2022.1.23 考试题

1、a) $h[n] = \delta[n] + \delta[n-4]$

$h[0]=1 \quad h[4]=1$

$h[n] = \{\underset{\substack{\uparrow \\ n=0}}{1}, 0, 0, 0, 1\}$

b) $\because H(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} h[n] e^{-j\omega n}$

$\cancel{= 1 + e^{-j3n}}$

$= 1 + e^{-j4\omega}$

$|H(e^{j\omega})| = \sqrt{2 + 2\cos(4\omega)}$

c) $\cancel{X(e^{j\omega}) = \sum_{k=-\infty}^{+\infty} x[n] e^{-j\omega n}}$

$y[n] = x[n] * h[n]$

$= x[n] * [\delta[n] + \delta[n-4]]$

$= x[n] + x[n-4]$

$Y(e^{j\omega}) = \pi \sum_{k=-\infty}^{+\infty} [\delta(\omega - \frac{\pi}{2} - 2\pi k) + \delta(\omega + \frac{\pi}{2} - 2\pi k) + \delta(\omega - \frac{\pi}{4} - 2\pi k) + \delta(\omega + \frac{\pi}{4} - 2\pi k)] + \pi e^{-j2\pi} \sum_{k=-\infty}^{+\infty} [\delta(\omega - \frac{\pi}{2} - 2\pi k) + \delta(\omega + \frac{\pi}{2} - 2\pi k)]$

$\cancel{\delta(\omega - \frac{\pi}{4}} + \pi e^{-j\pi} \sum_{k=-\infty}^{+\infty} [\delta(\omega - \frac{\pi}{4} - 2\pi k) + \delta(\omega + \frac{\pi}{4} - 2\pi k)]$

$= 2\pi \sum_{k=-\infty}^{+\infty} [\delta(\omega - \frac{\pi}{2} - 2\pi k) + \delta(\omega + \frac{\pi}{2} - 2\pi k)]$

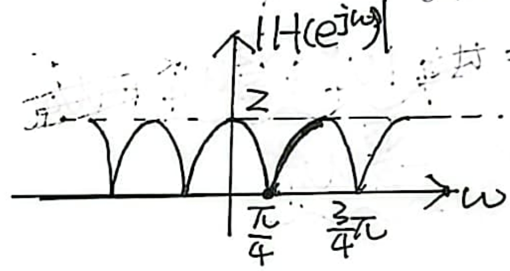
↓IDFT

$y[n] = 2\cos(\frac{\pi}{2} n)$

d) $\cancel{x[n] = \cos(\frac{\pi}{2} n) + \cos(\frac{\pi}{4} n)}$

$\cancel{= \frac{1}{2} e^{j(\frac{\pi}{2} n)} + \frac{1}{2} e^{-j(\frac{\pi}{2} n)} + \frac{1}{2} e^{j(\frac{\pi}{4} n)} + \frac{1}{2} e^{-j(\frac{\pi}{4} n)}}$

$\cancel{X(e^{j\omega}) =}$



c) 法二：

$x[n] = \cos(\frac{\pi}{2} n) + \cos(\frac{\pi}{4} n)$

$= \frac{1}{2} e^{j\frac{\pi}{2} n} + \frac{1}{2} e^{-j\frac{\pi}{2} n} + \frac{1}{2} e^{j\frac{\pi}{4} n} + \frac{1}{2} e^{-j\frac{\pi}{4} n}$

视作特征函数输入到系统中

$y[n] = |H(e^{j\omega_1})| \cos[\frac{\pi}{2} n + \angle H(e^{j\omega_1})] + |H(e^{j\omega_2})| \cos[\frac{\pi}{4} n + \angle H(e^{j\omega_2})]$

$\omega_1 = \frac{\pi}{2}, \quad \omega_2 = \frac{\pi}{4}$

$|H(e^{j\omega_2})| = 0 \quad |H(e^{j\omega_1})| = 2 \quad \angle H(e^{j\omega_1}) = 0$

故 $y[n] = 2\cos(\frac{\pi}{2} n)$

2. a) $N=4$

$$X[k]=\sum_{n=0}^{N-1} x[n]\,W_N^{kn}$$

$$=\sum_{n=0}^{3} x[n]\,W_4^{kn}$$

$$=2W_4^{k}+W_4^{2k}+4W_4^{3k}$$

b) $N=4$ $h[n]=\{\underset{\uparrow\, n=0}{1},\frac{\sqrt{2}}{2},0,-\frac{\sqrt{2}}{2}\}$

$$H[k]=\sum_{n=0}^{3} h[n]\,W_4^{kn}$$

$$=\cancel{W_4}\,W_4^{0}+\frac{\sqrt{2}}{2}W_4^{k}\cancel{+W_4}-\frac{\sqrt{2}}{2}W_4^{3k}$$

c) $\cancel{X[k]}$

$$Y[k]=X[k]\cdot H[k]$$

$$\cancel{=2W_4^{k}+(\tfrac{\sqrt{2}}{2}+1)W_4^{2k}}$$

$$\cancel{+(8-\tfrac{3}{2}\sqrt{2})W_4^{3k}+2\sqrt{2}}$$

$$\cancel{y[n]=2\sqrt{2}\delta[n]+2\delta[n-1]+}$$

$$\cancel{(\tfrac{\sqrt{2}}{2}+1)\delta[n-2]+(8-\tfrac{3}{2}\sqrt{2})\delta[n-3]}$$

$$Y[k]=\sqrt{2}+(2-\frac{\sqrt{2}}{2})W_4^{k}+(1-\sqrt{2})W_4^{2k}+(4+\frac{\sqrt{2}}{2})W_4^{3k}$$

$$y[n]=\sqrt{2}\delta[n]+(2-\frac{\sqrt{2}}{2})\delta[n-1]+(1-\sqrt{2})\delta[n-2]+(4+\frac{\sqrt{2}}{2})\delta[n-3]$$

3.

$$\cancel{H(z)=\frac{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}{1+\frac{1}{3}z^{-1}}}$$

$$H(z)=\frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

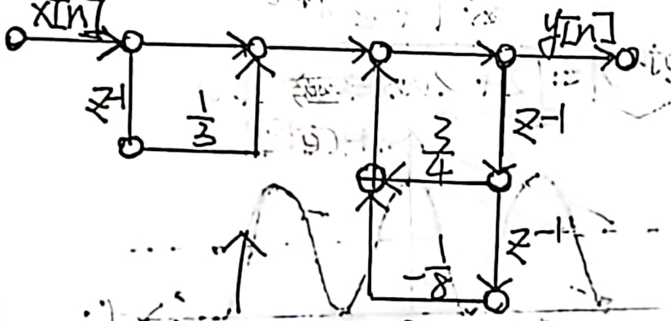
a)

$$V(z)=(1+\tfrac{1}{3}z^{-1})X(z)$$

$$Y(z)=\left(\frac{1}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}\right)V(z)$$

$$v[n]=x[n]+\tfrac{1}{3}x[n-1]$$

$$y[n]=\tfrac{3}{4}y[n-1]-\tfrac{1}{8}y[n-2]+v[n]$$



Direct I form

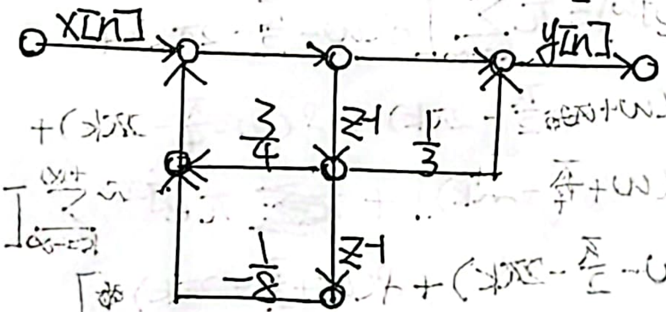
Direct II form

$$W(z)=\left(\frac{1}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}\right)X(z)$$

$$Y(z)=(1+\tfrac{1}{3}z^{-1})W(z)$$

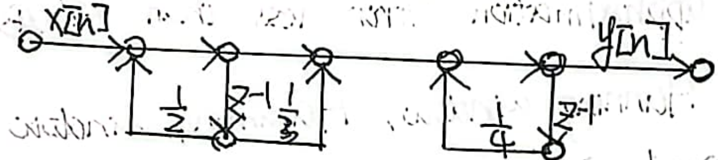
$$w[n]=\tfrac{3}{4}w[n-1]-\tfrac{1}{8}w[n-2]+x[n]$$

$$y[n]=\cancel{w}\,w[n]+\tfrac{1}{3}w[n-1]$$

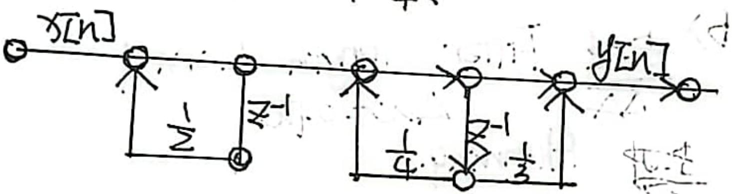


b) $H(z)=\dfrac{1+\frac{1}{3}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$

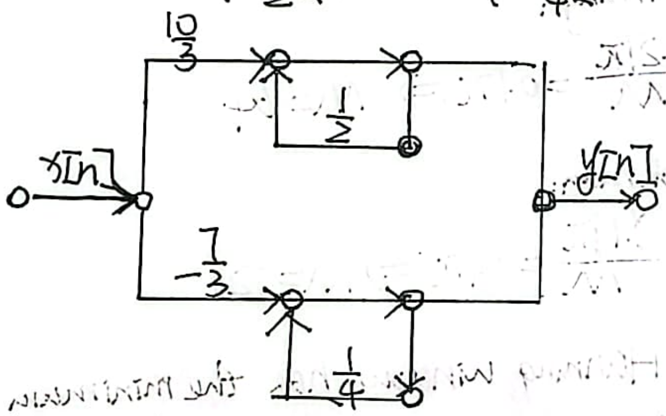
① $H(z)=\dfrac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{2}z^{-1}}\cdot\dfrac{1}{1-\frac{1}{4}z^{-1}}$



② $H(z)=\dfrac{1}{1-\frac{1}{2}z^{-1}}\cdot\dfrac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{4}z^{-1}}$



c) $H(z)=\dfrac{\frac{10}{3}}{1-\frac{1}{2}z^{-1}}-\dfrac{\frac{7}{3}}{1-\frac{1}{4}z^{-1}}$



4.

a) $h[n]=\{\underset{\substack{\uparrow\\ n=0}}{1},0,0,\cancel{\ },-1\}$

h[n] is anti-symmetric, so it is linear phase. M=4 is even, it is type-4 linear phase FIR. ~~$\alpha=\frac{M-1}{2}$~~

~~$\angle H(e^{jw})=$~~

Group delay $\alpha=\frac{M-1}{2}=\frac{3}{2}$

$\angle H(e^{jw})=\frac{\pi}{2}-\frac{3}{2}w$

b) $h[n]=\{\underset{\substack{\uparrow\\ n=0}}{1},0,0,0,-1\}$

h[n] is anti-symmetric, M=5 is odd, so it is linear phase FIR. It is type-3 FIR.

$\alpha=2$ ~~$\beta=0$~~ $\beta=\frac{\pi}{2}$

$\angle H(e^{jw})=$ ~~$2w$~~ $\frac{\pi}{2}-2w$

c) $h[n]=\{\underset{\substack{\uparrow\\ n=0}}{1},1,1,1,1,1,1\}$

h[n] is symmetric, ~~M=6 is even~~ M=7 is odd. so it is ~~linear phase~~ Type-1 linear phase FIR.

$\alpha=3$ $\beta=0$

$\angle H(e^{jw})=-3w$

5.

$$H_a(s) = \frac{\frac{3}{2}}{s+1} - \frac{\frac{3}{2}}{s+3}$$

$$h_a(t) = \frac{3}{2}e^{-t} - \frac{3}{2}e^{-3t}$$

$$h[n] = T_s \cdot h_a(nT_s) = \frac{3}{4}e^{-0.5n} - \frac{3}{4}e^{-1.5n}$$

$$H(z) = \frac{\frac{3}{4}}{1-e^{-0.5}z^{-1}} - \frac{\frac{3}{4}}{1-e^{-1.5}z^{-1}}$$

6.

a) $\cancel{h_d[n] = \int_{-w_c}^{w_c} e^{-jw\alpha}}$

$$h_d[n] = \frac{1}{2\pi}\int_{-w_c}^{w_c} e^{-jw\alpha} e^{jwn}\, dw$$

$$= \frac{-j}{2\pi(n-\alpha)}\int_{-w_c}^{w_c} e^{jw(n-\alpha)}\, d[w(n-\alpha)]$$

$$= \frac{\sin[w_c(n-\alpha)]}{\pi(n-\alpha)}$$

b) $$h[n] = h_d[n]\cdot w[n] = \begin{cases} \frac{\sin[w_c(n-\alpha)]}{\pi(n-\alpha)}, & 0 \le n \le N-1 \\ 0, & \text{otherwise.} \end{cases}$$

The filter order is M. $\Delta w = \frac{\pi}{8}$.

$\frac{1.81\pi}{M} = \frac{\pi}{8}$, ~~so $M=16 \Rightarrow$ Filter length $N=16$ $N=17$~~

So $M=16 \Rightarrow$ Filter length $N=17$.

$$\alpha = \frac{N-1}{2} = 8$$

1. a) $\delta_p = 0.05$ $\delta_s = 0.1$

$e_p = 20\lg(0.05) = -26\text{dB}$

$e_s = 20\lg(0.1) = -20\text{dB}$

This requires a window with a peak approximation error less than -26dB

Hanning window, Hamming window and Blackman window can be considered

b) Filter length $L = M+1$

$\Delta w = 0.1\pi$

Hanning:

$$\frac{5.01\pi}{M} = 0.1\pi \Rightarrow M = 52$$

Hamming:

$$\frac{6.27\pi}{M} = 0.1\pi \Rightarrow M = 64$$

Blackman:

$$\frac{8.19\pi}{M} = 0.1\pi \Rightarrow M = 82$$

Hanning window has the minimum filter length. $L=53$

c) Blackman window has the minimum passband / stopband ripples

$$w[n] = \begin{cases} 0.42 - 0.5\cos\left(\frac{2\pi n}{M}\right) + \cancel{0.08} \\ 0.08\cos\left(\frac{4\pi n}{M}\right) & , 0 \le n \le M \\ 0 & , \text{otherwise.} \end{cases}$$

$w_c = 0.3\pi \quad \alpha = \frac{N-1}{2} = 46$

~~$h[n] = h_d[n]$~~

$$h_d[n] = \frac{\sin[0.3\pi(n-46)]}{\pi(n-46)}$$

$$h[n] = \begin{cases} \frac{\sin[0.3\pi(n-46)]}{\pi(n-46)} \cdot [0.42 - \cdots] & 0 \le n \le 92 \\ 0 & , \text{otherwise.} \end{cases}$$

d) $w_c = 0.3\pi \quad N = 93 \quad \alpha = 46.$

$$h_d[n] = \delta[n-46] - \frac{\sin[0.3\pi(n-46)]}{\pi(n-46)}$$

$$h[n] = h_d[n] \cdot w[n]$$

$$= \begin{cases} \delta[n-46] - \frac{\sin[0.3\pi(n-46)]}{\pi(n-46)} \cdot \wedge & 0 \le n \le 92 \\ 0 , & \text{otherwise.} \end{cases}$$