

Lab2 Report

TERM:_	2022-2023
Module:	EE2622 Fundamentals of Signals and Systems
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Part 2: Modelling and Test of Control Systems Using Control Theory Experiment Box

1. Introduction

1.1. Aim

To explore and learn the relationship between control systems and system functions. To investigate the impact of control element parameters on output dynamic performance, understand time domain response and parameter measurement of first-order systems, and analyze transient response of second-order systems.

1.2. Objective

- (1) Review the knowledge what we have learned in lectures, include feedback control systems, 1st and 2nd order differentiation functions and system functions.
- (2) Use THKKL-1 experiment box to complete the 6 control segments.
- (3) Record the results and analysis them.

2. Experimental Equipment and Components

- (1) THKKL-1 experiment box.
- (2) Ultra-low frequency slow scanning oscilloscope.
- (3) Software: MATLAB R2022b.

3. Experimental Steps and Results

(1) Proportion segment G1(s) = 1, G2(s) = 2

Proportional segment is one of the simplest and fundamental components in a control system. It directly generates a control output that is proportional to the magnitude of the error signal. The primary purpose of the proportional control is to adjust the system's response speed and gain, enabling the output to track the changes in the input signal.

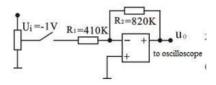


Fig. 3.1.1 Circuit diagram

Follow the instructions in the lab manual, the circuit is connected as shown in figure 3.1.1. The circuit is a closed-loop inverted amplification circuit composed of an operational amplifier and two resistors. As I learned before, $\frac{v_0}{v_i} = -\frac{R_2}{R_1}$. Thus, the transfer function is:

$$G(s) = \frac{R_2}{R_1} = K$$

For G1(s)=1, it means $R_2=R_1$. Therefore, I choose $R_1=820k\Omega$, $R_2=820k\Omega$. For G2(s)=2, it means $R_2=2R_1$. Therefore, I choose $R_1=410k\Omega$, $R_2=820k\Omega$.

Finally, I tested it on oscilloscope.

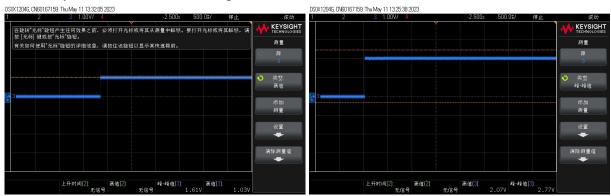


Fig. 3.1.2. The scanned measurement results.

As shown in figure 3.1.2. Each grid the oscilloscope represents 1V. The output is still a step signal, but the peak value undergoes a transformation. The left figure shows G1(s) = 1, K=,1 the peak value is 1V and the right figure shows G2(s) = 2, K=2, the peak value is 2V.

(2) Integration segment $G_1(s) = \frac{1}{s}$ and $G_2(s) = \frac{1}{0.5s}$

The integration segment is an important component in a control system used to eliminate steady-state error by integrating the error signal. It gradually reduces or eliminates steady-state error by continuously accumulating the error signal.

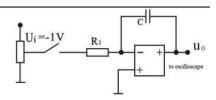


Fig. 3.2.1 Circuit diagram

Follow the instructions in the lab manual, the circuit is connected as shown in figure 3.2.1. The transfer function is

$$G(s) = \frac{1}{R_1 C s} = \frac{1}{T s}$$

For G1(s)=1/s, T=1, $R_1C=1$. Therefore, I choose $R_1=1M\Omega$, $C=1\mu F$. For G2(s)=1/0.5s, T=0.5, $R_1C=0.5$. Therefore, I choose $R_1=510k\Omega$, $C=1\mu F$

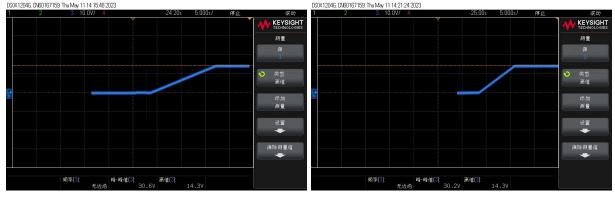


Fig. 3.2.2 The scanned measurement results.

As shown in figure 3.2.2. Each grid the oscilloscope represents 10V. The output signal is a linear function. The left figure shows G1(s)=1/s, the right figure shows G2(s)=1/0.5s. When $t \to \infty$, the output voltage of both transfer functions is 15V. The slope of G1(s) is smaller than G2(s).

(3) Proportional differentiation (PD) segment $G_1(s) = 2 + s$, $G_2(s) = 1 + 2s$

Proportional differentiation segment is a commonly used control element in control systems. It combines proportional control and derivative control to generate a control signal based on the error signal's proportionality and rate of change. The Proportional-Derivative Control generates the control signal based on both the magnitude and rate of change of the error signal. It combines the sensitivity of proportional control and the responsiveness of derivative control, balancing system stability and dynamic response.

The transfer function of a Proportional differentiation segment is typically represented as $G(s) = K_p + K_d s$, where K_p is the proportional gain, K_d is the derivative gain, and s is the Laplace variable.

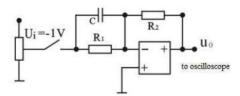


Fig. 3.3.1 Circuit diagram

Follow the instructions in the lab manual, the circuit is connected as shown in figure 3.3.1. The transfer function is

$$G(s) = \frac{R_2}{R_1} \cdot (R_1 C s + 1) = K(T_D s + 1)$$

For G1(s)=2+s, K=2,
$$T_D = 0.5$$
, $\frac{R_2}{R_1} = 2$, $R_1C = 0.5$, $R_1 = 510k\Omega$, $R_2 = 1M\Omega$, $C = 1\mu F$

For G2(s)=2s+1, K=1,
$$T_D=2$$
, $\frac{R_2}{R_1}=1$, $R_1C=2$, $R_1=1$ M Ω , $R_2=1$ M Ω , $C=2\mu$ F

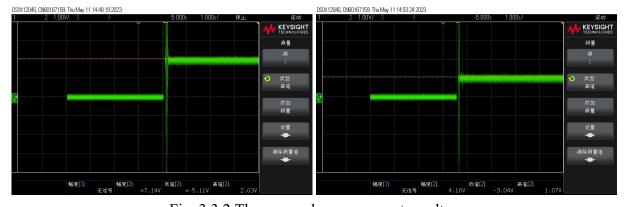


Fig. 3.3.2 The scanned measurement results.

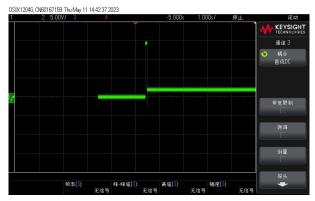


Fig. 3.3.3 The scanned measurement of overshoot $\delta(t)$

As shown in figure 3.3.2. Each grid the oscilloscope represents 1V. The output signal is like a unit impulse function add a unit step function, which have different coefficients. The left figure shows G1(s)=s+2, K=2, after $\delta(t)$, the output voltage is stable at 2V. The right figure shows G2(s)=2s+1, K=1, after $\delta(t)$, the output voltage is stable at 1V. Figure 3.3.3 shows the maximum value of overshoots, it is nearly +15V.

(4) Inertial segment
$$G_1(s) = \frac{1}{s+1}$$
, $G_2(s) = \frac{1}{0.5s+1}$

An inertial segment is an essential component in control systems that represents the inertia characteristics of a system or a signal. It simulates the inertia effect observed in the physical world, capturing the response speed and delay of the system or signal to changes. The inertial element can be seen as a low-pass filter, smoothing the signal, and introducing a delayed response. It exhibits lag and slow response to rapidly changing input signals, resembling the inertia and time delay exhibited by physical objects subjected to external forces or disturbances.

In control systems, the transfer function of an inertial element is typically represented as $G(s) = \frac{K}{Ts+1}$, where K is the gain, T is the time constant, and s is the Laplace variable.

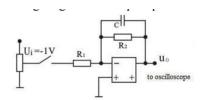


Fig. 3.4.1 Circuit diagram

Follow the instructions in the lab manual, the circuit is connected as shown in figure 3.4.1. The transfer function is

$$G(s) = \frac{R_2}{R_1} \cdot \frac{1}{R_2 C s + 1} = \frac{K}{T s + 1}$$

For G1(s)=1/(s+1),
$$\frac{R_2}{R_1}$$
 = 1, R_2C = 1, K = 1, T = 1. $R_1 = R_2 = 1M\Omega$, $C = 1\mu F$.

For G2(s)=1/(0.5s+1),
$$\frac{R_2}{R_1}$$
 = 1, R_2C = 0.5, K = 1, T = 0.5. $R_1 = R_2 = 510k\Omega$, $C = 1\mu F$.

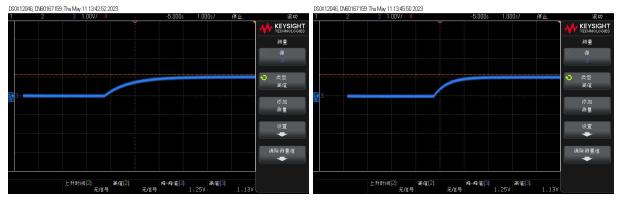


Fig. 3.4.2 The scanned measurement results.

As shown in figure 3.4.2. Each vertical grid the oscilloscope represents 1.00V, each horizontal grid represents 1.00s. The output signal is a graph with a gradually increasing curve. The left figure shows G1(s), and the right figure shows G2(s). The maximum value of the figure corresponding to two transfer functions is 1V, which means that the value of K of them are 1. However, the rising speed of G1(s) and G2(s) is different, G1(s) is much faster than G2(s) obviously.

For G1(s), the measurement time for complete charging is close to 6s. For G2(s), the measurement time for complete charging is close to 3s.

(5) Proportional integral segment $G_1(s) = 1 + \frac{1}{s}$, $G_2(s) = 2\left(1 + \frac{1}{2s}\right)$

Proportional integral segment is a common control element that combines proportional control and integral control to adjust the steady-state error and dynamic response of a system. In a proportional-integral segment, the generation of the control signal considers both the magnitude of the error (proportional action) and the accumulation of the error (integral action). Its output is a weighted sum of the proportional gain and the integral gain applied to the error signal.

The transfer function of a proportional-integral segment is typically represented as $G(s) = K_p + \frac{K_i}{s}$, where K_p is the proportional gain, K_i is the integral gain, and s is the Laplace variable.

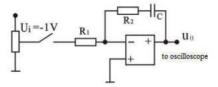


Fig. 3.5.1 Circuit diagram

Follow the instructions in the lab manual, the circuit is connected as shown in figure 3.5.1. The transfer function is

$$G(s) = \frac{R_2}{R_1} \cdot \left(1 + \frac{1}{R_2 C s}\right) = K\left(1 + \frac{1}{T_2 s}\right)$$

For G1(s)=1+1/s,
$$\frac{R_2}{R_1}$$
 = 1, R_2C = 1. $R_1 = R_2 = 1M\Omega$, $C = 1 \mu F$

For G2(s)=2(1+1/2s),
$$\frac{R_2}{R_1} = 2$$
, $R_2C = 2$. $R_1 = 510k\Omega$, $R_2 = 1M\Omega$, $C = 2 \mu F$

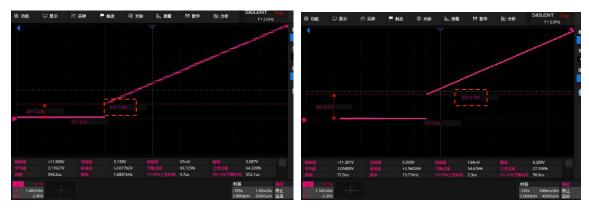


Fig. 3.5.2 The scanned measurement results.



Fig. 3.5.3 The scanned results when $t \rightarrow \infty$

As shown in figure 3.5.2. The left figure shows G1(s), when inputting unit step signal, the output jumps from 0V to 1V, so K=1. The right figure shows G2(s), when inputting unit step signal, the output jumps from 0V to 2V, so K=2. The slopes of the functions in both graphs are the same. Subsequently increasing in a linear manner. From figure 3.5.3, when $t \to \infty$, the output voltage is stable at Y1=14.96V, nearly +15V.

(6) Oscillation segment

An oscillatory element in a control system introduces periodic oscillations in the output through feedback or transfer functions. It is used in control system designs or specialized applications like oscillators or pendulum systems.

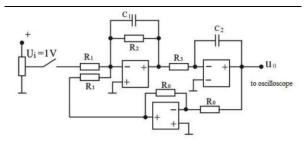


Fig. 3.6.1 Circuit diagram

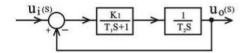


Fig. 3.6.2 Block diagram of the oscillation segment

Follow the instructions in the lab manual, the circuit is connected as shown in figure 3.6.1 and the corresponding block diagram is shown in figure 3.6.2, the diagram shows the simplified feedback system. Use this two figure can get the transfer function:

$$[U_{i}(s) - U_{0}(s)] \cdot \frac{K_{1}}{T_{1}T_{2}s^{2} + T_{1}s} = U_{0}(s)$$

$$G(s) = \frac{U_{0}(s)}{U_{i}(s)} = \frac{K_{1}}{T_{1}T_{2}s^{2} + T_{2}s + K_{1}}$$
For $G(s) = \frac{10}{0.1s^{2} + s + 10}$, $K_{1} = 10$, $T_{1} = 0.1$, $T_{2} = 1$

$$\frac{K_{1}}{T_{1} + s} = \frac{10}{0.1s + 1} \Rightarrow R_{1} = 100k\Omega, R_{2} = 1M\Omega, C_{1} = 0.1\mu F$$

$$\frac{1}{T_{2}s} = \frac{1}{s} \Rightarrow R_{3} = 510k\Omega, C_{2} = 2\mu F$$

Fig. 3.6.3 The scanned measurement results.

As shown in figure 3.6.3. The result is an oscillating image. The measured time to first peak $t_p = \Delta X = 0.43s$, the measured time of percentage overshoot PO is: $\frac{\Delta Y \times 100\%}{1V} = 12.3\%$. After some time of oscillation, the output gradually approaches a stable value of 1V.

4. Results Analysis

To analyze the experimental results, it is necessary to convert each transfer function from the s domain to the time domain. This is because the transfer function is originally defined in the s domain, while the experimental results are measured and represented in the time domain.

In system function, $Y(s) = H(s) \cdot E(s)$. The corresponding time-domain function is y(t) = h(t) * e(t).

However, it is not convenient to use convolution to calculate time-domain results. Therefore, I can use Laplace inverse transform convert the Y(s) to y(t). In Laplace inverse transform, if

 $Y(s) = \frac{k_1}{s - p_1} + \frac{k_2}{s - p_2} + \dots + \frac{k_n}{s - p_n}, \text{ the corresponding inverse transform is } y(t) = \mathcal{L}^{-1} \left[\frac{k_1}{s - p_1} \right] + \dots + \mathcal{L}^{-1} \left[\frac{k_n}{s - p_n} \right] = k_1 e^{p_1 t} + k_2 e^{p_2 t} + \dots + k_n e^{p_n t}$

$$Y(s) = \sum_{i=1}^{n} \frac{k_i}{s - p_i} \quad \leftrightarrow \quad y(t) = \sum_{i=1}^{n} k_i e^{p_i t}$$

(1) Proportional segment

Simulation

Overall,

Based on the two transfer functions, I firstly simulate them on MATLAB. The input signal is always unit step signal u(t).



Fig. 4.1.1 Simulation G1(s) = 1(left), G2(s) = 2(right)

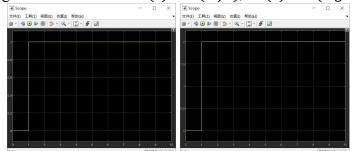


Fig. 4.1.2 Simulation results on MATLAB G1(s) = 1(left), G2(s) = 2(right)

The simulation results are shown in figure 4.1.2. It can find that the simulation results are consistent with the actual measured results.

Theoretical analysis

The input signal is u(t). Therefore, if t > 0, u(t) = 1. In s-domain, $E(s) = \frac{1}{s}$. $Y(s) = E(s) \cdot H(s) = \frac{K}{s}$. The corresponding unit step signal response is y(t) = K. Therefore, if G1(s)=1, y(t)=1, and if G2(s)=2, y(t)=2. The theoretical results are consistent with the actual results. The output is K times the input, which increases proportionally, so it is called the proportional segment.

(2) Integration segment



Fig. 4.2.1 Simulation G1(s)=1/s (left), G2(s)=1/0.5s (right).

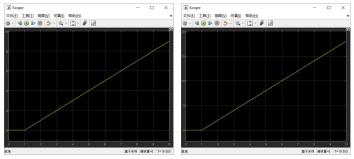


Fig. 4.2.2 Simulation result G1(s)=1/s (left), G2(s)=1/0.5s (right).

The simulation results are shown in figure 4.2.2. The slope of the right figure is twice the slope of the left figure. Therefore, the simulation results are consistent with the actual measured results.

Theoretical analysis

This segment is called integration segment. Thus, the output variable y(t) is the integral of the input variable x(t). The formula is: $y = \frac{1}{T} \int x \, dt$. Use Laplace transform, $Y(s) = \mathcal{L}[y(t)] = \frac{1}{T} \int x \, dt$.

$$\mathcal{L}\left[\frac{1}{T}\int x\,dt\right] = \frac{X(s)}{Ts}$$
, therefore, $G(s) = \frac{Y(s)}{X(s)} = \frac{1}{Ts}$. In this lab, $X(s)=1/s$. Thus, $Y(s) = \frac{1}{Ts^2}$, use

Laplace inverse transform to calculate the unit step signal response, $y(t) = \frac{tu(t)}{T}$. If G1(s)=1/s, T=1, $y_1(t) = tu(t)$, if G2(s)=1/0.5s, T=0.5, $y_2(t) = 2tu(t)$. This can explain why the slope of G2 (s) that of G1 (s) is twice. In addition, from $y_1(t)$ and $y_2(t)$, it can find that the function is

a proportional linear function. In this lab, I use $\pm 15V$ DC stabilized power supply. Therefore, the power supply for Op-Amp is $\pm 15V$, the maximum voltage output is 15V. That's why when $t \to \infty$, the output

voltage of both transfer functions is 15V.

The theoretical and actual results are generally consistent. The voltage consistently increases in a linear fashion over time, which is referred to as the integration segment.

(3) Proportional differential (PD) segment

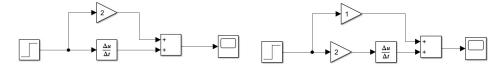


Fig. 4.3.1 Simulation G1(s)=s+2 (left), G2(s)=2s+1 (right)

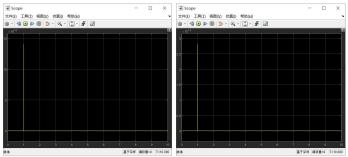


Fig. 4.3.2 Simulation result G1(s)=s+2 (left), G2(s)=2s+1 (right)

The simulation results are shown in figure 4.3.2. In MATLAB, it is not existing the maximum voltage, so $\delta(t)$ can be infinity. However, the simulation results are consistent with the actual measured results.

• Theoretical analysis

The segment is called proportional differential segment. The transfer function,

$$G(s) = K(T_D s + 1) = K_p + K_d s$$

Use Laplace inverse: because $\mathcal{L}^{-1}[1] = \delta(t), \mathcal{L}^{-1}\left[\frac{1}{s}\right] = u(t), Y(s) = G(s) \cdot E(s) = U(s)$

 $\frac{K(T_Ds+1)}{s}$. Therefore, the unit step signal response $y(t)=\mathcal{L}^{-1}[Y(s)]=KT_D\delta(t)+K=K_D\delta(t)+K_P$. For G1(s)=s+2, $y_1(t)=\delta(t)+2$, so if 't' is after $\delta(t)$, $y_1(t)=2$. For G2(s)=2s+1, $y_2(t)=2\delta(t)+1$, so if 't' after $\delta(t)$, $y_2(t)=1$. In addition, the DC power supply is $\pm 15V$, so the maximum value of overshoot is +15V.

The theoretical and actual results are generally consistent. For general forms, assume that the input signal is e(t), the output signal in time domain is $y(t) = K_p e(t) + K_D \frac{de(t)}{dt}$. The output signal appears in the form of a differential equation, which is why this circuit is called a proportional differential segment.

(4) Inertial segment

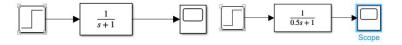


Fig. 4.4.1 Simulation G1(s)=1/(s+1), G2(s)=1/(0.5s+1)

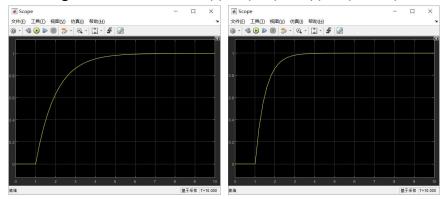


Fig.4.4.2 Simulation results
$$G1(s)=1/(s+1)$$
, $G2(s)=1/(0.5s+1)$

The simulation results are shown in figure 4.4.2. Through simulation, it can find that the complete charging time of G1(s)=6.00s and the time of G2(s)=3.00s. The simulation results are consistent with the actual measured results.

Theoretical analysis

The segment is called inertial segment, the transfer function is: $G(s) = \frac{K}{T_{S+}}$.

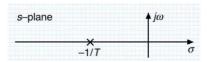


Fig. 4.4.3 The pole plot of G(s)

$$Y(s) = G(s) \cdot E(s) = \frac{K}{s(Ts+1)} = \frac{K}{s} + \frac{-KT}{Ts+1}$$

Hence, the unit step response $y(t) = \left(K - KTe^{-\frac{1}{T}t}\right)u(t)$. For G1(s), $y_1(t) = (1 - e^{-t})u(t)$. For G2(s), $y_2(t) = (1 - 0.5e^{-2t})u(t)$. " e^{-x} " is an attenuation function, so if $t \to \infty$, $y_1(t) = y_2(t) = 1$.

The meaning of time constant is the time required for a current turned into a circuit under a steady electromotive force to reach to (e-1)/e or 0.632 of its final strength (where e is the base of natural logarithms) [1]. Therefore, for G1(s) let $y_1(t) = 0.632 \Rightarrow t = 1s \Rightarrow$ time constant $\tau = 1s$. For G2(s), let $y_2(t) = 0.632 \Rightarrow t = 0.5s \Rightarrow$ time constant $\tau = 0.5s$. It is generally believed that after 5-time constants, a stable state is reached. Therefore, for G1(s), $T = 5\tau = 5s$. For G2(s), $T = 5\tau = 2.5s$.

Overall, the theoretical results are very close to the actual measurement results.

(5) Proportional integral segment

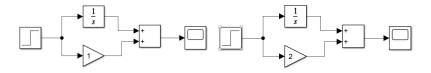


Fig. 4.5.1 Simulation G1(s)=1+1/s (left), G2(s)=2(1+1/2s)

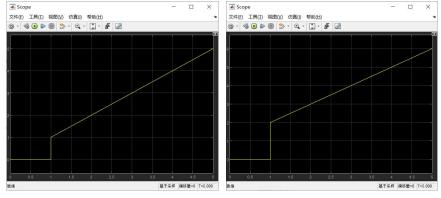


Fig. 4.5.2 Simulation results G1(s)=1+1/s (left), G2(s)=2(1+1/2s)

The slope of both G1(s) and G2(s) is the same, with a value of K equal to 1 for G1(s) and 2 for G2(s). The simulation results are consistent with the actual measured results.

Theoretical analysis

The segment is called proportional integral segment, the transfer function is $G(s) = K\left(1 + \frac{1}{T_2 s}\right)$.

$$Y(s) = G(s) \cdot E(s) = \frac{K}{s} + \frac{K}{T_2 s^2}$$

Hence, the unit step response is:

$$y(t) = Ku(t) + \frac{K}{T_2}tu(t)$$

For G1(s), $y_1(t) = u(t) + tu(t)$. For G2(s), $y_2(t) = 2u(t) + tu(t)$. From $y_1(t)$ and $y_2(t)$, it can explain why the jump values of the response are different, but the slope is the same. In addition, the DC power supply is +15V, thus, when $t \to \infty$, the maximum voltage output is nearly +15V.

Overall, theoretical analysis is consistent with actual results. For general forms, let e(t) as input and u(t) as response, $u(t) = K_p e(t) + K_1 \int_0^T e(t) dt$. That's why it's called the proportional integral segment.

(6) Oscillation segment

Simulation

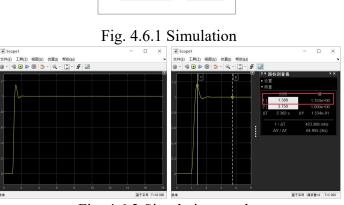


Fig. 4.6.2 Simulation results

The waveforms shown by the simulation results are very similar to the actual measured results. In simulation result, $t_p = 1.388s$, $PO = \frac{\Delta Y}{1V} \times 100\% = 15.3\%$, very close to the actual measured value. Therefore, the simulation results are consistent with the actual measured results.

Theoretical analysis

In general, a simple 2nd order TF can be written as:

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

In this situation,

$$H(s) = G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{100}{s^2 + 10s + 10}$$

Therefore, $\omega_n = 10$, $\zeta = 0.5$. The frequency of oscillations is: $\omega_d = \omega_n \sqrt{1 - \zeta^2} = 5\sqrt{3}$.

The time to the first peak $t_p = \frac{\pi}{\omega_n \times \sqrt{1-\zeta^2}} = \frac{\pi}{10 \times \frac{\sqrt{3}}{2}} = 0.363s$, and the percentage overshoot $PO = \frac{\pi}{10 \times \frac{\sqrt{3}}{2}} = \frac{\pi}{10 \times \frac{\sqrt{3}}{2}} = 0.363s$

 $exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right) \times 100\% = 16.3\%$. The results for these two values are very close to those measured. Therefore, it can be proved that the experimental results are accurate.

(7) Summary

i. When the typical segments are simulated with op-amp, under what assumptions are the transfer functions approximately derived?

When simulating the typical segments with an op-amp, the transfer functions are approximately derived under the following assumptions: (1) Ideal op-amp: The op-amp is assumed to be an ideal device with infinite open-loop gain, infinite input impedance, zero output impedance, and zero offset voltage. (2) Ideal components: Other components in the circuit, such as resistors and capacitors, are assumed to be ideal without any non-ideal effects.

- ii. What is the main difference between the integral segment and the inertial segment? Under what conditions can the inertial segment be approximately treated as an integral segment? Under what conditions can it be treated as a proportional segment?
 - (1) In the integral element, the output increases linearly with time as it represents the integration of the input over time. When the input is a unit step signal, the output exhibits a straight-line growth over time without any transient response, continuously accumulating the input signal until it reaches the maximum value. On the other hand, in the inertial element, the output exhibits a delayed and exponential transient response to changes in the input signal. When the input undergoes a step change, the output does not immediately reach the steady-state value but instead gradually approaches it following an exponential pattern. This is because the inertial element has a certain inertia or lag in responding to the input signal's variations.
 - (2) In inertial segment, $G(s) = \frac{K}{Ts+1}$. When T is large enough, $G(s) \approx \frac{1}{Ts}$, it is integral segment. When $T \to 0$, $G(s) \approx K$, it is proportional segment. Therefore, when the time constant τ is very large, the inertial segment be approximately treated as an integral segment, when τ is very small, it can be treated as a proportional segment.
- iii. How to determine the time constant of the integration and inertia segments based on the waveform of the step response?
 - (1) Inertial Segment: The time constant τ corresponds to the point at which the waveform reaches 0.632 times the maximum value.
 - (2) Integral Segment: The numerical value of the integration time constant τ is equal to the time it takes for the output signal to change by an amount equal to the step change in the input signal.

5. Conclusion

This lab experiment successfully simulated and analyzed six different control segments using an op-amp and various components. The transfer functions were approximately derived under the assumptions of an ideal op-amp and ideal components. The experimental results were accurate and consistent with theoretical analysis. The conversion of transfer functions from the s domain to the time domain was necessary for analyzing the experimental results. The integral and inertial segments were compared, and the conditions under which the inertial segment can be treated as an integral or proportional segment were discussed. The simulation results were also consistent with the actual measured results.

Overall, this lab experiment provided a comprehensive understanding of the behavior and characteristics of different control segments, which is essential for designing and analyzing control systems.

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