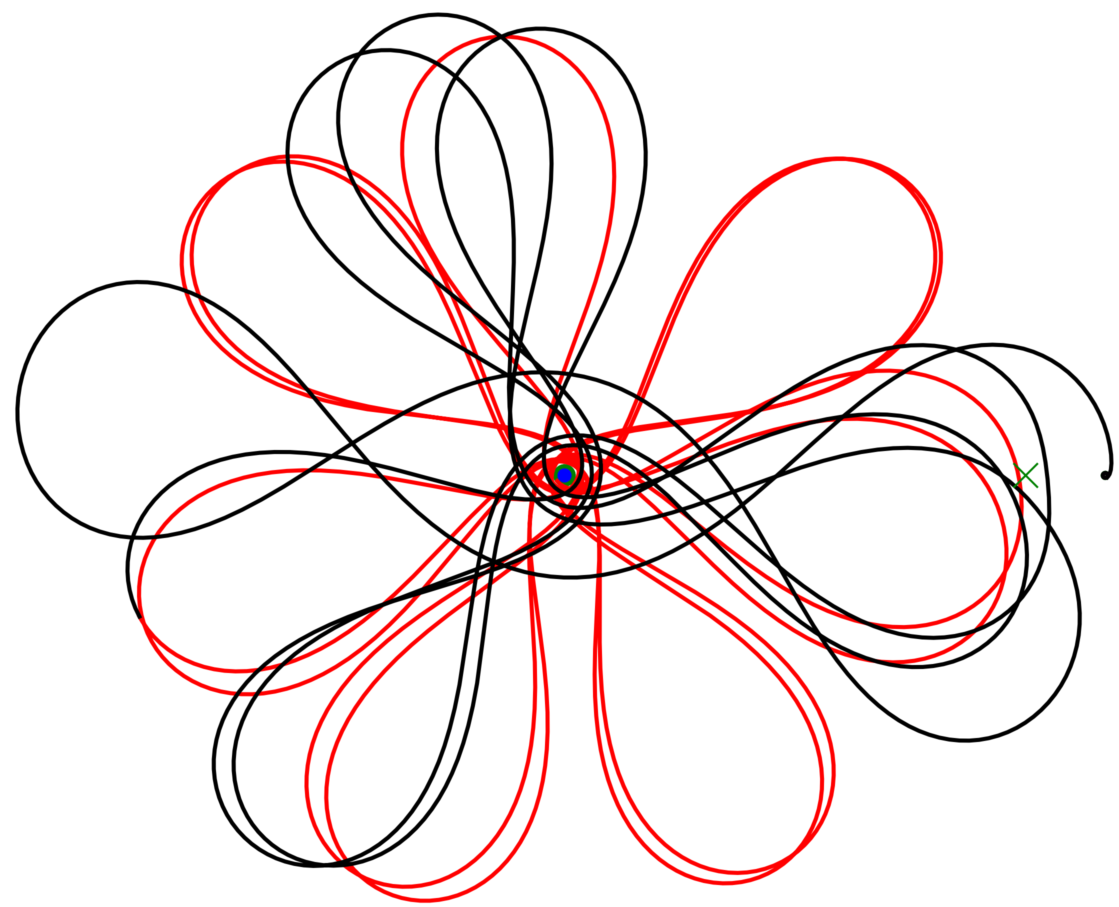
Low-Energy Transfer Orbits



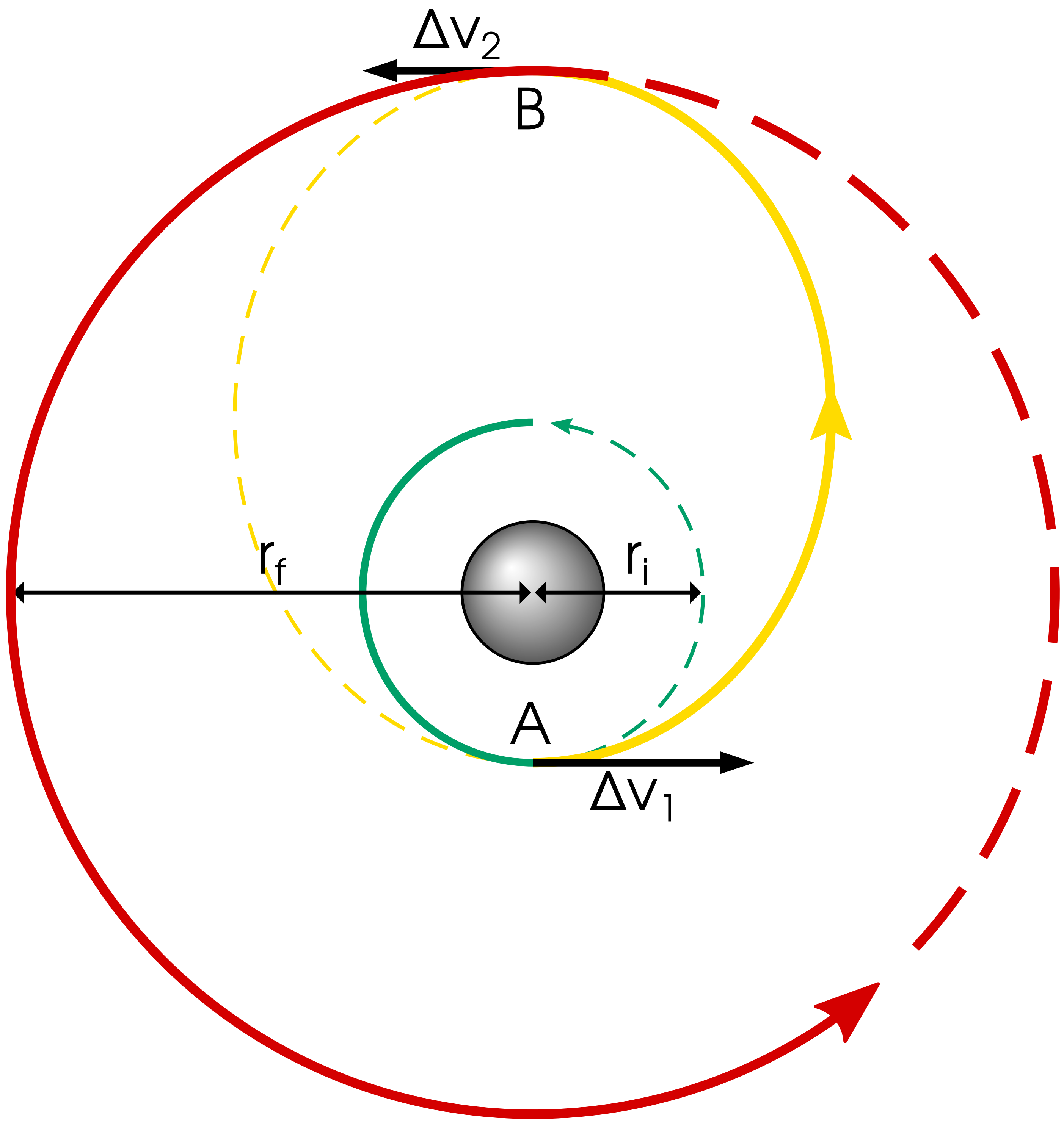
A Theoretical and Numerical Study

Gandalf Saxe, DTU

What are transfer orbits?

A way of getting from A to B in space

Hohmann



Why Are Transfer Orbits Interesting?

NASA manned spacecraft: Orion
Manned Mars Mission: 2030's


(NASA)

"The first crewed mission — called EM-2 — is now scheduled for April 2023; the flight was originally scheduled for August 2021"

- The Verge, September 16th

Mars One manned spacecraft: Dragon
Manned Mars Mission: ~2026

(SpaceX)

"Elon Musk argues that we must put a million people on Mars if we are to ensure that humanity has a future"

- Interview with aeon.co, 30 September 2014

Why Are Transfer Orbits Interesting?

Answer: Fuel efficiency

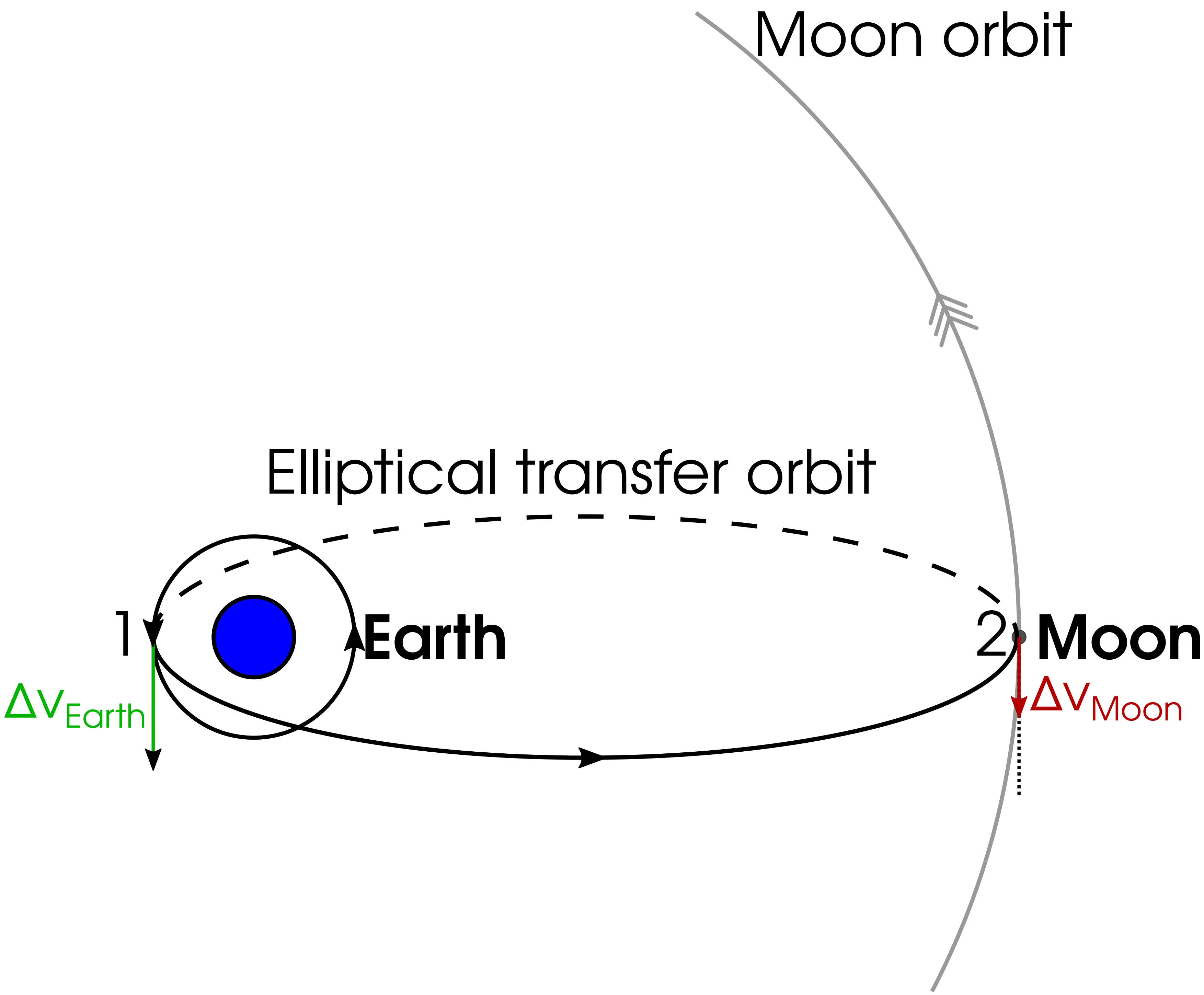
Cost of low Moon orbit: ~ $100,000 per kg
Cost of moon landing: ~ $1,000,000 per kg

Low-energy transfer orbit can double the payload to Moon!

Sources:

- https://www.astrobotic.com/lunar-delivery
- Jacob Akira Okada. Painting the Way to the Moon. 2015.
- Edward a Belbruno and John P Carrico. "Calculation of Weak Stability Boundary Ballistic Lunar Transfer Trajectories". In: Astrodynamics specialist conference. Denver, Colorado, 2000. doi: doi:10.2514/6.2000-4142.

Hohmann Transfer Orbit to the Moon



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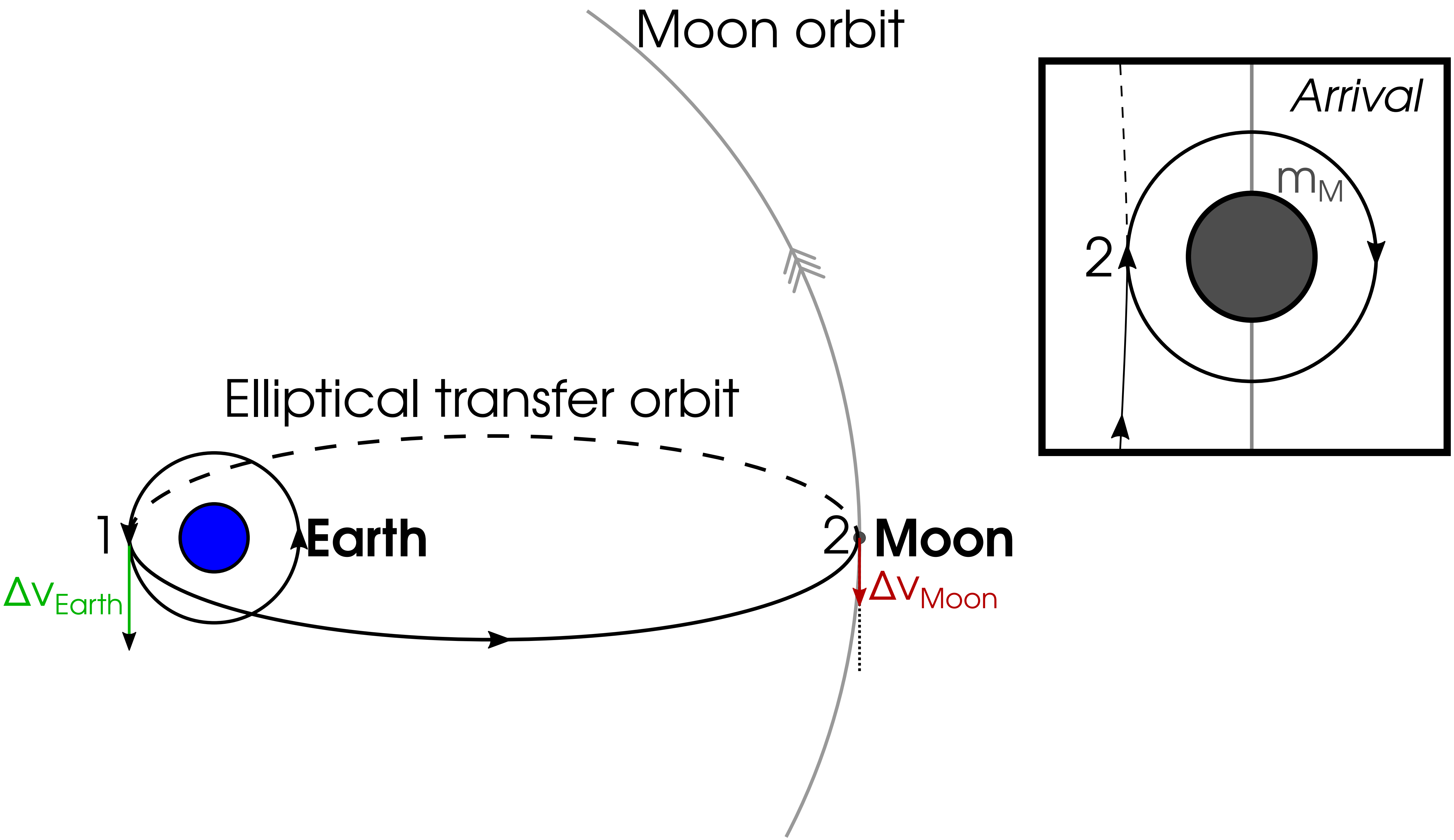

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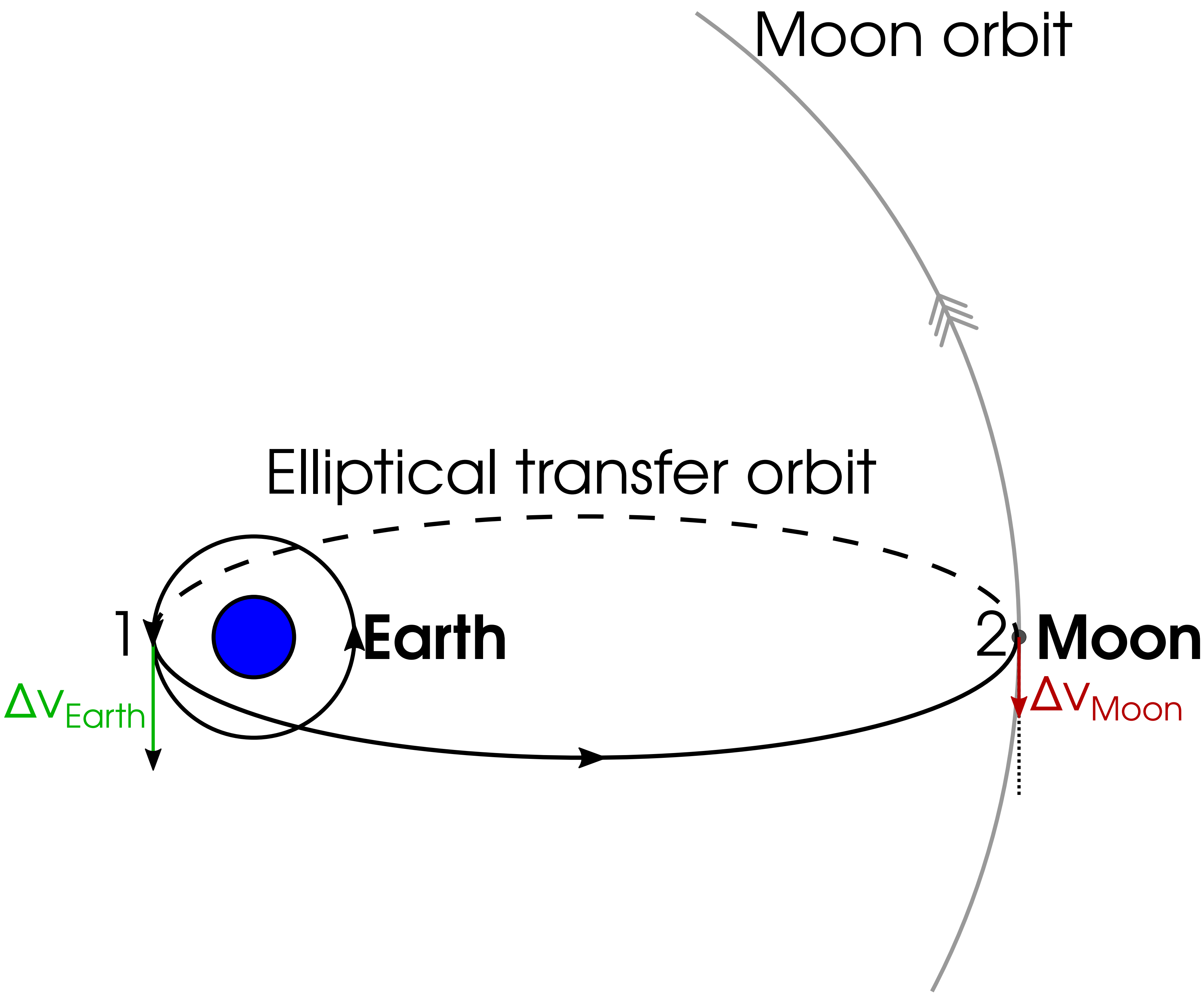

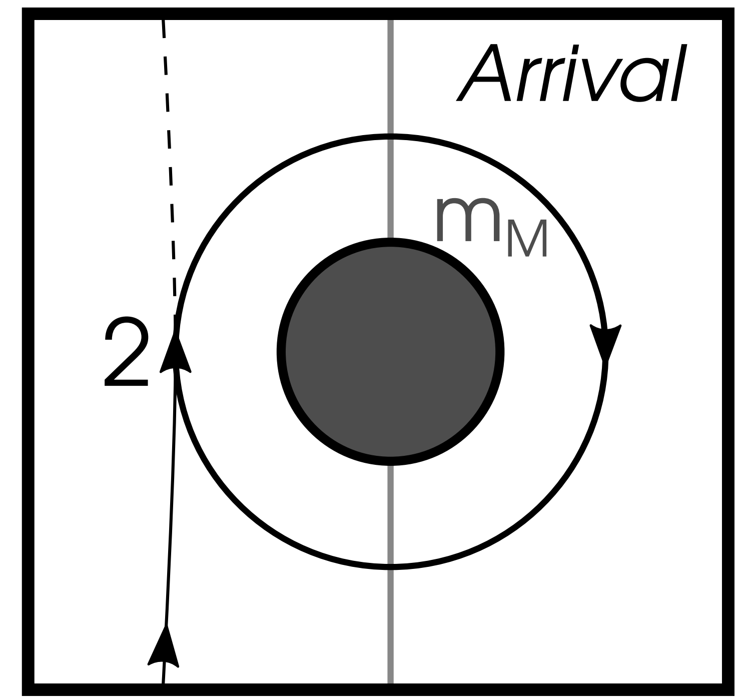

$t_H = 5.0$ days

$\Delta v_{\text{Earth}} = 3144$ m/s

$\Delta v_{\text{Moon}} = 802$ m/s

$\Delta v_{\text{Total}} = 3946$ m/s

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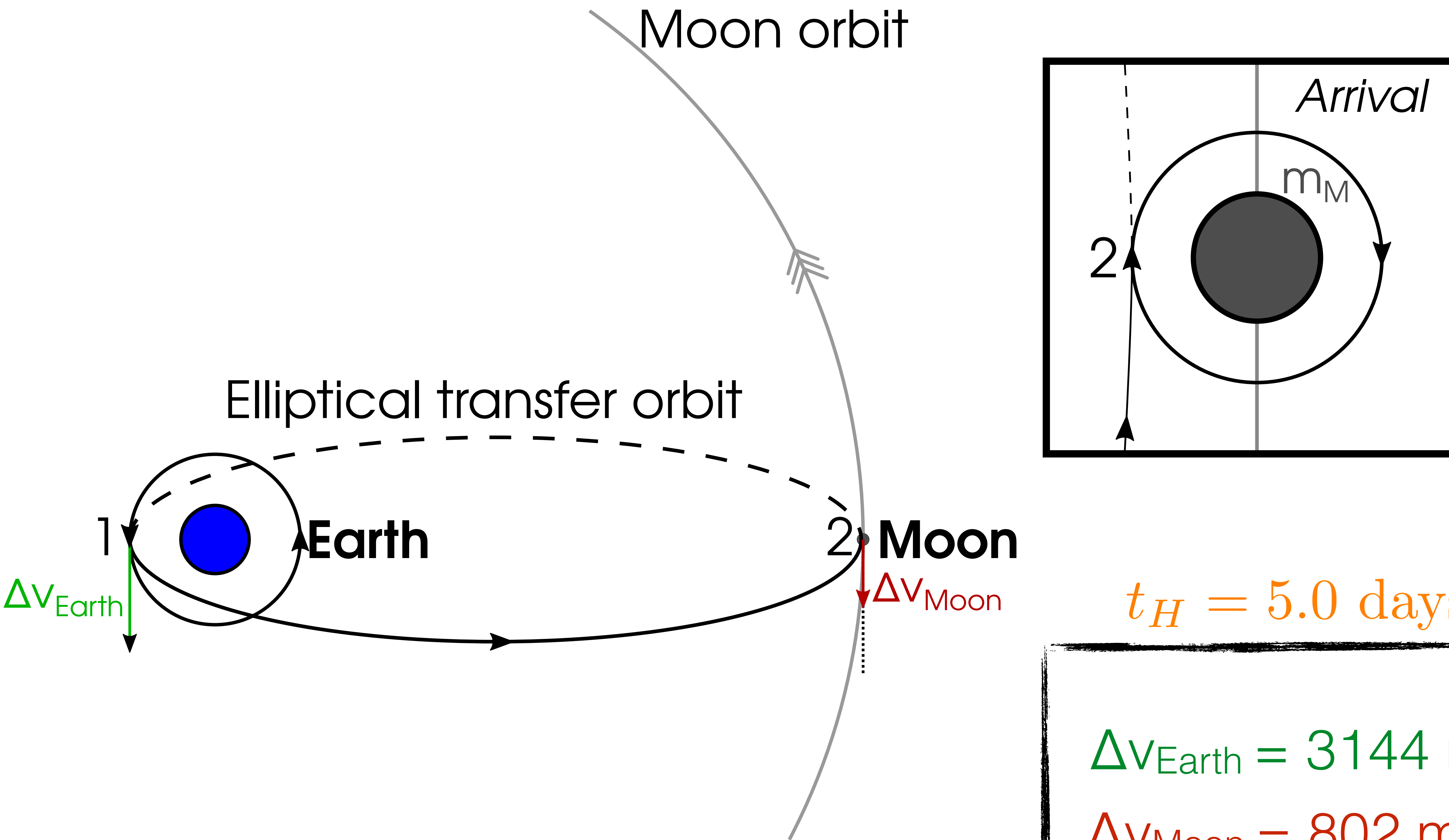

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It's all about low Δv

$t_H = 5.0$ days

Δv_{Earth} = 3144 m/s

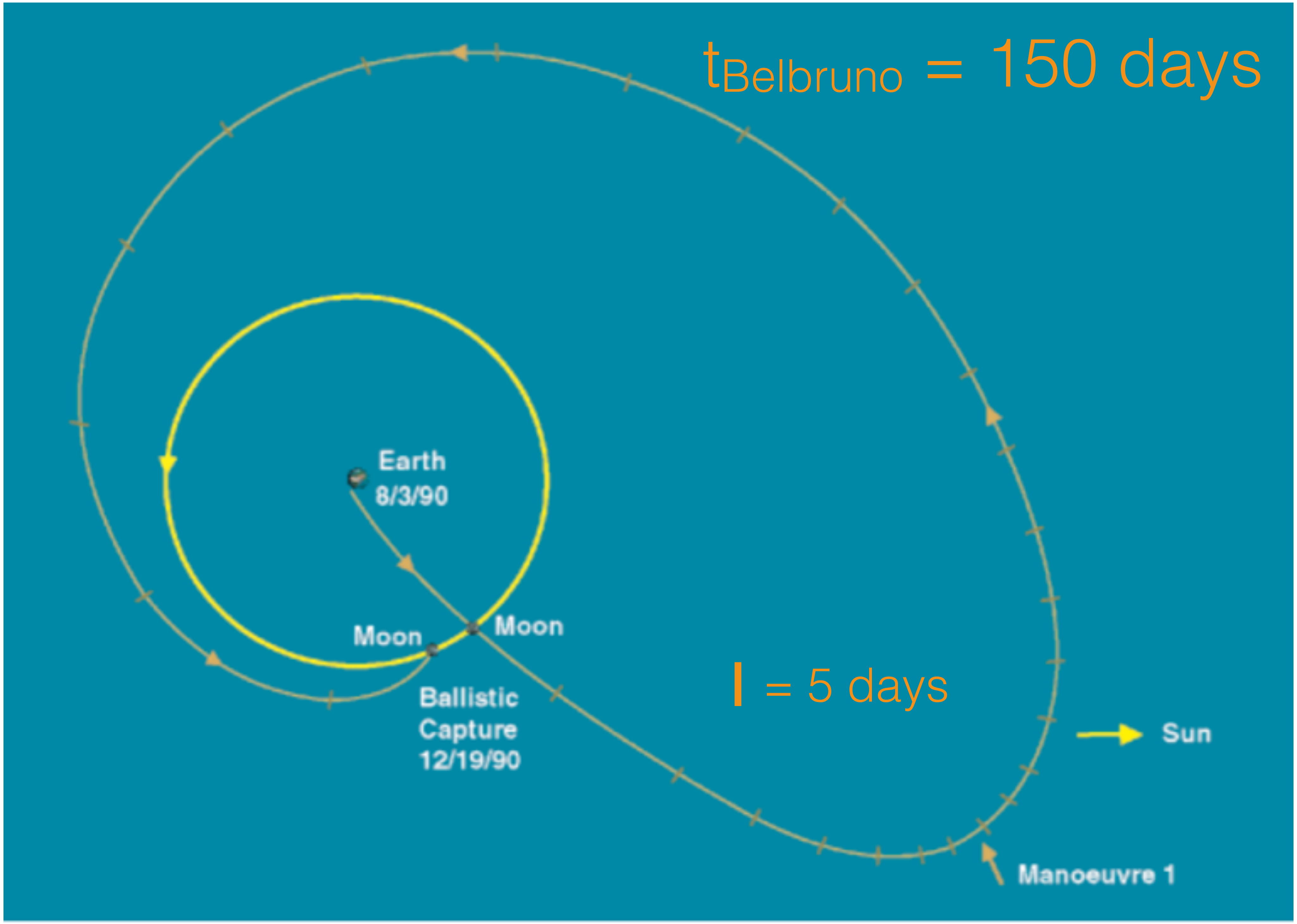
Δv_{Moon} = 802 m/s

Δv_{Total} = 3946 m/s

Find transfer orbits with Δv low as possible!

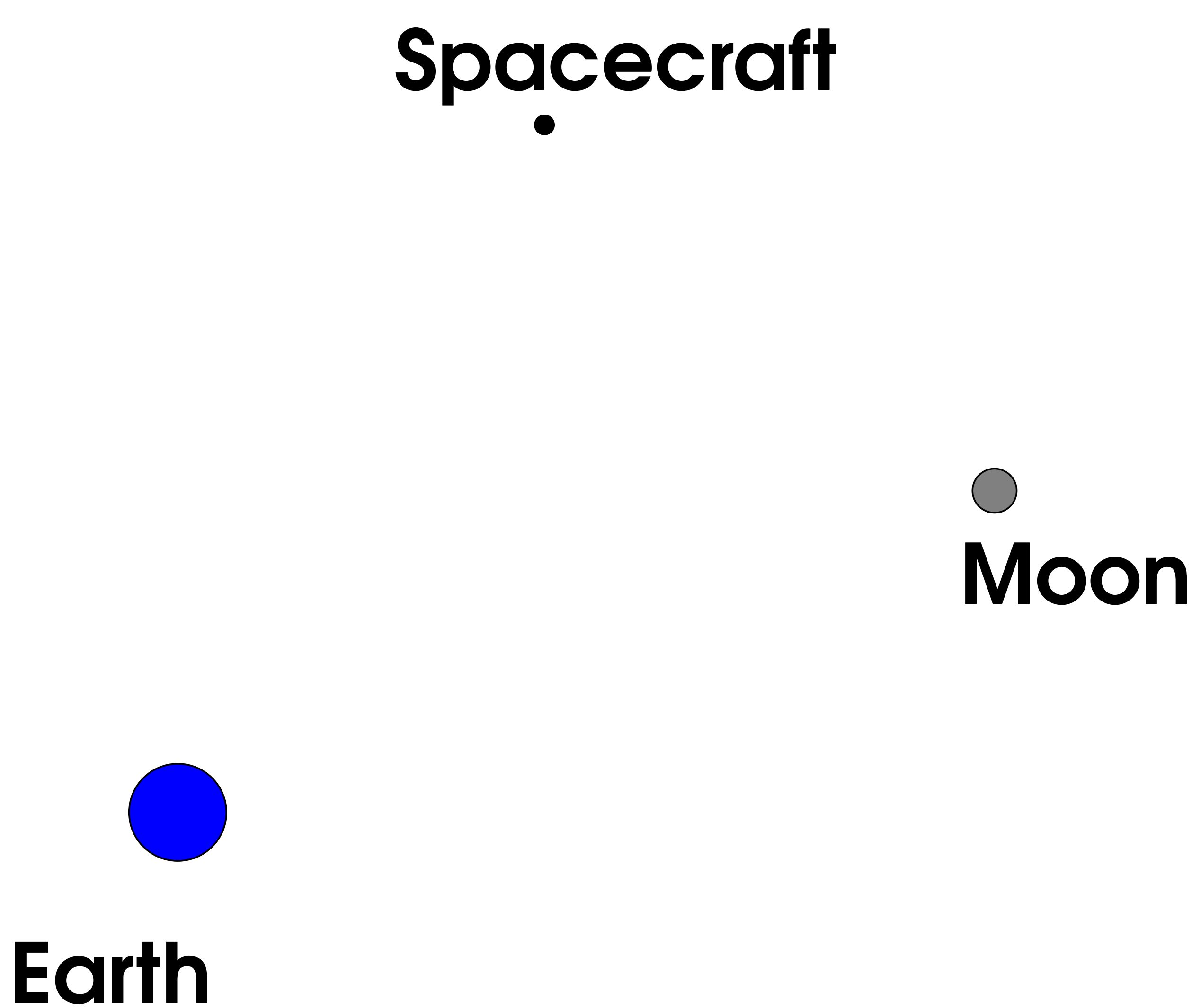
Δv_{Moon} can be reduced up to ~25%

Low Energy Transfer Orbits



Hiten: Japanese Spacecraft, 1990

Restricted 3-body problem



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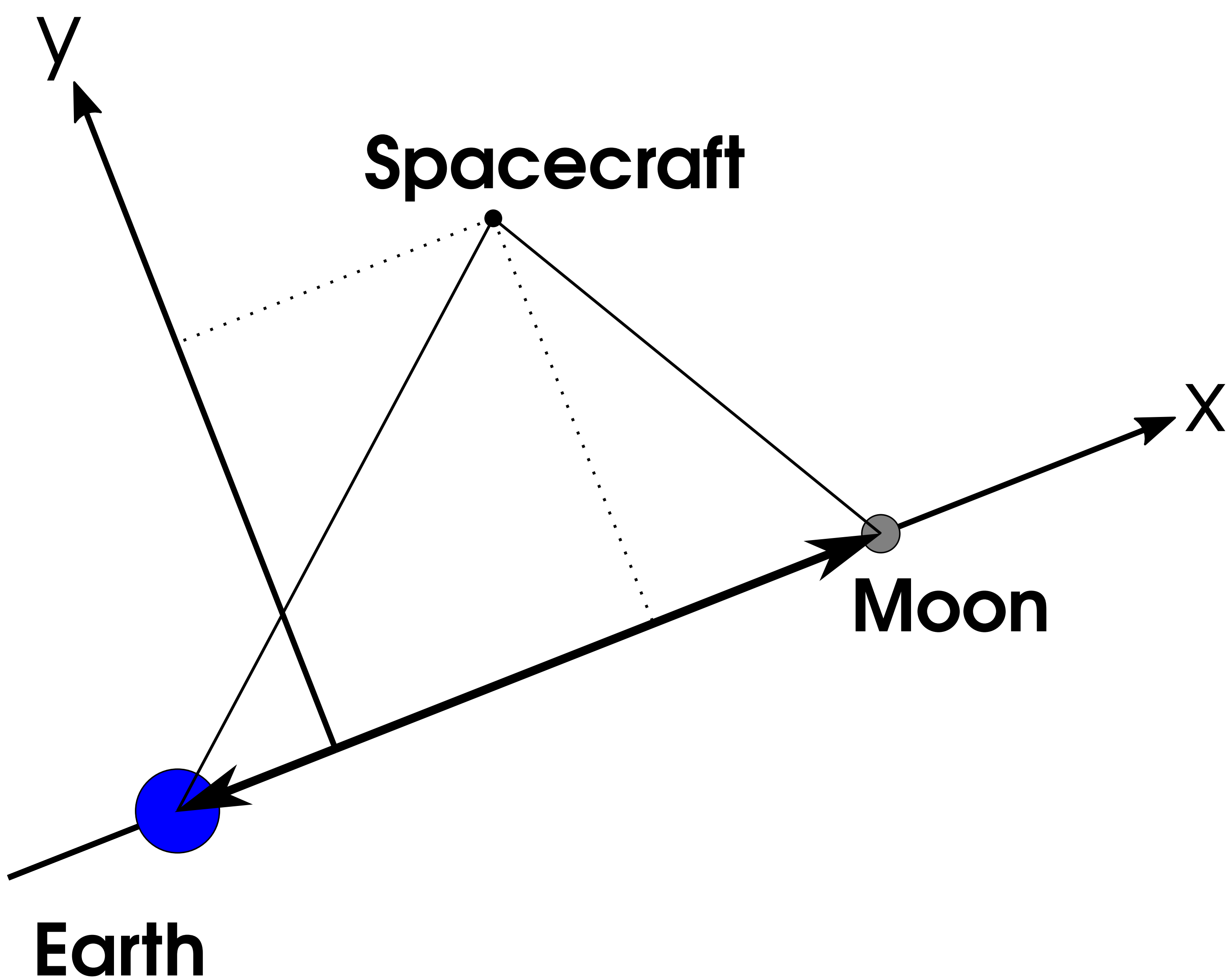

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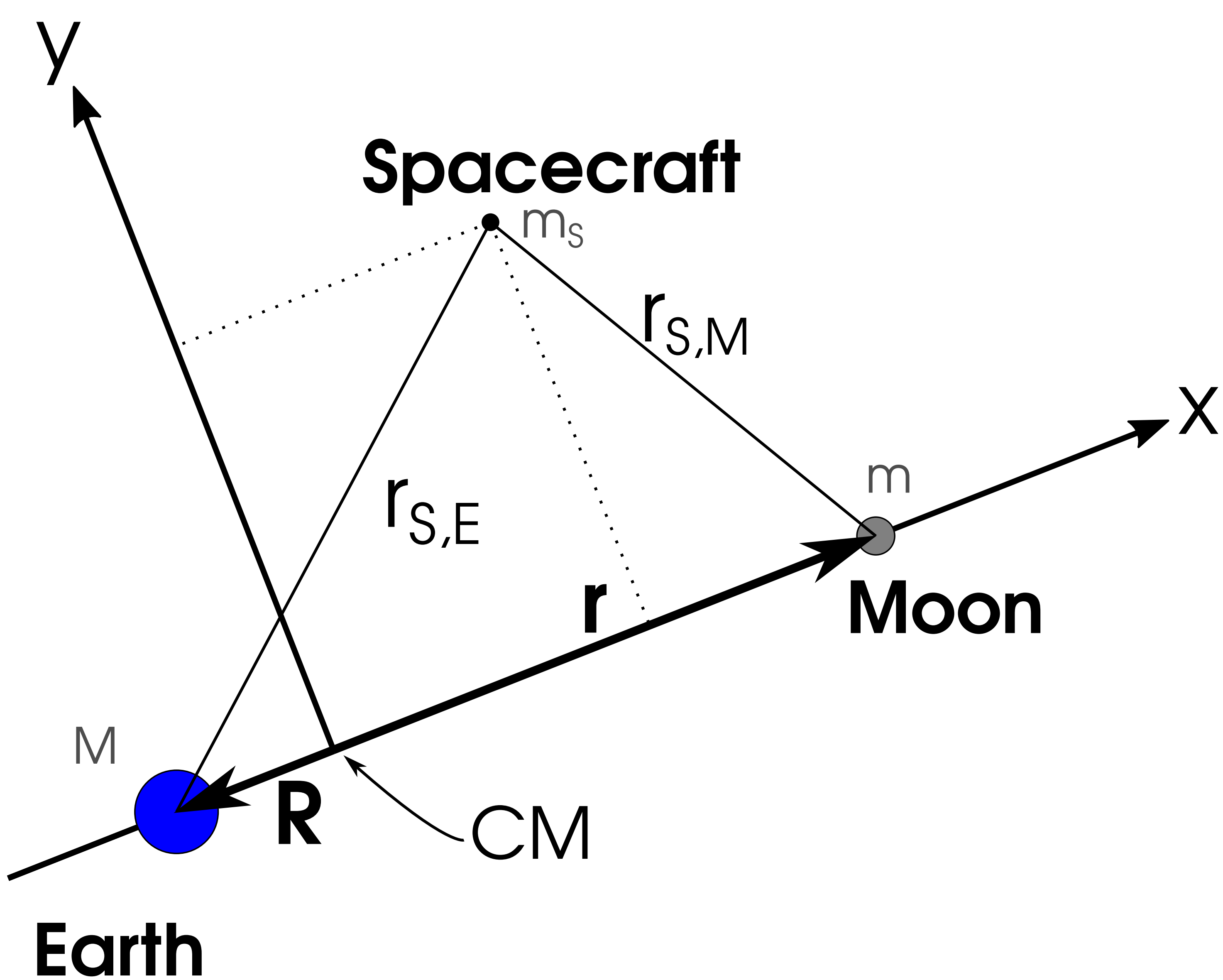

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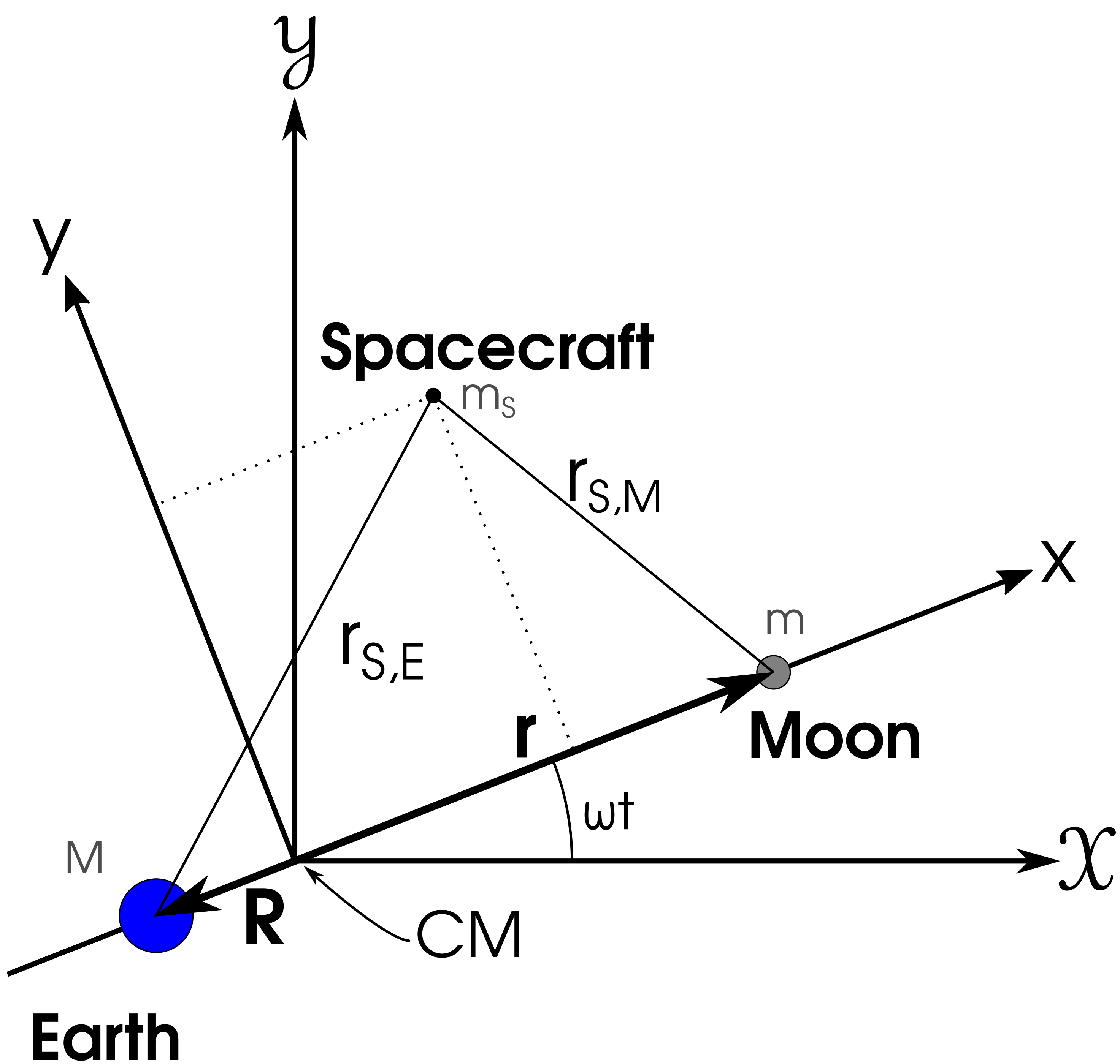

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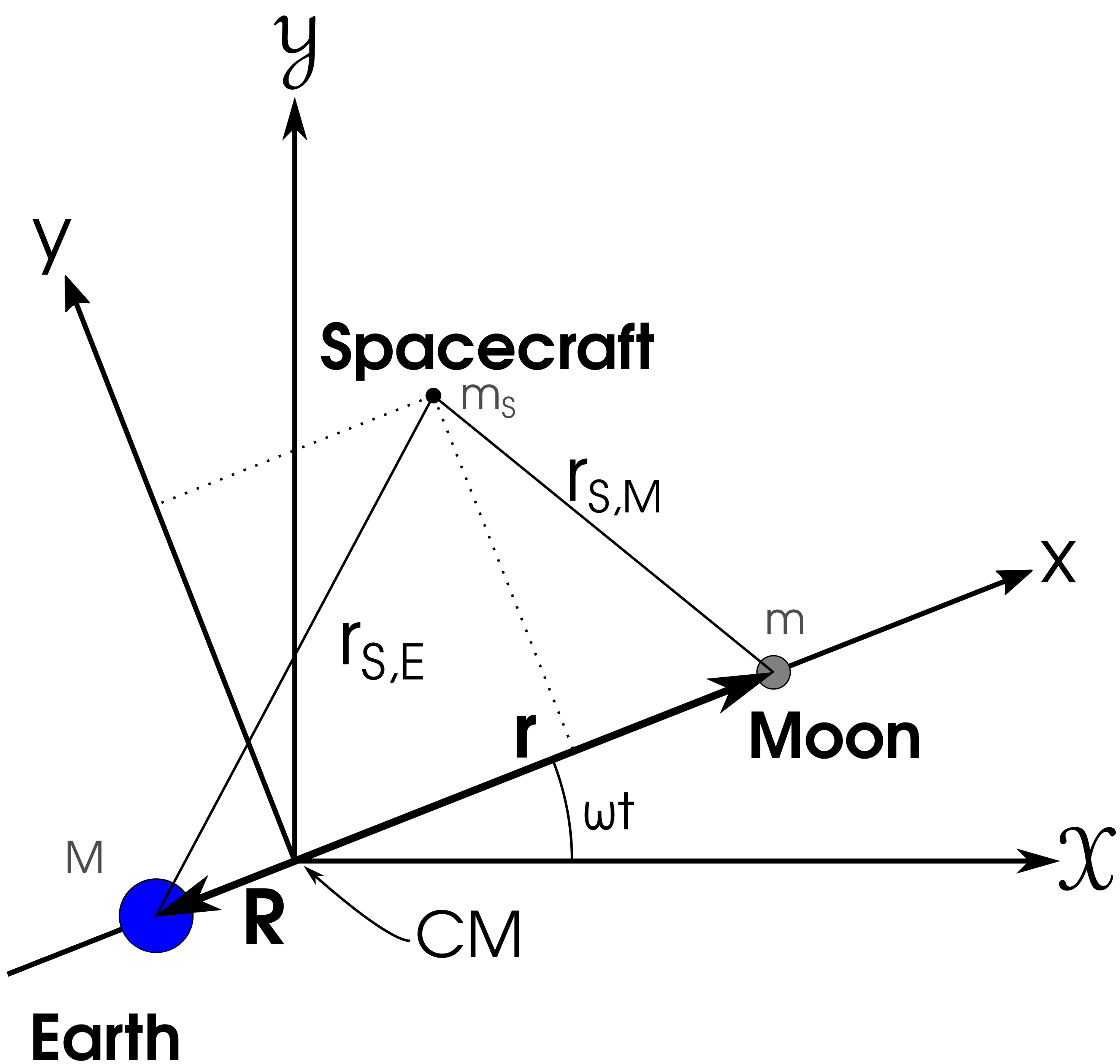
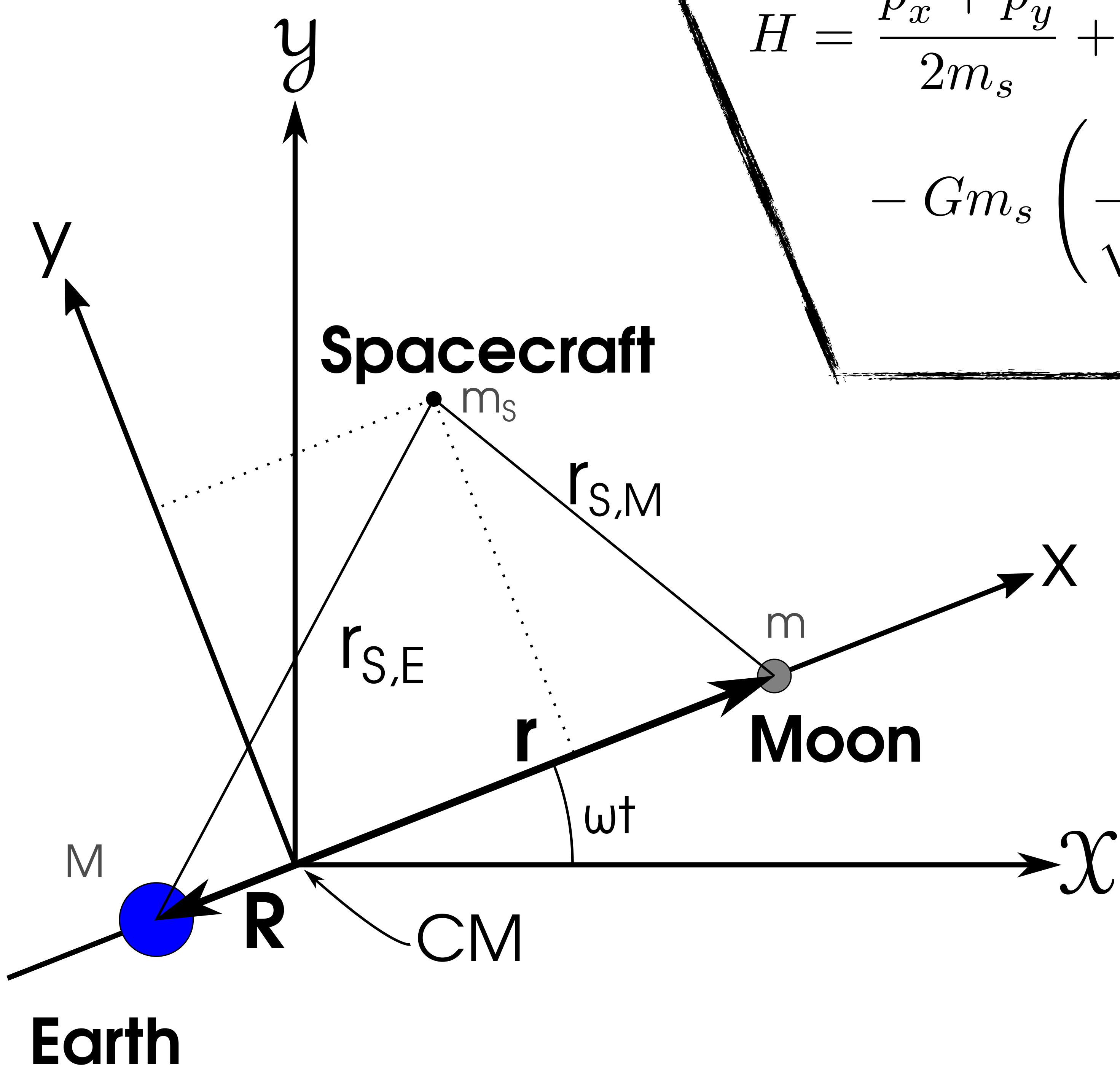


Assumptions:
- 2D
- 2-body

Restricted 3-body problem

$$H = \frac{p_x^2 + p_y^2}{2m_s} + p_x \omega y - p_y \omega x$$
$$- G m_s \left(\frac{M}{\sqrt{(x+R)^2 + y^2}} + \frac{m}{\sqrt{(x-r)^2 + y^2}} \right)$$



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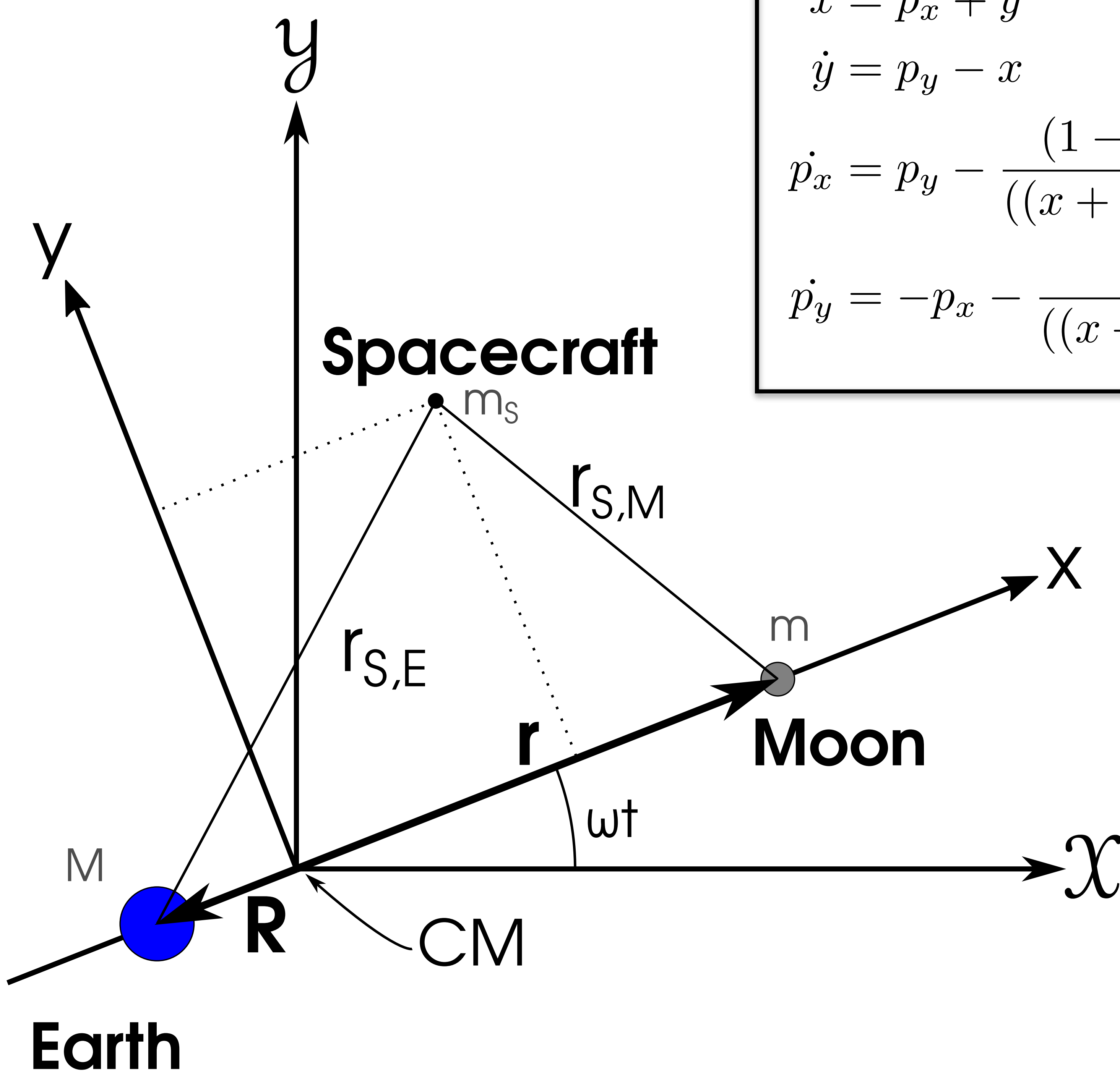


Equations of Motion

$$\dot{x} = p_x + y$$

$$\dot{y} = p_y - x$$

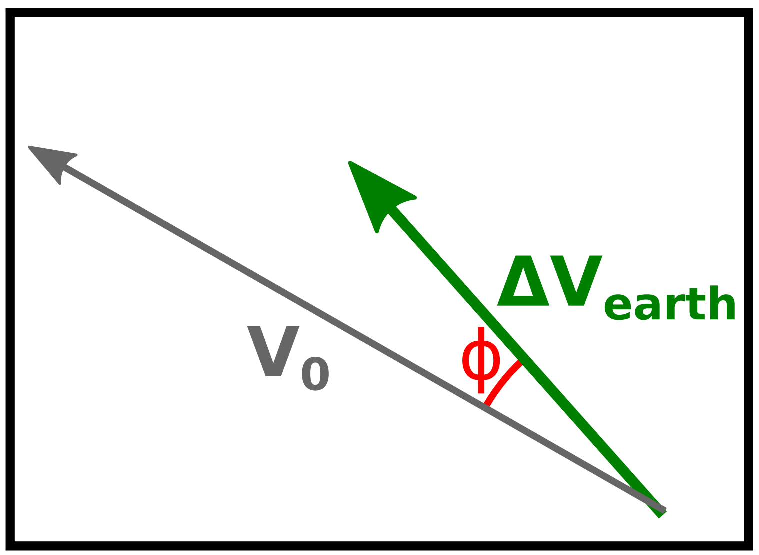
$$\dot{p_x} = p_y - \frac{(1-k)(x+k)}{((x+k)^2+y^2)^{3/2}} - \frac{k(x-1-k)}{((x-1-k)^2+y^2)^{3/2}}$$

$$\dot{p_y} = -p_x - \frac{(1-k)y}{((x+k)^2+y^2)^{3/2}} - \frac{ky}{((x-1-k)^2+y^2)^{3/2}}$$

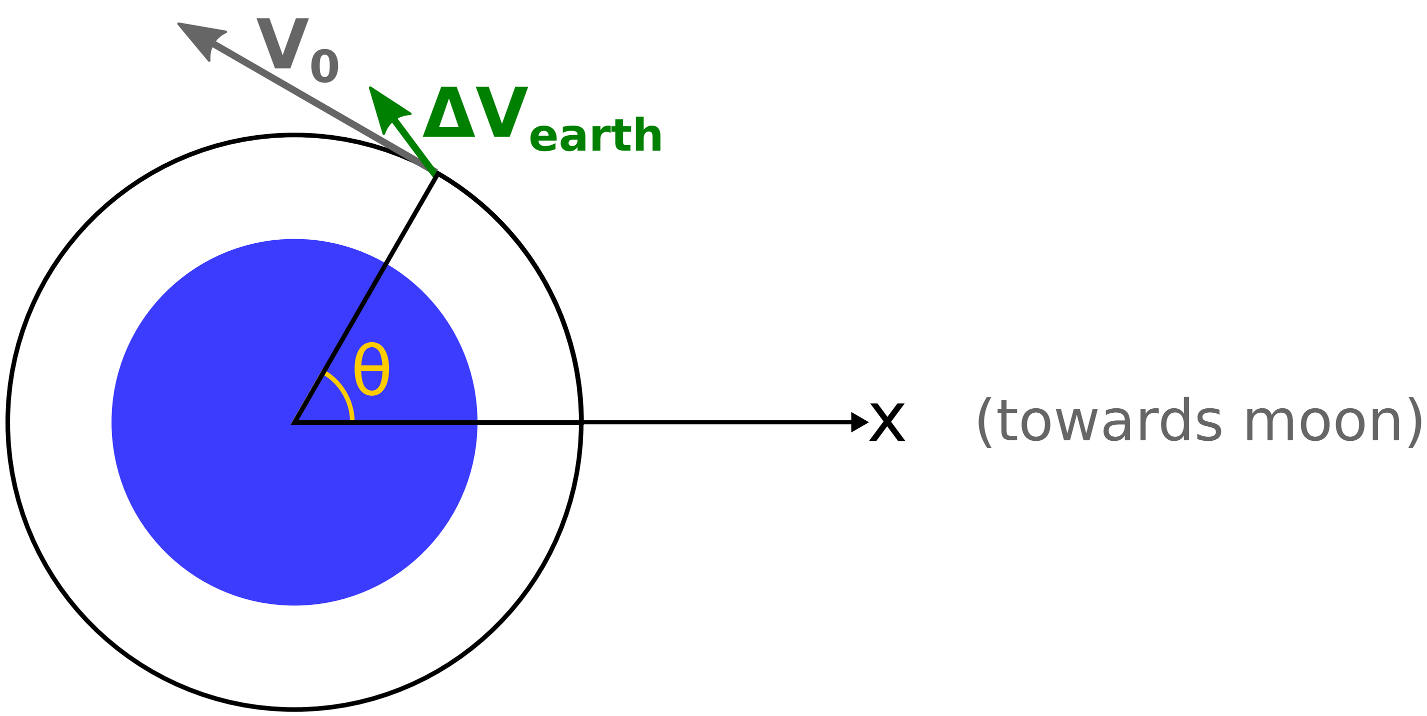
Assumptions:

- 2D
- 2-body

Searching for Transfer Orbits

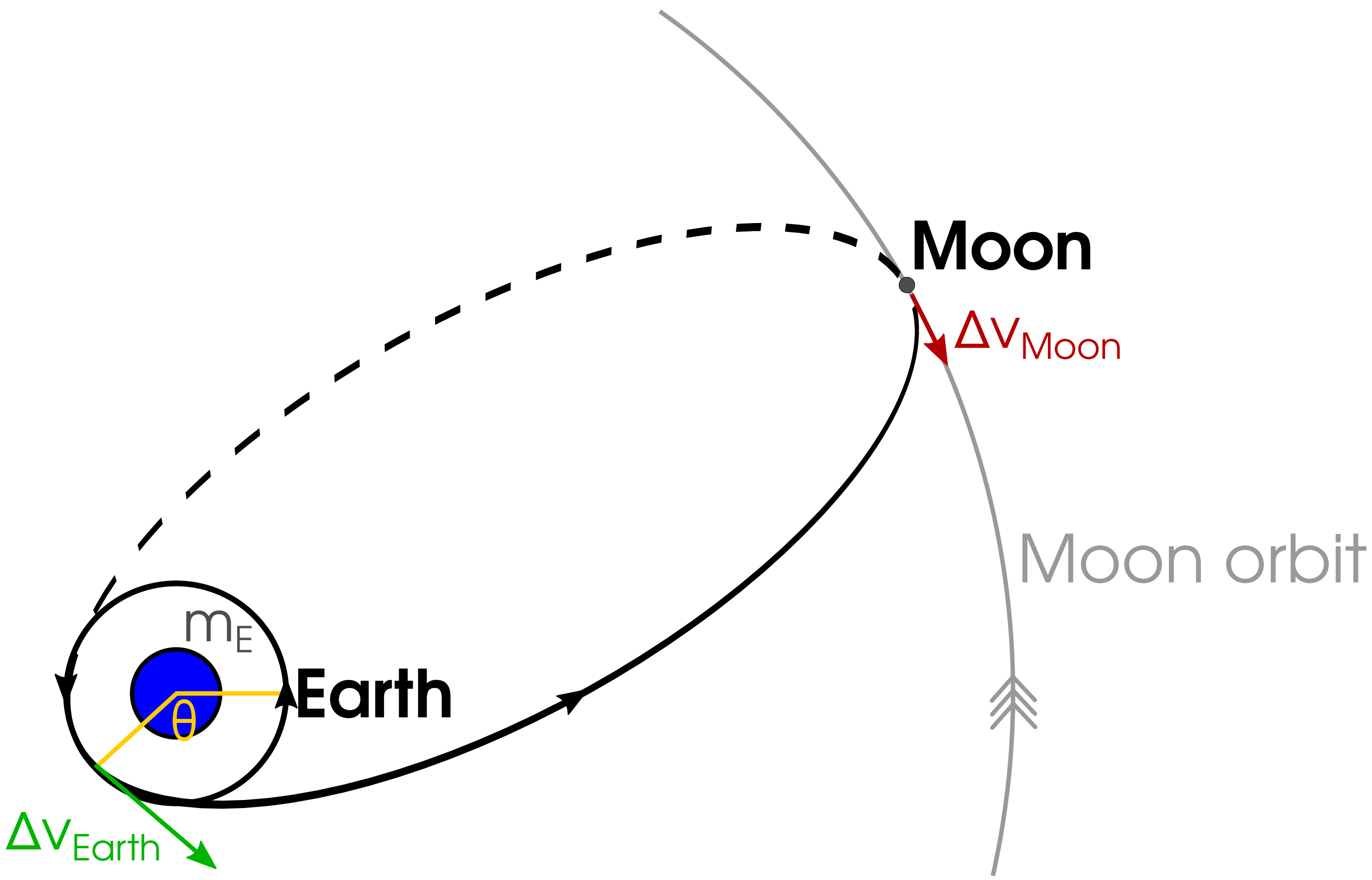


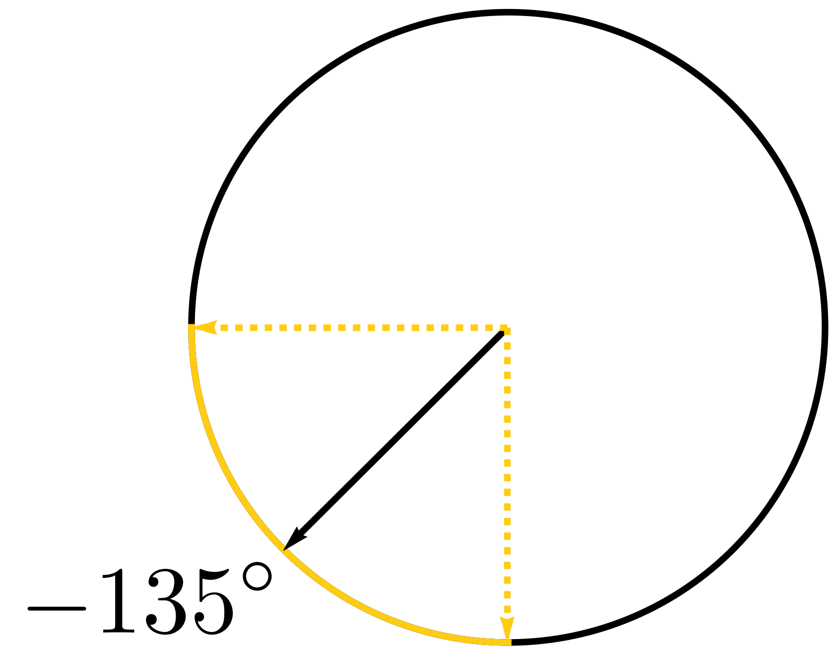
1. θ: Position in orbit
2. Δv_{earth}: Velocity change
3. ϕ: Angle to velocity vector



Searching for Transfer Orbits

Hohmann Transfer Orbit to the Moon





ϕ: 0° constant

First search

100 positions · 200 velocities = 20,000 simulations

Searching for Transfer Orbits

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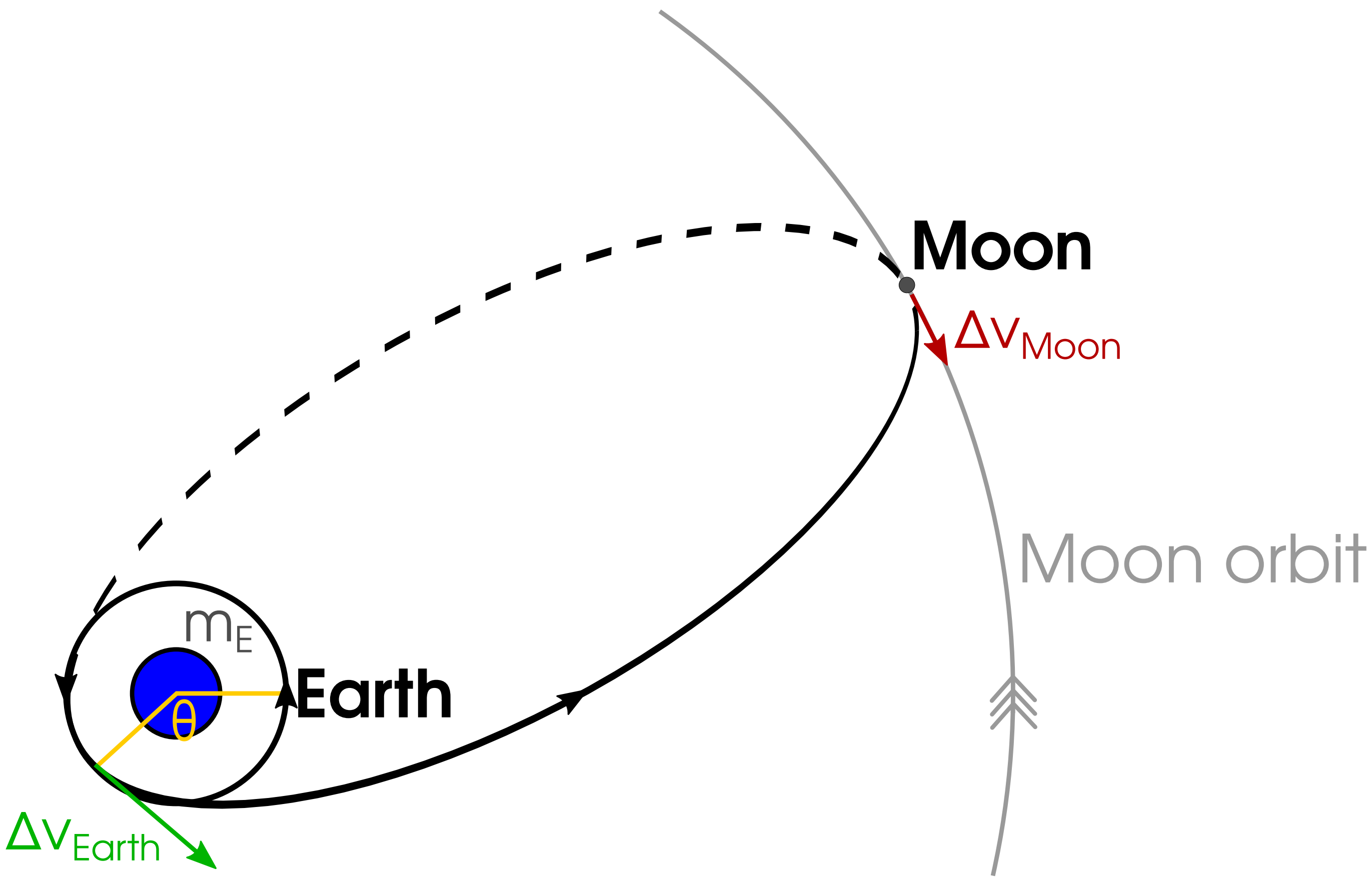
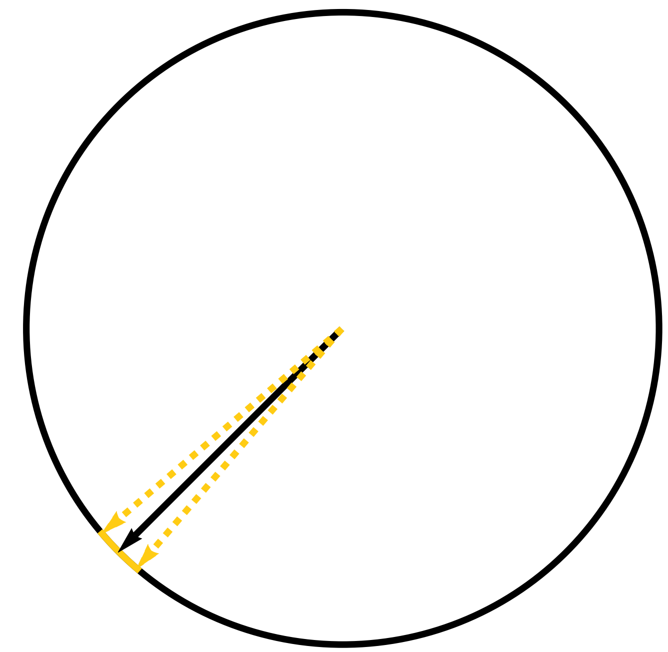




First search

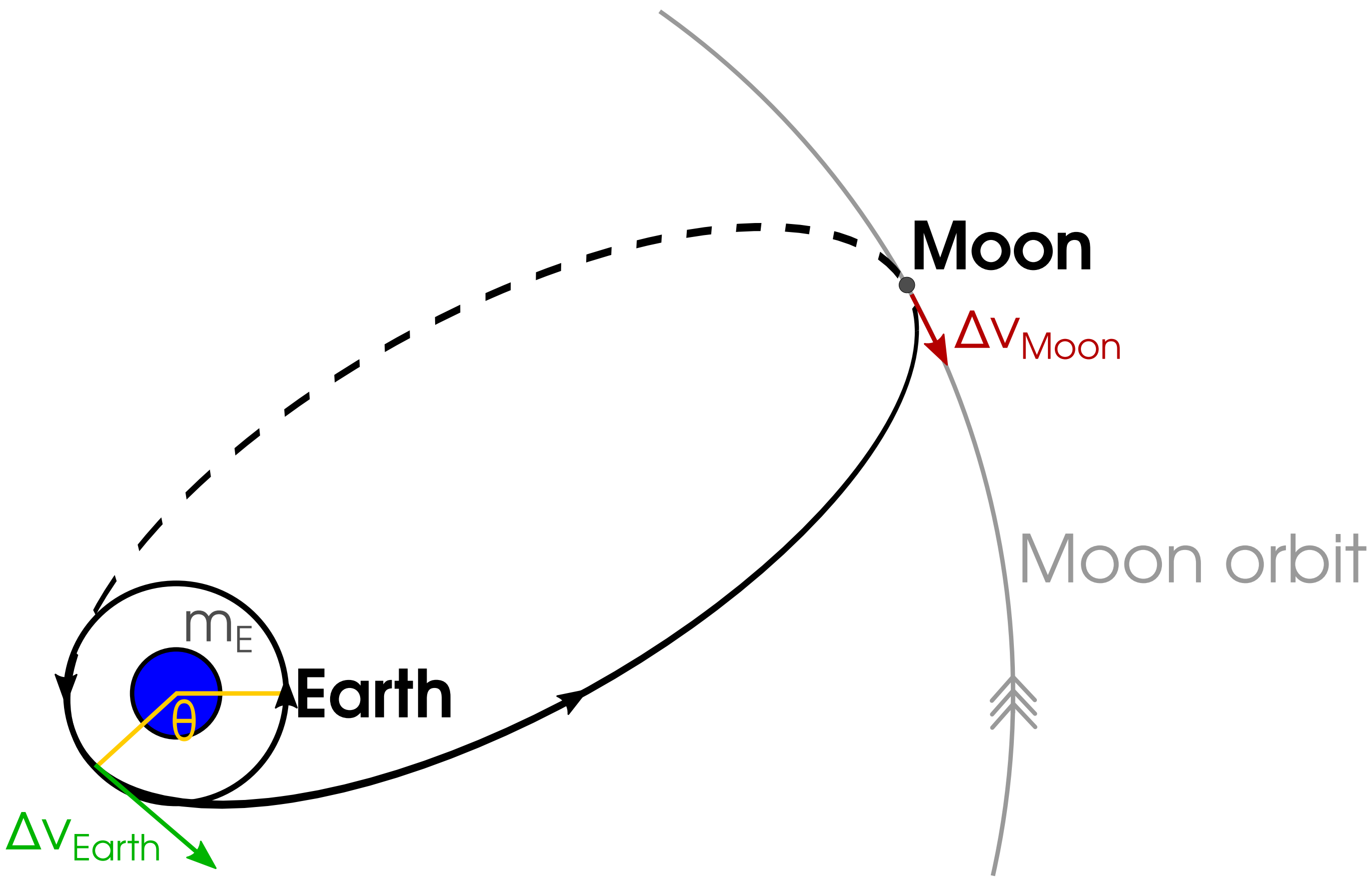
100 positions · 200 velocities = 20,000 simulations

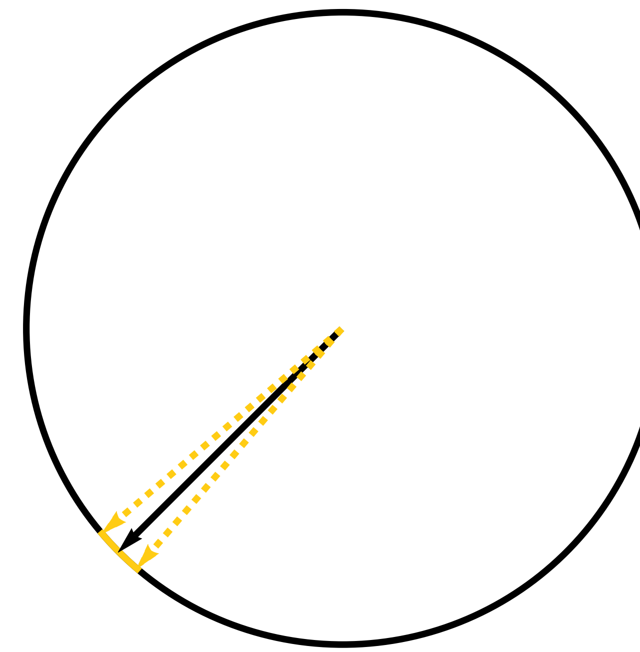
Refinement

15 positions · 15 velocities · 15 angles = 3375 simulations

Searching for Transfer Orbits

Hohmann Transfer Orbit to the Moon





First search

---

100 positions · 200 velocities = 20,000 simulations

Refinement

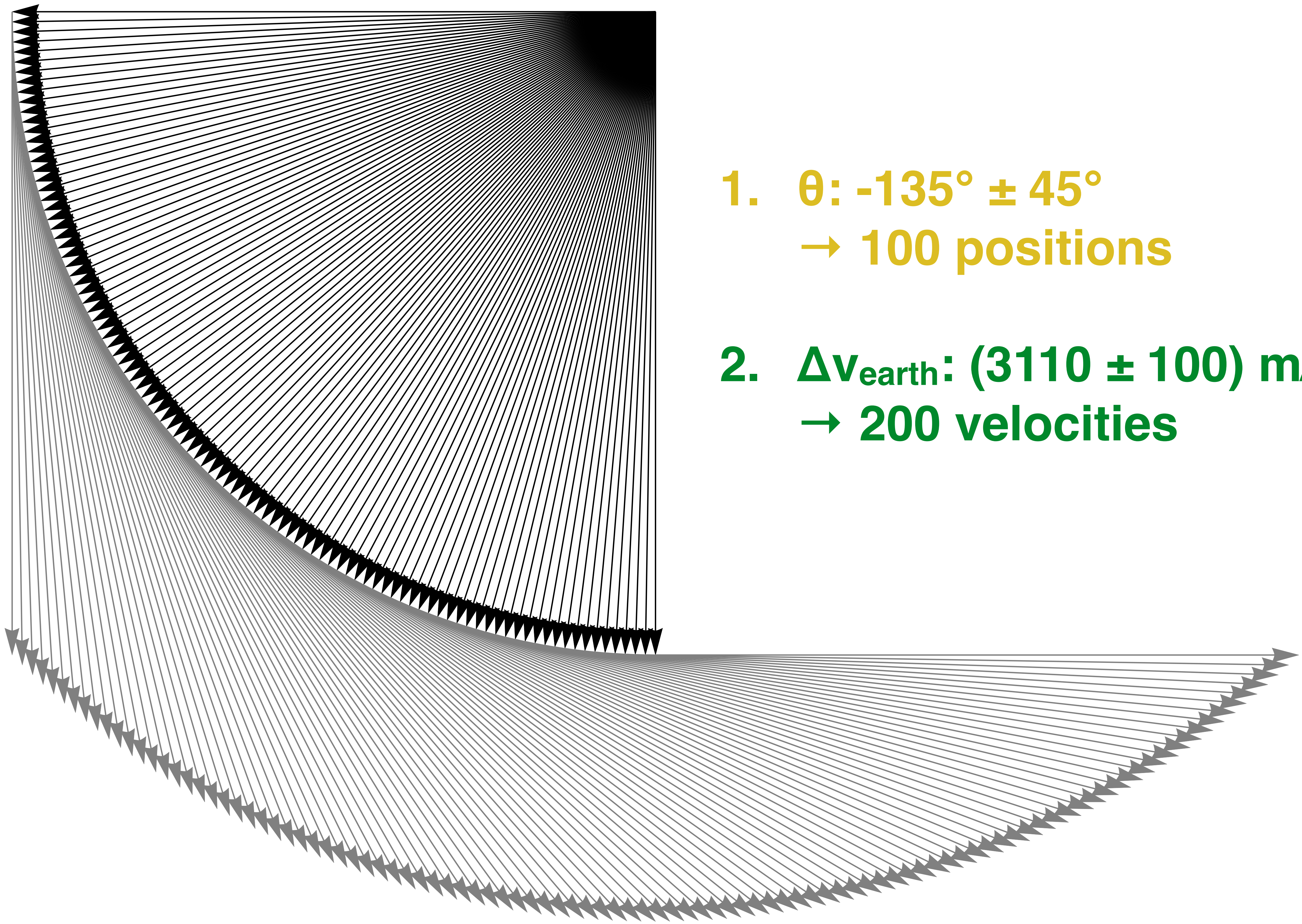
---

15 positions · 15 velocities · 15 angles = 3375 simulations

Total: 20,000 + 3375 = 23,375 simulations

Searching for Transfer Orbits

Hohmann Transfer Orbit to the Moon

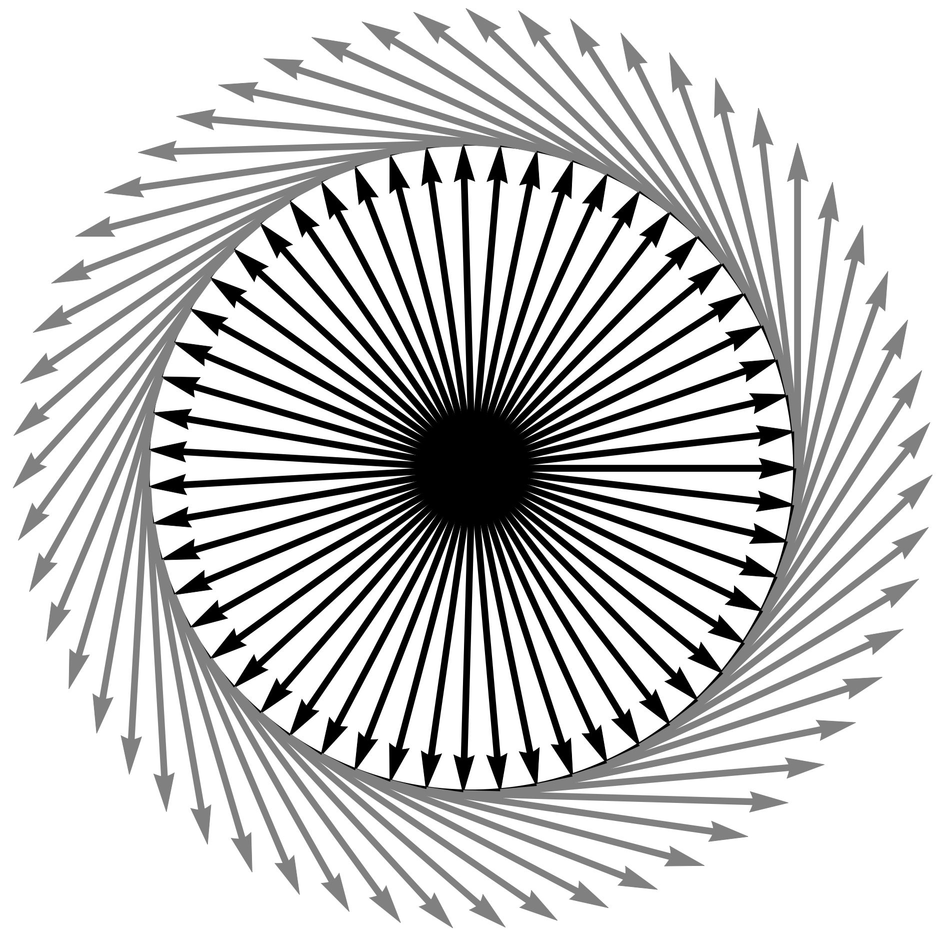


1. **θ: -135° ± 45°**
   → 100 positions

2. **Δv_{earth}: (3110 ± 100) m/s**
   → 200 velocities

Searching for Transfer Orbits

Low Energy Transfer Orbit to the Moon



First search + 7 refinements

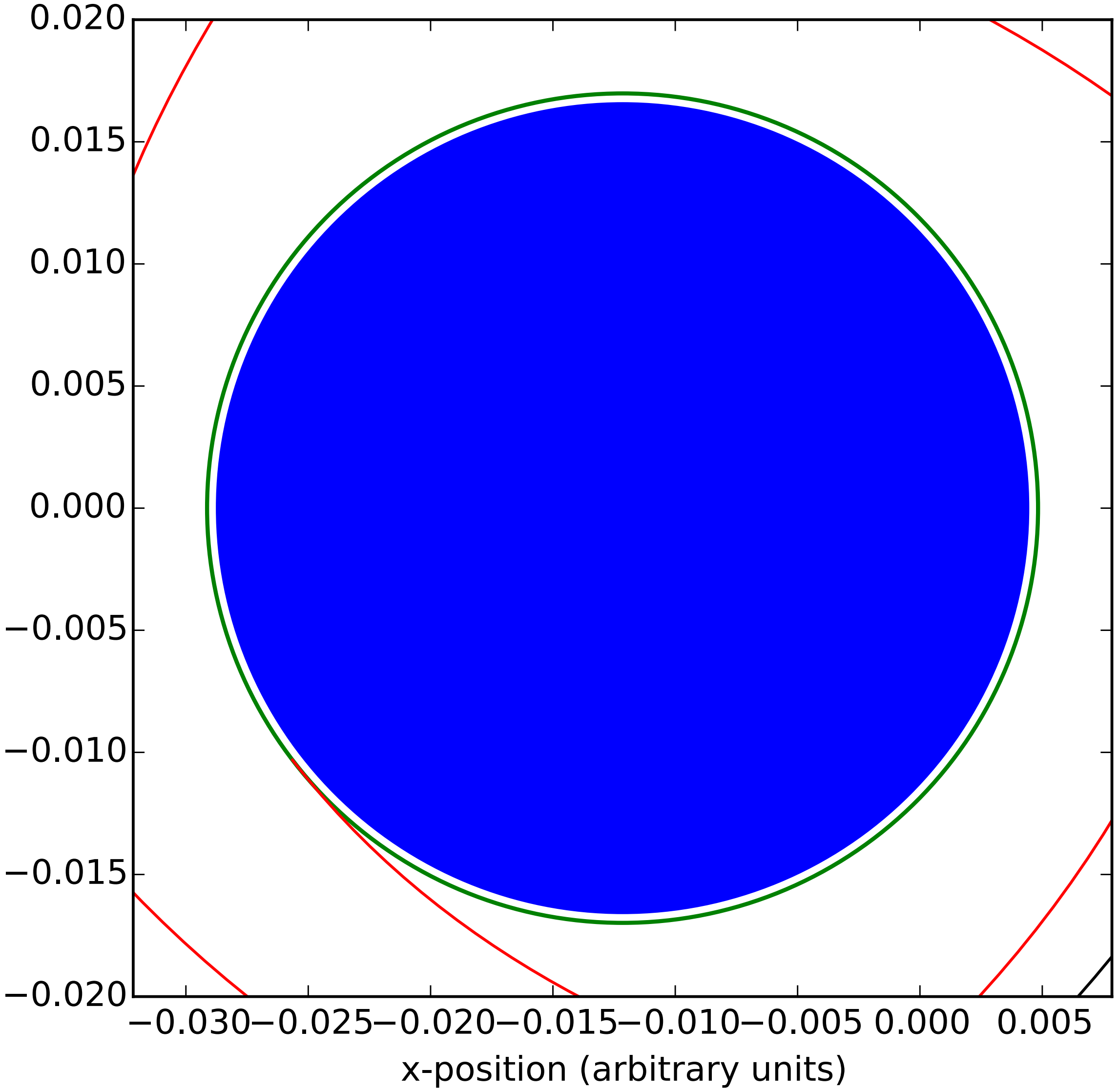
1. **θ: 0 ± 180° → 55 positions**
2. **Δv_{earth}: (3120 ± 100) m/s → 55 velocities**
3. **ϕ: 0° ± 1.8° → 55 angles**

TOTAL $55 \cdot 55 \cdot 55 \cdot 8$ = **1,331,000**

Searching for Transfer Orbits

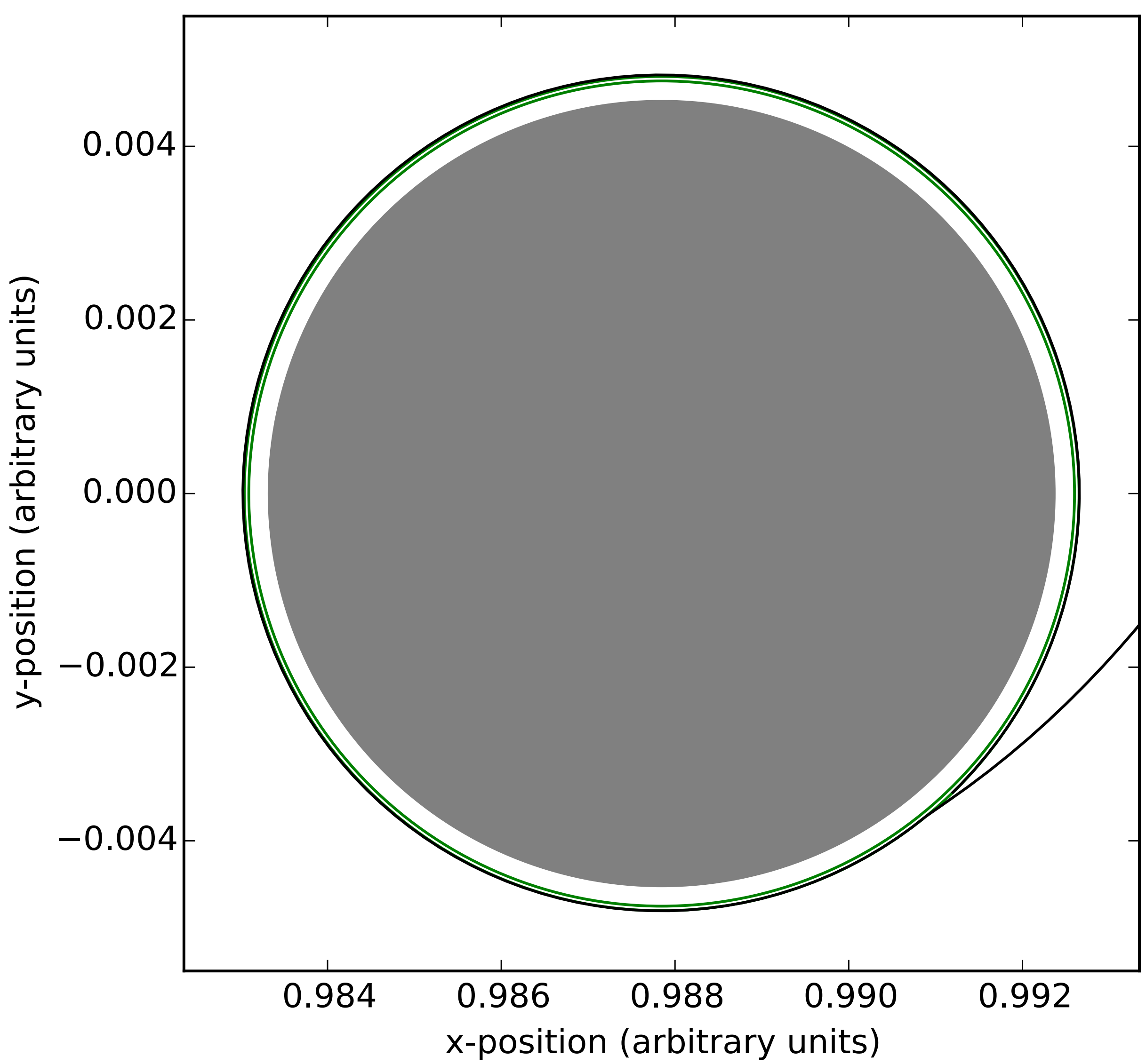
Entering Moon Orbit

Δv_{earth}



Exit from Earth orbit

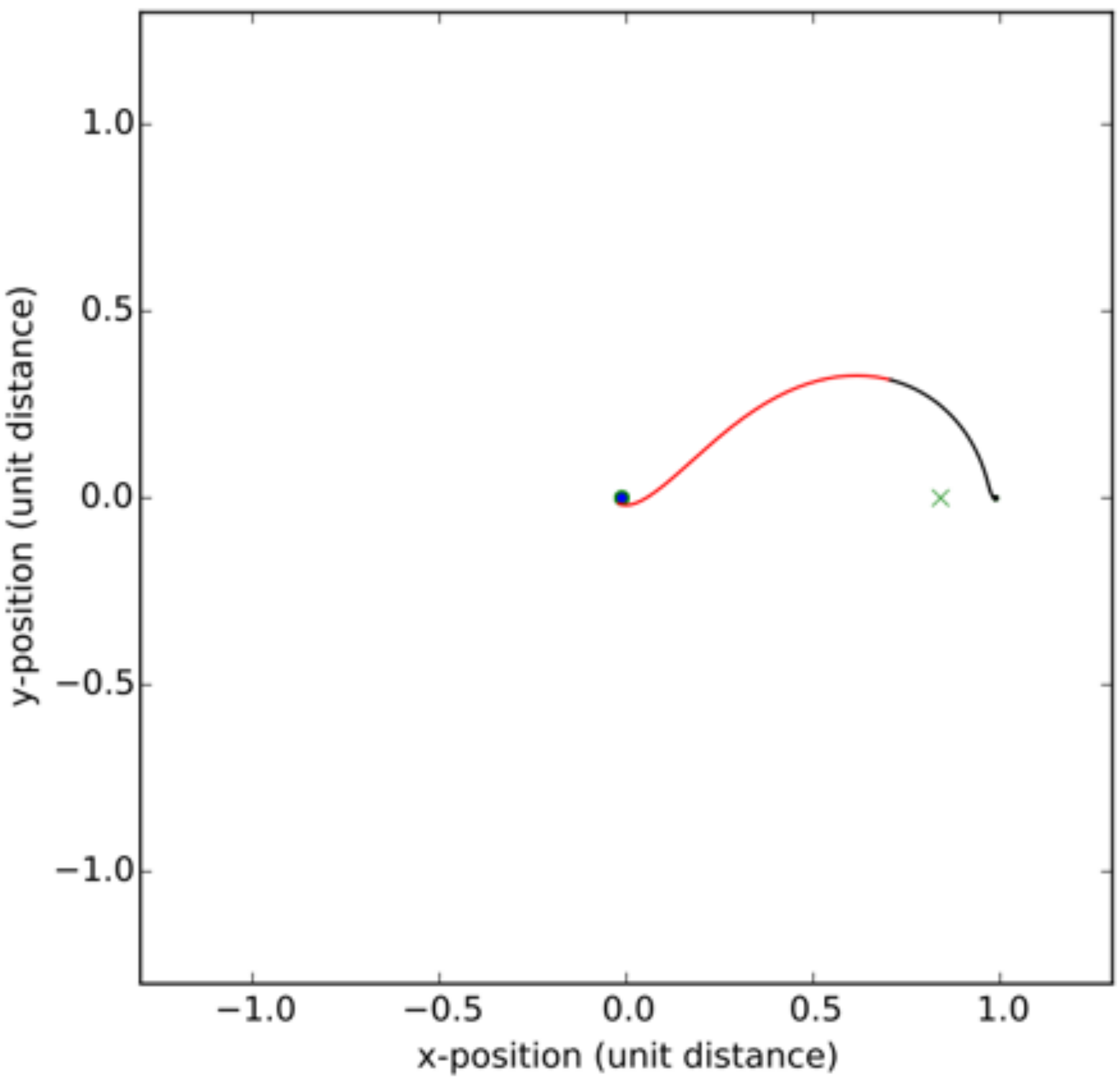
Δv_{moon}



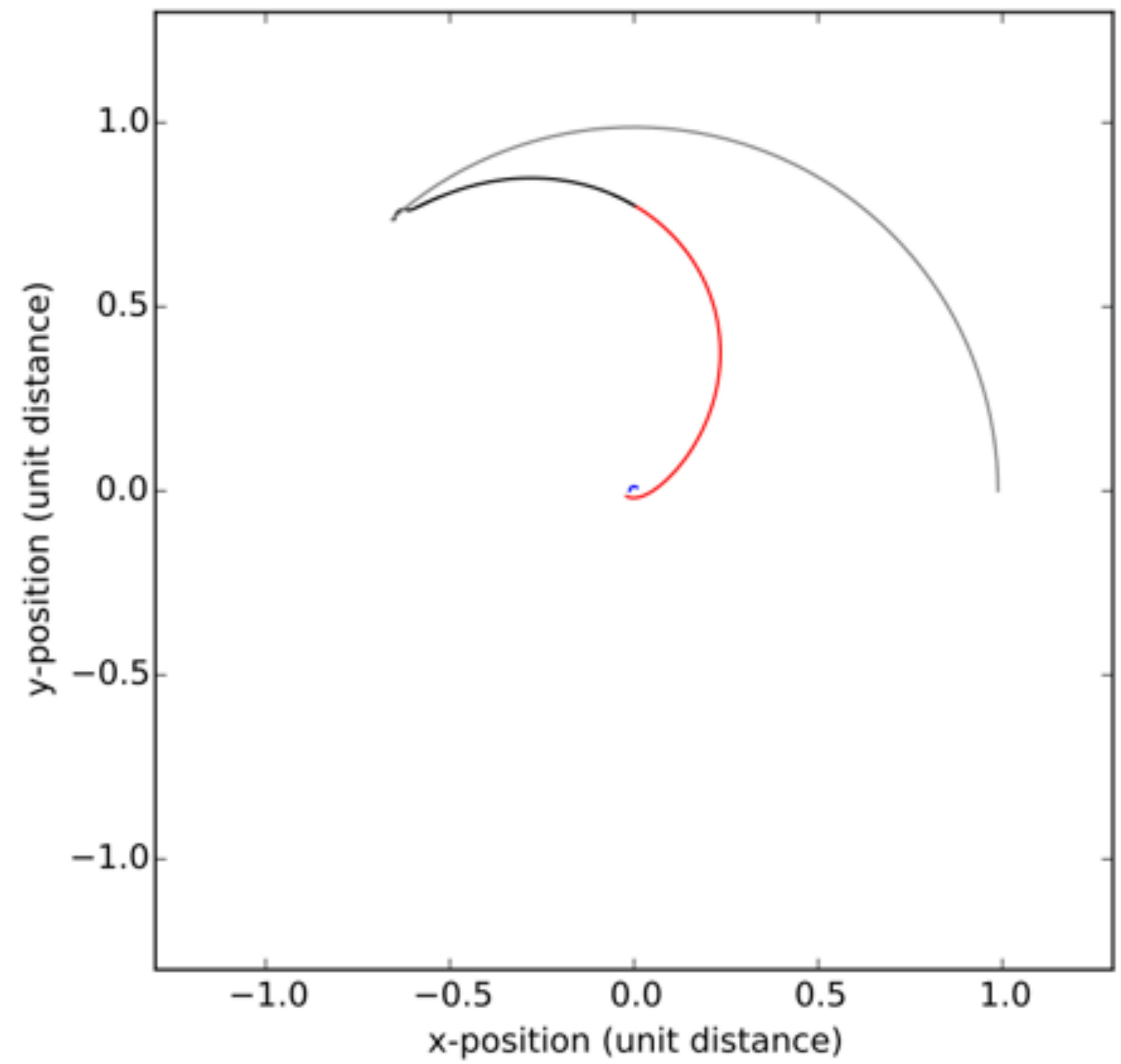
Entry to Moon orbit (100±10 km)

Searching for Transfer Orbits

Hohmann Transfer Orbit to the Moon



(x, y)



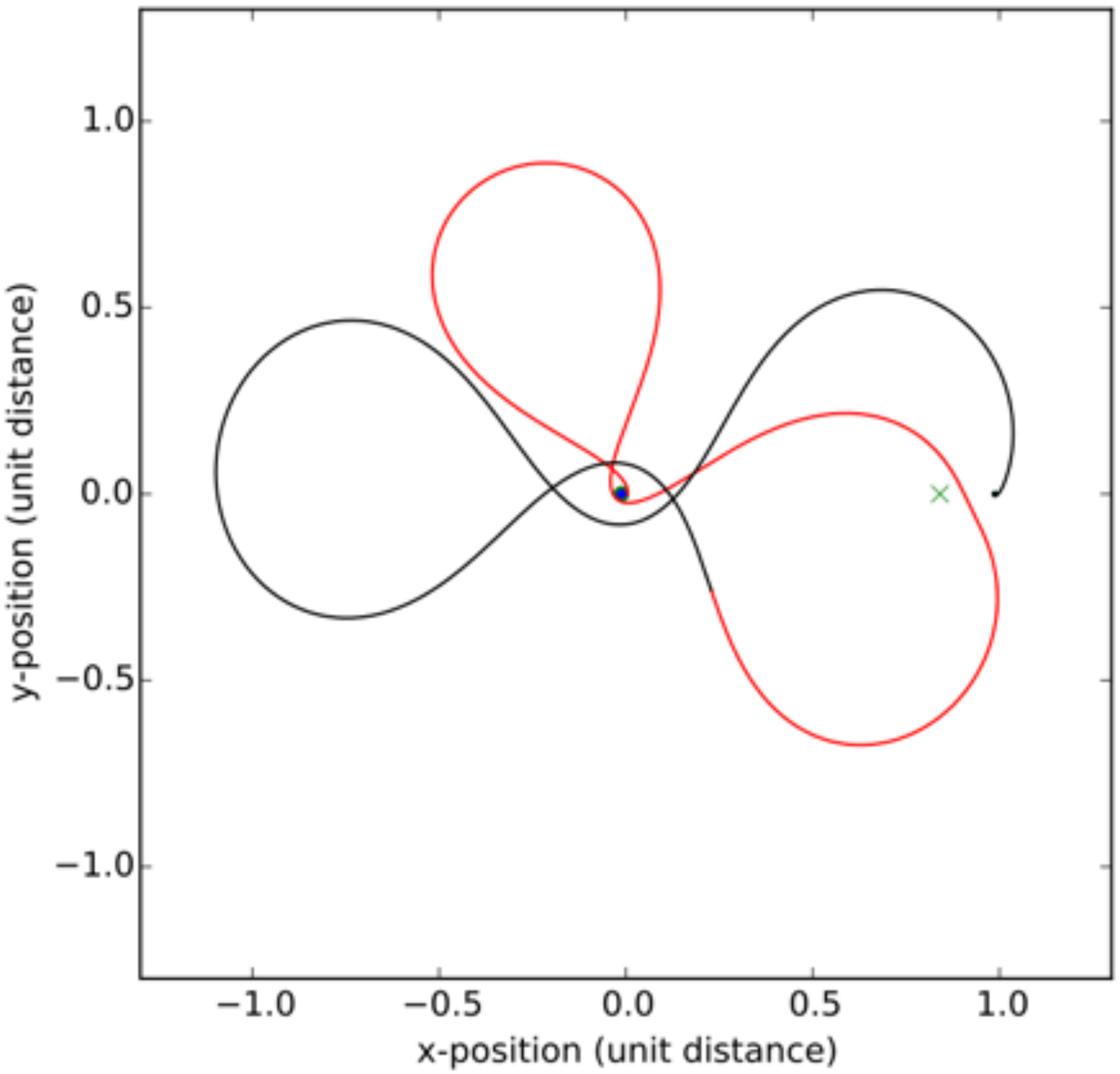
$(\mathcal{X}, \mathcal{Y})$

Δv_{total} = 3912 m/s

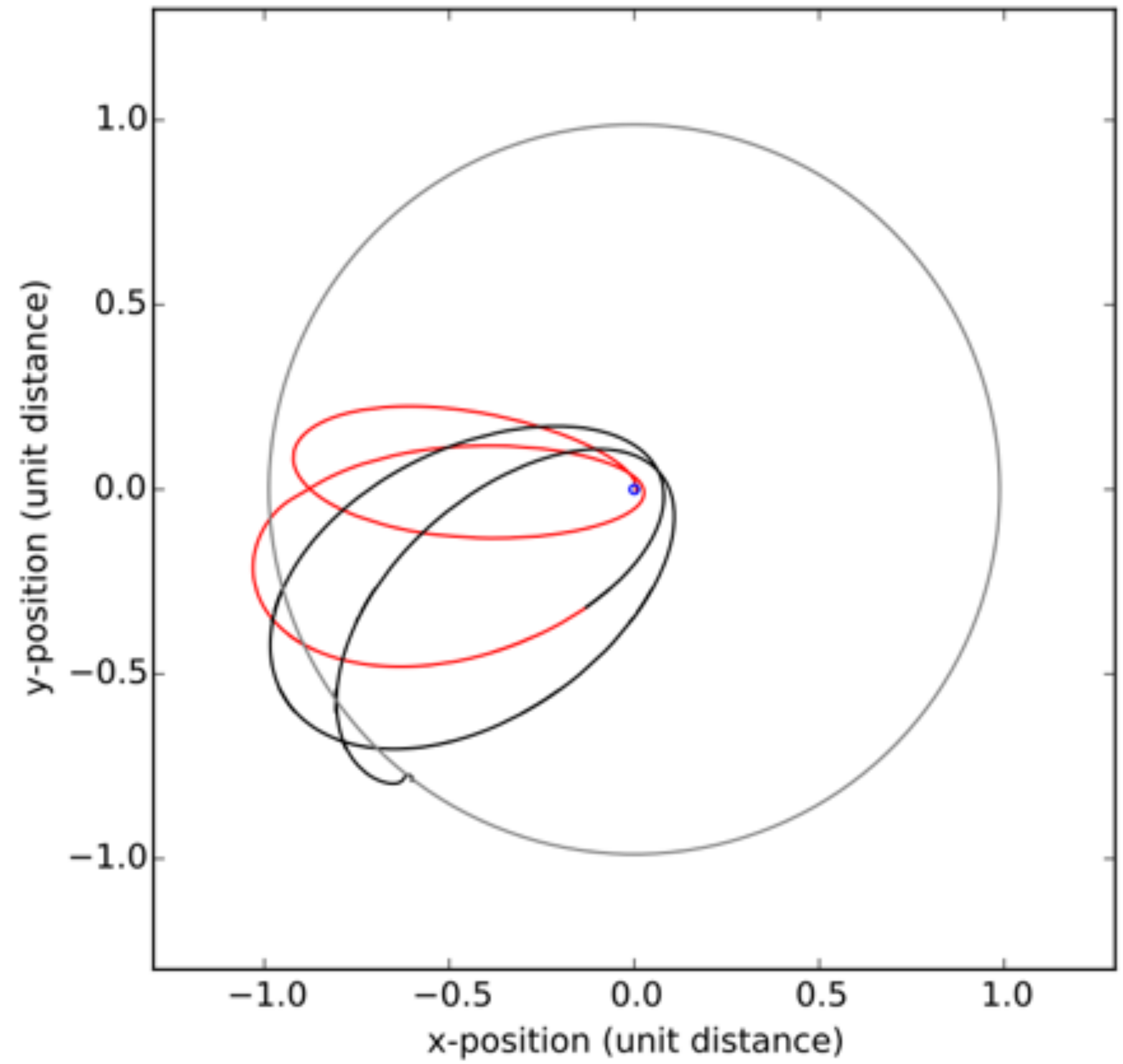
t_H = 4.3 days

Searching for Transfer Orbits

Low-Energy Transfer Orbit (short)



(x, y)



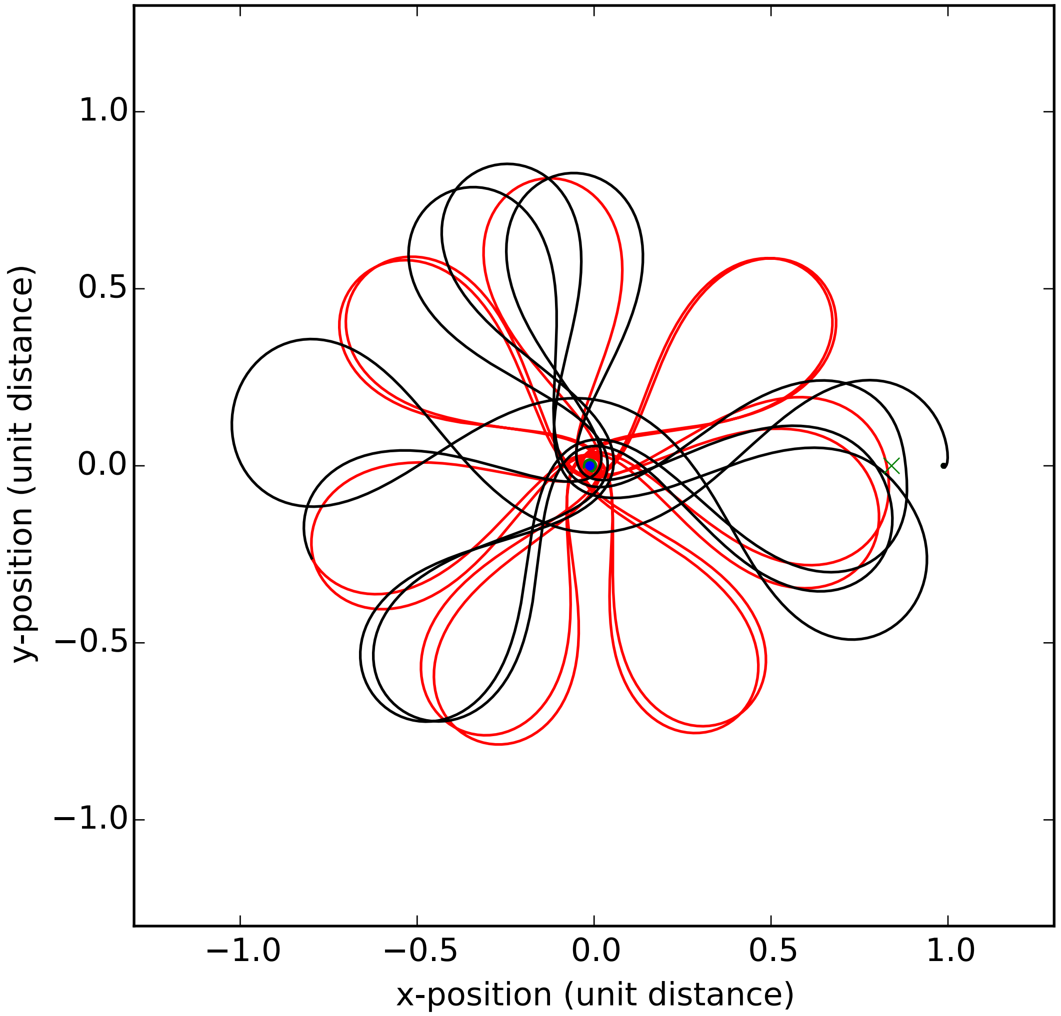
$(\mathcal{X}, \mathcal{Y})$

Δv_{total} = 3896 m/s

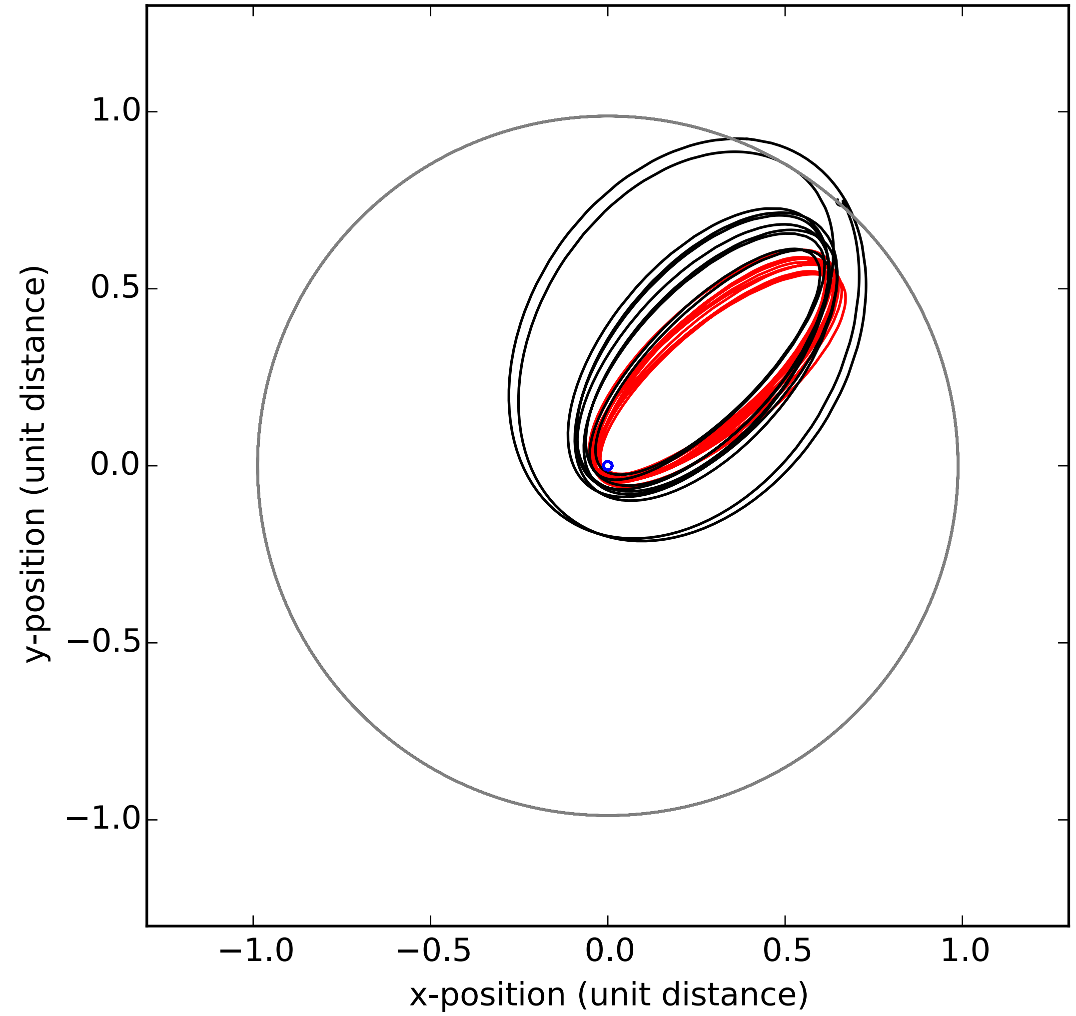
t_{short} = 41 days

Searching for Transfer Orbits

Low-Energy Transfer Orbit (long)



(x, y)



$(\mathcal{X}, \mathcal{Y})$

Δv_{total} = 3795 m/s

t_{long} = 194 days

Results Summarized

All transfer orbits

Trajectory	Flight time	Δv_{total} (km/s)	Δv_{earth} (km/s)	Δv_{moon} (km/s)
Minimum	N/A	3.721	3.099	0.622
Long LETO	194 days	3.795	3.091	0.704
Belbruno-Miller	3 months	3.838	3.187	0.651
Topputo	8 months	3.895	3.265	0.630
Short LETO	41 days	3.896	3.127	0.769
Hohmann - long (sim)	4.3 days	3.912	3.111	0.801
Hohmann - (model)	5.0 days	3.946	3.144	0.802
Hohmann - medium (sim)	3.00 days	4.015	3.136	0.880
Apollo (Hohman)	3.05 days	4.115	3.048	1.067
Hohmann - short (sim)	1.00 days	6.823	3.809	3.014

Conclusion

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- **We can go to Moon much cheaper than traditionally!**
  (If we’re willing to accept longer flight time)

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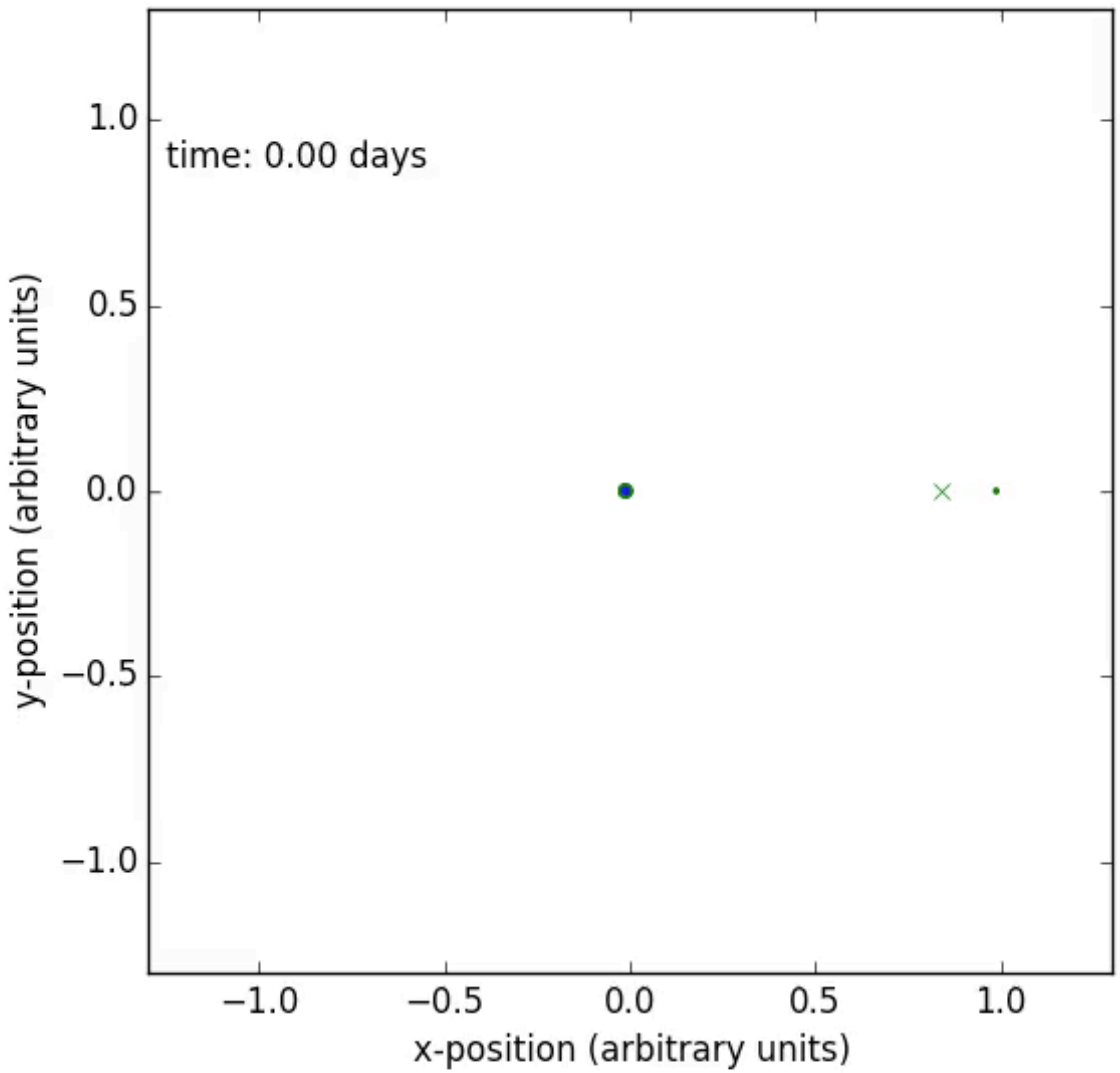
- **Brute force search** —> Useful low-energy transfer orbits

- **Brute force search made feasible by:**
  1. Effective algorithms (adaptive)
  2. Parallelization
  3. GPU acceleration

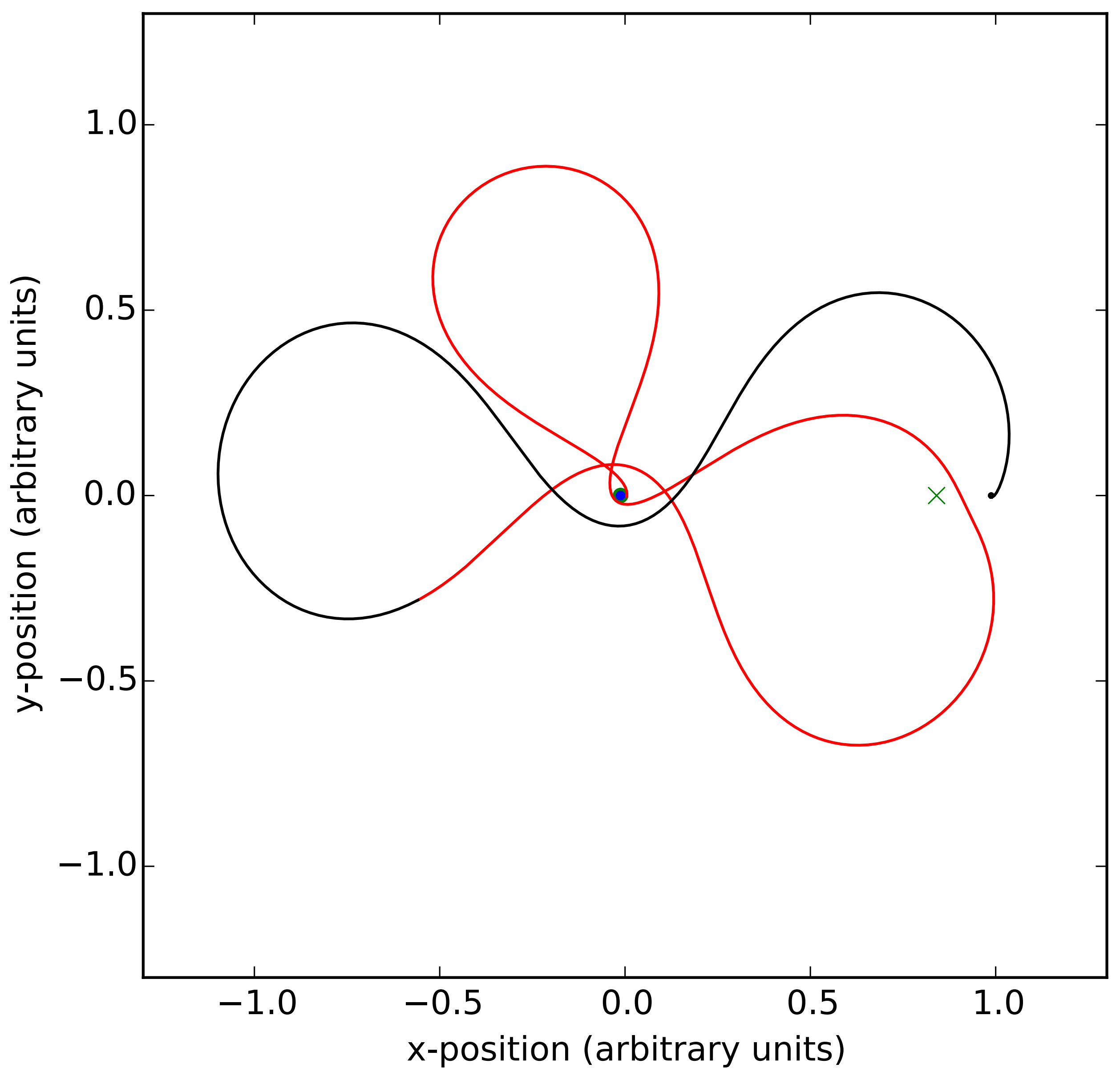
LETO Short Animation

0.043484 days/frame, 60 FPS



Numerical Method

Adaptive Störmer-Verlet (symplectic)



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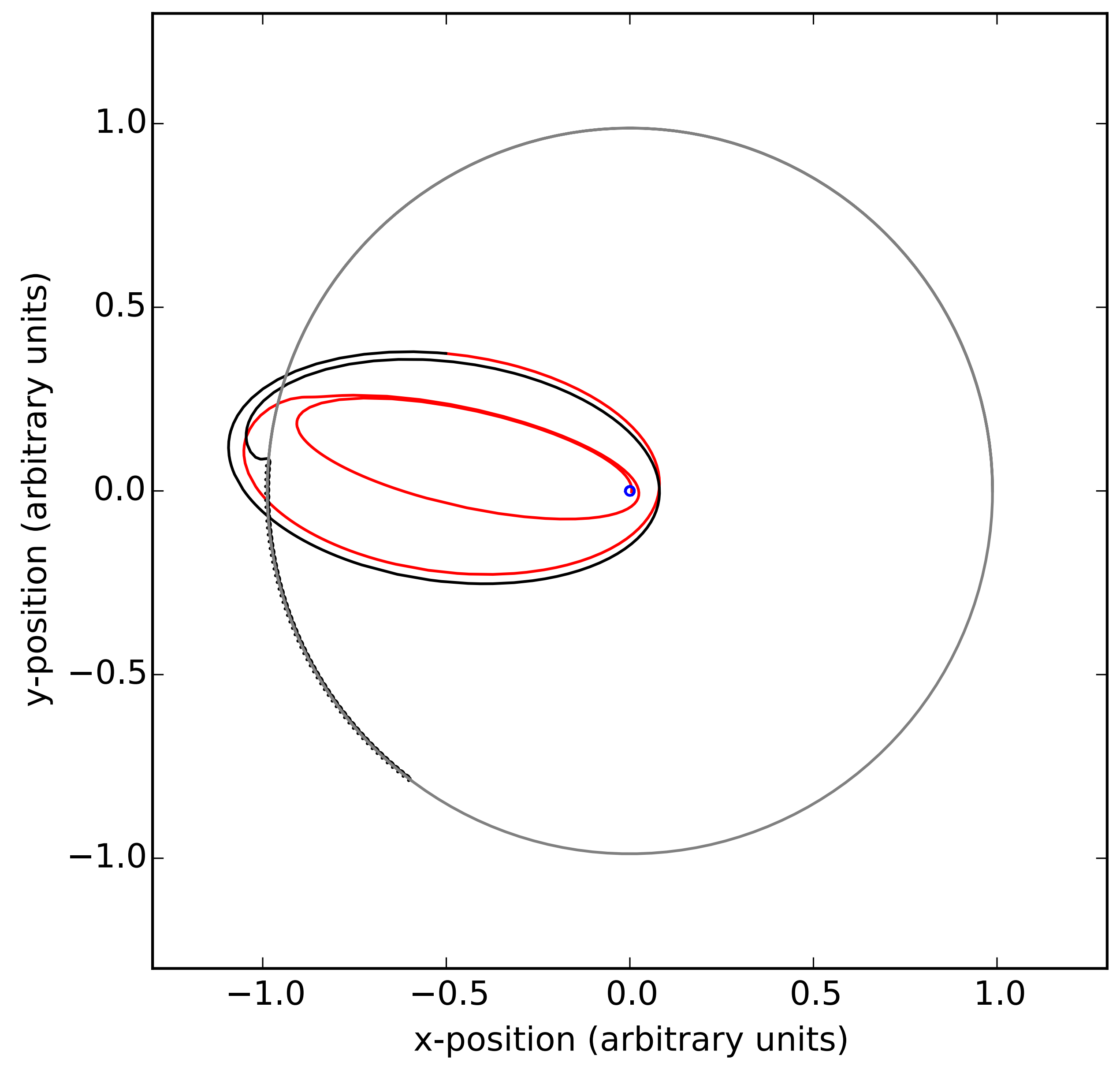

$(\mathcal{X}, \mathcal{Y})$

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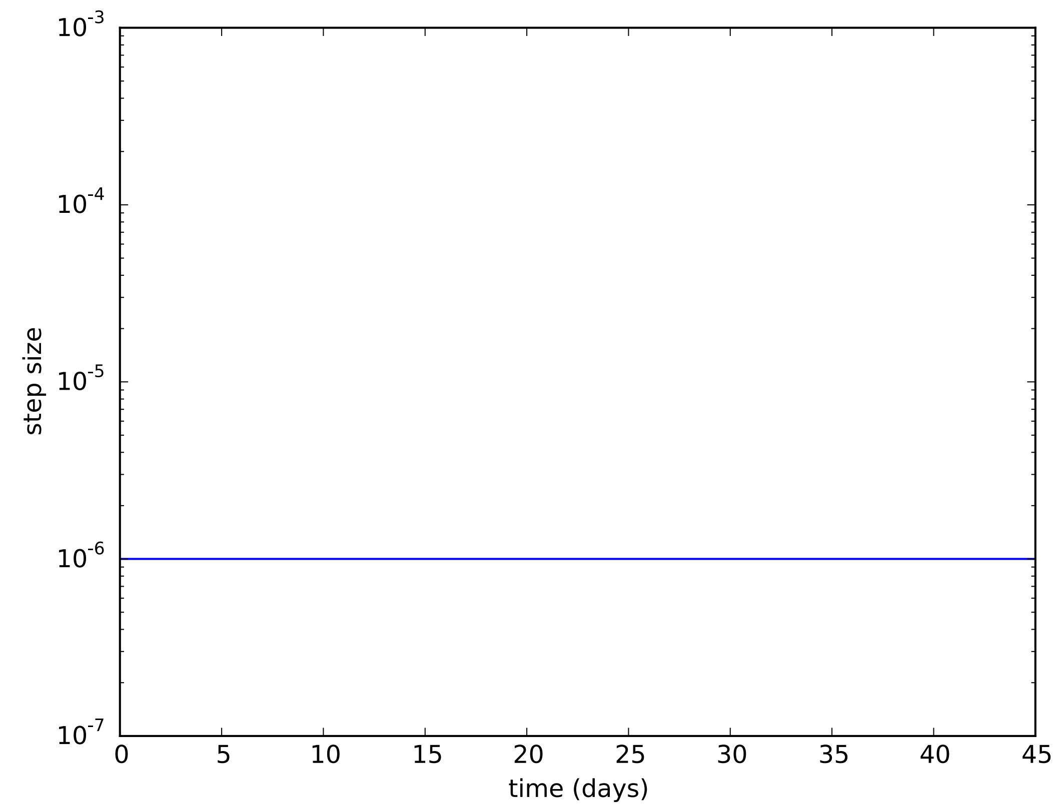
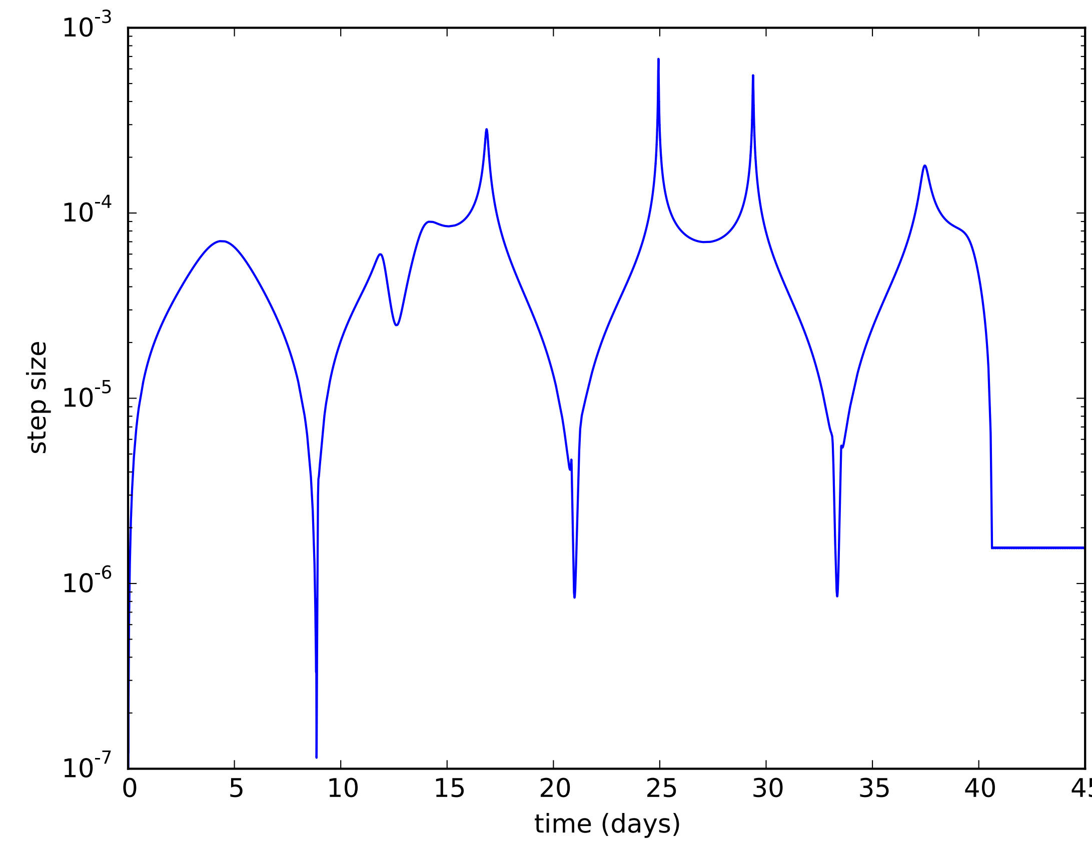
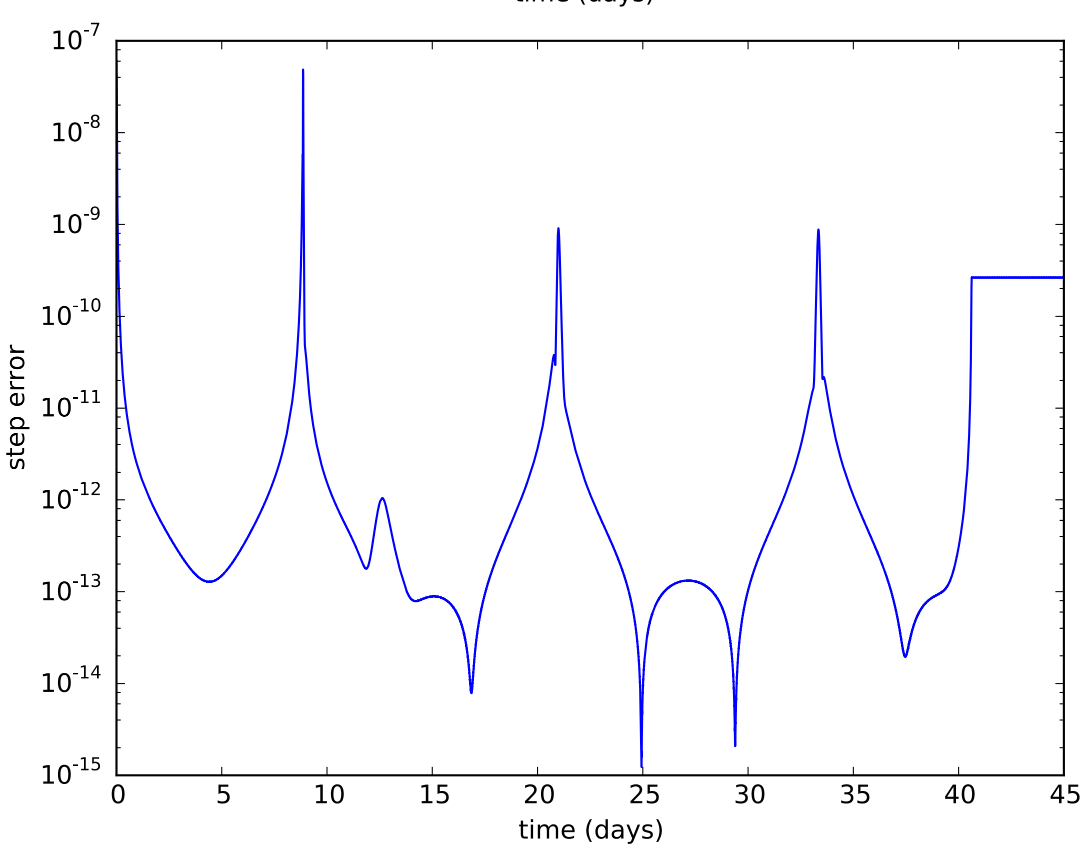
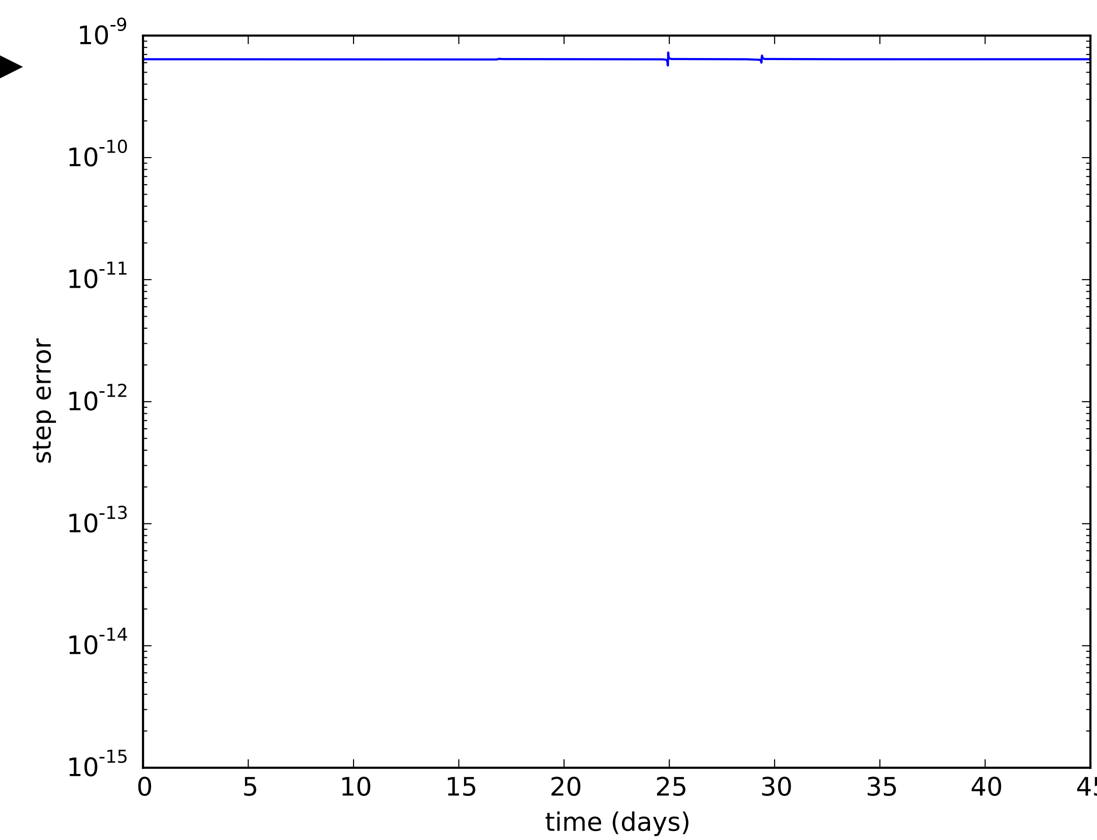


Adaptive







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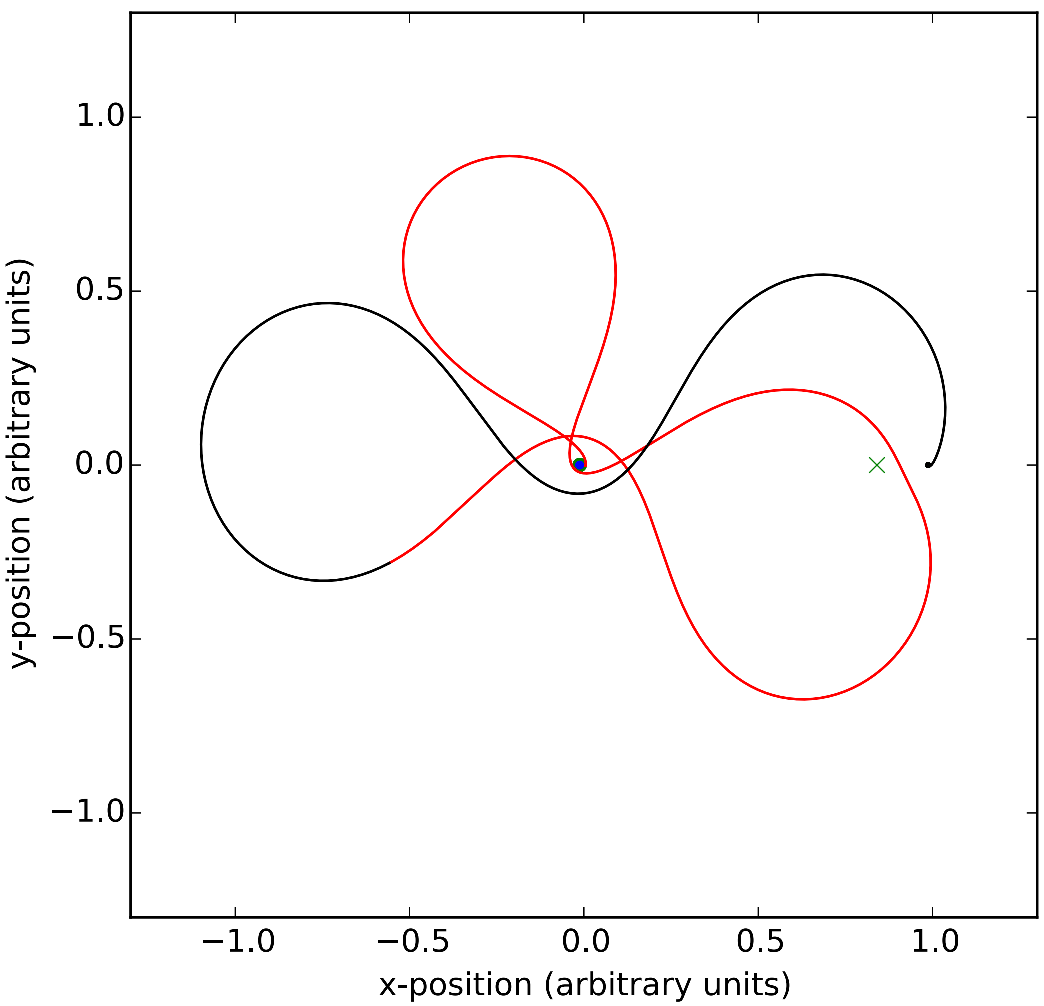

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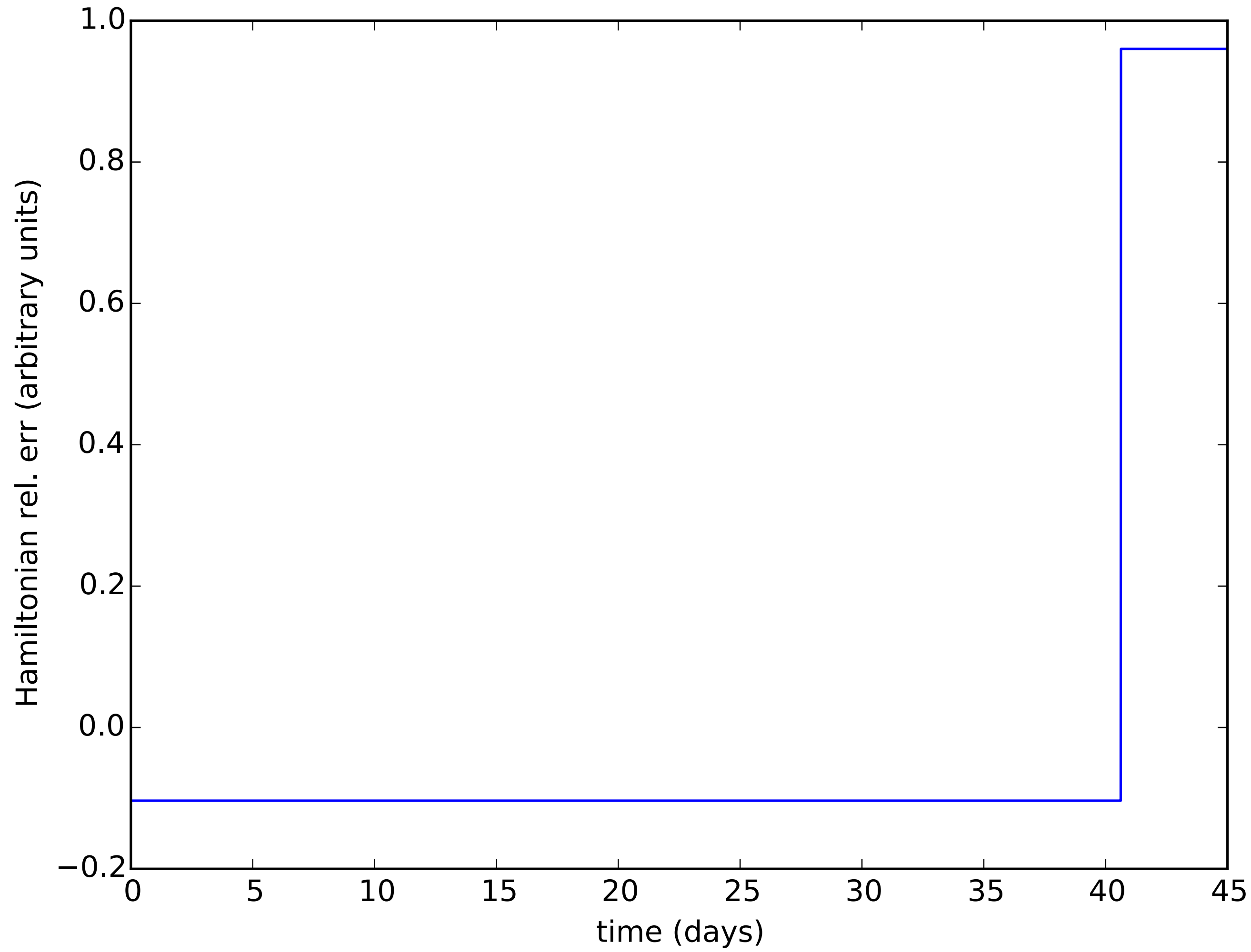
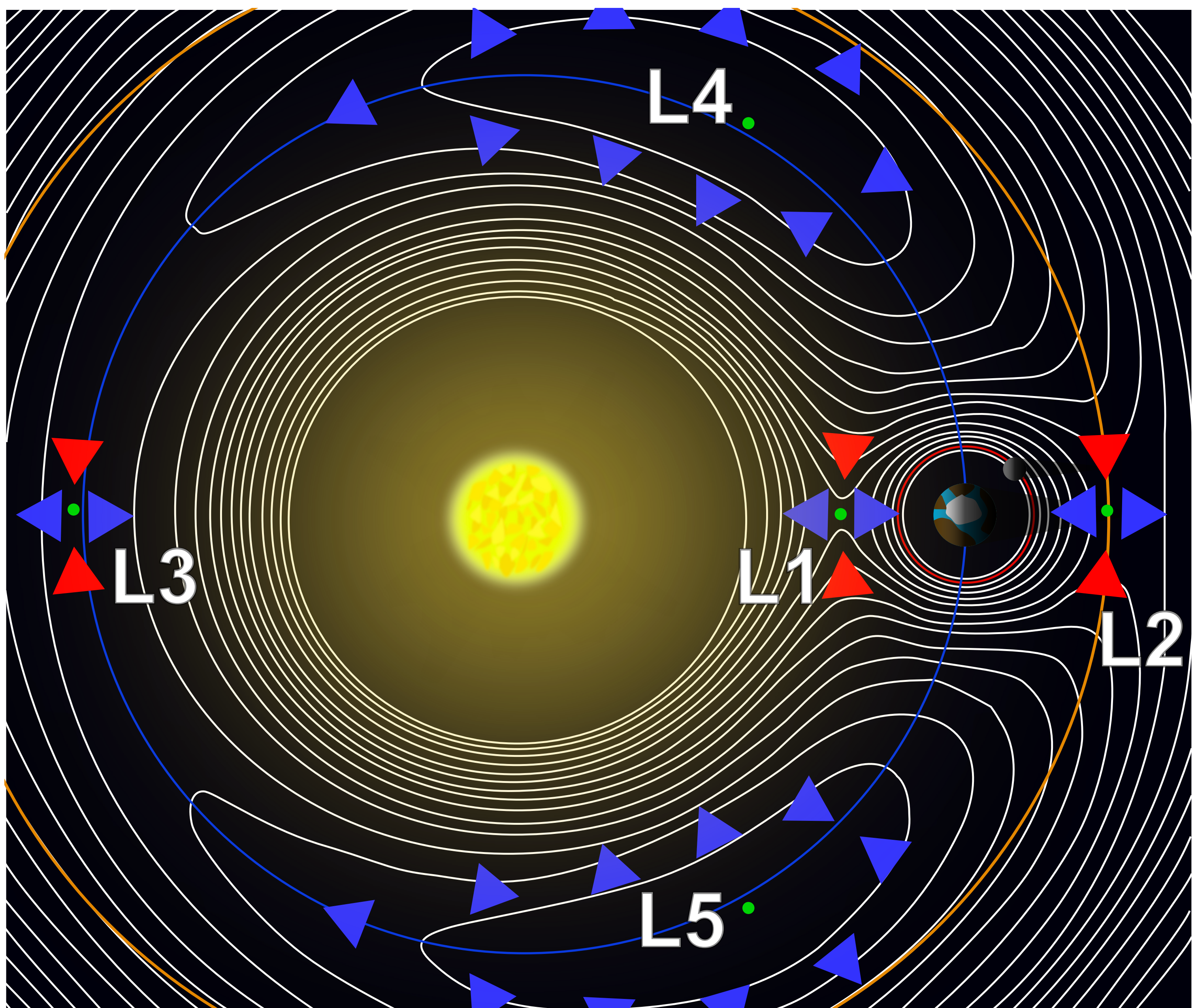
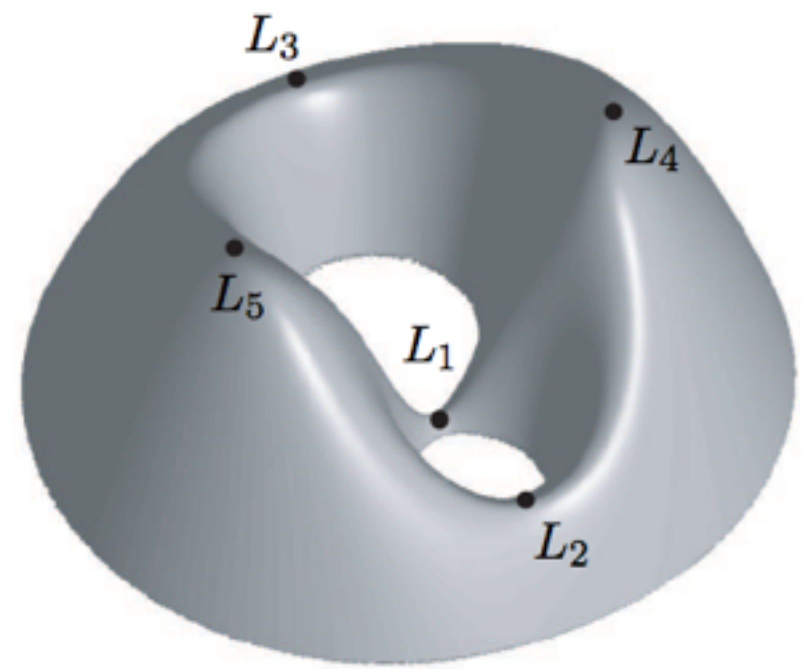
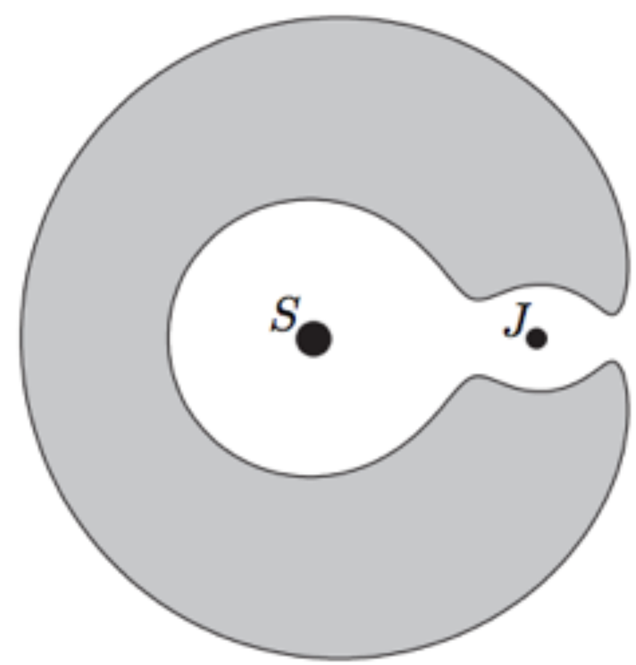


$\mathrm{Hamiltonian}$

Gravitational Potential and Lagrange Points







Wishlist

- Earth$\rightarrow L_1$ (forward) + $L_1 \rightarrow$Moon (backward)
- 2D $\rightarrow$ 3D
- Include Sun's potential
- Higher-order integrator:
  4.-5.-order symplectic Runge-Kutta