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A survey of models and algorithms for winter road maintenance. Part II: system design for snow disposal

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Abstract

This is the second part of a four-part survey of optimization models and solution algorithms for winter road maintenance planning. The first part addresses system design problems for spreading and plowing operations. The aim of this paper is to provide a comprehensive survey of optimization models and solution methodologies for the design of systems for snow disposal operations. These problems include partitioning a region or road network into sectors, locating snow disposal sites, allocating sectors to snow disposal sites, and allocating sectors to private companies or governmental agencies. The two last parts of the survey mainly concentrate on vehicle routing for winter road maintenance.

Keywords: Winter road maintenance; Snow removal; Snow disposal; Snow hauling; Operations research

0. Introduction

This is the second part of a four-part survey of optimization models and solution algorithms for winter road maintenance problems. Winter road maintenance planning involves a variety of decision-making problems relating to the system design and to the routing and scheduling of vehicles and crews. *System design* in winter road maintenance includes determining the level of service policy, partitioning

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the geographic region into sectors for efficient operations, locating vehicle depots, materials storage facilities, and disposal sites, assigning the sectors obtained from the partitioning to various facilities, allocating contracts for various operations to private organizations, and sizing and replacing vehicle fleets. Most commonly, decisions relating to the design of winter road maintenance systems belong to the *strategic* or *tactical planning levels*, while decisions concerning the routing and scheduling of vehicles and crews pertain to the *operational planning level* and *real-time control*. The distinction between the three levels of strategic, tactical, and operational planning, and the real-time control is explained in detail in the first part of the survey [1].

Winter road maintenance problems are very difficult and site specific because of the diversity of factors influencing the conduct of winter road maintenance operations, including geographical location, climatic and weather conditions, demographics, economics, technological innovations (for materials application, mechanical removal, and weather monitoring), legislative requirements, interagency agreements, variations of traffic rate, and information on the status of personnel, equipment, and materials. Also, winter road maintenance planners have a multi-criteria environment in which they have to address problems in terms of the three conflicting criteria of efficiency, effectiveness, and equity. For example, the fundamental tradeoff in determining the level of service policy is that a higher level of service requires greater costs for ensuring that roadways (and sidewalks) are safe for travel, but reduces costs for travelers, and for lost production and lost sales when travel is restricted. Furthermore, the benefits and costs of winter road maintenance range from some factors that are easy to quantify (for example, the expenditures for winter road maintenance and the effects of de-icing chemicals on the environment, infrastructure, and motor vehicles) to others that are difficult to quantify but are likely to be important in individual situations (for example, the safety effects and impacts on local economies, such as savings in accident costs, in delay costs, and in lost wages and productivity costs).

Spreading and plowing operations are usually performed on a regular basis in almost all rural and urban regions with frozen precipitation or significant snowfall. However, in urban areas with large snowfalls and prolonged subfreezing temperatures, the large volumes of snow plowed from roadways and walkways generally exceed the available space along roads for snow storage, and therefore require disposal by some means. The most common solution is to load snow into trucks for transport to disposal sites. Conversely, in rural regions, snow is often simply pushed to the sides of roadways without being removed and hauled.

The aim of this paper is to provide a comprehensive survey of optimization models and solution methodologies for the design of systems for snow disposal operations. These problems include partitioning a region or road network into sectors, locating snow disposal sites, allocating sectors to snow disposal sites, and allocating sectors to private companies or governmental agencies. The level of service policy and the design of sectors for spreading and plowing operations were reviewed in the first part of the survey [1]. Vehicle depot and materials depot location and fleet sizing and replacement problems are discussed along with the routing of vehicles for winter road maintenance in the two last parts of the survey [2,3].

The paper is organized as follows: Section 1 describes the operations of snow disposal and the system design problems related to those operations. Models for the assignment of sectors to snow disposal sites are described in Section 2. Models that address the assignment of sectors to private companies or governmental agencies are reviewed in Section 3. Section 4 focuses on disposal site location problems for snow disposal. Models dealing with the partitioning of a region or road network into sectors for snow disposal are presented in Section 5. Conclusions and future research paths in winter road maintenance planning are presented in the last section.

1. Snow disposal for winter road maintenance

Winter road maintenance operations include spreading of chemicals and abrasives, snow plowing, loading snow into trucks, and hauling snow to disposal sites. State and local governments spend about $2 billion in the United States [4] and approximately $4 to $6 billion in Japan and Europe [5] each year on these operations. Small percentage savings in these expenditures through optimization could result in substantial total savings over a number of years. The following section contains a brief description of snow loading and hauling operations. System design problems related to snow disposal that have been addressed with operations research methodologies are then discussed. A detailed review of the available technology for winter road maintenance is presented in the book by Minsk [4].

1.1. Snow disposal operations

Snow disposal operations involve loading snow into trucks for hauling to disposal sites. These operations are generally post-storm operations, although they may be required during a snowfall to remove snow from areas, such as alleys or narrow channelled sections, with insufficient space for snow storage. Loading and hauling of snow are usually performed in urban areas with significant snowfalls and prolonged subfreezing temperatures. However, many metropolitan areas may undertake snow disposal following uncommon but very large storms. During snow disposal, parking regulations are generally put into effect to facilitate loading snow into trucks for hauling to disposal sites.

Snow disposal sites are the destinations for snow hauling trucks originating in each sector, and must be visited many times during the snow disposal operations. There are several different types of disposal sites that may be considered, including surface sites, quarry sites, sewer chutes, snow melting machines, and water sites. With every disposal sites are associated a fixed location cost, an operating cost, and an annual capacity due to the limited space available to store snow. Each disposal site may also have an hourly capacity for unloading trucks depending on the configuration of the disposal site and the available equipment and manpower. Surface sites typically require large plots of open land and may have very large capacities. They also may have other uses when snow is not present. Melters, in contrast, can be small mobile machines, but are typically quite expensive. Disposal in rivers or lakes represents the most economical disposal method, although disposal sites that allow melted snow to be processed in wastewater treatment facilities provide environmental benefits.

Snow disposal operations require a fleet of trucks to haul snow to disposal sites and a fleet of snow-blowers, rotary plows, or other types of snow loaders to transfer snow from the roadway into the trucks. In order to minimize the completion time for winter road maintenance, snowblowers (or other types of snow loaders) generally operate in a continuous process loading trucks. In practice, there may be several empty trucks moving slowly in a queue alongside each snowblower to ensure the snowblowers are never idle. As soon as a truck is filled with snow, it departs for the assigned disposal site while another truck takes its place to begin being filled. The truck that departed for the disposal site will travel to the disposal site, dumps its load of snow, possibly after waiting in line, and then return to the end of the queue alongside the assigned snowblower. This closed cyclic continuous system is illustrated in Fig. 1. There may be more than one such cyclic closed system if a sector contains more than one snowblower or if a sector can be assigned to multiple disposal sites.

Snow disposal operations involve a range of fixed and variable labor, materials, and equipment costs. Variable costs of snow disposal include fuel costs, crew costs, vehicle maintenance costs, and variable

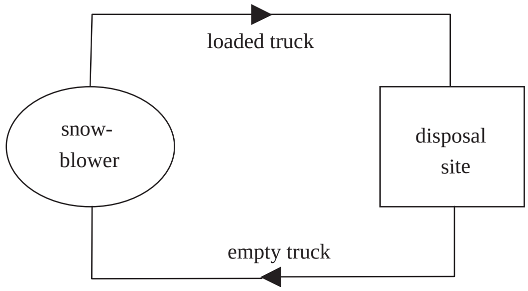

Fig. 1. The snowblower–truck–disposal site cyclic closed system.

costs of operating vehicle depots and disposal sites. Fixed costs of snow disposal include fixed costs of snow removal equipment and fixed costs of acquiring vehicle depots and disposal sites.

1.2. System design problems for snow disposal

This section describes system design problems of snow disposal operations that have been addressed by operations research techniques. These problems include partitioning a region or road network into sectors, locating snow disposal sites, allocating sectors to snow disposal sites, and allocating sectors to private companies or governmental agencies.

Section 2 of this survey covers models for assigning sectors to disposal sites. The primary costs for hauling snow to disposal sites include variable costs for transporting snow from sectors to disposal sites and elimination costs for operating disposal sites. Therefore, a good assignment plan of sectors to disposal sites is desirable to minimize these costs. Disposal sites have annual capacities based on their physical size. Disposal sites may also have hourly capacities for processing snow based on the operating and unloading practices at the site. Some disposal sites, such as large quarries and river disposal sites, have relatively high hourly capacities due to multiple unloading stations. Other disposal sites, such as sewer chutes, have effectively unlimited annual capacities but the unloading capabilities are restricted by the limited size of the openings into the sewer system, and the requirement that the temperature of the water not fall too low. Furthermore, for contractual reasons, all the snow of a given sector may be hauled to a single disposal site. This is called the *single assignment* requirement, as opposed to the *multiple assignment* case where snow from a sector can be hauled to several disposal sites. The *snow disposal assignment problem* consists of assigning a set of sectors to a set of disposal sites at minimum cost while satisfying some side constraints such as disposal site capacities and single or multiple assignment requirements.

Section 3 of this review addresses models for assigning contracts for snow removal to a set of contractors. Winter road maintenance operations are generally the responsibility of municipal or regional government public works agencies. In regions with low-snowfall or mild climate, agencies usually have sufficient in-house manpower and equipment to perform winter road maintenance operations for a light snowfall. When moderate to heavy snowfall occurs, contract maintenance forces are brought in to supplement the in-house maintenance capability. The contract maintenance forces may be integrated with in-house forces or assigned to particular sectors. Also, winter road maintenance can be fully contracted through private companies, other municipalities, in-house employees (contract with union), or a combination of these options. The *contract assignment problem* consists of assigning a set of sectors to a set of contractors, so as to minimize the total cost bid by the contractors.

Section 4 of this survey is devoted to disposal site location problems in the context of winter road maintenance. These problems are generally formulated as network location problems, in which disposal sites can be located only on the nodes or links of the network. In large cities, there are usually several snow disposal sites, possibly of different types, including surface sites, unused quarries, sewer chutes (openings into the storm sewer system), and water sites. Snow disposal sites may generate a number of environmental, social, and economic impacts such as soil and water contamination, noise, increased traffic, and infringements on the aesthetics. The environmental impacts of "waste" snow, which is contaminated with deicing chemicals, as well as roadway pollutants (heavy metals, oil and fats, abrasives) is becoming a major concern, and is limiting options for snow disposal (Quebec prevented dumping snow in rivers in 1996). This entails the redesign of the snow disposal system and gives an impetus to develop economical and environmentally sound snow disposal methods. Snow disposal sites can also present risks to the safety of the neighboring residents, so locating such sites in population centers can be undesirable. However, locating disposal sites far away from the sectors increases the costs due to the longer travel distances and times. Thus, in locating snow disposal sites, the key tradeoff is between minimizing transportation costs and minimizing the number of people adversely affected by the disposal sites. The *snow disposal site location problem* consists of locating disposal sites and assigning a set of sectors to the operative disposal sites at minimum cost while satisfying disposal site capacities and single or multiple assignment requirements.

Section 5 of this review is devoted to the sector design problem. Given the large geographic extent of most winter road maintenance operations, an agency generally partitions its service region (and transportation network) into subregions (subnetworks), called *sectors*. All sectors are treated simultaneously by separate crews to facilitate the organization of the operations. A sector is thus a bounded, organizational or administrative subarea in a larger geographical region. The *sector design problem* consists of partitioning a region or transportation network into a mutually exhaustive and exclusive collection of small sectors while satisfying side constraints such as contiguity, size or workload, and compactness or shape. A sector is contiguous if every pair of its basic units is connected. *Basic units* are the units of analysis used to partition the road network into sectors. A basic unit can be defined either as a single street segment or as a small geographic zone that contains a collection of neighboring street segments. Sectors are balanced in workload if they are approximately the same size and are assigned equivalent resources. Finally, sectors may either be compact or elongated in a direction perpendicular to the direction to the disposal site depending on the number of sectors and disposal sites. These criteria are explained in greater detail in the first paper [1]. Common criteria to design sectors also include the need to conform to existing infrastructure, geography, and jurisdictional boundaries. In sector design models, the number of sectors to construct may be either determined endogenously or given as an input. Also, the sector design process is most commonly performed on a network rather than in the plane.

2. Snow disposal assignment problems

Urban areas are generally partitioned into geographic sectors that are cleared of snow simultaneously by loading the snow into trucks which then haul the snow to assigned disposal sites. With every sector is usually associated an hourly removal rate and an annual volume of snow to remove. The hourly removal rate in a sector is the rate at which snow is sent out of the sector (in trucks) to a disposal site. This rate is usually expressed as cubic meters of snow per hour and depends on the snowblower and truck fleet sizes, as well as vehicle types. The annual volume of snow generated in a sector depends on the snowfall accumulation and the length of streets and sidewalks to be cleared of snow in the sector. This volume can be estimated based on historical data. Similarly, with every disposal site is associated an hourly capacity and an annual capacity for receiving snow. The hourly receiving rate capacity of a disposal site is usually expressed as cubic meters of snow per hour and depends on the logistics and configuration of unloading facilities at the disposal site. The annual capacity of a site depends on the finite space available for storing snow throughout the winter season.

The snow disposal assignment problem consists of assigning a set of sectors to the snow disposal sites so as to respect the hourly and annual capacities of the disposal sites. Since private contractors may be used for snow loading and hauling, for managerial, and contractual reasons the assignment of each sector may be restricted to a single disposal site. The objective is usually to minimize the sum of transportation and operating costs associated with hauling snow and operating disposal sites. Snow disposal assignment plans are usually updated every winter season, but monthly adjustments can be made during the winter season to account for snowfall variability.

Models for the assignment of sectors to snow disposal sites are now reviewed. The case where each disposal site has only an annual capacity and each sector can be assigned to multiple disposal sites is discussed first, followed by models for addressing the more realistic case where disposal sites have annual and hourly capacities and each sector must be assigned to a single site.

2.1. Multiple assignment models with annual disposal site capacities

A transportation formulation for the snow disposal assignment problem with annual disposal site capacity constraints and possibly multiple disposal sites per sector was proposed by the Bureau of Management Consulting, Transport Canada [6]. Let I be the set of sectors and J be the set of disposal sites. For every sector $i \in I$ and for every disposal site $j \in J$, let x_{ij} be a nonnegative variable representing the number of cubic meters of snow from sector i sent to disposal site j, and let c_{ij} represent the transportation cost per cubic meter for hauling snow from sector i to site j. For every site $j \in J$, define b_j as the variable cost per cubic meter of snow to operate site j, and V_j as the annual capacity of site j in cubic meters. For every sector $i \in I$, let v_i represent the annual volume of snow in sector i in cubic meters. The formulation for the snow disposal assignment problem with multiple assignment and annual disposal site capacities can be stated as follows:

Minimize

$$\sum_{i \in I} \sum_{j \in J} (b_j + c_{ij}) x_{ij}, \tag{2.1}$$

subject to

$$\sum_{j \in J} x_{ij} = v_i \quad (i \in I), \tag{2.2}$$

$$\sum_{i \in I} x_{ij} \leqslant V_j \quad (j \in J), \tag{2.3}$$

$$x_{ij} \geqslant 0 \quad (i \in I,\ j \in J). \tag{2.4}$$

The objective function (2.1) minimizes the sum of the disposal site variable costs and the total transportation cost. Constraints (2.2) ensure that all the snow for each sector is sent to some disposal site. Constraints (2.3) ensure that the capacity of each disposal site is not exceeded. Finally, all x_{ij} variables must assume nonnegative values.

The model was solved using IBM's MPSX mathematical programming package. Results on a real-life instance from a large Canadian city with 80 sectors and seven disposal sites produced cost savings on the order of 5% over the current assignment plan for the city. The transportation formulation (2.1)–(2.4) for the snow disposal assignment problem was also proposed by Leclerc [7], who suggested, in a subsequent paper [8], two procedures for post-optimal analysis of a degenerate optimal solution. In order to obtain a feasible solution to the single assignment case where each sector is restricted to be assigned to a single site, Leclerc [9] also proposed an interactive heuristic procedure that modifies the optimal solution to the transportation problem by slightly adjusting the annual capacity of the disposal sites. The approach was tested on data from the city of Montreal. Finally, a decision support system for the single assignment case has been developed by Leclerc et al. [10]. The system incorporates the stepping stone solution method [11], along with the heuristic capacity adjustment procedure.

2.2. *Single assignment models with annual and hourly disposal site capacities*

The most difficult version of the snow disposal assignment problem occurs when each disposal site has an annual capacity as well as an hourly capacity, and each sector must be assigned to a single disposal site. A model dealing with this version of the problem was proposed by Campbell and Langevin [12]. Their formulation is a linear 0–1 integer program. Let I be the set of sectors and J be the set of disposal sites. For every sector $i \in I$ and for every disposal site $j \in J$, let x_{ij} be a binary variable equal to 1 if and only if sector i is assigned to site j, and let d_{ij} represent the distance from the centroid of sector i to site j. For every site $j \in J$, define R_j as the maximum hourly capacity for receiving snow at site j. For every sector $i \in I$, let r_i represent the hourly snow removal rate in sector i. The hourly capacity of a disposal site and the hourly removal rate in a sector are expressed as cubic meters of snow per hour. Finally, define V_j and v_i as the annual capacity of disposal site j and the annual volume of snow in sector i, respectively, as above. The formulation is given next.

Minimize

$$\sum_{i \in I} \sum_{j \in J} d_{ij} v_i x_{ij}, \tag{2.5}$$

subject to

$$\sum_{i \in I} v_i x_{ij} \leqslant V_j \quad (j \in J), \tag{2.6}$$

$$\sum_{i \in I} r_i x_{ij} \leqslant R_j \quad (j \in J), \tag{2.7}$$

$$\sum_{j \in J} x_{ij} = 1 \quad (i \in I), \tag{2.8}$$

$$x_{ij} \in \{0, 1\} \quad (i \in I,\ j \in J). \tag{2.9}$$

The objective function (2.5) minimizes the total volume-weighted distance. Constraints (2.6) and (2.7) limit the assignments of sectors to disposal sites according to the annual and hourly receiving

1. *Penalty-based assignment*
   a. For each unassigned sector $i \in I$, let $d_{ik} = \min_{j \in J}\{d_{ij} \mid v_i \leq V_k \text{ and } r_i \leq R_k\}$ denote the distance from the centroid of sector i to the closest site $k \in J$ and let $d_{il} = \min_{j \in J \setminus \{k\}}\{d_{ij} \mid v_i \leq V_l \text{ and } r_i \leq R_l\}$ represent the distance from the centroid of sector i to the second closest site $l \in J \setminus \{k\}$. Compute $penalty(i) = v_i\,(d_{il} - d_{ik})$.
   b. Select an unassigned sector $i \in I$ such that $penalty(i) = \max_{i \in I}\{v_i\,(d_{il} - d_{ik})\}$. Assign sector i to site k and set $V_k = V_k - v_i$ and $R_k = R_k - r_i$.
   c. If all sectors are assigned to a site, go to step 2. Otherwise, for any unassigned sector $i \in I$, if $v_i > V_k$ or $r_i > R_k$ or if $v_i > V_l$ or $r_i > R_l$, recalculate $penalty(i)$ as defined above. Return to b.

2. *Two-opt exchange*
   a. For each pair of sectors i and j with assigned sites m and n, respectively, let $c(i, j) = v_i d_{im} + v_j d_{jn} - v_i d_{ik} - v_j d_{jl}$, $k, l \in J$, $k \neq m$ or $l \neq n$, represent the objective function improvement realized by reassigning sectors i and j to sites k and l, respectively. If $c(i, j) > 0$ and the hourly and annual capacities of sites k, l, m, and n are satisfied, reassign sector i to site k, reassign sector j to site l, and adjust hourly and annual capacity utilizations of sites k, l, m, and n accordingly.
   b. Return to the beginning of step 2 until no improvement is obtained.

Fig. 2. The two-phase heuristic for the snow disposal assignment problem [12].

capacity of the disposal sites. The single assignment constraints (2.8) ensure that each sector is assigned to exactly one disposal site. Finally, all x_{ij} variables are restricted to be binary. This model can be viewed as a two-resource generalized assignment problem, a particular case of the multi-resource generalized assignment problem. As defined by Gavish and Pirkul [13], the multi-resource generalized assignment problem involves the identification of a minimum-cost assignment of tasks to agents in a way that permits assignment of multiple tasks to an agent subject to the availability of a set of multiple resources consumed by that agent. Note that the coefficients v_i and r_i are the same for all sites in formulation (2.5)–(2.9), whereas the amount of a resource used by an agent in performing a task can differ from one agent to another in a two-resource generalized assignment problem. Since the well-known generalized assignment problem is a special case of the two-resource generalized assignment problem, it follows that the two-resource generalized assignment problem is NP-hard. The multi-resource generalized assignment problem has important applications in database allocation in distributed computing systems [14], in large distributed-computer-system design problems [15,16], in vehicle routing [17], and in flexible manufacturing systems in a material requirements planning environment [18].

Campbell and Langevin [12] proposed a two-phase heuristic to solve model (2.5)–(2.9). Their heuristic is a modification of the two-stage procedure proposed by Martello and Toth [19] for the generalized assignment problem. In the first phase, a feasible assignment that satisfies the annual and hourly capacities of the disposal sites is obtained by iteratively considering all unassigned sectors and determining the sector with maximum difference in the objective function value between the closest and second closest disposal site. This sector with maximum difference is then assigned to its closest disposal site and the remaining hourly and annual capacities for this site are adjusted accordingly. This phase is repeated until all sectors are assigned. In the second phase, interchanges are performed to improve the solution by considering sectors two at a time and reassigning if a new assignment decreases the total value of the objective function (2.5). The heuristic algorithm is outlined in Fig. 2.

Tests performed with data from the city of Montreal involving 60 sectors and 20 disposal sites showed an improvement of 4.2% over the solution in use by the city. The two-phase heuristic achieved the optimal solution in a few seconds, while using CPLEX to optimally solve model (2.5)–(2.9) required a few hours. Moreover, the heuristic can be used to calculate minimum-cost solutions for sensitivity analyses performed to evaluate changes in the total snowfall amount, decreasing use of the river disposal sites, and closing of disposal sites. This model was extended in Campbell and Langevin [20] to include both operating and fixed costs for disposal sites. The extended model is presented in Section 4.2.

3. Contract assignment models

Because of the seasonal nature of winter road maintenance operations and the magnitude of the manpower and equipment to be deployed, agencies may rely fully or partly on contract service for winter road maintenance operations. There are a variety of contract options for obtaining resources to perform winter road maintenance operations. There may be single or multi-year contracts with private companies or other governmental agencies, and contractor forces may be mixed with in-house forces or given complete responsibility for particular sectors. These choices depend on financial analysis and assessment, political issues, climate, level of service requirements, in-house resources, etc. Maintenance management systems have been developed to assist planners in selecting the least-cost contract options for maintenance activities such as winter road maintenance operations. Such systems were described, for example, by Bauman and Jorgenson [21] and Blaine [22]. The contract assignment problem consists of assigning a given number of sectors to a set of contractors, so as to minimize the total cost bid by the contractors. Some contractors may require that they be awarded a minimum number of sectors or none at all. Contractors may also quote a price for a specific collection of sectors. Finally, an agency may want to use its own forces to provide winter maintenance in some sectors.

The contract assignment problem can be represented as a simple network flow problem, even with constraints on the minimum number of sectors to be awarded to each contractor, or with bids allowed on collections of sectors. A transportation formulation of this problem for winter road maintenance was proposed by the Bureau of Management Consulting, Transport Canada [6]. Let I be the set of sectors and H be the set of contractors. For every sector $i \in I$ and for every contractor $h \in H$, let x_{ih} be a binary variable equal to 1 if and only if sector i is assigned to contractor h, and let c_{ih} represent the cost of the bid for sector i by contractor h. For each contractor $h \in H$, define N_h as the maximum number of sectors that can be awarded to contractor h. Define C as the number of sectors to assign. The basic model for the contract assignment problem can be stated as follows:

Minimize
$$\sum_{i\in I}\sum_{h\in H} c_{ih}x_{ih}, \tag{3.1}$$
subject to
$$\sum_{h\in H} x_{ih} \leqslant 1 \quad (i \in I), \tag{3.2}$$
$$\sum_{i\in I} x_{ih} \leqslant N_h \quad (h \in H), \tag{3.3}$$

$$\sum_{i\in I}\sum_{h\in H} x_{ih} = C, \tag{3.4}$$

$$x_{ih} \geqslant 0 \quad (i \in I, h \in H). \tag{3.5}$$

The objective function (3.1) minimizes the sum of all bidding costs. Constraints (3.2) require that each sector be assigned to at most one contractor. Constraints (3.3) impose a limit on the maximum number of sectors assigned to each contractor. The total number of contracts to grant is satisfied via constraint (3.4). Finally, all x_{ih} variables must assume nonnegative values. This is an unbalanced transportation model. Computational tests using IBM's MPSX mathematical programming package were performed on data from a major Canadian city containing 51 sectors and 27 contractors with 40 contracts to grant. The model was also used to analyze a variety of scenarios for extensions to the basic model, including constraints on the minimum number of sectors to be awarded to each contractor, and bids on collections of sectors.

4. Snow disposal site location models

Given a set of planned sectors, the snow disposal site location problem consists of locating disposal sites and assigning sectors to the operating disposal sites so as to respect the hourly and annual capacities of the disposal sites. Like the snow disposal assignment problem, the assignment of each sector may also be restricted to a single disposal site. The objective is to minimize the sum of the fixed disposal site location costs, the variable costs to operate the disposal sites, and the transportation costs for hauling snow from sectors to the disposal sites. Models for the disposal site location problem can also be used to determine the most economical disposal method at the strategic level. Since snow disposal sites generate a large volume of truck traffic and around-the-clock activities during snow disposal operations, snow disposal sites may be considered as seasonally obnoxious facilities. Good disposal site locations will minimize the transportation costs for hauling snow from sectors to the disposal sites and be in isolated or nonresidential locations far from population centers. However, in urban areas, nonresidential locations are rare, or require great travel distances, and cost. In an effort to estimate traffic impacts around disposal sites, Braaksma et al. [23] developed a three-phase procedure. The first phase solves the snow disposal assignment problem with annual capacities (not hourly capacities), where each sector may be assigned to more than one site. The second phase estimates the number of truck trips for each site by dividing the total amount of snow assigned to the site by the average capacity of the trucks. The truck traffic at a disposal site is then estimated by distributing the number of trips over the time period for the snow hauling operation and multiplying by a peak hour factor. Truck routing can also be accomplished at that stage and scenarios analysis can be examined. Analysis techniques to determine the traffic impacts for the time period are used last. The truck traffic is converted to an equivalent volume of passenger cars and the impacts on the roads and intersections are then determined. The procedure was tested with data from the regional municipality of Ottawa-Carleton, Canada.

Models for the location of snow disposal sites are now reviewed. We first discuss models that include annual, but not hourly, disposal site capacities and allow multiple assignment (the snow from a sector can be hauled to several disposal sites). We then present models addressing the more realistic case where disposal sites have annual as well as hourly capacities and each sector must be assigned to a single site.

4.1. Multiple assignment models with annual disposal site capacities

One of the first models for snow disposal site location belongs to the Bureau of Management Consulting, Transport Canada [6] which suggested a linear, mixed-integer programming formulation of the problem. The model, which is a capacitated fixed charge facility location problem, simultaneously determines the optimal subset of the locations at which to place disposal sites and the optimal assignment of given sectors to these disposal sites. Let I be the set of sectors and J be the set of disposal sites. For every disposal site $j \in J$, let y_j be a binary variable equal to 1 if and only if site j is operative, define V_j as the annual capacity of site j, and let b_j and f_j represent the variable cost per unit volume of snow to operate site j and the fixed cost to locate at candidate site j, respectively. For every sector $i \in I$ and for every site $j \in J$, define x_{ij} as the volume of snow transported from sector i to site j, and let c_{ij} represent the transportation cost per unit volume of snow from sector i to site j. Finally, for every sector $i \in I$, v_i corresponds to the annual volume of snow in sector i. The capacitated fixed charge formulation for the snow disposal site location problem can be stated as follows:

Minimize

$$\sum_{i \in I} \sum_{j \in J} (b_j + c_{ij}) x_{ij} + \sum_{j \in J} f_j y_j, \tag{4.1}$$

subject to

$$\sum_{j \in J} x_{ij} = v_i \quad (i \in I), \tag{4.2}$$

$$\sum_{i \in I} x_{ij} \leqslant V_j y_j \quad (j \in J), \tag{4.3}$$

$$x_{ij} \geqslant 0 \quad (i \in I,\ j \in J), \tag{4.4}$$

$$y_j \in \{0, 1\} \quad (j \in J). \tag{4.5}$$

The objective function (4.1) minimizes the sum of three costs: the variable cost to operate the disposal sites, the transportation cost for hauling snow from sectors to the disposal sites, and the fixed disposal site cost. Constraints (4.2) stipulate that each sector be cleared of snow. Constraints (4.3) limit the assignments of sectors to operating disposal sites according to the annual capacity of the disposal sites. Finally, all x_{ij} variables must assume nonnegative values, while y_j variables are restricted to be binary. Note that this model allows a sector to send snow to more than one disposal site, which is often prohibited in practice. Also, we note that if we are given values y_j for the location variables, then the snow disposal site location problem reduces to the snow disposal assignment problem of the sort discussed in Section 2.1. This formulation for the snow disposal site location problem was also proposed by Audette [24], who developed a two-phase heuristic to solve the model. The first phase finds an initial solution by solving a relaxation of model (4.1)–(4.5) obtained by relaxing the integrality requirements on the y_j location variables and by replacing the cost function in (4.1) by the linear approximation (4.6):

$$\sum_{i \in I} \sum_{j \in J} \left(b_j + c_{ij} + \frac{f_j}{V_j} \right) x_{ij}. \tag{4.6}$$

A starting value for the cost function can be calculated by fixing the x_{ij} variables to their optimal values in this initial solution and by setting $y_j = 1$ if and only if any sectors are assigned to site j in this solution.

The second phase then tries to reduce the value of the cost function by closing disposal sites selected by inspection. By iteratively fixing $V_j = 1$ for a given disposal site $j \in J$ in the model obtained from the relaxation and solving it at each iteration, an improved solution can be found. Tests performed on data from the city of Montreal involving 76 sectors and 28 sites showed that the second phase allowed an improvement of less than 1.5% over the initial solution, but the number of sites decreased by more than 18%.

4.2. Single assignment models with annual and hourly disposal site capacities

Depending on the logistics and configuration of unloading facilities at the disposal sites, the previous formulation may not be very realistic by disregarding the maximum snow receiving rate of the disposal sites. For example, sewer chutes have high annual capacities, but relatively small hourly capacities due to the small opening into the sewer system. Moreover, according to operating rules, the assignment of each sector may also be restricted to a single site. Campbell and Langevin [20] extended the basic model (4.1)–(4.5) to include both annual and hourly capacities as well as assignment of each sector to a single site. This model is formulated as a single-source capacitated facility location problem in which each facility has two capacities: an hourly capacity for receiving snow and an annual capacity for storing snow. In a single-source capacitated facility location problem, each customer has a demand which must be satisfied by a single facility. In the snow disposal site location problem considered here, each sector must be assigned to a single disposal site.

Let I be the set of sectors and J be the set of disposal sites. For every sector $i \in I$, let r_i represent the snow removal rate in sector i, expressed as cubic meters per hour. For every disposal site $j \in J$, define R_j as the hourly snow receiving rate capacity of site j, also expressed as cubic meters per hour. Then, the snow disposal site location problem can be formulated as a single-source capacitated facility location problem with two capacities for each facility as follows:

Minimize

$$\sum_{i \in I} \sum_{j \in J} (b_j + c_{ij}) v_i x_{ij} + \sum_{j \in J} f_j y_j, \tag{4.7}$$

subject to

$$\sum_{i \in I} v_i x_{ij} \leqslant V_j \quad (j \in J), \tag{4.8}$$

$$\sum_{i \in I} r_i x_{ij} \leqslant R_j \quad (j \in J), \tag{4.9}$$

$$\sum_{j \in J} x_{ij} = 1 \quad (i \in I), \tag{4.10}$$

$$x_{ij} \leqslant y_j \quad (i \in I,\ j \in J), \tag{4.11}$$

$$x_{ij}, y_j \in \{0, 1\} \quad (i \in I,\ j \in J). \tag{4.12}$$

In this model, the binary variables x_{ij} indicate the assignment of sectors to disposal sites. The objective function (4.7) minimizes the sum of the volume-weighted variable cost to operate the disposal sites, the volume-weighted transportation cost for hauling snow from sectors to the disposal sites, and the fixed disposal site cost. Constraint sets (4.8) and (4.9) ensure that the annual and hourly capacities of each disposal site are not exceeded. Constraint sets (4.10) and (4.11) ensure that each sector is assigned to exactly one operative disposal site. As noted by Campbell and Langevin [20], when the location variables y_j are known, then the snow disposal site location problem with single assignment constraints and hourly and annual site capacities reduces to a two-resource generalized assignment problem of the sort discussed in Section 2.2. Thus, the authors suggested solving model (4.7)–(4.12) using a heuristic that incorporates the two-phase heuristic of Campbell and Langevin [12] for the snow disposal assignment problem presented in Section 2.2. However, they do not provide an algorithm.

Maréchal [25] proposed a model and a column generation algorithm for the snow disposal site location problem with single assignment constraints and two capacities for each site. The model is a linear, 0–1 integer, set-partitioning problem with additional side constraints that limit the assignments of sectors to disposal sites according to the annual and hourly receiving capacity of the disposal sites. To present the formulation, let I be the set of sectors and J be the set of disposal sites. Given the sets I and J, let $M = I \cup J$ and let M_k for $k \in K = \{1, \ldots, |J|(2^{|I|} - 1)\}$ be a subset of M consisting of one disposal site and at least one sector to be assigned to site j. For any disposal site $j \in J$, define also $K_j = \{k \in K \mid j \in M_k\}$ as the set of all subsets of M associated with site j. For every sector $i \in I$ and for every subset $k \in K$, define the binary constant a_{ik} equal to 1 if and only if sector $i \in M_k$. For every subset $k \in K$, let z_k be a binary variable equal to 1 if and only if subset M_k is selected. The operational parameters v_i, r_i, V_j and R_j are defined as above. The cost parameters $f_{j(k)}$ and $b_{j(k)}$ are now defined for each subset $M_k \in K$ and for each site $j \in M_k$ and the costs $c_{ij(k)}$ are defined for each subset $M_k \in K$, for each sector $i \in M_k$ and for each site $j \in M_k$. Then the snow disposal site location problem amounts to choosing a minimum-cost collection of subsets of M such that each sector is assigned to exactly one disposal site according to the annual and hourly receiving capacities of the disposal sites. The formulation is given next.

Minimize

$$\sum_{k \in K} \left(f_{j(k)} + \sum_{i \in I} (b_{j(k)} + c_{ij(k)}) v_i a_{ik} \right) z_k, \tag{4.13}$$

subject to

$$\sum_{k \in K} a_{ik} z_k = 1 \quad (i \in I), \tag{4.14}$$

$$\sum_{k \in K_j} \left(\sum_{i \in I} v_i a_{ik} \right) z_k \leqslant V_j \quad (j \in J), \tag{4.15}$$

$$\sum_{k \in K_j} \left(\sum_{i \in I} r_i a_{ik} \right) z_k \leqslant R_j \quad (j \in J), \tag{4.16}$$

$$z_k \in \{0, 1\} \quad (k \in K). \tag{4.17}$$

The objective function (4.13) minimizes the sum of all subset costs. For the correctness of the objective function, Maréchal showed that there is always at most one subset M_k selected for each disposal site $j \in J$ in an optimal solution to model (4.13)–(4.17). Constraints (4.14) require that each sector be part of exactly one subset and constraints (4.15) and (4.16) are the disposal sites annual and hourly capacity constraints, respectively. Maréchal showed that model (4.13)–(4.17) is equivalent to Campbell and Langevin formulation (4.7)–(4.12). The Maréchal formulation contains $(|I| \cdot |J|)$ fewer constraints than Campbell and Langevin formulation, but the number of variables is exponential. Maréchal proposed to decompose the model into a master problem and a set of $|J|$ different independent subproblems.

The master problem is obtained by relaxing the capacity constraints (4.15) and (4.16) in the original formulation (4.13)–(4.17) and by imposing a limit of one subset $M_k, k \in K_j$, for each disposal site $j \in J$. The model for the master problem can be stated as follows:

Minimize

$$\sum_{k \in K} \left(f_{j(k)} + \sum_{i \in I} (b_{j(k)} + c_{ij(k)}) v_i a_{ik} \right) z_k, \tag{4.18}$$

subject to

$$\sum_{k \in K} a_{ik} z_k = 1 \quad (i \in I), \tag{4.19}$$

$$\sum_{k \in K_j} z_k \leqslant 1 \quad (j \in J), \tag{4.20}$$

$$z_k \in \{0, 1\} \quad (k \in K). \tag{4.21}$$

Moreover, for every site $j \in J$, the subproblem is of the following form:

Minimize

$$f_j + \sum_{i \in I} [(b_j + c_{ij}) v_i - u_i] a_i - u_{m+j}, \tag{4.22}$$

subject to

$$\sum_{i \in I} v_i a_i \leqslant V_j, \tag{4.23}$$

$$\sum_{i \in I} r_i a_i \leqslant R_j, \tag{4.24}$$

$$a_i \in \{0, 1\} \quad (i \in I), \tag{4.25}$$

where a_i is equal to 1 if and only if sector i is assigned to site j. For each site $j \in J$, the objective function (4.22) corresponds to the reduced cost of variable z_k, $k \in K_j$. Model (4.18)–(4.21) is solved with a branch-and-bound procedure using linear programming relaxations that are solved by column generation. Columns of the master problem are generated by solving, for each site $j \in J$, subproblem (4.22)–(4.25) with an objective that is iteratively updated to reflect the new values of the dual variables u_i. Branching is performed on the binary variables indicating whether a subset is chosen or not. The subproblem (4.22)–(4.25) for each disposal site is a bidimensional knapsack problem that is solved using a simple branch-and-bound method. Computational experiments on instances with up to 80 sectors and 25 disposal sites produced optimal solutions within 30 minutes. However, to address the slow convergence of the column generation algorithm, Maréchal suggested applying the stabilized column generation method proposed by du Merle et al. [26] to stabilize and accelerate the procedure. Computational experiments were also performed to evaluate the impact of increasing the fixed costs and the problem size on performance measures such as computation times.

5. Sector design models for snow disposal

The traditional approach for the sector design problem consists in partitioning the road network into sectors by assigning basic units to their closest facility. Muyldermans et al. [27,28] and Kandula and

Wright [29,30] used this approach for designing sectors for spreading and plowing operations. Similar approaches for solving the sector design problem in the context of snow disposal operations are described in this section. Sector design models can be classified according to the winter road maintenance operations considered. Optimization models that address the sector design related to spreading and plowing operations were reviewed in the first part of the survey [1]. In this section, sector design issues for snow disposal operations are discussed first, followed by compound models that integrate sector design, fleet sizing, and snow disposal assignment decisions for loading trucks and hauling snow to disposal sites.

5.1. Sector design issues for snow disposal

As highlighted by Perrier et al. [1], the compactness or shape criterion for designing sectors for winter road maintenance depends on the number of sectors, the number and type of facilities, and the type of winter road maintenance operations (spreading, plowing, loading snow into trucks, hauling snow to disposal sites). When the number of sectors to be designed corresponds to the number of facilities, compact sectors with centrally located facilities (vehicle depots, materials depots, disposal sites) lead to more efficient routing of vehicles. However, when the number of sectors exceeds the number of facilities, the appropriate shape of a set of sectors for efficient routing thus depends on the type of operations. Typically, for efficient routing of spreaders and snow plows, sectors should be elongated towards the vehicle depot or materials depot to reduce travel distance in each route. This is the general guideline for forming sectors for a vehicle routing problem.

As explained in Section 1.1, snowblowers generally operate in a continuous process loading trucks to minimize the completion time for snow disposal operations. For hauling snow to disposal sites, the travel time depends on the location of the truck relative to the assigned disposal site when it departs from, and returns to the snowblower. If the snowblower is far from the disposal site, then a truck must travel a long distance to and from the disposal site. Therefore, the number of trucks assigned to the sector must be large enough to ensure that an empty truck will always be available to be filled by the snowblower. However, if the snowblower is near the disposal site, then only a small number of trucks are required to prevent the snowblower to become idle. For efficient routing of snowblowers and trucks, sectors should thus be elongated in a direction perpendicular to the direction to the disposal site to reduce the number of trucks required [31].

5.2. Compound sector design, snow disposal assignment, and fleet sizing models for snow disposal

The combined sector design, snow disposal assignment, and fleet sizing problem addressed in this section involves partitioning an urban area into sectors, assigning the sectors to disposal sites, and determining the number of trucks assigned to each sector for snow loading and hauling operations. As was highlighted by Campbell and Langevin [20], the sector design and snow disposal assignment problems are interdependent. Indeed, the size and shape of a sector may influence its assignment and vice versa. In an effort to integrate both sector design and snow disposal assignment decisions into a single optimization model, Labelle et al. [31] proposed a formulation for the combined problem of sector design, snow disposal assignment, and truck fleet sizing. The model is based on a set of geographic zones each containing a collection of neighboring street segments and incorporates a limit on sector size, hourly and annual disposal site capacities, as well as contiguity constraints.

To present the Labelle et al. formulation, we first define the decision variables and the operational and cost parameters. Let I, J, and K be the sets of geographic zones, sectors, and disposal sites, respectively.

For every zone $i \in I$ and for every sector $j \in J$, let x_{ij} be a binary variable equal to 1 if and only if zone i is assigned to sector j. The number of snowblowers for loading snow into trucks is given and each sector must contain exactly one snowblower. Thus, the cardinality of J corresponds to the number of snowblowers available and there may be sectors to which no zones are assigned in a feasible solution. For every sector $j \in J$ and for every site $k \in K$, let y_{jk} be a binary variable equal to 1 if and only if sector j is assigned to site k. For each zone $i \in I$, define v_i as the annual volume of snow in zone i to be hauled to a disposal site, expressed as cubic meters of snow per year. The annual volume of snow in a zone is estimated based on the historical amount of snow per linear meter of street. Thus, the total length of streets in a zone determines the annual volume of snow generated by the sector to be sent to a disposal site. For every zone $i \in I$ and every site $k \in K$, define d_{ik} as the distance from the farthest part of zone i to site k, expressed as kilometers, and C_{ik} as the operational cost per cubic meter for hauling snow from zone i to site k. The distances d_{ik} are calculated with a shortest path algorithm specifically developed for this application that uses a reduced network of major roadways likely to be traveled by the heavy trucks hauling snow. For details, see Labelle [32] and Campbell et al. [33].

For each disposal site $k \in K$, let V_k and R_k be the annual and hourly capacities of site k, respectively, and let CV_k represent the variable operating cost for disposal site k. For every sector $j \in J$, let r_j be the snow removal rate in sector j, expressed as cubic meters of snow per hour, and let N_j be the number of snow hauling trucks assigned to sector j. The number of snow hauling trucks assigned to a sector is defined so that there should always be a truck available to be filled by the snowblower, while other trucks are traveling to and from the disposal site. This allows snowblowers to operate continuously and will minimize the time required to clear the streets of snow. Thus,

$$N_j = \left\lceil \frac{2 \max_{i,k}\{d_{ik} x_{ij} y_{jk}\}}{t_{\mathrm{s}}} \times \frac{r_j}{t_{\mathrm{v}}} \right\rceil + \tau, \tag{5.1}$$

where t_{v} is the truck size, t_{s} is the truck speed, and τ is the number of "additional" trucks assigned to a sector. The hourly removal rate from sectors is estimated based on the capabilities of snowblowers for filling trucks (described below). The first ratio in N_j is the time taken by a truck to travel from the farthest zone in sector j to its assigned disposal site and back. The second ratio is the snow removal rate in trucks per hour. The product of these two ratios provides the number of trucks, possibly fractional, that would be filled by a continuously operating snowblower during the longest trip to and from the disposal site. The additional number of trucks τ helps to account for variability in the truck travel time. If $\tau = 0$, then the blower would be idle whenever the actual travel time is greater than the average travel time to the farthest zone. Values of τ greater than zero allow the blower to stay busy when travel times exceed the average value.

Finally, let CT be the fixed cost for trucks and M the maximum number of zones in a sector. Then, Labelle et al. [31] formulated the combined sector design, snow disposal assignment, and truck fleet sizing problem as a nonlinear 0–1 integer program as follows:

Minimize

$$\sum_{i\in I}\sum_{j\in J}\sum_{k\in K} C_{ik} v_i x_{ij} y_{jk} + \sum_{k\in K} CV_k \sum_{i\in I}\sum_{j\in J} v_i x_{ij} y_{jk} + CT \sum_{j\in J} N_j, \tag{5.2}$$

subject to

$$\sum_{j\in J} x_{ij} = 1 \quad (i \in I), \tag{5.3}$$

$$x_{ij} \leqslant \sum_{k\in K} y_{jk} \quad (i \in I,\ j \in J), \tag{5.4}$$

$$\sum_{i\in I} x_{ij} \leqslant M \quad (j \in J), \tag{5.5}$$

$$\sum_{i\in I} \sum_{j\in J} v_i x_{ij} y_{jk} \leqslant V_k \quad (k \in K), \tag{5.6}$$

$$\sum_{i\in I} \sum_{j\in J} r_i x_{ij} y_{jk} \leqslant R_k \quad (k \in K), \tag{5.7}$$

$$\text{each sector is a contiguous collection of zones} \tag{5.8}$$

$$x_{ij},\ y_{jk} \in \{0, 1\} \quad (i \in I,\ j \in J, k \in K). \tag{5.9}$$

The nonlinear objective function (5.2) minimizes the sum of three costs: the transportation cost for hauling snow from the sectors to the disposal sites, the variable cost to operate the disposal sites, and the fixed cost for the trucks. Constraint set (5.3) assures that each zone is assigned to exactly one sector. Constraint set (5.4) links the zone and snow disposal assignments. It states that a zone can be assigned to a sector only if this sector is assigned to some site. Recall that there may be sectors to which no zones are assigned given that the number of snowblowers available is specified and that exactly one snowblower must be operative in each sector. Constraint set (5.5) limits the size of the sectors to at most M zones. Nonlinear constraint sets (5.6) and (5.7) limit the assignment of sectors to disposal sites according to the annual and hourly receiving capacity of each disposal site. The snow removal rate r_i in (5.7) is defined for each zone $i \in I$ rather than for each sector as in (5.1). Constraints (5.8) require that each sector is composed of a contiguous set of zones. Labelle [32] proposed a set of linear constraints requiring that each zone assigned to a disposal site must be contiguous to at least two other zones assigned to the same disposal site. For every pair of zones $i, h \in I$, $i \neq h$, define the binary constant a_{ih} equal to 1 if and only if zone i is adjacent to zone h. Then,

$$2x_{ij} - \sum_{h} a_{ih} x_{hj} \leqslant 0 \quad (i \in I,\ j \in J) \tag{5.10}$$

requires that every sector contains at least three zones. Note that this simple constraint set does allow noncontiguous sectors, but each subsector will have at least three zones. Finally, all x_{ij} and y_{jk} variables are restricted to be binary. Note that model (5.2)–(5.9) allows each sector to be assigned to several disposal sites. However, for operational reasons in many cities, the assignment of each sector is restricted to a single site.

Labelle [32] proposed a linear 0–1 integer program for the combined sector design and snow disposal assignment problem. The nonlinearities in the objective function (5.2) and in the constraint sets (5.6) and (5.7) of the Labelle et al. formulation are removed by eliminating the fixed cost for the trucks and by replacing the two groups of assignment variables by a single composite variable. For every zone $i \in I$, for every sector $j \in J$ and for every site $k \in K$, let x_{ijk} be a binary variable equal to 1 if and only if zone i is assigned to sector j and to site k. Define C as the cost per cubic meter-weighted distance for hauling snow. All other operational and cost parameters as well as constants are defined as above. The formulation is given next.

Minimize

$$C \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} v_i d_{ik} x_{ijk} + \sum_{k\in K} CV_k \sum_{i\in I}\sum_{j\in J} v_i x_{ijk}, \tag{5.11}$$

subject to

$$\sum_{j\in J}\sum_{k\in K} x_{ijk} = 1 \quad (i \in I), \tag{5.12}$$

$$\sum_{i\in I}\sum_{k\in K} x_{ijk} \leqslant M \quad (j \in J), \tag{5.13}$$

$$\sum_{i\in I}\sum_{j\in J} v_i x_{ijk} \leqslant V_k \quad (k \in K), \tag{5.14}$$

$$\sum_{i\in I}\sum_{j\in J} r_i x_{ijk} \leqslant R_k \quad (k \in K), \tag{5.15}$$

$$2\sum_{k\in K} x_{ijk} - \sum_{h\in I} a_{ih} \sum_{k\in K} x_{hjk} \leqslant 0 \quad (i \in I,\, j \in J), \tag{5.16}$$

$$x_{ijk} \in \{0,1\} \quad (i \in I,\, j \in J,\, k \in K). \tag{5.17}$$

The objective function (5.11) minimizes the sum of the transportation annual cost for hauling snow from sectors to disposal sites and the variable cost to operate the disposal sites. In this model, constraint sets (5.3) and (5.5) of the Labelle et al. [31] formulation are modified by replacing each variable x_{ij} by the sum of x_{ijk} for $k \in K$. In addition, contiguity constraints (5.10) are modified by replacing each variable x_{hj} by the sum of x_{hjk} for $k \in K$ and constraint sets (5.6) and (5.7) are modified by replacing each product $x_{ij} y_{jk}$ by the variable x_{ijk}. Though model (5.11)–(5.16) is linear and contains $(|I| \cdot |J|)$ fewer constraints than the nonlinear model (5.2)–(5.7) with the contiguity constraints (5.10), the number of variables increases rapidly as the number of zones, sectors, and sites increases.

In contrast to the sequential approach that consists of first partitioning a road network into sectors, and then assigning the sectors to disposal sites, Labelle [32] and Labelle et al. [31] developed a two-phase heuristic of "assign first, partition second" for the combined problem of sector design and snow disposal assignment. In the first phase (assign), the assignment of zones to disposal sites is determined to define each disposal site's "area of influence". In the second phase (partition), sectors are designed for each area of influence by agglomerating neighboring zones into sectors. The objective for the "assign" phase is to minimize relevant operational costs, while the "partition" phase seeks to minimize the number of trucks for the given zone assignments. The problem of assigning zones to disposal sites is solved using an adaptation of a composite heuristic proposed by Campbell and Langevin [12] for the snow disposal assignment problem. This composite heuristic is presented in detail in Section 2.2. In the constructive phase, zones are assigned to disposal sites based on a penalty calculation. Then, interchanges are performed to improve the solution by considering reassignment of every pair of zones to different sites.

The "partition" phase for aggregating zones into sectors considers each area of influence separately. Recall that the number of snow hauling trucks required in a sector depends on the maximum travel time between a sector and its assigned site. Under the assumption that minimizing distance minimizes

1. Set $k = 1$.
2. Repeat the following steps until $k = |K|$:
   a. Let d_{ik} be the distance between the zone i centroid and site k. For each pair of adjacent zones i and j assigned to site k, compute *savings* $s_{ij} = \min\{d_{ik}, d_{jk}\}$.
   b. If a zone assigned to site k has only one adjacent zone assigned to site k and their union will not exceed the sector size limit, then join the two zones. Repeat step b while there are zones assigned to site k with only one adjacent zone assigned to site k.
   c. Order the savings from largest to smallest.
   d. Starting at the top of the savings list, join two adjacent zones i and j whose union will not exceed the sector size limit. If the sector size constraint is not satisfied, move to the next largest savings in the list.
   e. Repeat steps a, b, c, and d until all zones assigned to site k belong to a sector.
   f. Set $k = k + 1$.

Fig. 3. The sector aggregation algorithm for the sector design problem [31,32].

travel time, the number of trucks can thus be minimized by designing sectors to minimize the sum of the maximum distances from sectors to disposal sites. The basic idea behind the sector aggregation algorithm is to combine two zones that satisfy the sector size constraint and whose union results in the greatest decrease in the sum of the maximum distances from the zones to the disposal site. This ideally produces sectors that are circular arcs centered on the disposal site. In practice, it tends to produce sectors elongated in the direction perpendicular to the direction to the disposal site. Combining two zones produces a “savings” corresponding to the trip to the closer of the two zones. (This is somewhat analogous to the Clarke and Wright [34] savings procedure for the capacitated vehicle routing problem.) The Labelle, Langevin and Campbell sector aggregation algorithm is presented in Fig. 3. The term “zone” is used to refer to the original set of geographic zones assigned to disposal sites and to the agglomeration of several of these original zones. The truck travel distances d_{ik} are estimated with a hybrid distance approximation developed by Campbell et al. [33].

The “assign first, partition second” heuristic was imbedded in a geographical information system to form a decision support system allowing manual adjustments to address selected geographic, political and economic concerns. The system was tested on a real-life instance from the city of Montreal involving 390 zones and 20 disposal sites. Results showed that the system produced sectors having the desired shape in less than 15 seconds and was useful in analyzing a variety of scenarios related to the modification of transportation and elimination costs as well as disposal site capacities. The solution produced by the heuristic had one disposal site with only one isolated zone assigned to it. Such a situation is addressed by manual adjustments or by taking into account the fixed costs of the disposal sites in the “assign” phase.

6. Conclusions

This paper is the second part of a four-part survey of optimization models for winter road maintenance. (The first part of the survey [1] discusses system design models for spreading and plowing operations. The two last parts of the review [2,3] mainly address vehicle routing, depot location, and fleet sizing models for winter road maintenance problems.) Table 1 summarizes the characteristics of the reviewed system design models related to snow disposal operations.

Table 1
Characteristics of system design models for snow disposal

Authors	Problem type	Planning level	Problem characteristics	Objective function	Model structure	Solution method
Transport Canada [6]	Snow disposal assignment	Tactical	Annual disposal site capacities and multiple assignment	Min transport costs and disposal site variable costs	Transportation problem	MPSX mathematical programming
Leclerc [7]	Snow disposal assignment	Tactical	Annual disposal site capacities and multiple assignment	Min transport costs and disposal site variable costs	Transportation problem	Stepping stone
Leclerc [9] Leclerc et al. [10]	Snow disposal assignment	Tactical	Annual disposal site capacities and single assignment	Min transport costs and disposal site variable costs	Transportation problem	Constructive heuristic
Campbell and Langevin [12]	Snow disposal assignment	Tactical	Hourly and annual disposal site capacities, and single assignment	Min total snow volume-weighted distance	Two-resource generalized assignment problem	Composite heuristic
Transport Canada [6]	Contract assignment	Tactical	Annual disposal site capacities and multiple assignment	Min bidding costs	Transportation problem	MPSX mathematical programming
Braaksma et al. [23]	Disposal site traffic impacts	Tactical	Annual disposal site capacities and multiple assignment	Min transport costs and disposal site variable costs	Snow disposal assignment problem	Heuristic
Transport Canada [6]	Disposal site location	Strategic	One contractor per sector and maximum number of sectors per contractor	Min transport costs, and disposal site variable and fixed costs	Capacitated facility location problem	MPSX mathematical programming
Audette [24]	Disposal site location	Strategic	Annual disposal site capacities and multiple assignment	Min transport costs, and disposal site variable and fixed costs	Capacitated facility location problem	Heuristic

Campbell and Langevin [20]	Disposal site location	Strategic	Hourly and annual disposal site capacities, and single assignment	Min transport costs, and disposal site variable and fixed costs	Single-source capacitated facility location problem	Heuristic
Maréchal [25]	Disposal site location	Strategic	Hourly and annual disposal site capacities, and single assignment	Min transport costs, and disposal site variable and fixed costs	Linear 0–1 IP	Branch-and-bound
Labelle [32]	Combined sector design and snow disposal assignment	Strategic	Contiguity, elongated sectors, maximum sector size, grouping of street segments, hourly and annual disposal site capacities, and multiple assignment	Min transport costs and disposal site variable costs	Linear 0-1 IP	Heuristic assign first, partition second
Labelle et al. [31]	Combined sector design, snow disposal assignment, and truck fleet sizing	Strategic	Contiguity, elongated sectors, maximum sector size, grouping of street segments, hourly and annual disposal site capacities, and multiple assignment	Min transport costs, disposal site variable costs, and truck fixed costs	Nonlinear 0–1 IP	Heuristic assign first, partition second

As mentioned in the introduction, these problems are often site specific and highly difficult because of the many significant differences in operating conditions surrounding the winter road maintenance operations. Hence, most research contributions have been case study oriented. Early works usually proposed simplified models of special structure (linear programming or network optimization) that often neglected to incorporate the characteristics of applications arising in practice. Later research generally focused on the design of heuristics to solve more realistic problems.

However, the use of operations research methodologies for winter road maintenance problems is still in its infancy. Even though most problems studied grew out of applications, and the proposed models were tested on real-life instances, few of them have been applied in practice. A survey carried out by Gupta [35] in 50 U.S. state departments of transportation and other agencies shows that most departments still rely in large part on decision rules dictated by experience when making vehicle and materials depot location and relocation decisions. Operations research holds great promise for improving winter road maintenance and efforts towards reducing the gap between theory and practice must be made. Also, to deal with almost any real-world applications, proposed models need to be extended in a variety of ways. However, advances in computing power now allow near-optimal solution of problems of realistic size. Thus, some promising directions for future research in winter road maintenance planning are the development of more realistic mathematical formulations, the use of multi-objective analysis, and the development of more comprehensive models that integrate multiple decisions.

The development of more realistic mathematical formulations is crucial not only to take into consideration the characteristics of applications arising from practice but also to exhibit problem structures that may be readily utilized to design heuristic algorithms. Since winter road maintenance operations lead to large and complex problems, heuristics may continue to be the prevailing approach to such problems. These heuristics could then use the structure revealed by new mathematical models.

As mentioned in the introduction, a common characteristic of nearly all winter road maintenance problems is that multiple and often conflicting objectives need to be addressed. While a few researchers have started to consider multiple objective models for certain problems in winter road maintenance, most problems still await a multi-objective analysis. In particular, in locating snow disposal sites, the tradeoff between minimizing transportation costs and minimizing the number of people affected by the disposal sites remains largely unexplored. Also, in determining the truck fleet size for hauling snow to disposal sites, there may be a tradeoff between minimizing the fixed and variable costs for the trucks and minimizing the length of time for the snow loading and hauling operations. Therefore, another direction worth pursuing involves the use of multi-objective analysis to assist in quantifying these sorts of tradeoffs.

Another promising direction of research is the development of models that address the integration of various decisions in system design. Given the high difficulty of winter road maintenance problems and the large size of the problems encountered in practice, the traditional approach has been to deal with these problems sequentially. Commonly, disposal sites are located first, sectors are then designed and assigned to disposal sites, and vehicle routes and schedules are determined last. This approach simplifies the analysis, but is likely to produce a suboptimal system. The integration of system design models with vehicle routing decisions for winter road maintenance may prove to have a great impact on the future use of operations research in winter road maintenance.

Also, one interesting question might be the potential savings from a dynamic assignment plan of sectors to disposal sites. Generally, a static assignment plan is made at the beginning of the season based on anticipated snowfall totals. However, there might be savings from changing the sector assignments dynamically during the season. This would entail additional administrative costs at the operational planning level, but if savings were substantial, it might be worthwhile.

Finally, new technologies in the application of road weather information systems and weather forecasting services are being implemented in many highway agencies in North America, Europe, and Japan. These technologies are likely to have major repercussions on winter road maintenance operations. More accurate temporal and spatial knowledge of weather and pavement conditions both in real time and for the near future can lead to better use of scarce winter road maintenance resources. New problems are arising from the implementation of these new technologies in winter road maintenance. Hence, there will be plenty of challenging problems in the field of winter road maintenance for many years to come.

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