Assignment Report

TERM: **2023-2024**

Module: **EE3628 Control System**

CLASS: 34092102

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Contents

1 Introduction

A control system is an interconnection of components forming a system configuration that will provide a desired system response. The aim of this assignment is to design a control system to meet the desired requirements. The control system is designed to achieve the desired performance, such as the gain crossover frequency, phase margin, overshoot, and settling time. The control system is designed to meet the requirements of the system and improve the system's performance.

Moreover, the control system is designed to achieve the desired performance using different methods, such as the open-loop system, compensator, state feedback controller, observer, and PID controller. The control system is designed to meet the requirements of the system and improve the system's performance.

2 Section 1

2.1 a) Test the data for the open-loop transfer function

In this section, I design an open-loop system to measure the response at different frequencies and calculate the gain. In addition, the compensator is designed to make the whole closed-loop system achieve the desired requirements.

(a) (b) (c)

Figure 1: Image of oscilloscope in the experiment

Figure 1 shows the image of the oscilloscope in our experiment. The circuit was built according to the experimental requirements. Set the peak-to-peak voltage of the input signal to 1V, and the input signal is sinusoidal wave.

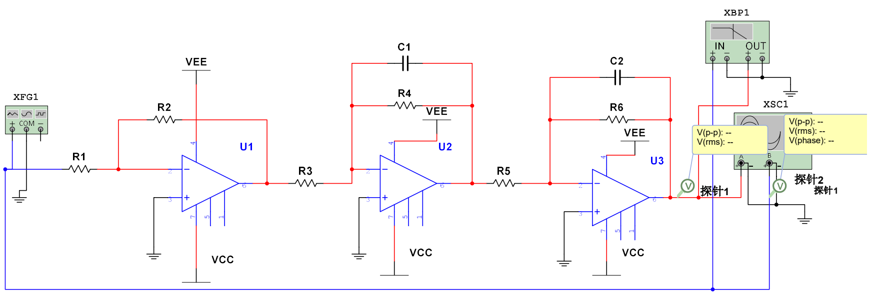


Figure 2: Circuit diagram of the experiment 1

Figure 2 shows the circuit diagram of the experiment 1. The frequency was gradually decreased from 30Hz to 0.25Hz, and the data were recorded and recorded in Table 1.

Table 1: Test data for different frequencies

Frequency $f(Hz)$	$Y_0(mv)$ (peak-to-peak amplitude of output signal)	$Y_i(V)$ (peak-to-peak amplitude of input signal)	Gain, $\lvert G(j\omega)\rvert = \frac{Y_0}{Y_i}$	Gain in dB, $20log(\frac{Y_0}{Y_i})$	Phase in difference between input and output signal, Φ
0.25	4810.0		4.81	13.6429	-67.08
0.4	3210.0		3.23	10.1841	-82.866
0.6	2150.0		2.15	6.6488	-95.98
0.8	1570.0		1.57	3.9180	-105.6
1	1200.0		1.2	1.5836	-113.72
2	444.0	1	0.444	-7.0523	-137.46
4	132.0		0.132	-17.5885	-156.1
6	61.0		0.061	-24.2934	-163.8
8	34.7		0.0347	-29.1934	-167.7
10	22.4		0.02	-32.995	-170.105
20	5.6		0.00564	-44.9744	-175.05
30	2.5		0.00252	-51.972	-176.68

Table 1 shows the test data of open loop amplitude-frequency and phase-frequency characteristics. Then, manually drawing the Bode diagram of the open-loop system.

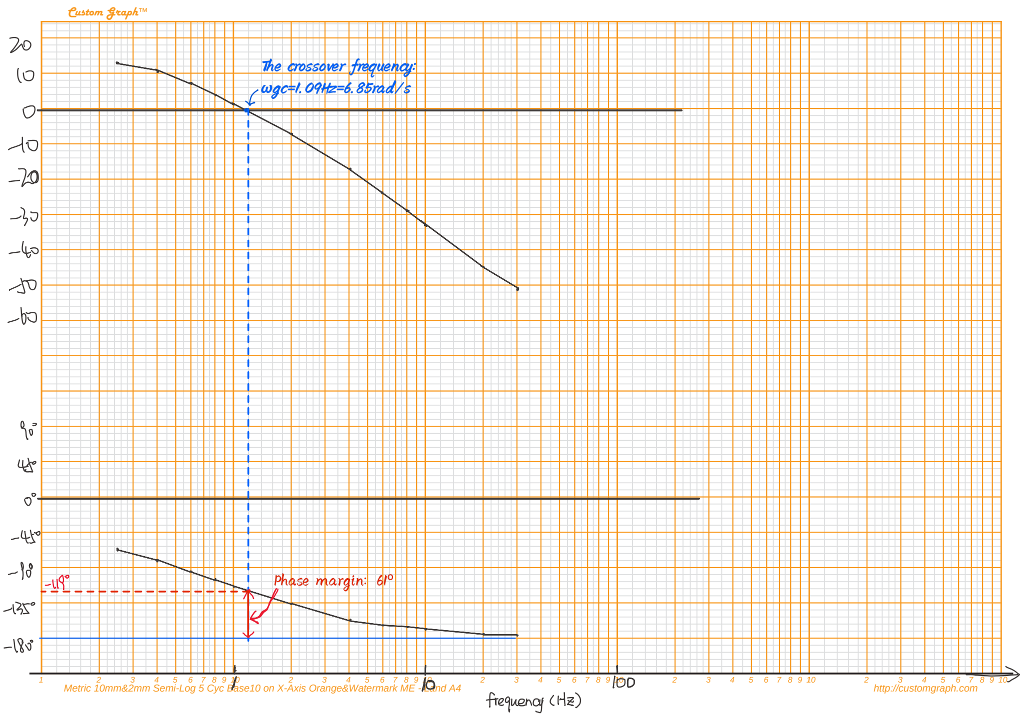


Figure 3: Manually drawing the Bode diagram of the open-loop system

Figure 3 shows the manually drawing the Bode diagram of the open-loop system. From this figure, the **gain crossover frequency** ω_{gc} and **phase margin** PM can be obtained. The gain crossover frequency means the frequency at which the gain of the open-loop system is 0dB. The phase margin is the difference between the phase of the open-loop system and -180 degrees at the gain crossover frequency [1].

$$\omega_{gc} = 1.09Hz = 6.85rad/s, \quad PM = 61^\circ$$

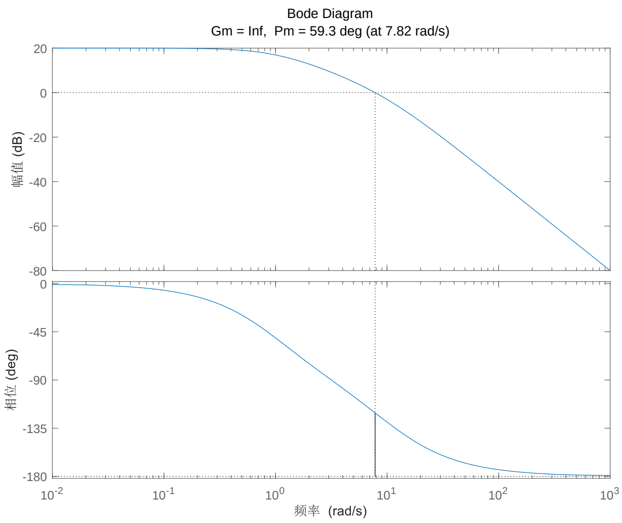


Figure 4: Matlab simulate the Bode diagram

Then I use Matlab to simulate the Bode diagram of the open-loop system. The result is shown in Figure 4. The gain crossover frequency and phase margin is

$$\omega_{gc} = 7.82rad/s, \quad PM = 59.3^\circ$$

It can be found that there is a certain gap between the actual and theoretical values. The difference between the theoretical and actual measured gain crossover frequencies is $0.97rad/s$ and the phase margin is 1.7°.

One possible reason is that the resistance values set in the experiment will cause the transfer function may not be the desired one. In this experiment, we are asked to use the resistance values of $R_0 = 51k\Omega$, $R_1 = 470k\Omega$, $R_2 = 510k\Omega$, $R_3 = 100k\Omega$. Use these data recalculated the parameter K, T_1 and T_2.

$$K = \frac{R_1}{R_0} = \frac{470K}{51K} = 9.21, \quad T_1 = R_2C_1 = 1.02sec, \quad T_2 = R_3C_2 = 0.1sec$$

Therefore, the actual transfer function is:

$$G_{actual}(s) = \frac{9.21}{(1+0.1s)(1+1.02s)} \tag{1}$$

Using equation 1, re-simulation to get the gain crossover frequency and phase margin.

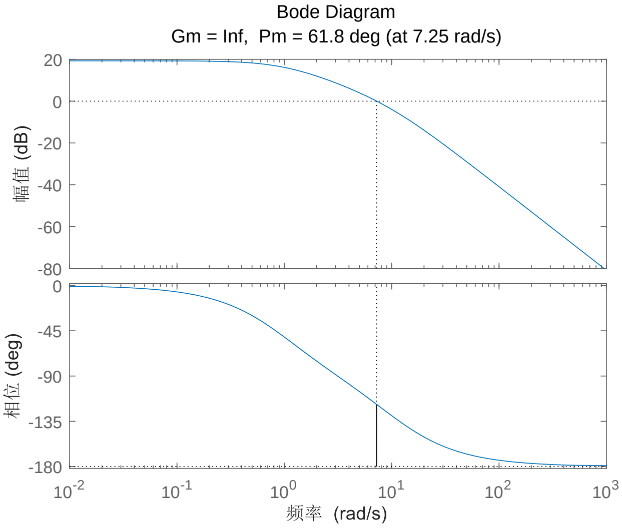


Figure 5: Matlab simulate the Bode diagram of the actual transfer function

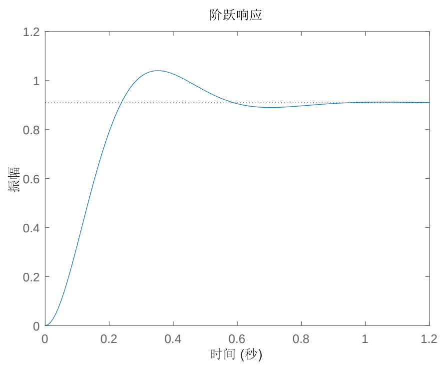
From figure 5, it can be found that the difference between the ideal and actual parameters are obviously reduced.

Another reason for the error is that the hand-drawn bode plot is inherently inaccurate, and the error caused by the drawing and the error caused by the reading together contribute to the difference between the actual and theoretical values.

2.2 b) Design the Compensator

Before the compensator is added in the system, the closed-loop system is:

$$H(s) = \frac{G(s)}{1+G(s)} = \frac{10}{0.1s^2 + 1.1s + 11}$$




```
     RiseTime: 0.1609
TransientTime: 0.7324
 SettlingTime: 0.7324
  SettlingMin: 0.8244
  SettlingMax: 1.0404
    Overshoot: 14.4446
   Undershoot: 0
         Peak: 1.0404
     PeakTime: 0.3517
```

(a) Step response of the closed-loop system (b) Data of step response

Figure 6: Step response of the closed-loop system before adding the compensator

From figure 6, it has been observed that both the percentage overshoot and the settling time are significantly large. In this section, I designed the phase-lead compensator to make the **percentage overshoot M_p** is less than 11% and the 2% **settling time T_s** is less than 0.5s.

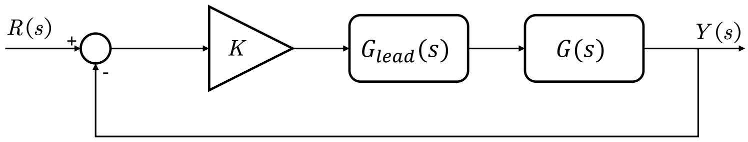


Figure 7: Block diagram of the designed closed-loop system

Figure 7 shows the block diagram of unit feedback closed-loop system. $G(s)$ is an open-loop transfer function. I just need design phase-lead compensator $G_{lead}(s)$ and gain K [2].

2.2.1 Phase-lead compensator design using the root locus

The transfer function of the phase-lead compensator is [3]:

$$G_{lead}(s) = \frac{s + \frac{1}{\alpha\tau}}{s + \frac{1}{\tau}} = \frac{s+z}{s+p} \tag{2}$$

where α and τ are defined for the RC network, and $|z| < |p|$. Using the requirements, the desired root location for the dominant roots can be found.

$$M_p = e^{-(\frac{\zeta}{\sqrt{1-\zeta^2}})\pi} \times 100\% \Rightarrow \zeta = \frac{|ln(\frac{M_p}{100})|}{\sqrt{\pi^2 + ln^2(\frac{M_p}{100})}} \Rightarrow \zeta = 0.575$$

$$T_s = \frac{4}{\zeta\omega_n} < 0.5 \Rightarrow \omega_n > 13.93 rad/s$$

Thus, I set the damping ratio $\zeta = 0.575$ and the natural frequency $\omega_n = 14 rad/s$.

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 11.454 rad/s, \ \sigma = -\omega_n\zeta = -8.05 rad/s$$

Therefore, **the dominant roots are $s = \sigma \pm j\omega_d = -8.05 \pm 11.454j$**.

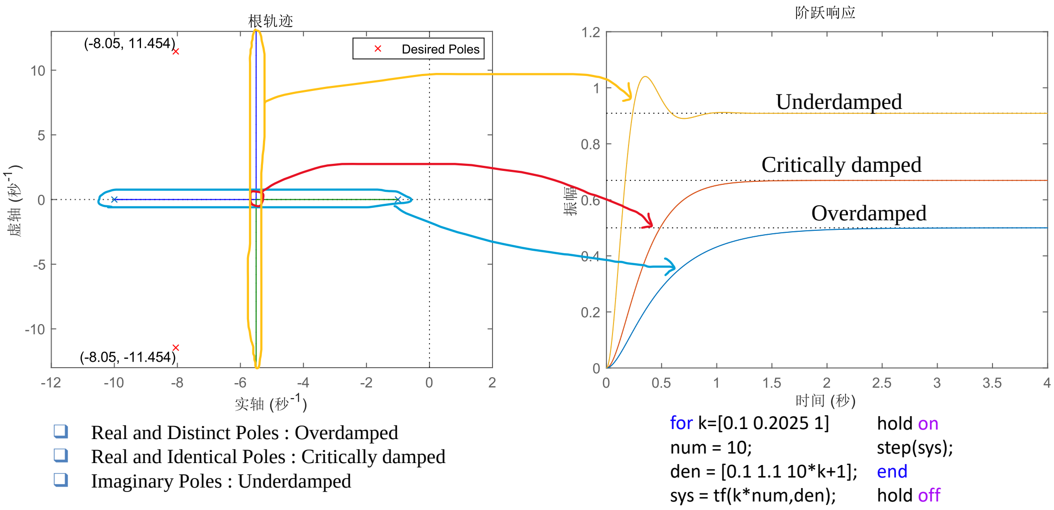


Figure 8: Root locus of the initial closed-loop system

Figure 8 shows the closed-loop system before add the compensator. The left figure is the root locus, the red point is the dominant roots. The right figure is the step response of the different value of K. When K is small, the step response is overdamped. As K gradually increases, the root locus shifts from the real axis to the imaginary axis, causing the step response to become underdamped.Since only the case where the step response is underdamped is desired, consideration is limited to when K is large ($K > 0.2025$), indicating that the root locus lies on the imaginary axis.

From this figure, it can be found that the dominant roots are located to the left of the root locus (imaginary axis). **The gain K affects the system's overshoot**; Specifically, a higher K increases the overshoot. Therefore, adjusting K allows for control over the system's overshoot. In contrast, **the setting time is influenced by the root locus**; The further left the root locus, the shorter the system's setting time.

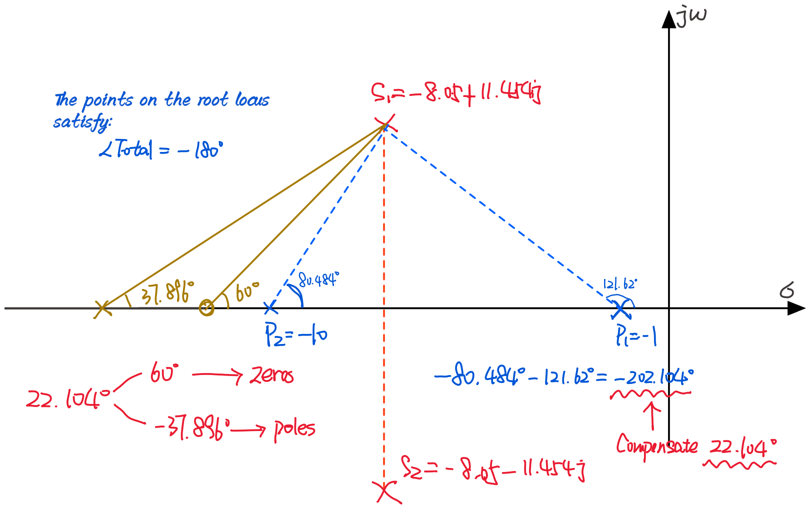


Figure 9: Design goals for the root of the system

Figure 9 shows the design goals for the root of the system. For the closed-loop system, **the total angle at the desired root location is 180°** and therefore is on the compensated root locus. For the open-loop transfer function $G(s)$, the poles are $\boldsymbol{p_1 = -1}$ and $\boldsymbol{p_2 = -10}$, respectively. The angle between the dominant roots and poles are

$$\theta = arctan(\frac{y_2 - y_1}{x_2 - x_1}) \tag{3}$$

where x_1, y_1 and x_2, y_2 are the real and imaginary parts of the poles and dominant roots, respectively. **The poles contribute negative angles, while the zeros contribute positive angles.**

$$\theta_{p_1} = arctan(\frac{11.454 - 0}{-8.05 - (-1)}) = 121.62°$$

$$\theta_{p_2} = arctan(\frac{11.454 - 0}{-8.05 - (-10)}) = 80.484°$$

Therefore, the compensator need to compensate

$$(-\theta_{p_1}) + (-\theta_{p_2}) + \theta_{compensator} = -180° \Rightarrow \theta_{compensator} = 22.104°$$

$$\theta_{compensator} = 22.104° = 60° - 37.896°$$

Using $60°$ as the zero of the compensator and $-37.896°$ as the pole of the compensator.

$$\frac{11.454 - 0}{-8.05 + z} = tan(60°) \Rightarrow z = 14.668$$

$$\frac{11.454 - 0}{-8.05 + p} = tan(37.896°) \Rightarrow p = 22.772$$

Therefore, the transfer function of the phase-lead compensator is:

$$G_{lead}(s) = \frac{s + 14.668}{s + 22.772} \tag{4}$$

2.2.2 Simulation of the designed closed-loop system

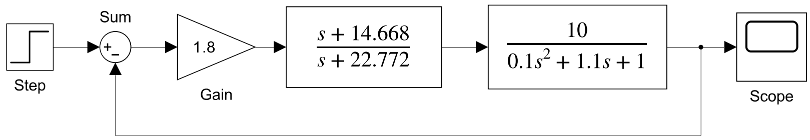
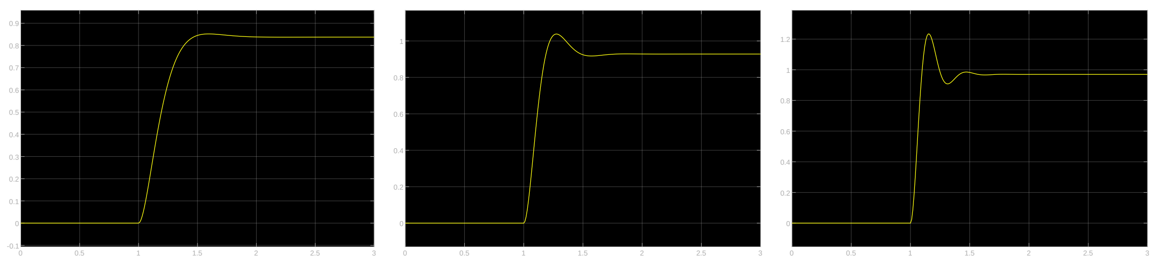


Figure 10: Simulation model

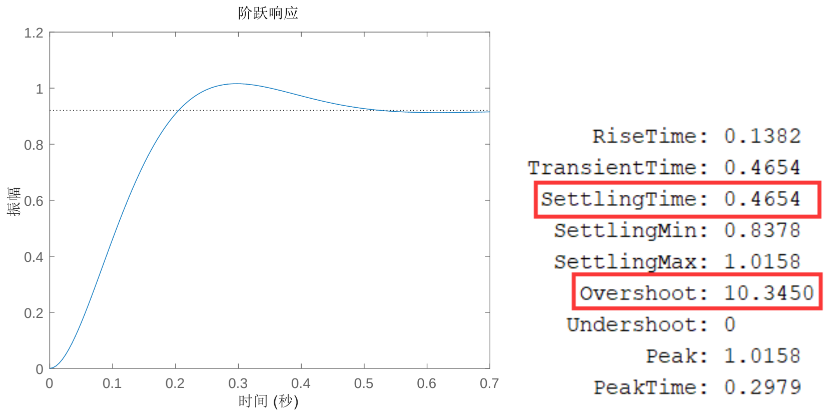
Figure 10 shows the simulation model on the MATLAB/Simulink. The setting time has met the requirements, and the experiment's needs can be satisfied by continuously adjusting K, K need larger than 0.2025.



(a) K=0.8 (b) K=2 (c) K=5

Figure 11: Step response for different value of gain K

Figure 11 shows the step response for different value of gain K. It can be observed that the overshoot increases as K increases. After repeated trials, it was found that the overshoot at $\boldsymbol{K = 1.8}$ meets the requirements.



(a) Step response of the closed-loop system (b) Data of step response

Figure 12: Step response of the closed-loop system after adding the compensator

Figure 12 shows the step response and corresponding data of the closed-loop system after adding the compensator. It can be found that **the setting time is 0.456s < 0.5s**, **the overshoot is 10.345 % < 11%**, which meets the requirements.

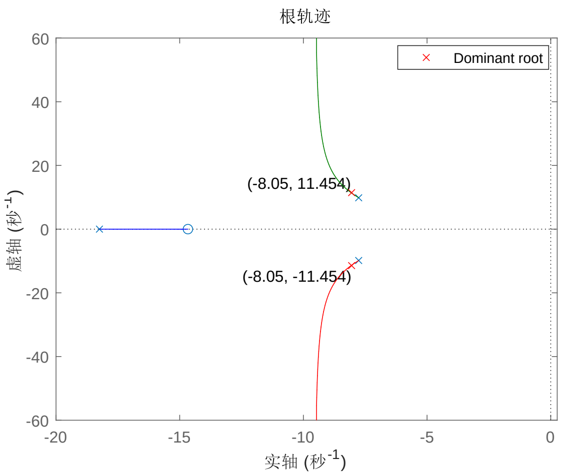


Figure 13: Root locus after adding the compensator

Figure 13 shows the root locus after adding the compensator. It can be observed that the root locus shifts to the left as a whole, and the dominant root remains on the root locus. This theoretical verification supports the correctness of my design.

The phase-lead compensation network is a useful compensator for altering the performance of a control system. However, lead networks are not suitable for providing high steady-state accuracy in systems requiring very high error constants. To provide large error constants, we must consider the use of integration-type compensation networks [3]. **The phase-lead compensator is equivalent to a PD controller, but it lacks an integral term, so the steady-state error cannot be eliminated.**

2.3 c) Design the state feedback controller and observer

In this section, I designed the state feedback controller and observer for the system.

2.3.1 State space model

$$G(s) = \frac{10}{0.1s^2 + 1.1s + 11} = \frac{100}{s^2 + 11s + 10} \tag{5}$$

From equation 5, $b_0 = 0,\ b_1 = 0,\ b_2 = 100,\ a_1 = 11,\ a_2 = 10$. Using **Phase Variable Form**, the corresponding matrix and vector are:

$$A = \begin{bmatrix} 0 & 1 \\ -10 & -11 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 100 & 0 \end{bmatrix}, D = 0$$

Thus, the state space model is:

$$\dot{x} = Ax + Bu = \begin{bmatrix} 0 & 1 \\ -10 & -11 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \quad y = Cx = \begin{bmatrix} 100 & 0 \end{bmatrix} x$$

2.3.2 State feedback controller design

Firstly, check whether the system is controllable. The controllability matrix is:

$$C_{Mx} = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -11 \end{bmatrix}$$

$det(C_{Mx}) = -1 \neq 0$. Thus, the rank of the controllability matrix is 2, which means the system is controllable. Let $K = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$, the closed-loop system matrix is:

$$A_c = A - BK = \begin{bmatrix} 0 & 1 \\ -10 & -11 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -10 - k_1 & -11 - k_2 \end{bmatrix}$$

The characteristic equation is:

$$det(sI - (A - BK)) = det \begin{bmatrix} s & -1 \\ 10 + k_1 & s + 11 + k_2 \end{bmatrix} = s^2 + (11 + k_2)s + (10 + k_1) = 0$$

According to section 2.2.1, $\zeta = 0.575, \omega_n = 14rad/s$. The desired characteristic equation is:

$$s^2 + 16.1s + 196 = s^2 + (11 + k_2)s + (10 + k_1) = 0 \Rightarrow k_1 = 186,\ k_2 = 5.1$$

$$K = \begin{bmatrix} 186 & 5.1 \end{bmatrix}$$

Finally, the state-space closed-loop system with the state feedback controller can be expressed as:

$$\begin{cases} \dot{x} = (A - BK)x + Br \\ y = Cx \end{cases} \Rightarrow \begin{cases} \dot{x} = \begin{bmatrix} 0 & 1 \\ -196 & -16.1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \\ y = \begin{bmatrix} 100 & 0 \end{bmatrix} x \end{cases}$$

Find the transfer function of the state-space model:

$$G(s) = C(sI - (A - BK))^{-1}B = \frac{100}{s^2 + 16.1s + 196} \tag{6}$$

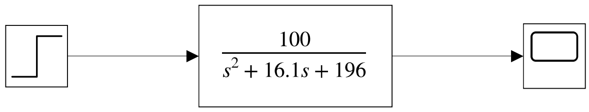


Figure 14: Simulation model of the state feedback controller

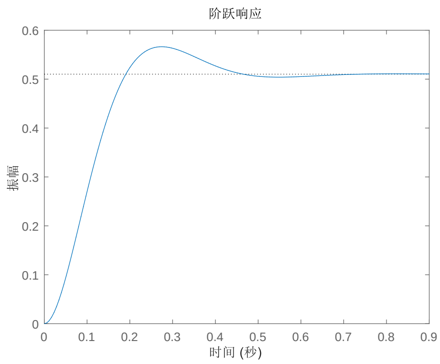


Figure 15: Step response of the state feedback controller

Figure 14 using Matlab/Simulink shows the simulation model of the state feedback controller. Figure 15 shows the step response of the state feedback controller.

2.3.3 State observer design

The state observer is designed to estimate the state of the system, in this section, I designed a **Luenberger Observer**. Using the data calculated in Section 2.3.1 and **Observable Canonical Form**. The observer matrix is:

$$A = \begin{bmatrix} -11 & 1 \\ -10 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 100 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}, D = 0$$

Then, check whether the system is observable. The observability matrix is:

$$O_{Mx} = \begin{bmatrix} C^* & A^*C^* \end{bmatrix} = \begin{bmatrix} 1 & -11 \\ 0 & 1 \end{bmatrix}$$

$det(O_{Mx}) = 1 \neq 0$. Thus, the rank of the observability matrix is 2, which means the system is observable. Let $L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}$. The state transition matrix is:

$$\Phi = A - LC = \begin{bmatrix} -11 & 1 \\ -10 & 0 \end{bmatrix} - \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} -11 - l_1 & 1 \\ -10 & 0 \end{bmatrix}$$

The characteristic equation of $(A - LC)$ is:

$$det(sI - (A - LC)) = det \begin{bmatrix} s + 11 + l_1 & -1 \\ 10 + l_2 & s \end{bmatrix} = s^2 + (11 + l_1)s + 10 + l_2 = 0$$

According to the Section 2.2.1, $\zeta = 0.575, \omega_n = 14rad/s$. The desired characteristic equation is $s^2 + 16.1s + 196 = 0$

$$s^2 + 16.1s + 196 = s^2 + (11 + l_1)s + 10 + l_2 = 0 \Rightarrow l_1 = 5.1,\ l_2 = 186$$

$$L = \begin{bmatrix} 5.1 \\ 186 \end{bmatrix}$$

2.3.4 Compare the state feedback controller with the phase lead compensator

Table 2: Step response measurement results

Step response information	Phase lead compensator	State feedback controller
RiseTime	0.1382s	0.1283s
Settling Time	0.4654s	0.4208s
Overshoot	10.3450%	10.9928%
Peak Time	0.2979s	0.2746s
Peak	1.0158	0.5663
Steady state error	0.0794	0.4764

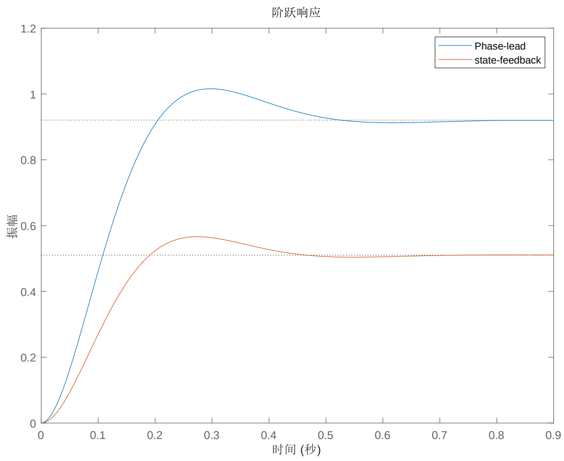


Figure 16: Comparison of the state-feedback controller and the phase lead compensator

Table 2 shows the step response data of the phase lead compensator and the state feedback controller. Figure 16 shows the step response of the two controllers. It can be observed that the phase-lead controller exhibits superior performance in terms of minimizing overshoot and steady-state error. However, the state feedback demonstrates higher efficiency in reducing rise time and setting time.

In the evaluation of second-order system performance parameters, it is necessary to comprehensively evaluate the stability, steady-state error and Comprehensive performance of the system [4].

Statility and Steady-State Error

Obviously, both systems are stable, but the steady-state error of the phase compensator is smaller.

Comprehensive performance index - ISE and ITSE

Let the overshoot and natural frequency of state feedback and phase lead be: $\zeta_1, \omega_{n1}, \zeta_2, \omega_{n2}$, respectively.

$$\zeta_1 = 0.5855,\ \omega_{n1} = 14.6793 rad/s, \quad \zeta_2 = 0.5750,\ \omega_{n2} = 16.5317 rad/s$$

The Integrated Square Error (ISE) and Integrated Time-weighted Square Error (ITSE) are:

$$ISE = \frac{1}{2\omega_n}(\frac{1}{2\zeta} + 2\zeta), \quad ITSE = \frac{1}{2\omega_n^2}(\frac{1}{4\zeta^2} + 2\zeta^2) \tag{7}$$

$$ISE_{\text{phase-lead}} = 0.0003, \quad ITSE_{\text{phase-lead}} = 0.0002$$

$$ISE_{\text{state-feedback}} = 0.0004, \quad ITSE_{\text{state-feedback}} = 0.0003$$

Smaller values of ISE and ITSE indicate better system performance. Therefore, comprehensive performance index of the phase-lead compensator is better than that of the state feedback controller.

System Robustnedd Analysis

The robustness of the systems can be assessed by their respective phase margins, with larger phase margins indicating greater robustness.

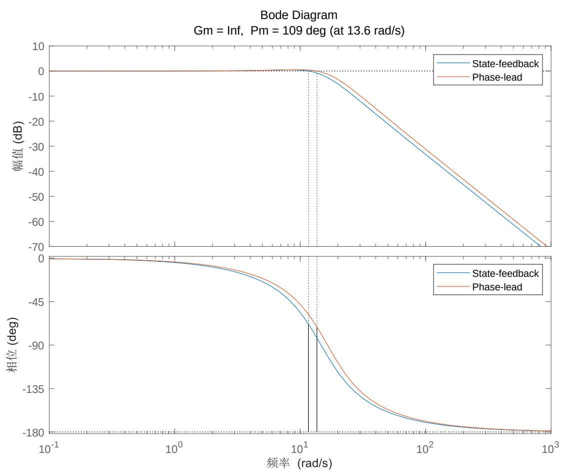


Figure 17: Phase margin of the two controllers

Through MATLAB calculated. The phase margin of the phase-lead compensator is $PM_{\text{phase-lead}} = 112°(at 11.6 rad/s)$, and the phase margin of the state feedback controller is $PM_{\text{state-feedback}} = 109°(at 13.6 rad/s)$. The phase margin of the phase-lead compensator is larger than the state feedback controller, which means the phase-lead compensator has better robustness.

Conclusions

Table 3: Performance parameters of the two controllers

Performance parameters	Phase lead compensator	State feedback controller
Stability	✓	✓
Steady-State Error	✓	
ISE	✓	
ITSE	✓	
Robustness	✓	

Table 3 shows the different performance parameters of the two controllers. It can be observed that the phase-lead compensator exhibits better performance in many performance parameters. **Therefore, the phase lead compensator has better performance and reliability than the state feedback controller.**

3 Section 2

In this section, I analyze the dynamic response of a third-order closed-loop system. Additionally, I experimentally observe the step response at various gain levels.

The open-loop transfer function of the system is:

$$G(s) = \frac{K}{T_3 s(T_s + 1)(T_2 s + 1)} = \frac{K}{0.5s(0.2s + 1)(0.1s + 1)} = \frac{K}{0.01s^3 + 0.15s^2 + 0.5s}$$

Because the system uses unit feedback, the closed-loop transfer function is:

$$CLTF(s) = \frac{G(s)}{1 + G(s)} = \frac{K}{0.01s^3 + 0.15s^2 + 0.5s + K} \tag{8}$$

The characteristic equation is $1 + G(s) = 0.01s^3 + 0.15s^2 + 0.5s + K = 0$.

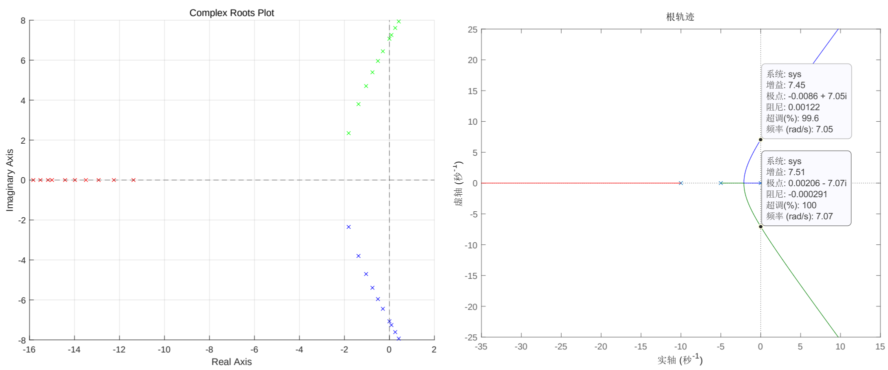
3.1 a) Properties of 3^{rd} system

3.1.1 Find the closed-loop poles for different gain levels

Table 4: Test data for different gain levels

K	p_1	p_2	p_3
1	-11.3780	-1.8110+2.3472i	-1.8110-2.3472i
2	-12.2513	-1.3744+3.7995i	-1.3744-3.7995i
3	-12.9274	-1.0363+4.7045i	-1.0363-4.7045i
4	-13.4915	-0.7542+5.3925i	-0.7542-5.3925i
5	-13.9816	-0.5092+5.9584i	-0.5092-5.9584i
6	-14.4184	-0.2908+6.4443i	-0.2908-6.4443i
7.5	-15.0000	0.0000+7.0711i	0.0000-7.0711i
8	-15.1783	0.0892+7.2594i	0.0892-7.2594i
9	-15.5159	0.2580+7.6117i	0.2580-7.6117i
10	-15.8316	0.4158+7.9368i	0.4158-7.9368i

Table 4 shows the test data for different gain levels, where p_1, p_2, p_3 are the poles of the system.



(a) Roots of the system (b) Root locus of the system

Figure 18: Roots and root locus of the system

Figure 18 shows the roots and root locus of the system for different gain levels. After determining the roots of the transfer function, the circuit is constructed to simulate the transfer function.

3.1.2 Circuit Implementation of the Third-Order System

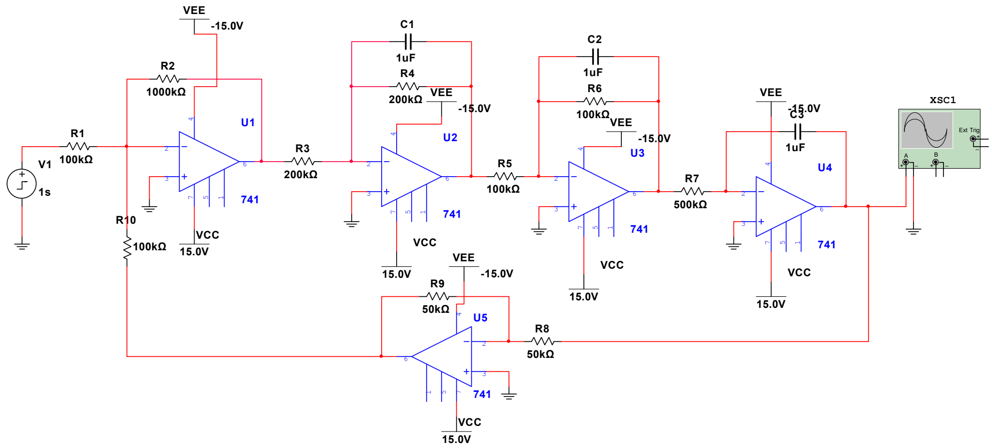
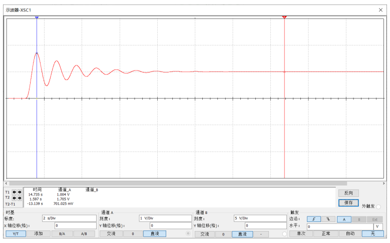


Figure 19: Circuit diagram of the 3^{rd} system

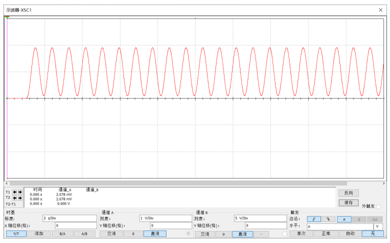
Figure 19 shows the circuit diagram of the 3^{rd} system given in experiment. Let $C_1 = 1\mu F, C_2 = 1\mu F, C_3 = 1\mu F$,

$$T_1 = R_3C_1 = 0.2s,\ T_2 = R_2C_1 = 0.1s,\ T_3 = R_1C_1 = 0.5s$$

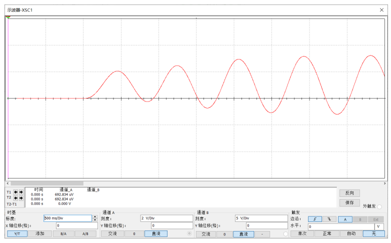
Therefore, $R_3 = 200K\Omega,\ R_4 = 100K\Omega,\ R_5 = 500K\Omega$.



(a) K=5



(b) K=7.5



(c) K=10

Figure 20: Step response of the 3^{rd} system for different gain levels

Figure 20 shows the step response of the 3^{rd} system for different gain levels.

3.1.3 i. System response for different gain levels

(1) Establish time-domain mathematical model for analysis

Equation 8 described a closed-loop transfer function of the system. The third-order system can be decomposed into a 1^{st} order system and a 2^{nd} order system.

$$G(s) = \frac{K}{0.01s^3 + 0.3s^2 + 0.1s + K} = \frac{C_1}{s - p_1} + \frac{C_2}{(s - p_2)(s - p_3)} = X_1(s) + X_2(s) \tag{9}$$

where $X_1(s)$ is the first-order system and $X_2(s)$ is the second-order system, p_1 is the pole of the first-order system, and p_2, p_3 are the poles of the second-order system. The coefficients C_1, C_2

can be obtained by partial fraction decomposition. Let $p_1 = a_1$, by the conjugate symmetry property of the roots $p_2 = a_2 + jb$, $p_3 = a_2 - jb$. For this 2^{nd} order system:

$$X_2(s) = \frac{C_2}{(s-p_2)(s-p_3)} = \frac{C_2}{(s-a_2-jb)(s-a_2+jb)} = \frac{C_2}{b} \cdot \frac{b}{(s-a_2)^2+b^2}$$

Using Inverse Laplace Transform, the time-domain response of the system is:

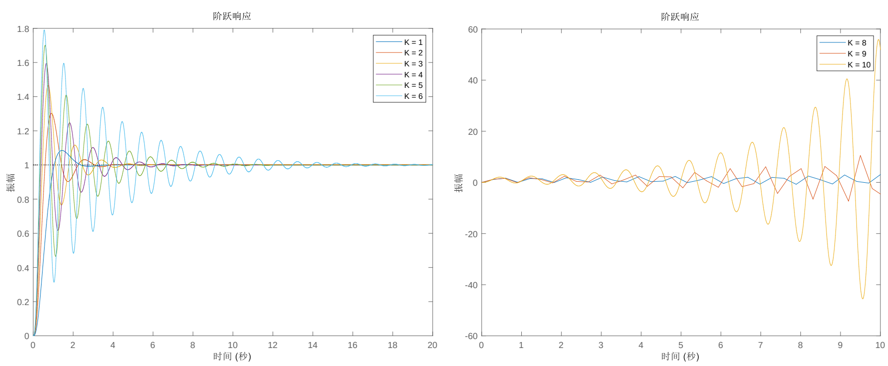
$$x_2(t) = \mathcal{L}^{-1}[X_2(s)] = \frac{C_2}{b} \cdot e^{a_2 t} sin(bt), \quad x_1(t) = \mathcal{L}^{-1}[X_1(s)] = C_1 e^{a_1 t}$$

$$x(t) = x_1(t) + x_2(t) = C_1 e^{a_1 t} + \frac{C_2}{b} \cdot e^{a_2 t} sin(bt) \tag{10}$$

From equation 10, the time-domain response of the system can be obtained. The system include a first-order response and a second-order response. The first-order response is exponential, and the second-order response is sinusoidal. This explains why **the output time domain waveform is similar to a sine function**.

It can be seen from Table 4 and Figure 18a that as K increases, the real pole move to left and the two complex conjugate poles move to the right on the root locus plot. However, from Equation 10, it is clear that the time domain's convergence and rate mainly depend on the term with the slower convergence rate. This explains why **the system transitions from convergent to non-convergent as K increases, even if the real pole shifts left.**

(2) Simulate the step response of the system for analysis



(a) K=1,2,3,4,5,6 (b) K=8,9,10

Figure 21: Step response of the system for different gain levels

Figure 21 shows the step responses obtained by MATLAB simulation for different gains. Combined with the step response obtained by circuit simulation in Figure 20. It can be observed that **as K increases, the system transitions from stable to critically stable, and eventually to unstable**.

(3) Conclusion for the system response with the change of gain

The system's response is analyzed in both the root locus and time-domain. The real pole primarily determines the system's response, while convergence and rate depend on the term with the slower convergence rate. As K increases, the real pole shifts left and the complex conjugate poles move right on the root locus plot. Despite the leftward shift of the real pole, the system transitions from stable to critically stable, and then to unstable.

3.1.4 ii. Critical gain and Critical frequency

Critical gain means the critical value of the feedback control gain at which the system just remains stable, exceeding this gain value can lead to system instability. The critical frequency is the frequency corresponding to the position where the amplitude curve of the system crosses 0dB in the frequency response.

According to Figure 18b, the point where the root locus intersects the imaginary axis represents both the critical gain and critical frequency. Based on the combination of Figure 18b and Table 4, it can be inferred that **the critical gain** is:

$$K_{cr} = 7.50$$

The **critical frequency** is:

$$\omega_{cr} = 7.07 rad/s$$

In PID controller tuning, the critical gain and critical frequency can be tuned by using the method of Ziegler Nichols. For a parallel PID controller form, the transfer function is:

$$G_c(s) = K_p + K_i \frac{1}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s} \tag{11}$$

The three parameters of PID can be obtained by critical gain and critical frequency:

$$K_p = 0.6 K_c,\ K_i = K_p \cdot \frac{\omega_c}{\pi},\ K_d = K_p \cdot \frac{\pi}{4\omega_c} \tag{12}$$

3.2 b) Design the PID controller

3.2.1 PID controller design using Ziegler Nichols method

In this section, I designed the PID controller for the system. The transfer function of the parallel PID controller form in shown in equation 11. The parameters of PID controller can be obtained by equation 12. In this section, I choose to use a **parallel PID controller form** for my design. Using Ziegler Nichols closed loop method, the parameters of PID controller are:

$$K_p = 0.6 \times 7.50 = 4.50,\ K_i = 4.50 \times \frac{7.07}{\pi} = 10.13,\ K_d = 4.50 \times \frac{\pi}{4 \times 7.07} = 0.50$$

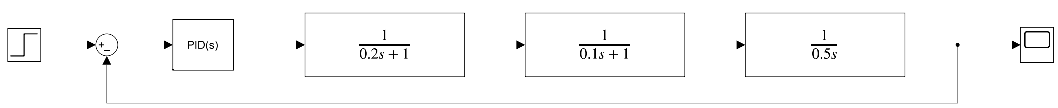


Figure 22: Simulation model of the PID controller

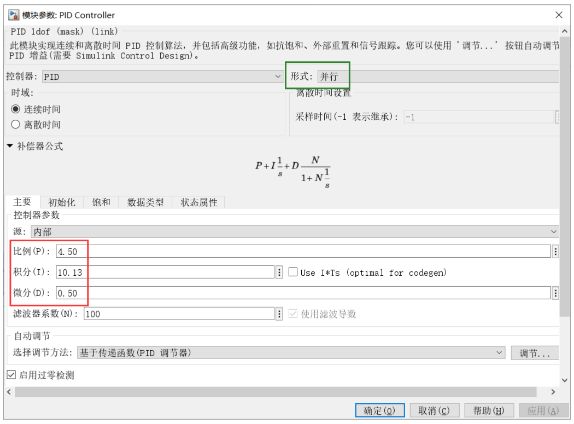
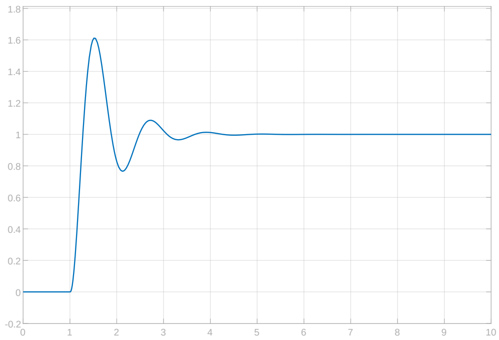
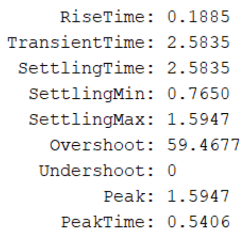


Figure 23: PID controller parameters

Figure 22 shows the simulation model of the PID controller. Figure 23 shows the parameters of the PID controller.



(a) Step response



(b) Data of step response

Figure 24: Step response of the PID controller

Figure 24 shows the step response and corresponding parameters of the PID controller. It's evident that the system is an underdamped stable system, and it's notable that the steady-state error can be entirely eliminated.

3.2.2 PID performance evaluation

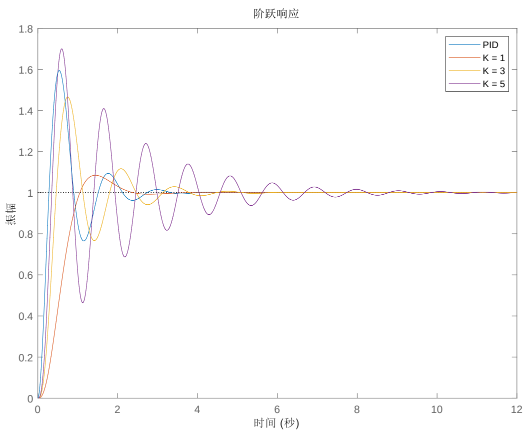


Figure 25: Comparison of PID system performance

Figure 25 illustrates the disparity between the step responses of the system employing PID control and those using only gain. It's apparent that the system with a PID controller exhibits a shorter rise time and settling time compared to the system without one, although there's a slightly larger overshoot present.

However, PID control offers overwhelming advantages compared to systems relying solely on gain. Within PID control, three parameters—proportional, integral, and derivative—play distinct roles. The proportional term reduces rise time, the integral term eliminates steady-state error, and the derivative term minimizes overshoot and settling time [5]. By adjusting these three parameters, PID control enables achieving desired outcomes effectively.

4 Discussions and Conclusions

In this experiment, I designed a phase-lead compensator, state feedback controller, and PID controller for a second-order system and a third-order system. The phase-lead compensator is used to improve the system's transient response, while the state feedback controller and PID controller are used to improve the system's steady-state response. The experimental results show that the phase-lead compensator can effectively improve the system's transient response, while the state feedback controller and PID controller can effectively improve the system's steady-state response.

References

[1] Hu S S. (2001). Principles of Automatic Control (4th edition). Science Press.

[2] Li, Z. (2002). Design of Lead Compensator in MATLAB Environment. Computer and Modernization, (11), pp.73-75.

[3] Dorf, R.C. & Bishop, R.H. (2017), Modern control systems, Global 13th edn, Pearson, Harlow, England, pp. 757-764.

[4] Lu, S. & Hou, Y. (2002). Research on the performance parameters of second-order systems. Journal of Changchun University, (01), pp.1-4.

[5] He, Z. (2005). PID controller parameter tuning methods and its application research (Master's thesis). Zhejiang University, China.