2021/22 考试题

1. a)

$h[n]-\frac{1}{2}h[n-1]=\delta[n]+\frac{1}{2}\delta[n-1]$

~~let n=0~~

$h[0]=1$

$h[1]=\frac{1}{2}+\frac{1}{2}=1$

$h[2]=\frac{1}{2}$

$h[3]=\frac{1}{4}$ $\quad h[n]=(\frac{1}{2})^{n-1},\ n\geq 1$

$h[n]=\{\underset{\substack{\uparrow\\ n=0}}{1},1,\frac{1}{2},\frac{1}{4},\dots\}$

~~b) Since the system is a causal system, so if n<0, y[n]=0.~~

~~n=0 If n=0 y[0]=~~

b) $H(e^{j\omega})=1+\sum_{k=1}^{+\infty}(\frac{1}{2})^{k-1}e^{-j\omega k}$

The input x[n] is the eigenvalue of the system.

$y[n]=e^{j\omega n}\cdot H(e^{j\omega})$

$=e^{j\omega n}\cdot\left[1+\sum_{k=1}^{+\infty}(\frac{1}{2})^{k-1}e^{-j\omega k}\right]$

c) $H(e^{j\omega})=1+\sum_{k=1}^{+\infty}(\frac{1}{2})^{k-1}e^{-j\omega k}$

d) $x[n]=\cos(\frac{\pi}{2}n+\frac{\pi}{8})$

$=\frac{1}{2}e^{j\frac{\pi}{8}}e^{j(\frac{\pi}{2}n)}+\frac{1}{2}e^{-j\frac{\pi}{8}}e^{-j(\frac{\pi}{2}n)}$

~~y[n]=~~ $x_1[n]=\frac{1}{2}e^{j\frac{\pi}{8}}e^{j(\frac{\pi}{2}n)}$

$x_2[n]=\frac{1}{2}e^{-j\frac{\pi}{8}}e^{-j(\frac{\pi}{2}n)}$

~~y[n]=~~ $y_1[n]=H(e^{j\frac{\pi}{2}})e^{j\frac{\pi}{8}}\cdot e^{j(\frac{\pi}{2}n)}\cdot\frac{1}{2}$

$y_2[n]=H(e^{-j\frac{\pi}{2}})e^{-j\frac{\pi}{8}}\cdot e^{-j(\frac{\pi}{2}n)}\cdot\frac{1}{2}$

$y[n]=y_1[n]+y_2[n]$

$=H(e^{j\frac{\pi}{2}})\frac{1}{2}e^{j\frac{\pi}{8}}e^{j(\frac{\pi}{2}n)}+H(e^{-j\frac{\pi}{2}})\frac{1}{2}e^{-j\frac{\pi}{8}}\cdot e^{-j(\frac{\pi}{2}n)}$

法二： $x[n]=e^{j\omega n}$

$y[n]=\sum_{k=-\infty}^{+\infty}h[k]x[n-k]$

$=\sum_{k=-\infty}^{+\infty}h[k]e^{j\omega(n-k)}$

$=e^{j\omega n}\sum_{k=-\infty}^{+\infty}h[k]e^{-j\omega k}$

$=e^{j\omega n}\left[1+\sum_{k=1}^{+\infty}(\frac{1}{2})^{k-1}e^{-j\omega k}\right]$

2. $N=4$

a) $X[k]=\sum_{n=0}^{3}\cos(\frac{\pi}{2}n)W_4^{kn}$

$=W_4^{k0}-W_4^{k\cdot 2}$

$=1-e^{-j\pi k}=1-\cos(\pi k)$

$(0<k\leq 3)$

b) $H[k]=\sum_{n=0}^{3}2^nW_4^{kn}$

$=1+2W_4^{k}+4W_4^{2k}+8W_4^{3k}$

$=1+2e^{-j(\frac{\pi}{2})k}+4e^{-j(\pi k)}+8e^{-j(\frac{3\pi}{2})k}$

$(0\leq k\leq 3)$

c)

$$\tilde{Y}[k] = \tilde{X}[k]\cdot\tilde{H}[k]$$

$$= (1-e^{-j\pi k})(1+2e^{-j(\frac{\pi}{2})k}+4e^{-j(\pi)k}+8e^{-j(\frac{3}{2}\pi)k})$$

$$= -3-6e^{-j(\frac{\pi}{2})k}+3e^{-j\pi k}+6e^{-j(\frac{3\pi}{2})k}$$

$$\cancel{y[n] = \frac{1}{4}\sum_{k=0}^{3}\tilde{Y}[k]W_4^{-kn}}$$

$$\cancel{\frac{1}{4}\sum_{k=0}^{3}(-3-6W_4^{k}+3W_4^{2k}+6W_4^{3k})W_4^{-kn}}$$

$$\cancel{=\frac{1}{4}\sum_{k=0}^{3}[-3W_4^{-kn}-6W_4^{k(1-n)}+3W_4^{k(2-n)}+6W_4^{k(3-n)}]}$$

使用循环移位性质

$$x[(n-m)\bmod(N)] \xleftrightarrow{DFT} W_N^{kn}X[k]$$

$$y[n] = -3\delta[n]-6\delta[n-1]+3\delta[n-2]+6\delta[n-3]$$

3、a)

$$\cancel{Y(z)-\frac{3}{4}Y(z^{-1})}$$

$$(1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2})Y(z) = (1+\frac{1}{3}z^{-1})X(z)$$

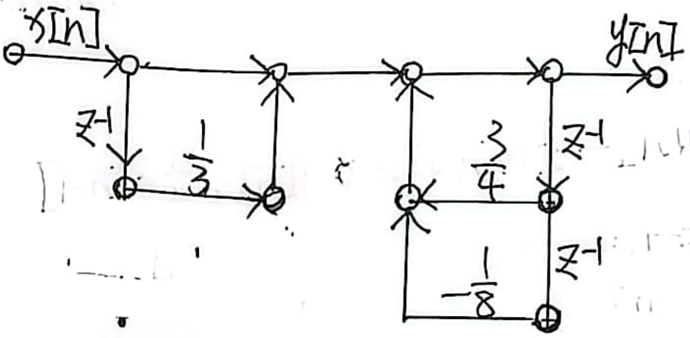
$$H(z) = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

$$V(z) = (1+\frac{1}{3}z^{-1})X(z)$$

$$Y(z) = \left(\frac{1}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}\right)V(z)$$

$$v[n] = x[n]+\frac{1}{3}x[n-1]$$

$$y[n] = v[n]+\frac{3}{4}y[n-1]-\frac{1}{8}y[n-2]$$



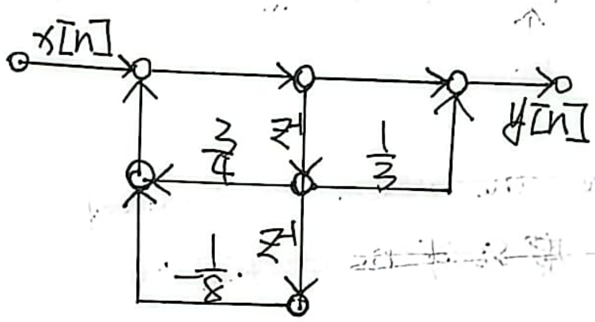
b)

$$W(z) = \left(\frac{1}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}\right)X(z)$$

$$Y(z) = (1+\frac{1}{3}z^{-1})W(z)$$

$$w[n] = \frac{3}{4}w[n-1]-\frac{1}{8}w[n-2]+x[n]$$

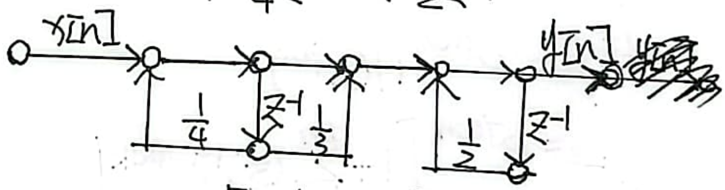
$$y[n] = w[n]+\frac{1}{3}w[n-1]$$



c)

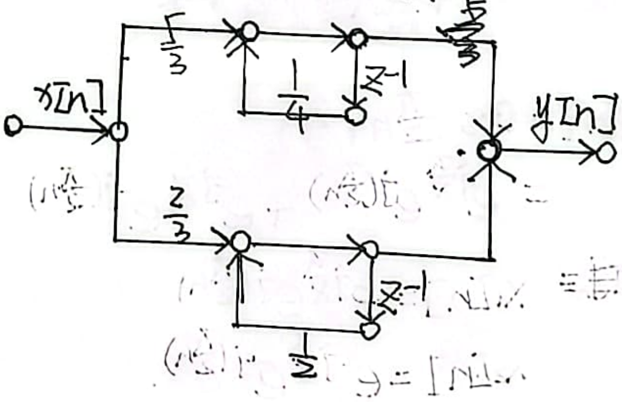
$$H(z) = \frac{1+\frac{1}{3}z^{-1}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{2}z^{-1})}$$

$$= \frac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{4}z^{-1}}\cdot\frac{1}{1-\frac{1}{2}z^{-1}}$$



d)

$$H(z) = \frac{\frac{5}{3}}{1-\frac{1}{4}z^{-1}}+\frac{\frac{2}{3}}{1-\frac{1}{2}z^{-1}}$$



4.

		M Odd	M Even
Impulse response $h[n]$	Symmetric	$\beta=0$, $\alpha=\frac{M-1}{2}$	$\beta=0$, $\alpha=\frac{M-1}{2}$
	Anti-Symmetric	$\beta=\frac{\pi}{2}$, $\alpha=\frac{M-1}{2}$	$\beta=\frac{\pi}{2}$, $\alpha=\frac{M-1}{2}$

5. $$H_a(s)=\frac{2}{s+2}-\frac{1}{s+1}$$

~~ha(t)~~

$$h_a(t)=2e^{-2t}-e^{-t}$$

$$h[n]=T_s\cdot h[nT_s]=T_s\cdot\left[2e^{-2(nT_s)}-e^{-(nT_s)}\right]$$

$$H(z)=\frac{2T_s}{1-e^{-2(T_s)}z^{-1}}-\frac{T_s}{1-e^{-T_s}z^{-1}}$$

6. a) $\omega_c=\frac{1}{4}\pi$ ~~M=5~~

~~α=~~ Filter length $M=5$

$$\alpha=\frac{M-1}{2}=2$$

$$\cancel{h_d[n]=\frac{\sin[\frac{1}{4}\pi(n-2)]}{\pi(n-2)}}$$

$$\cancel{w[n]=\begin{cases}\frac{1}{2}n, & 0\le n\le 2\\ 2-\frac{1}{2}n, & 2<n\le 4\end{cases}}$$

$\cancel{h[n]=\{}$ ~~h[n] = hd[n]·w[n]~~

$$\cancel{h[n]=\{0,\frac{\sqrt{2}}{4\pi},\frac{1}{4},\frac{\sqrt{2}}{4\pi},0\}}.$$

b) $\omega_c=0.5\pi$ $M=21$ $\alpha=\frac{M-1}{2}=10$

$$h[n]=\delta[n-\alpha]-\frac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}$$

~~δ[n-α]~~

$$=\delta[n-10]-\frac{\sin[0.5\pi(n-10)]}{\pi(n-10)}$$

c) $\delta=0.005$ $\varepsilon_p=20\lg(\delta)=-46.02\text{dB}$

Only Hamming window and Blackman window can be used. We want use minimum filter length. ~~so~~ ~~要~~ I choose Hamming window.

$$\Delta\omega=0.05\pi=\frac{6.27\pi}{M}\Rightarrow$$

$M=126\Rightarrow$ Filter length is 127

$$\cancel{h[n]=\frac{\sin[0.5\pi(n-126)]}{\pi(n-126)}}\quad \alpha=63$$

$$\cancel{\alpha=\frac{M-1}{2}=63}$$

$$h[n]=\frac{\sin[0.5\pi(n-63)]}{\pi(n-63)}\cdot w[n] \quad 0\le n\le 126.$$

6.(a) $$h_d[n-2]=\frac{\sin[\frac{1}{4}\pi(n-2)]}{\pi(n-2)}$$

Bartlett 窗第一项和最后一项均为0，故不要取，取中间5项。所以滤波器要长度为7 $\{0,\frac{1}{3},\frac{2}{3},1,\frac{2}{3},\frac{1}{3},0\}$.

$$w[n]=\begin{cases}\frac{1}{3}n, & 0\le n\le 3\\ 2-\frac{1}{3}n & 3<n\le 6.\end{cases}$$

$$h[n]=h_d[n-2]\cdot w[n]$$

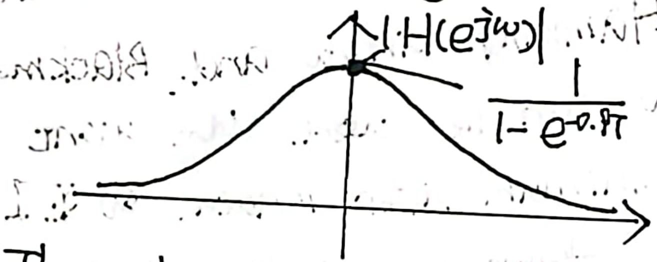
$$=\{\frac{1}{6\pi},\frac{\sqrt{2}}{3\pi},\frac{1}{4},\frac{\sqrt{2}}{3\pi},\frac{1}{6\pi}\}.$$

7.

$$h[n] = h_a(nT)$$
$$= (e^{-0.9nT})\, u[n] = (e^{-0.9T})^n u[n]$$

$$H(z) = \frac{1}{1 - e^{-0.9T} z^{-1}}$$

$$H(e^{j\omega}) = \frac{1}{1 - e^{-0.9T} e^{-j\omega}}$$



Thus the digital filter approximates a low pass filter. Since the pole is located at $z = e^{-0.9T}$, the pole is inside the ~~circuit~~ circle and hence the system is stable. ~~at~~ for $T>0$.

$$H(e^{j\omega}) = \frac{1}{(1 - \cos\omega\, e^{-0.9T}) + j\sin\omega\, e^{-0.9T}}$$

$$|H(e^{j\omega})| = \sqrt{\frac{1}{1 + e^{-1.8T} - 2\cos\omega\, e^{-0.9T}}}$$