

Section A

You must complete **ALL** questions in this section.

1. Find the tangent plane and normal line of the surface

$$z = x^2 + y^2 - 1$$

at the point (2,1,4)

[5 marks]

2. Find the area of the triangle with vertices $P(1,4,3), Q(-2,5,1), R(1,2,1)$.

[5 marks]

3. Find $\frac{dy}{dx}$ given

$$y^2 - x^2 - \sin(xy) = 0$$

[5 marks]

4. Find the general solution to the differential equation

$$y'' - 7y' + 10y = 0$$

[5 marks]

5. Find the general solution to the differential equation

$$y'' = 2e^{2x} + \sin x$$

[5 marks]

6. Given that the function $z = z(x, y)$ is determined by the equation

$$xyz + x^2 + y^2 + z^2 = 6$$

Find

$$\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$$

[5 marks]

7. Evaluate the line integral

$$\int_C 8x^2yzdx + 5zdy - 4xydz$$

along the curve C, which is defined by $x = t, y = t^2, z = t^3, 0 \leq t \leq 1$

[6 marks]

8. Use Green's theorem to evaluate the line integral

$$\oint_C (3y - e^{\cos x})dx + (6x + \sqrt{y^2 + 1})dy$$

along the curve C, where C is the circle $x^2 + y^2 = 4$, traversed in the counterclockwise direction.

[6 marks]

9. Find the extreme values of the following function

$$f(x, y) = xy - x^2 - y^2 - 2x - 2y + 5.$$

[6 marks]

10. Find the Fourier series for the square 2π -periodic wave defined on the interval $[-\pi, \pi]$;

$$f(x) = \begin{cases} -1, & -\pi \leq x < 0 \\ 1, & 0 \leq x < \pi \end{cases}$$

[8 marks]

11. Find $\frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}$ when

$$\begin{cases} x = e^u + u^2 \sin v \\ y = e^u - u^2 \cos v \end{cases}$$

[8 marks]

12. Evaluate

$$\iint_S (z + 3y - x^2) dS$$

where S is the portion of the plane $z = 2 - 3y + x^2$ that lies over the triangle in the xy -plane with vertices $(0,0), (2,0), (2,-4)$.

[10 marks]

13. Evaluate

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$

Where $\mathbf{F} = 4x\mathbf{i} + 4y\mathbf{j} + 2\mathbf{k}$, and the surface S is the end of the paraboloid $z = x^2 + y^2$ below the plane $z = 1$ with orientation outward.

[10 marks]

Section B

You must complete **EITHER** part(a) **OR** part(b) of each question in this section.

14. Complete **EITHER** part(a) **OR** part(b) of this question:

(a) Determine whether the following limit exists. If it does, give the value of the limit:

$$\lim_{(x,y) \rightarrow (0,0)} f(x,y) = \frac{2xy}{x^2 + y^2}.$$

[5 marks]

(b) Determine whether the following limit exist. If it does, give the value of the limit:

$$\lim_{(x,y) \rightarrow (0,0)} f(x,y) = \frac{4xy^2}{x^2 + y^4}.$$

[5 marks]

15. Complete **EITHER** part(a) **OR** part(b) of this question:

(a) Test the series

$$\sum_{n=0}^{\infty} \frac{n^4}{5^n}$$

for convergence or divergence. Explain the reason for your answer.

[5 marks]

(b) Test the series

$$\sum_{n=1}^{\infty} \left(\frac{n+3}{4n+2} \right)^n$$

for convergence or divergence. Explain the reason for your answer.

[5 marks]