Jasiak (2006) model the return quantile directly rather than specifying a complete data generating process. Finally, note that Manganelli (2004) suggests certain univariate models for approximate portfolio allocation by variance sensitivity analysis.

3 Multivariate, Base–Asset Return Methods

The univariate methods discussed in Section 2 are useful if the main purpose of the risk model is risk measurement. If instead the model is required for active risk management including deciding on optimal portfolio allocations, or VaR sensitivities to allocation changes, then a multivariate model may be required. In this section, we build on the model in Section 2 to develop a fully specified large-scale multivariate risk model.

We will assume that the risk manager knows his set of assets of interest. This set can either contain all the assets in the portfolio or a smaller set of so-called base assets which are believed to be the main drivers of risk in the portfolio. Base asset choices are, of course, portfolio-specific, but typical examples include equity indices, bond indices, and exchange rates as well as more fundamental economic drivers such as oil prices and real estate prices. Regression analysis can be used to assess the relationship between each individual asset and the base assets.

Once the set of assets has been determined, the next step in the multivariate model is to estimate a dynamic volatility model of the type in Section 1 for each of the n assets. When this is complete, we can write the n base asset returns in vector form

$$R_t = D_t Z_t,$$

where D_t is an n by n diagonal matrix containing the GARCH standard deviations on the diagonal, and zeros on the off diagonal. The n by 1 vector Z_t contains the shocks from the GARCH models for each asset.

Now, define the conditional covariance matrix of the returns as

$$Var_{t-1}(R_t) = \Sigma_t = D_t \Gamma_t D_t,$$

where Γ_t is an n by n matrix containing the base asset correlations on the off diagonals and ones on the diagonal. The next step in the multivariate model is to develop a tractable model for Γ_t.

3.1 The dynamic conditional correlation model

We wish to capture time variation in the correlation matrix of base asset returns without having to estimate many parameters. The correlation matrix has $n(n-1)/2$ unique elements but the dynamic conditional (DCC) model offers a convenient framework for modeling these using only two parameters that require numerical estimation methods.

The correlation dynamics are modeled through past cross products of the shocks in Z_t

$$\begin{aligned} Q_t &= \Omega + \alpha \left(Z_{t-1} Z'_{t-1} \right) + \beta Q_{t-1} \\ &= Q \left(1-\alpha-\beta\right) + \alpha \left(Z_{t-1} Z'_{t-1} \right) + \beta Q_{t-1} \\ &= Q_{t-1} + \kappa \left(Q - Q_{t-1} \right) + \alpha \left(Z_{t-1} Z'_{t-1} - Q_{t-1} \right), \end{aligned} \tag{5}$$

where we have used

$$E\left[Q_t\right] \equiv Q = \Omega / \left(1-\alpha-\beta\right) \equiv \Omega / \kappa.$$

The unconditional sample covariance matrix of Z_t provides an estimate of Q, leaving only κ and α to be estimated by numerical optimization. Forcing $\kappa > 0$ in estimation ensures correlation stationarity.

The conditional correlations in Γ_t are given by standardizing the relevant elements of the Q_t matrices. Let $\rho_{ij,t}$ be the correlation between asset i and asset j on day t. Then we have

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} q_{jj,t}}}, \tag{6}$$

where $q_{ij,t}$, $q_{ii,t}$, and $q_{jj,t}$ are elements of Q_t.

The dynamic correlation parameters κ and α can now be estimated by maximizing the QMLE criterion on the multivariate sample

$$QMLE\left(\kappa, \alpha\right) = -\frac{1}{2} \sum_{t=1}^{T} \left(\log\left(\|\Gamma_t\| \right) + Z'_t \Gamma_t^{-1} Z_t \right),$$

where $\|\Gamma_t\|$ denotes the determinant of Γ_t.

3.2 Multivariate filtered historical simulation

Based on the stylized facts in Section 1, we do not want to assume that the shocks to the assets are normally distributed. Nor do we wish to assume that they stem from the same distribution. Instead, we will simulate from historical shocks asset by asset to compute forward-looking VaRs and other risk measures.

We first create a database of historical dynamically uncorrelated shocks from which we can resample. We create the dynamically uncorrelated historical shock as

$$Z_t^D = \Gamma_t^{-1/2} Z_t,$$

where, $\Gamma_t^{-1/2}$ is the inverse of the matrix square-root of the conditional correlation matrix Γ_t. The matrix square root, $\Gamma_t^{1/2}$, can be computing using the spectral decomposition of Γ_t. In their chapter in this Handbook, Patton and Sheppard (2008) recommend the spectral decomposition over the standard Cholesky decomposition because the latter is not invariant to the ordering of the return variables in the vector Z_t.

When calculating the multi-day conditional VaR and other risk measures from the model, we need to simulate daily returns forward from today's (day T's) forecast of tomorrow's matrix of volatilities, D_{T+1} and correlations, Γ_{T+1}. The returns are computed from the GARCH and DCC models above.

From the data base of uncorrelated shocks $\left\{Z_t^D\right\}_{t=1}^T$, we can draw a random vector of historical uncorrelated shocks, called $Z_{i,T+1}^D$. It is important to note that in order to preserve asset-specific characteristics and potential extreme correlation in the shocks, we draw an entire vector representing the same day for all the assets.

From this draw, we can compute a random return for day $T+1$ as

$$\begin{aligned} R_{i,T+1} &= D_{T+1} \Gamma_{T+1}^{1/2} Z_{i,T+1}^D \\ &= D_{T+1} Z_{i,T+1}. \end{aligned}$$

Using the simulated shock vector, $Z_{i,T+1}$, we can now update the volatilities and correlations using the GARCH model in (2) and the DCC model in (5) and (6). We thus obtain $D_{i,T+2}$ and $\Gamma_{i,T+2}$. Drawing a new vector of uncorrelated shocks, $Z_{i,T+2}^D$, enables us to simulate the return for the second day as

$$\begin{aligned} R_{i,T+2} &= D_{i,T+2} \Gamma_{i,T+2}^{1/2} Z_{i,T+2}^D \\ &= D_{i,T+2} Z_{i,T+2}. \end{aligned}$$

We continue this simulation for K days, and repeat it for $i = 1, ..., M$ simulated shocks.

The cumulative K-day log returns are calculated as

$$R_{i,T:K} = \sum_{k=1}^{K} R_{i,T+k}.$$

The portfolio Value-at-Risk (VaR) is calculated by computing the user-specified percentile of the M simulated returns for each horizon as in

$$VaR^p_{T,K} = Percentile\left\{\{W'_T R_{i,T:K}\}_{i=1}^M, 100p\right\},$$

where W_T is the vector of portfolio weights at the end of day T.

The Expected Shortfall (ES) is computed by taking the average of those simulated returns which are worse than the VaR

$$ES^p_{T,K} = \frac{1}{p * M}\sum_{i=1}^{M} W'_T R_{i,T:K} * \mathbf{1}\left(W'_T R_{i,T:K} < VaR^p_{T,K}\right).$$

The advantages of the multivariate FHS approach tally with those of the univariate case: It captures current market conditions by means of dynamic variance and correlation models. It makes no assumption on the conditional multivariate shock distributions. And, it allows for the computation of any risk measure for any investment horizon of interest.

3.3 Multivariate extensions and alternatives

The DCC model in (5) is due to Engle (2002). See also Tse and Tsui (2002). Extensions to the basic model are developed in Capiello, Engle and Sheppard (2004). For alternative multivariate GARCH approaches, see the surveys in Andersen, Bollerslev, Christoffersen and Diebold (2006a and b), and Bauwens, Laurent, and Rombouts (2006). Jorion (2006) discusses the choice of base assets.

Parametric alternatives to the filtered historical simulation approach include specifying a multivariate normal or Student's t distribution for the GARCH shocks. See, for example Pesaran and Zaffaroni (2004). The multivariate normal and Student's t asset distributions offer the advantage that they are closed under linear transformations so that the portfolio returns will be normal and Student's t, respectively, as well.

The risk manager can also specify parametric conditional distributions for each asset and then link these marginal distributions together to form a multivariate distribution by using a copula function. See, for example, Demarta and McNeil (2005), Patton (2004, 2006), and Jondeau and Rockinger (2005). The results in Joe (1997) suggest that the DCC model itself can be viewed as a copula approach. Multivariate versions of the extreme value approach have also been developed. See, for example, Longin and Solnik (2001), and Poon, Rockinger, and Tawn (2004).

4 Summary and Further Issues

In this chapter, we have built first a univariate and then a multivariate filtered historical simulation model for financial risk management. The models are constructed to capture the stylized facts in daily asset returns, they are simple to estimate, and they enable the risk manager to easily compute Value-at-Risk and other risk measures including Expected Shortfall for various investment horizons conditional on current market conditions. The univariate model is suitable for passive risk management or risk measurement whereas the multivariate model is useful for active risk management such as optimal portfolio allocation. We also discuss various alternatives to the suggested approach.

Because our focus has been on the modeling of *market risk*, that is the risk from fluctuations in observed market prices, other important types of risk have been left unexplored.

We have focused on applications where a relatively long history of daily closing prices is available for each asset or base asset. In practice, portfolios often contain assets where daily historical market prices are not readily observable. Examples include derivatives, bonds, loans, new IPOs, private placements, hedge funds, and real estate investments. In these cases, asset pricing models are needed to link the unobserved asset prices to prices of other liquid assets. The use of pricing models to impute asset prices gives rise to an additional source of risk, namely *model risk*. Hull and Suo (2002) suggest a method to assess model risk.

In loan portfolios, nontraded *credit risk*is the main source of uncertainty. Lando (2004) provides tools for credit risk modeling and credit derivative valuation.

Illiquidity can itself be a source of risk. Historical closing prices may be available for many assets but if little or no trade was actually conducted at those prices then the historical information may not properly reflect risk. In this case, *liquidity risk* should be accounted for. See Persaud (2003) for various aspects of liquidity risk.

Even when computing the VaR and ES for readily observed assets, the use of parametric models implies estimation risk which we have not accounted for here. Christoffersen and Goncalves (2005) show that estimation risk can be substantial, and suggest ways to measure it in dynamic models.

References

Andersen, T.G., Bollerslev, T., Christoffersen, P. and Diebold, F.X. (2006a): Volatility and Correlation Forecasting. *In: Elliott, G., Granger, C. and Timmermann, A. (Eds.): Handbook of Economic Forecasting.* North-Holland, Amsterdam.

Andersen, T.G., Bollerslev, T., Christoffersen, P. and Diebold, F.X. (2006b): Practical Volatility and Correlation Modeling for Financial Market Risk Management. *In:*

Carey, M. and Stulz, R. (Eds.): The Risks of Financial Institutions. University of Chicago Press.

Barone-Adesi, G., Bourgoin, F. and Giannopoulos, K. (1998): Don't Look Back. *Risk* **11**, 100–104.

Bauwens, L., Laurent, S. and Rombouts, J. (2006): Multivariate GARCH Models: a Survey. *Journal of Applied Econometrics* **21**, 79–109.

Bodoukh, J., Richardson, M., and Whitelaw, R. (1998): The Best of Both Worlds. *Risk* **11**, 64–67.

Bollerslev, T. (1986): Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics* **31**, 307–327.

Bollerslev, T. (1987): A Conditionally Heteroskedastic Time Series Model for Speculative Prices and Rates of Return. *Review of Economics and Statistics* **69**, 542–547.

Cappiello, L., Engle, R.F. and Sheppard, K. (2004): Asymmetric Dynamics in the Correlations of Global Equity and Bond Returns. *Manuscript, Stern School of Business New York University.*

Christoffersen, P. (2003): *Elements of Financial Risk Management.* Academic Press, San Diego.

Christoffersen, P. and Diebold, F. (2000): How Relevant is Volatility Forecasting for Financial Risk Management? *Review of Economics and Statistics* **82**, 1–11.

Christoffersen, P. and Goncalves, S. (2005): Estimation Risk in Financial Risk Management. *Journal of Risk* **7**, 1–28.

Christoffersen, P., Diebold, F. and Schuermann, T. (1998): Horizon Problems and Extreme Events in Financial Risk Management. *Economic Policy Review* Federal Reserve Bank of New York, October, 109–118.

Demarta, S., and McNeil, A. J. (2005): The t Copula and Related Copulas. *International Statistical Review* **73**, 111–129.

Diebold, F.X., Schuermann, T. and Stroughair, J. (1998): Pitfalls and Opportunities in the Use of Extreme Value Theory in Risk Management. *In: Refenes, A.-P. N., Burgess, A.N. and Moody, J.D. (Eds.): Decision Technologies for Computational Finance*, 3–12. Kluwer Academic Publishers, Amsterdam.

Duffie, D. and Pan, J. (1997): An Overview of Value at Risk. *Journal of Derivatives* **4**, 7–49.

Engle, R. (1982): Autoregressive Conditional Heteroskedasticity With Estimates of the Variance of U.K. Inflation. *Econometrica* **50**, 987–1008.

Engle, R. (2002): Dynamic Conditional Correlation - A Simple Class of Multivariate GARCH Models. *Journal of Business and Economic Statistics* **20**, 339–350.

Engle, R. and Manganelli, S. (2004): CAViaR: Conditional Autoregressive Value at Risk by Quantile Regression. *Journal of Business and Economic Statistics* **22**, 367–381.

Engle, R. and Ng, V. (1993): Measuring and Testing the Impact of News on Volatility. *Journal of Finance* **48**, 1749–1778.

Engle, R. F. and Sheppard, K. (2001): Theoretical and Empirical properties of Dynamic Conditional Correlation Multivariate GARCH. *NBER Working Paper* **8554**.

Gourieroux, C. and Jasiak, J. (2006): Dynamic Quantile Models. *Manuscript, University of Toronto.*

Hansen, B. (1994): Autoregressive Conditional Density Estimation. *International Economic Review* **35**, 705–730.

Harvey, C.R. and Siddique, A. (1999): Autoregressive Conditional Skewness. *Journal of Financial and Quantitative Analysis* **34**, 465–488.

Hull, J. and Suo, W. (2002): A methodology for assessing model risk and its application to the implied volatility function model. *Journal of Financial and Quantitative Analysis* **37**, 297–318.

Hull, J. and White, A. (1998): Incorporating Volatility Updating into the Historical Simulation Method for VaR. *Journal of Risk* **1**, 5–19.

Joe, H. (1997): *Multivariate Models and Dependence Concepts.* Chapman Hall, London.

Jondeau, E. and Rockinger, M. (2005): The Copula-GARCH Model of Conditional Dependencies: An International Stock-Market Application. *Journal of International Money and Finance* forthcoming.

Jorion, P. (2006): Value-at-Risk: The New Benchmark for Managing. *Financial Risk*. McGraw Hill, New York.

Morgan, J.P. (1996): *RiskMetrics – Technical Document* 4th Edition. New York.

Lando, D. (2004): *Credit Risk Modeling: Theory and Applications* Princeton University Press, New Jersey.

Longin, F. and Solnik, B. (2001): Extreme Correlation of International Equity Markets. *Journal of Finance* **56**, 649–676.

Manganelli, S. (2004): Asset Allocation by Variance Sensitivity Analysis. *Journal of Financial Econometrics* **2**, 370–389.

McNeil, A. and Frey, R. (2000): Estimation of Tail-Related Risk Measures for Heteroskedastic Financial Time Series: An Extreme Value Approach. *Journal of Empirical Finance* **7**, 271–300.

Patton, A. (2004): On the Out-of-Sample Importance of Skewness and Asymmetric Dependence for Asset Allocation. *Journal of Financial Econometrics* **2**, 130–168.

Patton, A. (2006): Modeling Asymmetric Exchange Rate Dependence. *International Economic Review* **47**, 527–556.

Patton, A.J. and Sheppard, K. (2008): Evaluating volatility and Correlation forecasts. *In: Andersen, T.G., Davis, R.A., Kreiss, J.-P. and Mikosch, T. (Eds.): Handbook of Financial Time Series*, 801–838. Springer Verlag, New York.

Persaud, A. (2003): *Liquidity Black Holes: Understanding, Quantifying and Managing Financial Liquidity Risk*. Risk Books, London.

Pesaran, H. and Zaffaroni, P. (2004): Model Averaging and Value-at-Risk based Evaluation of Large Multi Asset Volatility Models for Risk Management. *Manuscript, University of Cambridge*.

Poon, S.-H., Rockinger, M. and Tawn, J. (2004): Extreme Value Dependence in Financial Markets: Diagnostics, Models and Financial Implications. *Review of Financial Studies* **17**, 581–610.

Pritsker, M. (2001): The Hidden Dangers of Historical Simulation. *Finance and Economics Discussion Series* **2001-27**. *Washington: Board of Governors of the Federal Reserve System*.

Tse, Y.K. and Tsui, K.C. (2002): A Multivariate Generalized Autoregressive Conditional Heteroscedasticity Model with Time-varying Correlations. *Journal of Business and Economic Statistics* **20**, 351–362.