CHONGQING UNIVERSITY OF POSTS AND TELECOMMUNICATIONS

Lab1 Report

TERM: **2022-2023**

Module: **EE2622 Fundamentals of Signals and Systems**

CLASS: 34092102

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TUTOR:

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Part1: Time-domain Analysis of Discrete-time Signals

1. Introduction

1.1. Aims

- To become familiar with and comprehend the time domain representation and analysis of discrete signals.
- To acquire proficiency in using MATLAB for coding and simulating various discrete signals.
- To gain an understanding of the representation and properties of different special discrete signals.
- To comprehend the representation and properties of various special discrete signals.

1.2. Objectives

(1) Review and understand the representation of some special discrete signals.
(2) Use MATLAB to write code to simulate some discrete signals.
(3) The simulation results are analyzed, and the theoretical results are compared with the simulation results to verify whether the simulation results are correct.

2. Experimental Equipment and Components

Computer, software: MATLAB R2022b

3. Experimental Steps and Results

(1) Task 1: Unit impulse sequence

```
%question 1
function plot_unit_sequence(k1,k2,k0)
k=k1:k2;
n=length(k);
f=zeros(1,n);
f(1,k0-k1+1)=1;
stem(k,f,'filled')
axis([k1,k2,0,1.5])
title('unit sequence δ(k)')
end
```

Fig. 3.1.1 Revised code

I corrected some syntax error in the code. Secondly, I find that when I change the value of k_0, the corresponding t_0 will change in the opposite direction. Therefore, I removed the negative sign before 'k0' in 'f (1, - k0-k1+1)=1' and changed it to 'f (1, k0-k1+1)=1'.

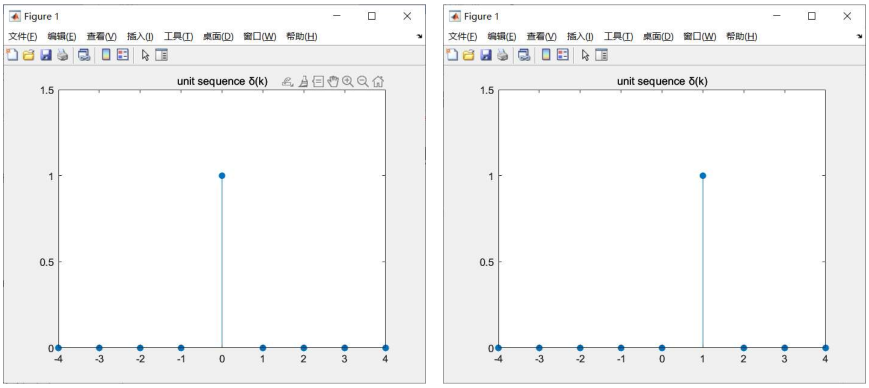


Fig. 3.1.1 Simulation results $t_0 = 0$ (left), $t_0 = 1$ (right)

The function name is changed to “function plot_unit_sequence(k1,k2,k0)”. I firstly enter the instruction “function plot_unit_sequence(-4,4,0)”, the result is shown in figure 3.1.1 (left). The range of simulation results is $-4 < t < 4$. The figure shows the unit impulse sequence, when $t = t_0$, $\delta(k) = 1$, otherwise, $\delta(k) = 0$. If I change the value of k_0 in MATLAB, the corresponding t_0 will also be changed. If I set $k_0 = 0, t_0 = 0$(left), if I set $k_0 = 1, t_0 = 1$(right).

(2) Task 2: Unit step sequence

```
%question 2
function unit_step_sequence(k1,k2,k0)
k=k1:-k0-1;
kk=-k0:k2;
n=length(k);
nn=length(kk);
u=zeros(1,n);
uu=ones(1,nn);
stem(kk,uu,'filled')
hold on
stem(k,u,'filled')
hold off
title('Unit Step Sequence')
axis([k1 k2 0 1.5])
```

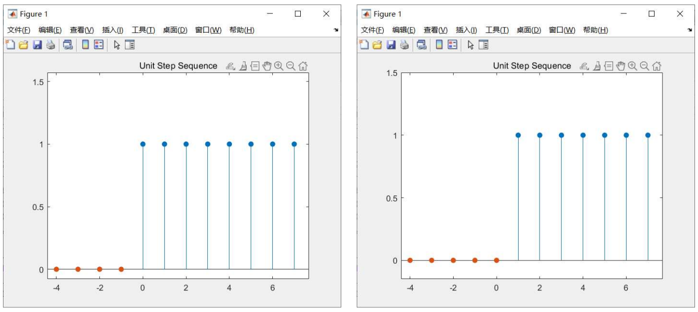


Fig. 3.2.1 Code section and simulation results k0=0 (left), k0=1 (right)

From code section, I added some missing functions. There are two 'hold on' in the source code, and I will modify the second one to 'hold off'. After trying, I found that the input result of k0 was opposite to the expected result, so I removed the negative sign before k0.

If I enter “unit_step_sequence(-4,7,0)”, the result is shown in left figure. The range of the result is from -4 to 7. When t<0, the output is ‘0’ and when t>0, the output is ‘1’. If I revise the value of t0, entering “unit_step_sequence(-4,7,1)”, the result is shown in right figure. The moment when a sequence changes from 0 to 1.

(3) Task 3: Cosine sequence

```
%question 3
k=0:40;
subplot(2,1,1)
stem(k,cos(k*pi/8),'filled')
title('cos(1*pi/8)')
subplot(2,1,2)
stem(k,cos(2*k),'filled')
title('cos(2*k)')
```

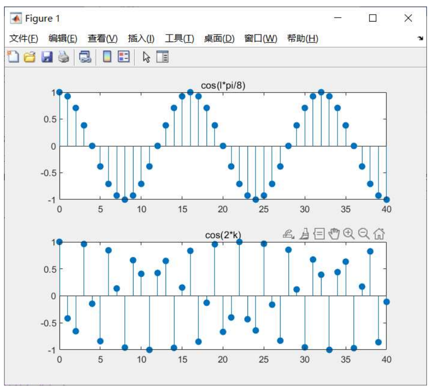


Fig. 3.3.1 Code section and simulation results

In this section, I use ‘subplot()’ to divide the 2 functions into 2 images. I can directly use “cos()” to draw the waveform. From the results, “$cos\left(\frac{k\pi}{8}\right)$” is a obvious periodic function, the function reaches its maximum every 16 points, so the period of the function is 16. But “cos(2k)” is non periodical function. The function ranges from 0 to 40.

(4) Task 4: Exponential sequence (1)

```
function expontional_sequence_1(c,a,k1,k2)
%c: The amplitude of sequence
%a: The base of sequence
%k1: The starting number for drawing the sequence
%k2: The stop number for drawing the sequence
k=k1:k2;
x=c.*(a.^k);
stem(k,x,'filled')
hold on
plot([k1,k2],[0,0])
hold off
end
```

```
%f_2(k)=(-3/4)^k
c=1;
a=(-3/4);
k1=-10;
k2=10;
expontional_sequence_1(c,a,k1,k2)

%f_1(k)=(5/4)^k*u(k)
k1=-10;k2=10;
c=[zeros(1,10), ones(1,11)];
a=5/4;
expontional_sequence_1(c,a,k1,k2)
```

Fig. 3.4.1 Code section and call function section

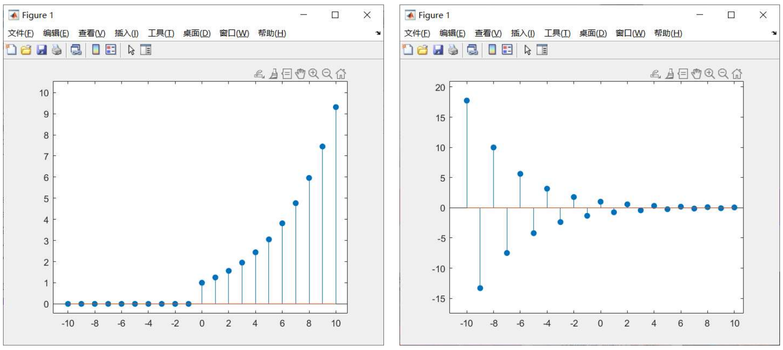


Fig. 3.4.2 Simulation results, $f_1(k)=\left(\frac{5}{4}\right)^k \cdot u(k)$ (left), $f_2(k)=\left(-\frac{3}{4}\right)^k$ (right)

This task requires drawing functions of exponential type. Code section is shown in figure 3.4.1. The general form of the exponential function is $x = c \cdot a^k$, I just need change the value of ‘c’ and ‘a’ to create different waveforms. When I call $f_1(k)$, I need to multiply by the unit step sequence, use “c=[zeros(),ones()]” can realized it.

The simulation results are shown in figure 3.4.2. $f_1(k)$ has unit step sequence, when $n < 0, f_1(k) = 0$, when $n \geq 0, f_1(k) = \left(\frac{5}{4}\right)^k$, $f_1(k)$ is a monotonically increasing sequence and $f_1(k)$ is always greater than 0. $f_2(k)$ is a convergent sequence, when $t \to \infty$, $f_2(k) = 0$. In addition, $f_2(k)$ is one that alternates between positive and negative and decreases gradually.

(5) Task 5: Exponential sequence (2)

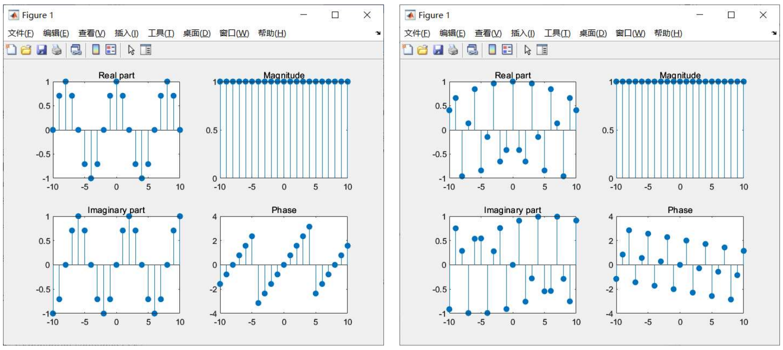


Fig. 3.5.1 Simulation results, $f_1(k) = e^{j\frac{k\pi}{4}}$ (left), $f_2(k) = e^{j2k}$ (right)

For $f_1(k) = e^{j\frac{k\pi}{4}}$, the real part is a cosine function of magnitude 1 and period 8, the imaginary part is a sine function of magnitude 1 and period 8. Magnitude is always 1. The range of phase is $[-\pi, \pi]$ and the slope is $\frac{\pi}{4}$. For $f_2(k) = e^{j2k}$, its real and imaginary parts are aperiodic functions. The magnitude is always 1. The range of phase is $[-\pi, \pi]$ and the slope is 2.

(6) Task 6: Exponential sequence (3)

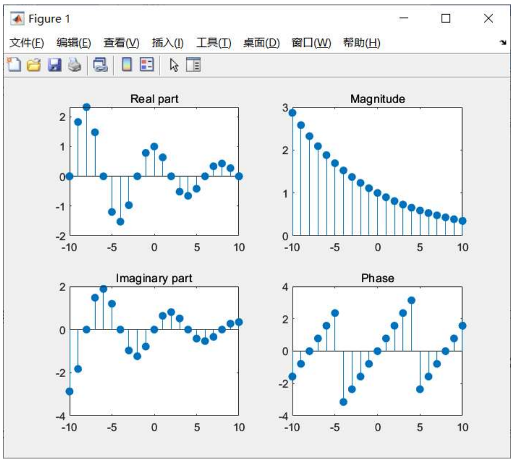
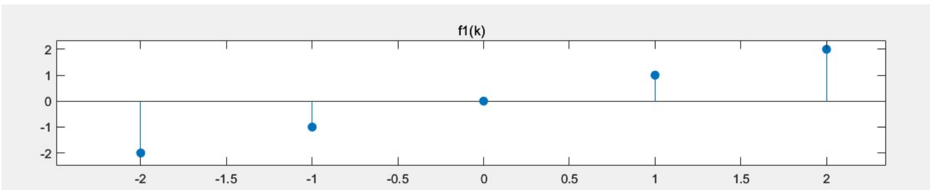
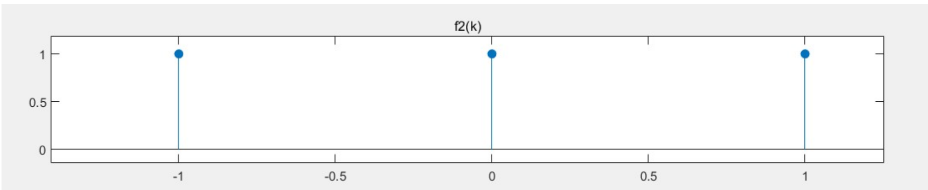


Fig. 3.6.1 Simulation result

This sequence is $f(k) = 0.9^k \cdot e^{j\frac{k\pi}{4}}$. The real part is a decreasing amplitude cosine function, and the imaginary part is a decreasing amplitude sine function. The amplitude is an exponentially decreasing function. The phase is like 'task 5', the slope is also $\frac{\pi}{4}$.

(7) Task 7: Addition of sequences





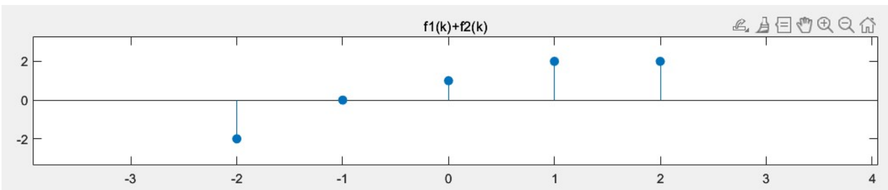


Fig. 3.7.1 Simulation result

Figure 3.7.1 shows the sequence of $f_1(k), f_1(k)$ and $f_1(k) + f_1(k)$.

(8) Task 8: Reversal of sequence

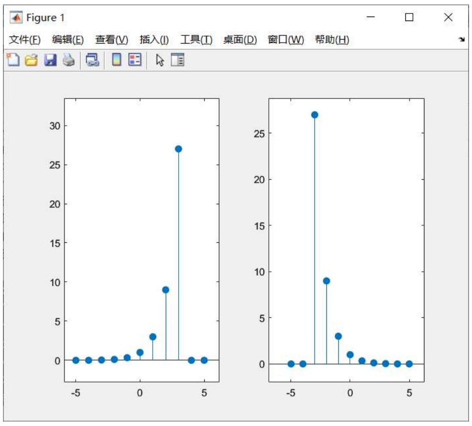


Fig. 3.8.1 Simulation result

From figure 3.8.1, the left image shows the $f(k)$, the right image shows its inverse sequence. It can find that if $k \in [-3,3]$, $f(k) > 0$, otherwise $f(k) = 0$. The two graphs are symmetric about the y axis and have the same pattern.

```
%question 8
function[f,k]=reversal_of_sequence(f1,k1,c)
f=c.*fliplr(f1);
k=-fliplr(k1);
stem(k,f,'filled')
axis([min(k)-1,max(k)+1,min(f)-0.5,max(f)+0.5])

% Function call part
% k1=-5:5;
% f1=3.^k1;
% c=[zeros(1,2), ones(1,7),zeros(1,2)];
% subplot(1,2,1)
% stem(k1,c.*f1,"filled");
% subplot(1,2,2)
% reversal_of_sequence(f1,k1,c);
```

Fig. 3.8.2 Code section

To show the piecewise function, I add a parameter to the function declaration and defined a variable 'c' in the code. If $-3 \leq k \leq 3, c = 1$, otherwise, $c = 0$. Therefore, if k is not in the range of [-3,3], c is 0, it's going to be 0 times any value. In this way, the segmentation effect can be achieved.

(9) Task 9: Time shift

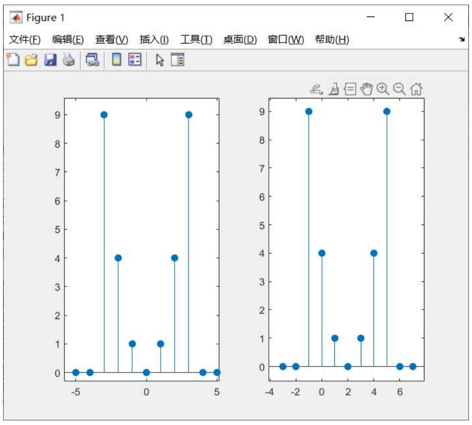


Fig.3.9.1 Simulation result

The left image shows $f(k)$ and the right image shows $f(k-2)$. It can find that if $k \in [-3,3]$, $f(k) > 0$, otherwise $f(k) = 0$. The 2 images have the same shape, but the one on the right has shifted to the right by 2 units overall.

(10) Task 10: Phase inversion

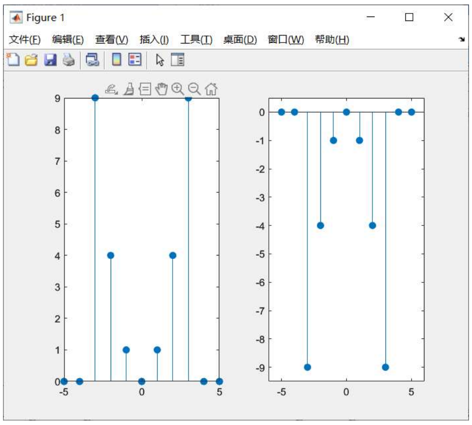


Fig. 3.10.1 Simulation result

The left image shows $f(k)$ and the right image shows $-f(k)$. The left and right graphs are symmetric about the k-axis.

4. Results Analysis

(1) Task1: Unit impulse sequence

For discrete unit impulse sequences, the function can be written as:

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}, or\ \ \delta(n-k) = \begin{cases} 1, & n = k \\ 0, & n \neq k \end{cases}$$

From the function, it can find that the result shown in figure 3.1.1 is correct, the theoretical results are consistent with the actual results.

In code section, “k=k1:k2” means this line creating a sequence of integers from ‘k1’ to ‘k2’ and stores it in the variable ‘k’. “n=length(k)” means creating the length of the sequence k, which corresponds to the number of elements in the sequence. “f=zeros(1,n)” means creating a zero-filled vector of length n to represent the function f(k). “f(1,k0-k1+1)=1” means that from the 1^{st} point to the "k0-k1+1" point, the value is 1. “stem(k,f,‘filled’)” is a very important function which can draw discrete image, it means drawing the terms in f based on the corresponding terms in k, ‘filled’ means drawing the solid circle.

(2) Task 2: Unit step sequence

For discrete unit step sequence, the function can be written as:

$$u(n) = \begin{cases} 1, & n > 0 \\ 0, & n < 0 \end{cases}$$

From the function, it can find that the result shown in figure 3.2.1 is correct, the theoretical results are consistent with the actual results.

This task need display the sequence of ‘0’ and ‘1’ in the same image. Therefore, in code section, it requires ‘hold on’ and ‘hold off’.

(3) Task 3: Cosine sequence

Periodic discrimination for discrete signals, $x[n] = cos(\omega_0 n)$. If $\frac{\omega_0}{2\pi}$ is a rational number, this signal can be considered as a periodic signal. The period is $N = \frac{2\pi n}{\omega_0}, (N \in Z)$.

In this task, for "$cos\left(\frac{k\pi}{8}\right)$", $\omega_0 = \frac{\pi}{8}, \frac{\omega_0}{2\pi} = \frac{1}{16}$ is a rational number. Therefore, $cos\left(\frac{k\pi}{8}\right)$ is a periodic function, the period is $N = 16$. For "cos(2k)", $\omega_0 = 2, \frac{\omega_0}{2\pi} = \frac{1}{\pi}$ is an irrational number. Therefore, cos(2k) is a non-periodical function.

As for the judgment of periodicity, the theoretical result is consistent with the practical result, so the judgment is correct.

(4) Task 4: Exponential sequence (1)

In exponential sequence, $x = c \cdot a^k$. When $|a| < 1$, the sequence monotonically decreases. When $|a| > 1$, the sequence monotonically increases. When $a < 0$, the sequence will alternate up and down the X-axis. The theoretical result is consistent with the practical result.

In code section, "c=[zeros(1,10),ones(1,11)]" can create unit step sequence, "zeros(1,10)" means creating 10 points with a value of 0 firstly, "ones(1,11)" means creating 11 points with a value of 1 again.

(5) Task 5: Exponential sequence (2)

In complex functions, complex numbers can be expressed in terms of exponents. Use Euler's formula: $e^{jx} = cosx + jsinx$ [1].

Therefore, $f_1(k) = e^{j\frac{k\pi}{4}} = cos\left(\frac{k\pi}{4}\right) + isin\left(\frac{k\pi}{4}\right)$. The real part is $cos\left(\frac{k\pi}{4}\right)$, imaginary part is $sin\left(\frac{k\pi}{4}\right)$. $\omega_0 = \frac{\pi}{4}$, $\frac{\omega_0}{2\pi} = \frac{1}{8}$ is a rational number, $N = \frac{2\pi n}{\omega_0} = 8$, so, the period of real part and imaginary part is 8. $|f_1(k)| = 1$, so, the magnitude of $f_1(k)$ is 1. The phase is $\varphi = arctan\left(\frac{sin\left(\frac{k\pi}{4}\right)}{cos\left(\frac{k\pi}{4}\right)}\right) = \frac{k\pi}{4}$.

$f_2(k) = e^{j2k} = cos(2k) + isin(2k)$. The real part is $cos(2k)$, imaginary part is $sin(2k)$. $\omega_0 = 2, \frac{\omega_0}{2\pi} = \frac{1}{\pi}$ is an irrational number, so $f_2(k)$ is an aperiodic function. $|f_2(k)| = 1$, so, the magnitude of $f_2(k)$ is 1. The phase is $\varphi = arctan\left(\frac{sin(2k)}{cos(2k)}\right) = 2k$. After analyzing the relationship between the periodicity and the angular frequency of the real part, the imaginary part and the phase, it can prove that the simulation results are correct.

(6) Task 6: Exponential sequence (3)

Compared with task 5, the function of this task has an additional effect factor of 0.9^k on the amplitude. $f(k) = 0.9^k \cdot e^{j\frac{k\pi}{4}} = 0.9^k\left[cos\left(\frac{k\pi}{4}\right) + isin\left(\frac{k\pi}{4}\right)\right]$. The real part is $0.9^k \cdot cos\left(\frac{k\pi}{4}\right)$, the imaginary part is $0.9^k \cdot sin\left(\frac{k\pi}{4}\right)$. The period is 8 and the phase is $\frac{k\pi}{4}$, the same as 'task 5'. $|f(k)| = 0.9^k$, 0.9<1, so, the magnitude is a monotonically decreasing function. After theoretical analysis, it can prove that the simulation results are correct.

(7) Task 7: Addition of sequences

This is a simulation of adding two discrete sequences. $f_1(k) = \{-2, -1, \underbrace{0}_{k=0}, 1, 2\}$, $f_2(k) = \{1, \underbrace{1}_{k=0}, 1\}$. Add the terms of two discrete sequences, $f_1(k) + f_2(k) = \{-2, 0, \underbrace{1}_{k=0}, 2, 2\}$. According to the theoretical results, the simulation results are correct.

(8) Task 8: Reversal of sequence

$$f(k) = \begin{cases} 3^k, & -3 \le k \le 3 \\ 0, & k = other\ values \end{cases}$$

Reverse the function, it can get:

$$f(-k) = \begin{cases} 3^{(-k)}, & -3 \le k \le 3 \\ 0, & k = other\ values \end{cases}$$

As shown in figure 4.8.1, by replacing the independent variable k in signal $f(k)$ with $-k$, its geometric meaning is to reverse the signal $f(\cdot)$ along the ordinate axis.

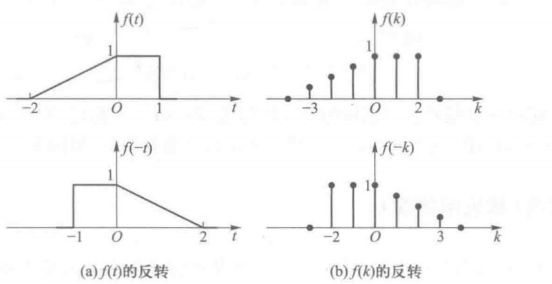


Fig. 4.8.1 Reversal of signal [2].

The calculation of this theory is the same as the actual result, so the simulation result can be considered correct.

(9) Task 9: Time shift

$$f(k) = \begin{cases} k^2, & -3 \le k \le 3 \\ 0, & k = other\ values \end{cases}$$

$$f(k-2) = \begin{cases} (k-2)^2, & -1 \le k \le 5 \\ 0, & k = other\ values \end{cases}$$

As shown in figure 4.9.1, for discrete signal $f(k)$, if there is integral constant $k_0 > 0$, the delayed signal $f(k - k_0)$ is the translation of the original sequence along the positive direction of the k axis by k_0 units. And $f(k + k_0)$ is the translation of the original sequence by k_0 units along the negative k-axis.

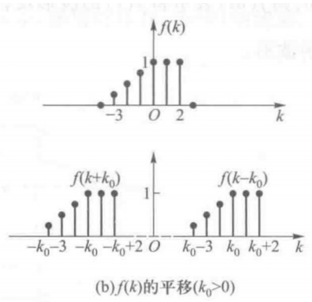


Fig. 4.9.1 Translation of signal [2].

The calculation of this theory is the same as the actual result, so the simulation result can be considered correct.

(10) Task 10: Phase inversion

$$f(k)=\begin{cases} k^2, & -3\le k\le 3 \\ 0, & k = other\ values \end{cases}$$

$$-f(k)=\begin{cases} -k^2, & -3\le k\le 3 \\ 0, & k = other\ values \end{cases}$$

The calculation of this theory is the same as the actual result, so the simulation result can be considered correct.

5. Conclusion

In this report, I have explored various tasks related to signals and systems, including exponential sequences, addition of sequences, reversal of sequences, time shift and phase inversion, and translation of signals. Theoretical analysis and practical simulations were conducted for each task, and the results were found to be consistent. The report also includes code sections for each task, providing a clear understanding of the implementation process.

Overall, this report provides a comprehensive overview of fundamental concepts in signals and systems and serves as a valuable resource for anyone interested in this field.

REFERENCE

[1] Wikipedia contributors. (2023, May 16). Euler's formula. In *Wikipedia, The Free Encyclopedia*. Retrieved 06:43, May 24, 2023, from https://en.wikipedia.org/w/index.php?title=Euler%27s_formula&oldid=1155159190

[2] W. D. Zheng, "Analysis of Signals and Linear Systems (5th Edition)," Higher Education Press, Beijing, China, 2019.

[3] R. Wu, ‘EE2622 Signals and Systems Lecture Notes’, Brunel University London, 2020.

Appendix

All code parts in Lab1 are submitted to *wiseflow* and placed in the appendix section.