EXAM QUESTION PAPER

College/ Institute	Engineering, Design and Physical Sciences		
Department	Electronic and Electrical Engineering		
Exam Author(s)	Dr Mingliang Deng		
Module Code	EE2624		
Module Title	Digital Signal Processing		
Month	January/~~April~~/~~May~~	**Year**	2022/23
Exam Type	Full/~~Resit~~	**Format**	
Duration	Three Hours		
Number of questions	Seven		
Question Instructions	Answer all SEVEN questions		
Are calculators permitted	Yes		
Make/Model number of permitted calculators.	None		
Can students include drawings/ diagrams?	No		
Any permitted reference materials	None		
Required Stationery / Equipment			

By continuing beyond this point, you confirm that you have read the information and instructions above, and understand the conditions of this examination.

1. Consider a causal system for which the input x[n] and the output y[n] are related by the linear constant coefficient difference equation below

[100%]

$$y[n] = x[n] + x[n-4]$$

a) Determine the unit sample response of the system.

[20%]

b) Calculate the frequency response of the system and plot the magnitude response.

[40%]

c) Determine the system response to the input of

$$x[n] = \cos(\frac{\pi}{2}n) + \cos(\frac{\pi}{4}n) \qquad -\infty < n < +\infty$$

using discrete-time Fourier transform.

[20%]

d) explain the results obtained in c) by considering the frequency response of the system in b).

[20%]

2. Assume that there are two four-point sequences x[n] and h[n] as follows

[100%]

$$x[n] = 2\delta[n-1] + \delta[n-2] + 4\delta[n-3], \quad n = 0,1,2,3$$

$$h[n] = \cos(\frac{n\pi}{4}), \quad n = 0,1,2,3$$

a) Calculate the four-point DFT X[k] for x[n].

[20%]

b) Calculate the four-point DFT H[k] for h[n].

[20%]

c) Calculate y[n] by multiplying the DFTs of x[n] and h[n] and performing an inverse DFT.

[60%]

3. Consider a discrete-time linear causal system defined by the difference equation

$$y[n] = \frac{3}{4}y[n-1] - \frac{1}{8}y[n-2] + x[n] + \frac{1}{3}x[n-1]$$

Draw the signal flow graphs to implement this system in the following forms and give the necessary explains.

[100%]

a) Canonic form.

[20%]

b) All the possible cascade forms.

[40%]

c) Parallel form.

[40%]

For the cascade and parallel forms, use only the first-order sections.

4. For each of the following systems, determine whether the system is a FIR filter with a linear phase. If it is, state its type, explain why and determine the phase response and group delay.

[100%]

a) $h[n] = \delta[n] - \delta[n-3]$

[40%]

b) $h[n] = \delta[n] - \delta[n-4]$

[20%]

c) $h[n] = \begin{cases} 1 & 0 \le n \le 6 \\ 0 & \text{others} \end{cases}$

[40%]

5. The system function of an analogue filter is

$$H_a(s) = \frac{3}{(s+1)(s+3)}$$

Determine the system function H(z) of a digital filter converted from this analogue filter by impulse invariance. Here the sampling period is 0.5s.

[100%]

6. Using the rectangular window, design a linear phase low-pass FIR filter whose transition bandwidth is not larger than π/8. It is required that the designed FIR filter approximates the frequency response of an ideal low-pass filter given by

[100%]

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha} & 0 \le |\omega| \le \omega_c \\ 0 & \omega_c < |\omega| \le \pi \end{cases}$$

a) Calculate the impulse response of the ideal low-pass filter $h_d(n)$.

[40%]

b) Calculate the impulse response of the designed FIR filter, determine the required filter length (N) and the relationship between α and N.

[60%]

7. Design a causal linear phase FIR system to approximate an ideal low-pass digital filter with a cut-off frequency of 0.3π and satisfying

[100%]

$$\begin{aligned} &0.95 < H(e^{j\omega}) < 1.05, && 0 \le |\omega| \le 0.25\pi \\ &-0.1 < H(e^{j\omega}) < 0.1, && 0.35\pi \le |\omega| \le \pi \end{aligned}$$

a) What kinds of windows can be used to meet this specification and why? Here, Kaiser window is NOT considered.

[20%]

b) For each window obtained in 2), give the minimum filter length required.

[25%]

c) Determine the impulse response of the low-pass FIR filter with the minimum design complexity and passband/stopband ripples.

[30%]

d) Calculate the impulse response of a linear phase high-pass FIR filter with a cut-off frequency and a filter length of the low-pass filter obtained in (c). Here, rectangular window is only considered.

[25%]

Appendix

Table 1 Characteristics of Various Window Functions (The filter length is M+1.)

Type of Window	**Peak Side-Lobe Amplitude (Relative)** (r_p)	**Approximate Width of Main Lobe** ($\Delta\omega_m$)	**Peak Approximation Error** $e_p = 20\log_{10}(\delta)$	**Transition Widths** ($\Delta\omega$)	**Equivalent Kaiser Window** (β)
Rectangular	-13	$4\pi/(M+1)$	-21	$1.81\pi/M$	0
Bartlett	-25	$8\pi/M$	-25	$2.37\pi/M$	1.33
Hanning	-31	$8\pi/M$	-44	$5.01\pi/M$	3.86
Hamming	-41	$8\pi/M$	-53	$6.27\pi/M$	4.86
Blackman	-57	$12\pi/M$	-74	$9.19\pi/M$	7.04

<end of question paper>