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A survey of models and algorithms for winter road maintenance. Part I: system design for spreading and plowing

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Abstract

Winter road maintenance operations involve a host of decision-making problems at the strategic, tactical, operational, and real-time levels. Those operations include spreading of chemicals and abrasives, snow plowing, loading snow into trucks, and hauling snow to disposal sites. As the first of a four-part survey, this paper reviews optimization models and solution algorithms for the design of winter road maintenance systems for spreading and plowing operations. System design problems for snow disposal operations are discussed in the second paper. The two last parts of the survey mainly address vehicle routing, depot location, and fleet sizing models for winter road maintenance. The present paper surveys research on determining the level of service policy and partitioning a region or road network into sectors for spreading and plowing operations. We also describe the applied setting in which these problems arise.

Keywords: Winter road maintenance; Snow removal; Snow disposal; Snow hauling; Operations research

0. Introduction

Planning the operations of winter road maintenance involves a host of decision problems that can be addressed with operations research methodologies. The importance of these problems is obvious from

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the magnitude of the expenditures required to conduct winter road maintenance operations, as well as the indirect costs from both the lost productivity due to decreased mobility and the effects of chemicals (especially salt) and abrasives on infrastructure, vehicles and the environment. In the United States, winter road maintenance operations account for a substantial portion of annual operating budgets in many northern jurisdictions, costing state and local governments an estimated $2 billion each year [1]. Expenditures for winter road maintenance in Japan and Europe can be nearly two to three times higher than those in the United States [2]. In addition to the direct expenditures are substantial indirect costs associated with deterioration of infrastructure and vehicles, corrosion, water quality degradation, and other environmental impacts resulting from the use of chemicals for winter road maintenance. These indirect costs are estimated to total more than $5 billion a year in the United States [2]. Additional indirect costs incurred by motorists and businesses include accident costs, delay costs, and lost wages and productivity costs. These costs are difficult to quantify monetarily but can be significant in specific situations. For example, the “Blizzard of ‘96” (January 1996) in the northeastern United States shut down businesses (and federal government offices) and cost an estimated $20 billion in lost production and lost sales [3].

As highlighted by Kuemmel [4], in the past few decades there have been many important developments in winter road maintenance technologies to improve operations and minimize environmental impacts. These developments include use of alternative deicing materials, anti-icing methods, improved snow removal equipment, more accurate spreaders, better weather forecasting models and services, road weather information systems, etc. The microcomputer revolution has also had an immense impact on winter road maintenance practices by enhancing management systems, improving access to weather forecasting information, and aiding in the development of control systems in vehicles for spreading salt and other materials. These advances, and their growing use by state and local government agencies, have improved the effectiveness and efficiency of winter maintenance operations, benefiting government agencies, users, and the general public.

Unfortunately, advances in system design for winter road maintenance have not matched improvements in technology, as indicated by the relatively small number of contributions in the operations research literature. A study by Nixon and Foster [5] to determine the current state of practice in winter road maintenance in the United States found that several needs are not being met, particularly in effectively utilizing new technologies to better plan and manage winter road maintenance operations. This suggests that mathematical optimization techniques, leading to even a small increase in efficiency or effectiveness, could result in substantial savings, improved mobility, and reduced environmental and societal impacts.

In fact, the limited progress of operations research in winter road maintenance highlights the considerable difficulty of the problems studied. Problems facing winter road maintenance planners are especially complex and site specific because of the tremendous differences in initial conditions such as geography, meteorology, demographics, economics, and technology. Winter road maintenance operations are incredibly diverse as they are affected by population density, the topography of a region, the road network, the level of service policies, and the climatic factors of snowfall rate, temperature profile, hours of daylight, and wind. Differences in these conditions necessitate differences in the planning and operation of winter road maintenance.

Winter road maintenance presents a variety of decision-making problems at the strategic, tactical, operational, and real-time levels. The *strategic level* involves the acquisition or construction of long-lasting resources intended to be utilized over a long time period. Decisions related to the partitioning of a region or road network into sectors, the scheduling of fleet replacement, and the location of facilities such as snow disposal sites and vehicle depots may be viewed as strategic. The *tactical level* includes medium and short term decisions that are usually updated every few months. For example, the assignment of sectors to snow disposal sites and the sizing of vehicle fleets could be termed tactical. The *operational level* is related to the winter tasks that require ongoing attention on a day-to-day basis. Various decisions concerning the routing and scheduling of vehicles and the staffing of such vehicles with crews belong to the operational level. Finally, the *real-time level* involves decision-making situations in which operations must be undertaken or altered in a very short time frame (e.g., minutes) in response to the sudden change of the system (equipment breakdowns, weather change, etc.). The modification of routes based on new weather and pavement information is an example of real-time control. Some decision problems may be viewed at different levels according to the planning horizon considered. For example, decisions related to the assignment of sectors to snow disposal sites could be termed tactical since they are usually updated every winter season. However, these decisions belong to the operational level when monthly adjustments must be made to account for snowfall variability.

This paper is the first part of a four-part survey of optimization models and solution algorithms for winter road maintenance problems. The aim of the two first parts is to provide a comprehensive survey of optimization models and solution methodologies for the design of winter road maintenance systems. These problems include determining the level of service policy, partitioning a region or road network into sectors, locating winter maintenance facilities (snow disposal sites, vehicle depots, and materials depots), allocating sectors to snow disposal sites, allocating sectors to private companies or governmental agencies, and sizing and replacing vehicle fleets. In this paper, the contributions dealing with the level of service policy and the design of sectors with regard to spreading and plowing operations are reviewed. System design models for snow disposal operations are discussed in the second part of the survey [6]. The two last parts [7,8] mainly concentrate on vehicle routing, depot location, fleet sizing, and fleet replacement problems but several other important winter road maintenance problems are also considered.

The paper is organized as follows. Section 1 describes the techniques and operations of winter road maintenance, the system design problems related to spreading and plowing operations, particularly the level of service policy and the sector design problem, and the critical factors that have an impact on winter road maintenance operations. Models dealing with the level of service policy are reviewed in Section 2. Models that address the partitioning of a region or road network into sectors for spreading and plowing operations are described in Section 3. Conclusions and directions for future research are presented in the last section. The terms roadways, highways, roads, and streets are used interchangeably throughout the text with the intent that the models discussed in the survey are usually applicable to all levels of jurisdiction.

1. Winter road maintenance

The following section contains a brief description of winter road maintenance techniques and operations. A more detailed review of the available technology for winter road maintenance, and the scientific underpinnings of that technology, is presented in the book by Minsk [1]. For further details on winter road maintenance state-of-the-art technologies and processes in North America, Europe and Japan, see The American Association of State Highway and Transportation Officials [9], Pöyry [10], Kuemmel [4,11], and Nixon and Foster [5]. Following the description of winter road maintenance techniques and operations, this section describes the system design problems that have been addressed by operations researchers. This section concludes with a discussion of the operating conditions that have a strong impact on winter road maintenance operations.

1.1. Winter road maintenance operations

As was highlighted by Minsk [1], winter road maintenance techniques to clear a roadway of snow can be classified into three categories: chemical, mechanical, or thermal. Chemical methods include the application of a freezing-point depressant on a surface and incorporation of the freezing-point depressant within the surface itself. Chemicals are applied to pavements to melt ice that has formed on pavement (deicing), to prevent formation of ice (anti-icing), and to prevent the building of pack, namely snow compacted by traffic action that becomes nearly as tightly bonded to pavement as ice, and that is frequently much thicker and more irregular. Several chemicals are available, but salt is most commonly used because of its low cost, ready availability, ease of application, high solubility in water, and effectiveness as a melting agent at temperatures near 0 °C. The determination of the application rate depends on the level of service required, weather conditions, form (liquid or solid) and characteristics of the chemicals used, time of application, traffic density at the time of, and subsequent to, chemical application, as well as topography and the type of road surface.

The objective of mechanical removal is to pick up the snow that is loose or not bonded to the pavement surface from the road, shearing it from the road if necessary, and cast it to a storage area off the road. Mechanical methods include plowing and brooming. The role of snow plowing in either deicing or anti-icing operations is to remove as much snow and loose ice as possible before applying chemicals. Brooming can significantly reduce the need for chemical methods and is most effective on areas that receive little or no traffic between broomings.

Thermal methods involve applying heat to the roadway surface from either above or below to remove snow and ice or to prevent its formation. The purpose of heating a pavement is to reduce traffic delays, personal injury, and property damage from accidents caused by black ice, glaze ice, or packed snow. The cost of installing a fixed heating system and operating it, or of making and operating a mobile heating apparatus, is too high for general use. Critical locations where heating systems are usually installed include bridge decks, toll plazas, on and off ramps, and steep grades.

Chemicals assume a major role for winter road maintenance because of their effective performance and relatively low cost in comparison to alternatives. However, most of the adverse effects on infrastructure, vehicles and the environment from winter road maintenance operations stem from the use of salt and other materials for deicing. The main side effects (and indirect costs) of salting are motor vehicle corrosion, infrastructure damage, degradation of the roadside vegetation, and sodium infiltration of drinking water. Environmental problems arising from the use of ice control chemicals are reviewed by Minsk [1].

Sand and other abrasives are also commonly used in winter road maintenance. Although they are often mixed with salt and other chemicals for deicing purposes, abrasives are used primarily to improve traction, particularly when pavement temperatures are too low for chemical treatments to be effective. However, abrasives are not chemicals and neither prevent or break the bond between pavement and ice. According to Ketcham et al. [12], application of abrasives provides no significant increase in friction or improvement in pavement condition on a road receiving properly timed anti-icing treatments. Abrasives are inexpensive, but they can be difficult to apply and they have several potential negative consequences (damage to cars, need for additional highway cleanup operations, and airborne dust problems).

The response to winter precipitation on a road depends on the type and magnitude of the precipitation. If the frozen precipitation is snow of depth sufficient to impede pedestrian and vehicular traffic, plowing is the appropriate response. If the precipitation is ice or a thin snow layer, spreading of chemicals and abrasives may be the best method. Effective spreading of chemicals and abrasives at the proper time, in the proper manner, and at proper locations is a critical operation. The decision must take into account such factors as type of snow (wet or dry), expected temperature conditions at the time of, and following, application, anticipated variations at the critical freeze-thaw point, methods of application, and types of material. Ice may be removed from pavement through a combination of plowing, chemical treatment, natural melting, and traffic action. However, because ice adherence to pavement is often quite strong, removal by mechanical means becomes very difficult and often impossible, even with repeated passes by plows. Weakening the ice-pavement bond by heat or by a deicing chemical becomes necessary, so that the resulting ice sheets can then be removed mechanically by plowing or traffic action.

In urban areas, the large volumes of snow cleared from roads and walkways may exceed the available space along roadways and walkways for snow storage, and therefore require disposal by some means. Loading snow into trucks for hauling to disposal sites is the most common solution. Loading and hauling of snow are usually post-storm operations, although they may be required during the precipitation to remove snow from alleys, narrow channelled sections, and other areas where there is no space for snow storage. Snowblowers or rotary plows are commonly used to pick up snow to load into trucks. The trucks may be adjacent to, or in some cases following, the vehicle loading it, though adjacent trucks will further restrict traffic during the operation [1]. Hauled snow is often dumped into rivers and other water bodies, or on large open fields. Where there is sufficient flow in the sewer system and the volume of snow is not too great, dumping the snow into sewers is a practicable snow disposal method (this method is in use in Montréal, Canada). Generally, however, the snow must be melted or reduced to slush before it is discharged into a sewer system. Fixed melting pits are installed in some cities, notably in eastern Canada [1], or mobile snow melters can be transported to a problem area. Surface and underground flowing water systems are also used to carry snow away. In Sapporo, Japan, channels are constructed along streets, and river water or treated sewage diverted to them carries away snow dumped in them, rather than hauling the snow to disposal sites [13]. These snow disposal methods all present different costs and benefits. Some of the issues are haul distances, traffic patterns into and out of disposal areas, the effect of snow melt on stream flows, and the impact of contaminants in the snow and ice plowed from roadways on the environment when the accumulations melt and flow into surface waters or seep into the groundwater.

Many agencies have parking regulations to facilitate winter road maintenance operations. Examples are regulations that prohibit street parking at all times on designated snow routes, allow alternate side street parking, prescribe alternate times for parking, and ban overnight parking. These restrictions are often limited to critical roads and streets that carry large traffic volume during the winter months or only during a snow emergency. The principal advantage of the winter-long set of parking restrictions is that road maintenance operations can quickly begin after a storm without encountering delays waiting for a snow emergency declaration and for people to move their vehicles.

Not all jurisdictions perform all winter road maintenance operations each winter. Spreading of chemicals and abrasives and snow plowing are operations common to almost all road networks that experience significant snowfall or frozen precipitation. Snow loading and hauling are generally performed on a regular basis only in urban areas with large snowfalls and prolonged subfreezing temperatures. However, many metropolitan areas may undertake snow loading and hauling in response to infrequent but very heavy winter storms.

Table 1
Operating costs in the context of winter road maintenance

Costs	Operations		
	Spreading	Plowing	Loading and hauling
Variable costs	Fuel costs Crew costs Vehicle maintenance Material costs Variable costs of vehicle depots Variable costs of materials depots	Fuel costs Crew costs Vehicle maintenance Variable costs of vehicle depots	Fuel costs Crew costs Vehicle maintenance Variable costs of vehicle depots Variable costs of disposal sites
Fixed costs	Fixed costs of spreaders Fixed costs of vehicle depots Fixed costs of materials depots	Fixed costs of plowing equipment Fixed costs of vehicle depots	Fixed costs of loading and hauling equipment Fixed costs of vehicle depots Fixed costs of disposal sites

The fundamental operations in winter road maintenance involve a range of different benefits and cost. Safety effects and impacts on local economies that should be considered in valuing the benefits of winter road maintenance programs include savings in accident costs, in delay costs, and in lost wages and productivity costs. Adverse effects and indirect costs of winter road maintenance programs include the effects of deicing chemicals on the environment, infrastructure, and motor vehicles. The most direct cost of winter road maintenance is the expenditure required (generally of regional and local governments) to conduct winter road maintenance operations. Table 1 presents the major fixed and variable labor, materials, and equipment costs for each operation.

Since some vehicles, equipment and infrastructure may be shared among different winter road maintenance operations, and some may be used for non-winter operations, the allocation of costs for winter road maintenance is a challenging task. Over the years, many analytical models were proposed to help planners in properly allocating winter maintenance funds. For example, Miller [14] described a multiple regression model to predict labor, equipment and material costs for snow and ice control on various roadway types for each county of the Ohio state highway system for given snowfall amounts and traffic volumes. Dunlay [15] proposed a simultaneous equation stochastic model to explain or predict U.S. county expenditures for winter road maintenance based on selected measures of a county's need for spreading and plowing operations. Duaas et al. [16] described an analytical model to help planners at the directorate of public works in Norway secure a fair distribution of the national funds allocated to winter maintenance among the 19 counties. For each winter road maintenance operation and for each road section, the maintenance cost is calculated as a product of three elements. The first element is a quantity estimating the number of passes multiplied by the road length. The second element is the frequency of the maintenance operation during a year. The unit cost for providing personnel and equipment for the operation is the last element. The model also serves as a tool for analysing the impact of changes in factors that affect the unit cost. Finally, Thornes [17] reviewed the literature on the benefits and costs of the use of salt on roads and proposed a benefit/cost model for the salting of roads in the United Kingdom.

1.2. System design problems for spreading and plowing

System design in winter road maintenance includes determining the level of service policy, partitioning the geographic region into sectors for efficient operations, locating the needed facilities (vehicle depots, materials storage facilities, and disposal sites), assigning the sectors obtained from the partitioning to various facilities, allocating contracts for various operations to private organizations, and replacing and sizing the vehicle fleet. This section describes two system design problems related to spreading and plowing operations that have been addressed by operations research techniques: the level of service policy and the sector design problem. Operations research models for those two types of problems are reviewed in Sections 2 and Section 3, respectively. System design problems related to snow disposal operations are described in the second part of the survey [6].

The level of service for spreading and plowing operations is generally defined in terms of snow conditions on the roadway, including evenness and wetness, and pavement skid characteristics. These conditions and characteristics are controlled by the type and frequency of winter road maintenance operations. A high level of service is characterized by little snow accumulation before plowing, absence of an ice-pavement bond during precipitation, and a rapid return to near normal road surface conditions after the precipitation ends. A low level of service may involve snow plowing only once after the precipitation is over. In this case, the road surface may be very slippery and very uneven with possible projecting bumps, making passage difficult in some places. In general, the higher the level of service, the greater the resource investment. Since agencies have finite resources that generally do not allow the highest level of service on all roads, they must then prioritize their response efforts. The most common criterion for prioritizing response efforts is traffic volume. Typically, the roads of a network are partitioned into classes based on traffic volume which induce a *service hierarchy*, namely all roads carrying the heaviest traffic are given the highest level of service in order to provide safe roads for the greatest number of motorists, followed by medium-volume roads, and so on. *Class continuity* requires that each winter maintenance vehicle route be homogeneous in class to allow for a clear hierarchy in the importance of a particular route. In urban areas, streets serving bus routes, hospitals, firehouses, schools, and similar important places are often given a high priority. In rural areas, school bus routes also rate a high priority. Note that priority treatment locations for spreading operations may not be the same as priority plowing locations.

Because of the difficulty and impracticability of organizing winter road maintenance operations in a wide geographical region, the spreading and plowing operations are generally carried out concurrently by separate crews and equipment in many small subareas. The *sector design problem* consists of partitioning a large geographic region or road network into non-overlapping subregions or subnetworks, called *sectors*, according to several criteria related to the operational effectiveness and the geographical layout. It is often more convenient to partition a transportation network in a region instead of the region itself. In that case, the sectors correspond to subnetworks in the larger network. The number of sectors to design may be given or may be part of the design. Moreover, one or several physical facilities such as disposal sites, vehicle depots, or chemical and abrasive depots are generally specified.

Common criteria for designing sectors for winter road maintenance include *compactness* or *shape*, *balance in workload*, and *contiguity*. The compactness or shape criterion depends on the number of sectors, the number and type of facilities, and the type of winter road maintenance operations. If the number of sectors to be designed corresponds to the number of facilities, then compact sectors with centrally located facilities lead to more efficient routing of vehicles for winter road maintenance operations. However, if the number of sectors exceeds the number of facilities, then the appropriate shape of a set of sectors depends on the type of winter road maintenance operations. The general guideline for forming sectors for efficient routing of vehicles for chemical spreading and snow plowing is that sectors should be elongated towards the vehicle depot or materials depot to reduce travel distance in each route. Conversely, for loading and hauling snow to disposal sites, sectors should be elongated in a direction perpendicular to the direction to the disposal site to reduce the number of trucks required [18].

To balance the workload across sectors, they are often approximately the same size and are assigned equivalent resources (equipment and manpower). This helps ensure that operations will be completed at the same time in all sectors. The size of the sectors is usually determined by the level of service required and the operating capabilities of the equipment and manpower. Size and workload can be measured in terms of the number of basic entities, length of streets, or annual amount of snow. Basic entities or *basic units* are the units of analysis used to design sectors and can be defined either as single street segments or as small geographic zones. A micro approach to design sectors is to use a single street segment as the unit of analysis. However, for large road networks, this approach may lead to long computation times. One tractable approach is to define the basic units as geographic zones containing a collection of neighboring street segments. The contiguity criterion requires that sectors do not include distinct parts separated by other sectors. Non-contiguous sectors are undesirable from an administrative standpoint and from an operational standpoint given that deadheading trips would be necessary between the disjoint collections of street segments of each non-contiguous sector. *Deadheading* occurs when a vehicle must traverse a road segment without servicing it. Sector design may also need to conform to existing infrastructure (roadways, rail lines, bridges, etc.), geography (for example, rivers, hills, etc.), and jurisdictional boundaries. At a strategic planning level, the same sectors should usually be used for different types of winter road maintenance operations. Consequently, sectors should be robust and not influenced by minor changes in the characteristics of the operations performed within the sectors. Conversely, at the tactical and operational levels, different guidelines should be used to design sectors for spreading, snow plowing, loading trucks, and hauling snow to disposal sites.

1.3. Operating conditions

Many factors influence the conduct of winter road maintenance operations. Winter road maintenance methods vary widely according to geographical location and climatic conditions. Mechanical methods are the primary method used in North America whereas chemical treatment is the most effective in the British Isles, where the main problems arise from very light snowfalls or from thin layers of ice which form after water condenses on the roads and freezes. Also, in areas without prolonged sub-freezing temperatures, snow that is plowed from streets and sidewalks may quickly melt and need not be hauled away.

In addition to the climatic characteristics of the area, accurate and timely information on current weather conditions and forecasts of emerging conditions are critical to ensuring efficient and effective winter road maintenance operations. With access to good weather information, agencies can better time chemical treatments and more effectively deploy winter road maintenance equipment, which can lead to reducing expenditures and adverse environmental effects by eliminating unnecessary applications of salt and other chemicals and abrasives. For example, future warm weather may obviate the need for snow disposal, while a forecast for a second soon-to-arrive snowstorm may cause an agency to wait until after the subsequent storm to begin loading snow into trucks and hauling it to disposal sites.

Technological innovations can potentially improve the effectiveness and efficiency of winter road maintenance operations and, in some cases, reduce costs associated with that maintenance. Nixon and Foster

[5] present a review of new technologies related to winter road maintenance and methods for integrating these technologies into current practice. New technologies include materials application (deicing, anti-icing), mechanical methods, and weather information systems. Weather information capabilities such as road weather information systems, radiometers, and thermal mapping can lead to accurate lead-time for mobilization of resources to specific points of the road network. This results in less patrolling by maintenance crews, more timely spreading of chemicals and abrasives, and guidance on when winter road maintenance operations can begin to wind down. Geographical information systems and geographical positioning systems are also valuable tools for planning winter road maintenance operations. Geographical information systems can show what roads have been chemically treated or plowed, and geographical positioning systems can show the location of vehicles in real time. Advances in weather forecasting are also important and it is becoming more common to use "nowcasting" rather than forecasting to predict weather conditions. Nowcasting refers to the use of real-time information for short-term forecasting of the probable weather and pavement conditions or temperatures within one or two hours. Nowcasting is a valuable tool for deciding when to call-in and discharge personnel. A more detailed account of technology now available to provide real-time information about the microclimate of road segments to meteorologists can be found in the book by Minsk [1].

Winter road maintenance operations are increasingly influenced by legislative requirements. Agencies in some states and local jurisdictions in North America are required by law to curtail use of deicing chemicals. Many reports have appeared during the last several years describing the effects of deicing chemicals on soils, vegetation, and structural materials. The legislation has led to experimentation with reduced salting programs and alternative chemicals and application methods, including anti-icing. Demands that highway agencies reduce salt use to control adverse environmental effects have also raised concerns about the potential effects on service levels, safety, and operations.

Many other factors influence the conduct of winter road maintenance operations, including demographics, economics, interagency cooperation, traffic information, and resource information. A region with a large population and a number of high-traffic roads has different priorities and requirements, than a very rural region with low population density. Information such as the variation of traffic rate throughout a 24-hour period is important for making operational decisions. Plowing and spreading of chemicals during storms should be made prior to peak traffic intervals. Information on the status of personnel, equipment and materials provides the basis for decisions relative to treatment and resource allocation options such as reallocation of personnel and equipment, interagency agreements to provide increased capability, and identification of materials depots and disposal sites that can be accessed from neighboring agencies.

This diversity of operating conditions dictates the wide variety of winter road maintenance methods and operations. Fig. 1 summarizes the decisions and the operating conditions surrounding the winter road maintenance operations.

2. Service level models

The level of service for spreading and plowing operations is generally defined in terms of desired results, resources to be used, or both. As explained in Section 1.2, roadways are generally classified into several categories based on traffic volume and the level of service is actually a set of policies for each class of roadways. *Results-oriented policies* specify how different classes of roadway should appear after winter road maintenance operations, namely bare pavement, bare wheel paths, slush-covered roadways,

OPERATING CONDITIONS →	DECISIONS →	OPERATIONS
Geography	Level of service	Spreading of chemicals and abrasives
Climate	Snow disposal assignment	Snow plowing
Meteorology	Contract assignment	Loading snow into trucks
Demographics	Sector design	Hauling snow to disposal sites
Economics	Vehicle depot location	
Technology	Materials depot location	
Environment	Snow disposal site location	
Legislation	Fleet sizing and replacement	
Interagency cooperation	Crew assignment	
Traffic information	Crew scheduling	
Resource information	Vehicle routing	

Fig. 1. Winter road maintenance operating conditions, decisions, and operations.

or snow-packed roadways. *Resource-oriented policies* specify the level of equipment usage, chemical and abrasive application rates, and hours of coverage that should be provided on each roadway class. Agencies may also have service policies that specify both desired results and resources to be used according to roadway class. For example, level of service guidelines may define several highway types based on traffic levels, each with different equipment coverage times and recommended pavement conditions. The fundamental tradeoff in establishing level of service policies is between minimizing the total costs of winter road maintenance, including the direct and indirect costs, and maximizing the benefits of winter road maintenance for safer travel and smaller delays.

Since level of service policies constitute an important strategic component of winter road maintenance operations (affecting sector design, depot location, and fleet sizing), models developed to assess and quantify level of service policies for spreading and plowing operations are reviewed in this section. Analytical and optimization models dealing primarily with resource-oriented policies are reviewed first, followed by optimization models that address results-oriented policies.

2.1. Resource-oriented models

Several models were proposed to assess and quantify resource-oriented level of service policies by road segment or highway class. Early work was contributed by Ross and Miller [19] who proposed a multiple regression model to predict the maximum time to service a road for given values of storm duration, snowfall amount, traffic volume, temperature, percentage of daylight, wind velocity, route priority, and time interval between start of storm and initiation of maintenance operation on the road. Lindsey and Seely [20] described a multicriteria approach to the problem of classifying the road segments of a network into various categories for snow plowing. Each road segment is evaluated on six criteria: average annual snowfall, estimated highest elevation of the road, annual average daily traffic, pavement type, functional classification (interstate, national highway system or state road), and number of lanes. A weighted additive multicriteria value is calculated for each road segment, and these values are grouped into separate categories. Each category covers a specific range of values, and all road segments falling into a single category receive the same level of service. A method to survey citizens to assess the appropriate level of service was described by Hayashiyama et al. [21]. Results are based on how much respondents indicate they would pay to achieve an improved level of service, and how much compensation they would require if the level of service was reduced. Finally, Adams et al. [22] described a comprehensive set of resource-oriented performance measures implemented in the state of Wisconsin, United States, for analyzing winter road maintenance level of service policies, for evaluating performance of materials, labor, and equipment, and for developing reliable evidence of compliance with standards and policies. The measures are computed from data collected by differential global positioning system receivers and sensors on winter maintenance vehicles.

Sage [23] proposed an optimization model to determine a cost minimizing service level. Let x be the snow removal rate expressed as centimeters per hour. Let b and m represent two delay cost parameters (detailed in [24]), and let h and g represent two winter road maintenance cost parameters estimated by simulation. Define also $L_{\max}$ as the maximum allowable service level. The optimization model is a nonlinear programming model stated as follows.

$$\text{Minimize} \quad bx^m + h \ln x + g, \tag{2.1}$$

$$\text{subject to} \quad 0 < x \leqslant L_{\max}. \tag{2.2}$$

The objective function (2.1) seeks to minimize the sum of the total cost defined by delay costs for travel and winter road maintenance costs. Constraint (2.2) requires that variable x assumes a positive value not larger than the maximum allowable service level. Sage [23] gave the second-order sufficient conditions for a strict relative minimum service level of the total cost function.

In order to balance the workload among various facilities or sectors, Decker et al. [25] developed a measure of a maintenance facility's efficiency in performing winter road maintenance operations. The approach is to normalize winter road maintenance costs for labor, equipment, and spreading materials for a specific maintenance facility by considering the number of lane-kilometers of given service levels in the service area and a storm severity index. The storm severity index is calculated from weighted daily snowfall totals and minimum and maximum temperatures.

Decision support systems have also been developed to help planners in determining the appropriate level of service. For example, McBride [26] described a decision support system to aid planners in the selection of level of service for a specific storm, based on incremental cost-benefit ratio analysis. The benefits considered when choosing a high level of service over a lower one are defined by the cost savings of traffic delay and safety. The additional costs required to achieve the higher level of service are defined by the additional equipment costs, material costs and manpower costs. Also, Keranen [27] described an operations management system to help planners in the Minneapolis-St. Paul metropolitan area in evaluating times to regain bare pavement according to three highway types based on traffic levels (annual average daily traffic). For each class of roads, thresholds for acceptable "regain" times for bare pavement are defined based on previous data on regain times, decision rules dictated by experience, sample surveys of citizens, and weather conditions. The system, coupled with data from road weather information systems, is used for both planning and controlling winter maintenance operations by providing guidelines for service level performance.

Some agencies have level of service policies that emphasize both desired results and resources to be used. For example, Roussel [28] described service policies implemented in France that contain both performance and coverage guidelines. The country has defined thresholds for acceptable snow levels and pavement conditions during a snowstorm, and times to regain bare pavement. Also, Baroga [29] proposed two performance measures that emphasize both desired resources and results for rating the

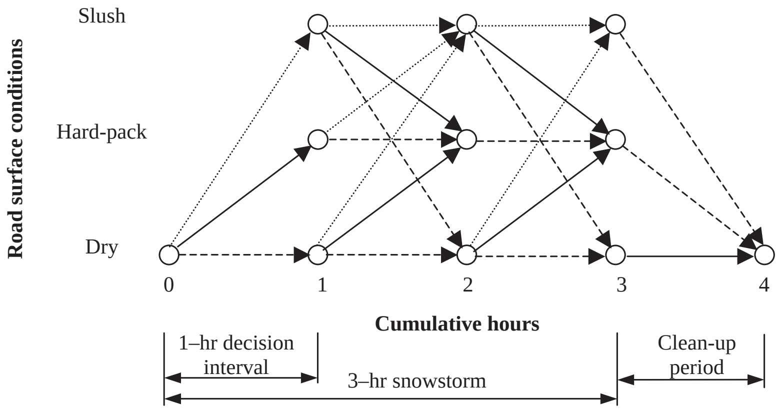

Fig. 2. Possible road surface condition (states) for each stage.

level of service provided on a road segment: the condition of the road segment and the elapsed time from the end of precipitation to attainment of bare pavement. Scores for different pavement conditions and different hour thresholds are aggregated to assess the service level delivered on each road segment.

2.2. Results-oriented models

Ungerer [30] described a solution approach to select the sequence of winter road maintenance operation alternatives that gives the lowest maintenance cost over a storm duration planning horizon considering the given snowstorm type and roadway class, while ensuring that the road surface condition be dry by the end of the storm. The type of a snowstorm depends on many variables such as snowfall rate, storm duration, snow accumulation on the pavement, temperature, and visibility. The problem is modeled as a deterministic dynamic program. In this model, a stage is defined as the cumulative total number of hours completed and the decision variable at a given stage is the winter road maintenance operation to begin. The states associated with each stage are defined in terms of the road surface condition. Thus, the model tends to be results-oriented. Fig. 2, taken from Ungerer [30], provides an acyclic graph depicting the possible road surface condition (states) for each stage of a winter storm of three hour duration and one clean-up period. At each node, at most three decisions can be imposed. These decisions, which are "do nothing", "spread sand", and "plow snow", use arcs represented by full lines, short dotted lines, and long dotted lines, respectively. With every arc in the network is associated a cost which depends upon the cumulative number of hours completed and whether dry, hard-pack or slush-covered pavement states are associated with the two arc endpoints. The initial pavement condition and final desired state are both specified as dry.

Every path in the network from the starting node (initial state) at stage 0 to the end node (final state) at stage 4 specifies a strategy indicating which winter road maintenance operation to begin in each time interval. Since the cost of a strategy sums the cost at each stage, the total cost corresponds to the length of a path from the starting node to the ending node. The minimum-cost strategy then is just the shortest path.

The author also discussed the use of the model in a multiobjective arena where many possibly conflicting or competing objectives need to be considered such as the minimization of maintenance cost, road user cost or congestion, and the maximization of vehicle volume-to-capacity ratio or safety index. The multiobjective version of the strategy selection problem is solved using compromise programming. Let I be the set of objectives. For each individual objective $i \in I$, the best shortest paths (or longest paths as appropriate) in the acyclic directed network associated with the strategy selection problem are identified first. Let P be the set of paths. The set $J \subseteq P$ of noninferior strategies is then identified. For each objective $i \in I$, define λ_i and z_i^* as the nonnegative weight for objective i, $\sum_{i \in I} \lambda_i = 1$, and the normalized length of a shortest path (or a longest path as appropriate) in the acyclic network in terms of the individual objective i, respectively. For each objective $i \in I$ and for each strategy $j \in J$, define z_{ij} as the normalized length of path j in terms of objective i in the acyclic network. For each strategy $j \in J$, let D_j be the square root of the total weighted sum of square deviations for strategy j calculated according to (2.3). The strategies with the smallest criteria D_j represent the best compromise strategies. The method was tested on a small hypothetical problem based on the network of Fig. 2.

$$D_j = \sqrt{\sum_{i \in I} \lambda_i^2 (z_i^* - z_{ij})^2}. \tag{2.3}$$

In the same paper, Ungerer [30] extended his solution methodology for simultaneously deciding the optimal strategy for various expected annual roadway classes and snowstorm types subject to a budgetary constraint as well as material, equipment and manpower availabilities. This strategic problem is formulated as a 0–1 linear integer program with an objective function that maximizes the total hourly vehicular traffic rate, which is defined as the total number of vehicles per hour circulating through a section of each roadway class during each snowstorm type. This objective function is weighted to provide superior maintenance to certain roadway-storm combinations, such as high-volume roads during severe storms. Let I be the set of roadway-storm combinations. For every roadway-storm combination $i \in I$, let G_i be the acyclic directed network associated with the strategy selection problem for roadway-storm combination i, and let S_i be the set of noninferior strategies identified by solving the multiobjective problem in the associated network G_i. The multiobjective version of the strategy selection problem is solved using the method of compromise programming described above. Also, for every roadway-storm combination $i \in I$, define W_i as the weight associated with roadway-storm combination i. For every roadway-storm combination $i \in I$ and for every strategy $j \in S_i$, let x_{ij} be a binary variable equal to 1 if and only if strategy j is assigned to roadway-storm combination i. Also, for every roadway-storm combination $i \in I$ and for every strategy $j \in S_i$, let M_{ij}, c_{ij}, g_{ij}, and n_{ij} represent the hourly vehicular traffic rate, the maintenance cost per lane-kilometer per storm, the material utilization per lane-kilometer per storm and the number of workers per snowplow and spreader of applying strategy j to roadway-storm combination i, respectively. Let K be the set of regions considered. For each roadway-storm combination $i \in I$ and for each region $k \in K$, define e_{ik} and l_{ik} as the expected yearly number of storms and the number of roadway lane-kilometers associated with roadway-storm combination i in region k, respectively. For every roadway-storm combination $i \in I$, for every strategy $j \in S_i$, and for every region $k \in K$, let t_{ijk} be the number of snowplows and spreaders required per lane-kilometer per storm of applying strategy j to roadway-storm combination i in region k. Finally, let C, G, T, and N be the annual budget level, the annual material availability, the number of snowplows and spreaders available per storm, and the number of workers available per storm, respectively. The extended model for the strategy selection problem can be stated

1. *Strategy selection phase*
   a. For each roadway-storm combination $i \in I$, find the maximum-vehicular traffic rate strategy by solving a longest path problem in G_i. If the resource availabilities are satisfied, then STOP. The strategy assignment is optimal.
   b. For each roadway-storm combination $i \in I$, select the best compromise strategy $j \in S_i$ with the smallest criteria D_j.
   c. If the resource availabilities are satisfied, go to step 2. Otherwise, go to step d.
   d. Order the criteria from the largest to the smallest and select the roadway-storm combination i with assigned strategy j at the top of the criteria list.
   e. Select the best compromise strategy $s \in S_i$, $s \neq j$, $D_s \geq D_j$. If such a strategy exists, reassign strategy s to roadway-storm combination i and return to step c. Otherwise, move down the criteria list and repeat step e.
2. *Improvement phase*
   a. For each roadway-storm combination i with assigned strategy j, let $c(i) = \max_s \{W_iM_{is} - W_iM_{ij} \mid j, s \in S_i, s \neq j\}$ represent the best objective function improvement realized by reassigning strategy s to roadway-storm combination i while satisfying resource availabilities.
   b. Select the roadway-storm combination i such that $c(i) = \max_{l \in I} \{c(l)\}$. If $c(i) > 0$, reassign strategy s to roadway-storm combination i.
   c. Return to the beginning of step 2 until no improvement is obtained.

Fig. 3. The two-phase heuristic for the strategy selection problem [30].

as follows:

$$\text{Maximize} \quad \sum_{i \in I} \sum_{j \in S_i} W_i M_{ij} x_{ij}, \tag{2.4}$$

$$\text{subject to} \quad \sum_{j \in S_i} x_{ij} \leqslant 1 \quad (i \in I), \tag{2.5}$$

$$\sum_{i \in I} \sum_{j \in S_i} \sum_{k \in K} e_{ik} l_{ik} c_{ij} x_{ij} \leqslant C, \tag{2.6}$$

$$\sum_{i \in I} \sum_{j \in S_i} \sum_{k \in K} e_{ik} l_{ik} g_{ij} x_{ij} \leqslant G, \tag{2.7}$$

$$\sum_{i \in I} \sum_{j \in S_i} \sum_{k \in K} l_{ik} t_{ijk} x_{ij} \leqslant T, \tag{2.8}$$

$$\sum_{i \in I} \sum_{j \in S_i} \sum_{k \in K} l_{ik} t_{ijk} n_{ij} x_{ij} \leqslant N, \tag{2.9}$$

$$x_{ij} \in \{0, 1\} \quad (i \in I,\ j \in S_i). \tag{2.10}$$

The objective function (2.4) maximizes the total weighted hourly vehicular traffic rate. Constraints (2.5) require that at most one strategy be applied to each roadway-storm combination. Annual budget level and annual material availability are respected via constraints (2.6) and (2.7), respectively. Constraints (2.8) and (2.9) ensure that the number of snowplows, spreaders and workers available per storm be respected. Finally, all x_{ij} variables are restricted to be binary.

This model was applied to a very small instance and solved with the two-phase heuristic detailed in Fig. 3. The first phase finds an initial, possibly infeasible, solution by assigning the best compromise strategy to each roadway-storm combination. A feasible solution is then obtained by iteratively considering the next best compromise strategy for the roadway-storm combination with the largest criteria that satisfies the resource availabilities. Note that if the maximum-vehicular traffic rate strategies found by solving a longest path problem in each of the $|I|$ acyclic directed networks associated with the roadway-storm combinations constitute a feasible solution to model (2.4)–(2.10), then this solution is optimal. The second phase is an exchange procedure that tries to improve the solution by iteratively identifying the combination-strategy pair that yields the best improvement. This is done as long as the total value of the objective function (2.4) increases.

3. Sector design models for spreading and plowing

The design of sectors consists in partitioning a region or transportation network into a mutually exhaustive and exclusive collection of small sectors according to several criteria related to the operational effectiveness and the geographical layout. As explained in Section 1.2, each sector must be a contiguous collection of basic units, balanced in workload, and appropriately shaped according to the operations. Several criteria may also be used to assess the quality of the sector design. One important criterion is that physical facilities such as materials and vehicle depots should be centrally located (relative to sectors they serve) for efficient routing of spreaders and plows. Kandula [31] studied the impact of the location of depots on deadheading. Computational experiments indicated that centrally located depots appear to result in routes with less deadheading than non-centrally located depots. Alternatively, a depot may merely serve as a home base, but not necessarily as a sector center. Other commonly used criteria for sector design include: the unicursality of the graph generated by the arcs and edges of each sector to enable routes with less deadheading, the minimization of the fleet size, the assignment of both directions of a two-way segment to the same sector, respect of natural boundaries, respect of some existing administrative or political subsectors, and similarity to the existing network partitioning plan. Typically, sector design problems involve medium and long-term planning decisions. Thus, sector design plans may have long-term consequences on the overall efficiency and effectiveness of the operations to be performed within the sectors.

The sector design problem in spreading and plowing operations is similar to the arc partitioning problem studied by Bodin and Levy [32] in the context of postal delivery. The sector design problem also shares several characteristics with districting problems for arc routing applications such as the design of service regions among one or more vehicle depots when each vehicle can visit several clients in a tour [33–35] and the design of sectors for refuse collection [36–38]. In comparison with other districting applications such as political districting, sales territory alignment, health care districting, school district design, and emergency services, the problem of districting in connection with vehicle routing for collection or distribution services has received very little attention. One of the main difficulties is to incorporate aspects related to the efficiency of vehicle routing within the solution method. This difficulty is also present in sector design problems for winter road maintenance operations.

Optimization models for sector design can be grouped into categories according to the winter road maintenance operations considered. Sector design models for spreading operations are reviewed first, followed by compound sector design models that address the sector design, vehicle depot location, and fleet sizing for both spreading and plowing operations. Compound models that integrate sector design, fleet sizing, and snow disposal assignment decisions for loading trucks and hauling snow to disposal sites are discussed in the second paper [6].

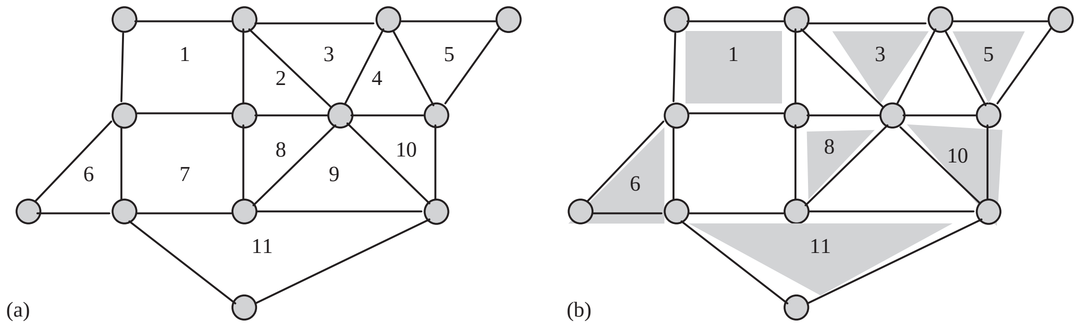

Fig. 4. Example of seven cycles designated on an Eulerian graph: (a) Eulerian graph G', (b) Checkerboard pattern.

3.1. Sector design models for spreading

Muyldermans et al. [39] proposed a solution method to design sectors for spreading operations that produces contiguous, balanced and geographically compact subnetworks with centralized depots. The objective is to minimize the number of spreader trucks, while ensuring that the graph generated by the edges of each sector is Eulerian to enable tours with less deadheading. The locations of the depots are given and they coincide with nodes of an undirected road network G. Furthermore, every edge of G has a length and a weight corresponding to the demand for chemicals or abrasives. The authors proposed a four-phase composite heuristic that builds all sectors simultaneously by assigning basic entities represented by small cycles to the depots. The number of sectors to be designed equals the number of depots. The first phase corresponds to the cycle decomposition approach suggested by Male and Liebman [36] in the context of districting and routing for solid waste collection. If G is not even, then edges of least total cost are added to G to create an even graph. As shown by Edmonds and Johnson [40], these edges can be determined by solving a perfect matching problem on an auxiliary graph whose vertices are the odd-degree vertices of G. The basic entities are then determined by partitioning the Eulerian graph G' into small cycles using a "checkerboard pattern" to obtain a set of faces with associated cycles as illustrated in Fig. 4.

In the second phase, the sector building process is initiated by assigning first the cycles that are adjacent to a depot and next, the cycles that are very close to a depot. Only eligible cycles are assigned to ensure the contiguity of the partially built sectors. A cycle is *eligible* for a certain depot if it is not yet assigned to a depot, and if it has at least one node in common with a cycle already assigned to that depot. Isolated cycles are then checked for and assigned to the nearest depot. In the third phase, all sectors are built simultaneously in order to balance the total workload. The first part of the third phase operates as a simple bin packing heuristic and is reminiscent of the partitioning algorithm of Levy and Bodin [41]. The second part of the third phase uses a multicriteria approach for the assignment of the remaining cycles. Each candidate cycle-to-depot assignment is evaluated on three criteria: sector balance, compactness and fleet size. All criteria are treated through the calculation of a weighted additive multicriteria score for each cycle-depot pair. The candidate pair with the lowest score is then assigned. In the fourth phase, cycle shifts or interchanges between the depots are performed so as to decrease the number of required spreader trucks, while satisfying the contiguity constraints. No specific exchange procedure was developed. However, the authors suggested a user interactive procedure to allow sectors to be adjusted to incorporate topographic, climatic or other constraints that are difficult to quantify. Users

1. *Preprocessing*
   a. Partition the road network into elemental cycles. Let J be the set of cycles and let I be the set of depots or sectors.
   b. For each cycle $j \in J$, let CN_j represent the set of nodes on cycle j. For every depot $i \in I$ and for every node $k \in CN_j$, define D_{ik} as the length of a shortest path from depot i to node k. For each depot $i \in I$ and for each cycle $j \in J$, calculate the ratio R_{ij} as follows:

$$R_{ij} = \frac{\overline{D}_{ij}}{\min_{\substack{i' \in I \\ i' \neq i}} \{\overline{D}_{i'j}\}} \text{ where } \overline{D}_{ij} = \frac{\sum_{k \in CN_j} D_{ik}}{|CN_j|}$$

2. *Initial partial assignment*
   a. For each cycle $j \in J$, if cycle j is adjacent to a depot, then assign cycle j to that depot.
   b. For each depot $i \in I$ and for each eligible cycle $j \in J$, if $R_{ij} \leq 0.5$, then assign cycle j to depot i.
   c. For each isolated cycle $j \in J$, assign cycle j to the nearest depot.
3. *Two-part iterative assignment*
   Part I
   a. Select the sector $i \in I$ with the smallest workload assigned to it.
   b. For each cycle $j \in J$, let the cycle weight CW_j represent the amount of chemicals or abrasives to service the roads in cycle j. For each depot $i \in I$, let BR_i be a ratio such that $0.5 < BR_i < 1$. Select the eligible cycle $j \in J$ with the largest weight such that $R_{ij} < BR_i$. If no such cycle is available, go to Part II. Otherwise, assign cycle j to sector i.
   c. For each isolated cycle $j \in J$, assign cycle j to the nearest depot. Return to the beginning of Part I.
   Part II
   d. For each depot $i \in I$, select the largest weight eligible cycle $j \in J$ with $R_{ij} < BR_i$. If no such cycle is available for a depot i, select the eligible cycle j with the smallest ratio R_{ij}.
   e. For every depot $i \in I$, define CD_i as the set of cycles in sector i. Define also Q as the vehicle capacity and N as an estimate for the number of vehicles required in the final partition calculated as follows:

$$N = \sum_{i \in I} \left\lceil \frac{\sum_{j \in CD_i} CW_j}{Q} \right\rceil$$

   f. For each candidate cycle-depot pair, let N_1 and N_2 be two estimates for the number of vehicles when assuming that the cycle-to-depot assignment is carried out and dividing the remaining unassigned cycles over the sectors such that N is minimized and maximized, respectively.
   g. Let S_1, S_2, and S_3 be the score of the compactness measure, the sum of both estimates for the number of vehicles required and the score of the balance measure of the partially built sectors, respectively. Let also $W_1 = 1$, $W_2 = 3$, and $W_3 = 2$ be three weights. For each candidate cycle j depot i pair, calculate the score $S_{ij} = W_1S_1 + W_2S_2 + W_3S_3$ where $S_1 = 0$ if $\max\{BR_i, R_{ij}\} \leq BR_i$, $S_1 = 1$ if $BR_i < \max\{BR_i, R_{ij}\} \leq 1$, $S_1 = 2$ if $1 < \max\{BR_i, R_{ij}\} \leq 1.5$, $S_1 = 3$ if $1.5 < \max\{BR_i, R_{ij}\} < 2$, $S_2 = N_1 + N_2$, $S_3 = 0$ if the addition of cycle j to depot i decreases the imbalance, and $S_3 = 1$ if the assignment increases the imbalance.
   h. Select the assignment with the smallest total score and assign cycle j to sector i.
   i. For each isolated cycle $j \in J$, assign cycle j to the nearest depot. Return to the beginning of Part II.
4. *Improvement and user interaction*
   Shift or interchange cycles between the depots so as to decrease the number of required trucks, while ensuring the contiguity of the sectors.

Fig. 5. The four-phase heuristic for the sector design problem [39].

can also modify parameter values, determine new basic entities and define new criteria and weights in the multicriteria approach. The four-phase heuristic procedure is presented in detail in Fig. 5.

The heuristic was tested on a real network from the province of Antwerp in Belgium with 244 edges, 154 nodes, and four depots. The new sectors found by the heuristic were not as well balanced as those

1. Define the basic entities in G by the cycle decomposition approach and the exchange heuristic. Construct the adjacency graph H.
2. Let sc_{ij} be the length of the shortest chain linking vertex v_i to vertex v_j in G. For every vehicle depot $w_i \in W_{\mathrm{D}}$ and for every cycle $w_j \in W_{\mathrm{C}}$, define D_{ij} as the sum of shortest deadheading distances in G for servicing each edge of cycle c_j separately from depot v_i. For every vehicle depot $w_i \in W_{\mathrm{D}}$ and for every cycle $w_j \in W_{\mathrm{C}}$, calculate the ratio R_{ij} as follows:

$$R_{ij} = \frac{D_{ij}}{\min_{\substack{v_k \in X \\ v_k \neq v_i}} \{D_{kj}\}} \quad \text{where } D_{ij} = \sum_{(v_r, v_s) \in c_j} (sc_{ir} + sc_{is})$$

3. If all cycles $w_j \in W_{\mathrm{C}}$ are assigned, go to step 5.
4. For each vehicle depot $w_i \in W_{\mathrm{D}}$, select the unassigned cycle $w_j \in W_{\mathrm{C}}$ with the lowest ratio value R_{ij}, adjacent to w_i or to a cycle w_k already assigned to vehicle depot w_i. Among these candidate depot-cycle assignments (w_i, w_j), allocate the cycle to the vehicle depot in the pair with the lowest R_{ij} value. Return to step 3.
5. Translate the cycle allocations in H into a sector design in G.

Fig. 6. Greedy heuristic for the sector design problem [43].

actually used by the province. However, solving the capacitated arc routing problem for spreader trucks for these new sectors with the Pearn [42] augment-insert algorithm allowed deadheading cost savings of about 14% over the solution produced with the same algorithm in the sectors actually used by the province.

In a subsequent paper, Muyldermans et al. [43] suggested and compared three heuristics for generating contiguous, balanced, and geographically compact sectors with centralized vehicle depots for spreading operations and road maintenance. The first two heuristics differ only in the definition of the basic units. Let $G = (V, E)$ be a connected, undirected, and planar graph with vertex set V and edge set E. With every edge $(v_r, v_s) \in E$ are associated a positive length c_{rs} and a positive demand q_{rs} for service. Define $X \subset V$ as the set of vehicle depots, each depot housing identical vehicles with a finite capacity Q. The first heuristic aggregates individual edges in E into sectors by allocating them to the nearest vehicle depot. The second heuristic aggregates groupings of edges into sectors in the same greedy manner as the first heuristic. Groupings of edges in G are defined by the cycle decomposition approach described previously by Muyldermans et al. [39] and an edge exchange heuristic [44] that we have not described in detail here. Let $H = (W, F)$ be an adjacency graph such that the vertex set W has a vertex w_j for each cycle and for each vehicle depot in the Eulerian graph G' and the edge set F contains an edge f_{ij} between w_i and w_j if the corresponding cycles c_i and c_j in G' have a vertex in common or if the vehicle depot associated with w_i is located on the cycle represented by w_j. Let $W_{\mathrm{C}} \subset W$ and $W_{\mathrm{D}} \subset W$ be two vertex subsets corresponding to cycles and vehicle depots, respectively. For every vertex $w_j \in W$, define the constant $q(w_j)$ equal to the total demand on cycle c_i if and only if $w_j \in W_{\mathrm{C}}$ and zero otherwise. The second heuristic is described in Fig. 6.

In the third heuristic, basic units obtained by the cycle decomposition are allocated to the vehicle depots through the solution of a linear mixed integer program. Prior to solving the model, the adjacency graph H is reduced in size. The idea is to allocate immediately the cycles that are considered as very near the depots, and further to merge some cycles by exploiting structural properties in H. The adjacency graph reduction is described in detail by Muyldermans et al. [43]. To present the formulation, the following notations apply. For every cycle $w_j \in W_C$ and for every vehicle depot $w_i \in W_D$, let x_{ij} be a binary variable equal to 1 if and only if cycle w_j is assigned to depot w_i. For every vehicle depot $w_i \in W_D$, let y_i be a nonnegative integer variable corresponding to the continuous lower bound $\Sigma_i \lceil q(E_i)/Q \rceil$ on the number of vehicles scheduled from w_i, with $q(E_i)$ being the total demand in G serviced by vehicle depot w_i. Muyldermans [44] showed that by partitioning G into p sectors, in the worst case this lower bound can grow to $\lceil q(E)/Q \rceil + p - 1$ where $q(E)$ is the total demand in G. For each vehicle depot $w_i \in W_D$, let $R_{\max,i}$ be a nonnegative variable representing the maximum R_{ij} value among the cycles assigned to depot w_i, and let $H_i = (W_i \cup \{w_i\}, F_i)$ be the subgraph induced by w_i and the cycles $w_j \in W_i$ that can be assigned to vehicle depot w_i. A cycle w_j can be assigned to a vehicle depot w_i if $R_{ij} \leqslant R_{\lim,i}$ where $R_{\lim,i}$ is a limit value for the ratios R_{ij} associated to depot w_i. For each cycle $w_j \in W_C$, let W_j be the set of vehicle depots that can receive cycle $w_j (R_{ij} \leqslant R_{\lim,i})$. Finally, for every cycle $w_j \in W_C$ and for every vehicle depot $w_i \in W_D$, let $W_{ij} = \{w_k \in W_i : n_i(w_j) = n_i(w_k) + 1$ and $(w_k, w_j) \in F_i\}$ be the set of vertices in H_i to express the sector connectivity when cycle $w_j \in W_i$ is assigned to depot w_i with $n_i(w_j)$ corresponding to the minimum number of edges needed for reaching w_j from w_i in H_i. The formulation is then:

$$\text{Minimize} \quad \sum_{w_i \in W_D} y_i + \alpha \sum_{w_i \in W_D} R_{\max,i}, \tag{3.1}$$

$$\text{subject to} \quad \sum_{w_i \in W_j} x_{ij} = 1 \quad (w_j \in W_C), \tag{3.2}$$

$$\sum_{w_j \in W_i} q(w_j)x_{ij} + q(w_i) \leqslant Q y_i \quad (w_i \in W_D), \tag{3.3}$$

$$x_{ij} \leqslant \sum_{w_k \in W_{ij}} x_{ik} \quad (W_{ij} \neq \emptyset, w_i \in W_D, w_j \in W_i), \tag{3.4}$$

$$R_{ij} x_{ij} \leqslant R_{\max,i} \quad (w_i \in W_D, w_j \in W_i), \tag{3.5}$$

$$x_{ij} \in \{0, 1\} \quad (w_i \in W_D, w_j \in W_i), \tag{3.6}$$

$$y_i \geqslant 0 \text{ and integer} \quad (w_i \in W_D), \tag{3.7}$$

$$R_{\max,i} \geqslant 0 \quad (w_i \in W_D). \tag{3.8}$$

The objective function (3.1) minimizes the sum of the lower bounds on the number of vehicles to be used from the vehicle depots and the ratio values of the most distant cycles that are assigned to the depots. Minimizing these ratio values penalizes the non-compactness of each sector. The scale factor α is chosen suitably small in order to make the contribution of the second term in (3.1) less than one so that it does not affect the minimum value of the first term. Constraints (3.2) require that each cycle be assigned to exactly one vehicle depot. The total capacity of the vehicles is respected at any vehicle depot via constraint set (3.3). Constraints (3.4) ensure network connectivity within a sector. For each vehicle depot $w_i \in W_D$, the cycles $w_j \in W_i$ are partially ordered according to the minimum number of edges $n_i(w_j)$ needed for reaching w_j from w_i in H_i. Constraints (3.4) require that at least one cycle of the set W_{ij} be allocated to w_i, before w_j can be assigned to w_i. Constraints (3.5) state that the maximum ratio value within each sector must be greater than the ratio value of any cycle in the sector and the vehicle depot to which it is assigned. Finally, all x_{ij} variables are restricted to be binary, while y_i and $R_{\max,i}$ variables must assume nonnegative integer values and nonnegative values, respectively. Two simplified versions of this model are also proposed. In the first version, the second term in the objective function

(3.1) is removed as well as constraint sets (3.5) and (3.8). In the second version, the objective function (3.1) and constraint sets (3.5) and (3.8) are modified by replacing the variables $R_{\max,i}$ by a single variable $R_{\max}$. Some valid inequalities are added to the formulations and the resulting models are solved by the standard branch-and-bound algorithm of CPLEX 6.5. For details, see Muyldermans [44].

The three heuristics were tested on a large set of graphs constructed from the road network in Flanders (Belgium) with up to 27 vehicle depots and 1692 edges. The larger test problem required 598 vertices in the adjacency graph H defined by the cycle decomposition and 216 cycles after reducing H. The quality of the partitions generated by the three heuristics is evaluated by solving capacitated arc routing problems in the sectors by a local search heuristic [45] and by comparing the solution values with a cutting plane lower bound based on the supersparse formulation for the capacitated arc routing problem [46]. The authors concluded that the first heuristic is most effective when Q is very small, whereas the second heuristic performs better for average Q values. The three versions of the linear mixed integer program based heuristic are particularly suitable when Q is large, but these require more time than the other heuristics.

3.2. Compound sector design, depot location, and fleet sizing models for spreading and plowing

As was highlighted by Cattrysse et al. [47] and Van Oudheusden et al. [48], there are strong interactions between the design of sectors, the location of disposal sites and depots, and the routing of vehicles for plowing, spreading, loading, and hauling operations. However, these interdependent problems are most often solved separately. Typically, disposal sites are first located, sectors are then designed and assigned to disposal sites, and routes are determined last. Obviously, this sequential approach may lead to suboptimal decisions at all planning levels. Compound models reviewed in this section integrate the three components of sector design, depot location and fleet sizing for winter road maintenance.

Kandula and Wright [49] proposed a linear mixed integer programming formulation for the combined sector design, depot location and fleet sizing problem. Their model addresses the combined problem for both spreading and plowing operations in a largely rural area with a sparse road network, but can also serve to model other districting problems in the context of arc routing. A solution to their model indicates the depot sites to open, the assignment of each road segment to an opened depot as well as the number of vehicles required to service simultaneously all road segments. Thus, each sector includes one opened depot at which a number of vehicles are based. The formulation incorporates contiguity and compactness constraints for each sector. Service hierarchy, class continuity for each vehicle, and constraints which place different limits on sector sizes are also enforced. The objective considered is a surrogate compactness measure.

Let $G = (V, E)$ be a connected undirected graph where $V = \{v_1, v_2, \ldots, v_n\}$ is the vertex set and $E = \{(v_i, v_j) : v_i, v_j \in V \text{ and } i \neq j\}$ is the edge set. With every edge (v_i, v_j) are associated a nonnegative length c_{ij} and a positive number of circulation lanes l_{ij}. Let sc_{ij} be the length of the shortest chain linking vertex v_i to vertex v_j in G. Let $D \subset V$ be a set of potential depot sites. For every depot $v_d \in D$, define dhf_d as the deadhead factor used for road segments associated with depot $v_d (dhf_d \geqslant 1$ for all $v_d \in D)$, cap_d as the maximum number of kilometers assigned to depot v_d, and $sumsc_d$ as the limit on the sum of the lengths of the shortest chains from depot v_d to both ends of road segments that are assigned to depot v_d. For every edge $(v_i, v_j) \in E$ and for every potential depot site $v_d \in D$, let x_{ijd} be a binary variable equal to 1 if and only if edge (v_i, v_j) is assigned to depot v_d. For every potential depot site $v_d \in D$, let y_d be a binary variable equal to 1 if and only if depot site v_d is opened.

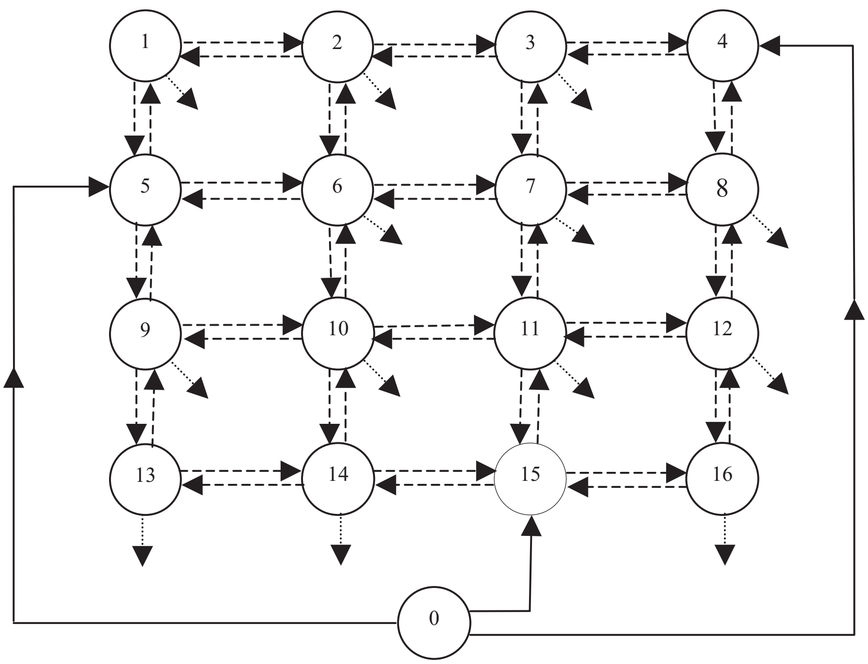

Fig. 7. Example of network G' with three potential depots v_4, v_5 and v_{15}.

Let $P_K = \{E_1, E_2, \ldots, E_K\}$ be a partition of E with $E_1 \cup E_2 \cup \ldots \cup E_K = E$ and $E_i \cap E_j = \emptyset$ for all $i, j \in \{1, 2, \ldots, K\}$, $i \neq j$. For every depot $v_d \in D$ and every class $E_k \subseteq P_K$, let n_{kd} be a nonnegative integer variable representing the number of vehicles based at depot v_d to service edges of class E_k assigned to depot v_d. For every depot $v_d \in D$ and every class $E_k \subseteq P_K$, define cl_{kd} as the maximum number of class k kilometers assigned to depot v_d. For every class $E_k \subseteq P_K$, define f_k as the frequency of service in hours that must be provided to road segments of class k. The vehicle speed, expressed as kilometers per hour, is denoted by s.

Kandula and Wright modeled the contiguity constraints as a circulation multi-commodity network flow problem with supplementary variables and constraints. Each commodity corresponds to a potential depot site and shares the same directed graph $G' = (V \cup \{v_0\}, A_1 \cup A_2 \cup A_3)$ constructed from G where v_0 is an artificial vertex and A_1, A_2 and A_3 are three sets of arcs defined as follows. The arc set A_1 contains arcs of opposite direction for each edge (v_i, v_j) in E, $A_2 = \{(v_0, v_i) : v_i \in D\}$ and $A_3 = \{(v_i, v_0) : v_i \in V \backslash D\}$. For every depot $v_d \in D$ and every arc $(v_i, v_j) \in A_1 \cup A_2 \cup A_3$, let w_{ijd} be a nonnegative real variable representing the flow on arc (v_i, v_j) assigned to depot v_d. An example of network G' with three potential depots v_4, v_5 and v_{15} is illustrated in Fig. 7. The three sets of arcs A_1, A_2 and A_3 are shown as dashed lines, solid lines, and dotted lines, respectively. For reasons of clarity, the arcs of A_3 are not shown as incoming arcs of vertex v_0.

Finally, define *numv* as the maximum number of vehicles to be used, *numd* as the maximum number of depots to be operative and *sumsc* as the limit on the sum of the lengths of the shortest chains from operative depots to both ends of road segments that are assigned to these depots. We present here a slightly simplified version of the Kandula and Wright formulation for the combined sector design, depot location and fleet sizing problem (we eliminate some variables used by Kandula and Wright to clarify the interpretation of results).

$$\text{Minimize} \quad \sum_{v_d \in D} \sum_{(v_i,v_j) \in E} (sc_{id} + sc_{jd}) x_{ijd}, \tag{3.9}$$

$$\text{subject to} \quad \sum_{(v_i,v_j) \in E_k} l_{ij} c_{ij} x_{ijd} \leqslant cl_{kd} \quad (v_d \in D, E_k \in P_K), \tag{3.10}$$

$$n_{kd} \geqslant \frac{dhf_d cl_{kd}}{f_k s} \quad (v_d \in D, E_k \in P_K), \tag{3.11}$$

$$\sum_{v_d \in D} \sum_{E_k \in P_K} n_{kd} \leqslant numv \tag{3.12}$$

$$\sum_{v_d \in D} y_d = numd \tag{3.13}$$

$$\sum_{(v_i,v_j) \in E} l_{ij} c_{ij} x_{ijd} \leqslant cap_d y_d \quad (v_d \in D), \tag{3.14}$$

$$\sum_{v_d \in D} x_{ijd} = 1 \quad ((v_i, v_j) \in E), \tag{3.15}$$

$$w_{ijd} \leqslant M x_{ijd} \quad ((v_i, v_j) \in E, v_d \in D), \tag{3.16}$$

$$w_{jid} \leqslant M x_{ijd} \quad ((v_i, v_j) \in E, v_d \in D), \tag{3.17}$$

$$\sum_{\{v_j : (v_i,v_j) \in A_1 \cup A_3\}} w_{ijd} - \sum_{\{v_j : (v_j,v_i) \in A_1\}} w_{jid} = 0 \quad (v_i \in V \backslash D, v_d \in D), \tag{3.18}$$

$$\sum_{\{v_j : (v_i,v_j) \in A_1\}} w_{ijd} - \sum_{\{v_j : (v_j,v_i) \in A_1 \cup A_2\}} w_{jid} = 0 \quad (v_i \in D, v_d \in D), \tag{3.19}$$

$$\sum_{v_d \in D} w_{ijd} \geqslant 1 \quad ((v_i, v_j) \in A_3), \tag{3.20}$$

$$\sum_{v_d \in D} \sum_{(v_i,v_j) \in A_2} w_{ijd} \geqslant |V \backslash D|, \tag{3.21}$$

$$w_{ijd} = 0 \quad ((v_i, v_j) \in A_2, v_d \in D, v_j \neq v_d), \tag{3.22}$$

$$x_{ijd} \leqslant w_{i0d} \quad ((v_i, v_j) \in E, v_d \in D), \tag{3.23}$$

$$x_{ijd} \leqslant w_{j0d} \quad ((v_i, v_j) \in E, v_d \in D), \tag{3.24}$$

$$(sc_{id} + sc_{jd}) x_{ijd} \leqslant sumsc \quad ((v_i, v_j) \in E, v_d \in D), \tag{3.25}$$

$$\sum_{(v_i,v_j) \in E} (sc_{id} + sc_{jd}) x_{ijd} \leqslant sumsc_d \quad (v_d \in D), \tag{3.26}$$

$$w_{ijd} \geqslant 0 \quad ((v_i, v_j) \in A_1 \cup A_2 \cup A_3, v_d \in D), \tag{3.27}$$

$$n_{kd} \geqslant 0 \text{ and integer} \quad (v_d \in D, E_k \in P_K), \tag{3.28}$$

$$x_{ijd} \quad \text{and} \quad y_d \in \{0, 1\} \quad ((v_i, v_j) \in E, v_d \in D). \tag{3.29}$$

The objective function (3.9) minimizes the sum of all lengths of the shortest chains between operative depots and both ends of road segments in G. Constraints (3.10) impose an upper bound on the number of kilometers of each roadway class assigned to each depot. Constraints (3.11) impose a lower bound on the number of vehicles based at every depot for each class. Equipment availability is respected at all depots for all roadway classes via constraints (3.12). Constraint sets (3.13)–(3.15) are identical to those of the P-median problem except that a capacity constraint is included. Constraint (3.13) imposes the number of depots to operate. The capacity constraints (3.14) link the location variables (y_d) and the allocation variables (x_{ijd}). They assure that each edge is assigned to a depot that is operative. Constraints (3.15) require each edge to be assigned to exactly one depot. Constraints (3.16)–(3.24) guarantee that each sector is contiguous. The linking constraints (3.16) and (3.17) ensure that the flow on arc (v_i, v_j) or arc (v_j, v_i) assigned to depot v_d is positive if the edge (v_i, v_j) is assigned to that depot. M is a sufficiently large positive number. Flow conservation at every node, except node v_0, for each potential depot is imposed by constraints (3.18) and (3.19). Constraints (3.20) assure that every non-depot node is part of at least one sector. Constraint (3.21) imposes a lower bound on the number of flow units out of vertex v_0 so that every vertex is part of at least one sector. Constraints (3.22) assure that every sector includes exactly one depot (operative or not). The linking constraints (3.23) and (3.24) ensure that an edge (v_i, v_j) is assigned to a depot v_d if both ends v_i and v_j of that edge are part of the sector including that depot. Constraints (3.25) and (3.26) assure that every sector is compact. Finally, all w_{ijd} variables must assume nonnegative values and all n_{kd} variables must assume nonnegative integers values, while x_{ijd} and y_d variables are restricted to be binary.

Kandula and Wright used the CPLEX Mixed Integer Optimizer with barrier code (version 2.1) to solve model (3.9)–(3.29) with a set of data from the La Porte district of Indiana with three priority classes of roadways and four depots on a network of 63 vertices and 79 edges. This gave 1554 variables and 2295 constraints. The optimal solution provided sectors that were more compact than the sectors in use by the district. However, the number of vehicles used increased by 16%, probably on account of the class continuity constraints. Moreover, the computational effort to solve the model was significant with 178 branch-and-bound nodes and 5780 iterations. To accelerate the computational time, the contiguity constraints (3.16)–(3.24) and the integrity constraints on the x_{ijd} variables were relaxed. Although the new solution was not integer, the resulting sectors were contiguous, with the exception of only one. Computational experiments performed by Kandula [31] on five other regions of Indiana with up to 62 nodes, 73 edges and three potential depots showed that the mixed integer model and its relaxation resulted in the same sectors in all cases.

In a subsequent paper, Kandula and Wright [50] extended the original model to introduce cost characteristics related to route configuration such as service and deadhead costs for road segments. This is done by introducing new variables and new constraint sets, as well as by modifying the objective function and some constraints. The directed graph G' constructed from G to formulate the contiguity constraints as a circulation multi-commodity network flow problem is augmented by introducing an arc from each depot node $v_d \in D$ to the artificial vertex v_0. For every depot $v_d \in D$ and every arc $(v_i, v_j) \in A_1 \cup A_2 \cup A_3 \cup A_4$ where A_1, A_2 and A_3 are defined as above and $A_4 = \{(v_i, v_0) : v_i \in D\}$, the flow variable w_{ijd} representing the flow on arc (v_i, v_j) assigned to depot v_d is now expressed as time units. With each edge $(v_i, v_j) \in E$ are now associated two traversal times: ts_{ij} is the time for servicing edge (v_i, v_j) and td_{ij} is the traversal time of (v_i, v_j) no matter if it has already been serviced or not. However, the service hierarchy of the network is no longer considered. Then, variable n_{kd} representing the number of vehicles based at depot v_d to service edges of class E_k assigned to depot v_d is now replaced by variable n_d representing the number of vehicles based at depot v_d. For every edge $(v_i, v_j) \in E$ and for every potential depot site $v_d \in D$, let q_{ijd} be a binary variable equal to 1 if and only if edge(v_i, v_j) assigned to depot v_d is traversed from

v_i to v_j. For every depot $v_d \in D$ and every edge $(v_i, v_j) \in E$, let p_{ijd} be a nonnegative real variable representing the number of deadhead time units traversing edge (v_i, v_j) assigned to depot v_d from v_i to v_j. For every depot $v_d \in D$, let $SMAX_d$ be the maximum sum of the lengths of the shortest chains from operative depot *d* to both ends of a road segment that is assigned to this depot. Finally, define $numv_d$ as the maximum number of vehicles based at depot v_d and *t* as the average time to service edges on one vehicle route. The time *t* does not include the deadheading time. The formulation is given next.

$$\begin{aligned}
\text{Minimize} \quad & \sum_{v_d \in D} SMAX_d + \sum_{v_d \in D} \sum_{(v_i,v_j)\in E} (p_{ijd} + p_{jid}) \\
& + \sum_{v_d \in D} \sum_{(v_i,v_j)\in A_1 \cup A_3 \cup A_4} w_{ijd} + \sum_{v_d \in D} n_d, && (3.30)\\
\text{subject to} \quad & \sum_{(v_i,v_j)\in E} (sc_{id} + sc_{jd}) x_{ijd} \leqslant SMAX_d \quad (v_d \in D), && (3.31)\\
& \sum_{v_d \in D} x_{ijd} = 1 \quad ((v_i, v_j) \in E), && (3.32)\\
& x_{ijd} \leqslant y_d \quad ((v_i, v_j) \in E, v_d \in D), && (3.33)\\
& \sum_{v_d \in D} y_d = numd && (3.34)\\
& (sc_{id} + sc_{jd}) x_{ijd} \leqslant sumsc \quad ((v_i, v_j) \in E, v_d \in D), && (3.35)\\
& n_d \geqslant \frac{\sum_{(v_i,v_j)\in E} l_{ij} c_{ij} x_{ijd}}{ts} \quad (v_d \in D), && (3.36)\\
& n_d \leqslant numv_d \quad (v_d \in D), && (3.37)\\
& w_{ijd} = t n_d \quad ((v_i, v_j) \in A_2, v_d \in D), && (3.38)\\
& w_{ijd} = 0 \quad ((v_i, v_j) \in A_2, v_d \in D, v_j \neq v_d), && (3.39)\\
& \sum_{v_d \in D} w_{ijd} \geqslant 0.05 \quad ((v_i, v_j) \in A_3 \cup A_4), && (3.40)\\
& x_{ijd} \leqslant 100 w_{i0d} \quad ((v_i, v_j) \in E, v_d \in D), && (3.41)\\
& x_{ijd} \leqslant 100 w_{j0d} \quad ((v_i, v_j) \in E, v_d \in D), && (3.42)\\
& w_{ijd} \leqslant M x_{ijd} \quad ((v_i, v_j) \in E, v_d \in D), && (3.43)\\
& w_{jid} \leqslant M x_{ijd} \quad ((v_i, v_j) \in E, v_d \in D), && (3.44)\\
& \sum_{v_d \in D} (w_{ijd} + w_{jid}) \geqslant \frac{ts_{ij}}{2} \quad ((v_i, v_j) \in E), && (3.45)\\
& w_{ijd} \leqslant M q_{ijd} \quad ((v_i, v_j) \in E, v_d \in D), && (3.46)\\
& w_{jid} \leqslant M q_{jid} \quad ((v_i, v_j) \in E, v_d \in D), && (3.47)\\
& \sum_{v_d \in D} (q_{ijd} + q_{jid}) = 1 \quad ((v_i, v_j) \in E), && (3.48)\\
& \sum_{\{v_j:(v_i,v_j)\in A_1 \cup A_3\}} w_{ijd} - \sum_{\{v_j:(v_j,v_i)\in A_1\}} w_{jid} = 0 \quad (v_i \in V \backslash D, v_d \in D), && (3.49)
\end{aligned}$$

$$\sum_{\{v_j:(v_i,v_j)\in A_1\cup A_4\}} w_{ijd} - \sum_{\{v_j:(v_j,v_i)\in A_1\cup A_2\}} w_{jid} = 0 \quad (v_i \in D, v_d \in D), \tag{3.50}$$

$$w_{ijd} \leqslant t + p_{ijd} \quad ((v_i, v_j) \in E, v_d \in D), \tag{3.51}$$

$$w_{jid} \leqslant t + p_{jid} \quad ((v_i, v_j) \in E, v_d \in D), \tag{3.52}$$

$$w_{ijd} \geqslant \sum_{\left\{\substack{v_k:(v_k,v_i)\in E \\ \text{or}(v_i,v_k)\in E}\right\}} \left(\frac{ts_{ki}}{2} x_{kid}\right) + \sum_{\{v_k:(v_k,v_i)\in E\}} \left(\frac{l_{ki} td_{ki}}{t} p_{kid}\right) \quad ((v_i, v_j) \in A_3 \cup A_4, v_d \in D), \tag{3.53}$$

$$w_{ijd} \geqslant 0 \quad ((v_i, v_j) \in A_1 \cup A_2 \cup A_3 \cup A_4, v_d \in D) \tag{3.54}$$

$$p_{ijd} \geqslant 0 \quad ((v_i, v_j) \in E, v_d \in D), \tag{3.55}$$

$$n_d \geqslant 0 \text{ and integer} \quad (v_d \in D), \tag{3.56}$$

$$x_{ijd}, y_d \quad \text{and} \quad q_{ijd} \in \{0, 1\} \quad ((v_i, v_j) \in E, v_d \in D). \tag{3.57}$$

In this formulation, the objective function (3.30) minimizes the total sum of the maximum sums of the lengths of the shortest chains from operative depots to both ends of road segments that are assigned to these depots, the total deadheading time, the total number of time units of all the commodities on each arc, except arcs of set A_2, and the total number of vehicles used. Minimizing the total flow of all the commodities on each arc ensures splitting of flows at nodes near the vehicle depots, so that most deadheading occurs on edges close to the vehicle depots. Constraints (3.31) state that the maximum sum of the lengths of the shortest chains between every operative depot and both ends of a road segment that is assigned to the depot must be greater than the sum of the lengths of the shortest chains from the depot to both ends of any road segment that is assigned to the depot. Constraint sets (3.32)–(3.34) are identical to those of the P-median problem. Constraint set (3.32) requires each edge to be assigned to exactly one depot. Constraints (3.33) state that an edge can only be assigned to a depot if it is operative. Constraint (3.34) states that exactly *numd* depots are to be located. Constraint set (3.35) is identical to its counterparts (3.25) of the model (3.9)–(3.29). Constraint sets (3.36) and (3.37) impose lower and upper bounds on the number of vehicles based at every depot. Constraints (3.38)–(3.53) ensure that each sector is contiguous. Constraint sets (3.38) and (3.39) assure that every sector includes exactly one depot (operative or not). Constraint sets (3.40)–(3.42) are identical to their respective counterparts (3.20), (3.23), (3.24) of the model (3.9)–(3.29), except that the constraint set (3.40) is now defined on the augmented graph G' and the right-hand sides of (3.40)–(3.42) are modified since flow units are now expressed as time units. Constraint sets (3.43) and (3.44) are identical to their respective counterparts (3.16) and (3.17) of the model (3.9)–(3.29). Constraint set (3.45) imposes lower flow bounds on all pairs of arcs of opposite direction associated with each edge. The time for servicing an edge is divided between the endpoints of that edge. Constraint sets (3.46)–(3.48) assure that the flow may be positive for exactly one depot on only one of the two arcs associated with each edge. Flow conservation at every node of the augmented graph G' for each disposal site is imposed by constraints (3.49)–(3.50). For each commodity, constraint sets (3.51)–(3.52) state that for every pair of arcs associated with each edge, any flow in excess of average time required to service edges on one vehicle route must be due to deadheading. For each commodity, constraint set (3.53) states that the total outflow of any node of the augmented graph G' must at least correspond to half of the time spent in servicing all edges assigned to the depot and incident to that node at the specified service speed, as well as deadhead travel into that node.

Again, the model was solved using CPLEX. Numerical experiments performed on five regions of Indiana with up to 62 nodes, 73 edges and three potential depots showed that the sectors created in every region were more compact than the existing sectors. However, the computation times were rather long with up to 1927 branch-and-bound nodes and 55,673 iterations. As mentioned by Kandula and Wright [50], the computation time may be reduced by exploiting the spatial nature of the problem. Each edge is considered a candidate for assignment to all sectors. Nevertheless, in practice, there are likely to be very few reasonable alternatives for each edge assignment. Fixing such obvious variables at zero or one, or eliminating some variables might help to limit the size of the branch-and-bound search tree and therefore reduce the computation time. The quality of the configuration of the sectors was also evaluated on the basis of the quality of the vehicle routes produced in each sector with both a lower-bound based composite heuristic suggested by Kandula and Wright [50] and a tabu search heuristic developed by Wang et al. [51]. Results indicated that the routes produced in the new sectors satisfied more constraints, such as service time intervals and class continuity, were less numerous and had fewer deadhead miles than the routes determined in the existing sectors. Kandula [31] compared the two models solved using CPLEX and showed that the extended model (3.30)–(3.57) produced better results than the basic model (3.9)–(3.29) in half the real-life instances described above, but required excessive computing time. Kandula [31] also demonstrated how the basic and extended models may be used to address location issues such as opening, closing, or relocating depots, as well as repartitioning areas of a network where depots are located near the boundaries so as to improve compactness.

4. Conclusions

This paper is the first of a four-part survey of optimization models and solution algorithms for winter road maintenance problems. (The second part of the survey [6] discusses system design models for snow disposal operations. The two last parts of the review [7,8] mainly address vehicle routing, depot location, and fleet sizing models for winter road maintenance problems.)

This paper addresses the level of service policy and the sector design problem for spreading and plowing operations. This represents an important part of the system design planning performed by regional and local government agencies. Table 2 summarizes the characteristics of the service level optimization models and the sector design models related to spreading and plowing operations.

The level of service policy related to spreading and plowing operations is usually given as an input in system design or vehicle routing models. Several analytical models (multiple regression and cost-benefit analyses) were proposed to assess and quantify the resource-oriented level of service policy for spreading and plowing operations. For the results-oriented policy, deterministic dynamic programming based algorithms, such as those proposed by Ungerer [30], are promising optimization solution methods. Multiobjective analysis is also showing much promise to assist planners in making results-oriented policy decisions. However, the service level policy should ideally be determined endogenously in system design and vehicle routing models to identify the most efficient way of, for example, designing sectors while satisfying a set of technological constraints on contiguity, size or workload, and compactness or shape. Future research directions in service level planning for spreading and plowing operations should thus be oriented towards the development of new mathematical formulations that fully integrate the results

Table 2
Characteristics of service level and sector design models for spreading and plowing

Authors	Problem type	Planning level	Problem characteristics	Objective function	Model structure	Solution method
Sage [23]	Resource-oriented service level	Strategic	Snow removal rate and maximum service level	Min delay and maintenance costs	Nonlinear	Analytical
Ungerer [30]	Results-oriented service level	Operational	Storm duration and surface road condition	Min maintenance costs	Shortest path	Dynamic programming
Ungerer [30]	Results-oriented service level	Operational	Storm duration and surface road condition	Multi-objective	Shortest paths	Compromise programming
Ungerer [30]	Results-oriented service level	Strategic	Service hierarchy, storm types, and resource availabilities	Max weighted vehicular traffic rate	Linear 0-1 IP	Composite heuristic
Muyldermans et al. [39]	Sector design for spreading	Strategic	Contiguity, compactness, balanced sectors, grouping of street segments, fixed depot location, and fixed number of sectors	Min spreader fleet size and deadheading	Assignment problems	Composite heuristic
Muyldermans et al. [43]	Sector design for spreading	Strategic	Contiguity, compactness, balanced sectors, grouping of street segments, vehicle capacity, fixed depot location, and fixed number of sectors	Min spreader fleet size and deadheading	Assignment problems and linear MIP	Construction heuristics
Kandula and Wright [49]	Combined sector design, depot location, and fleet sizing for spreading and plowing	Strategic	Contiguity, compactness, service hierarchy, class continuity, maximum sector sizes, and fixed number of depots and sectors	Min total distance	Linear MIP	CPLEX Mixed Integer Optimizer
Kandula and Wright [50]	Combined sector design, depot location, and fleet sizing for spreading and plowing	Strategic	Contiguity, compactness, and fixed number of depots and sectors	Min maximum distances, fleet size, and deadheading	Linear MIP	CPLEX Mixed Integer Optimizer

of service level models with the design of systems or the routing of vehicles for spreading and plowing operations.

The design of sectors for spreading and plowing operations is closely linked to the location of vehicle and materials depots and the routing of spreaders and plows. However, the design of sectors is most commonly treated as a separate problem. Very frequently, sectors are designed by assuming that depot location decisions are given. The traditional approach for the sector design problem has thus consisted in partitioning the road network into sectors by assigning basic units to their closest depot. Muyldermans et al. [39,43] used this approach for designing sectors for spreading and plowing operations. Since the quality of the vehicle routes produced in each sector is highly dependent on the size and shape of the sectors, this approach obviously leads to suboptimal routing decisions. A better approach could consist of designing sectors following the development of the vehicle routing plans similar to the “route first” approaches for the solution of multiple vehicle node routing problems [52].

In addition to the development of new sequential approaches, the sector design problem can be incorporated in compound models that address the integration of sector design with other decisions related to spreading and plowing operations. Models that integrate multiple interdependent subcomponents of the planning process can significantly help to improve benefits and reduce costs of spreading and plowing operations. The compound sector design, depot location, and fleet sizing models proposed by Kandula and Wright [49,50] are good examples of integrated models.

In short, as new mathematical formulations and solution strategies are developed for the service level policy and the sector design problem, the challenge of the future is to build broader models that address the integration of various subcomponents of the planning process related to spreading and plowing operations.

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References

[1] Minsk LD. Snow and ice control manual for transportation facilities. New York: McGraw-Hill; 1998.

[2] Transportation Research Board. Winter maintenance technology and practices—learning from abroad. NCHRP Research Results Digest 204. Washington, DC: Transportation Research Board; 1995.

[3] Lord B. Federal highway administrator in an address to the Eastern Winter Road Maintenance Symposium Equipment Expo. Washington, DC, 1996.

[4] Kuemmel DE. Managing roadway snow and ice control operations. NCHRP Synthesis of Highway Practice No 207. Washington, DC, 1994.

[5] Nixon WA, Foster NSJ. Strategies for winter highway maintenance. Report, Iowa University, 1996.

[6] Perrier N, Langevin A, Campbell JF. A survey of models and algorithms for winter road maintenance. Part II: system design for snow disposal. Computers & Operations Research 2004, doi:10.1016/j.cor.2004.07.007

[7] Perrier N, Langevin A, Campbell JF. A survey of models and algorithms for winter road maintenance. Part III: vehicle routing and depot location for spreading. Computers & Operations Research, 2004, forthcoming.

[8] Perrier N, Langevin A, Campbell JF. A survey of models and algorithms for winter road maintenance. Part IV: vehicle routing and fleet sizing for plowing and snow disposal. Computers & Operations Research, 2004, forthcoming.

[9] American Association of State Highway and Transportation Officials. Guide for snow and ice control. Washington, DC: The Association; 1999.
[10] Pöyry T. Technical exchange in the European northern periphery to identify best practices in winter maintenance. In: Proceeding of XIth International Winter Road Congress, New Challenges for Winter Road Service, France: World Road Association (PIARC); 2002.
[11] Kuemmel DA. Report on the 1998 scanning review of European winter service technology. NCHRP Research Results Digest No 238, Washington, DC, 1999.
[12] Ketcham SA, Minsk LD, Blackburn RR, Fleege EJ. Manual of practice for an effective anti-icing program: a guide for highway winter maintenance personnel. Report No FHWA-RD-95-202, Washington, DC, 1996.
[13] City of Sapporo. Snow-Sapporo-21 Project. City of Sapporo, Japan, 1991.
[14] Miller E. Models for predicting snow-removal costs and chemical usage. In: Proceedings of an International Symposium. Special Report No 115, Washington, DC: Highway Research Board; 1970. p. 267–78.
[15] Dunlay WJ. A simultaneous equation econometric model of winter maintenance cost categories. Highway Research Record 1971;359:37–48.
[16] Duaas LI, Gabestad KO, Solberg KA. A method for allocating funds for winter maintenance. In: Proceedings of Ninth PIARC International Winter Road Congress. Wien, Austria: Bundesministerium fur wirtschaftliche Angelegenheiten; 1994. p. 382–8.
[17] Thornes JE. UK road salting—an international benefit/cost review. Highways & Transportation 1999;46:22–4.
[18] Labelle A, Langevin A, Campbell JF. Sector design for snow removal and disposal in urban areas. Socio-Economic Planning Sciences 2002;36:183–202.
[19] Ross MA, Miller EL. Statistical approach to establishment of maintenance levels. Highway Research Record 1971;347: 113–23.
[20] Lindsey RK, Seely MS. Resource allocation study for snow removal. Transportation Research Record 1999;1672:23–7.
[21] Hayashiyama Y, Tanabe S, Hara F. Economic evaluation of snow-removal level by contingent valuation method. Transportation Research Record 2001;1741:183–90.
[22] Adams TM, Danijarsa M, Martinelli T, Stanuch G, Vonderohe A. Performance measures for winter operations. Transportation Research Record 2003;1824:87–97.
[23] Sage JD. The use of synthetic snowfalls in the evaluation of snow removal systems. Transportation Research Board 1979;184:121–8.
[24] Russell C. An application of the adjustment model. In: Baumann DD, Russell C, editors. Urban snow hazard: economic and social implications. Urbana: University of Illinois; 1971.
[25] Decker R, Bignell JL, Lambertsen CM, Porter KL. Measuring efficiency of winter maintenance practices. Transportation Research Record 2000;1741:167–75.
[26] McBride JC. Economic impact of highway snow and ice control. Transportation Research Record 1978;674:58–63.
[27] Keranen PF. Optimization of winter maintenance in the Minneapolis-St. Paul metropolitan area using performance targets. In: Proceedings of XIth International Winter Road Congress on new challenges for winter road service. France: World Road Association (PIARC); 2002.
[28] Roussel JJ. The new French methodology for quality objectives in winter maintenance. In: Ninth PIARC International Winter Road Congress. Wien, Austria: Bundesministerium fur wirtschaftliche Angelegenheiten; 1994. p. 174–80.
[29] Baroga EV. Performance measures for snow and ice control activities. In: Proceedings of the Ninth Maintenance Management Conference. Washington, DC: Transportation Research Board; 2001. p. 75–80.
[30] Ungerer RP. Decision models for winter highway maintenance. Final Report No. FHWA/NY/SR-89/95, New York State Department of Transportation, 1989.
[31] Kandula LNP. Network partitioning for edge-oriented route design. Ph.D. Dissertation, Purdue University, West Lafayette, IN, 1996.
[32] Bodin L, Levy L. The arc partitioning problem. European Journal of Operational Research 1991;53:393–401.
[33] Wong KF, Beasley JE, Vehicle routing using fixed delivery areas. Omega, The International Journal of Management Science 1984;12:591–600.
[34] Novaes AGN, Graciolli OD. Designing multi-vehicle delivery tours in a grid-cell format. European Journal of Operational Research 1999;199:613–34.
[35] Golden BL, Wasil EA. Computerized vehicle routing in the soft drink industry. Operations Research 1987;35:6–17.

[36] Male JW, Liebman JC. Districting and routing for solid waste collection. Journal of the Environmental Engineering Division ASCE 1978;104(EE1):1–14.
[37] Silva Gomes AC. Sur la collecte des ordures ménagères: le problème de sectorisation. Thesis, National Institution for Pure and Applied Science, Grenoble, France, 1983.
[38] Hanafi S, Freville A, Vaca P. Municipal solid waste collection: an effective data structure for solving the sectorization problem with local search methods. INFOR 1999;37:236–54.
[39] Muyldermans L, Cattrysse D, Van Oudheusden D, Lotan T. Districting for salt spreading operations. European Journal of Operational Research 2002;139:521–32.
[40] Edmonds J, Johnson EL. Matching, Euler tours and the chinese postman. Mathematical Programming 1973;5:88–124.
[41] Levy L, Bodin L. The arc oriented location routing problem. INFOR 1989;27:74–94.
[42] Pearn WL. Augment-insert algorithms for the capacitated arc routing problem. Computers Operations & Research 1991;18:189–98.
[43] Muyldermans L, Cattrysse D, Van Oudheusden D. District design for arc-routing applications. Journal of the Operational Research Society 2003;54:1209–21.
[44] Muyldermans L. Routing, districting and location for arc traversal problems. Ph.D. Thesis, Katholieke Universiteit Leuven, Belgium, 2003.
[45] Beullens P, Muyldermans L, Cattrysse D, Van Oudheusden D. A guided local search heuristic for the capacitated arc routing problem. European Journal of Operational Research 2003;147:629–43.
[46] Belenguer JM, Benavent E. The capacitated arc routing problem: valid inequalities and facets. Computational Optimization and Applications 1998;10:165–87.
[47] Cattrysse D, Van Oudheusden D, Lotan T. The problem of efficient districting. OR Insight 1997;10:9–13.
[48] Van Oudheusden D, Cattrysse D, Lotan T. On the importance of districting and its potential impact on routing. In: Proceedings of 8th World Conference on Transport Research. Amsterdam: Pergamon; 1999. p. 521–31.
[49] Kandula P, Wright JR. Optimal design of maintenance districts. Transportation Research Record 1995;1509:6–14.
[50] Kandula P, Wright JR. Designing network partitions to improve maintenance routing. Journal of Infrastructure Systems 1997;3:160–8.
[51] Wang JY, Kandula P, Wright JR. Evaluation of computer-generated routes for improved snow and ice control. Transportation Research Record 1995;1509:15–21.
[52] Beasley JE. Route-first cluster-second methods for vehicle routing. Omega 1983;11:403–8.