APPENDIX

F Computing the Difference Between Nearly Equal Numbers

Let **a**, **b**, and **c** be vectors such that $\mathbf{c} = \mathbf{b} - \mathbf{a}$ and $a << b$. Clearly, $c \approx b$. In order to calculate

$$F \equiv 1 - \frac{c^3}{b^3} \tag{F.1}$$

we may first define

$$q \equiv 1 - \frac{c^2}{b^2} \tag{F.2}$$

It follows that

$$F = 1 - \left(\frac{c^2}{b^2}\right)^{\frac{3}{2}} = 1 - (1-q)^{\frac{3}{2}} = \left[1 - (1-q)^{\frac{3}{2}}\right] \frac{1 + (1-q)^{\frac{3}{2}}}{1 + (1-q)^{\frac{3}{2}}} = \frac{1 - (1-q)^3}{1 + \left(\sqrt{1-q}\right)^3}$$

or

$$F(q) = \frac{q^2 - 3q + 3}{1 + (1-q)^{\frac{3}{2}}} q \tag{F.3}$$

Using this formula to compute F does not require finding the difference between nearly equal numbers, as in Eqn (F.1). However, that problem persists when using Eqn (F.2) to calculate q. We can work around that issue by observing that

$$q = \frac{b^2 - c^2}{b^2} = \frac{(\mathbf{b} - \mathbf{c}) \cdot (\mathbf{b} + \mathbf{c})}{b^2}$$

or, since $\mathbf{c} = \mathbf{b} - \mathbf{a}$,

$$q = \frac{\mathbf{a} \cdot (2\mathbf{b} - \mathbf{a})}{b^2} \tag{F.4}$$

Computing q by means of this formula and substituting the result into Eqn (F.3) avoids roundoff error that may occur by calculating F using Eqn (F.1) when $c/b \approx 1$ (Battin, 1999).