A survey of models and algorithms for winter road maintenance. Part III: Vehicle routing and depot location for spreading

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Abstract

Winter road maintenance planning involves a variety of decision-making problems related to the routing of vehicles for spreading chemicals and abrasives, for plowing roadways and sidewalks, for loading snow into trucks, and for transporting snow to disposal sites. These problems are very difficult and site specific because of the diversity of operating conditions influencing the conduct of winter road maintenance operations and the wide variety of operational constraints. As the third of a four-part survey, this paper reviews optimization models and solution algorithms for the routing of vehicles for spreading operations. We also review models for the location of vehicle and materials depots and for the assignment of crews to vehicle depots. The two first parts of the survey address system design problems for winter road maintenance. The fourth part of the survey covers vehicle routing problems for plowing and snow disposal operations.
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0. Introduction

This paper is the third part of a four-part survey of optimization models and solution algorithms for winter road maintenance problems. This paper surveys optimization models and solution methodologies

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for vehicle routing and depot location problems for spreading chemicals and abrasives. It also addresses related problems for crew assignment, fleet sizing, and fleet replacement. The fourth part of the survey [1] addresses vehicle routing, fleet sizing, and fleet replacement models for plowing and snow disposal operations. The two first parts of the survey [2,3] address system design models for winter road maintenance.

Winter road maintenance operations involve a large number of interesting and expensive problems that can be modeled and solved using operations research techniques. These operations include spreading of chemicals and abrasives, snow plowing, loading snow into trucks, and hauling snow to disposal sites. In the United States, winter road maintenance operations consume over $2 billion in direct costs each year [4]. Application of chemicals, including material, labor, and equipment costs, accounts for about one-third of the direct expenditures [4]. Expenditures on sand and other abrasives account for more than 10 percent of winter maintenance budgets, excluding application costs [4]. The use of chemicals and abrasives is also linked with many indirect costs, including damage to motor vehicles, infrastructure, and the environment, totaling more than $5 billion per year in the United States [4].

In recent years, new technologies in the application of road weather information systems, weather forecasting services, and thermal mapping have been implemented in many agencies in Europe, Japan, and the Unites States to help control the total costs associated with spreading materials, while enhancing their effectiveness. These developments are improving the timeliness and accuracy of weather information at the regional and road network levels, thus facilitating the use of more efficient and effective techniques for spreading operations, including anti-icing and prewetting techniques. New technologies can also make the use of chemicals alternatives more feasible by permitting use of smaller amounts of chemicals and targeted applications in environmentally sensitive areas.

However, progress in the development of optimization models for the routing of vehicles and the location of depots for winter road maintenance has experienced a slow growth compared to improvements in new technologies. Until recently, most contributions addressed simulation methods, simple constructive methods, and simplified models with little consideration of practical characteristics. Previous surveys by Gupta [5] and Campbell and Langevin [6] suggest that many agencies still rely in large part on decision rules dictated by field experiences when making vehicle routing and depot location decisions. The limited progress in the use of optimization models is somewhat surprising given that even a small increase in efficiency or effectiveness through optimization could result in significant savings, improved mobility, and reduced environmental and societal impacts.

In fact, the slow progress in the development of optimization models for the routing of vehicles and the location of depots for winter road maintenance highlights the difficulty of the problems studied. Vehicle routing and depot location problems related to winter road maintenance are especially complex and site specific because of the tremendous diversity in operating conditions such as geography, meteorology, demographics, economics, and technology (These operating conditions are described in detail in the first part of the survey [2]). In addition, winter road maintenance operations are incredibly diverse as they are affected by different operational constraints, which depend on the level of service policies and on the characteristics of the transportation network, road segments, sectors, depots, vehicles, and drivers.

In spite of the difficulty of winter road maintenance vehicle routing problems, recently proposed models tend to take into account a larger variety of characteristics of the problems arising in real-world applications, and the proposed solution methods are often based on local search techniques. Recent developments in modeling and algorithmic tools, the increased performance of computers, and the increased pressure

facing state and local agencies to reduce expenditures on winter road maintenance operations, while maintaining or enhancing service levels and minimizing environmental impacts, all motivate the more widespread use of optimization models.

Vehicle routing and depot location problems facing state and local agencies can be classified into four categories according to the planning horizon considered: strategic level, tactical level, operational level, and real-time control. The *strategic* level involves the acquisition or construction of long-lasting resources intended to be utilized over a long time period. The *tactical* level includes medium and short term decisions that are usually updated every few months. The *operational* level is related to the winter tasks that require ongoing attention on a day-to-day basis. Finally, the *real-time* level involves decision-making situations in which operations must be undertaken or altered in a very short time frame (e.g., minutes) in response to the sudden change of the system (equipment breakdowns, weather change, etc.). Though each storm is unique in duration, intensity, and composition, vehicle routes are generally fixed at the beginning of the winter season. A 1995 survey in Minnesota reported that 62% of counties and 55% of cities re-evaluate or change their routes for plowing operations every year [7]. However, vehicle routes can be modified based on real-time weather and pavement information. Decisions relating to the location of depots for winter road maintenance may be viewed as strategic or tactical.

The paper is organized as follows. Section 1 describes the operations of spreading chemicals and abrasives, the vehicle routing and depot location problems related to those operations, and the crew assignment problem. Models dealing with the routing of vehicles for spreading operations are reviewed in Section 2. Models that address the location of vehicle and materials depots for spreading operations are described in Section 3. Models for the assignment of crews to vehicle depots are presented in Section 4. Conclusions and directions for future research are presented in the last section. The term roadway used in this paper refers to any highway, road street, or other pavement surface that carries motor vehicles.

1. Operations context and decision problems

The following section contains a brief description of spreading operations for winter maintenance. More detailed information on the state of the practice in managing winter road maintenance operations, including spreading operations, is presented in the synthesis report by Kuemmel [8]. That report also includes additional information on estimating winter maintenance benefits and costs; personnel and management issues; weather information systems; and materials, equipment, and facilities for winter road maintenance. Also, a detailed review of the available technology for winter road maintenance, and the scientific underpinnings of that technology, is presented in the book by Minsk [9]. Finally, guidelines for selecting winter road maintenance strategies and tactics for a wide range of operating conditions found in the United States are provided in the report by Blackburn et al. [10]. These strategies and tactics refer to the combinations of materials, equipment, and techniques (both chemical and mechanical) used in winter road maintenance to achieve a defined level of service. They also include road weather information systems and weather forecasting.

Following the description of spreading operations, this section describes the characteristics of vehicle routing problems related to spreading operations. This section concludes with a brief discussion on vehicle and materials depot location problems, followed by crew assignment problems that have been addressed by operations researchers.

1.1. Spreading operations

Spreading operations are directed at achieving three specific goals in winter road maintenance: anti-icing, deicing, and traction enhancement. Anti-icing operations involve the application of chemicals in advance of precipitation to prevent the bonding of snow and ice to the pavement surface. The idea is to use ultimately less chemicals by preventing ice from forming, rather than melting it once it has formed, and to remove snow that has not frozen and bonded to the road surface by less plowing. Anti-icing is most effective when most or all of a lane is treated uniformly. Spraying a liquid chemical permits good control of application rate and coverage and results in much less material loss during application and from traffic compared to spreading a solid chemical. The two major pieces of information to consider when deciding on the start time of anti-icing operations are pavement surface condition and weather forecast. Pavement surface temperature and its trend indicate the probability of snow or ice freezing on the pavement, and the weather forecast indicates the likelihood of precipitation and its most likely form. The advent of road weather information systems and thermal maps of highway segments now provides the means for assessing the pavement surface condition in real time.

Deicing operations involve allowing the bonding of snow and ice to the pavement surface during the precipitation and periodically weakening it with chemicals until the ice-pavement bond is broken and the resulting ice sheets can be removed mechanically by plowing or traffic action. Deicing operations require greater quantities of chemicals than anti-icing operations as a result of dilution of the particles as they bore through the snow and ice layer to reach the ice-pavement bond. Prewetting of solid chemicals improves adhesion to the roadway and speeds the melting into the ice. This reduces bounce and scatter and accelerates deicing operations.

The most widely used chemical is salt because of its low price, ready availability, ease of application, and reliable ice-melting performance. However, over the years, evidence has grown that salt has many negative side effects on infrastructure, vehicles and the environment. The literature in this area was reviewed and evaluated by a special TRB study committee in 1991 [4]. The committee reported that damage attributable to salt include accelerated corrosion of metals in bridges, parking structures, and motor vehicles, increased sodium levels in drinking water, and injury to roadside vegetation. Environmental problems arising from the use of deicing chemicals are also reviewed by Minsk [9]. Salt storage facilities are usually located at highway maintenance yards as well as at other intermediate points along highways.

The most common technique for enhancing traction on thick snow-packed and ice surfaces when temperatures are too low for chemicals to be effective is to spread abrasive materials such as sand, cinders, ash, tailings, and crushed stone and rock. These materials may be applied alone or with varying amounts of chemicals in a mixture. Abrasives are mixed with small quantities of chemicals to keep depots of abrasives from freezing or chunking. They are also mixed with sufficient quantities of chemicals to support both anti-icing and deicing operations. Abrasives are inexpensive, offer some immediate traction on slippery surfaces, and, where cinders and other dark materials are used, provide visible evidence of action by crews. However, the excessive use of abrasives can have several negative consequences, including problems of clogging of drainage channels and sewers, significant cleanup efforts following storms and the winter season, adverse effects on cars, and the pollution resulting from airborne fine particles. A further problem results from the reduced distance a truckload of abrasive can cover compared to a load of salt or other chemical, thus requiring frequent reloading.

The selection of the appropriate spreading operation is based on economics, environmental constraints, climate, level of service, material availability, and application equipment availability. The level of service

policies determine the extent of the resource investment. The environmental constraints influence the choice of a chemical or nonchemical operation.

1.2. Vehicle routing problems for spreading

The operations of spreading chemicals and abrasives concern the service of a set of road segments by a fleet of vehicles, which are based at one or more depots located in one or more sectors, and travel over an appropriate transportation network. In particular, vehicle routing problems for spreading operations consist of determining a set of routes, each performed by a vehicle that starts and ends at its own depot, such that all road segments are serviced, all the operational constraints are satisfied, and the global cost is minimized. This section describes the typical characteristics of vehicle routing problems related to spreading operations by considering their main components (transportation network, road segments, sectors, vehicle and materials depots, vehicles, and drivers), the different operational constraints that can be imposed on the configuration of the routes, and the possible objectives to be achieved in the optimization process. These characteristics are summarized in Table 1. Models and algorithms proposed for the solution of vehicle routing problems related to spreading operations are reviewed in Section 2. Vehicle routing problems related to plowing and snow disposal operations are described in the fourth part of the survey [1].

The transportation network is generally described through a graph, whose arcs and edges represent the one-way streets and two-way streets to be serviced, respectively, and whose nodes correspond to the road junctions and to the vehicle and materials depot locations. The corresponding graph can be *directed*, *undirected*, or *mixed* depending on the topology of the transportation network and on the operating policies involved. If the two sides of some streets can be serviced at the same time, as is often the case in spreading operations, the mixed graph can be the appropriate representation. Conversely, if the two sides of the street must be serviced separately, as is the case in plowing and snow disposal operations, arcs may have to be duplicated and edges are replaced by two arcs of opposite direction. The resulting graph is then directed. Associated with the transportation network is a *maximum time* for completing spreading operations based on political and economic considerations. Since agencies have finite resources that generally do not allow the highest level of service on all roads, they must then prioritize their response efforts. The most common criterion for prioritizing response efforts is traffic volume. Typically, the roads of a network are partitioned into classes based on traffic volume which induce a *service hierarchy*, namely all roads carrying the heaviest traffic are given the highest level of service in order to provide safe roads for the greatest number of motorists, followed by medium-volume roads, and so on. Associated with each class of roads can also be a *maximum time* for spreading completion.

Most policies for winter road maintenance define levels of service for classes of highways based on their priority. *Level of service policies* tend to be *results-oriented* (e.g., bare pavement), *resource-oriented* (e.g., 24-hour equipment coverage), or a combination of both. Associated with each road segment is a cost, which generally represents its *length*, and three *traversal times*, which are possibly dependent on the vehicle type: the time required to service the road segment, the time of deadheading the road segment if it has not yet been serviced, and the time of deadheading the road segment if it has already been serviced. *Deadheading* occurs when a vehicle must traverse a road segment without servicing it. In general, the longest operation consists of spreading materials on a road segment, followed by deadheading an unserviced road segment, followed by deadheading a serviced road segment. However, the time of deadheading an unserviced road segment can exceed the time of servicing it if, for example, traversing an

Table 1
Characteristics of vehicle routing problems for spreading

Components	Characteristics
Transportation network	Undirected, directed or mixed
	Service hierarchy
	Maximum time for spreading completion
Road segments	Resource-oriented or results-oriented level of service policies
	Length
	Service and deadhead traversal times
	Service time windows
	Service frequencies
	Required road segments
	One or multiple passes per road segment
	One or two lanes in a single pass
Sectors	Sector design
	Number of sectors
	Compactness or shape
	Balance in sector size or workload
	Contiguity
	Basic units
Vehicle and materials depots	Vehicle and materials depot locations
	Centrally located depots relative to sectors
	Number of vehicle and materials depots
	Materials depot capacities
	Variable costs of vehicle and materials depots
	Fixed costs of vehicle and materials depots
Vehicles	Home depot
	Spreader capacity
	Spreader type and road segment dependencies
	One or multiple routes per spreader
	Variable costs of spreaders
	Fixed costs of spreaders
Drivers	Maximum duration of driving and working periods
	Number and duration of breaks
	Overtime
	Variable driver costs
Routes	Start and end locations of routes
	Start times of routes
	Load balancing
	Class continuity
	Class upgrading
	Both-sides service
	Turn restrictions
	Alternations between deadheading and servicing
	Service connectivity
	Sector boundaries
Objectives	Minimize deadheading and fixed costs of spreaders and depots
	Minimize fleet size
	Minimize alternations between deadheading and servicing
	Minimize operational constraint violations

unserviced road segment is extremely difficult or impossible. Associated with each road segment is also a time interval, called *service time window*, during which the road segment can be spread with materials, which is possibly dependent on the hierarchy of the network, and a *service frequency* (e.g., the road segment should be covered at least once every two hours). Road segments that require spreading at least once according to the level of service policy are called *required* road segments. Lane configurations and road segment widths may necessitate *multiple passes* to service some road segments. Finally, chemicals and abrasives are often spread onto the road segment through a spinner which can be adjusted so that *two lanes* are treated *on a single pass*.

Given the large geographic extent of most winter road maintenance operations, an agency generally partitions its transportation network into subnetworks, called *sectors*. All sectors are treated simultaneously by separate crews to facilitate the management of the operations. A sector is thus a bounded, organizational or administrative subarea in a larger geographical region. Details on the design of sectors for winter road maintenance are given in the first part of the survey [2].

The routes performed for spreading operations start and end at one or more *vehicle depots*, located at the vertices of the graph. Associated with every vehicle depot is a given number of vehicles of each type. In spreading operations, vehicle routes often consist of materials spreading and intermediary trips to intermediate facilities, called *materials depots*. These facilities contain chemicals and abrasives to provide opportunities for vehicles to refill with materials without returning to the original starting point. Vehicle and materials depots should be *centrally located relative to sectors they serve* to reduce the distance covered by deadheading trips [11]. *Costs* associated with vehicle and materials depots include variable costs of operating vehicle and materials depots and fixed costs of acquiring vehicle and materials depots.

Spreading operations are performed using a fleet of vehicles, called *spreaders*, whose size and composition can be fixed or can be defined according to the level of service policies, the configuration of the streets and sidewalks, land use (e.g., residential or commercial) and density of development, and times for spreading completion for each class. A spreader may end service at a depot other than its *home depot*. The *capacity* of the spreader is expressed as the maximum quantity of chemicals or abrasives the spreader can discharge. Application rates for chemicals and abrasives are usually specified in kilogram-per-lane-kilometer. The subset of road segments of the transportation network which can be traversed by the spreader is *dependent* on road segment widths and on the spreader type. Large road segments may require large spreaders. Narrow road segments may require small or medium-sized spreaders. In some applications, each spreader can cover *multiple routes* in the considered time period. Finally, each vehicle type is associated with a *fixed* leasing or acquiring *cost* and a *variable cost* that is proportional to the distance traveled. The variable cost component encompasses the costs of fuel, materials, and maintenance.

Drivers operating the spreaders must satisfy several constraints laid down by union contracts and agency regulations. Examples are *working periods* during the day, maximum duration of working and *driving periods*, number and duration of *breaks* during service, and *overtime*. *Costs* associated with drivers depend on the pay structure (e.g., regular or premium time, single or dual working periods).

The routes must satisfy several operational constraints, which depend on the level of service policies, and on the characteristics of the transportation network, road segments, sectors, spreaders, and drivers. The routes can *start* and *end* at one or more depot *locations* and each route can end service at a depot other than the original starting depot. In anti-icing operations, routes must *start* at the proper *time* for effective spreading of chemicals. The decision must take into account such factors as type of snow (wet or dry), expected temperature conditions at the time of, and following, application, anticipated variations at the

critical freeze-thaw point, methods of application, and types of chemical. To *balance* the *workload* across routes, they are often approximately the same length or duration. This helps ensure that all spreading operations will be completed in a timely fashion. Since most arterial roads have multiple lanes that require separate passes, the total workload is usually measured in lane-kilometers. *Class continuity* requires that each route services road segments with the same priority classification. Thus, if a lower-class road is included in a route servicing higher-class roads, its service level may be *upgraded*. Sometimes, it is desirable that both sides of a two-lane, two-way road (one lane each way) be serviced by the same vehicle in a single route. However, *both-sides service* constraints usually arise in plowing operations. Also, the impact of undesirable *turns*, such as U-turns and turns across traffic lanes, is generally greater in routing snow plows as compared to spreading operations. Certain locations may be pre-specified as turn-around locations while turns may be simply prohibited in some regions. Good routes typically have long stretches of spreading and long stretches of deadheading. Too many *alternations between deadheading and servicing* must be avoided form an operational standpoint. *Service connectivity* requires that the subgraph induced by the set of road segments serviced by a spreader is connected. The configuration of routes may also need to conform to existing *sector boundaries*. Routes crossing these boundaries must be avoided from an administrative standpoint. Some operational constraints can be treated as hard constraints and others as soft requirements or as *terms* in an objective function. For example, it may be reasonable to exceed the spreading completion time deadlines for lower-class roads if this leads to reducing the fleet size.

Finally, several, and often conflicting, objectives can be considered for the routing of spreaders. Typical objectives are minimization of the global cost, dependent on the distance covered by *deadheading* trips (or on the deadhead travel time) and on the *fixed costs* associated with the used spreaders and depots; minimization of the *number of spreaders* required to service all the required road segments; minimization of the *alternations* between deadheading and servicing; minimization of the *terms* penalizing the violation of some operational constraints; or any weighted combination of these objectives. Note that minimizing deadhead distance and minimizing total distance are equivalent objectives since all required road segments must be serviced.

1.3. Vehicle and materials depot location problems for spreading

Section 3 of this survey is devoted to depot location problems in the context of winter road maintenance. A number of different depots are needed for spreading operations, including vehicle depots or equipment repair locations, and materials depots. Depot location problems in the context of winter road maintenance are generally formulated as network location problems, in which facilities can be located only on the nodes or links of the network. Since the same type of vehicles may be used for winter road maintenance as for other maintenance activities, the vehicle depot locations may need to be based on year-round operations. Some material depots may be associated with vehicle depots, while other material re-supply points may be placed strategically in a region to allow spreaders to re-supply without returning to the home depot. Many designs of buildings are used for storage of chemicals and abrasives, from simple sheds with fixed roofs, to sheds with slide-back roofs, to large dome-shaped structures, to silos. Ground-level storage facilities require some device for loading mechanized spreader trucks, such as a front-end loader. The silos enable faster filling of spreaders because they permit several trucks to be filled simultaneously and drivers can load without additional help or equipment.

1.4. Crew assignment problems

Section 4 of this survey discusses crew assignment models in the context of winter road maintenance. Agencies in areas that frequently experience severe weather year-round sometimes choose to have extra manpower capability to assure timely recovery. When winter road maintenance equipment and vehicles are idle, personnel are used for other tasks such as litter patrol, shop maintenance, equipment repair, or highway facility maintenance. However, in most agencies, the number of workers required for winter road maintenance operations typically exceeds the amount of permanent staff available. Options for obtaining staff to perform winter road maintenance operations include reassigning staff from other agencies and temporary or seasonal employees, and part time workers. Assigning specific highway sections to staff from these other sources usually involves disruption of existing permanent staff assignment plans. Given planned vehicle routes for winter road maintenance operations, the *crew assignment problem* consists of assigning a set of crews to vehicle depots from which emanate the planned routes, so as to satisfy the demand for crews for vehicle routing while minimizing travel costs.

2. Vehicle routing models for spreading

Vehicle routing problems related to spreading operations are generally formulated as arc routing problems. Assad and Golden [12] presented an extensive review of the literature in arc routing with special emphasis on applications. In a series of two papers, Eiselt et al. [13,14] presented an integrated overview of the most relevant operations research literature on arc routing. More recently, a book on the subject was edited by Dror [15].

Over the years, many decision support systems using optimization methods have been developed to assist planners in making vehicle routing decisions for spreading operations. Such systems were described, for example, by Durth and Hanke [16], Jaquet [17], Blesik [18], McDonald [19], and Anderson et al. [20]. These authors did not, however, provide a description of the optimization methods on which their systems rely. A survey of decision support systems for spreader and plow routing is presented in the report by Gini and Zhao [21]. Also, Turchi [22] identified existing decision support systems for winter road maintenance and proposed a taxonomy scheme to categorize them.

Several rule-based decision support tools have also been developed to help planners in selecting chemical applications appropriate to winter weather conditions. For example, Malmberg and Axelson [23] described a project initiated by the Swedish National Road Administration in the early 1990s to develop an expert system, called VVEXP, to recommend the starting time for salting, the salt quantity and the width of application. The project was a comprehensive plan for an integrated road weather information system network, including development of methods for interpolation of road weather information between stations, and automated selection of chemical applications based on rules and facts gathered from field interviews and meetings with experts. However, for financial reasons, further development of the system was stopped [24]. Ketcham et al. [25] described a table-based menu developed by the US Federal Highway Administration (FHWA) to suggest maintenance actions to field personnel in the selection and applications of chemicals for anti-icing operations according to current and forecast pavement temperature, snowfall and dew point conditions. Most of the maintenance actions involve the application of chemicals in either a dry solid, liquid, or prewetted solid form. Application rates are given for each chemical form where appropriate. The guidance is based upon the results of

four years of anti-icing field testing conducted by 15 US state highway agencies and supported by the Strategic Highway Research Program (SHRP) and the FHWA. It has also been augmented with practices developed outside the US. Later, the De-icing Anti-icing Response Treatment (DART) program was developed for the Ministry of Transportation of Ontario, Canada, to adapt the FHWA menu to a computerized format and to evaluate the effectiveness of the recommended treatments [26]. A more detailed review of published information on the VVEXP, FHWA, and DART systems is presented in the report by Perchanok et al. [27].

Several heuristics procedures have been proposed for the routing of vehicles for spreading operations. These can be broadly classified into three categories: constructive methods, composite methods, and adaptation of metaheuristics. These three classes of methods are reviewed in the next three sections, respectively. The characteristics of the contributions are then summarized in Table 2 at the end of the section.

2.1. Constructive methods

Constructive methods for the routing of vehicles for spreading operations can be divided into four classes: sequential route construction methods, parallel route construction methods, cluster first, route second methods, and optimization methods based on capacitated arc routing or capacitated minimum spanning tree formulations. These four classes of methods are discussed in the next four sections, respectively.

2.1.1. Sequential constructive methods

Evans [28] and Evans and Weant [29] described a decision support system, called SnowMaster, to assist planners in constructing feasible routing plans for salt spreading operations that satisfy maximum time constraints for spreading completion, maximum route duration, and vehicle capacities. The time for spreading completion does not include the return time to the depot after salting the last road segment on the route. Road segments that need to be serviced can be specified as requiring one single pass from one direction (as with narrow county roads) or two passes from both directions (as in the case of divided highways). Multiple depots can also easily be taken into account. The problem is solved using the path-scanning algorithm developed by Golden et al. [30] for the capacitated arc routing problem. The path-scanning algorithm uses several selection rules to extend a route under construction. Golden et al. described five such rules. However, other selection rules can be preferred. In particular, Evans [28] proposed the "1-degree rule" that tries to insert into the emerging route the road segment with one endpoint with degree 1. With this rule, one tends to avoid the formation of isolated required road segments that are likely to increase deadheading if they are inserted into the emerging route at a later stage. The system can also be used to aid planners in the development of routing plans for plowing operations. Results on the road network of Butler County, Ohio involving 185 road segments and a total of 284 lane miles indicated that the system reduced the fleet size by 30% and cut the spreading completion time by 40% over the routing plan in use by the county. The system was useful at both the operational and strategic levels in analyzing a variety of scenarios regarding travel speeds, salt spreading rates, response/coverage times, general fleet requirements, fleet size and mix, and the scheduling of fleet replacement. Gupta [5] used SnowMaster at the strategic level to analyze various scenarios related to the opening or closing of vehicle or materials depots. The system has also been used successfully by several different US counties. Details on these applications are given by Waddell [31] who also mentioned the use of SnowMaster to

Table 2
Characteristics of vehicle routing models for spreading

Authors	Problem type	Planning level	Problem characteristics	Objective function	Model structure	Solution method
Evans [28], Evans and Weant [29]	Salt spreader truck routing	Operational	Vehicle capacities, maximum time for spreading completion, maximum route duration, and one or two lanes in a single pass	Min total distance	Capacitated arc routing problem	Sequential constructive method
Li and Eglese [32]	Salt spreader truck routing	Operational	Service hierarchy, vehicle capacities, multiple vehicle and materials depots, and two lanes in a single pass	Min fixed spreader costs and transport costs	Capacitated arc routing problem	Sequential constructive method
Soyster [34]	Salt and abrasives spreader truck routing	Operational	Service hierarchy and maximum route durations	Min service hierarchy violations and deadheading	Capacitated arc routing problem	Parallel constructive method
Cook and Alprin [35]	Salt spreader truck routing	Operational	Vehicle capacity, load balancing, and both-sides service	Min total spreading time	Capacitated arc routing problem	Parallel constructive method
Ungerer [36]	Salt spreader truck routing	Real-time	Vehicle capacities and maximum route lengths	Min total distance	Capacitated vehicle routing problem	Parallel constructive method
Liebling [38]	Combined sector design and spreader routing	Strategic	Vehicle capacities, working periods, and service frequencies	Min deadheading	Directed Chinese postman problem	Cluster first, route second
England [39,40]	Combined sector design and vehicle routing	Strategic	Compactness, workload balance, service frequencies, and turn restrictions	Min deadheading and alternations between servicing and deadheading	Arc routing problem	Cluster first, route second
Soyster [34]	Salt spreader truck routing	Operational	Vehicle capacity and two lanes in a single pass	Min total distance	Linear 0–1 IP	Optimization-based method
Haghani and Qiao [41]	Salt spreader truck routing	Operational	Service connectivity, vehicle capacity, and both lanes of two-lane, two-way roads in a single route	Min fleet size and/or deadheading	Capacitated minimum spanning tree	Optimization-based method

Table 2 (*continued*)

Authors	Problem type	Planning level	Problem characteristics	Objective function	Model structure	Solution method
Xin and Eglese [43]	Salt spreader truck routing	Operational	Vehicle capacities, time limit for spreading completion, multiple routes per vehicle, and multiple depots	Min fleet size and deadheading	Capacitated arc routing problem	Composite method
Haghani and Qiao [46]	Salt spreader truck routing	Operational	Service hierarchy, vehicle capacities, maximum route durations, and level of service policies	Min deadheading	Linear MIP	Composite method
Eglese [33]	Salt spreader truck routing	Operational	Service hierarchy, vehicle capacities, multiple depots, grouping of street segments, class upgrading, and two lanes in a single pass	Min fleet size	Spanning trees	Simulated annealing
Benson et al. [51]	Salt spreader routing	Operational	Vehicle capacities, service/ deadhead speeds, times for service completion, and one or multiple passes per road	Min vehicle capacities and time limits violations and deadheading	Nonlinear MIP	Tabu search
Qiao [49]	Salt spreader truck routing	Operational	Service connectivity, vehicle capacity, and level of service policies	Min deadheading	Linear MIP	Tabu search and elite route pool

help planners in making decisions about fleet size and mix, materials depot location, and materials type and usage.

Li and Eglese [32] proposed a three-stage heuristic for the salt spreader routing problem of Lancashire County Council, UK. This problem may be characterized as a multi-depot capacitated arc routing problem with a heterogeneous fleet, roadways of a rural mixed network $G = (V, E \cup A)$ partitioned into classes, a set of planned sectors with centrally located vehicle depots, fixed materials depot locations, and with both sides of a roadway spread in a single pass. The vehicle capacity constraints translate into maximum spreading distances. Associated with each roadway class is a completion time for salting treatment. A route can cover roadways of different classes but the time by which roadways in each class are treated must not be exceeded. The time constraint does not consider the deadheading trip to the depot. Each route must start and end at the same depot location. The objective is to minimize the sum of fixed costs of spreaders and transport costs. Considering a given sector and its centrally located depot v_0, the heuristic determines feasible routes one at a time in three stages. The first stage chooses the farthest non-serviced required link (v_i, v_j) from the depot v_0. Let v_i be the nearest vertex from the depot among the two endpoints v_i and v_j. The second stage finds a feasible path $P_1 = (v_0, \ldots, v_i)$ starting from v_i (in the reverse direction) using the following link-selection rule: if v_{start} is the current last vertex of the partial path P_1, extend P_1 with the next link (v_k, v_{start}) that fits within capacity and time constraints and minimizes the distance from the depot v_0 to v_k. The last stage determines another feasible path $P_2 = (v_j, \ldots, v_0)$ starting from v_j using the farthest link-selection rule with additional decision rules to determine if the spreader should head back towards the vehicle depot or the nearest materials depot to refill with salt. Given a partial path P_2 that ends at vertex v_{end}, the farthest link-selection rule chooses the link (v_{end}, v_k) that fits within capacity and time and maximizes the distance from v_k to the depot v_0. When either no non-serviced required link(v_{end}, v_k) of current class can be found or the time constraint would be exceeded by including (v_{end}, v_k), the third stage tries to insert a link of a lower roadway class following the farthest link-selection rule. The time left is then set to the time remaining until the deadline for the lower roadway class. Links of current or lower class with one endpoint with degree 1 are always chosen first in the two last stages to avoid the formation of isolated required roadways. A route ends when no more non-serviced required links of a suitable roadway class can be found. The heuristic, called *Time constraint two phases algorithm*, is described in Fig. 1. The following notation is used. Given two paths $P' = (v_i, \ldots, v_j)$ and $P'' = (v_j, \ldots, v_k)$ having a common endpoint v_j, the union of the links of these two paths is a longer path $P = (v_i, \ldots, v_j, \ldots, v_k)$ which is denoted as $P' + P''$.

The authors compared the performance of the Time constraint two phases algorithm and the greedy algorithm proposed by Eglese [33] (see Section 2.3) on salt spreading problems for three service areas in the County with 77, 140 and 254 nodes and 111, 203 and 380 road segments, respectively. With some manual intervention, this algorithm produced better routes than the greedy algorithm, in terms of both the number of spreaders required and the total distance travelled. This algorithm has also been used successfully by two counties in the northwest of England.

2.1.2. Parallel constructive methods

Soyster [34] described a computerized system for the routing of trucks for spreading of salt and abrasives. The system starts by generating feasible routing plans that satisfy the maximum route durations constraints using the heuristic described in Fig. 2.

1. Determine the farthest non-serviced required link (v_s, v_t) from the depot v_0. Set $h = \alpha(v_s, v_t)$ where $\alpha(v_s, v_t)$ denotes the priority index of link (v_s, v_t) in G, $\alpha(v_s, v_t) \in \{1, ..., p\}$ with 1 being the highest priority. If the length of the shortest path SP_{0s} from v_0 to v_s is shorter than the length of the shortest path SP_{0t} from v_0 to v_t in G, set $v_{start} := v_s$ and $v_{end} := v_t$ (v_{start} and v_{end} denote the endpoints at the start and end of the route, respectively). Otherwise, set $v_{start} := v_t$ and $v_{end} := v_s$. Set $P_1 := \varnothing$ and $P_2 := \varnothing$.

2. *Phase I*
   a. Let $d(v_i)$ be the number of links incident to v_i. Choose a non-serviced required link (v_{start}, v_i) such that $\alpha(v_{start}, v_i) \geq h$ and $d(v_i) = 1$. If the capacity and time limit constraints permit,
      set $P_1 := P_1 + (v_{start}, v_i) + SP_{i,start}$ (the link (v_{start}, v_i) must be serviced only once).
   b. Choose a non-serviced required link (v_i, v_j) such that v_i is adjacent to v_{start} in G, $\alpha(v_{start}, v_i) \geq h$, $\alpha(v_i, v_j) \geq h$, and $d(v_j) = 1$. If the capacity and time limit constraints permit, set $P_1 := P_1 + (v_{start}, v_i, v_j) + SP_{j,start}$ (the links (v_{start}, v_i) and (v_i, v_j) are serviced only once).
   c. Choose two non-serviced required links (v_i, v_j) and (v_j, v_k) such that v_i is adjacent to v_{start} in G, $\alpha(v_{start}, v_i) \geq h$, $\alpha(v_i, v_j) \geq h$, $\alpha(v_j, v_k) \geq h$, and $d(v_k) = 1$. If the capacity and time limit constraints permit, set $P_1 := P_1 + (v_{start}, v_i, v_j, v_k) + SP_{k,start}$ (the links (v_{start}, v_i), (v_i, v_j), and (v_j, v_k) are serviced only once).
   d. Repeatedly apply Steps a, b, and c until no such links can be found.
   e. Choose the non-serviced required link (v_i, v_{start}) of priority h in G such that v_i is the nearest vertex to the depot. If the capacity and time limit constraints permit, set $P_1 := (v_i, v_{start}) + P_1$ and $v_{start} := v_i$. Otherwise, set $P_1 := SP_{0,start} + P_1$ (the links of the shortest path $SP_{0,start}$ are not serviced) and $v_{start} := v_0$.
   f. If $v_{start} := v_0$, go to Step 3. Otherwise, return to Step a.

3. *Phase II*
   a. Apply Steps a, b, c, and d of Phase I but by replacing v_{start} by v_{end} and P_1 by P_2.
   b. If the vehicle is less than half full and the remaining distance it can spread is less than the distance to the nearest materials depot location (possibly v_0), choose the non-serviced required link (v_{end}, v_j) of priority h in G such that v_j is the nearest vertex to the materials depot. Otherwise, choose a non-serviced required link (v_{end}, v_j) of priority h in G such that v_j is the farthest vertex from the depot v_0. If no non-serviced required link (v_{end}, v_j) of priority h can be found according to one of these two rules, go to Step e. If there is no non-serviced required link of any priority incident to v_{end}, go to Step f.
   c. If the capacity and time limit constraints permit, set $P_2 := P_2 + (v_{end}, v_j)$, $v_{end} := v_j$, and return to Step a. If the capacity constraint would be exceeded by including (v_{end}, v_j), go to Step d. If the time limit constraint would be exceeded by including (v_{end}, v_j), go to Step e.
   d. Choose the nearest materials depot location v_d (possibly v_0) to v_{end}. If the time limit permits, set $P_2 := P_2 + SP_{end,d}$ (the links of the shortest path $SP_{end,d}$ are not serviced), $v_{end} := v_d$ (the vehicle is refilled with salt at depot v_d), and return to Step a. Otherwise, go to Step e.
   e. If all required links of priority index higher than or equals to h are serviced, go to Step g. Otherwise, set $h = h + 1$ (the time left is the time remaining until the deadline for roadways of priority h) and return to Step a.
   f. Choose the nearest non-serviced required link (v_i, v_j) of priority h to v_{end}. If no such link exists, go to Step g. Otherwise, set $P_2 := P_2 + SP_{end,i} + (v_i, v_j)$ (the links of the shortest path $SP_{end,i}$ are not serviced), $v_{end} := v_j$, and return to Step a.
   g. Set $P_2 := P_2 + SP_{end,0}$ (the links of the shortest path $SP_{end,0}$ are not serviced) and $v_{end} := v_0$. $P_1 + (v_s, v_t) + P_2$ is a feasible vehicle route. If all required links are serviced, stop. Otherwise, return to Step 1.

Fig. 1. Time constraint two phases algorithm [32].

This heuristic is embedded into a discrete event simulation approach to model spreader movements and interactions. Each feasible routing plan is then evaluated on the number of deadheading trips and on the respect of the service hierarchy. These two criteria are treated through the calculation of a weighted

1. For each spreader truck $k \in K$, start with any edge incident to the depot in G as the beginning of route k.
2. For each spreader truck $k \in K$, choose any edge incident to the last edge added to route k that has not been serviced. If such an edge exists and if the maximum route duration t_k permits, add this edge for servicing to route k. Otherwise, choose any edge incident to the last edge added to route k and add this deadheading edge to route k if the maximum route duration t_k permits.
3. If all edges in G have been serviced, for each spreader truck $k \in K$, add the shortest chain between the endpoint of route k and the depot in G to route k in order to get a set of tours and stop.
4. For each spreader truck $k \in K$, if the maximum route duration t_k has elapsed, add the shortest chain between the endpoint of route k and the depot in G to route k and return to step 1.
5. Return to step 2.

Fig. 2. The heuristic procedure for the spreader routing problem [34].

additive score S_p for each routing plan R_p defined as

$$S_p = \sum_{(v_i, v_j) \in R_p} \left(n_{ij} \cdot f_{ij}(t) \right),$$

where n_{ij} is the priority of roadway $(v_i, v_j) \in E$, $n_{ij} \in \{0, 1, 2, 3, 4, 5\}$ with 5 being the highest priority, and $f_{ij}(t)$ is the value of a benefit function corresponding to the time t at which service begins on roadway (v_i, v_j). If roadway (v_i, v_j) is deadheaded, then $n_{ij} = 0$ for the corresponding units of time. The benefit to the public from servicing a particular roadway decreases as a function of the time span between the onset of the storm and the beginning of the service. The routing plans with the highest scores then represent the best routing plans. The author reports that the system performed well on theoretical networks with about a dozen or so intersections and 30 to 40 road segments. The author did not, however, provide a model for the determination of a valid benefit-time function.

Cook and Alprin [35] considered the salt spreader routing problem in the city of Tulsa, Oklahoma. The authors proposed a parallel route construction heuristic to balance the total workloads assigned to the different spreader trucks while satisfying the vehicle capacity and the both-sides service constraint. The heuristic starts by determining, for each spreader truck, the nearest street segment (v_i, v_j) from the materials depot such that the amount of salt to service both sides of the street segment in two separate passes does not exceed the capacity of the vehicle. For each spreader truck, a feasible vehicle route servicing both sides of the street segment (v_i, v_j) in two separate passes is then created (i.e. a route made of a shortest path between the materials depot and v_i, the street segment serviced from v_i to v_j, the street segment serviced from v_j to v_i, and a shortest path between v_i and the materials depot). This process is repeated until all street segments are contained in a route. The heuristic is embedded into a discrete event simulation model of the salt spreading operations for Tulsa. The simulation model incorporates waiting times incurred by trucks when queueing for the operational safety check of spreaders or for reloading at the materials depot. Tests performed with data from the city of Tulsa showed a 36% reduction in total spreading time over the solution in use by the city. The simulation model was also useful to evaluate the benefits of increasing the number of materials depots, the fleet size, and the vehicle capacity.

Ungerer [36] studied a salt spreading problem where bridge decks and some critical highway locations require immediate salting so as to minimize total distance traveled, while satisfying vehicle capacities and

maximum route lengths. This real-time problem is formulated as a capacitated vehicle routing problem with customers representing bridge decks and critical highway locations. It is solved with a parallel version of the Clarke and Wright [37] savings algorithm in which the merge step is repeated until no further improvement is possible.

2.1.3. Cluster first, route second methods

Liebling [38] developed a traditional cluster-first, route-second approach for the salt spreader routing problem in the city of Zurich. The approach takes into account vehicle capacities, working periods, and service frequencies. Basic units are first aggregated into a minimum number of feasible sectors, and a spreader route is constructed for each sector by solving a directed Chinese postman problem. The two-phase heuristic can also serve to solve routing problems encountered in other road maintenance operations such as snow plowing or street sweeping. The approach has been successfully used by the city of Zurich. Indeed, the author reports considerable savings in the amount of salt required over the solution used by the city. Finally, the author suggested formulating the spreader routing problem on sidewalks as an undirected Chinese postman problem.

England [39] described a decision support system to assist planners at the South Yorkshire County Council's department of engineering in developing feasible routes for road maintenance operations such as gully emptying, road sign cleaning, grass cutting, road sweeping, street lighting, and salt spreading. The system first organizes road segments into balanced and compact sectors by using cluster analysis and an interchange method that tries to produce the desired workload [40]. Several vehicle routes are generated for each sector by using an insertion procedure that takes into account service frequencies and turn restrictions while minimizing deadhead travel time and the number of alternations between deadheading and servicing. The insertion procedure is embedded into a discrete event simulation approach to model vehicle movements for selecting the best identified vehicle route for each sector.

2.1.4. Optimization-based methods

Soyster [34] treated a spreader truck routing problem in which all two-lane, two-way road segments (one-lane each way) must be serviced in one single pass from either direction so as to minimize the total distance traveled, while satisfying vehicle capacities. Let $G = (V, E)$ be an undirected graph where V is the vertex set and E is the edge set and let K be the set of heterogeneous spreader trucks. For each node $v_i \in V$, let $E(v_i) = \{v_j \in V : (v_i, v_j) \in E\}$ be the set of nodes adjacent to node v_i. With every edge $(v_i, v_j) \in E$ is associated a nonnegative length c_{ij}. For each spreader truck $k \in K$, define b_k as the maximum distance truck k can cover depending on its capacity. For each edge $(v_i, v_j) \in E$ and for each spreader truck $k \in K$, let x_{ijk} be a binary variable equal to 1 if and only if edge (v_i, v_j) is serviced from v_i to v_j by truck k. Then the problem can be formulated as a linear 0–1 integer program as follows.

Minimize

$$\sum_{k \in K} \sum_{(v_i, v_j) \in E} c_{ij}(x_{ijk} + x_{jik}) \tag{2.1}$$

subject to

$$\sum_{k \in K} (x_{ijk} + x_{jik}) \geqslant 1 \quad ((v_i, v_j) \in E), \tag{2.2}$$

$$\sum_{(v_i, v_j) \in E} c_{ij}(x_{ijk} + x_{jik}) \leqslant b_k \quad (k \in K), \tag{2.3}$$

$$\sum_{\{v_j : (v_j, v_i) \in E\}} x_{jik} - \sum_{\{v_j : (v_i, v_j) \in E\}} x_{ijk} = 0 \quad (v_i \in V, k \in K), \tag{2.4}$$

$$\sum_{\{v_j : (v_0, v_j) \in E\}} x_{0jk} \geqslant 1 \quad (k \in K), \tag{2.5}$$

$$\sum_{\{v_j : (v_j, v_0) \in E\}} x_{j0k} \geqslant 1 \quad (k \in K), \tag{2.6}$$

$$x_{ijk} \in \{0, 1\} \quad ((v_i, v_j) \in E, k \in K). \tag{2.7}$$

The objective function (2.1) minimizes the total distance traveled. Constraints (2.2) require that each road segment be serviced at least once from either direction by a spreader truck. Constraints (2.3) impose a limit on the distance each truck can travel. Flow conservation at every node for each spreader truck type is imposed by constraints (2.4). Constraints (2.5) and (2.6) ensure that each truck starts and ends its route at the depot v_0, respectively. Finally, all x_{ijk} variables are restricted to be binary. Note that this model does not prevent the formation of disconnected subtours, which is often undesirable from an administrative standpoint given that deadheading trips would be necessary between the disconnected subtours associated to each truck. The model (2.1)–(2.7) was applied to a very small hypothetical problem. The LP relaxation of the model was solved using IBM's MPS system. The constraints

$$\sum_{k \in K} \sum_{\{v_j : (v_j, v_i) \in E\}} x_{jik} \geqslant \begin{cases} (|E(v_i)| + 1)/2 & \text{if } |E(v_i)| \text{ is odd} \\ |E(v_i)|/2 & \text{if } |E(v_i)| \text{ is even} \end{cases} \quad (v_i \in V), \tag{2.8}$$

$$\sum_{k \in K} \sum_{\{v_j : (v_i, v_j) \in E\}} x_{ijk} \geqslant \begin{cases} (|E(v_i)| + 1)/2 & \text{if } |E(v_i)| \text{ is odd} \\ |E(v_i)|/2 & \text{if } |E(v_i)| \text{ is even} \end{cases} \quad (v_i \in V), \tag{2.9}$$

$$x_{ijk} \geqslant p \quad ((v_i, v_j) \in E, k \in K), \tag{2.10}$$

were added to the LP relaxation in an effort to obtain integrality. Constraints (2.8) and (2.9) ensure that the number of trucks entering and leaving each node, respectively, satisfies the minimum requirement depending on the degree of the node. Integer solutions are also searched for by varying the value of the parameter p in constraints (2.10).

Haghani and Qiao [41] proposed a heuristic decomposition approach for routing salt spreader trucks in Calvert County, Maryland. The approach takes into account service connectivity and vehicle capacity. Moreover, all road segments are treated as two-lane, two-way segments and both lanes can be serviced by the same spreader truck. The approach uses a decomposition of the problem into two subproblems: allocation of road segments to salt spreader trucks and routing. This approach is similar to cluster first, route second methods for vehicle routing problems. In a first step, the problem of assigning road segments to salt spreader trucks is formulated as a capacitated minimum spanning tree problem. Consider the undirected graph $G = (V, E)$ where $V = \{v_0, v_1, \ldots, v_n\}$ is the vertex set and $E = \{(v_i, v_j) : v_i, v_j \in V \text{ and } i \neq j\}$ is the edge set. Let $H = (W \cup \{w_r\}, F_1 \cup F_2)$ be an auxiliary graph such that the vertex set W has a vertex w_j for each edge in the undirected graph G, w_r is the root node, the edge set F_1 contains an edge between w_i and w_j if the corresponding edges in G have a vertex in common, and the edge set F_2 contains an edge
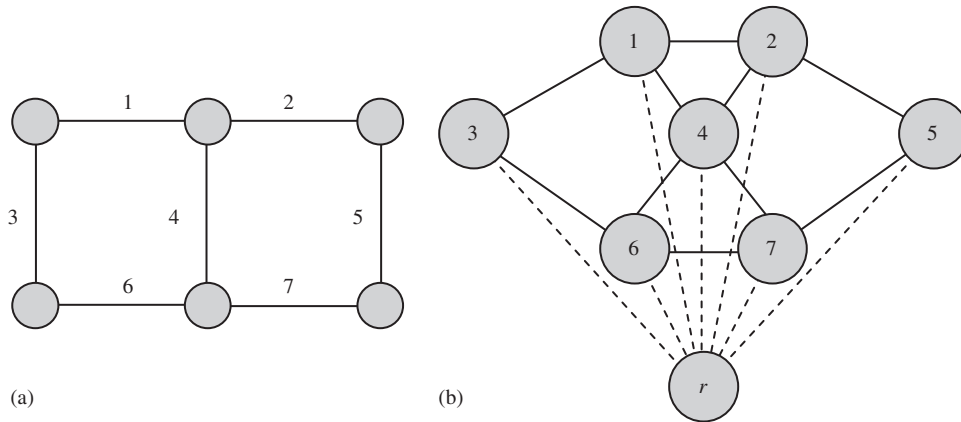
Fig. 3. Example of a network transformation: (a) a graph G; (b) the corresponding graph H.

between the root w_r and each vertex $w_i \in W$. An example of such a network transformation is illustrated in Fig. 3. The numbers in G correspond to edge numbers. The two sets of edges F_1 and F_2 are shown as solid lines and dashed lines, respectively.

Every tree rooted at vertex w_r in graph H specifies a set of road segments to service in the same route that satisfies the service connectivity requirement. With every vertex $w_i \in W$ is associated a nonnegative amount of required salt q_i equal to the total amount of salt to spread on both sides of the corresponding edge in G. Then the subproblem of allocating road segments to salt spreader trucks amounts to choosing a spanning tree rooted at vertex w_r in H such that the total amount of required salt in each subtree does not exceed the truck salting capacity Q. The objective is to minimize the required truck fleet size. For every edge $(w_i, w_j) \in F_1 \cup F_2$, let z_{ij} be a binary variable equal to 1 if and only if edge (w_i, w_j) is included in the optimal spanning tree, and let also f_{ij} be a nonnegative real variable representing the flow on edge (w_i, w_j) from w_i to w_j, expressed as salt units. The formulation is then:

Minimize

$$\sum_{w_i \in W} z_{ir} \tag{2.11}$$

subject to

$$\sum_{\{w_j : (w_i, w_j) \in F_1 \cup F_2\}} z_{ij} = 1 \quad (w_i \in W), \tag{2.12}$$

$$\sum_{\{w_j : (w_i, w_j) \in F_1 \cup F_2\}} f_{ij} - \sum_{\{w_j : (w_j, w_i) \in F_1\}} f_{ji} = q_i \quad (w_i \in W), \tag{2.13}$$

$$f_{ir} \leqslant Q z_{ir} \quad (w_i \in W), \tag{2.14}$$

$$z_{ij} \in \{0, 1\} \quad ((w_i, w_j) \in F_1 \cup F_2), \tag{2.15}$$

$$f_{ij} \geqslant 0 \quad ((w_i, w_j) \in F_1 \cup F_2). \tag{2.16}$$

The objective function (2.11) seeks to minimize the total number of edges incident to the root node w_r, which corresponds to the number of subtrees and translates into the number of trucks. (Note that the

objective followed in a capacitated minimum spanning tree problem is usually to minimize the sum of costs of the edges in the spanning tree.) Constraints (2.12) assure that each node in H (except the root node) is connected to some other node. Summing up these constraints implies that a spanning tree of H contains exactly $|W|$ edges. Flow conservation at every node (except the root node) is guaranteed by constraints (2.13). Constraints (2.14) impose a limit on the flow on each edge incident to the root node. Finally, all z_{ij} variables are restricted to be binary, while f_{ij} variables must assume nonnegative values. The authors also considered the following objectives for the allocation of road segments to spreader trucks: the minimization of the distance covered by deadheading trips (for a fixed truck fleet size), and a weighted combination of the number of trucks needed and the distance covered by deadheading trips. A distance approximation requiring that each truck traverses a road segment (with or without servicing it) twice from both directions is used to estimate the distance covered by deadheading trips.

Once the set R of required road segments belonging to a vehicle route is determined, routes are then constructed by solving a series of directed rural postman problems. Let $G' = (V, A_1)$ be a directed graph constructed from G where the arc set A_1 contains arcs of opposite direction for each edge (v_i, v_j) in E. With every arc $(v_i, v_j) \in A_1$ is associated a nonnegative length c_{ij}. Let $A_2 = \{(v_i, v_j), (v_j, v_i) \in A_1 : (v_i, v_j) \in R \text{ or } (v_j, v_i) \in R\}$ be a set of two-lane, two-way roads (one lane each way) that can be serviced only once from one direction, and let also $A_3 = \{(v_i, v_j), (v_j, v_i) \in A_1 : (v_i, v_j) \in R \text{ and } (v_j, v_i) \in R\}$ be a set of two-lane, two-way roads that need to be serviced twice from both directions, $A_2 \cup A_3 = R$, $A_2 \cap A_3 = \emptyset$. For every arc $(v_i, v_j) \in A_1$, let x_{ij} be a binary variable equal to 1 if and only if arc (v_i, v_j) is traversed (with or without servicing it), and let f_{ij} be a nonnegative real variable representing the flow on arc (v_i, v_j). Haghani and Qiao [41] proposed the following linear, mixed integer program for the problem of determining a minimum cost covering tour for the set R of required road segments assigned to a given salt spreader truck.

Minimize

$$\sum_{(v_i, v_j) \in A_1} c_{ij} x_{ij} \tag{2.17}$$

subject to

$$\sum_{\{v_j : (v_j, v_i) \in A_1\}} x_{ji} - \sum_{\{v_j : (v_i, v_j) \in A_1\}} x_{ij} = 0 \quad (v_i \in V), \tag{2.18}$$

$$x_{ij} + x_{ji} = 1 \quad ((v_i, v_j), (v_j, v_i) \in A_2), \tag{2.19}$$

$$x_{ij} = 1 \quad ((v_i, v_j) \in A_3), \tag{2.20}$$

$$\sum_{\{v_j : (v_i, v_j) \in A_1\}} f_{ij} - \sum_{\{v_j : (v_j, v_i) \in A_1\}} f_{ji} = \sum_{\{v_j : (v_i, v_j) \in A_1\}} x_{ij} \quad (v_i \in V \setminus \{v_0\}), \tag{2.21}$$

$$f_{ij} \leqslant |V|^2 x_{ij} \quad ((v_i, v_j) \in A_1), \tag{2.22}$$

$$f_{ij} \geqslant 0 \quad ((v_i, v_j) \in A_1) \tag{2.23}$$

$$x_{ij} \in \{0, 1\} \quad ((v_i, v_j) \in A_1). \tag{2.24}$$

The objective function (2.17) minimizes the total distance traveled. Constraints (2.18) ensure that the indegree of every vertex is equal to its outdegree. Constraints (2.19) and (2.20) assure that each arc is traversed as required. Constraints (2.21)–(2.23) are the subtour elimination constraints [42]. Constraints (2.21) state that the outflow minus inflow of a node must equal the number of outgoing arcs of the node

that are traversed. Constraints (2.22) specify that the flow on an arc can be positive only if the arc is in the route. Finally, all f_{ij} variables must assume nonnegative values, while x_{ij} variables are restricted to be binary. The model (2.11)–(2.16) for the problem of assigning road segments to spreader trucks was solved using CPLEX. The authors did not, however, solve the model (2.17)–(2.24). Computational tests on three subnetworks of the existing road network of salting operations in Calvert County showed that the model with the objective function of minimizing the distance covered by deadheading trips obtained the best solutions within 15 min.

2.2. Composite methods

When the vehicle capacity constraints are dominated by the time limit constraint for spreading completion, each route based at a depot location should be covered by a different spreader to complete the salting treatment within the time limit at minimum cost. Conversely, when the time limit constraint is dominated by the capacity constraints, each spreader should cover more than one route within the time limit to minimize the fleet size. A heuristic dealing with this version of the problem was proposed by Xin and Eglese [43]. The problem considered is to design a set of spreader routes so as to minimize the number of spreaders and the distance covered by deadheading trips, while satisfying the capacities of the spreaders and the time limit for spreading completion. The capacities of the spreaders are again given as maximum distances which can be spread in one route. Each required road segment must be spread exactly once. The heuristic also considers the presence of multiple depots but each route and each spreader must start and end at the same depot. This problem is solved with a heuristic decomposition strategy that iterates between two subproblems until a given number of iterations have been executed or until no further improvement is possible. The first subproblem determines a set of spreader routes based at each depot location such that the total distance of all required road segments to service in a route emanating from a depot does not exceed the maximum distance that can be covered by a spreader based at the same depot, while minimizing the distance covered by deadheading trips. Given the planned spreader routes, the second subproblem partitions the set of routes based at a given depot location into a mutually exhaustive and exclusive collection of subsets of routes so that each subset of routes can be covered within the time limit and the routes included in a subset are associated with the same spreader type, while minimizing the number of spreaders. The first subproblem for each depot location is solved with a heuristic, called *cycle-node scanning algorithm*, analogous to the path-scanning algorithm proposed by Golden et al. [30] for the capacitated arc routing problem. The authors formulated the second subproblem as a bin packing problem with items representing spreader routes and bins representing spreaders or subsets of routes. The second subproblem for each depot location is solved using a heuristic similar to the first-fit decreasing heuristic for the bin packing problem, called the *merging-combining algorithm*. The heuristic for the salt spreading problem studied by Xin and Eglese is summarized in Fig. 4.

This heuristic is based on the construction of an auxiliary graph, called *cyclenode graph*, suggested by Male and Liebman [44] to solve the routing problem of waste collection vehicles. Let G be an undirected graph. In a first step, required road segments in G are specified as the ones of highest priority and the time window during which these road segments must be spread is replaced by a time limit for service completion (not counting the time to return to the depot after finishing spreading). If G is not Eulerian, then copies of some edges must be added to G so that the augmented graph G' becomes Eulerian. The Eulerian graph G' can be obtained from G by solving a perfect matching problem on a graph whose vertices are the odd-degree vertices of G [45]. A lower class road segment is treated as a required road

1. *Cycle-node scanning algorithm*
   a. Let G be an undirected graph. If G is not Eulerian, transform G into an Eulerian graph by solving a minimum cost matching problem. Construct the cyclenode network G'' from G.
   b. For each depot vertex $v_d \in V_2$ in G'', execute the following.
      i) Choose a cycle vertex v_i adjacent to the depot vertex v_d in G'' that is not yet added to a subtree. If no such cycle exists, go to Step iv). Add (v_d, v_i) to T and set $v_k := v_i$ (T denotes a subtree rooted at depot vertex v_d in G'').
      ii) Choose a cycle vertex v_j adjacent to v_k in G'' that is not yet added to a subtree, according to one of the above mentioned cycle-selection rules such that the vehicle capacity is satisfied. If no such cycle vertex exists, go to Step iii). Add (v_k, v_j) to T, set $v_k := v_j$, and repeat Step ii).
      iii) Choose a cycle vertex v_j of T according to one of the above mentioned searching modes. If no such cycle vertex exists, go to Step i). Set $v_k := v_j$, and go to Step ii).
      iv) Choose another depot vertex and go to Step i).

2. *Merging-combining algorithm*
   a. Select two route trees T and T' rooted at the same depot or not.
   b. Combine T and T' if the capacity and time limit constraints permit (If these constraints are broken slightly, move the service of a leaf node serviced in T or T' to another route tree, while satisfying the capacity and time limit constraints. Repeat this "pruning" process until T and T' can feasibly be combined or no leaf node serviced in T or T' can be moved to another route).
   c. Repeat Steps a and b until no more route trees can be combined.
   d. For each depot vertex $v_d \in V_2$ in G'', execute the following.
      i) Let R be the set of routes based at depot v_d. Let R_H be the route of highest duration in R. Set $i = 1$ and $S_i := \{R_H\}$ (S_i denotes the ith subset of routes based at depot v_d). Remove R_H from R.
      ii) Choose the route R_L of lowest duration in R. If $R := \varnothing$, stop. If the time limit permits and if R_L is based at the same depot as the routes of S_i, add R_L to S_i and go to Step iii). Otherwise, set $i = i + 1$ and go to Step iv).
      iii) Remove R_L from R and return to Step ii).
      iv) Choose the route R_k of highest duration in R. If $R := \varnothing$, stop. Otherwise, set $R_H := R_k$, $S_i := \{R_H\}$, remove R_H from R, and return to Step ii).

Fig. 4. Heuristic for the salt spreading problem [43].

segment if it is included in the set of edges added to G to create the Eulerian graph G'. Road segments are then grouped into cycles by partitioning the Eulerian graph G' into small cycles using a "checkerboard pattern" to obtain a set of faces with associated cycles. An algorithm for decomposing G into such cycles is given by Eglese [33]. The cyclenode graph $G'' = (V_1 \cup V_2, E_1 \cup E_2)$ is constructed from the Eulerian graph G' where the vertex sets V_1 and V_2 have a vertex for each cycle and for each vehicle depot in the Eulerian graph G', respectively, and E_1 and E_2 are two edge sets defined as follows. The edge set E_1 contains an edge between two vertices in V_1 if the corresponding cycles in G' have a vertex in common and the edge set E_2 contains an edge between two vertices in $V_1 \cup V_2$ if the depot associated with the vertex in V_2 is the closest depot to any node on the cycle associated with the vertex in V_1. All edges in E_1 have zero cost. With every edge $(v_d, v_i) \in E_2$ is associated a cost c_{di} equal to twice the length of a shortest chain in the original graph G between the depot v_d and the closest node in the cycle to the depot. The construction of the cyclenode graph is illustrated in Fig. 5. Vehicle depots correspond to black vertices and the two sets of edges E_1 and E_2 are shown as solid lines and dashed lines, respectively.

Xin and Eglese use two cycle-selection rules in Step (ii) for attempting to construct a feasible route. Given a subtree T and a leaf node v_i of T, one can choose the cycle vertex v_j adjacent to v_i in the cyclenode
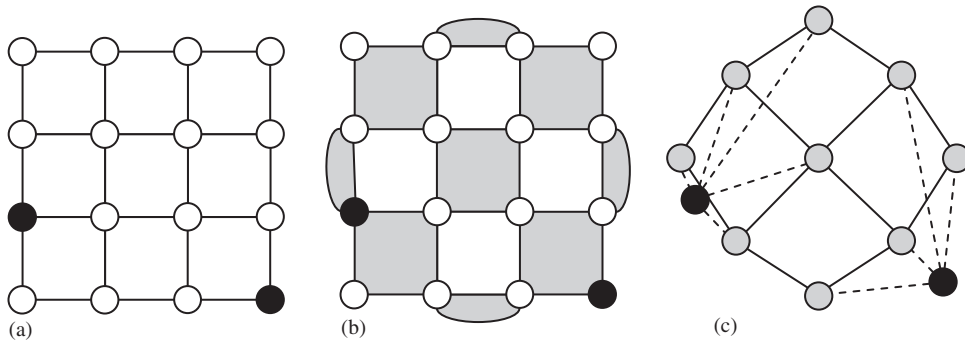
Fig. 5. An example of a cyclenode construction. (a) Original graph G, (b) Enterian graph G', and (c) Cyclenod graph G''.

network G'' so that the remaining vehicle capacity in T is either minimized or maximized. The authors also use two searching modes in Step (iii) to identify a cycle vertex in T from which the route can be extended. They select the cycle vertices in either a first-in, first-out order or in a last-in, first-out order.

The authors proposed a modified version of the cycle-node scanning algorithm where the cycle-selection rule is chosen randomly at each step and the types of spreaders based at each depot location are chosen randomly at each run of the algorithm, following given probability distributions. The authors also developed a somewhat more sophisticated merging-combining algorithm that treats the time limit constraint as a soft constraint by adding to the objective function a term penalizing time limit constraint violations. A *soft time limit constraint* allows the spreaders to complete the salting treatment before or after the time limit. The soft time limit is chosen at random from a uniform distribution which is within a given percentage of the time limit. The heuristic was tested on actual data from the County of Lancashire in England. Computational tests showed that the modified versions of the two algorithms obtained the best solutions but required more time than the original heuristic. The largest instance solved contained 380 road segments, 141 cycle vertices, and three depots.

Haghani and Qiao [46] proposed a model and a heuristic for routing salt spreader trucks in Calvert County, Maryland. The model is a linear, mixed integer, capacitated arc routing problem with time windows and additional side constraints. Besides flow conservation equations and vehicle capacities, the model incorporates priority classes of roadways, maximum route durations, and level of service policies. In particular, each two-lane, two-way segment (one lane each way) is associated with two time intervals, one for each direction, called *time windows*, within which salting must be completed. High-class road segments are given strict service time windows that must be before the time intervals associated with low-class road segments. The number of times required for servicing each two-lane, two-way segment from each direction is also taken into consideration and guarantees that the appropriate level of service is achieved. Finally, most multi-lane, two-way road segments with more than two lanes in each direction must be serviced more than once, but spreader trucks can usually make one single pass over a two-lane, two-way road segment in either direction to spread salt covering both lanes.

The model is based on the formulation proposed by Golden and Wong [42] for the undirected capacitated arc routing problem. Let $G = (V, E)$ be an undirected graph where $V = \{v_0, v_1, \ldots, v_n\}$ is the vertex set and $E = \{(v_i, v_j) : v_i, v_j \in V \text{ and } i \neq j\}$ is the edge set. The depot is represented by the node v_0. Let K be the set of vehicles of multiple types. For every vehicle $k \in K$, let W_k and t_k be the salting capacity and

the maximum route duration of vehicle k, respectively. Let $G' = (V, A_1)$ be a directed graph constructed from G where the arc set A_1 contains arcs of opposite direction for each edge (v_i, v_j) in E. With every arc $(v_i, v_j) \in A_1$ are associated a nonnegative length c_{ij}, a nonnegative number of times n_{ij} arc (v_i, v_j) should be spread, a nonnegative amount of required salt q_{ij} for each pass, and a time window $[0, b_{ij}]$ within which spreading must be completed. Note that since $n_{ij} \geqslant 0$, the arc routing problem considered is a rural postman problem. For every arc $(v_i, v_j) \in A_1$ and for every vehicle $k \in K$, let x_{ijk} be a binary variable equal to 1 if and only if arc (v_i, v_j) is *traversed* by vehicle k while deadheading, let y_{ijk} be a binary variable equal to 1 if and only if arc (v_i, v_j) is *serviced* by vehicle k, and let t^s_{ijk} be a nonnegative variable representing the starting time of service or traversal of arc (v_i, v_j) by vehicle k. Note that an arc cannot be serviced or deadheaded more than once by the same vehicle. In addition, with every arc $(v_i, v_j) \in A_1$ and with every vehicle $k \in K$ are associated two positive durations t_{ijk} and t'_{ijk} for the service and traversal of arc (v_i, v_j) by vehicle k, respectively. The formulation is then as follows.

Minimize

$$\sum_{k \in K} \sum_{(v_i, v_j) \in A_1} c_{ij} x_{ijk} \tag{2.25}$$

subject to

$$\sum_{\{v_j : (v_j, v_i) \in A_1\}} x_{jik} - \sum_{\{v_j : (v_i, v_j) \in A_1\}} x_{ijk} + \sum_{\{v_j : (v_j, v_i) \in A_1\}} y_{jik} - \sum_{\{v_j : (v_i, v_j) \in A_1\}} y_{ijk} = 0$$
$$(v_i \in V, k \in K), \tag{2.26}$$

$$\sum_{k \in K} (y_{ijk} + y_{jik}) = 1 \quad ((v_i, v_j), (v_j, v_i) \in A_2), \tag{2.27}$$

$$\sum_{k \in K} y_{ijk} = n_{ij} \quad ((v_i, v_j) \in A_1 \backslash A_2), \tag{2.28}$$

$$\sum_{(v_i, v_j) \in A_1} q_{ij} y_{ijk} \leqslant W_k \quad (k \in K), \tag{2.29}$$

$$t^s_{ijk} + t_{ijk} y_{ijk} + t'_{ijk} x_{ijk} \leqslant t^s_{jlk} + M(1 - x_{ijk} - y_{ijk})$$
$$((v_i, v_j), (v_j, v_l) \in A_1, k \in K), \tag{2.30}$$

$$t^s_{ijk} \leqslant b_{ij} + M(1 - y_{ijk}) \quad ((v_i, v_j) \in A_1, k \in K), \tag{2.31}$$

$$\sum_{(v_i, v_j) \in A_1} (t_{ijk} y_{ijk} + t'_{ijk} x_{ijk}) \leqslant t_k \quad (k \in K), \tag{2.32}$$

$$\sum_{\{v_j : (v_0, v_j) \in A_1\}} (x_{0jk} + y_{0jk}) = 1 \quad (k \in K), \tag{2.33}$$

$$\sum_{\{v_i : (v_i, v_0) \in A_1\}} (x_{i0k} + y_{i0k}) = 1 \quad (k \in K), \tag{2.34}$$

$$\sum_{v_i, v_j \in S} (x_{ijk} + y_{ijk}) \leqslant |S| - 1 + |V|^2 u_k^S \quad (S \subseteq V \setminus \{v_0\}, \ S \neq \emptyset, \ k \in K), \qquad (2.35)$$

$$\sum_{v_i \in S} \sum_{v_j \notin S} (x_{ijk} + y_{ijk}) \geqslant 1 - w_k^S \quad (S \subseteq V \setminus \{v_0\}, \ S \neq \emptyset, k \in K), \qquad (2.36)$$

$$u_k^S + w_k^S \leqslant 1 \quad (S \subseteq V \setminus \{v_0\}, \ S \neq \emptyset, \ k \in K), \qquad (2.37)$$

$$u_k^S, w_k^S \in \{0, 1\} \quad (S \subseteq V \setminus \{v_0\}, \ S \neq \emptyset, k \in K), \qquad (2.38)$$

$$t_{ijk}^s \geqslant 0 \quad ((v_i, v_j) \in A_1, \ k \in K), \qquad (2.39)$$

$$x_{ijk}, y_{ijk} \in \{0, 1\} \quad ((v_i, v_j) \in A_1, k \in K). \qquad (2.40)$$

The objective function (2.25) minimizes the total distance covered by deadheading trips. Constraints (2.26) are flow conservation equations for each vehicle. Define $A_2 = \{(v_i, v_j), (v_j, v_i) \in A_1 : n_{ij} = 1/2$ and $n_{ji} = 1/2\}$ as the set of two-lane, two-way roads (one lane each way) that can be serviced only once from one direction. Constraints (2.27) and (2.28) state that each arc is serviced as required. Constraints (2.29) guarantee that the capacity of each vehicle is never exceeded. Constraints (2.30) ensure that the time when each vehicle completes the service or traversal of an arc is ahead of the starting time of service or traversal of the next arc (M is a sufficiently large positive number). Constraints (2.31) are the time window restrictions. Maximum route duration for each vehicle is imposed by constraints (2.32). Constraints (2.33) and (2.34) force all vehicles to start and end at the depot, respectively. Constraints (2.35)–(2.38) prohibit the formation of disconnected subtours but allow tours that include two or more closed cycles. These constraints are explained in detail by Golden and Wong [42]. Finally, all t_{ijk}^s variables must assume nonnegative values, while x_{ijk} and y_{ijk} variables are restricted to be binary.

Haghani and Qiao [46] developed a four-stage solution procedure for a less difficult version of the problem where time windows are not considered and all vehicles have the same capacity. The four-stage heuristic procedure is presented in detail in Fig. 6.

In the first stage, feasible vehicle routes are constructed one at a time as follows. Initial service directions are randomly selected for two-lane, two-way road segments (one lane each way) that can be serviced from one direction. An initial route is created by first determining the furthest required arc from the depot. Then, the nearest required arcs from the route are sequentially inserted into the route as long as vehicle capacity and maximum route duration permit. The arc insertion procedure is analogous to the ADD algorithm for the undirected rural postman problem [47]. The route generation process is repeated until all required arcs are part of one route or more according to the level of service policy. The other stages are improvement procedures that attempt to reduce the total distance traveled while satisfying the vehicle capacity and maximum route duration constraints. The service directions of the two-lane, two-way road segments can be changed in the improvement stages.

The second stage tries to improve the solution obtained at the first stage by applying three post-optimization procedures successively. The first procedure, called Augment, selects two routes and discards the shorter route if its required arcs can be serviced by the longer route. The second procedure, called Merge, combines two routes at the common node that provides the best objective function improvement. The Augment and Merge procedures are clearly analogous to the approaches used in the "Augment" and "Merge" phases of the augment-merge algorithm developed by Golden et al. [30] for the capacitated Chinese postman problem. The last post-optimization procedure, called Delete and Insert, deletes a required arc from a route, reduces the length of the route by covering the same set of required arcs, but

1. *Initial solution*
    a. For each pair of arcs (v_i, v_j), $(v_j, v_i) \in A_2$, select a service direction arbitrarily (say from v_i to v_j), set $n_{ij} = 1$, and set $n_{ji} = 0$.
    b. Let sc_{ij} be the length of the shortest path between vertex v_i and vertex v_j in G'. Determine the farthest required arc $(v_i, v_j) \in A_1$ from the depot v_0 with $n_{ij} \geq 1$. The arc (v_i, v_j) is the farthest arc from the depot if it yields the maximum value $sc_{0i} + sc_{j0}$.
    c. Create a feasible vehicle route servicing (v_i, v_j) (i.e. a route made of a shortest path between the depot and v_i, the required arc (v_i, v_j), and a shortest path between v_j and the depot). Set $n_{ij} = n_{ij} - 1$.
    d. Let SP_{ij} be the shortest path between vertex v_i and vertex v_j in G'. If every arc on the route is serviced, then identify a required arc (v_k, v_l) that does not appear on the route and a vertex v_i on the route yielding the minimum value of $sc_{ik} + sc_{li}$, add the circuit $SP_{ik} \cup \{(v_k, v_l)\} \cup SP_{li}$ to the route, and set $n_{kl} = n_{kl} - 1$. Otherwise, if some arcs on the route are not serviced, identify a path $P = (v_s, \ldots, v_t)$ of non-required arcs on the route and a required arc (v_k, v_l) that does not appear on the route yielding the best objective function improvement by replacing P by the path $SP_{sk} \cup \{(v_k, v_l)\} \cup SP_{lt}$, replace P by this path, and set $n_{kl} = n_{kl} - 1$.
    e. Repeat step d as long as vehicle capacity and maximum route duration constraints permit.
    f. If $n_{ij} = 0$ for each required arc $(v_i, v_j) \in A_1$, go to step 2. Otherwise, return to step b.

2. *First improvement*
    a. Augment algorithm
        i) Select two routes R_1 and R_2.
        ii) Let R_1 be the longest route. As long as vehicle capacity and maximum route duration permit, change the status of traversed arcs on R_1, from non-serviced to serviced, if these arcs are serviced by the shorter route R_2. Remove R_2 if all serviced arcs on R_2 are now serviced by the longer route R_1.
        iii) Return to the beginning of step a until no improvement can be obtained.
    b. Merge algorithm
        i) Select two routes R_1 and R_2.
        ii) Identify a common vertex v_C of R_1 and R_2 yielding the best objective function improvement by merging R_1 and R_2 at v_C while satisfying vehicle capacity and maximum route duration constraints. Combine the routes R_1 and R_2.
        iii) Return to the beginning of step b until no improvement can be obtained.
    c. Delete and Insert algorithm
        i) Select two routes R_1 and R_2.
        ii) Identify a serviced arc (v_i, v_j) in R_1 yielding the best objective function improvement by deleting (v_i, v_j) from R_1 (by means of the "Delete" phase described in step iii) and by inserting (v_i, v_j) into R_2 (by means of the "Insert" phase described in step iv) while satisfying vehicle capacity and maximum route duration constraints.
        iii) Delete (v_i, v_j) from R_1 and then try to get a shorter route R_1 by means of the two following strategies:
            ▪ a path P of non-required arcs in R_1 can be reduced by replacing P by a shortest path between the endpoints of P;
            ▪ if a serviced arc is traversed twice in route R_1 in the same direction, interchange the service and deadhead of that arc and try to reduce the length of R_1 by applying the former strategy.
        iv) Insert (v_i, v_j) into R_2 by means of the following procedure. If every arc on R_2 is serviced, then identify a vertex v_k on R_2 yielding the minimum value of $sc_{ki} + s_{jk}$ and add the circuit $SP_{ki} \cup \{(v_i, v_j)\} \cup SP_{jk}$ to R_2. Otherwise, if some arcs on R_2 are not serviced, identify a path $P = (v_s, \ldots, v_t)$ of non-required arcs on R_2 yielding the best objective function improvement by replacing P by the path $SP_{si} \cup \{(v_i, v_j)\} \cup SP_{jt}$ on R_2 and replace P by this path.
        v) Return to the beginning of step c until no improvement can be obtained.

3. *Second improvement*
    a. Try to improve the solution by means of the Merge algorithm.
    b. Try to improve the solution by means of the Delete and Insert algorithm.
    c. Link Exchange algorithm
        i) Select two routes R_1 and R_2.
        ii) Identify a serviced arc (v_i, v_j) in R_1 and a serviced arc (v_k, v_l) in R_2 yielding the best objective function improvement by deleting (v_i, v_j) from R_1 and (v_k, v_l) from R_2 (by means of the "Delete" phase of the Delete and Insert algorithm) and by inserting (v_i, v_j) into R_2 and (v_k, v_l) into R_1 (by means of the "Insert" phase of the Delete and Insert algorithm) while satisfying vehicle capacity and maximum route duration constraints.
        iii) Delete (v_i, v_j) from R_1 and (v_k, v_l) from R_2 and insert (v_i, v_j) into R_2 and (v_k, v_l) into R_1.
        iv) Return to the beginning of step c until no improvement can be obtained.

4. *Third improvement*
    a. Try to improve the solution by means of the Delete and Insert algorithm.
    b. Try to improve the solution by means of the Link Exchange algorithm.

Fig. 6. The four-stage heuristic [46].

not necessarily in the same order, and inserts the arc into another route. This procedure is similar to the DROP-ADD algorithm used for the undirected rural postman problem [47]. However, the DROP-ADD algorithm removes a required edge from a route, shortens the solution, and reinserts it into the same route.

The third stage then tries to improve the solution obtained at the second stage by applying the Merge, Delete and Insert, and Link Exchange algorithms consecutively. The Link Exchange algorithm is similar to the Delete and Insert algorithm except that a required arc is removed from each route and inserted in the other route. A detailed analysis of the performance of the Augment, Merge, Insert, and Link Exchange algorithms and combinations of these was performed by Qiao [48]. As a complement of the Link Exchange algorithm, Qiao [49] proposed two improvement procedures in which the number of arcs to remove or to insert into a route is randomly generated (Random Link Exchange) or the arcs to delete or to insert are randomly chosen and the number of arcs removed or inserted into a route can be more than two (Multiple Link Exchange).

Finally, the fourth stage tries to improve the solution obtained at the third stage by applying the Delete and Insert and Link Exchange algorithms successively. The heuristic was tested on three subnetworks of the existing road network of salting operations in Calvert County, Maryland, with up to 42 nodes and 104 edges. The system reduced the distance covered by deadheading trips by 15–54% over the solution in use by the County with fewer vehicles used (up to three) and with computing times less than 2 min.

2.3. Metaheuristics

We are aware of three types of metaheuristic that have been applied to vehicle routing problems related to spreading operations: simulated annealing, tabu search, and elite route pool.

A simulated annealing approach was used by Eglese [33] to address the salt spreader routing problem of Lancashire County Council, UK. The approach takes into account the service hierarchy of the network, given in terms of time windows, as well as the capacities of the spreaders and the salt application rates, given in terms of maximum distances which can be spread in one route. For each road segment, both sides must be spread in one single pass. The approach can also deal with multiple depot locations but each route must start and end at the same depot. The simulated annealing approach is again based on the concept of cyclenode network (see Fig. 5). Each subtree (tree) rooted at a depot node in the cyclenode graph G'' corresponds to a route (set of routes) in the original graph G. A solution of the salt spreader routing problem is thus defined as a spanning collection of trees of the graph G'' rooted at the depot nodes. An initial solution is obtained by means of a greedy algorithm similar to the Clarke and Wright [37] savings procedure for the capacitated vehicle routing problem. When two subtrees or trees, one containing cycle vertex v_i incident to depot v_d and the other containing cycle vertex v_j adjacent to v_i in the graph G'', can be combined so that the maximum spreading distance and time limit constraints are satisfied, a distance saving $s_i = c_{di}$ is generated. The algorithm is described in Fig. 7.

Solutions violating the maximum spreading distance and time limit constraints are allowed during the simulated annealing search. These intermediate infeasible solutions are however penalized through the minimization of an artificial objective function $f(S) = R(S) + \alpha_D E_D(S) + \alpha_T E_T(S)$, where $R(S)$ is the total number of routes in solution S, $E_D(S)$ is the sum over all routes of S of the excess distance with respect to maximum spreading distance, $E_T(S)$ is the sum over all routes of S of the time exceeding time limit for service completion, and α_D and α_T are the corresponding penalty parameters. A neighbor solution S' is obtained from a solution S by moving the service of a leaf node v_i from a route tree T to another route tree T', by replacing the edge incident to v_i in T by the edge joining v_i to the closest depot,

1. *Savings computation*
   a. For each cycle vertex $v_i \in V_1$, compute the *saving* $s_i = c_{di}$.
   b. For each cycle vertex $v_i \in V_1$, create a route made of the edge (v_d, v_i). The resulting spanning collection of trees corresponds to the subgraph induced by the set of edges in E_2 (At this stage, if the maximum spreading distance and time limit constraints are not satisfied, partition the graph G into two or three smaller subgraphs, construct a cyclenode network for each subgraph, compute the savings, and create a route for each cycle vertex).
   c. Order the savings from largest to smallest.

2. *Best feasible combination*
   a. Starting at the top of the savings list and moving downwards, execute the following. Given a saving s_i, determine whether there exists two subtrees or trees, one containing cycle vertex v_i incident to the depot vertex v_d and the other containing cycle vertex v_j adjacent to v_i in the graph G'' that can be combined so that the maximum spreading distance and time limit constraints are satisfied. If so, combine these two subtrees or trees by deleting (v_d, v_i) and introducing (v_i, v_j). If the maximum spreading distance and time limit constraints are not satisfied, move to the next largest saving in the list.

Fig. 7. Greedy algorithm for the salt spreading problem [33].

or by combining two route trees. The service of v_i is removed from T by deleting the edge incident to v_i while it is introduced into T' by adding to T' the edge joining v_i to a cycle vertex serviced in T' that is adjacent to v_i in the graph G''. Two route trees T and T' are combined by adding to S the edge joining v_i to a cycle vertex serviced in T' that is adjacent to v_i in G'' and by removing from S the highest-cost edge of the two edges incident to the root nodes. The cooling schedule employed by Eglese is typical of what is commonly done in simulated annealing. The temperature is decreased as a step function: initially, the temperature is set equal to a value chosen experimentally and is multiplied by a constant factor α $(0 < \alpha > 1)$ after every K iterations, where K is a fixed multiple of the size of the neighborhood. The search ends when the number of routes has not decreased for at least five consecutive cycles of K iterations and the number of accepted moves for the more recent cycle of K iterations has been less than a given percentage of K. To account for the service hierarchy of the network, the problem for each roadway class is solved using the cyclenode construction as well as the simulated annealing approach. Routes are then assigned to spreaders so that each covers a route of highest priority first, followed by a route of lower priority, and so on.

The heuristic decreased the number of depots by more than 50% over the solution in use by the County without increasing the fleet size or the number of routes. The heuristic was also used to evaluate the impacts of fleet size and depot location changes on service levels and costs. Eglese and Li [50] evaluated the efficiency of the heuristic by computing a ratio of salting distance to the total distance travelled. This ratio varied from 56% to 80% in 31 County divisions. An upper bound on this ratio can be obtained by evaluating it for the unconstrained Chinese postman solution. Computational tests on four subsets of roads in Lancashire showed that even the upper bound rarely exceeded 70%, a result that the authors explained by noting the large incidence of "T-junctions" (nodes at which three roads meet) in the rural areas considered. Such nodes of degree 3 generate important deadheading trips reflected in the low efficiency ratios. In contrast, urban areas forming a square grid pattern have odd nodes only on the borders of the area and, as they are close to each other, the artificial links added to join them in pairs are relatively short, thus yielding higher ratios. In practice, in addition to the topology of the network, the sector design can also affect the routing efficiency measure.

Benson et al. [51] proposed the SIRMM system (Snow and Ice Removal Monitoring and Management) to assist planners in developing feasible routes for salt spreading maintenance vehicles. The system can deal with different vehicle capacities, service and deadhead speeds, different times for service completion, and one or multiple passes per road segment depending on lane configurations and road widths. The problem is modeled as a nonlinear mixed integer program in which vehicle capacities and time limit constraints are treated as soft constraints. Soft time limit constraints allow a vehicle to complete spreading road segments before or after its time limit. Similarly, soft vehicle capacity constraints allow a spreader vehicle to be assigned to a route requiring more salt than the vehicle can contain. As a result, the vehicle incurs a lower level of service.

Let $G = (V, A)$ be a directed graph where $V = \{v_1, \ldots, v_n\}$ is the vertex set and $A = \{(v_i, v_j) : v_i, v_j \in V, i \neq j\}$ is the arc set and let $R \subseteq A$ be the subset of required arcs. It is convenient to also denote an arc by a. With every arc $(v_i, v_j) \in R$ are associated a traversal time t_{ij} of servicing arc (v_i, v_j) and a positive number of passes l_{ij} required to service arc (v_i, v_j). For every pair of required arcs $(v_i, v_j), (v_p, v_q) \in R$, let sp_{ijpq} be the traversal time of the shortest path between arc (v_i, v_j) and arc (v_p, v_q) in G. Let H be the set of spreader routes. The number of spreader vehicles is given and each route must be covered by exactly one spreader. Thus, the cardinality of H corresponds to the number of spreader vehicles available and there may be routes to which no required arcs are assigned in a feasible solution. For every route $h \in H$, define a_h, z_h, d_h, and r_h as the home depot of the vehicle assigned to route h, the destination depot at which the vehicle assigned to route h ends service, the time required by deadheading trips on route h, and the rate at which salt is applied by the vehicle assigned to route h, expressed as kilograms per unit of time, respectively. Let $K = \{1, 2, \ldots, |R|\}$ be the set of order indices in which a required arc serviced in a route can be visited with 1 being the first order position in the sequence of required arcs serviced in the route, 2 being the second order position, and so on. For every arc $(v_i, v_j) \in R$, for every route $h \in H$, and for every order index $k \in K$, let x_{ijk}^h be a binary variable equal to 1 if and only if required arc (v_i, v_j) appears on route h in order position k. For every route $h \in H$, let t_h be a nonnegative variable representing the time for service completion of route h and let T_h denote the time limit of route h. The time limits T_h, $h \in H$, can be violated at a cost and by an unlimited length of time. Unlimited time limits are defined together with the following penalty functions:

$$f_1^h(t_h) = \begin{cases} 0 & \text{if } t_h \leqslant T_h, \\ w_1(t_h - T_h) & \text{if } t_h - T_h \leqslant \alpha_1 T_h, \\ \exp(w_1^+(t_h - T_h)) & \text{if } t_h - T_h > \alpha_1 T_h, \end{cases}$$

where w_1 and w_1^+ are two positive constants and $0 \leqslant \alpha_1 \leqslant 1$. For every route $h \in H$, let s_h be a nonnegative variable representing the amount of salt applied to the roads assigned to route h and let C_h denote the salt capacity of the vehicle servicing route h. Similarly, the vehicle capacities C_h, $h \in H$, can be violated at a cost and by an unlimited amount of salt. Unlimited vehicle capacities are defined together with the following penalty functions:

$$f_2^h(s_h) = \begin{cases} 0 & \text{if } s_h - C_h < \alpha_2 C_h, \\ w_2(s_h - C_h)^2 & \text{if } s_h - C_h \geqslant \alpha_2 C_h, \end{cases}$$

where w_2 is a positive constant and $0 \leqslant \alpha_2 \leqslant 1$. The problem is then formulated as follows:

Minimize
$$\sum_{h \in H} \left(f_1^h(t_h) + f_2^h(s_h) + w_3 d_h \right) \tag{2.41}$$

subject to

$$\sum_{\left\{ \substack{v_j:(v_i,v_j) \in A, \\ v_i = a_h} \right\}} x_{ij1}^h = 1 \quad (h \in H), \tag{2.42}$$

$$\sum_{\left\{ \substack{v_i:(v_i,v_j) \in A, \\ v_j = z_h} \right\}} x_{ij1}^h = 1 \quad (h \in H), \tag{2.43}$$

$$\sum_{h \in H} \sum_{k=1}^{|R|} x_{ijk}^h = l_{ij} \quad ((v_i, v_j) \in R), \tag{2.44}$$

$$\sum_{(v_i,v_j) \in R} x_{ijk}^h \leqslant 1 \quad (h \in H, k \in K), \tag{2.45}$$

$$d_h = \sum_{k=1}^{|R|-1} \sum_{\substack{(v_i,v_j) \in R, \\ (v_p,v_q) \in R}} sp_{ijpq} x_{ijk}^h x_{pq(k+1)}^h \quad (h \in H), \tag{2.46}$$

$$s_h = r_h \sum_{k=1}^{|R|} \sum_{(v_i,v_j) \in R} t_{ij} x_{ijk}^h \quad (h \in H), \tag{2.47}$$

$$t_h = \sum_{k=1}^{|R|} \left(\sum_{(v_i,v_j) \in R} t_{ij} x_{ijk}^h \right) + d_h \quad (h \in H), \tag{2.48}$$

$$x_{ijk}^h \in \{0, 1\} \quad ((v_i, v_j) \in R, h \in H, k \in K), \tag{2.49}$$
$$d_h, s_h, t_h \geqslant 0 \quad (h \in H), \tag{2.50}$$

where w_3 is a positive constant. The objective function (2.41) minimizes a weighted combination of time limits constraints violations, vehicle capacities constraints violations, and the deadhead traversal time. Constraints (2.42) and (2.43) ensure that each vehicle starts service at its home depot and ends service at its destination depot, respectively. Constraints (2.44) guarantee that the appropriate number of passes is achieved on each required arc. Constraints (2.45) impose that at most one required arc appears in each order position of a route. Constraints (2.46), (2.47), and (2.48) define, for each route, the time required by deadheading trips, the amount of salt consumed, and the time for service completion, respectively. Finally, all x_{ijk}^h variables are restricted to be binary while d_h, s_h, and t_h variables must assume nonnegative values. The model is solved using a tabu search heuristic. The authors report an implementation in Wayne County, Michigan. They did not, however, provide an algorithm or numerical results.

Qiao [49] extended the original model (2.25)–(2.40) for routing salt spreader trucks in Calvert County, Maryland, to incorporate service connectivity and suggested two metaheuristic methods to solve the model. The extension is also an attempt at integrating both allocation of road segments to salt spreader

trucks and routing decisions into a single optimization model. However, the service hierarchy of the transportation network and the maximum route duration constraints are no longer imposed. For each edge $(v_i, v_j) \in E$ and for every vehicle $k \in K$, let y_{ijk} be a binary variable equal to 1 if and only if edge (v_i, v_j) is serviced from v_i to v_j by vehicle k. Let $H = (W \cup \{w_r\}, F_1 \cup F_2)$ be an auxiliary graph defined as in Section 2.1.4 (see Fig. 3). For each edge $(w_m, w_n) \in F_1 \cup F_2$ and for each vehicle $k \in K$, let z_{mnk} be a binary variable equal to 1 if and only if edge (w_m, w_n) is in the subtree associated with vehicle k, and let also f_{mnk} be a nonnegative real variable representing the flow on edge (w_m, w_n) from w_m to w_n, expressed as salt units, in the subtree associated with vehicle k. For each node $w_m \in W$ and for each vehicle $k \in K$, let s_{mk} (also represented by $s_{m(ij),k}$) be a binary variable equal to 1 if and only if vertex w_m (corresponding to edge (v_i, v_j) in E) is serviced by vehicle k. Finally, for each edge $(w_m, w_n) \in F_1$ and for each vehicle $k \in K$, let r_{mnk} be a binary variable equal to 1 if and only if edge (w_m, w_n) is in the subgraph induced by the nodes in the adjacency graph H that are serviced by vehicle k. Then, the constraints

$$y_{ijk} + y_{jik} \geqslant s_{m(ij),k} \quad ((v_i, v_j) \in A_1, w_m \in W, k \in K), \tag{2.51}$$

$$y_{ijk} + y_{jik} \leqslant M s_{m(ij),k} \quad ((v_i, v_j) \in A_1, w_m \in W, k \in K), \tag{2.52}$$

$$M(s_{mk} + s_{nk} - 2) + (1 - r_{mnk}) \leqslant 0 \quad ((w_m, w_n) \in F_1, k \in K), \tag{2.53}$$

$$(s_{mk} + s_{nk} - 2) + M(1 - r_{mnk}) \geqslant 0 \quad ((w_m, w_n) \in F_1, k \in K), \tag{2.54}$$

$$z_{mnk} \leqslant r_{mnk} \quad ((w_m, w_n) \in F_1, k \in K), \tag{2.55}$$

$$\sum_{\{w_n : (w_m, w_n) \in F_1\}} z_{mnk} = s_{mk} \quad (w_m \in W, k \in K), \tag{2.56}$$

$$\sum_{w_m \in W} z_{mrk} \leqslant 1 \quad (k \in K), \tag{2.57}$$

$$\sum_{\{w_n : (w_m, w_n) \in F_1 \cup F_2\}} f_{mnk} - \sum_{\{w_n : (w_n, w_m) \in F_1\}} f_{nmk} = s_{mk} \quad (w_m \in W, k \in K), \tag{2.58}$$

$$f_{mnk} \leqslant Q z_{mnk} \quad ((w_m, w_n) \in F_1 \cup F_2, k \in K), \tag{2.59}$$

$$r_{mnk}, z_{mnk} \in \{0, 1\} \quad ((w_m, w_n) \in F_1 \cup F_2, k \in K), \tag{2.60}$$

$$s_{mk} \in \{0, 1\} \quad (w_m \in W, k \in K), \tag{2.61}$$

$$f_{mnk} \geqslant 0 \quad ((w_m, w_n) \in F_1 \cup F_2, k \in K), \tag{2.62}$$

are added to model (2.25)–(2.40) to impose service connectivity. Constraints (2.51) and (2.52) ensure that if either or both directions of a road segment in the original graph G are serviced by a vehicle, then the corresponding node in the adjacency graph H should also be serviced by the vehicle (selected in the associated subtree). Constraints (2.53) and (2.54) define the set of edges in the adjacency graph H. These constraints ensure that an edge is included in the subgraph induced by the nodes in H that are serviced by a vehicle only if the two endpoints of the edge are serviced by the vehicle. Constraints (2.55) link the spanning tree variables z_{mnk} and the adjacency graph variables r_{mnk}. They state that an edge can be part of the subtree associated with a vehicle only if this edge is in the subgraph induced by the nodes that are serviced by the vehicle. Constraints (2.56)–(2.59) are similar to their counterparts (2.12)–(2.14) of the capacitated minimum spanning tree model. Constraints (2.56) assure that each node in H (except the root node) is connected to some node (except the root node) in the optimal spanning tree only if both nodes are serviced by the same vehicle. Constraints (2.57) guarantee that at most one

node in each subtree associated with a vehicle should be connected to the root node. Constraints (2.58) ensure that flow conservation is satisfied for each vehicle at all nodes of the adjacency graph H (except the root node). Finally, the vehicle salting capacity Q is respected at every edge in each subtree associated with a vehicle via constraints (2.59). Qiao also showed how load balancing constraints can be introduced in the formulation and how multiple salt depots can be taken into account. The model is solved with a classical tabu search algorithm and an elite route pool procedure. The *elite route pool* procedure is similar to the technique of genetic algorithms. The population is forgebrea pool of good routes found in the best solutions, called the elite route pool. Associated with every route in the elite route pool is a weight corresponding to the frequency with which the route appears in the best solutions. New offspring routes are produced by selecting the routes with the highest weights in the elite route pool while avoiding duplications of serviced required arcs. Mutations are then obtained by applying the improvement methods described in Fig. 6 (see Section 2.2). Tests showed that the various heuristics were useful in analyzing a variety of scenarios related to the modification of load balancing parameters and depot or salt dome locations as well as vehicle capacities. The thesis by Qiao [49] provided an interesting comparison of the various heuristics (Merge, Delete and Insert, Link Exchange, Random Link Exchange, Multiple Link Exchange, tabu search, and elite route pool) and four popular constructive methods for the capacitated arc routing problem: Pearn's algorithm [52], augment-merge [30], path-scanning [30], and construct-strike [53]. Computational tests on 23 networks derived from the test problems used by Pearn [52] showed that the elite route pool procedure obtained the largest number of best solutions on sparse networks with $7 \leqslant |V| \leqslant 27$ and arc densities between 13% and 40%. On dense networks, Pearn's algorithm produced the best solutions in most cases.

3. Vehicle and materials depot location models

A number of different depot location problems arise in the context of winter road maintenance. These include determining the locations of vehicle depots and materials depots. Vehicle depots serve as starting and ending points for spreader trucks, as well as plows and snow loaders. Because vehicle depots are usually used year-round for the various road maintenance activities, the vehicle depot location problem is not exclusive to winter road maintenance. However, as indicated by Gupta [5], winter road maintenance is the most resource intensive activity that impacts the decision to establish a new vehicle depot or close an existing one.

Materials depots are intermediate facilities containing chemicals and abrasives to provide opportunities for spreader vehicles to refill with materials without returning to the original starting point. Materials depots should be located so as to minimize non-productive travel time, maximize use by multiple crews, minimize possible environmental damage, and not create a nuisance to adjoining properties. The number and locations of materials depots depend on many considerations such as capacity of the spreaders, maximum time allowed for a spreading operation, level of service of the road segments to be treated, and special treatment features such as bridges, tunnels, and intersections. The number and locations of materials depots are usually reviewed periodically to incorporate new technology. For example, the number of materials depots may be reduced through the use of anti-icing operations.

We now review optimization models aimed specifically at the location of vehicle and materials depots in the context of winter road maintenance. Models that are mainly concerned with the efficient location of vehicle and materials depots are discussed first, followed by compound models which integrate depot location and routing or fleet sizing decisions. A summary of these models is presented in Table 3 at the end of the section.

Table 3
Characteristics of vehicle depot and materials depot location models

Authors	Problem type	Planning level	Problem characteristics	Objective function	Model structure	Solution method
Korhonen et al. [54]	Vehicle depot location	Strategic	Service hierarchy and fixed location of new vehicle depots	Min transport and fixed vehicle depot costs	Uncapacitated facility location problem	Constructive method
Rahja and Korhonen [56]	Materials depot location	Tactical	Grouping of road segments into fixed and balanced zones	Min demand-weighted distance	P-median problem	Improvement method
Gupta [5]	Opening or closing vehicle and materials depots	Strategic	Vehicle capacities and maximum route times	Min vehicle and materials depot variable and fixed costs	Capacitated arc routing problem	Composite method
Hayman and Howard [58]	Combined vehicle and materials depot location, and fleet sizing	Strategic	Service hierarchy, homogeneous fleet, and maximum spreading route times	Min transport, depot depreciation, and material costs	Linear IP	Simplex algorithm
Reinert et al. [59]	Combined materials depot location and route assignment	Tactical	Maximum number of materials depots, materials depot capacities, and fixed spreader routes	Min total length-weighted distance	P-median problem	MPSX mathematical programming
Lotan et al. [60]	Combined depot location and spreader routing	Strategic	Service hierarchy, vehicle capacity, and times for service completion	Min fleet size, deadheading, and number of vehicle depots	Location-arc routing problem	Constructive method

3.1. Depot location models

Since the location of vehicle and materials depots influences the cost for spreading and plowing operations, models for vehicle and materials depot location include costs for different aspects of the operations, such as vehicle routes, materials or the vehicles themselves.

Korhonen et al. [54] described a decision support system developed to assist planners in Finland in locating vehicle depots for winter road maintenance. The objective is to minimize transport costs and fixed vehicle depot costs. For economical and administrative reasons, all vehicle depots located for 10 years or less are forced into their location in the solution and transport costs are calculated for high-class roadways only. The system incorporates a construction heuristic similar to the "add heuristic" devised by Kuehn and Hamburger [55] for the solution of the uncapacitated facility location problem. The construction heuristic opens vehicle depots sequentially until it fails to find a vehicle depot whose addition will result in a decrease in the total cost. However, the author did not provide a rule for selecting the vehicle depot to add to the solution at each step. The effects of vehicle depot location on accidents and travel delays are also considered. The system yielded an annual cost reduction of 11% over the solution in use by the Finnish National Road Administration.

Rahja and Korhonen [56] described a computerized tool to assist planners in Finland in locating sand and salt storage facilities for anti-icing. The system is based on the national road network divided into balanced geographic zones defined as groups of neighboring road segments. Each zone has a demand given in terms of sand and salt consumption. The system relies on the exchange heuristic proposed by Teitz and Bart [57] for the *P*-median problem. This heuristic tries to find improved locations based on an objective of minimizing the demand-weighted distance between each demand zone and the nearest storage facility. The system was also used to select Finnish seaports and inland ports for importing salt by ship, while minimizing the sum of the cost of transporting salt from the ports to the storage facilities, the port charges, and the cost for unloading and shipping. The authors also proposed a transshipment model to determine transportation and storage decisions for imported salt. In this model, salt from a port may progress through several storage facilities before reaching a road segment. The objective considered is the minimization of salt purchase costs, port charges, unloading and shipping costs, transportation costs, and storageing costs, as well as the cost of salt spreading defined by equipment cost, salt solution production cost, loading cost, and crew cost.

For spreading operations, Gupta [5] described a GIS-based computerized tool to assist planners in analyzing the opening or closing of vehicle or materials depots. The routing of spreader trucks is dealt with by the Snowmaster decision support system [28] described in Section 2.1.1, which helps planners in generating feasible spreader truck routes using the constructive path-scanning heuristic proposed by Golden et al. [30] for the capacitated arc routing problem. The objectives guiding the opening of a new depot, or the closing of an existing one, include operating costs (vehicle costs, crew wages and road materials costs), fixed costs associated with the depot (depreciation of permanent structures, maintenance cost of permanent infrastructure, and depreciation and maintenance costs of trucks), and costs for opening a new depot or closing an existing depot (salvage value of closed depot, plant and machinery reassignment cost, personnel reassignment cost, environmental treatment cost for closing an existing depot and construction cost of a new depot). The usefulness of the GIS-based computerized system is demonstrated using data from Hamilton County, Ohio involving 360 nodes and 855 arcs. Finally, Gupta suggested formulating the materials depot location problem as a *P*-median problem.

3.2. Compound depot location and routing or fleet sizing models

A combined model for vehicle depot location, materials storage facility location and spreader truck fleet sizing was described by Hayman and Howard [58]. The problem considered is to determine the spreader truck fleet size based at each depot and assigned to each intermediate facility containing chemical and abrasive materials that gives the lowest operational and depreciation costs for long-term planning, while ensuring that the total roadway system be covered within a specific time period following the beginning of a storm. Maximum service times are provided within each class of roadways to reflect priorities. In the optimal solution of the model, if no truck at all is required at a given depot, then no depot is located at this candidate site. Similarly, no intermediate facility is located at a candidate site if no spreader truck is assigned to this intermediate facility in the optimal solution.

To present the formulation, let I, J, and K be the sets of vehicle depots, materials depots, and roadway sections, respectively. For every vehicle depot $i \in I$ and for every materials depot $j \in J$, let x_{ij} be a nonnegative integer variable representing the number of trucks based at vehicle depot i and having to refill with materials at intermediate facility j, and let b_{ij} and d_{ij} represent the unit time cost in traveling from vehicle depot i to materials depot j and the travel distance from vehicle depot i to materials depot j, respectively. For each materials depot $j \in J$ and for every roadway section $k \in K$, let y_{jk} be a nonnegative integer variable representing the number of trucks assigned to materials depot j to effect the servicing of roadway k, and let also z_{jk} be a nonnegative integer variable representing the number of loads of chemical and abrasive product to be hauled from materials depot j and spread on roadway k. A single load of materials corresponds to 1 ton. For each materials depot $j \in J$ and for every roadway $k \in K$, let also c_{jk} and d_{jk} represent the unit time cost for trucks plus loaders involved in the spreading operation from materials depot j to roadway k and the travel distance from materials depot j to roadway k, respectively. For every depot $i \in I$, define a_i as the depot depreciation cost per storm or spreading event applied to each truck based at vehicle depot i. For every materials depot $j \in J$, let f_j and t_j represent the cost per ton of the spreading material delivered to materials depot j and the average time in traveling from a vehicle depot to materials depot j, respectively. For every roadway $k \in K$, define l_k, t_k, and n_k as the length of roadway k, the time available to complete the spreading of roadway k measured from the beginning of the storm, and the number of loads of material required to service roadway k, respectively. Finally, if we let s represent the average vehicle speed, then the formulation is as follows.

Minimize

$$\sum_{i \in I} \sum_{j \in J} \left(a_i + b_{ij} \frac{2d_{ij}}{s} \right) x_{ij} + \sum_{j \in J} \sum_{k \in K} c_{jk} \left(\frac{2z_{jk}(d_{jk} + 0.5l_k) + 6z_{jk}}{s} \right)$$

$$+ \sum_{j \in J} \sum_{k \in K} f_j z_{jk} \tag{3.1}$$

subject to

$$\frac{2z_{jk}(d_{jk} + 0.5l_k) + 6z_{jk}}{s} \leqslant (t_k - t_j)y_{jk} \quad (j \in J, k \in K), \tag{3.2}$$

$$\sum_{j \in J} z_{jk} \geqslant n_k \quad (k \in K), \tag{3.3}$$

$$\sum_{i \in I} x_{ij} \geqslant \sum_{k \in K} y_{jk} \quad (j \in J), \tag{3.4}$$

$$x_{ij}, y_{jk}, z_{jk} \geqslant 0 \text{ and integer} \quad (i \in I, j \in J, k \in K). \tag{3.5}$$

In this formulation, the objective function (3.1) minimizes the total cost per storm or spreading event. This objective includes three costs: the time cost in deployment of the trucks from the vehicle depots to the materials depots, including depot depreciation, the time cost of delivering the material from the materials depot to the roadways, and the cost of the material. The quantity $2d_{ij}/s$ in (3.1) is the time required to travel between vehicle depot i and materials depot j and back to the vehicle depot. The quotient $2z_{jk}(d_{jk} + 0.5l_k) + 6z_{jk}/s$ represents the time required to service roadway k when refilling the spreader vehicles z_{jk} times with materials at materials depot j, regardless of the actual number of vehicles involved. This quotient accounts for z_{jk} trips over the distance between materials depot j and the beginning of roadway section k to be serviced and is determined based on studies of the relationship between the total distance driven and the distance spread. If we assume that the total time required to service roadway k when reloading with materials at materials depot j may be equally divided among y_{jk} spreader vehicles, each traveling at the same speed s, then constraint set (3.2) ensures that the total time available measured from the storm beginning to service each roadway is respected. Constraint set (3.3) requires that each roadway be properly spread. Constraints (3.4) link the total number of trucks required at each materials depot to effect the spreading operation and the number of trucks dispatched from the various vehicle depots to each materials depot. Finally, all x_{ij}, y_{jk} and z_{jk} variables must assume nonnegative integer values.

In order to reduce the size of the problem, some x_{ij} variables are set to zero if t_j, the travel time from vehicle depot i to materials depot j, is excessively larger than t_k, the time allotted to service any roadway from material depot j. Similarly, some y_{jk} and z_{jk} variables are discarded if too much deadhead time is consumed by travelling from materials depot j to roadway k. Computational results were reported on a real-life instance with 15 potential vehicle depot sites, 21 materials depots and 41 roadway sections. The LP relaxation of the model (3.1)–(3.5) was solved using the simplex algorithm and the total number of trucks required at any materials depot was rounded up to the nearest integer value, as was the total number of trucks required from any vehicle depot.

Reinert et al. [59] proposed a model for the combined problem of locating materials storage depots and assigning predetermined spreader truck routes to these depots. The problem is formulated as a *P*-median problem with depot capacities. Let I be the set of vehicle routes and let J be the set of materials depots. For every vehicle route $i \in I$ and for every materials depot $j \in J$, let x_{ij} be a binary variable equal to 1 if and only if route i is assigned to materials depot j and let d_{ij} represent the distance between the median point of route i and the materials depot j. For every materials depot $j \in J$, let y_j be a binary variable equal to 1 if and only if a materials depot is located at potential site j and let k_j represent the capacity of a materials depot at candidate site j. Finally, for every route $i \in I$, define a_i and s_i as the length and the materials requirement associated with route i, respectively. The formulation is then

as follows.

Minimize

$$\sum_{i \in I} \sum_{j \in J} a_i d_{ij} x_{ij} \tag{3.6}$$

subject to

$$\sum_{j \in J} x_{ij} = 1 \quad (i \in I), \tag{3.7}$$

$$\sum_{j \in J} y_j \leqslant P, \tag{3.8}$$

$$x_{ij} \leqslant y_j \quad (i \in I, j \in J), \tag{3.9}$$

$$\sum_{i \in I} s_i x_{ij} \leqslant k_j \quad (j \in J), \tag{3.10}$$

$$x_{ij}, y_j \in \{0, 1\} \quad (i \in I, j \in J). \tag{3.11}$$

The objective function (3.6) minimizes the total length-weighted distance between each route median point and the nearest materials depot. Constraints (3.7) require each spreader truck route to be assigned to exactly one materials depot. Constraints (3.8) state that at most P materials depots are to be located. Constraints (3.9) link the location variables y_j and the assignment variables x_{ij}. They ensure that each route is assigned to a materials depot that is selected. Storage capacity is respected for every materials depot via constraints (3.10). Finally, all variables x_{ij} and y_j are restricted to be binary. The problem was solved using IBM's MPSX mathematical programming package. Tests performed with data from the District of Columbia involving 14 potential materials depot sites showed a 27% reduction in deadheading over the solution in use by the district.

Lotan et al. [60] proposed a three-stage procedure for the combined depot location and spreader routing problem in the province of Antwerp, Belgium. The procedure takes into account the service hierarchy, the spreader capacities (all the same), and the times for service completion for each class of roads. For high-priority roads, two lanes must be spread in one pass in each direction, whereas low priority roads can be spread either in two directions (servicing one lane at a time), or in one pass (servicing the two lanes together). The authors define a priority class network for each class of roads. The first stage simultaneously locates vehicle depots and constructs feasible routes in the network induced by the set of high-priority roads. The authors observed that the network induced by the set of high-priority roads is characterized by having a tree structure rooted around the ring of Antwerp. Given a fixed number of vehicle depots, feasible routes are thus defined by traversing the tree from its leaves towards the root and vehicle depots are located on high-priority roads so as to minimize the distance covered by deadheading trips. By varying the number of vehicle depots to locate and iteratively solving the location-routing problem on the tree structure, the proper tradeoff between minimizing the number of vehicle depots and minimizing the distance covered by deadheading trips can be identified. The second stage then allocates low priority roads to vehicle depots by assigning roads to their closest vehicle depot, while ensuring that the graph generated by the links assigned to each depot is Eulerian. This facilitates the creation of routes with less

deadheading during route construction. The last stage constructs feasible routes and locates materials storage silos for each depot independently. The AUGMENT-INSERT algorithm of Pearn [61] is used to produce a solution to the capacitated arc routing problem depot by depot. Silos are then located to improve the resulting solution. The three-stage procedure can be seen as a location-allocation-routing heuristic scheme typified by Laporte [62] for node location-routing problems. The procedure was tested on the network of Brecht, Belgium with 33 nodes and 43 road segments. Results indicated that the procedure reduced the total distance travelled by 28% and the fleet size by 17% over the solution in use by the district when allowing two lanes to be spread in one pass. These improvements increase to 34% and 33%, respectively, when including a silo, but the duration of the route that utilises the silo then increases.

4. Crew assignment models

For most northern countries, the extent of winter road maintenance operations involves seasonal reassignment of workers from summer maintenance activities to winter maintenance operations. This reassignment avoids seasonal hiring and firing, but may disrupt existing worker assignment plans by reassigning crews to work out of different depots for the winter season. This section contains a review of optimization models for the assignment of crews to vehicle depots. A summary of these models is presented in Table 4 at the end of the section.

The information related to the first optimization model for the crew assignment problem is derived from Wright et al. [63], Egly and Wright [64], and Wright and Egly [65] who developed a decision support system to assist planners at the Indiana Department of Highways in making worker assignment decisions at the operational planning level. Given a planned vehicle routing for plowing and spreading, the worker assignment problem considered consists of assigning a set of workers to the depots from which emanate the planned routes, so as to satisfy the demand for workers at each depot while respecting the availability of vehicles that can be issued to certain workers to travel from their residence to their respective depot. According to the Indiana Department of Highways' policy, a state-owned vehicle must be provided to a worker during the winter season if the distance between the residence of the worker and his/her assigned depot is higher than some maximum allowable distance, and if the assigned depot is not the nearest depot to the residence of the worker. Planners may also take into consideration worker seniority requirements.

The problem is formulated as a 0–1 integer program with two objective functions. The first objective function seeks to minimize the total distance between each residence and the nearest depot located within some maximum allowable distance. The second objective function is the minimization of the maximum distance between a worker and the closest depot located within some maximum allowable distance. Let I be the set of workers and let J be the set of vehicle depots. For every worker $i \in I$ and for every depot $j \in J$, let x_{ij} be a binary variable equal to 1 if and only if worker i is assigned to depot j and let d_{ij} represent the distance between the residence of worker i and depot j. For every worker $i \in I$, define $N_i = \{j \in J \mid d_{ij} \leqslant D\}$ as the set of candidate depots to which worker i may be assigned if the shortest path distance d_{ij} between the residence of worker i and depot j is less than or equal to the maximum travel distance D. For every worker $i \in I$ and for every depot $j \in J$, define the binary constant a_{ij} equal to 1 if and only if the assignment of worker i to depot j requires a vehicle. For every depot $j \in J$, define T_j as the total number of workers required at depot j. Finally, let *DMAX* and C be the maximum

Table 4
Characteristics of crew assignment models

Authors	Problem type	Planning level	Problem characteristics	Objective function	Model structure	Solution method
Wright et al. [63], Egly and Wright [64], Wright and Egly [65]	Assignment of workers to vehicle depots	Operational	Fixed vehicle depots location and fixed routes	Min total distance Min maximum distance	Linear 0–1 IP	Constraint method
Bogardi et al. [66]	Crew assignment for plowing or spreading	Operational and strategic	Fixed number of crews, fixed vehicle depots location, and fixed routes	Min total distance	Transportation problems	Scenario analysis
Bogardi et al. [66]	Multi-service crew assignment	Operational	Fixed number of crews, fixed vehicle depots location, and balanced depots	Min relative distance	Linear IP	—
Bogardi et al. [66]	Multi-service crew assignment	Operational and strategic	Fixed number of crews and fixed vehicle depots location	Multi-objective	Multiple criteria	Composite programming

distance between a worker and the nearest depot located within the maximum distance D and the number of vehicles available, respectively. The formulation is as follows.

Minimize

$$\sum_{i \in I} \sum_{j \in N_i} d_{ij} x_{ij} \tag{4.1}$$

$$DMAX \tag{4.2}$$

subject to $\quad \sum_{j \in N_i} x_{ij} \leqslant 1 \quad (i \in I), \tag{4.3}$

$$\sum_{i \in I} x_{ij} \geqslant T_j \quad (j \in J), \tag{4.4}$$

$$\sum_{i \in I} \sum_{j \in J \setminus N_i} a_{ij} x_{ij} \leqslant C, \tag{4.5}$$

$$DMAX \geqslant \sum_{j \in N_i} d_{ij} x_{ij} \quad (i \in I), \tag{4.6}$$

$$x_{ij} \in \{0, 1\} \quad (i \in I, j \in J). \tag{4.7}$$

Constraints (4.3) require that each worker be assigned to at most one depot located within the maximum distance D. Constraints (4.4) ensure that demand for workers is satisfied at each depot. Vehicle availability is respected via constraint (4.5). Constraints (4.6) state that the maximum distance between a worker and the nearest depot located within the maximum distance D must be greater than the distance between any worker and the depot located within the maximum distance D to which he or she is assigned.

The two-objective model is solved using a modified constraint method of multiobjective optimization. First, the method finds the endpoints of the noninferior solution set in objective space. The model with the objective function (4.1) for minimizing the total distance is solved as a single objective linear program using a branch-and-bound algorithm. In the same way, the formulation with the single objective function (4.2) for minimizing the maximum distance is solved. The two optimal solutions correspond to points A and B, respectively, in Fig. 8. Then, the method identifies the set of noninferior solutions by systematically varying the value of D between the smallest and the largest maximum distance measure (as provided by the initial endpoint solutions), and iteratively solving the model with the single objective function (4.1). A solution s is inferior if there exists some other solution s' that is as good as s in terms of the two objectives and s' is strictly better than s in terms of at least one objective. This method allows identification of convex-dominated solutions such as solution C in Fig. 8.

Numerical experiments showed that the dual simplex algorithm produced an integer solution to the linear programming relaxation of the model with the single objective function (4.1) in excess of 97% of tested instances. In fact, non-integer solutions occurred only when the constraint (4.5) on available vehicles was binding at optimality. The authors observed that when the availability of state-owned vehicles is relaxed, the linear programming relaxation of the model with the single objective function (4.1)
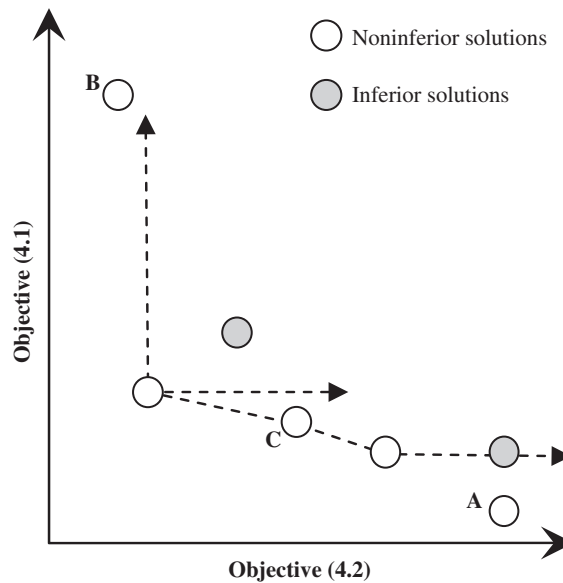
Fig. 8. Tradeoff curve for distance objectives.

reduces to that of a transportation problem. Consequently, the simplex algorithm is used to obtain the optimal solution to the linear programming relaxation of the model with the single objective function (4.1) followed by a rounding heuristic that consists of decreasing the number of available vehicles by one until an integer solution is obtained.

The system was tested on a problem involving 118 workers and 20 depots. Computational results indicated significant cost savings over the solution used by the Indiana Department of Highways. The system may also be used at the real-time level to aid planners in making decisions about modifications of existing routes and related demand for workers. Planners at the Indiana Department of Highways can solve the worker assignment problem via the Internet. Upon submission of a problem, the server activates CPLEX to solve the problem, and then reports the solution.

Bogardi et al. [66] studied the problem of assigning crews to vehicle depots for plowing and spreading operations in Lincoln, Nebraska. The time horizon considered is a single winter season. All possible scenarios of allocating the total number of crews among the vehicle depots are first generated. For each scenario, the optimal assignment of crew-days from vehicle depots to vehicle routes is then found by solving a transportation problem with supply nodes representing vehicle depots and demand nodes representing either plowing routes or spreading routes. Let I be the set of vehicle depots and let J be the set of plowing or spreading routes. For every vehicle depot $i \in I$ and for every route $j \in J$, let x_{ij} be a nonnegative variable representing the number of crew-days for servicing route j from vehicle depot i, and let c_{ij} represent the sum of the distance between vehicle depot i and midpoint of route j and back to the depot per crew-day, and the distance to service route j per crew-day. For spreading routes, the lengths of the round trips for reloading of materials per crew-day are also taken into account. For every vehicle depot $i \in I$, define s_i as the total number of crew-days provided by vehicle depot i. For every route $j \in J$, define d_j as the total number of crew-days required for servicing route j. For every scenario, the

following transportation problem is solved:

Minimize
$$\sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} \tag{4.8}$$

subject to
$$\sum_{j \in J} x_{ij} = s_i \quad (i \in I), \tag{4.9}$$

$$\sum_{i \in I} x_{ij} = d_j \quad (j \in J), \tag{4.10}$$

$$x_{ij} \geqslant 0 \quad (i \in I, j \in J). \tag{4.11}$$

The objective function (4.8) minimizes the sum of all distances. Constraints (4.9) stipulate that the total number of crew-days provided by each vehicle depot must be equal to the capacity of the depot. Similarly, constraints (4.10) ensure that each route be served by the required number of crew-days. The scenario with the lowest total distance gives the optimal crew assignment plan. The authors also applied this approach to other maintenance activities provided by the city such as concrete, asphalt, drainage, and traffic sign engineering. Travel distance reductions on the order of 3–5% were obtained over the solution in use by the city for snow plowing, asphalt service, and drainage. (An improvement on the order of 30% was obtained for traffic sign engineering.) We note that the crew assignment problem studied by Bogardi et al. [66] can simply be formulated as a network flow problem with one supply node, transhipment nodes representing vehicle depots, and demand nodes representing either plowing routes or spreading routes.

In the same paper, Bogardi et al. [66] extended the original transportation model to consider a multi-season planning horizon. For each planning season, maintenance service demands are predicted on the basis of city development trends. Since the single-period problems do not interact in any way, crew distribution scenarios are defined, and each single-period problem is solved separately using the transportation model for each scenario. The scenario with the lowest total distance in time corresponds to the optimal crew assignment plan.

As mentioned before, part of the winter road maintenance can be performed by workers from other maintenance services such as concrete, asphalt, or drainage. However, the allocation of crews to vehicle depots is often treated separately for each of these maintenance activities. This can lead to unbalanced vehicle depots having a shortage or excess of crews. Bogardi et al. [66] proposed to integrate crew assignment decisions for concrete, asphalt, drainage, curb-cut, plowing, and spreading operations into a single optimization model. Let I be the set of vehicle depots and let J be the set of road maintenance activities. For every vehicle depot $i \in I$ and for every maintenance service $j \in J$, let x_{ij} be a nonnegative integer variable representing the number of crews based at vehicle depot i for service j, and let $w_{ij} = c_{ij}/\max_{k \in I}\{c_{kj}\}$ represent the nonnegative weight associated with vehicle depot i for service j with c_{ij} corresponding to the total travel distance incurred by assigning all crews to vehicle depot i for service j. For each maintenance service, this entails placing a weight of 1 on the vehicle depots incurring the maximum travel distance and smaller weights on the others vehicle depots. For every maintenance service $j \in J$, define s_j, d_j, and D_j as the total number of worker-days associated with each crew for service j, the number of workers in each crew for service j, and the total number of workers required for service j,
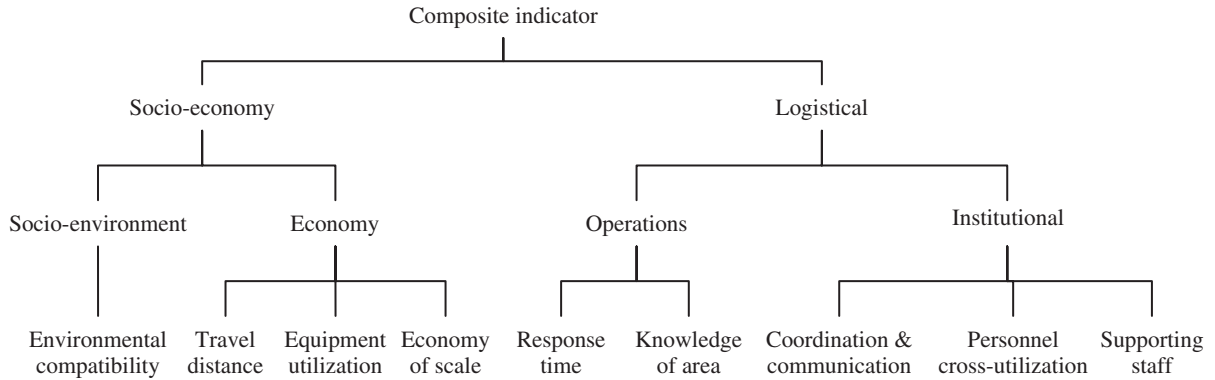
Fig. 9. Hierarchy of decision criteria.

respectively. The optimization model is an integer linear program stated as follows:

Minimize

$$\sum_{i \in I} \sum_{j \in J} w_{ij} x_{ij} \tag{4.12}$$

subject to

$$\sum_{j \in J} s_j x_{ij} \leqslant S \quad (i \in I), \tag{4.13}$$

$$\sum_{i \in I} d_j x_{ij} = D_j \quad (j \in J), \tag{4.14}$$

$$x_{ij} \geqslant 0 \text{ and integer} \quad (i \in I, j \in J). \tag{4.15}$$

The objective function (4.12) minimizes the total weighted number of crews. Constraints (4.13) limit the total number of worker-days associated with each vehicle depot to a maximum of S. Constraints (4.14) ensure that the appropriate number of workers is assigned to each service. Finally, all x_{ij} variables must assume nonnegative integer values. The authors did not propose a solution methodology to solve the model.

Finally, Bogardi et al. [66] studied the combined problem of assigning concrete, asphalt, curb-cut, drainage, spreading, and plowing crews to vehicle depots from a multiobjective perspective. In particular, the problem is concerned with the selection of a preferred crew assignment scenario for these maintenance activities among twelve possible such scenarios, while considering economic, social, environmental, and political criteria. The problem is solved using composite programming [67]. The method of composite programming identifies solutions which are closest to the ideal solution as determined by some measure of distance. The method starts with the selection of the decision criteria, called *basic indicators*, for constructing a hierarchy of criteria. The decision criteria are identified based on experience and constitute the first level of the structure. Based on their characteristics, these basic indicators are then grouped into successively broader clusters of higher level indicators, called *composite indicators*, until one final composite indicator is obtained at the highest level of the hierarchy. A composite index associated with this final composite indicator is calculated for each scenario. The crew assignment scenario with the highest composite index corresponds to the best scenario. Fig. 9 presents the hierarchy of decision criteria developed by Bogardi et al. for the crew assignment problem.
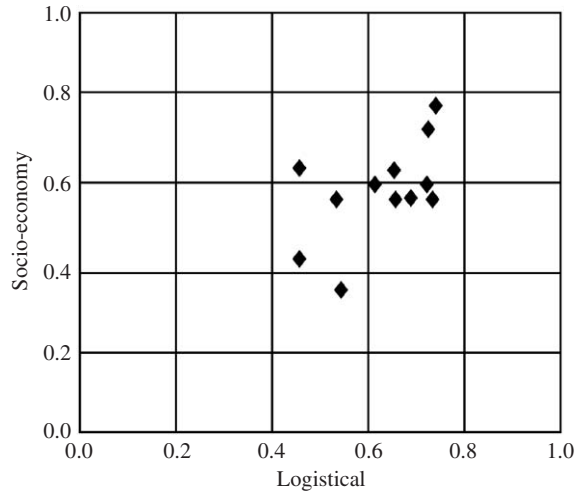
Fig. 10. Composite indices for the twelve scenarios.

For each scenario, the travel distance criterion is measured in kilometers while subjective and numerical values are assigned to the other criteria. Since the basic indicators are not expressed in commensurate terms, a scaling function is defined to ensure the same range for each basic indicator. This range corresponds to the interval [0, 1]. Let I be the set of basic indicators. For each basic indicator $i \in I$, define z_i as the numerical value of basic indicator i for a given scenario, and let z_i^* and z_i^{**} be the best and the worst numerical values of basic indicator i among all scenarios, respectively. For each basic indicator $i \in I$, the scaling function S_i is then calculated as follows:

$$S_i = \frac{z_i - z_i^{**}}{z_i^* - z_i^{**}}.$$

Let $P_J = \{I_1, I_2, \ldots, I_J\}$ be a partition of I with $I_1 \cup I_2 \cup \cdots \cup I_J = I$ and $I_k \cap I_l = \emptyset$ for all $k, l \in \{1, 2, \ldots, J\}, k \neq l$. For each group $I_j \subseteq P_J$ and for each basic indicator i in group I_j, let α_{ij} and S_{ij} be the weight value expressing the relative importance of basic indicator i in group I_j ($\sum_i \alpha_{ij} = 1$, $i \in I_j$, for each group $I_j \subseteq P_J$) and the normalized value of basic indicator i in group I_j, respectively. For each group $I_j \subseteq P_J$, define p_j as the balancing factor among indicators in group I_j. The parameter p_j reflects the importance of the maximal deviation. The measure of closeness used by Bogardi et al. to identify scenarios which are closest to the ideal solution is a family of L_j composite indices, defined as follows:

$$L_j = 1 - \left[\sum_{i \in I_j} \alpha_{ij} (1 - S_{ij})^{p_j} \right]^{1/p_j}.$$

For each scenario, the composite indices L_j are calculated iteratively for each level of the hierarchy of Fig. 9 until the final composite index is obtained. The composite programming method leads to (1, 1) as the best possible point, and (0, 0) as the worst possible point of the solution space depicted in Fig. 10.

The composite programming method was embedded in a decision support system. Computational tests on data from the city of Lincoln, Nebraska, showed that the scenario actually used by the city ranked eleventh best among the twelve scenarios. Bogardi et al. also extended the composite programming method to consider a multi-period planning horizon. Results on the same data indicated that the scenario actually used by the city ranked 13th best among the 17 scenarios considered for future maintenance services. Moreover, the multiple criteria approach appeared robust, namely, neither the best or worst scenarios changed under different weights α_{ij}. The authors concluded that the system can be used for other applications such as the selection of landfill sites, water resources planning, facility location studies, or transportation planning.

Computerized tools have also been developed to help planners in scheduling crews for winter road maintenance operations. Such tools were described, for example, by Rissel and Scott [68], Gagnon [69], and Sapling Corporation [70].

5. Conclusions

This paper is the third part of a four-part survey of optimization models and solution algorithms for winter road maintenance. (The two first parts of the survey [2,3] discuss system design models for winter road maintenance operations. The last part of the review [1] addresses vehicle routing, fleet sizing, and fleet replacement models for plowing and snow disposal operations.) This paper addresses vehicle routing, depot location, and crew assignment models for spreading operations. Vehicle routing problems in winter road maintenance are the most studied of any winter road maintenance problems. Because of the inherent difficulties of these problems, most solution methods that have been proposed are heuristics. Much early work for the routing of vehicles for spreading operations adapted or extended simple capacitated arc routing algorithms with little consideration of practical characteristics. Early attempts to apply simple heuristics, such as parallel route construction methods or cluster first, route second methods, produced nice results from simulation studies, but these heuristics were rarely implemented and used in spreading operations. Recent models are solved with more sophisticated local search techniques (e.g., composite methods and metaheuristics), and are showing much promise to assist planners in making routing decisions for spreading operations in practice. One interesting line of research would be the further development of compound models that address the integration of spreader routing with other decisions related to spreading operations.

In summary, despite the increasing realism of recent spreading operations applications and the important progress in solution methods, considerable work remains to be accomplished on the design of fast heuristic algorithms that produce good approximate solutions, and on the development of more comprehensive models that address the integration of depot location models with spreader routing decisions.

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