

# Measuring and Testing the Impact of News on Volatility

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## ABSTRACT

This paper defines the news impact curve which measures how new information is incorporated into volatility estimates. Various new and existing ARCH models including a partially nonparametric one are compared and estimated with daily Japanese stock return data. New diagnostic tests are presented which emphasize the asymmetry of the volatility response to news. Our results suggest that the model by Glosten, Jagannathan, and Runkle is the best parametric model. The EGARCH also can capture most of the asymmetry; however, there is evidence that the variability of the conditional variance implied by the EGARCH is too high.

THE ABILITY TO FORECAST financial market volatility is important for portfolio selection and asset management as well as for the pricing of primary and derivative assets. While most researchers agree that volatility is predictable in many asset markets (see for example the survey by Bollerslev *et al.* (1992)), they differ on how this volatility predictability should be modeled. In recent years the evidence for predictability has led to a variety of approaches, some of which are theoretically motivated, while others are simply empirical suggestions. The most interesting of these approaches are the “asymmetric” or “leverage” volatility models, in which good news and bad news have different predictability for future volatility. These models are motivated by the empirical work of Black (1976), Christie (1982), French, Schwert, and Stambaugh (1987), Nelson (1990), and Schwert (1990). Pagan and Schwert (1990) provide the first systematic comparison of volatility models. This paper builds on their results, focusing on the asymmetric effect of news on volatility. Specifically, we provide new diagnostic tests, a partially nonparametric model for discovering the empirical relations between news and volatility, and a metric for interpreting the differences between volatility models.

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The importance of a correctly specified volatility model is clear from the range of applications requiring estimates of conditional volatilities. In the valuation of stocks, Merton (1980) shows that the expected market return is related to predictable stock market volatility. French, Schwert, and Stambaugh (1987) and Chou (1988) also find empirical evidence for this relationship, although Chou, Engle, and Kane (1992) indicate that the relation may be more complex. Ferson and Harvey (1991) provide evidence that much of the predictability of a sample of monthly portfolio returns can be related to the predictability of risk premiums. Schwert and Seguin (1990) and Ng, Engle, and Rothschild (1992) show that individual stock return volatility is driven by market volatility, with individual stock return premiums affected by the predictable market volatility.

In the valuation of stock options, Wiggins (1987) and Hull and White (1987) suggest that stochastic stock return volatility might be the source of some documented pricing biases of the Black-Scholes option-pricing formula. Furthermore, the research of Day and Lewis (1992) shows that implied volatility from the Black-Scholes model cannot capture the entire predictable part of future volatility relative to some GARCH and EGARCH models. Harvey and Whaley (1992) also find some predictability in changes in implied volatilities, and profits can be earned by trading on this information, although only gross of transaction costs. Kuwahara and Marsh (1992) find predictable volatility models like the EGARCH useful in the valuation of warrants. Amin and Ng (1993) show that option valuation under predictable volatility is different from option valuation under unpredictable volatility.

Ross (1989) argues that volatility can be regarded as a measure of information flow, and Conrad, Gultekin, and Kaul (1991) find evidence that information flow is from large to small firms. Hamao, Masulis, and Ng (1990) find evidence of volatility spillover from the U.S. stock market to both the U.K. and the Japanese stock markets. However, Susmel and Engle (1992) and Lin, Engle, and Ito (1992) show that such spillover could be very short lived. Masulis and Ng (1992) find evidence of overnight information affecting daytime volatility.

Finally, the predictability of volatility is important in designing optimal dynamic hedging strategies for options and futures (Baillie and Myers (1991) and Engle, Hong, Kane, and Noh (1992)). The predictability of volatility might also affect the results of event studies (see, for example, Connolly (1989)).

In the next section, we discuss several models of predictable volatility and present the idea of a news impact curve which characterizes the impact of past return shocks on the return volatility implicit in a volatility model. In Section II, we suggest several new diagnostic tests based on the news impact curve. We also perform a small Monte Carlo experiment to examine the finite sample properties of the test statistics. In Section III, a partially nonparametric ARCH model is introduced. Using a Japanese stock return series, in Section IV we compare the GARCH (1, 1) model with several other volatility models that allow for asymmetry in the impact of news on volatility. To check

the adequacy of the models, we employ the diagnostic tests developed in Section II. In Section V, the partially nonparametric model is estimated and compared with the others. In Section VI, the best models are reestimated on a precrash sample period. Section VII concludes the paper.

### I. Models of Predictable Volatility

Given the importance of predicting volatility in many asset-pricing and portfolio management problems, many approaches of forecasting volatility have been proposed in the literature. The most popular one is the class of autoregressive conditional heteroskedasticity (ARCH) models originally introduced by Engle (1982). In a recent survey by Bollerslev *et al.* (1992) more than 200 papers are cited applying ARCH and related models to financial time series. In this section, we will review some of the more popular predictable volatility models in the ARCH class. We will define a news impact curve which can be used to evaluate and compare the properties of different volatility models in the ARCH class.

Let  $y_t$  be the rate of return of a particular stock or the market portfolio from time  $t - 1$  to time  $t$ . Also, let  $F_{t-1}$  be the past information set containing the realized values of all relevant variables up to time  $t - 1$ . Since investors know the information in  $F_{t-1}$  when they make their investment decision at time  $t - 1$ , the relevant expected return and volatility to the investors are the conditional expected value of  $y_t$ , given  $F_{t-1}$ , and the conditional variance of  $y_t$ , given  $F_{t-1}$ . We denote these by  $m_t$  and  $h_t$ , respectively. That is,  $m_t \equiv E(y_t|F_{t-1})$  and  $h_t \equiv \text{Var}(y_t|F_{t-1})$ . Given these definitions, the unexpected return at time  $t$  is  $\varepsilon_t \equiv y_t - m_t$ . In this paper,  $\varepsilon_t$  is treated as a collective measure of news at time  $t$ . A positive  $\varepsilon_t$  (an unexpected increase in price) suggests the arrival of good news, while a negative  $\varepsilon_t$  (an unexpected decrease in price) suggests the arrival of bad news. Further, a large value of  $|\varepsilon_t|$  implies that the news is “significant” or “big” in the sense that it produces a large unexpected change in price.

Engle (1982) suggests that the conditional variance  $h_t$  can be modeled as a function of the lagged  $\varepsilon$ 's. That is, the predictable volatility is dependent on past news. The most detailed model he develops is the  $p$ th order autoregressive conditional heteroskedasticity model, the ARCH( $p$ ):

$$h_t = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2, \quad (2)$$

where  $\alpha_1, \dots, \alpha_p$ , and  $\omega$  are constant parameters. The effect of a return shock  $i$  periods ago ( $i \leq p$ ) on current volatility is governed by the parameter  $\alpha_i$ . Normally, we would expect that  $\alpha_i < \alpha_j$  for  $i > j$ . That is, the older the news, the less effect it has on current volatility. In an ARCH( $p$ ) model, old news which arrived at the market more than  $p$  periods ago has no effect at all on current volatility.

Bollerslev (1986) generalizes the ARCH( $p$ ) model to the GARCH( $p, q$ ) model, such that

$$h_t = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^q \beta_i h_{t-i}, \quad (3)$$

where  $\alpha_1, \dots, \alpha_p$ ,  $\beta_1, \dots, \beta_p$ , and  $\omega$  are constant parameters. The GARCH model is an infinite order ARCH model. In the GARCH(1, 1) model, the effect of a return shock on current volatility declines geometrically over time. Empirically, the family of GARCH models has been very successful. Of these models, the GARCH(1, 1) is preferred in most cases (see the survey by Bollerslev *et al.* (1992)).

Despite the apparent success of these simple parameterizations, the ARCH and GARCH models cannot capture some important features of the data. The most interesting feature not addressed by these models is the leverage or asymmetric effect discovered by Black (1976), and confirmed by the findings of French, Schwert, and Stambaugh (1987), Nelson (1990), and Schwert (1990), among others.<sup>1</sup> Statistically, this effect occurs when an unexpected drop in price (bad news) increases predictable volatility more than an unexpected increase in price (good news) of similar magnitude. This effect suggests that a symmetry constraint on the conditional variance function in past  $\varepsilon$ 's is inappropriate. One method proposed to capture such asymmetric effects is Nelson's (1990) exponential GARCH or EGARCH model

$$\log(h_t) = \omega + \beta \cdot \log(h_{t-1}) + \gamma \cdot \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \alpha \left[ \frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}} - \sqrt{2/\pi} \right], \quad (4)$$

where  $\omega$ ,  $\beta$ ,  $\gamma$ , and  $\alpha$  are constant parameters. The EGARCH model is asymmetric because the level of  $\varepsilon_{t-1}/\sqrt{h_{t-1}}$  is included with a coefficient  $\gamma$ . Since this coefficient is typically negative, positive return shocks generate less volatility than negative return shocks, all else being equal.

A comparison between the GARCH(1, 1) model and the EGARCH(1, 1) suggests an interesting metric by which to analyze the effect of news on conditional heteroskedasticity. Holding constant the information dated  $t-2$  and earlier, we can examine the implied relation between  $\varepsilon_{t-1}$  and  $h_t$ . We call this curve, with all lagged conditional variances evaluated at the level of the unconditional variance of the stock return, the *news impact curve* because it relates past return shocks (news) to current volatility. This curve measures how new information is incorporated into volatility estimates. It is similar in spirit to Figure 2 in Pagan and Schwert (1990). In the GARCH model, this curve is a quadratic function centered on  $\varepsilon_{t-1} = 0$ . For the EGARCH, it has its minimum at  $\varepsilon_{t-1} = 0$ , and is exponentially increasing in both directions but with different parameters. In particular, the news impact

<sup>1</sup> It is not yet clear in the finance literature that the asymmetric properties of variances are due to changing leverage. The name "leverage effect" is used simply because it is popular among researchers when referring to such a phenomenon.

curve for the EGARCH model when the lagged conditional variance is evaluated at its unconditional level,  $\sigma^2$ , is given by

$$\begin{aligned} h_t &= A \cdot \exp \left[ \frac{(\gamma + \alpha)}{\sigma} \cdot \varepsilon_{t-1} \right], \quad \text{for } \varepsilon_{t-1} > 0, \quad \text{and} \\ h_t &= A \cdot \exp \left[ \frac{(\gamma - \alpha)}{\sigma} \cdot \varepsilon_{t-1} \right], \quad \text{for } \varepsilon_{t-1} < 0, \end{aligned}$$

where

$$A \equiv \sigma^{2\beta} \cdot \exp \left[ \omega - \alpha \cdot \sqrt{2/\pi} \right]. \quad (5)$$

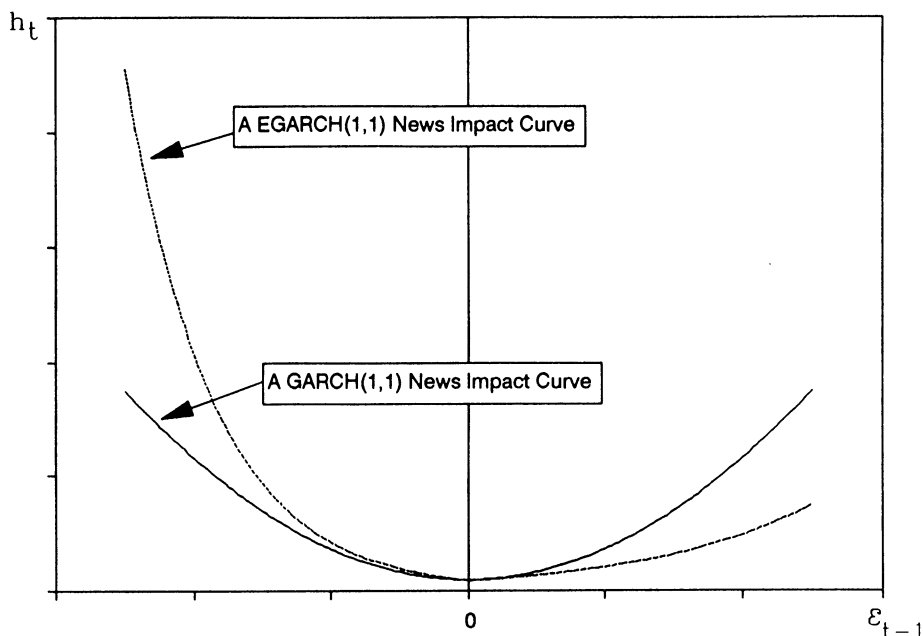
In Figure 1, the news impact curve of the EGARCH(1, 1) is compared with the news impact curve of the GARCH(1, 1) for  $\gamma < 0$  but  $\alpha + \gamma > 0$ . If the curves were extrapolated, the EGARCH would have higher variances in both directions because the exponential curve eventually dominates the quadratic. Thus, we can see from the news impact curve that the EGARCH model differs from the standard GARCH model in two main respects:

1. The EGARCH model allows good news and bad news to have a different impact on volatility, while the standard GARCH model does not, and
2. the EGARCH model allows big news to have a greater impact on volatility than the standard GARCH model.

The news impact curve can be constructed for many other models, some of which are outlined in Table I. While the functional form of these models is rather complicated, the qualitative differences between these models can be compared by contrasting their implied news impact curves.

The news impact curve of the nonlinear model of Engle and Bollerslev (1986) is symmetric. However, it implies a reduced response to extreme news if  $\gamma < 2$ . The news impact curve of the multiplicative ARCH model of Mihov (1987), Geweke (1986), and Pantula (1986), is symmetric and passes through the origin. Depending on the value of the  $\alpha_i$ 's, the two sides of the news impact curve can be either steeper or less steep than the GARCH(1, 1) news impact curve. The news impact curve of the autoregressive standard deviation model of Schwert (1990) is symmetric and centered at  $\varepsilon_{t-1} = 0$ . The news impact curve of the asymmetric GARCH model of Engle (1990), AGARCH, is asymmetric and centered at  $\varepsilon_{t-1} = -\gamma$ , which is to the right of the origin when  $\gamma < 0$ . The news impact curve of the GJR model of Glosten, Jagannathan, and Runkle (1989) is centered at  $\varepsilon_{t-1} = 0$ , but has different slopes for its positive and negative sides. The news impact curves of both the nonlinear asymmetric GARCH model (NGARCH) and the VGARCH model are symmetric and centered at  $\varepsilon_{t-1} = (-\gamma) \cdot \sqrt{h_{t-1}}$ . However, the slope of the two upward-sloping portions of the VGARCH is steeper than that of the NGARCH.

To summarize, the news impact curves of the above asymmetric volatility models capture the leverage or asymmetric effect by allowing either the slope of the two sides of the news impact curve to differ or the center of the news



**Figure 1. The news impact curves of the GARCH(1, 1) model and the EGARCH(1, 1) model.** The solid line is the GARCH(1, 1) news impact curve. The dashed line is the EGARCH(1, 1) news impact curve. The equation for the GARCH(1, 1) news impact curve is

$$h_t = A + \alpha \cdot \varepsilon_{t-1}^2,$$

where  $h_t$  is the conditional variance at time  $t$ ,  $\varepsilon_{t-1}$  is the unpredictable return at time  $t-1$ ,  $A \equiv \omega + \beta \cdot \sigma^2$ ,  $\sigma$  is the unconditional return standard deviation,  $\omega$  is the constant term, and  $\beta$  is the parameter corresponding to  $h_{t-1}$  in the GARCH variance equation. The shape of the above GARCH(1, 1) news impact curve is indicative of cases with  $\omega > 0$ ,  $0 \leq \beta < 1$ ,  $\sigma > 0$ ,  $0 \leq \alpha < 1$ , and  $\alpha + \beta < 1$ .

The equations for the EGARCH(1, 1) news impact curve are

$$h_t = A \cdot \exp \left[ \frac{(\gamma + \alpha)}{\sigma} \cdot \varepsilon_{t-1} \right], \quad \text{for } \varepsilon_{t-1} > 0, \quad \text{and}$$

$$h_t = A \cdot \exp \left[ \frac{(\gamma - \alpha)}{\sigma} \cdot \varepsilon_{t-1} \right], \quad \text{for } \varepsilon_{t-1} < 0,$$

where  $A \equiv \sigma^{2\beta} \cdot \exp[\omega - \alpha \cdot \sqrt{2/\pi}]$ ,  $\sigma$  is the unconditional return standard deviation,  $\omega$  is the constant term,  $\beta$  is the parameter for the  $\log(h_{t-1})$  term,  $\alpha$  is the parameter for the  $|\varepsilon_{t-1}|/\sqrt{h_{t-1}}$  term, and  $\gamma$  is the parameter for the  $\varepsilon_{t-1}/\sqrt{h_{t-1}}$  term in the EGARCH log-variance equation (as given in Table I). The shape of the above EGARCH(1, 1) news impact curve is indicative of cases with  $\omega > 0$ ,  $0 \leq \beta < 1$ ,  $\sigma > 0$ ,  $0 \leq \alpha < 1$ , and  $\alpha + \beta < 1$  and, importantly,  $\gamma < 0$ .

impact curve to locate at a point where  $\varepsilon_{t-1}$  is positive. In Figure 2, the news impact curve is plotted for the GJR model and the AGARCH model. We can see that the GJR news impact curve captures the asymmetry in the effect of news on volatility because it has a steeper slope in its negative side than on

Table I

**Some Alternative Predictable Volatility Models**

In the following model specifications,  $h_t$  is the conditional variance at time  $t$  and  $\varepsilon_{t-1}$  is the unpredictable return (the residual) at time  $t - 1$ .  $\omega$ ,  $\alpha$ ,  $\beta$ , and  $\gamma$  are constant parameters in each of the models.

Nonlinear ARCH model (Engle and Bollerslev (1986))

$$h_t = \omega + \alpha |\varepsilon_{t-1}|^\gamma + \beta h_{t-1}$$

Multiplicative ARCH (Mihoj (1987), Geweke (1986), Pantula (1986))

$$\log(h_t) = \omega + \sum_{i=1}^p \alpha_i \log(\varepsilon_{t-i}^2)$$

GJR model (Glosten, Jagannathan, and Runkle (1989), Zakoian (1990))

$$h_t = \omega + \beta h_{t-1} + \alpha \varepsilon_{t-1}^2 + \gamma S_{t-1}^- \varepsilon_{t-1}^2, \quad \text{where } S_t^- = 1 \text{ if } \varepsilon_t < 0, S_t^- = 0 \text{ otherwise}$$

EGARCH model (Nelson (1989))

$$\log(h_t) = \omega + \beta \cdot \log(h_{t-1}) + \gamma \cdot \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \alpha \left[ \frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}} - \sqrt{2/\pi} \right]$$

Autoregressive Standard Deviation model (Schwert (1990))

$$h_t = \left[ \omega + \sum_{i=1}^p \alpha_i |\varepsilon_{t-i}| \right]^2$$

Asymmetric GARCH model (Engle (1990))

$$h_t = \omega + \alpha (\varepsilon_{t-1} + \gamma)^2 + \beta h_{t-1}$$

Nonlinear Asymmetric GARCH model

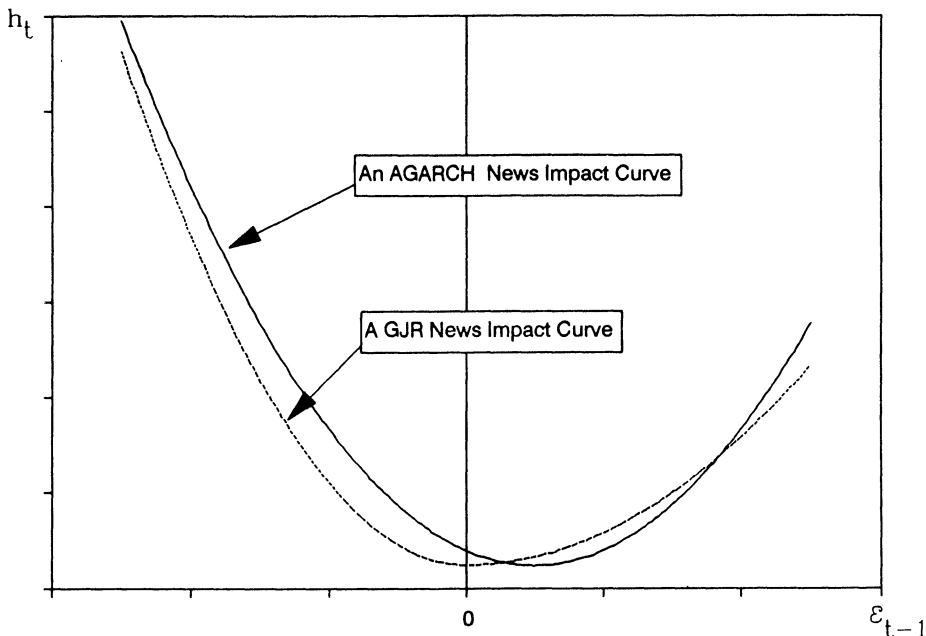
$$h_t = \omega + \beta h_{t-1} + \alpha (\varepsilon_{t-1} + \gamma \cdot \sqrt{h_{t-1}})^2$$

VGARCH model

$$h_t = \omega + \beta h_{t-1} + \alpha (\varepsilon_{t-1} / \sqrt{h_{t-1}} + \gamma)^2$$

its positive side. The AGARCH, on the other hand, captures the asymmetry by allowing its new impact curve to be centered at a positive  $\varepsilon_{t-1}$ .

These differences between the news impact curves of the models have important implications for portfolio selection and asset pricing. For instance, after a major unexpected price drop, like the 1987 crash, the predictable market volatilities given by the GARCH and the EGARCH are very different, as implied by their news impact curves. Since predictable market volatility is related to market risk premium, the two models imply very different market risk premiums, and hence different risk premiums for individual stocks under a conditional version of the capital asset-pricing model. The differences in the news impact curves implied by the two models also have important



**Figure 2. The news impact curves of the AGARCH(1, 1) model and the GJR model.** The solid line is the AGARCH(1, 1) news impact curve. The dashed line is the GJR news impact curve. The equation for the AGARCH(1, 1) news impact curve is

$$h_t = A + \alpha \cdot (\varepsilon_{t-1} + \gamma)^2$$

where  $h_t$  is the conditional variance at time  $t$ ,  $\varepsilon_{t-1}$  is the unpredictable return at time  $t - 1$ ,  $A \equiv \omega + \beta \cdot \sigma^2$ ,  $\sigma$  is the unconditional return standard deviation,  $\omega$  is the constant term,  $\beta$  is the parameter for the  $h_{t-1}$  term, and  $\alpha$  and  $\gamma$  are the parameters in the quadratic term,  $\alpha(\varepsilon_{t-1} + \gamma)^2$ , in the AGARCH variance equation (as given in Table I). The shape of the above AGARCH(1, 1) news impact curve is indicative of cases with  $\omega > 0$ ,  $0 \leq \beta < 1$ ,  $\sigma > 0$ ,  $0 \leq \alpha < 1$ ,  $\alpha + \beta < 1$  and  $\gamma < 0$ .

The equations for the GJR news impact curve are

$$h_t = A + \alpha \cdot \varepsilon_{t-1}^2, \quad \text{for } \varepsilon_{t-1} > 0, \quad \text{and}$$

$$h_t = A + (\alpha + \gamma) \cdot \varepsilon_{t-1}^2, \quad \text{for } \varepsilon_{t-1} < 0,$$

where  $A \equiv \omega + \beta \cdot \sigma^2$ ,  $\sigma$  is the unconditional return standard deviation,  $\omega$  is the constant term,  $\beta$  is the parameter for the  $h_{t-1}$  term,  $\alpha$  is the parameter for the  $\varepsilon_{t-1}^2$  term, and  $\gamma$  is the parameter for the  $S_{t-1}^- \varepsilon_{t-1}^2$  term in the GJR variance equation where  $S_{t-1}^-$  takes a value of 1 when  $\varepsilon_{t-1}$  is negative and a value of 0 when  $\varepsilon_{t-1}$  is positive or zero. The shape of the above GJR news impact curve is indicative of cases with  $\omega > 0$ ,  $0 \leq \beta < 1$ ,  $\sigma > 0$ ,  $0 \leq \alpha < 1$ ,  $\alpha + \beta < 1$  and  $\gamma > 0$ .

implications for option pricing. Stock return volatility is a major factor in determining option prices. A significant difference in predicted volatility after the arrival of some major news leads to a significant difference in the current option price. Furthermore, since the EGARCH and the GARCH models imply



very different volatilities following major bad news, the dynamic hedging strategies implied by the two sets of volatility estimates would be very different.

All these concerns point to the need to have a correct understanding of the impact of news on volatility. The news impact curve we have introduced is a convenient way to summarize the effect of news on volatility implied by a parametric model of predictable volatility. By comparing the news impact curves of alternative predictable volatility models, we can highlight the differences between the models. By testing whether the news impact curve of a model offers a good fit to the data, we can understand the quality of the model.

## II. Diagnostic Tests Based on the News Impact Curve

As discussed in Section I, implicit in any choice of a volatility model is a particular news impact curve. The standard GARCH model has a news impact curve which is symmetric and centered at  $\varepsilon_{t-1} = 0$ . That is, positive and negative return shocks of the same magnitude produce the same amount of volatility. Also, larger return shocks forecast more volatility at a rate proportional to the square of the size of the return shock. If a negative return shock causes more volatility than a positive return shock of the same size, the GARCH model underpredicts the amount of volatility following bad news and overpredicts the amount of volatility following good news. Furthermore, if large return shocks cause more volatility than a quadratic function allows, then the standard GARCH model underpredicts volatility after a large return shock and overpredicts volatility after a small return shock.

These observations suggest three new diagnostic tests for volatility models: the *Sign Bias Test*, the *Negative Size Bias Test*, and the *Positive Size Bias Test*. These tests examine whether we can predict the squared normalized residual by some variables observed in the past which are not included in the volatility model being used. If these variables can predict the squared normalized residual, then the variance model is misspecified. The sign bias test considers the variable  $S_{t-1}^-$ , a dummy variable that takes a value of one when  $\varepsilon_{t-1}$  is negative and zero otherwise. This test examines the impact of positive and negative return shocks on volatility not predicted by the model under consideration. The negative size bias test utilizes the variable  $S_{t-1}^- \varepsilon_{t-1}$ . It focuses on the different effects that large and small *negative* return shocks have on volatility which is not predicted by the volatility model. The positive size bias test utilizes the variable  $S_{t-1}^+ \varepsilon_{t-1}$  where  $S_{t-1}^+$  is defined as 1 minus  $S_{t-1}^-$ . It focuses on the different impacts that large and small *positive* return shocks may have on volatility, which are not explained by the volatility model. Since an important piece of bad news might have a very different impact on volatility than an important piece of good news, it is critical to distinguish between positive and negative return shocks while examining the effects of the magnitude of a piece of news. Using the same approach, we can create a number of closely related tests. For example, we can examine even

more extreme values of  $\varepsilon$  for particular biases by using variables, such as  $S_{t-1}^- \varepsilon_{t-1}^2$  or order statistics, such as  $D_t^\alpha(\varepsilon_{t-1})$ , where  $D_t^\alpha(\varepsilon_{t-1})$  is one if  $\varepsilon_{t-1}$  exceeds the  $\alpha$ th percentile of the set of  $\{\varepsilon_t\}$ . All these tests can be carried out individually or jointly. They can also be applied to volatility models which are not members of the GARCH family.

To derive the optimal form of these tests, we assume that the volatility model under the null hypothesis is a special case of a more general model of the following form:

$$\log(h_t) = \log(h_{ot}(\underline{\delta}_o' \underline{z}_{ot})) + \underline{\delta}_a' \underline{z}_{at}, \quad (6)$$

where  $h_{ot}(\underline{\delta}_o' \underline{z}_{ot})$  is the volatility model hypothesized under the null;  $\underline{\delta}_o$  is the  $k \times 1$  vector of parameters under the null;  $\underline{z}_{ot}$  is the  $k \times 1$  vector of explanatory variables under the null; and  $\underline{\delta}_a$  is the  $m \times 1$  vector of additional parameters corresponding to  $\underline{z}_{at}$ , which is the  $m \times 1$  vector of missing explanatory variables. This particular maintained hypothesis about the general form of the model is chosen over the commonly used linear form,  $h_t = h_{ot}(\underline{\delta}_o' \underline{z}_{ot}) + \underline{\delta}_a' \underline{z}_{at}$ , in our specification tests because it encompasses both the GARCH and EGARCH classes of variance models.<sup>2</sup>

Since the volatility model under the null hypothesis is obtained when the parameters for the additional explanatory variables,  $\underline{z}_{at}$ , are zeros, we can test the model by testing whether these parameter restrictions ( $\underline{\delta}_a = \underline{0}$ ) are satisfied.

Let  $v_t$  be the normalized residual corresponding to observation  $t$  under the volatility model hypothesized. That is,  $v_t \equiv \varepsilon_t / \sqrt{h_{ot}}$ . Following Engle and Kraft (1983), Engle (1984), and Bollerslev (1986), the Lagrange multiplier test statistic for  $H_0: \underline{\delta}_a = \underline{0}$  in equation (6) is simply a test of  $\underline{\delta}_a = \underline{0}$  in the auxiliary regression

$$v_t^2 = \underline{z}_{ot}^* \underline{\delta}_o + \underline{z}_{at}^* \underline{\delta}_a + u_t, \quad (7)$$

where  $\underline{z}_{ot}^* \equiv h_{ot}^{-1} \partial h_t / \partial \underline{\delta}_o$ ,  $\underline{z}_{at}^* \equiv h_{ot}^{-1} \partial h_t / \partial \underline{\delta}_a$ , and  $u_t$  is the residual. Both  $\partial h_t / \partial \underline{\delta}_o$  and  $\partial h_t / \partial \underline{\delta}_a$  are evaluated at  $\underline{\delta}_a = \underline{0}$  and  $\underline{\delta}_o$  (the maximum likelihood estimator of  $\underline{\delta}_o$  under  $H_0$ ). Theoretically,  $v_t^2$  is orthogonal to  $\underline{z}_{ot}^*$  by the first-order conditions for maximum likelihood. So, if the parameter restrictions are met, the right-hand side variables in (7) should have no explanatory

<sup>2</sup> For example, if  $h_{ot}(\cdot)$  is the usual GARCH(1, 1) form such that:  $h_{ot}(\underline{\delta}_o' \underline{z}_{ot}) \equiv \underline{\delta}_o' \underline{z}_{ot}$ ,  $\underline{\delta}_o \equiv [\omega, \beta, \alpha]'$  and  $\underline{z}_{ot} \equiv [1, h_{t-1}, \varepsilon_{t-1}^2]'$ ; and  $\underline{\delta}_a$  and  $\underline{z}_{at}$  are the EGARCH parameters and variables:  $\underline{\delta}_a \equiv [\beta^*, \gamma^*, \alpha^*]'$  and  $\underline{z}_{at} \equiv [\log(h_{t-1}), \varepsilon_{t-1} / \sqrt{h_{t-1}}, (|\varepsilon_{t-1}| / \sqrt{h_{t-1}} - \sqrt{2/\pi})]$  then the encompassing model is

$$\begin{aligned} \log(h_t) = & \log[\omega + \beta h_{t-1} + \alpha \varepsilon_{t-1}^2] + \beta^* \log(h_{t-1}) + \gamma^* \varepsilon_{t-1} / \sqrt{h_{t-1}} \\ & + \alpha^* (|\varepsilon_{t-1}| / \sqrt{h_{t-1}} - \sqrt{2/\pi}). \end{aligned}$$

So, when  $\beta = \alpha = 0$ , the model is the EGARCH. But when  $\beta^* = \gamma^* = \alpha^* = 0$ , the model is the standard GARCH(1, 1).

power at all. Thus, the test is often computed as

$$\xi_{LM} = T \cdot R^2, \quad (8)$$

where  $R^2$  is the squared multiple correlation of (7), and  $T$  is the number of observations in the sample. However, for highly nonlinear models, the numerical optimization algorithm generally does not guarantee exact orthogonality. Therefore, the procedure we use throughout this paper is to: (i) regress  $y_t^2$  on  $z_{ot}$  alone, and (ii) use the residuals from this regression (which are now guaranteed to be orthogonal to  $z_{ot}$ ) in place of  $v_t^2$  in (7). In a Monte Carlo experiment, this procedure provides a test size closer to the one given by asymptotic theory.

The LM test statistic is asymptotically distributed as chi-square with  $m$  degrees of freedom when the null hypothesis is true, where  $m$  is the number of parameter restrictions. It is asymptotically equivalent to the likelihood ratio test and hence is also asymptotically the most powerful test.

Under our maintained encompassing model (6),  $\partial h_t / \partial \underline{\delta}_a$  evaluated under the null is equal to  $h_{ot} z_{at}$ . Hence,  $\underline{z}_{at}^* = \underline{z}_{at}$ . The regression actually involves regressing  $v_t^2$  on a constant  $\underline{z}_{ot}^*$ , and  $\underline{z}_{at}$ . By selecting different measures of  $\underline{z}_{at}$ , we construct different tests. By testing one variable at a time, we formulate each test against a particular alternative. Finally, by allowing  $\underline{z}_{at}$  to include several variables, we can construct joint tests. In this paper, we consider the variables  $S_{t-1}^-$ ,  $S_{t-1}^- \varepsilon_{t-1}$ , and  $S_{t-1}^+ \varepsilon_{t-1}$ . Tests for higher order asymmetry can also be constructed based on the same principle by considering the variables  $S_{t-j}^-$ ,  $S_{t-j}^- \varepsilon_{t-j}$ , and  $S_{t-j}^+ \varepsilon_{t-j}$ ,  $j = 1, \dots, p$ .

The above discussion suggests that the optimal forms of the regressions for conducting the sign bias test, the negative size bias test, and the positive size bias test are respectively,

$$v_t^2 = a + b \cdot S_{t-1}^- + \underline{\beta}' \underline{z}_{ot}^* + e_t, \quad (9a)$$

$$v_t^2 = a + b \cdot S_{t-1}^- \varepsilon_{t-1} + \underline{\beta}' \underline{z}_{ot}^* + e_t, \quad (9b)$$

$$v_t^2 = a + b \cdot S_{t-1}^+ \varepsilon_{t-1} + \underline{\beta}' \underline{z}_{ot}^* + e_t, \quad (9c)$$

where  $a$  and  $b$  are constant parameters,  $\underline{\beta}$  is a constant parameter vector, and  $e_t$  is the residual. The *sign bias test statistic* is defined as the  $t$ -ratio for the coefficient  $b$  in regression equation (9a). The *negative size bias test statistic* is defined as the  $t$ -ratio of the coefficient  $b$  in regression equation (9b). The *positive size bias test statistic* is defined as the  $t$ -ratio of the coefficient  $b$  in regression equation (9c). To conduct these tests jointly, we can consider the regression

$$v_t^2 = a + b_1 S_{t-1}^- + b_2 S_{t-1}^- \varepsilon_{t-1} + b_3 S_{t-1}^+ \varepsilon_{t-1} + \underline{\beta}' \underline{z}_{ot}^* + e_t, \quad (10)$$

where  $a$ ,  $b_1$ ,  $b_2$ , and  $b_3$  are constant coefficients,  $\underline{\beta}$  is a vector of constant coefficients, and  $e_t$  is the residual. The  $t$ -ratios for  $b_1$ ,  $b_2$ , and  $b_3$  are the sign bias, the negative size bias, and the positive size bias test statistics, respec-

tively. The joint test is the LM test for adding the three variables in the variance equation under the maintained specification (6). The test statistic is equal to  $T$  times the  $R$ -squared from this regression. If the volatility model being used is correct, then  $b_1 = b_2 = b_3 = 0$ ,  $\beta = 0$  and  $e_t$  is i.i.d. Thus, the  $t$ -statistics and the LM test statistic have the standard limiting distributions. In particular, the LM test statistic follows a chi-square distribution with three degrees of freedom. If  $\underline{z}_{ot}^*$  is not included in (10), the test will be conservative; the size will be less than or equal to the nominal size, and the power may be reduced.

These diagnostic test statistics can also be used as summary statistics on the raw data to explore the nature of time-varying volatility in the data series, without first imposing a volatility model. In this case,  $\varepsilon_t$  and  $v_t$  would simply be defined as follows:

$$\varepsilon_t \equiv y_t - \mu \quad (11a)$$

$$v_t \equiv \varepsilon_t / \sigma, \quad (11b)$$

where  $\mu$  and  $\sigma$  are the unconditional mean and standard deviation of  $y_t$ , respectively. Since multiplying the dependent variable in the regression by a constant will not change the results when the null is constant variance, we can simply use  $\varepsilon_t^2$  instead of  $v_t^2$  as the dependent variable in the regression. The  $t$ -statistics and the LM test statistic, which are both scale invariant, will give us the three individual tests and the joint test.

To examine the performance of the test statistics, we conduct a small Monte Carlo experiment. From this experiment we find that the size is rather close to the nominal value and the power to detect departures is reasonable, at least for the larger (1000) sample size. Since the sample size of our daily stock return series is rather large (2532), we use the asymptotic critical values in our empirical section.

The Monte Carlo experiment for checking the size of the tests is based on a GARCH(1, 1) data-generating process, as in (12):

$$\begin{aligned} y_t &= \varepsilon_t \\ h_t &= \omega + \beta \cdot h_{t-1} + \alpha \cdot \varepsilon_{t-1}^2 \\ \varepsilon_t &= \sqrt{h_t} \cdot v_t, \end{aligned} \quad (12)$$

where  $v_t \sim \text{i.i.d. } N(0, 1)$ , and  $\omega$ ,  $\beta$ , and  $\alpha$  are constant parameters. Three sets of parameter values are considered: (1) model H (for high persistence), where  $(\omega, \beta, \zeta) = (0.01, 0.9, 0.09)$  and  $\alpha + \beta = 0.99$ ; (2) model M (for medium persistence), where  $(\omega, \beta, \alpha) = (0.05, 0.9, 0.05)$  and  $\alpha + \beta = 0.95$ ; and (3) model L (for low persistence), where  $(\omega, \beta, \alpha) = (0.2, 0.75, 0.05)$  and  $\alpha + \beta = 0.8$ . For each model, samples of size 100 and 1000 are drawn with 10,000 replications. For each replication, a GARCH(1, 1) model is estimated and the sign bias test, the negative size bias test, the positive size bias test, and the joint test are conducted. The actual rejection frequencies based on the

1, 5, and 10 percent critical values under the asymptotic distribution are reported in Table II.

As is usual in Monte Carlo experiments with nonlinear models, the convergence criterion is not satisfied in a fraction of cases.<sup>3</sup> In such cases, we add new replications to ensure that we have 10,000 converged replications. The results reported in Table II are based on the converged replications. Approximately 7.2 percent of all replications do not converge. The test statistics from these nonconverged replications are analyzed separately as if they had converged. The results, not reported, are similar to those for the converged replications.

The simulated size of the test is quite close to the proposed size under the asymptotic distribution of the test statistics for the larger sample size of 1000. The results for the smaller sample size of 100 are also reasonable. For the proposed 1 percent level, there are only one or two cases where the actual rejection frequency is about 0.6 percent. Since the actual rejection frequencies are quite close to the proposed ones for the 5 percent and the 10 percent levels, these exceptions might be due to the small number of observations from the tails of the distribution.

The Monte Carlo experiment for checking the power of the tests is based on an EGARCH(1, 1) data-generating process as in (13), and a data-generating process, as in (14), which corresponds to the GJR model. We call these processes model E and model G, respectively.

#### Model E

$$\begin{aligned} y_t &= \varepsilon_t \\ \ln h_t &= -0.23 + 0.9 \cdot \ln h_{t-1} + 0.25 \cdot [v_{t-1}^2 - 0.3 \cdot v_{t-1}] \\ \varepsilon_t &= \sqrt{h_t} \cdot v_t, \end{aligned} \quad (13)$$

where  $v_t \sim \text{i.i.d. } N(0, 1)$  and

#### Model G

$$\begin{aligned} y_t &= \varepsilon_t \\ h_t &= 0.005 + 0.7 \cdot h_{t-1} + 0.28 \cdot [|\varepsilon_{t-1}| - 0.23 \cdot \varepsilon_{t-1}]^2 \\ \varepsilon_t &= \sqrt{h_t} \cdot v_t, \end{aligned} \quad (14)$$

where  $v_t \sim \text{i.i.d. } N(0, 1)$ .

Again, for each model, samples of size 100 and 1000 are drawn with 10,000 replications. For each replication, a GARCH(1, 1) model is estimated and the tests are conducted. The actual rejection frequencies based on the 1, 5, and 10 percent critical values under the asymptotic distribution are reported in Table III for the joint test. As before, Table III is based on only the converged

<sup>3</sup> The convergence criterion we use in the simulation experiment is that the  $R$ -squared from a regression of a vector of ones on the scores is less than or equal to 0.0000001.

Table II  
Actual Rejection Frequencies When the Null is True  
(The Simulated Size)

To check the size of the tests, we perform a Monte Carlo experiment based on the GARCH(1, 1) data-generating process below.  $\varepsilon_t$  is the unpredictable return (the residual) at time,  $t$ ,  $h_t$  is the conditional variance at time  $t$ ,  $v_t$  is the normalized residual at time  $t$  which is generated from a standard normal random number generator,  $y_t$  is the simulated return at time  $t$  (which, for simplicity is assumed to be entirely unpredictable and hence is equal to  $\varepsilon_t$ ), and  $\omega$ ,  $\beta$ , and  $\alpha$  are constant parameters.

$$y_t = \varepsilon_t; \quad \varepsilon_t = \sqrt{h_t} \cdot v_t, \quad \text{where } v_t \sim \text{i.i.d. } N(0, 1)$$
$$h_t = \omega + \beta \cdot h_{t-1} + \alpha \cdot \varepsilon_{t-1}^2.$$

Three sets of parameter values are considered. They are: (1) model H (for high persistence), where  $(\omega, \beta, \alpha) = (0.01, 0.9, 0.09)$  and  $\alpha + \beta = 0.99$ ; (2) model M (for medium persistence), where  $(\omega, \beta, \alpha) = (0.05, 0.9, 0.05)$  and  $\alpha + \beta = 0.95$ ; and (3) model L (for low persistence), where  $(\omega, \beta, \alpha) = (0.2, 0.75, 0.05)$  and  $\alpha + \beta = 0.8$ . For each model, samples of size 100 and 1000 are drawn with 10,000 replications. For each replication, a GARCH(1, 1) model is estimated and the tests are conducted. This table reports the actual rejection frequencies based on the 1, 5, and 10 percent critical values under the asymptotic distribution.

Test		Model and Sample Size					
		H	H	M	M	L	L
		1000	100	1000	100	1000	100
Actual Rejection Frequencies (%)							
Sign bias	1%	1.11	1.04	0.92	1.00	1.11	1.18
	5%	4.94	4.67	4.70	4.34	5.22	5.21
	10%	9.99	10.19	9.78	8.93	10.07	9.19
Negative size bias	1%	0.72	1.40	1.03	1.47	0.87	0.93
	5%	4.27	5.11	5.25	5.70	4.79	4.80
	10%	8.79	10.58	10.35	10.52	9.45	9.26
Positive size bias	1%	0.96	0.90	0.96	0.64	0.95	0.94
	5%	4.33	5.18	4.51	5.01	5.41	5.08
	10%	8.95	10.18	9.26	10.61	10.11	9.74
Joint	1%	0.82	10.20	0.94	0.61	0.90	0.85
	5%	4.45	4.87	4.98	4.72	4.69	4.72
	10%	9.02	10.05	9.38	10.78	9.50	9.81

replications. The nonconverged replications, not reported, yield similar results.

The power of the joint test is reasonably good for the larger sample size of 1000. Based on the 5 percent asymptotic critical value, the test rejects the null hypothesis of no asymmetry 67.05 percent of the time with data generated from model E, and 41.5 percent of the time with data generated from model G. However, the power of the joint test is weak when the sample size is small. This weakness is expected as both asymmetric effects and time-varying variance are hard to detect in small samples. Furthermore, the power of the individual tests is weak due to collinearity between the misspecifica-

**Table III**  
**Actual Rejection Frequencies When the Null is Not True**  
**(The Simulated Power)**

To check the power of the joint test, we perform a Monte Carlo experiment based on the two data-generating processes below.  $\varepsilon_t$  is the unpredictable return (the residual) at time  $t$ ,  $h_t$  is the conditional variance at time  $t$ ,  $v_t$  is the normalized residual at time  $t$  which is generated from a standard normal random number generator, and  $y_t$  is the simulated return at time  $t$  (which, for simplicity is assumed to be entirely unpredictable and hence is equal to  $\varepsilon_t$ ).

*Model E*

$$y_t = \varepsilon_t; \quad \varepsilon_t = \sqrt{h_t} \cdot v_t, \quad \text{where } v_t \sim \text{i.i.d. } N(0, 1)$$

$$\log(h_t) = -0.23 + 0.9 \cdot \log(h_{t-1}) + 0.25 \cdot [|v_{t-1}^2| - 0.3 \cdot v_{t-1}].$$

*Model G*

$$y_t = \varepsilon_t; \quad \varepsilon_t = \sqrt{h_t} \cdot v_t, \quad \text{where } v_t \sim \text{i.i.d. } N(0, 1)$$

$$h_t = 0.005 + 0.7 \cdot h_{t-1} + 0.28 \cdot [|v_{t-1}| - 0.23 \cdot \varepsilon_{t-1}]^2.$$

for each model, samples of size 100 and 1000 are drawn with 10,000 replications. For each replication, a GARCH(1, 1) model is estimated and the tests are conducted. This table reports the actual rejection frequencies based on the 1, 5, and 10 percent critical values under the asymptotic distribution.

		Model and Sample Size			
		E 1000	E 100	G 1000	G 100
		Actual Rejection Frequencies (%)			
Joint Test	1%	40.29	1.70	19.59	1.61
	5%	67.05	7.51	41.50	6.04
	10%	78.64	14.42	55.18	12.83

tion indicators, particularly that of the sign bias test and the negative size bias test.

### III. A Partially Nonparametric News Impact Model

An alternative approach to estimating the news impact curve is to implement a nonparametric procedure which allows the data to reveal the curve directly. Several approaches are available in the literature, including notably, Pagan and Schwert (1990) and Gouriéroux and Monfort (1992). In both cases the methods are developed only for low order autoregressive variance equations and in the fourier case, only OLS estimation is considered. Gouriéroux and Monfort essentially specify a histogram for the response of volatility to lags of the news which they estimate by maximum likelihood. In their most successful model however, they introduce a GARCH term to capture the long memory aspects.

Here we introduce a new partially nonparametric model. This will allow consistent estimation of the news impact curve under a range of conditions. It is labeled partially nonparametric because the long memory in the variance equation is given by a parametric component.

Let the range of  $\{\varepsilon_t\}$  be divided into  $m$  intervals with break points  $\tau_i$ . Let  $m^-$  be the number of intervals in the range where  $\varepsilon_{t-1}$  is negative. Also, let  $m^+$  be the number of intervals in the range where  $\varepsilon_{t-1}$  is positive, so that  $m = m^+ + m^-$ . Denote these boundaries by the numbers  $\{\tau_{-m^-}, \dots, \tau_{-1}, \tau_0, \tau_1, \dots, \tau_{m^+}\}$ . These intervals need not be equal size, nor do we need the same number on each side of  $\tau_0$ . For convenience and the ability to test symmetry, we select  $\tau_0 = 0$ .

If we define

$$\begin{aligned} P_{it} &= 1 && \text{if } \varepsilon_t > \tau_i \\ &= 0 && \text{otherwise, and} \\ N_{it} &= 1 && \text{if } \varepsilon_t < \tau_{-i} \\ &= 0 && \text{otherwise,} \end{aligned} \quad (16)$$

then a piecewise linear specification of the heteroskedasticity function is

$$h_t = \omega + \beta h_{t-1} + \sum_{i=0}^{m^+} \theta_i P_{it-1}(\varepsilon_{t-1} - \tau_i) + \sum_{i=0}^{m^-} \delta_i N_{it-1}(\varepsilon_{t-1} - \tau_{-i}), \quad (17)$$

where  $\omega$ ,  $\beta$ ,  $\theta_i$  ( $i = 0, \dots, m^+$ ), and  $\delta_i$  ( $i = 0, \dots, m^-$ ) are constant parameters. This functional form, which is really a linear spline with knots at the  $\tau_i$ 's, is guaranteed to be continuous. Between 0 and  $\tau_1$  the slope is  $\theta_0$  while between  $\tau_1$  and  $\tau_2$  it is  $\theta_0 + \theta_1$ , and so forth. Above  $\tau_{m^+}$ , the slope is the sum of all the  $\theta$ 's. If the partial sums at each point are of the same sign, the shape of the curve is monotonic.

To obtain better resolution with larger samples, we increase  $m$ . This is an example of the method of sieves approach to nonparametric estimation. A larger value of  $m$  can be interpreted as a smaller bandwidth, which will give lower bias and higher variance to each point on the curve. If  $m$  is increased slowly as a function of sample size, the procedure should asymptotically give a consistent estimate of any news impact curve. However, the rate of convergence and the standard errors may both be different from standard maximum likelihood results. Conversely, if  $m$  is held fixed, the estimator produces a consistent estimate of the news impact curve only if (17) is correctly specified. In such cases, the standard errors are given in their usual form.

We should point out that although the specification in (17) is capable of generating a wide range of news impact curves, it is very simple with respect to the impact of older information. All information is assumed to decay in an exponential fashion with decay rate  $\beta$ . Other terms could be added to the model, but they would substantially increase the computational complexity.

Two simple approaches to choosing the  $\tau_i$ 's could be used. The  $\tau_i$ 's could be unequally spaced, based on the order statistics, or equally spaced. In our example, we use equally spaced bins with break points at  $\sigma \cdot i$  for  $i = 0, \pm 1$ ,



$\pm 2$ ,  $\pm 3$ ,  $\pm 4$ , where  $\sigma$  is the unconditional standard deviation of the dependent variable. Thus:

$$h_t = \omega + \beta h_{t-1} + \sum_{i=0}^{m^+} \theta_i P_{it-1}(\varepsilon_{t-1} - i\sigma) + \sum_{i=0}^{m^-} \delta_i N_{it-1}(\varepsilon_{t-1} + i\sigma). \quad (18)$$

With  $m^+ = m^- = 4$ , there are ten coefficients in the news impact curve. Figure 3 gives an example of the graph of a partially nonparametric, or PNP, news impact curve.

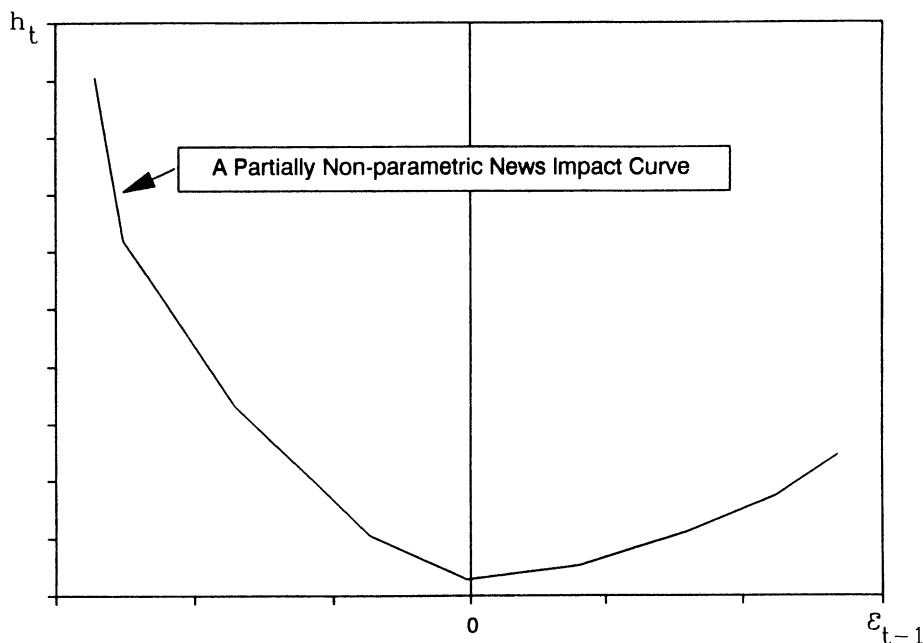
#### IV. Estimation of Japanese Stock Return Volatility: 1980 to 1988

To compare and demonstrate the empirical properties of the GARCH(1, 1) with some of the above-mentioned leverage or asymmetric volatility models, we apply the models to the daily returns series of the Japanese TOPIX index. The data were obtained from the PACAP Databases provided by the Pacific Basin Capital Market Research Center at the University of Rhode Island. In this section, we report our estimation and testing results for the parametric models for the full sample period from January 1, 1980 to December 31, 1988. In the next section, we estimate conditional volatility and the news impact curve using a nonparametric approach, and compare the news impact curve obtained from the nonparametric method to those obtained from the various parametric volatility models. In Section VI, we check the robustness of our results by reestimating some of our models using a shorter sample period from January 1, 1980 to September 30, 1987.

Since our focus is on the conditional variance, rather than the conditional mean, we concentrate on the unpredictable part of the stock returns, as obtained through a procedure similar to the one in Pagan and Schwert (1990). The procedure involves a day-of-the-week effect adjustment and an autoregressive regression which removes the predictable part of the return series.

Let  $y_t$  be the daily return of the TOPIX index for day  $t$ . We first regress  $y_t$  on a constant and five day-of-the-week dummies (for Tuesday through Saturday) to get the residual,  $u_t$ . The  $u_t$  is then regressed on a constant and  $u_{t-1}, \dots, u_{t-6}$  to obtain the residual,  $\varepsilon_t$ , which is our unpredictable stock return data.

The results for the above adjustment regressions and some summary statistics for our unpredictable stock return series are reported in Table IV. From the Ljung-Box test statistic for twelfth-order serial correlation for the levels, we find no significant serial correlation left in the stock returns series after our adjustment procedure. The coefficients of skewness and kurtosis both indicate that the unpredictable stock returns, the  $\varepsilon_t$ 's, have a distribution which is skewed to the left and significantly flat tailed. Furthermore, the Ljung-box test statistic for twelfth-order serial correlations in the squares strongly suggests the presence of time-varying volatility. The sign bias, negative size bias, and positive size bias test statistics introduced in the



**Figure 3. A partially nonparametric news impact curve.** The partially nonparametric news impact curve is a piecewise linear function. The equation for a typical partially nonparametric news impact curve is

$$h_t = A + \sum_{i=0}^{m^+} \theta_i P_{it-1}(\varepsilon_{t-1} - i\sigma) + \sum_{i=0}^{m^-} \delta_i N_{it-1}(\varepsilon_{t-1} + i\sigma).$$

where  $h_t$  is the conditional variance at time  $t$ ,  $\varepsilon_{t-1}$  is the unpredictable return at time  $t-1$ ,  $A \equiv \omega + \beta \cdot \sigma^2$ ,  $\sigma$  is the unconditional return standard deviation,  $\omega$  is the constant term, and  $\beta$  is the parameter for the  $h_{t-1}$  term in the PNP variance equation (equation 18 in the text).  $\theta_i (i = 0, \dots, m^+)$  and  $\delta_i (i = 0, \dots, m^-)$  are constant parameters. The above partially nonparametric news impact curve is indicative of cases with  $|\delta_i| > |\theta_i|$ ,  $|\delta_i| > |\delta_{i-1}|$ , and  $|\theta_i| > |\theta_{i-1}|$ , for all  $i$ .

previous section are also computed. The sign bias and negative size bias tests are both highly significant. The positive size bias test is not particularly significant, although if the size term were dropped it would be significant. These statistics strongly indicate that the value of  $\varepsilon_{t-1}$  influences current volatility: positive return shocks appear to increase volatility regardless of the size, while large negative return shocks cause more volatility than small ones.

Using the unpredictable stock returns series as the data series, we estimate the standard GARCH(1, 1) model, as well as five other parametric models from the first section which are capable of capturing the leverage and size effects. The five additional models are: the Exponential-GARCH(1, 1), the Asymmetric-GARCH(1, 1), the VGARCH(1, 1), the Nonlinear-Asymmetric-

Table IV

Mean Adjustment Regressions

This table reports the results of an adjustment procedure to remove the day-of-the-week effects from the daily return of the TOPIX index and to remove the predictable part of the return series. The procedure is analogous to the one in Pagan and Schwert (1990). First, the daily return  $y$  is regressed on a constant and five day-of-the-week dummies to get the residual  $u$ . Then  $u$  is regressed on a constant and the first six lags of  $u$  to obtain the residual  $\varepsilon$ , which is our unpredictable stock return data. The TOPIX index data are obtained from the PACAP Databases provided by the Pacific Basin Capital Market Research Center at the University of Rhode Island. The sample period is from January 1, 1980 to December 31, 1988. There are 2532 daily return observations in the sample period.

$y_t$  is the rate of return of the TOPIX index from day  $t - 1$  to day  $t$ . TUE $_t$ , WED $_t$ , THU $_t$ , FRI $_t$ , and SAT $_t$  are dummy variables for Tuesday, Wednesday, Thursday, Friday, and Saturday respectively. Each of these day-of-the-week dummies takes a value of 1 on the corresponding weekday and a value of 0 otherwise.  $u_t$  is the residual of the day-of-the-week adjustment regression (standard errors in parentheses). Ljung-Box (12) is the Ljung-Box statistics for twelfth-order serial correlation.

Day-of-the-Week Effect Adjustment		
$y_t =$	$-0.0119 - 0.0907 \cdot \text{TUE}_t + 0.2025 \cdot \text{WED}_t + 0.0953 \cdot \text{THU}_t + 0.1100 \cdot \text{FRI}_t + 0.1629 \cdot \text{SAT}_t + u_t$	
	(0.039) (0.055) (0.055) (0.055) (0.060)	
Autocorrelation Adjustment		
$u_t =$	$-0.0002 + 0.1231 \cdot u_{t-1} - 0.0802 \cdot u_{t-2} + 0.0456 \cdot u_{t-3} - 0.0526 \cdot u_{t-4} + 0.0772 \cdot u_{t-5} - 0.0706 u_{t-6} + \varepsilon_t$	
	(0.016) (0.02) (0.02) (0.02) (0.02) (0.02) (0.02)	
Summary Statistics for the Unpredictable Stock Returns		
Number of observations	2532	
Mean	0.0000	
Variance	0.6397	
Coefficient of skewness	-1.8927	
Coefficient of kurtosis	71.3511	
Ljung-Box (12) for the levels	1.3251	
Ljung-Box (12) for the squares	406.6700	
Sign bias test	-6.26	
Negative size bias test	-20.30	
Positive size bias test	1.56	
Joint test	139.00	
[ $p$ -Value]	[0.000]	

GARCH(1, 1), and the Glostten-Jagannathan-Runkle (GJR) model. The estimation is performed using the Bollerslev-Wooldridge quasi maximum likelihood approach. The adequacy of these volatility models is then checked using the sign bias, the negative size bias, and the positive size bias tests, as well as the commonly used Ljung-Box test for serial correlation in the squared normalized residuals. The estimation and diagnostic results for each of these models are presented in Table V. As a convention, the asymptotic standard errors are reported in parentheses ( $\cdot$ ) and the Bollerslev-Wooldridge robust standard errors are reported in brackets [ $\cdot$ ].

The estimation results in Table V indicate that the parameters corresponding to the  $\varepsilon_{t-1}/\sqrt{h_{t-1}}$  term in the EGARCH, the constant in the quadratic form in the AGARCH, the constant in the quadratic form in the VGARCH, and the  $\sqrt{h_{t-1}}$  term in the NGARCH, are all significant and negative using both standard and robust standard errors. The parameter corresponding to the  $S_{t-1}^-\varepsilon_{t-1}^2$  term in the GJR is significant and positive using both standard and robust standard errors. All these results are consistent with the hypothesis that negative return shocks cause higher volatility than positive return shocks. We can also see that the standard GARCH(1, 1) has a lower log-likelihood than most of the leverage or asymmetric models with the exception of the VGARCH. The GJR, the NGARCH, and the EGARCH yield the highest log-likelihood.

In diagnostic checks, the Ljung-Box statistic for twelfth-order serial correlations in the squared normalized residuals is significant at the 5 percent level for only the VGARCH. On the other hand, the negative size bias test statistics, as well as the joint test statistics, are significant for all models, with the EGARCH and the GJR being only marginally rejected by the joint test. All the models seem to have some problem in capturing the correct impact of news on volatility. Furthermore, the results indicate that the Ljung-Box test, which is commonly used as a specification check for volatility models, does not have much power in detecting misspecifications related to the leverage or asymmetric effects.

Overall, the Exponential GARCH model and the GJR model seem to outperform all other models in capturing the dynamic behavior of the Japanese stock returns, with the GJR model having a higher log-likelihood. To further our understanding of these different volatility models, some summary statistics, including the mean, standard deviation, minimum, maximum, skewness, and kurtosis, are produced for each of the estimated conditional variance series. They are reported in Table VI.

The conditional variance series produced by the best models, the EGARCH model and the GJR model, have the highest variation over time. The estimated conditional variance ranges from a low of 0.0491 to a high of 485.27, compared to 0.0842 and 90.83 under the standard GARCH model. The standard deviation of the EGARCH conditional variance, 10.555, is more than three times that of the standard GARCH model and twice that of the squared residual itself. The EGARCH conditional variance also has a much more skewed and flat tailed distribution than the other conditional variance

series and the squared return shocks. The fact that the unconditional variance of the EGARCH conditional variance is larger than the unconditional variance of the squared residual can actually be interpreted as evidence against the EGARCH model. To see this point, note that we can always write

$$\varepsilon_t^2 = h_t + (\varepsilon_t^2 - h_t). \quad (19)$$

Since  $h_t \equiv E_{t-1}(\varepsilon_t^2)$ , the two terms on the right-hand side of (19) are uncorrelated. So

$$\text{Var}(\varepsilon_t^2) = \text{Var}(h_t) + \text{Var}(\varepsilon_t^2 - h_t). \quad (20)$$

Hence,  $\text{Var}(\varepsilon_t^2) \geq \text{Var}(h_t)$  if  $h_t$  is correctly specified and if the unconditional variances exist. As we can see, the EGARCH model fails this test.<sup>4</sup>

## V. Partially Nonparametric ARCH Estimation

We now estimate the news impact curve by fitting a partially nonparametric model of the form given in (18). The exact specification and the estimation results are reported in Table VII. The specification is a piecewise linear model with kinks at  $\varepsilon_{t-1}$  equal to 0,  $\sigma$ ,  $2\sigma$ ,  $3\sigma$ , and  $4\sigma$ . If we compare the values of the coefficients corresponding to the terms  $P_{it}(\varepsilon_{t-1} - i\sigma)$ ,  $i = 0, 1, 2$ , to their counterparts  $N_{it}(\varepsilon_{t-1} + i\sigma)$ ,  $i = 0, 1, 2$ , we can see that negative  $\varepsilon_{t-1}$ 's cause more volatility than positive  $\varepsilon_{t-1}$ 's of equal absolute size. Moreover, the rate of increase in volatility, as we move towards  $\varepsilon_{t-1}$ 's with bigger absolute magnitude, is higher for the negative  $\varepsilon$ 's than for the positive ones. This finding suggests a sign or asymmetric effect, as well as a size effect, that differs for negative and positive  $\varepsilon$ 's. The estimated parameter values for the terms  $P_{it}(\varepsilon_{t-1} - i\sigma)$  and  $N_{it}(\varepsilon_{t-1} + i\sigma)$  for  $i = 3, 4$  have somewhat unexpected signs and magnitudes. Since these terms are for the extreme  $\varepsilon$ 's, they might be driven by only a few outliers. Indeed, even though they are significant using the traditional asymptotic standard errors, they are all insignificant using the Bollerslev-Wooldridge robust standard errors. Thus, the nonparametric estimation results indicate that the true slope of the news impact curve is probably steeper on the negative side.

To compare the news impact curve of the nonparametric model with those implied by the various volatility models, we compute the implied volatility level for each model at several prespecified values for  $\varepsilon_{t-1}$ , assuming that  $h_{t-1} = \sigma^2 = 0.63966$ . The results are summarized in Table VIII.

If we confine ourselves to  $\varepsilon_{t-1}$  in the range  $(-2.5, 2.5)$ , we see that, relative to the EGARCH model, the standard GARCH model tends to understate  $h_t$  for large negative  $\varepsilon_{t-1}$ 's and overstate  $h_t$  for large positive  $\varepsilon_{t-1}$ 's. These results are also true for the AGARCH, VGARCH, and NGARCH models. Of all six parametric models, the EGARCH and the GJR have news impact curves closest to the one suggested by the nonparametric estimation. However, if we consider the very extreme values for  $\varepsilon_{t-1}$ , we see that the

<sup>4</sup> We thank Rob Stambaugh for pointing this out to us.

**Table V**  
**Estimation Results and Diagnostics**

This table reports the estimation and diagnostic test results of various predictable volatility models for the daily return of the TOPIX index. Day-of-the-week effects and a predictable component in the daily return series have been removed. The estimation is performed by the method of quasi maximum likelihood using the BHHH numerical optimization algorithm. The sample period is from January 1, 1980 to December 31, 1988. In the estimation results part of the table, the numbers in parentheses (·) are the asymptotic standard errors and the numbers in brackets [·] are the Bollerslev-Wooldridge robust standard errors. In the test results part, Ljung-Box (12) is the Ljung-Box statistics for twelfth-order serial correlations in the squared normalized residuals. Also, one and two asterisks indicate significance at the 5 and 1 percent levels respectively.

$h_t$  is the conditional variance on day  $t$  and  $\varepsilon_{t-1}$  is the unpredictable return on day  $t - 1$ . The unpredictable return is obtained from the adjustment regressions in Table IV.

Estimation Results			
GARCH(1, 1)			
$h_t = 0.0238 + 0.6860 \cdot h_{t-1} + 0.3299 \cdot \varepsilon_{t-1}^2$			
	(0.003)	(0.011)	(0.008)
	[0.005]	[0.059]	[0.097]
logL = -2356.03			
EGARCH(1, 1)			
$\log(h_t) = -.0668 + 0.9012 \cdot \log(h_{t-1}) + 0.4927 \cdot \left[ \frac{ \varepsilon_{t-1} }{\sqrt{h_{t-1}}} - \sqrt{\frac{2}{\pi}} \right] - 0.1450 \cdot \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}$			
	(0.010)	(0.007)	(0.016)
	[0.020]	[0.022]	[0.104]
			(0.011)
			[0.048]
logL = -2344.03			
AGARCH(1, 1)			
$h_t = 0.0216 + 0.6896 \cdot h_{t-1} + 0.3174 \cdot (\varepsilon_{t-1} - 0.1108)^2$			
	(0.003)	(0.012)	(0.009)
	[0.005]	[0.055]	[0.088]
			(0.017)
			[0.030]
logL = -2345.12			
VGARCH(1, 1)			
$h_t = 0.0192 + 0.6754 \cdot h_{t-1} + 0.1508 \cdot (\varepsilon_{t-1}/\sqrt{h_{t-1}} - 0.1458)^2$			
	(0.005)	(0.014)	(0.004)
	[0.013]	[0.071]	[0.047]
			(0.031)
			[0.052]
logL = -2424.63			
NGARCH(1, 1)			
$h_t = 0.0199 + 0.7253 \cdot h_{t-1} + 0.2515 \cdot (\varepsilon_{t-1} - 0.2683/\sqrt{h_{t-1}})^2$			
	(0.002)	(0.010)	(0.008)
	[0.005]	[0.060]	[0.083]
			(0.036)
			[0.061]
logL = -2335.34			

Table V—Continued

Estimation Results					
GJR					
$h_t = 0.0241 + 0.7053 \cdot h_{t-1} + 0.1672 \cdot \varepsilon_{t-1}^2 + 0.2636 \cdot S_{t-1}^- \varepsilon_{t-1}^2$					
	(0.003)	(0.013)	(0.018)	(0.020)	
	[0.005]	[0.045]	[0.036]	[0.102]	
logL = -2333.11					
Diagnostic Test Results					
Model	Ljung-Box(12)	Sign Bias	Negative Size Bias	Positive Size Bias	Joint Test
GARCH(1, 1)	12.18	-0.30	-3.22*	-0.59	16.05**
EGARCH(1, 1)	12.14	-0.50	-2.79*	0.01	8.38*
AGARCH(1, 1)	11.43	-0.77	-3.21*	-0.58	12.57**
VGARCH(1, 1)	26.64*	-2.38*	-4.75*	-0.32	23.85**
NGARCH(1, 1)	11.19	-1.06	-3.22*	-0.61	11.92**
GJR	12.37	-0.67	-2.59*	-0.54	7.91*

Table VI

### Summary Statistics of the Conditional Variance Estimates

This table reports the summary statistics of the estimated conditional variances of the daily TOPIX returns from various predictable volatility models. The TOPIX data are obtained from the PACAP Databases provided by the Pacific Basin Capital Market Research Center at the University of Rhode Island. The sample period is from January 1, 1980 to December 31, 1988.

The statistic “Skew.” is the coefficient of skewness and the statistic “Kurto.” is the coefficient of kurtosis. For a standard normal random variable, the value of the coefficient of skewness is 0 and the value of the coefficient of kurtosis is 3.

$\varepsilon_t^2$  is the squared unpredictable return obtained from the adjustment regressions in Table IV.

$h_t$ GARCH,  $h_t$ EGARCH,  $h_t$ AGARCH,  $h_t$ VGARCH,  $h_t$ NGARCH, and  $h_t$ GJR, are conditional variances estimated from the GARCH(1, 1), EGARCH(1, 1), AGARCH(1, 1), VGARCH(1, 1), NGARCH(1, 1), and GJR models in Table V.

	Mean	Std. Dev.	Min.	Max.	Skew.	Kurto.
$\varepsilon_t^2$	0.6397	5.366	2.8e-8	236.60	37.038	1543.47
$h_t$ GARCH	0.7483	3.124	0.0842	90.83	21.173	523.20
$h_t$ EGARCH	0.8669	10.555	0.0491	485.27	40.843	1799.19
$h_t$ AGARCH	0.7367	3.047	0.0807	87.78	21.014	515.64
$h_t$ VGARCH	0.5243	0.674	0.0943	20.16	15.279	373.39
$h_t$ NGARCH	0.6961	2.574	0.0847	64.47	18.215	392.61
$h_t$ GJR	0.7561	3.492	0.0885	104.21	21.950	559.01

EGARCH and the GJR are indeed very different. In fact, because of its exponential functional form, the EGARCH model produces a ridiculously high  $h_t$  of 1225.1 for an  $\varepsilon_{t-1}$  equal to -10 which is about three thousand times the value of the unconditional variance. Since the Japanese stock market volatility after the 1987 crash was not that high, the EGARCH model might be too

Table VII  
Partially Nonparametric Estimation

This table reports the estimation results of a partially nonparametric ARCH volatility model for the daily return of the TOPIX index. Day-of-the-week effects and a predictable component in the daily return series have been removed. The estimation is performed by the method of quasi maximum likelihood using the BHHH numerical optimization algorithm. The raw data are obtained from the PACAP Database provided by the Pacific Basin Capital Market Research Center at the University of Rhode Island. The sample period is from January 1, 1980 to December 31, 1988. The numbers in parentheses (·) are the asymptotic standard errors and the numbers in brackets [·] are the Bollerslev-Wooldridge robust standard errors.

$h_t$  is the conditional variance on day  $t$  and  $\varepsilon_{t-1}$  is the unpredictable return on day  $t - 1$  which is obtained from the adjustment regressions in Table IV.  $\sigma$  is the unconditional standard deviation of  $\varepsilon_t$ ,  $P_{it}(i = 0, 1, 2, 3, 4)$  is a dummy variable that takes a value of 1 if  $\varepsilon_t$  is greater than  $i \cdot \sigma$  and a value of 0 otherwise, and  $N_{it}(i = 0, 1, 2, 3, 4)$  is a dummy variable that takes a value of 1 if  $\varepsilon_t$  is less than  $-i \cdot \sigma$  and a value of 0 otherwise.

Partially Nonparametric ARCH (PNP)	(logL = -2310.72)
$h_t = 0.0039 + 0.8015 \cdot h_{t-1}$ (0.002) (0.013) [0.012] [0.040]	
$+ 0.0897 \cdot P_{0t-1} \varepsilon_{t-1}$ (0.014) [0.043]	$+ 0.2269 \cdot P_{1t-1}(\varepsilon_{t-1} - \sigma)$ (0.088) [0.172]
$+ 0.6666 \cdot P_{2t-1}(\varepsilon_{t-1} - 2\sigma)$ (0.353) [0.720]	$- 3.7664 \cdot P_{3t-1}(\varepsilon_{t-1} - 3\sigma)$ (1.096) [1.991]
$+ 3.6915 \cdot P_{4t-1}(\varepsilon_{t-1} - 4\sigma)$ (1.540) [2.327]	
$- 0.1536 \cdot N_{0t-1} \varepsilon_{t-1}$ (0.014) [0.053]	$- 0.3312 \cdot N_{1t-1}(\varepsilon_{t-1} + \sigma)$ (0.093) [0.203]
$- 3.1194 \cdot N_{2t-1}(\varepsilon_{t-1} + 2\sigma)$ (0.278) [4.143]	$+ 7.3481 \cdot N_{3t-1}(\varepsilon_{t-1} + 4\sigma)$ (0.959) [8.699]
$- 5.4904 \cdot N_{4t-1}(\varepsilon_{t-1} + 4\sigma)$ (1.679) [5.769]	

extreme in the tails. Consequently, the GJR model, which also has a higher log-likelihood than the EGARCH, might be a more reasonable model to use.

VI. Subsample Robustness Check

To judge the sensitivity of our results to the extreme observations around the 1987 crash, we repeat part of our analysis for the subsample period from January 1, 1980 to September 30, 1987, excluding the crash. The results for



**Table VIII**  
**The News Impact Curves**

This table gives the value of the current volatility,  $h_t$ , as a function of the past return shock,  $\varepsilon_{t-1}$ , holding past conditional variance,  $h_{t-1}$ , fixed at its unconditional mean level. The values are given for various predictable volatility models for the daily return of the TOPIX Index. The TOPIX data are obtained from the PACAP Database provided by the Pacific Basin Capital Market Research Center at the University of Rhode Island. The sample period is from January 1, 1980 to December 31, 1988.

$\varepsilon_{t-1}$	GARCH $h_t$	EGARCH $h_t$	AGARCH $h_t$	VGARCH $h_t$	NGARCH $h_t$	GJR $h_t$	PNP $h_t$
-10.0	33.450	1225.100	32.910	24.580	26.730	43.550	12.793
-5.0	8.710	22.739	8.753	6.623	7.323	11.245	4.061
-2.5	2.524	3.098	2.626	2.065	2.337	3.167	3.533
-2.0	1.782	2.079	1.877	1.507	1.717	2.198	2.470
-1.0	0.793	0.937	0.854	0.745	0.855	0.906	0.736
-0.5	0.545	0.629	0.581	0.541	0.612	0.583	0.593
0.0	0.463	0.422	0.467	0.454	0.495	0.475	0.517
0.5	0.545	0.525	0.511	0.486	0.504	0.517	0.561
1.0	0.793	0.652	0.714	0.635	0.639	0.642	0.652
2.0	1.782	1.007	1.596	1.287	1.286	1.144	1.235
2.5	2.524	1.251	2.275	1.790	1.797	1.520	1.348
5.0	8.710	3.710	8.050	6.073	6.243	4.655	1.038
10.0	33.453	32.616	31.503	23.480	24.566	17.195	5.579

the day-of-the-week and autocorrelation adjustments, as well as some summary statistics for the residuals, are reported in Table IX. The Ljung-Box(12) statistic for the squared residuals strongly suggests the existence of autocorrelation in the squared residuals (and hence time-varying conditional volatility of the autoregressive type). The sign bias test statistic is significant and the two size bias test statistics are also highly significant, with the negative size bias test statistic having a higher value. These results indicate a size effect of news, which is stronger for bad news than for good news. Given the superiority of the EGARCH and the GJR model over the other asymmetric volatility models, we repeat our estimation for the standard GARCH, the EGARCH and the GJR models only. The results are reported in Table X.

Several results in Table X are worth special notice. First, the parameter corresponding to the  $\varepsilon_{t-1}/\sqrt{h_{t-1}}$  term in the EGARCH and the parameter corresponding to the  $S_{t-1}^2\varepsilon_{t-1}^2$  term in the GJR are both highly significant, even using the Bollerslev-Wooldridge  $t$ -test. Second, the joint test is significant for the standard GARCH model but not for the EGARCH and GJR models. The log-likelihoods of both the EGARCH and the GJR models are substantially higher than the log-likelihood of the standard GARCH model. All of these results point to the presence of a leverage effect in the data. In terms of the size effect, the positive size bias test is insignificant for all three models, indicating that there is not much size effect for positive return

Table IX  
Mean Adjustment Regressions for the Precrash Period

This table reports the results of an adjustment procedure to remove the day-of-the-week effects from the daily return of the TOPIX index and to remove the predictable part of the return series. The procedure is analogous to the one in Pagan and Schwert (1990). First, the daily return  $y$  is regressed on a constant and five day-of-the-week dummies to get the residual  $u$ . Then  $u$  is regressed on a constant and the first six lags of  $u$  to obtain the residual  $\varepsilon$ , which is our unpredictable stock return data. The TOPIX index data are obtained from the PACAP Databases provided by the Pacific Basin Capital Market Research Center at the University of Rhode Island. The sample period is from January 1, 1980 to September 30, 1987. There are 2192 daily return observations in this precrash subsample period.

$y_t$  is the rate of return of the TOPIX index from day  $t - 1$  to day  $t$ . TUE $_t$ , WED $_t$ , THU $_t$ , FRI $_t$ , and SAT $_t$  are dummy variables for Tuesday, Wednesday, Thursday, Friday, and Saturday respectively. Each of these day-of-the-week dummies takes a value of 1 on the corresponding weekday and a value of 0 otherwise.  $u_t$  is the residual of the day-of-the-week adjustment regression (standard errors in parentheses). Ljung-Box (12) is the Ljung-Box statistics for twelfth-order serial correlation.

Day-of-the-Week Effect Adjustment	
$y_t = 0.0162 - 0.1088 \cdot \text{TUE}_t + 0.1412 \cdot \text{WED}_t + 0.0682 \cdot \text{THU}_t + 0.1008 \cdot \text{FRI}_t + 0.1411 \cdot \text{SAT}_t + u_t$	
(0.035) (0.049) (0.049) (0.049) (0.049) (0.053)	
Autocorrelation Adjustment	
$u_t = -0.0002 + 0.2491 \cdot u_{t-1} - 0.0614 \cdot u_{t-2} - 0.0275 \cdot u_{t-3} + 0.0496 \cdot u_{t-4} + 0.0019 \cdot u_{t-5} - 0.0490 u_{t-6} + \varepsilon_t$	
(0.014) (0.02) (0.02) (0.02) (0.02) (0.02)	
Summary Statistics for $\varepsilon$	
Number of observations	2192
Mean	0.0000
Variance	0.4302
Coefficient of skewness	0.0947
Coefficient of kurtosis	8.7135
Ljung-Box (12) for the levels	5.9197
Ljung-Box (12) for the squares	540.8552
Sign bias test	-2.30
Negative size bias test	-13.90
Positive size bias test	5.64
Joint test	77.00
[ $p$ -Value]	[0.000]

Table X

Estimation Results and Diagnostics for the Precrash Period

This table reports the estimation and diagnostic test results of various predictable volatility models for the daily return of the TOPIX index. Day-of-the-week effects in the daily return series have been adjusted for and the predictable component in the daily return series has been removed. The estimation is performed by the method of quasi maximum likelihood using the BHHH numerical optimization algorithm. This precrash subsample period is from January 1, 1980 to September 30, 1987.

In the estimation results part of the table, the numbers in parentheses (·) are the asymptotic standard errors and the numbers in squared brackets [·] are the Bollerslev-Wooldridge robust standard errors. In the test results part, Ljung-Box(12) is the Ljung-Box statistics for twelfth-order serial correlations in the squared normalized residuals. Also, one asterisk indicates significance at the 5 percent level.

$h_t$  is the conditional variance on day  $t$  and  $\varepsilon_{t-1}$  is the unpredictable return on day  $t - 1$ . The unpredictable return is obtained from the adjustment regressions in Table IX.

Estimation Results					
GARCH(1, 1)					
$h_t = 0.0129 + 0.8007 \cdot h_{t-1} + 0.1829 \cdot \varepsilon_{t-1}^2$ <div>(0.002) (0.013) (0.014) [0.003] [0.025] [0.026]</div>					
logL = -1829.50					
EGARCH(1, 1)					
$\log(h_t) = -.0350 + 0.9579 \cdot \log(h_{t-1}) + 0.2955 \cdot \left[ \frac{ \varepsilon_{t-1} }{\sqrt{h_{t-1}}} - \sqrt{\frac{2}{\pi}} \right] - 0.0615 \cdot \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}$ <div>(0.007) (0.005) (0.019) (0.010) [0.014] [0.011] [0.037] [0.024]</div>					
logL = -1822.30					
GJR					
$h_t = 0.1093 + 0.8181 \cdot h_{t-1} + 0.1130 \cdot \varepsilon_{t-1}^2 + 0.1048 \cdot S_{t-1}^- \varepsilon_{t-1}^2$ <div>(0.002) (0.012) (0.014) (0.019) [0.003] [0.021] [0.023] [0.038]</div>					
logL = -1819.23					
Diagnostic Test Results					
Model	Ljung-Box(12)	Sign Bias	Negative Size Bias	Positive Size Bias	Joint Test
GARCH(1, 1)	13.23	-0.01	-1.98	-0.33	8.15*
EGARCH(1, 1)	21.30*	-0.46	-2.59*	0.47	7.38
GJR	15.47	-0.19	-1.78	0.61	3.97

shocks. However, the negative size bias test statistics are marginally significant for the standard GARCH and significant for the EGARCH, but insignificant for the GJR. The failure of the EGARCH model to capture the size effect is probably due to two factors: the quadratic function dominates the exponen-

tial for small  $\varepsilon$ 's, and the Japanese stock market was quite calm before the 1987 crash. The only model that seems to do well in both normal and abnormal times is the GJR model, which also has the highest log-likelihood in both periods.

## VII. Summary and Conclusion

This paper recommends the news impact curve as a standard measure of how news is incorporated into volatility estimates. In order to better estimate and match news impact curves to the data, several new candidates for modeling time-varying volatility are introduced and contrasted. These models allow several types of asymmetry in the impact of news on volatility. Furthermore, some new diagnostic tests are presented which are designed to determine whether the volatility estimates adequately represent the data. Finally, a partially nonparametric model is suggested which allows the data to determine the news impact curve directly.

These models are fitted to daily Japanese stock returns from 1980 to 1988. All the models find that negative shocks introduce more volatility than positive shocks, with this effect particularly apparent for the largest shocks. The diagnostic tests however, indicate that in many cases the modeled asymmetry is not adequate. The best model is the one proposed by Glosten, Jagannathan, and Runkle (GJR).

The partially nonparametric (PNP) ARCH model, when fitted to the data, confirm this behavior. For reasonable shock values, the volatilities forecast by EGARCH, GJR, and PNP are similar. However, for more extreme shocks, these forecasts differ dramatically. In fact, the standard deviation of the EGARCH estimated conditional variance is even higher than that of the squared residual itself. This result could be interpreted as evidence against the EGARCH, because the variability of the conditional variance, if correctly specified, should not be higher than that of the squared residual.

The results are similar, although less dramatic, when the same analysis is conducted excluding the October 1987 crash. Overall, these results show a greater impact on volatility of negative, rather than positive, return shocks. The results indicate that, of the variance parametric models, the GJR is the best at parsimoniously capturing this asymmetric effect. Finally, the PNP model successfully reveals the shape of the news impact curve and is a useful approach to modeling conditional heteroskedasticity.

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