EXAM QUESTION PAPER

College/ Institute	Engineering, Design and Physical Sciences		
Department	Electronic and Electrical Engineering		
Exam Author(s)	Dr Mingliang Deng		
Module Code	EE2624		
Module Title	Digital Signal Processing		
Month	January/~~April~~/~~May~~	**Year**	2021/22
Exam Type	Full/~~Resit~~	**Format**	
Duration	Three Hours		
Number of questions	Seven		
Question Instructions	Answer all SEVEN questions		
Are calculators permitted	Yes		
Make/Model number of permitted calculators.	None		
Can students include drawings/ diagrams?	No		
Any permitted reference materials	None		
Required Stationery / Equipment			

By continuing beyond this point, you confirm that you have read the information and instructions above, and understand the conditions of this examination.

1. Consider a causal system for which the input x[n] and the output y[n] are related by the linear constant coefficient difference equation below [15 marks]

$$y[n]-\frac{1}{2}y[n-1]=x[n]+\frac{1}{2}x[n-1]$$

   a) Determine the unit sample response of the system. [3 marks]
   b) Using the result obtained in a) and the convolution sum to determine the response to the input $x(n)=e^{jwn}$. [3 marks]
   c) Determine the frequency response of the system. [3 marks]
   d) Determine the response of the system to the input $x(n)=\cos\left(\frac{\pi}{2}n+\pi/8\right)$. [6 marks]

2. Assume that there are two four-point sequences x[n] and h[n] as follows [15 marks]

$$x[n]=\cos\left(\frac{\pi n}{2}\right),\quad n=0,1,2,3$$

$$h[n]=2^{n},\quad n=0,1,2,3$$

   a) Calculate the four-point DFT X[k] for x[n]. [5 marks]
   b) Calculate the four-point DFT H[k] for h[n]. [5 marks]
   c) Calculate y[n] by multiplying the DFTs of x[n] and h[n] and performing an inverse DFT. [5 marks]

3. Consider a discrete-time linear causal system defined by the difference equation [20 marks]

$$y[n]-\frac{3}{4}y[n-1]+\frac{1}{8}y[n-2]=x[n]+\frac{1}{3}x[n-1]$$

   Draw a signal flow graph to implement this system in each of the following forms
   a) Direct form I. [5 marks]
   b) Direct form II. [5 marks]
   c) Cascade. [5 marks]
   d) Parallel. [5 marks]

   For the cascade and parallel forms use only first-order sections.

4. Write down the features of four types of linear phase FIR filters. [5 marks]

5. The system function of an analog filter is

$$H_a(s) = \frac{s}{(s+2)(s+1)}$$

Determine the system function H(z) of the digital filter obtained from this analog filter by impulse invariance. [10 marks]

6. Design causal FIR systems to approximate ideal digital filters with corresponding specifications. [20 marks]

   a) Use the Bartlett window to design a 4th order FIR filter for approximating [5 marks]

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| < \pi / 4 \\ 0, & \text{otherwise} \end{cases}$$

   b) Use the rectangular window and a filter length of 21 to design an ideal highpass filter whose cutoff frequency is 0.5π. [5 marks]

   c) Design a lowpass filter whose cutoff frequency is 0.5π. The maximum allowable transition width $\Delta\omega$ is 0.05π and the maximum allowable tolerance δ is 0.005. What are the windows possible to be used? Explain the reasons. Use the possible window with the minimum filter length to design such a filter. [10 marks]

7. The impulse response of an analog filter is given by

$$h_a(t) = \begin{cases} e^{-0.9t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Let h[n] denote the unit sample response and H(z) denote the system function for the digital filter designed from this analog filter via impulse invariance, i.e. with

$$h[n] = h_a(nT)$$

Determine H(z), including T as a parameter, and show that for any positive values of T, the digital filter is stable. Indicate also whether the digital filter approximates a low pass filter or a high pass filter. [15 marks]

<end of question paper>