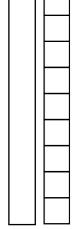
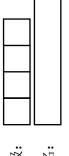


任课老师品





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重庆邮电大学 2024-2025 学年第一学期(期末)

Advanced Mathematics I Final Examination

INSTRUCTIONS

- (a) This examination paper consists of 15 questions, 2 pages for the total of 100 marks.
- (b) Write the DETAILED process for each question in the space provided on the answer sheet
- (c) You are required to answer each question in ENGLISH.
- 1. [5 marks] Let a > 0 and

$$y = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\ln(x + \sqrt{x^2 + a^2})$$

Find dy.

2. [5 marks] Find the limit

$$\lim_{x\to 0}\frac{6\sin x-6x+x^3}{x^5}$$

3. [5 marks] Find the limit

$$\lim_{x\to\infty} \left(\frac{x+1012}{x-1013}\right)^{x+2025}$$

4. [5 marks] Find $\frac{dy}{dx}$ from the equation

$$\arctan \frac{y}{x} = \ln \sqrt{x^2 + y^2}$$

5. [6 marks] Determine the values of the constants a, b, c such that the second derivative of the function f(x) at the point x = 0 does exist, where

$$f(x) = \begin{cases} ax^2 + bx + c, & x < 0 \\ \ln(1+x), & x \ge 0 \end{cases}$$

6. **[6 marks]** Let

$$\begin{cases} x = f'(t) \\ y = tf'(t) - f(t) \end{cases}$$

If $f''(t) \neq 0$, find $\frac{d^2y}{dx^2}$.

7. [6 marks] Find the 5th derivate of the function

$$f(x) = \sin^2 x$$

8. [6 marks] Find the local extreme values in the interval $[0, 2\pi]$ of the function

$$f(x) = sinx + cosx$$

9. [6 marks] If $\frac{\sin x}{x}$ is a primitive function of f(x), find the integral

$$\int x f'(x) dx$$

10. [6 marks] Find the integral

$$\int \arcsin^2 x \ dx$$

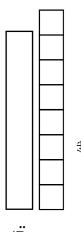
11. [6 marks] Find the integral

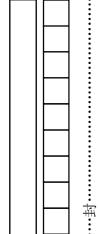
$$\int_{1}^{1} x^{3} \ln^{2} x \ dx$$

12. [6 marks] Find the integral

$$\int \frac{x-5}{x^2-4x-12} \, dx$$

13. **[6 marks]** Find the volume of the solid obtained by rotating the curve $y = \sqrt{x}$ between x = 1 and x = 4 about the x - axis.





14. [8 marks] Find the limit

$$\lim_{x\to 0} \frac{\displaystyle\int_{\cos x}^1 t \ln t \, dt}{e^{x^2}-1}$$

15. [8 marks] Let a, b > 0. If f(x) is continuous on the closed interval [a, b], and f(x) is differentiable in the open interval (a, b). Prove that the equation

$$2x[f(b) - f(a)] = (b^2 - a^2)f'(x)$$

has at least one zero in (a, b).

16. [10 marks] The sequence $[x_n]$ is defined by $x_1 = a$ with a > 0, and

$$x_{n+1}\!=\!rac{1}{2}\!\left(\!x_{n}\!+\!rac{a}{x_{n}}\!
ight)\!,\,n\!=\!1\,,2\,,...$$

- (1) [2 marks] Prove that $x_n \ge \sqrt{a}$ for n = 2,3,...(2) [2 marks] Prove that $x_n \ge x_{n+1}$ for n = 2,3,...
- (3) [6 marks] Prove that $\lim_{n\to\infty} x_n$ does exist and find it.