In[129]:=
```
SetOptions[SelectedNotebook[],
  PrintingStyleEnvironment -> "Printout", ShowSyntaxStyles -> True]
```

$q_{i+1/2}$ equations

In[130]:=
```
Quiet@Remove["`*"]
```

In[131]:= $eqn1 = xh == x0 + \frac{h}{2} (px0 + yh);$

In[132]:= $eqn2 = yh == y0 + \frac{h}{2} (py0 - xh);$

In[133]:=
```
Solve[{eqn1, eqn2}, {xh, yh}] // Simplify
```

Out[133]= $\left\{\left\{xh \to \frac{h^2\, py0 + 4\, x0 + 2\, h\, (px0 + y0)}{4 + h^2},\ yh \to \frac{-h^2\, px0 + 2\, h\, (py0 - x0) + 4\, y0}{4 + h^2}\right\}\right\}$

We recognize that:

$px0 + y0 = \dot{\boldsymbol{p}}_{\boldsymbol{x}}$

So we can rewrite:

$$xh \to \frac{h^2\, py0 + 4\, x0 + 2\, h\, (xdot0)}{4 + h^2}$$

p_{i+1} equations

In[134]:=
```
Quiet@Remove["`*"]
```

In[135]:= $eqn1 = px1 == px0 - \frac{h}{2} (-(py0 + pdxR) - (py1 + pdxR));$

In[136]:= $eqn2 = py1 == py0 - \frac{h}{2} (-(-px0 + pdyR) - (-px1 + pdyR));$

In[137]:=
```
Solve[{eqn1, eqn2}, {px1, py1}] // FullSimplify
```

Out[137]= $\left\{\left\{px1 \to \frac{h^2\, (2\, pdyR - px0) + 4\, px0 + 4\, h\, (pdxR + py0)}{4 + h^2},\ py1 \to \frac{4\, py0 - h\, (-4\, pdyR + 4\, px0 + h\, (2\, pdxR + py0))}{4 + h^2}\right\}\right\}$

where **pdxR** and **pdyR** are the second (*rest*) terms of $\dot{p}_x$ and $\dot{p}_y$ respectively.

We recognize that:

$2\, pdyR - px0 = \mathbf{2}\, \dot{\boldsymbol{p}}_{\boldsymbol{y}} \mathbf{+ px}$

$pdxR + py0 = \dot{\boldsymbol{p}}_{\boldsymbol{x}}$

So we can rewrite, also ordering terms:

$$px1 \to \frac{h^2\, (2\, pdot\_y0 + p\_x0) + 4\, h\, pdot\_x0 + 4\, p\_x0}{4 + h^2}$$