




任课老师:

班级:


专业:

学号:


年级:

姓名:

线

封

密

## 重庆邮电大学 2024-2025 学年第一学期（期末）

## Advanced Mathematics I Final Examination

## INSTRUCTIONS

- (a) This examination paper consists of 15 questions, 2 pages for the total of 100 marks.  
(b) Write the DETAILED process for each question in the space provided on the answer sheet  
(c) You are required to answer each question in ENGLISH.

1. [5 marks] Let  $a > 0$  and

$$y = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\ln(x + \sqrt{x^2 + a^2})$$

Find  $dy$ .

2. [5 marks] Find the limit

$$\lim_{x \rightarrow 0} \frac{6\sin x - 6x + x^3}{x^5}$$

3. [5 marks] Find the limit

$$\lim_{x \rightarrow \infty} \left( \frac{x + 1012}{x - 1013} \right)^{x + 2025}$$

4. [5 marks] Find  $\frac{dy}{dx}$  from the equation

$$\arctan \frac{y}{x} = \ln \sqrt{x^2 + y^2}$$

5. [6 marks] Determine the values of the constants  $a, b, c$  such that the second derivative of the function  $f(x)$  at the point  $x = 0$  does exist, where

$$f(x) = \begin{cases} ax^2 + bx + c, & x < 0 \\ \ln(1 + x), & x \geq 0 \end{cases}$$

6. [6 marks] Let

$$\begin{cases} x = f'(t) \\ y = tf'(t) - f(t) \end{cases}$$

If  $f''(t) \neq 0$ , find  $\frac{d^2y}{dx^2}$ .

7. [6 marks] Find the 5<sup>th</sup> derivate of the function

$$f(x) = \sin^2 x$$

8. [6 marks] Find the local extreme values in the interval  $[0, 2\pi]$  of the function

$$f(x) = \sin x + \cos x$$

9. [6 marks] If  $\frac{\sin x}{x}$  is a primitive function of  $f(x)$ , find the integral

$$\int x f'(x) dx$$

10. [6 marks] Find the integral

$$\int \arcsin^2 x \, dx$$

11. [6 marks] Find the integral

$$\int_e^1 x^3 \ln^2 x \, dx$$

12. [6 marks] Find the integral

$$\int \frac{x - 5}{x^2 - 4x - 12} dx$$

13. [6 marks] Find the volume of the solid obtained by rotating the curve  $y = \sqrt{x}$  between  $x = 1$  and  $x = 4$  about the  $x$ -axis.






任课老师：  
班级：


专业：  
学号：


年级：  
姓名：

.....线.....封.....密.....

14. [8 marks] Find the limit

$$\lim_{x \rightarrow 0} \frac{\int_{\cos x}^1 t \ln t \, dt}{e^{x^2} - 1}$$

15. [8 marks] Let  $a, b > 0$ . If  $f(x)$  is continuous on the closed interval  $[a, b]$ , and  $f(x)$  is differentiable in the open interval  $(a, b)$ . Prove that the equation

$$2x[f(b) - f(a)] = (b^2 - a^2)f'(x)$$

has at least one zero in  $(a, b)$ .

16. [10 marks] The sequence  $[x_n]$  is defined by  $x_1 = a$  with  $a > 0$ , and

$$x_{n+1} = \frac{1}{2} \left( x_n + \frac{a}{x_n} \right), \, n = 1, 2, \dots$$

- (1) [2 marks] Prove that  $x_n \geq \sqrt{a}$  for  $n = 2, 3, \dots$
- (2) [2 marks] Prove that  $x_n \geq x_{n+1}$  for  $n = 2, 3, \dots$
- (3) [6 marks] Prove that  $\lim_{n \rightarrow \infty} x_n$  does exist and find it.

