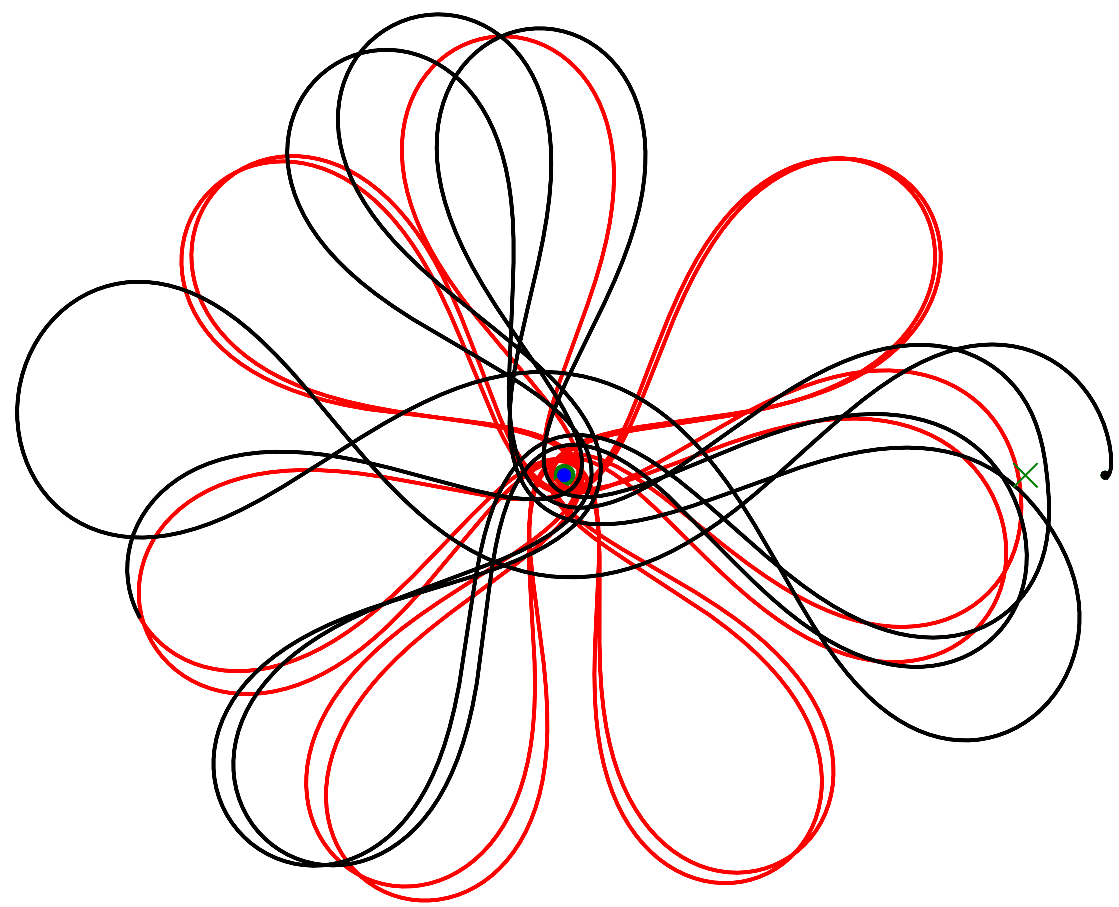
Low-Energy Transfer Orbits



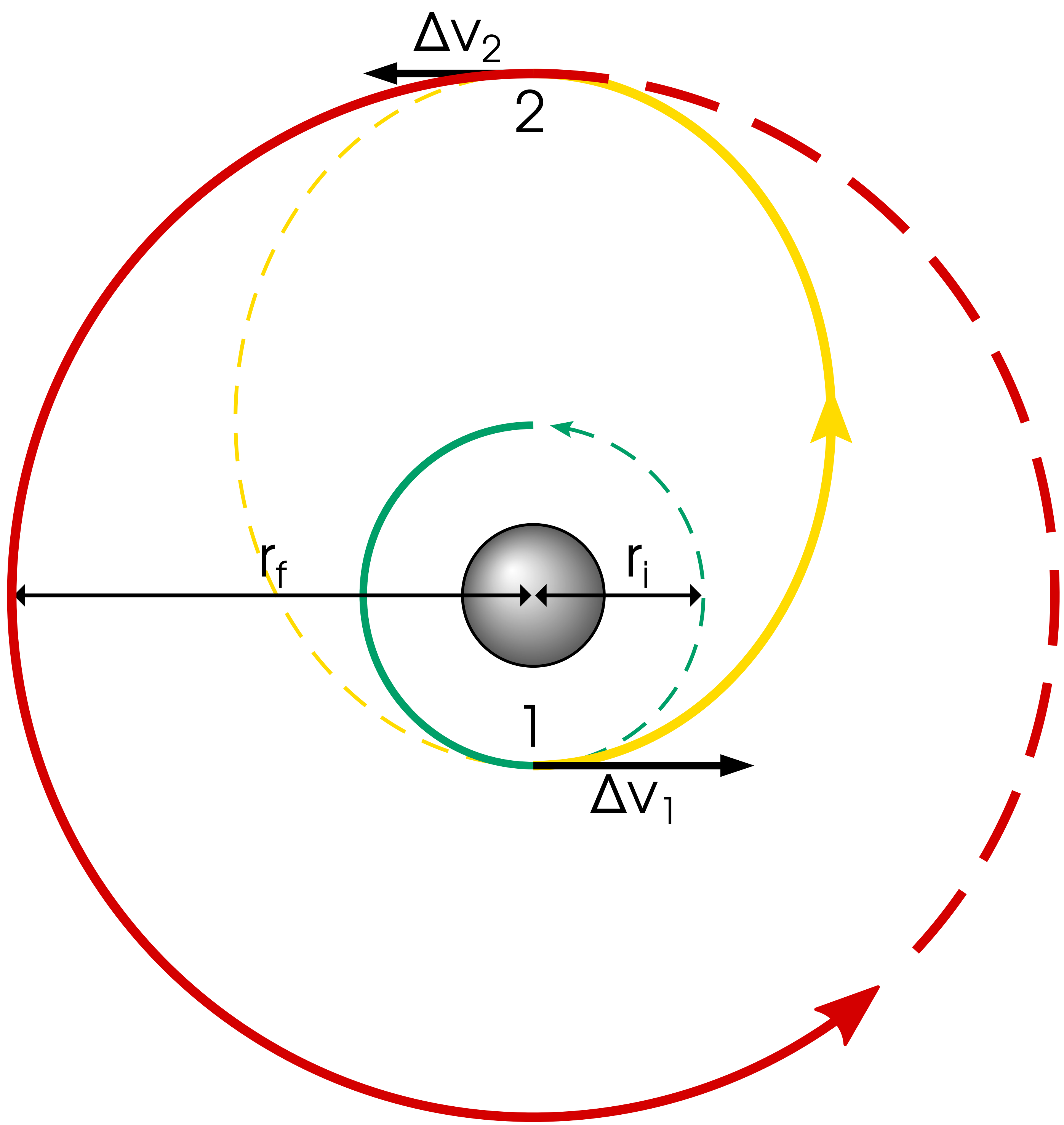
A Theoretical and Numerical Study

Gandalf Saxe, DTU

What are transfer orbits?

A way of getting from A to B in space

Hohmann



Why Are Transfer Orbits Interesting?

NASA manned spacecraft: Orion
Manned Mars Mission: 2030's

(NASA)

"The first crewed mission — called EM-2 — is now scheduled for April 2023; the flight was originally scheduled for August 2021"

- The Verge, September 16th

Why Are Transfer Orbits Interesting?

Mars One manned spacecraft: Dragon
Manned Mars Mission: ~2026

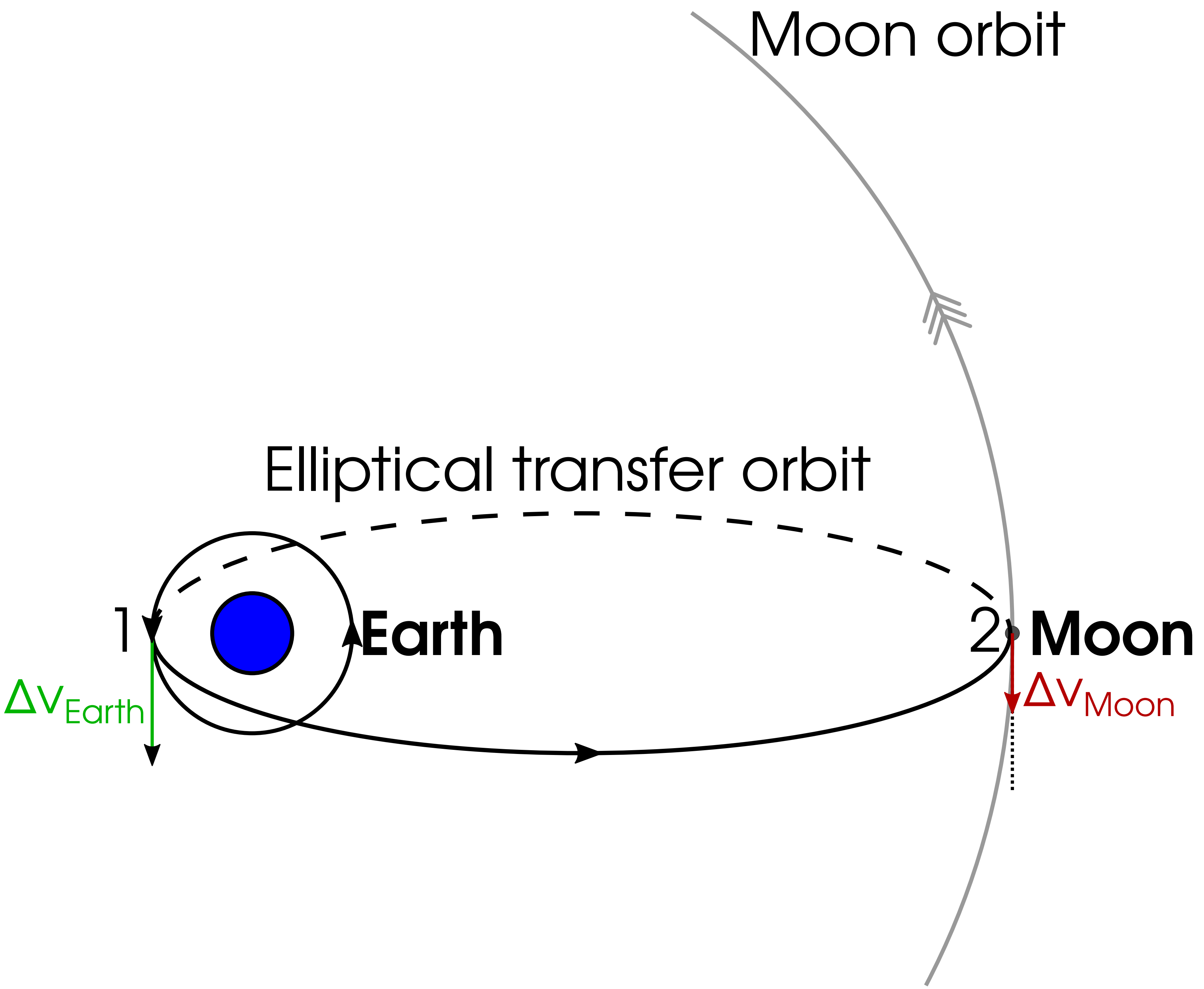
"Elon Musk argues that we must put a million people on Mars if we are to ensure that humanity has a future"

- Interview with aeon.co, 30 September 2014

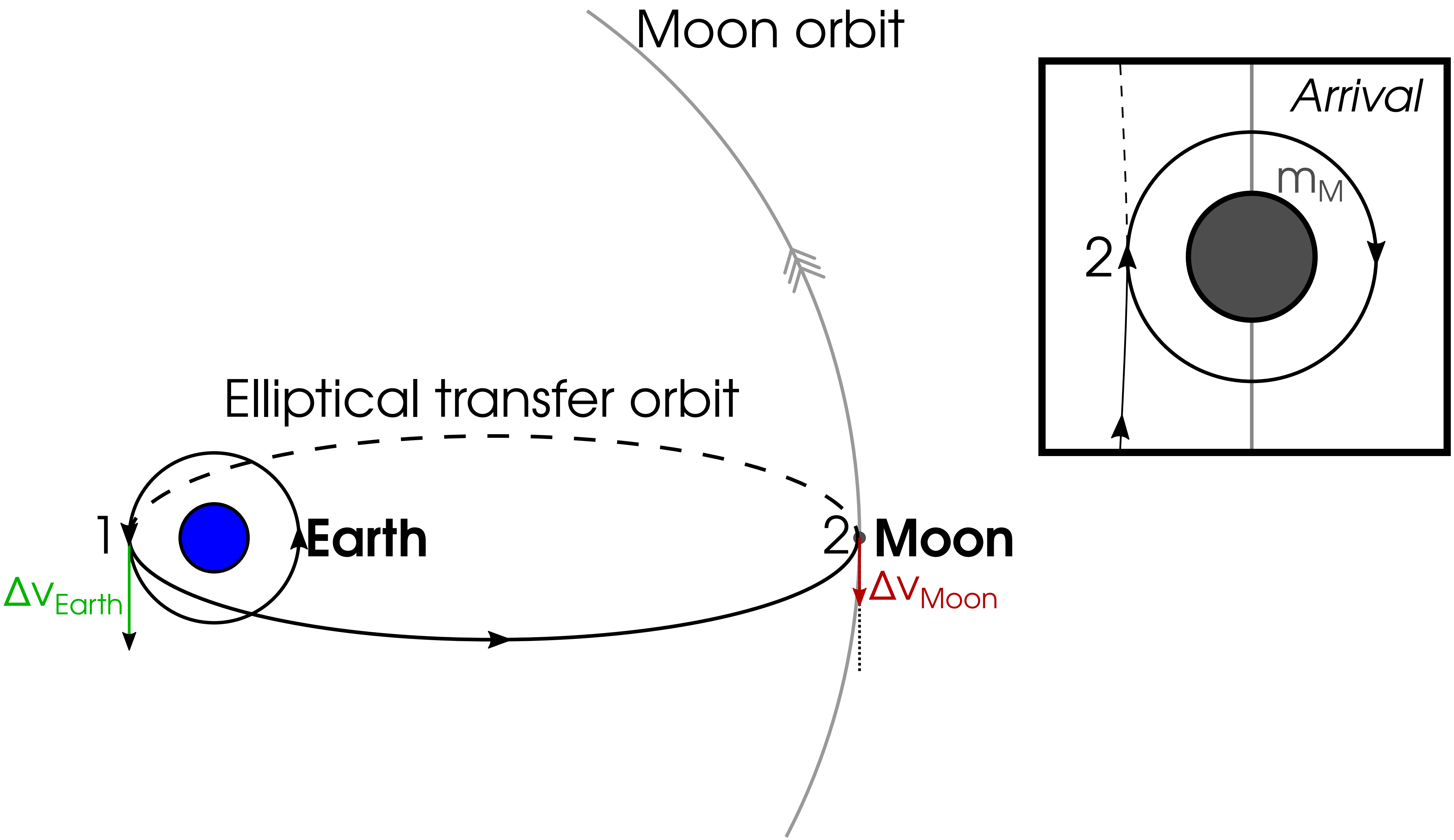
Why Are Transfer Orbits Interesting?

Answer: Fuel efficiency

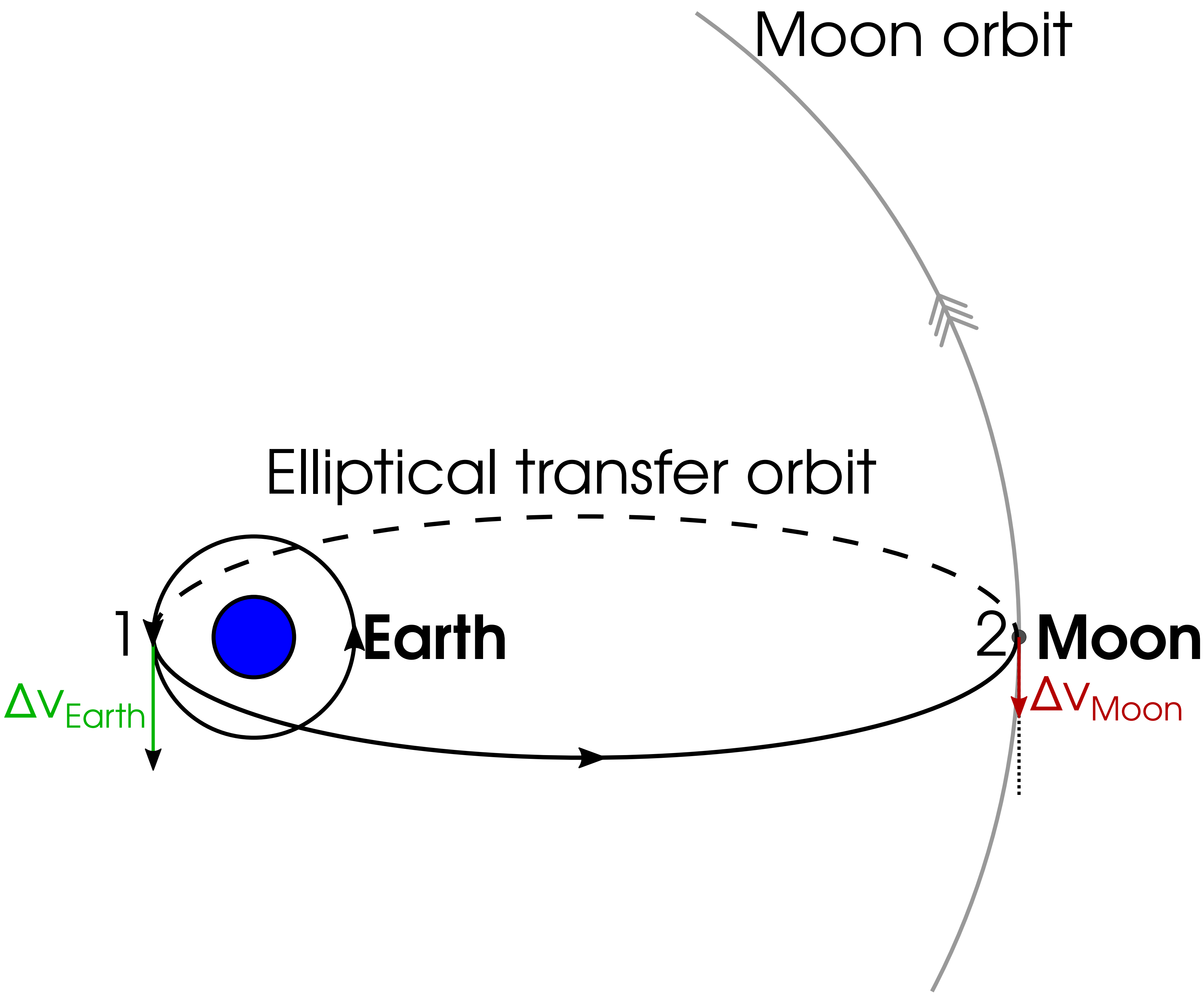
Hohmann Transfer Orbit to the Moon

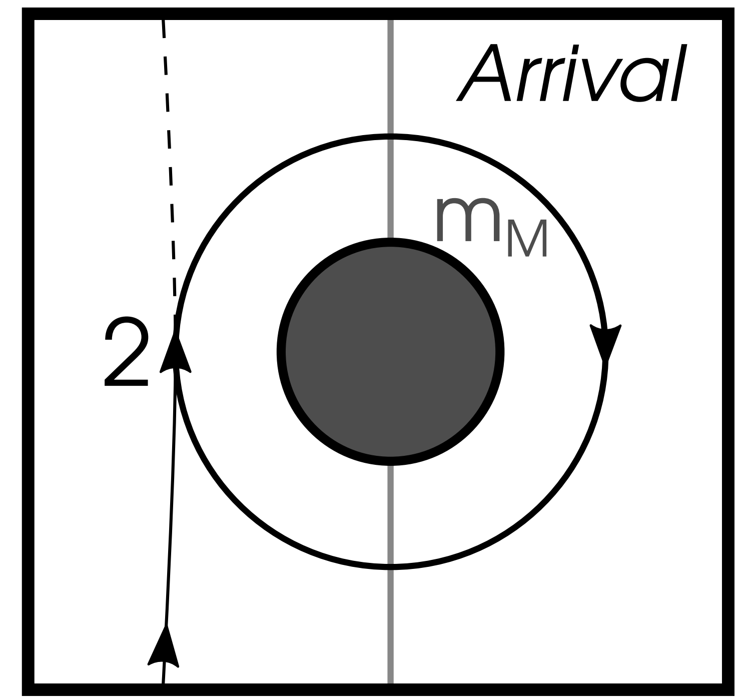


Hohmann Transfer Orbit to the Moon



Hohmann Transfer Orbit to the Moon





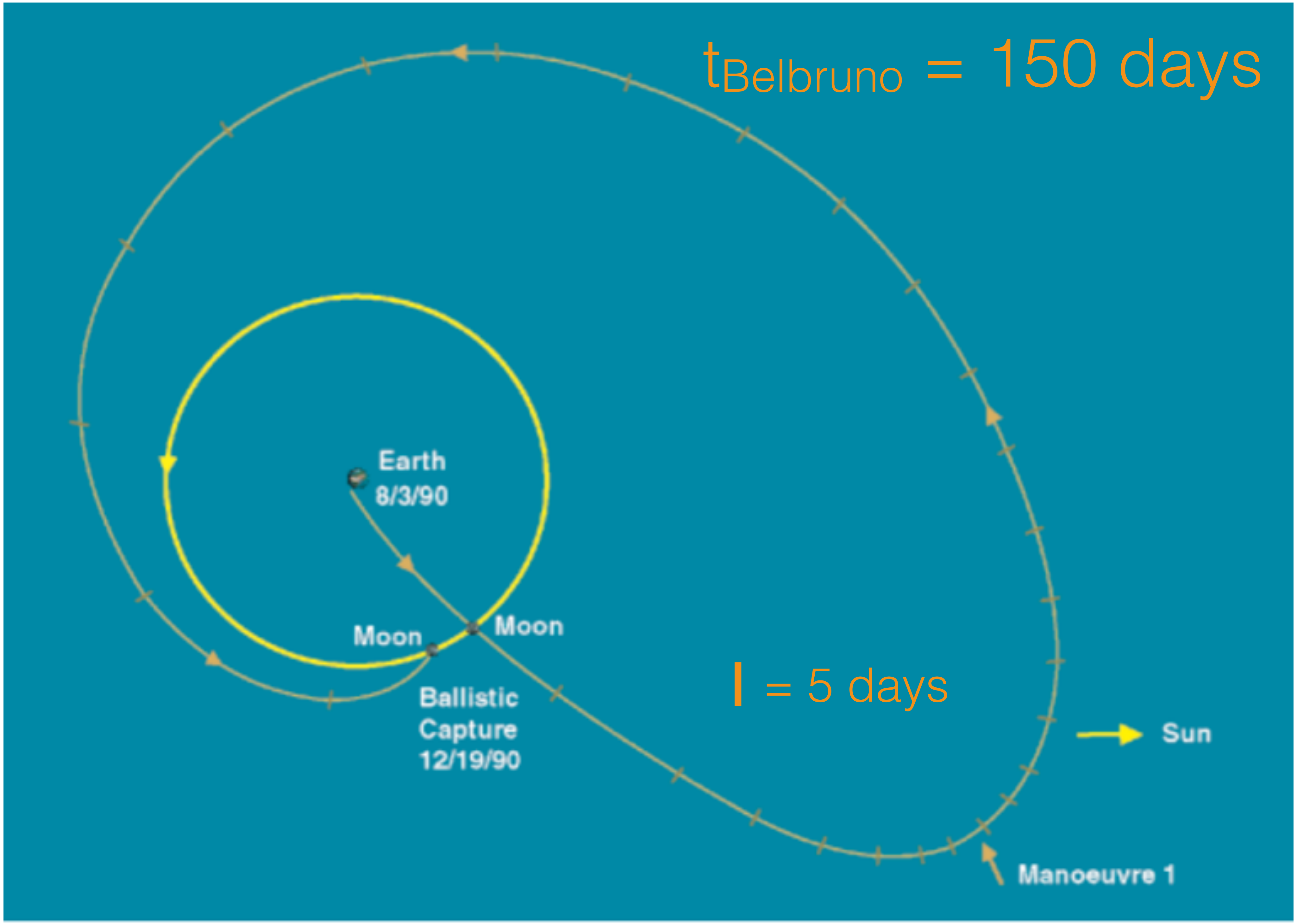
$t_H = 5.0$ days

Δv_{Earth} = 3144 m/s

Δv_{Moon} = 802 m/s

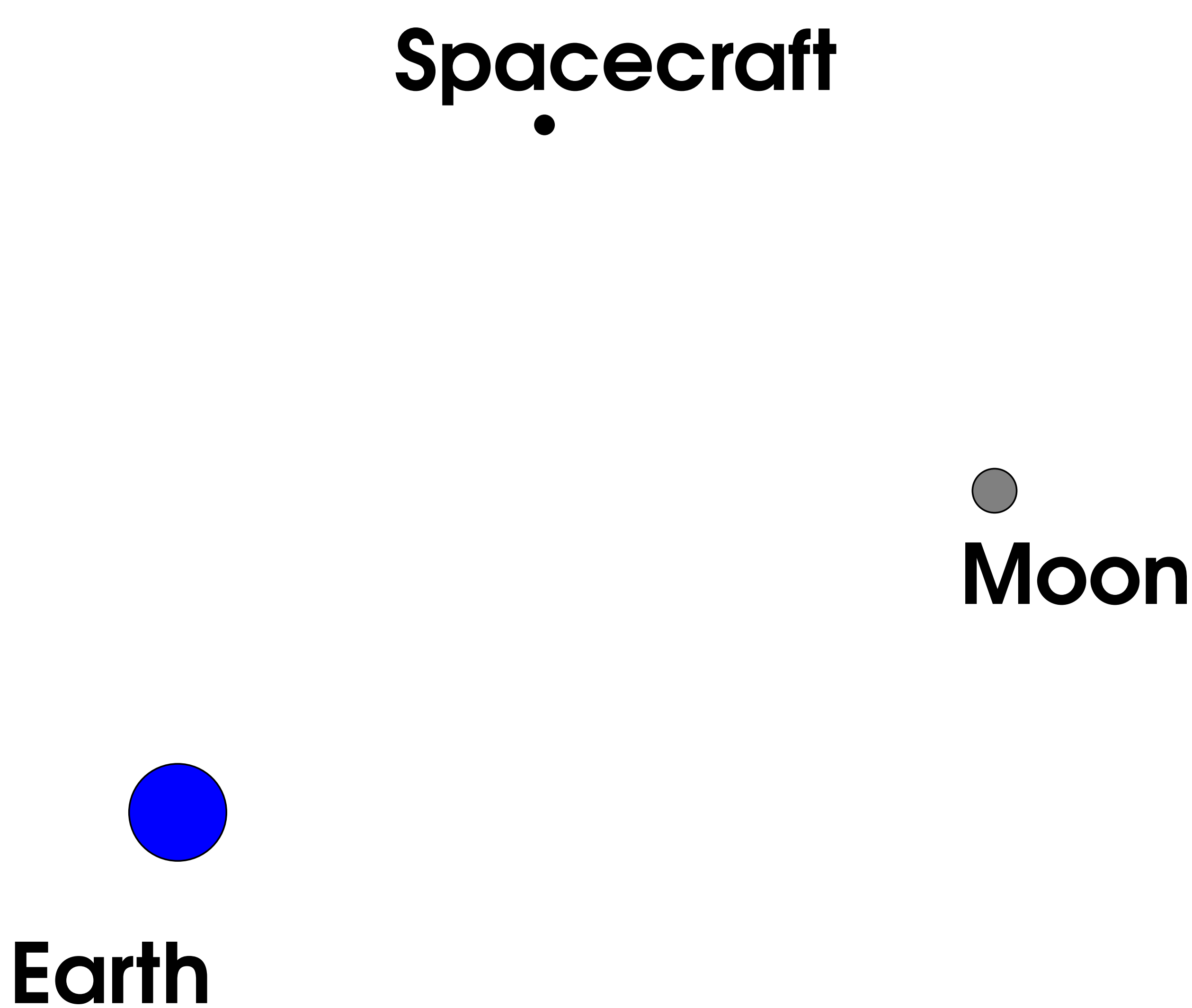
Δv_{Total} = 3946 m/s

Low Energy Transfer Orbits

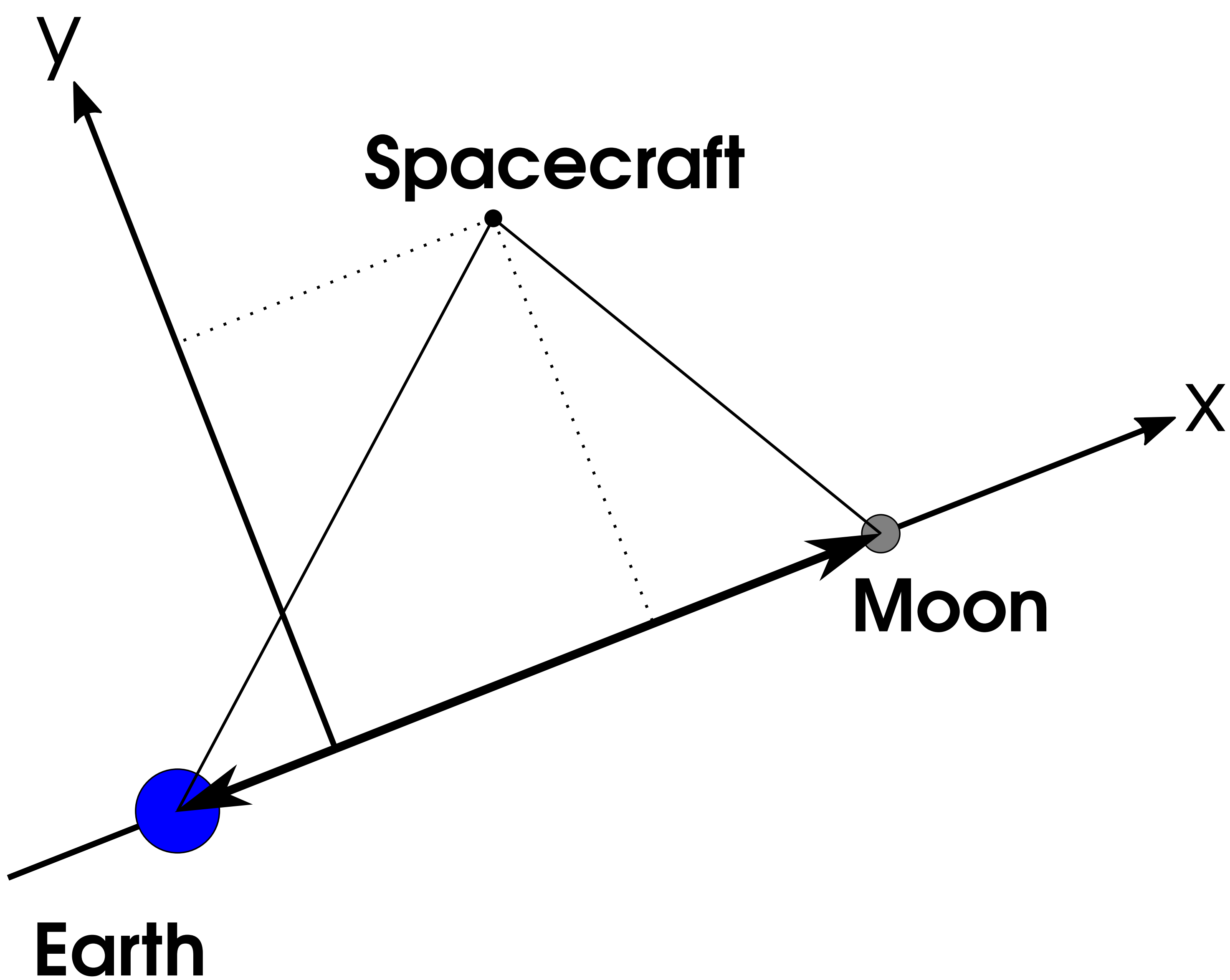


Hiten: Japanese Spacecraft, 1990

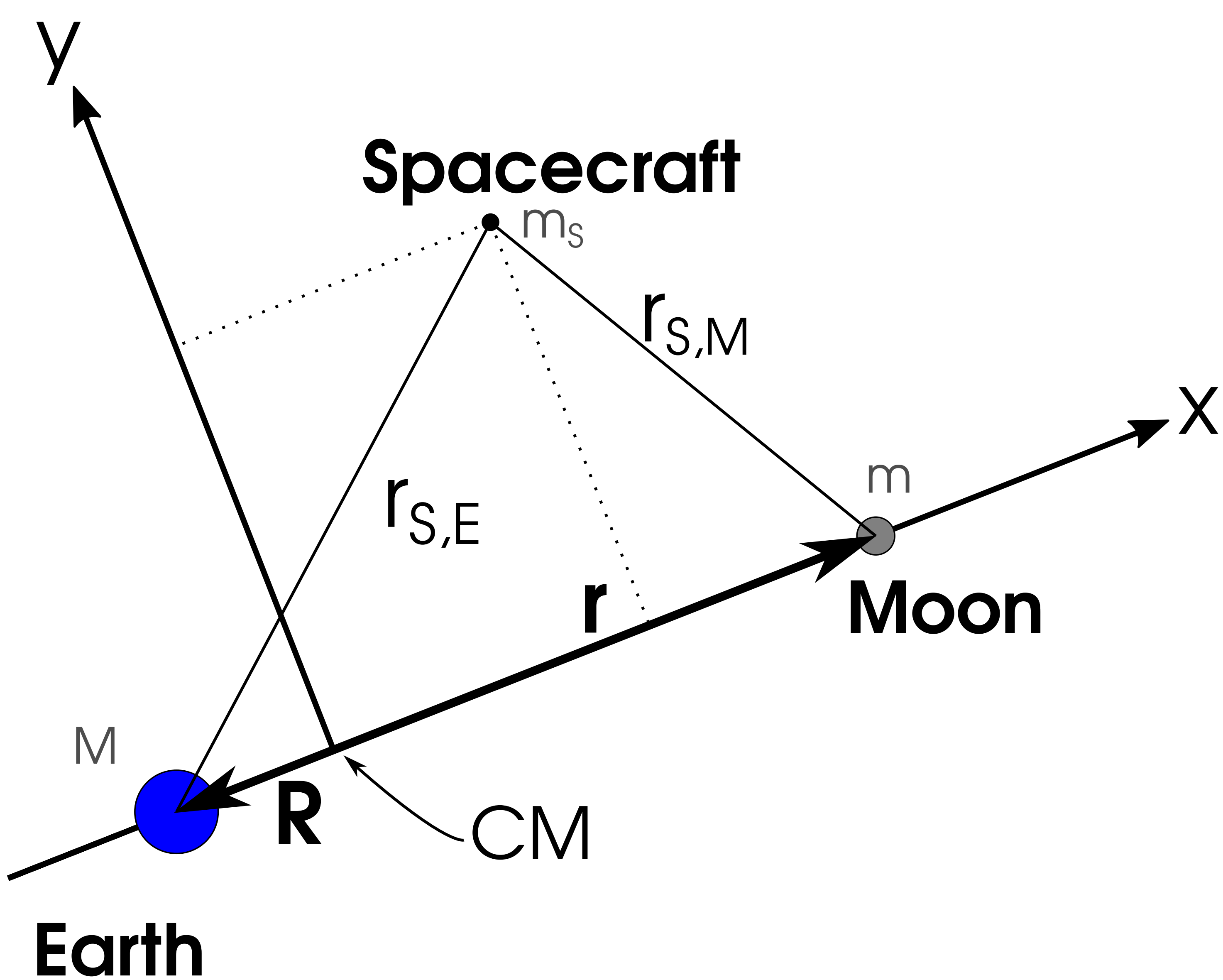
Restricted 3-body problem



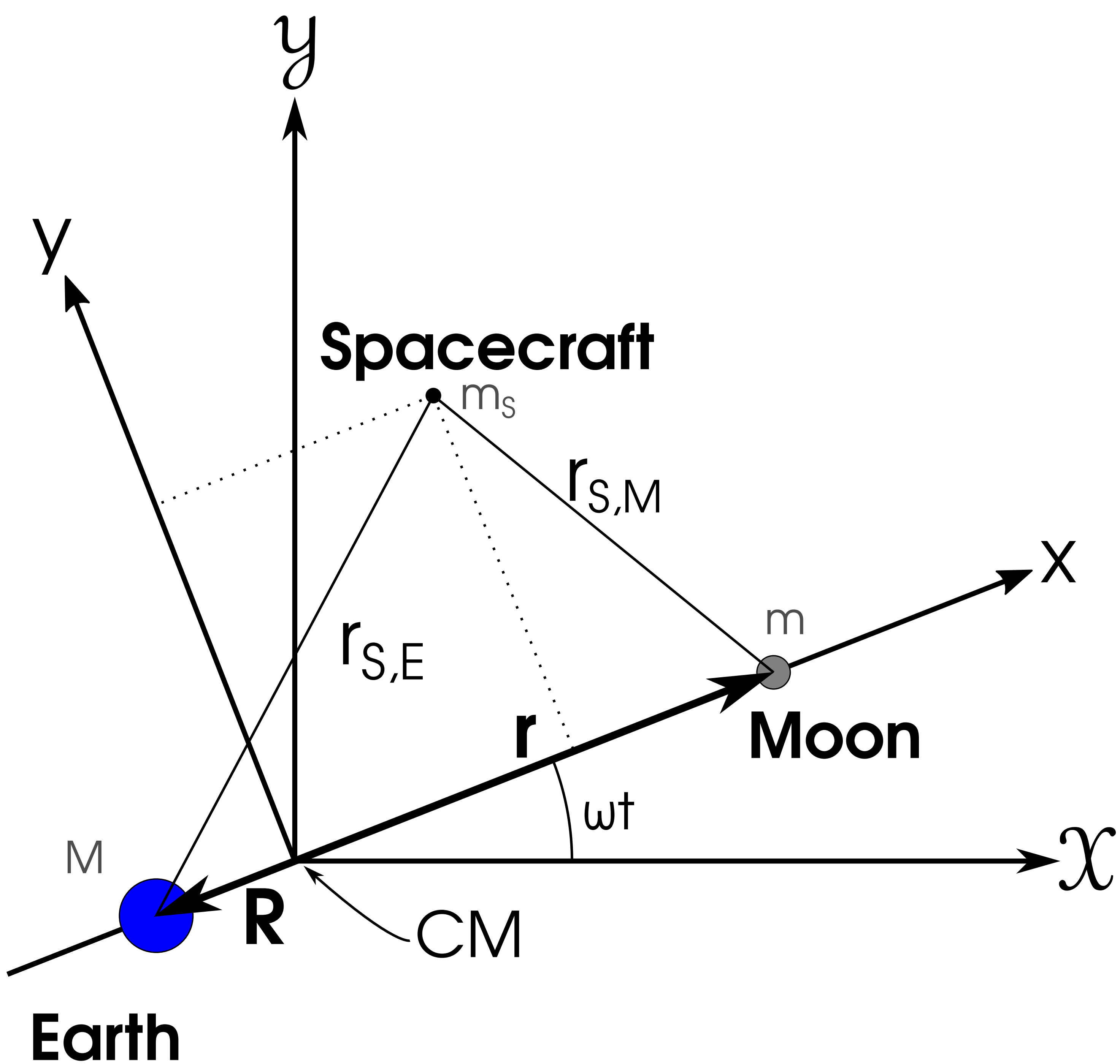
Restricted 3-body problem



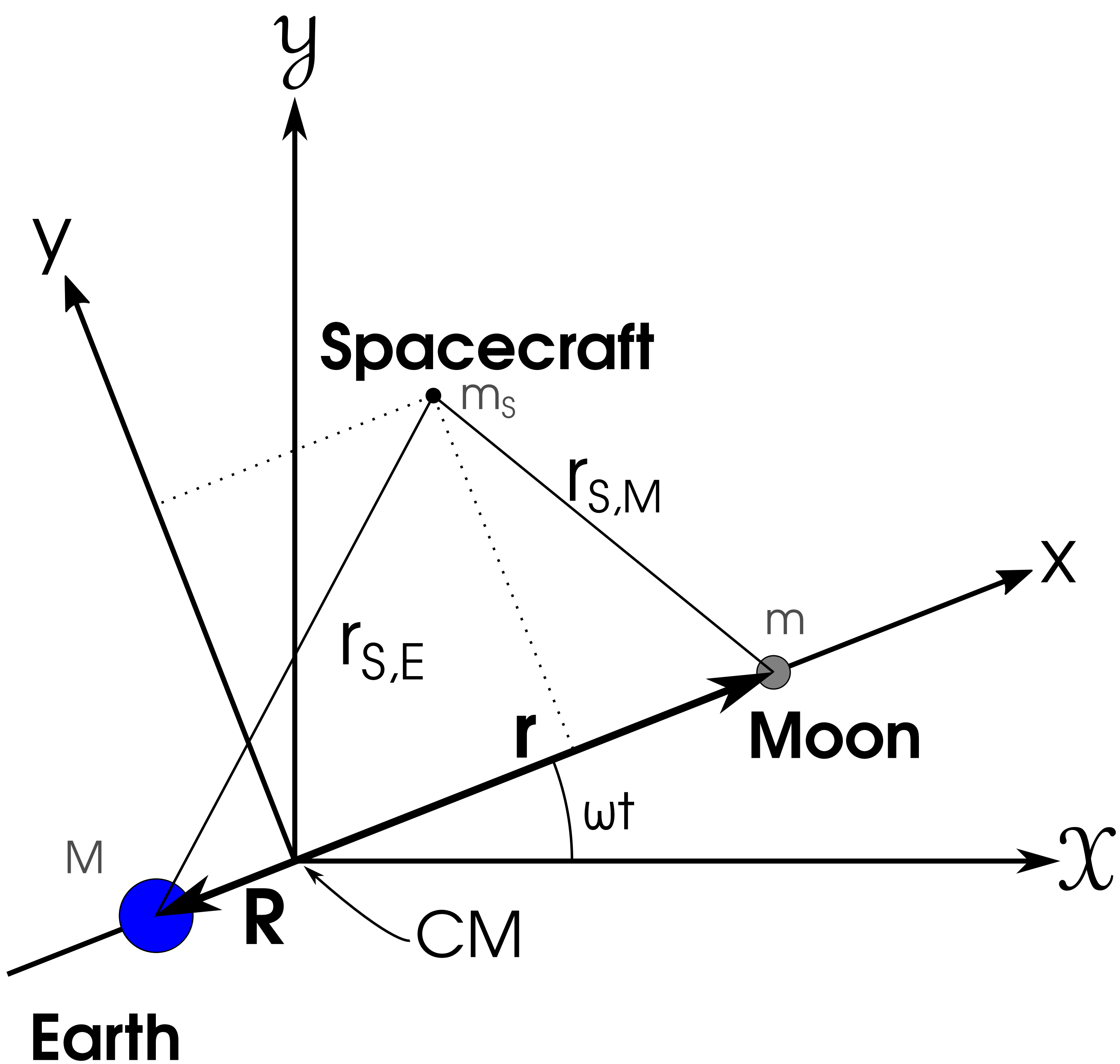
Restricted 3-body problem



Restricted 3-body problem



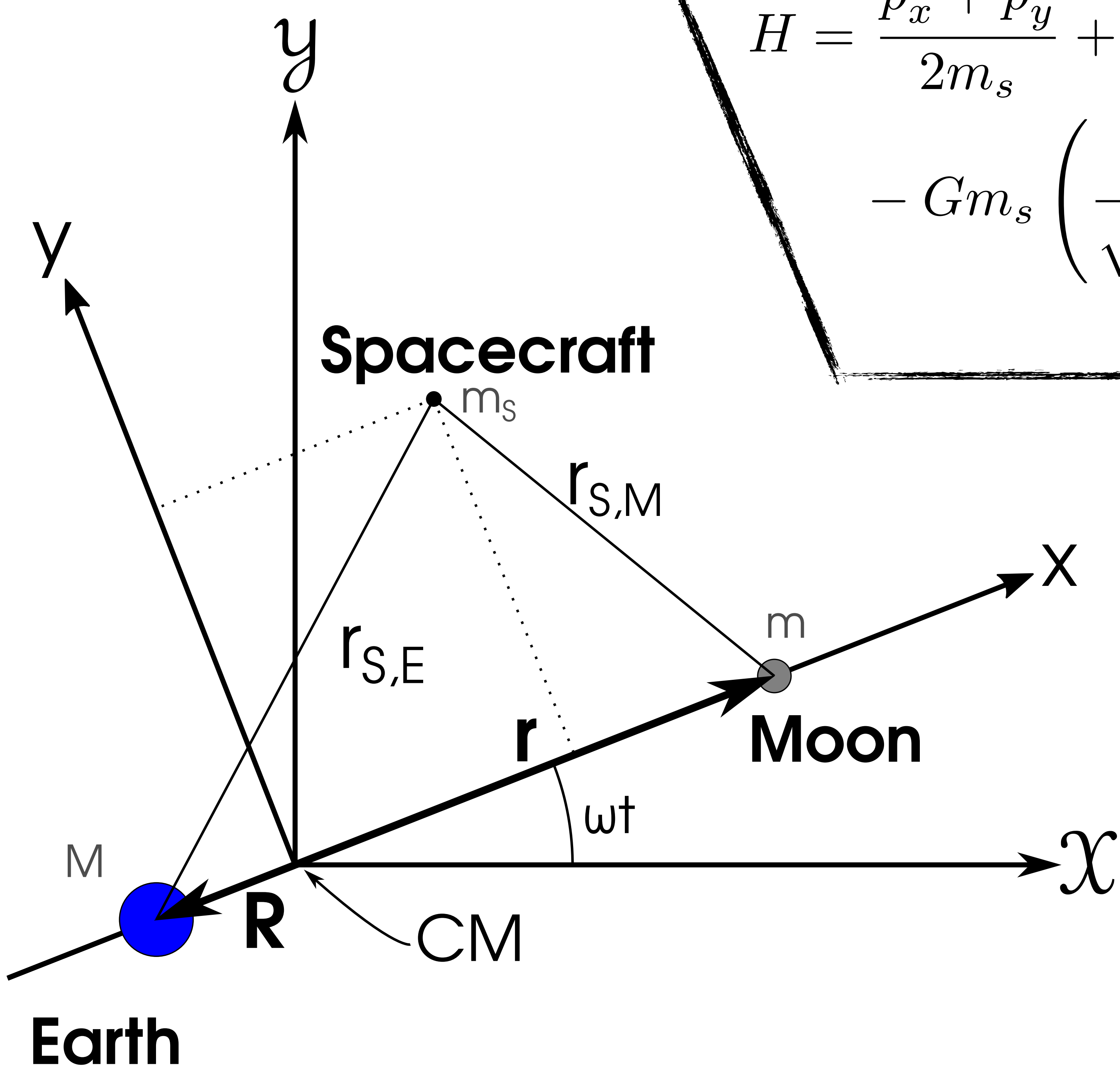
Restricted 3-body problem



Assumptions:

- 2D
- 2-body

Restricted 3-body problem

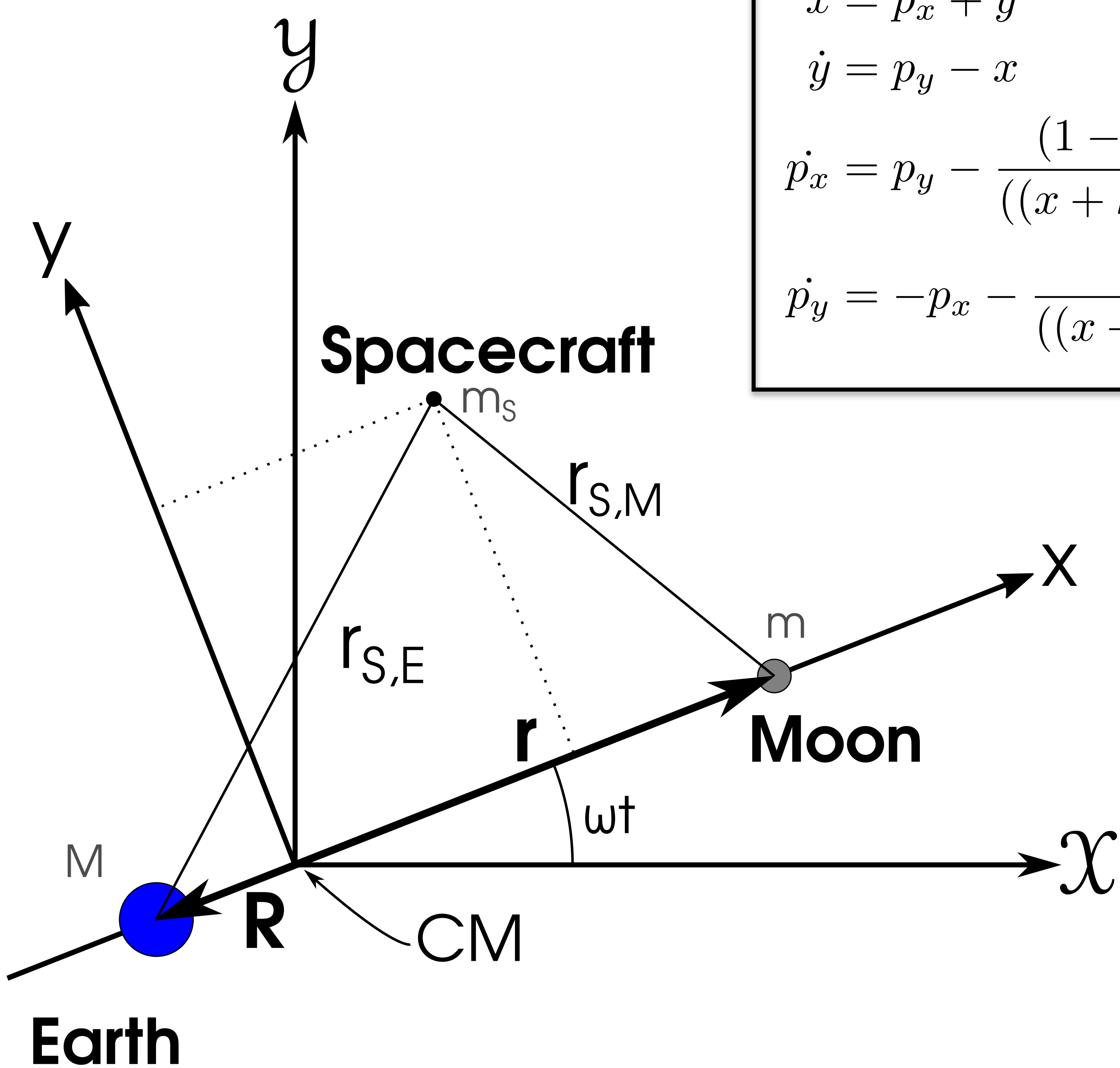


$$H = \frac{p_x^2 + p_y^2}{2m_s} + p_x \omega y - p_y \omega x - G m_s \left(\frac{M}{\sqrt{(x+R)^2 + y^2}} + \frac{m}{\sqrt{(x-r)^2 + y^2}} \right)$$

Assumptions:

- 2D
- 2-body

Restricted 3-body problem



Equations of Motion

$$\dot{x} = p_x + y$$

$$\dot{y} = p_y - x$$

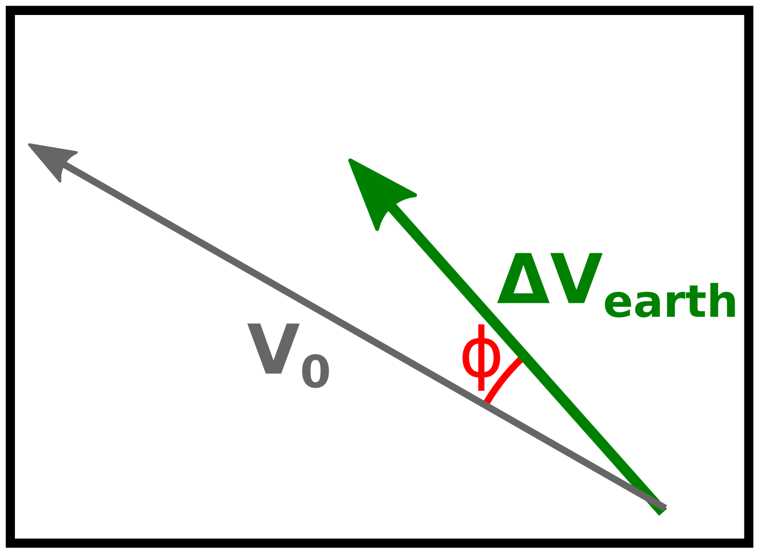
$$\dot{p_x} = p_y - \frac{(1-k)(x+k)}{((x+k)^2+y^2)^{3/2}} - \frac{k(x-1-k)}{((x-1-k)^2+y^2)^{3/2}}$$

$$\dot{p_y} = -p_x - \frac{(1-k)y}{((x+k)^2+y^2)^{3/2}} - \frac{ky}{((x-1-k)^2+y^2)^{3/2}}$$

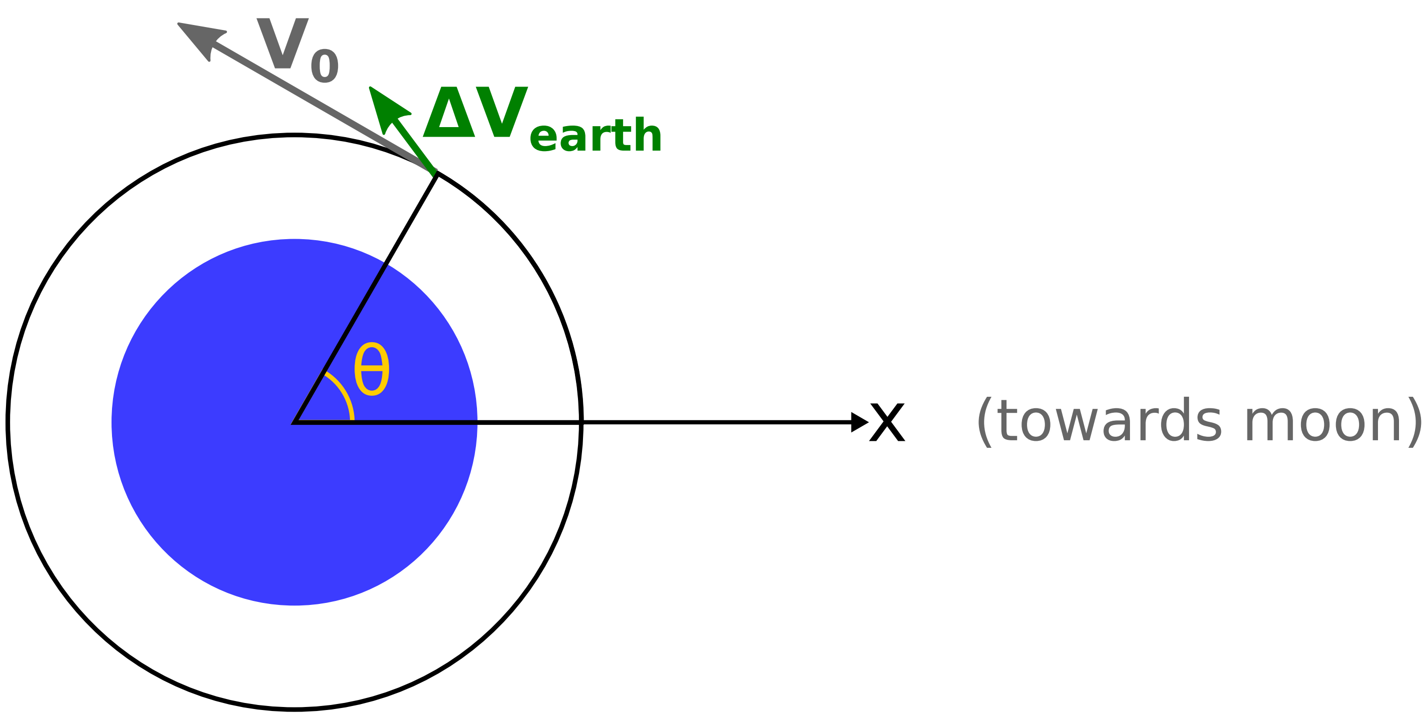
Assumptions:

- 2D
- 2-body

Searching for Transfer Orbits

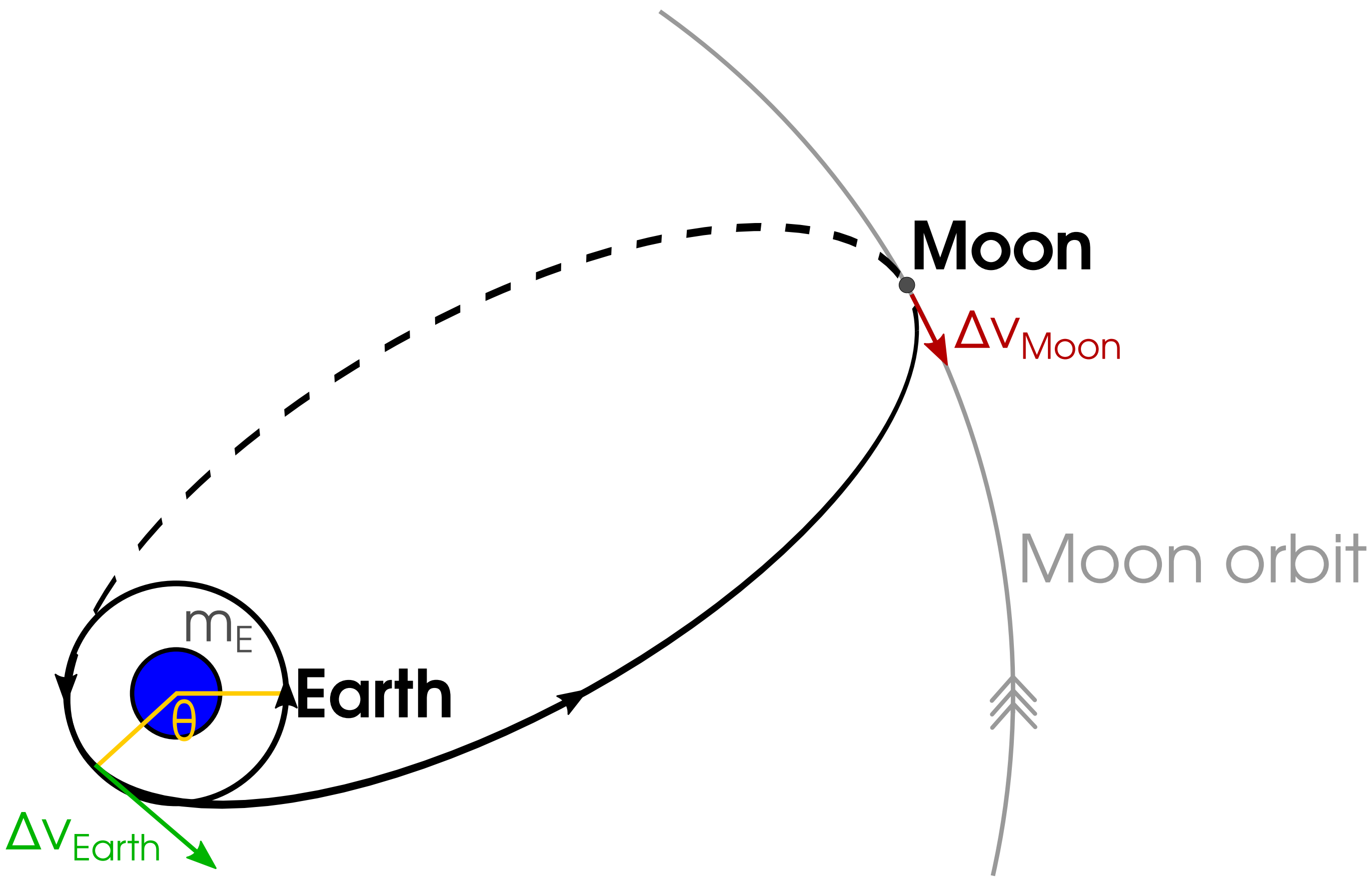


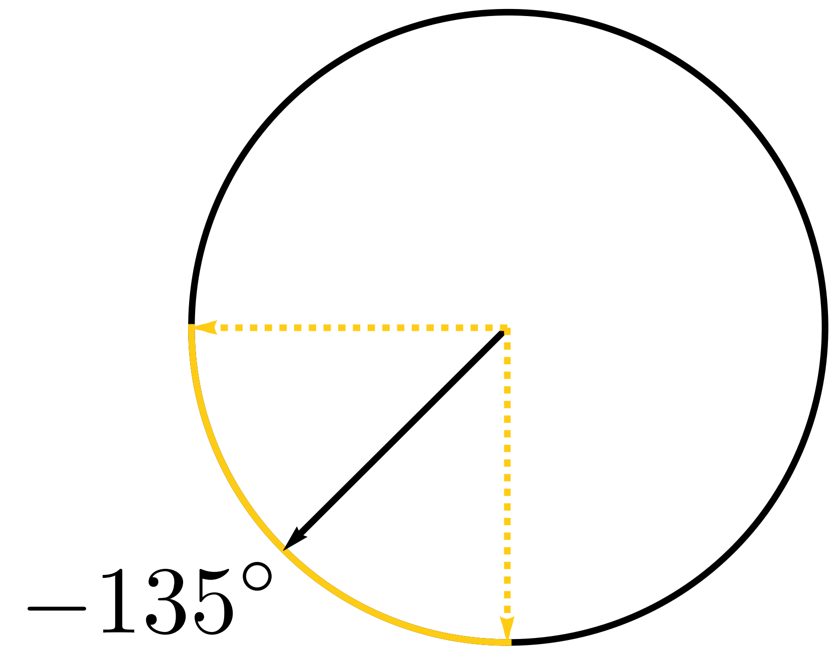
1. θ: Position in orbit
2. Δv_{earth}: Velocity change
3. ϕ: Angle to velocity vector



Searching for Transfer Orbits

Hohmann Transfer Orbit to the Moon





φ: 0° constant

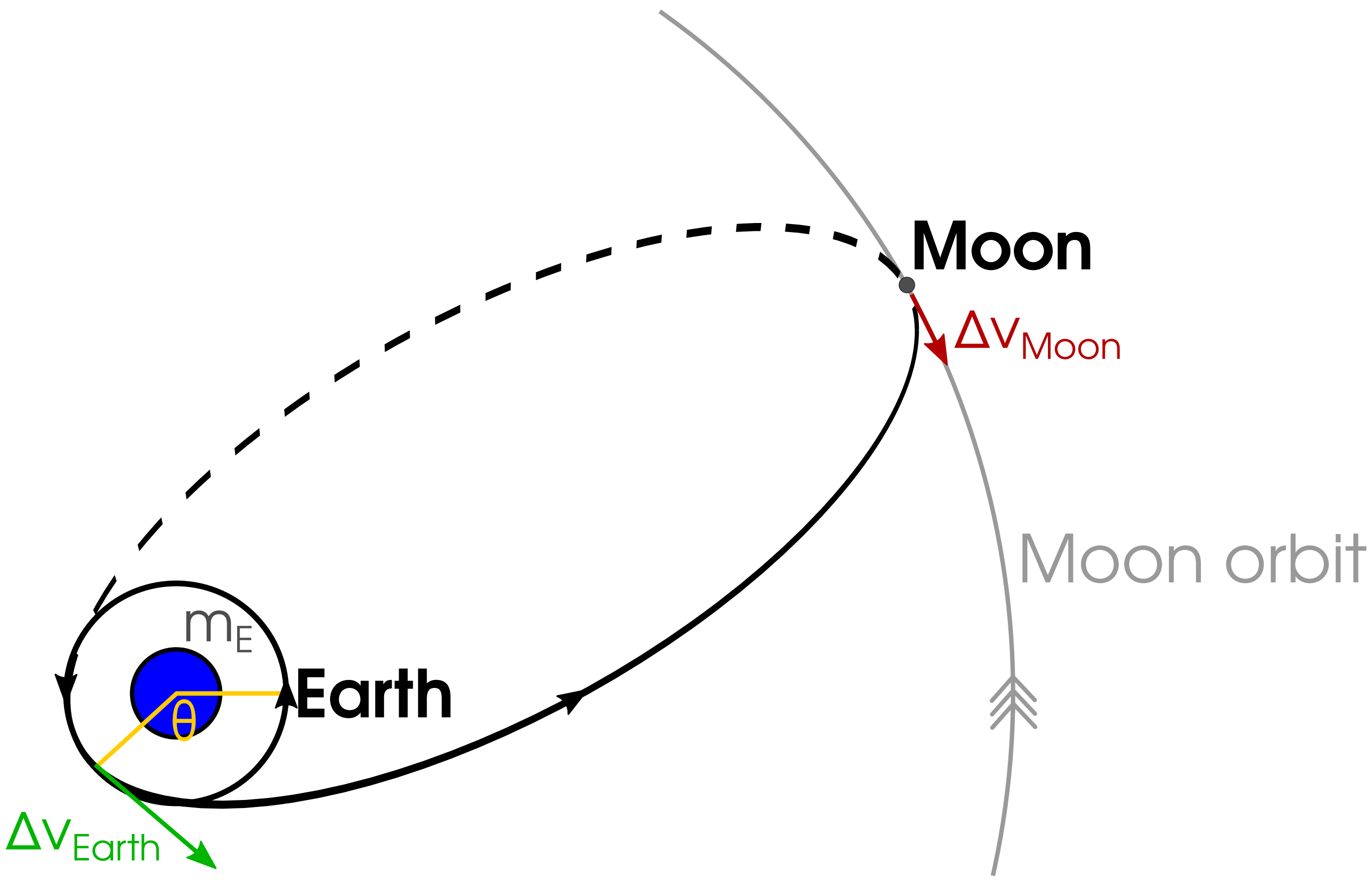
First search

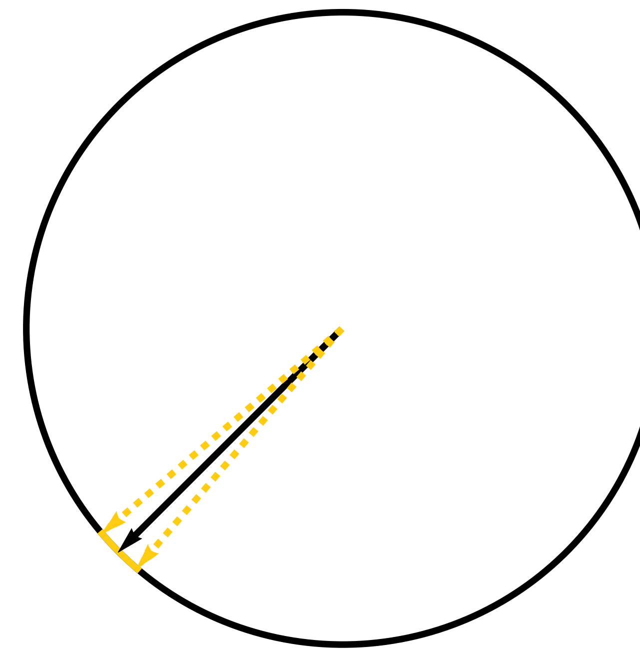
---

100 positions · 200 velocities = 20,000 simulations

Searching for Transfer Orbits

Hohmann Transfer Orbit to the Moon





First search

---

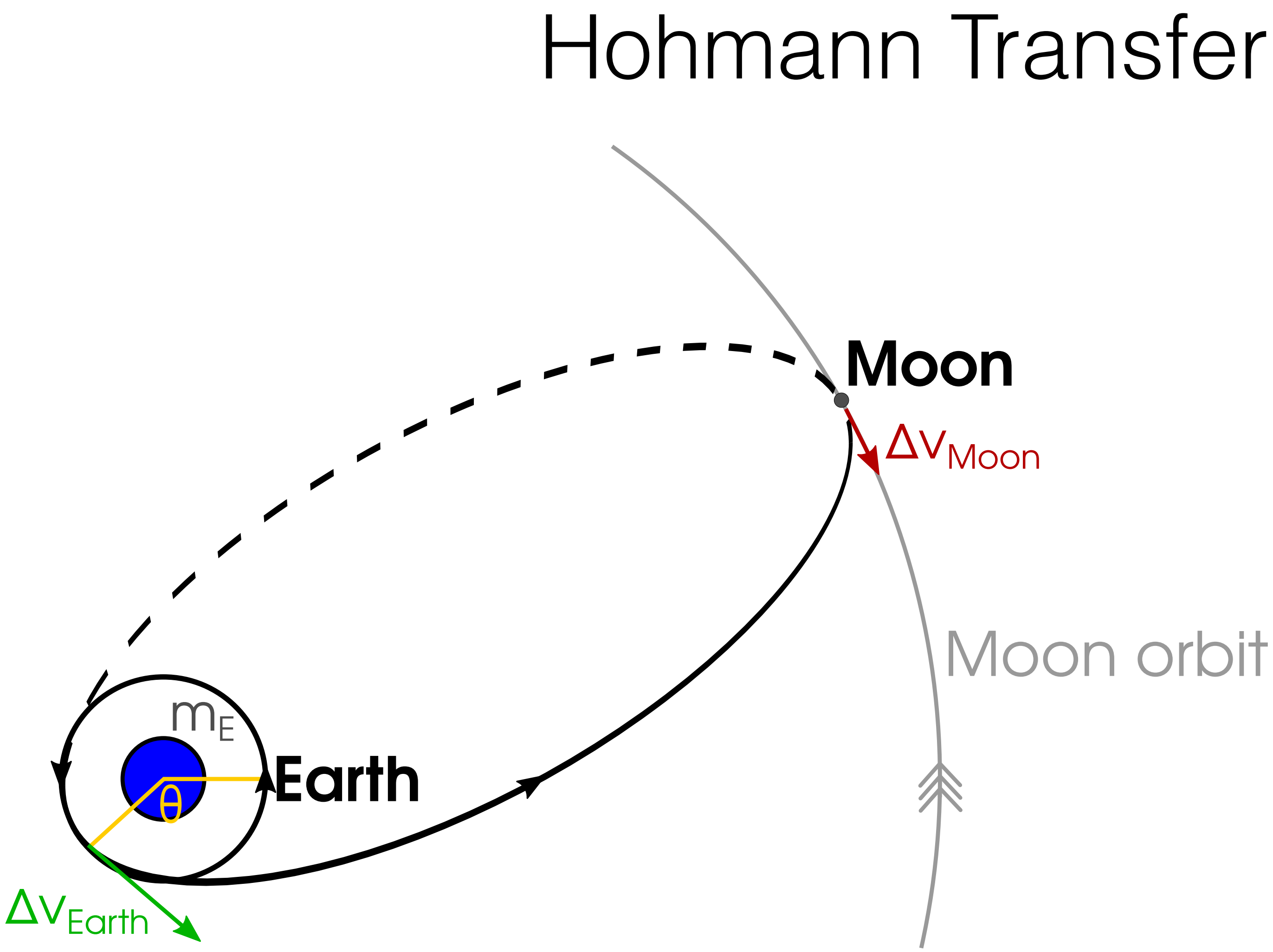
100 positions · 200 velocities = 20,000 simulations

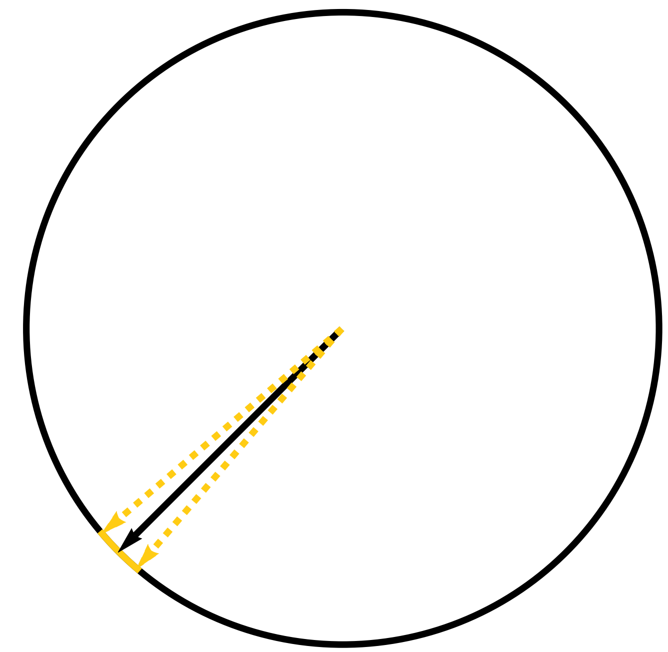
Refinement

---

15 positions · 15 velocities · 15 angles = 3375 simulations

Searching for Transfer Orbits





First search

---

100 positions · 200 velocities = 20,000 simulations

Refinement

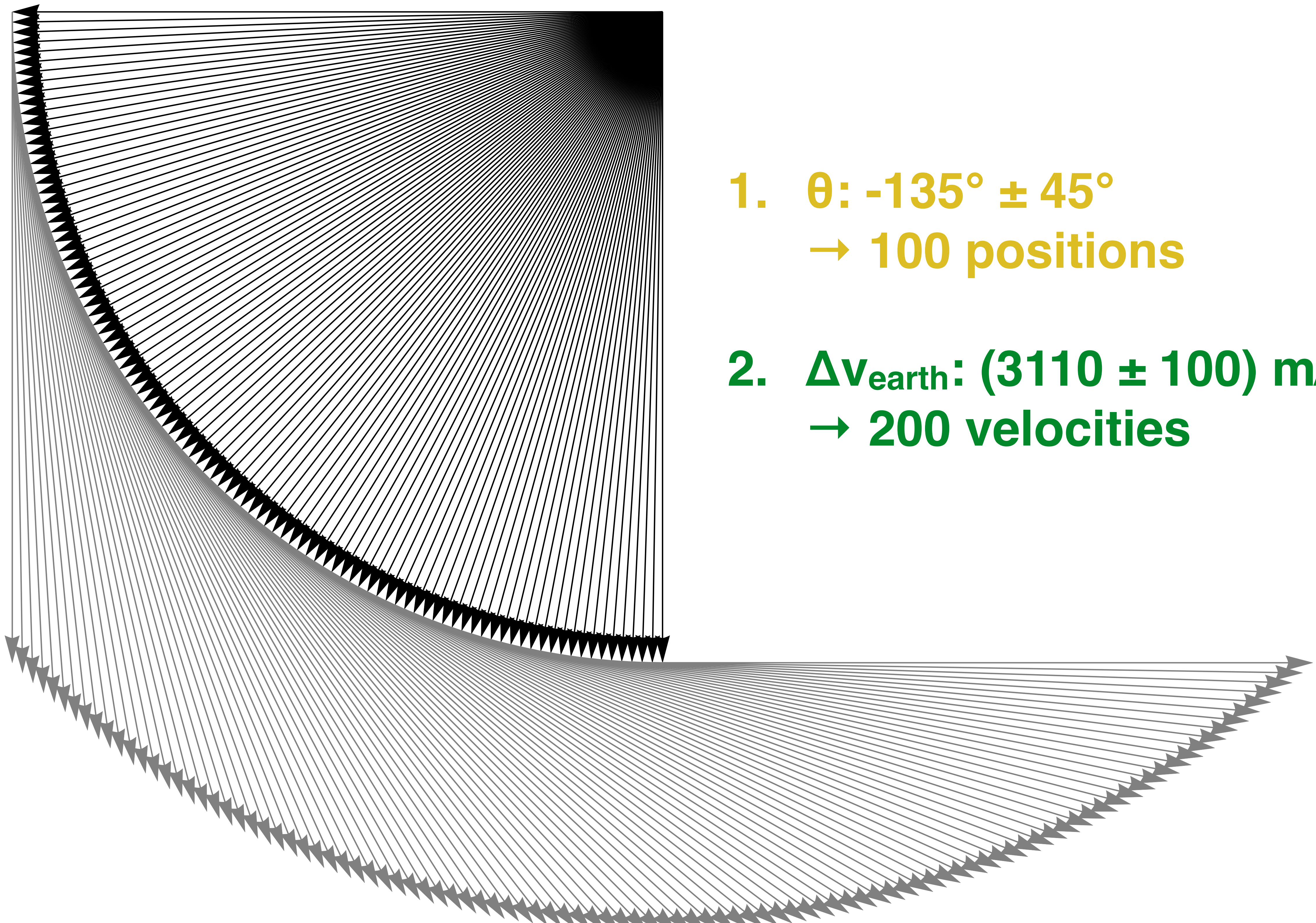
---

15 positions · 15 velocities · 15 angles = 3375 simulations

Total: 20,000 + 3375 = 23,375 simulations

Searching for Transfer Orbits

Hohmann Transfer Orbit to the Moon

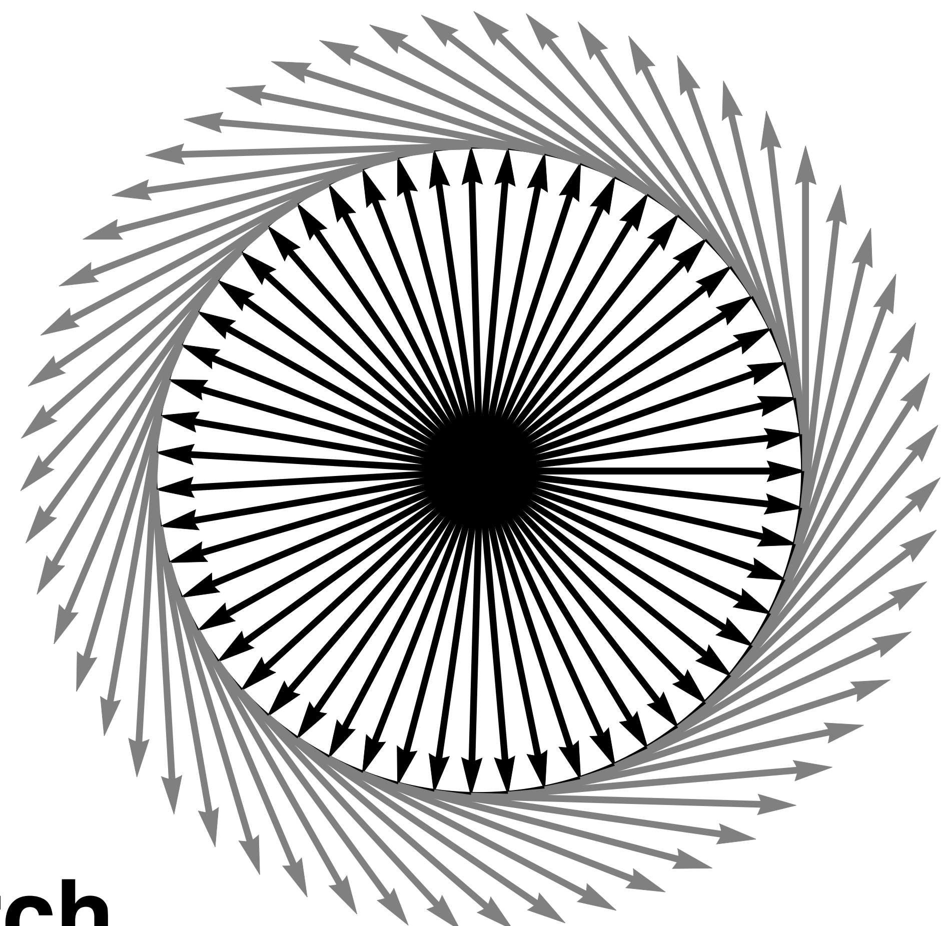


1. θ: -135° ± 45°
   → 100 positions

2. Δv_{earth}: (3110 ± 100) m/s
   → 200 velocities

Searching for Transfer Orbits

Low Energy Transfer Orbit to the Moon



First search

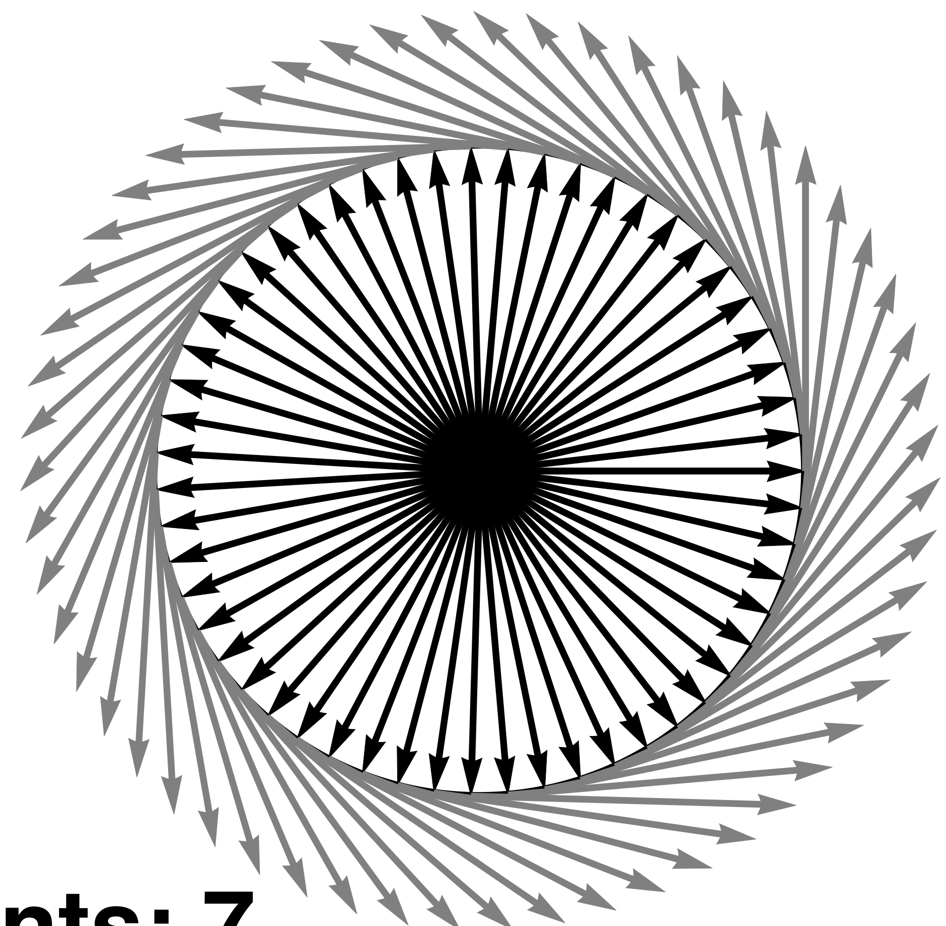
---

1. **θ: 0 ± 180° → 55 positions**
2. **Δv_{earth}: (3120 ± 100) m/s → 55 velocities**
3. **ϕ: 0° ± 1.8° → 55 angles**

SUBTOTAL $55 \cdot 55 \cdot 55 = 166{,}375$

Searching for Transfer Orbits

Low Energy Transfer Orbit to the Moon



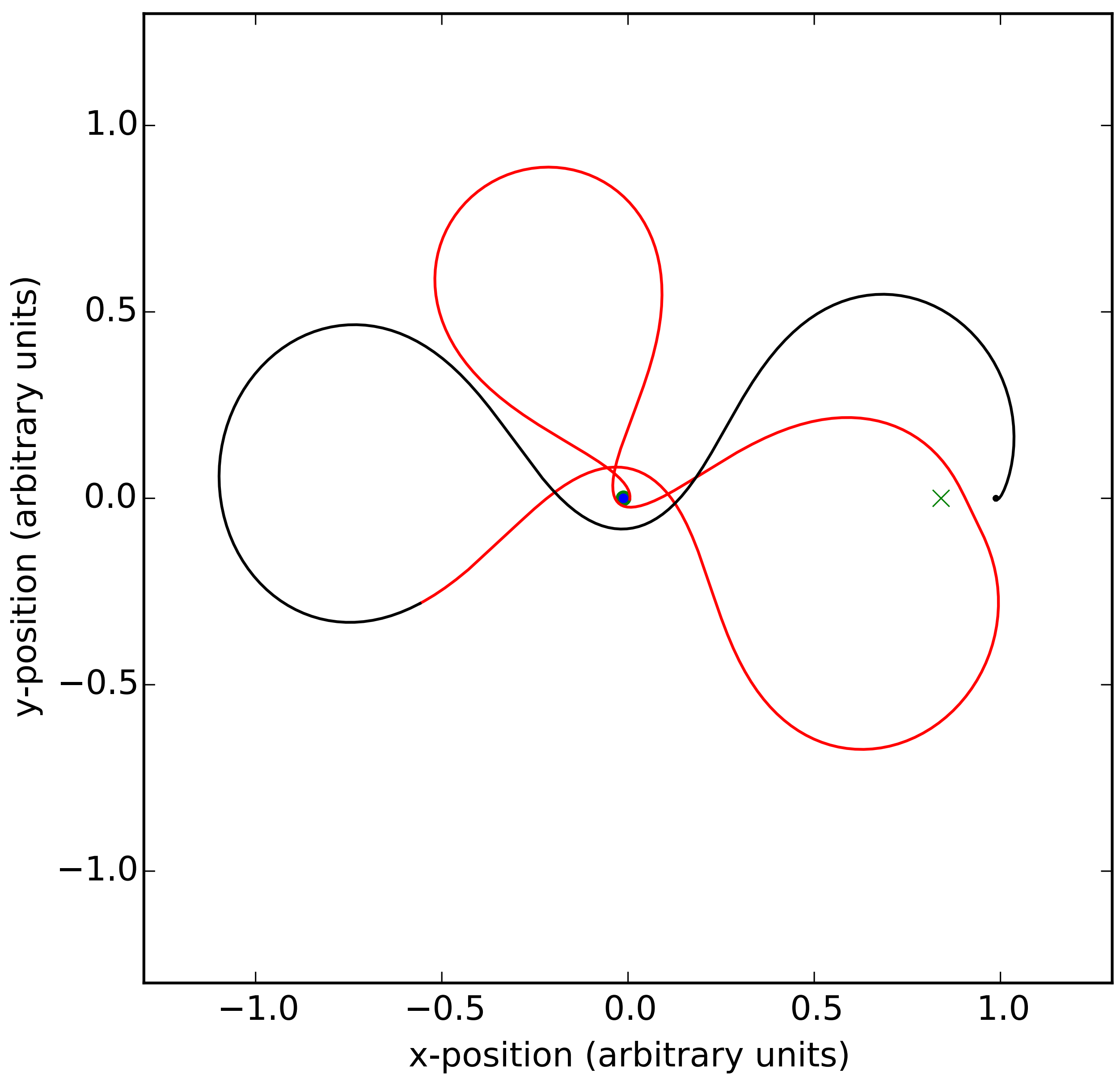
Refinements: 7

1. **θ: best ± 180°/10 → 55 positions**
2. **Δv_{earth}: (best ± 100/10) m/s → 55 velocities**
3. **ϕ: best ± 1.8°/10 → 55 angles**

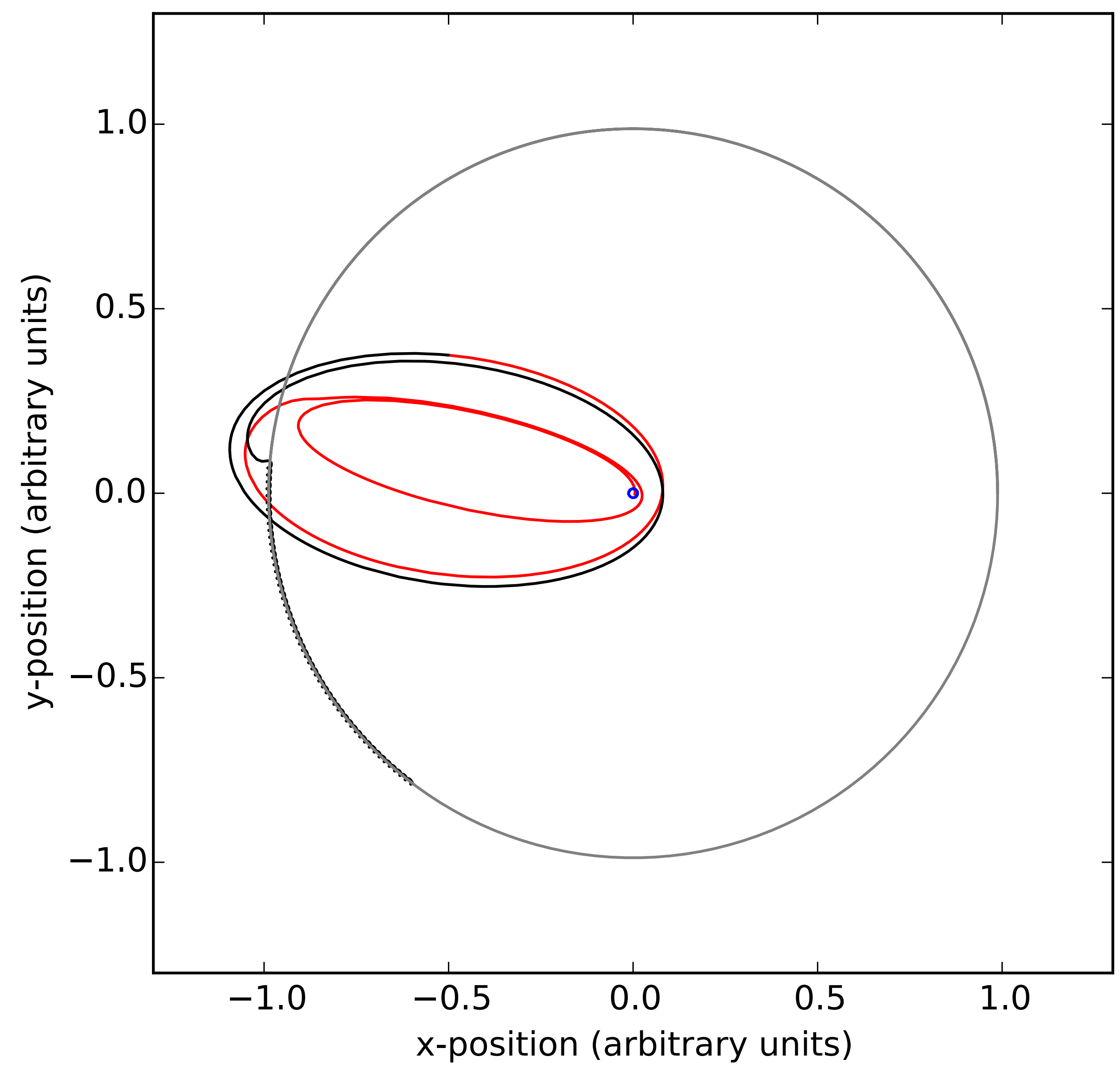
TOTAL $55 \cdot 55 \cdot 55 \cdot 8$ = **1,331,000**

Numerical Method

Adaptive Störmer-Verlet (symplectic)



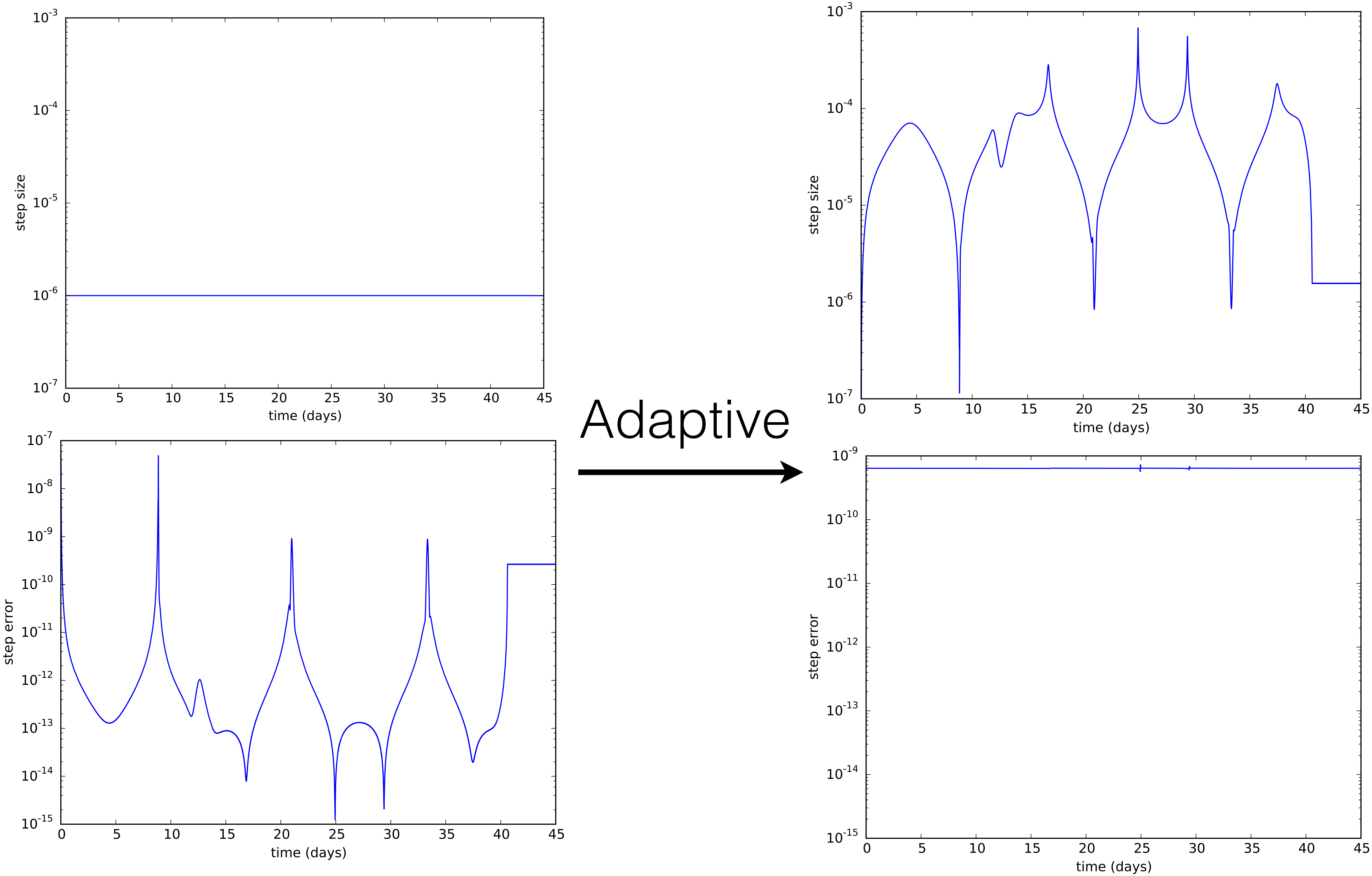
(x, y)



$(\mathcal{X}, \mathcal{Y})$

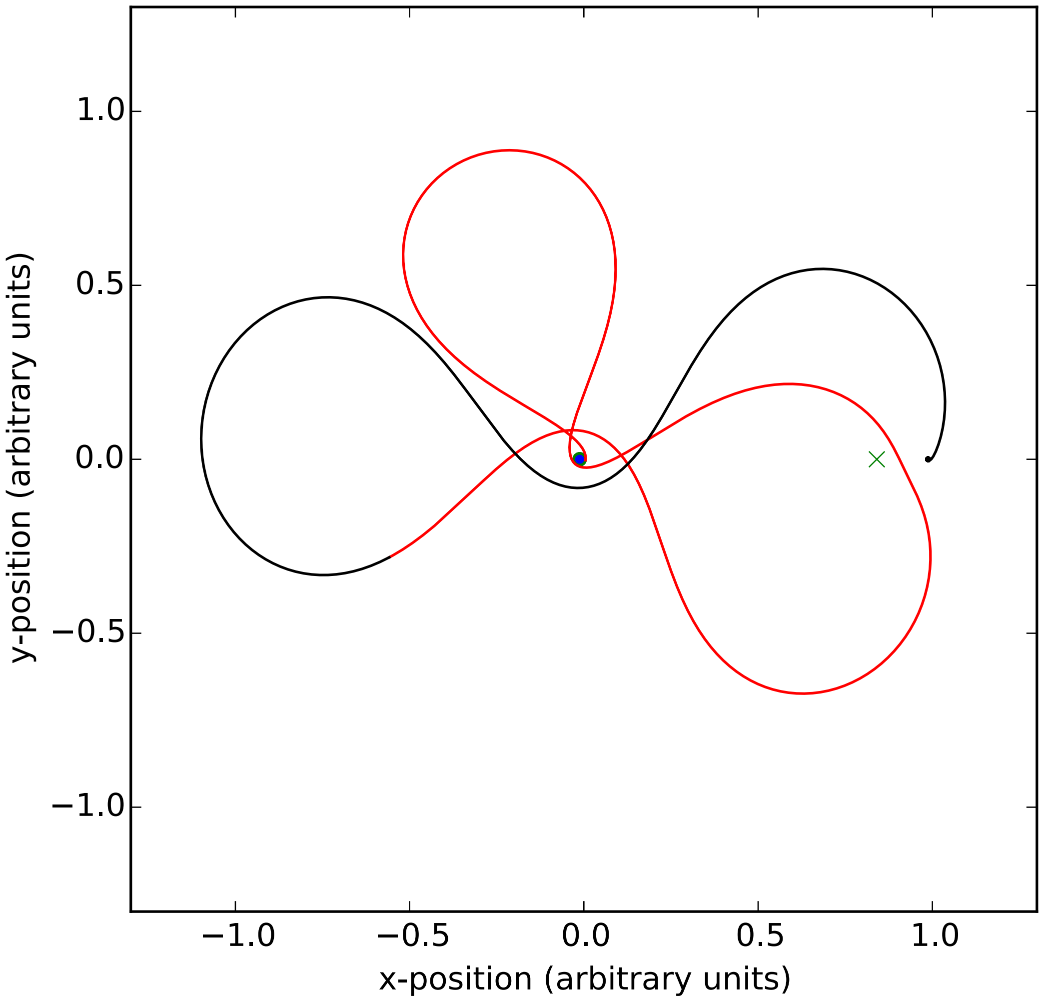
Numerical Method

Adaptive Störmer-Verlet (symplectic)

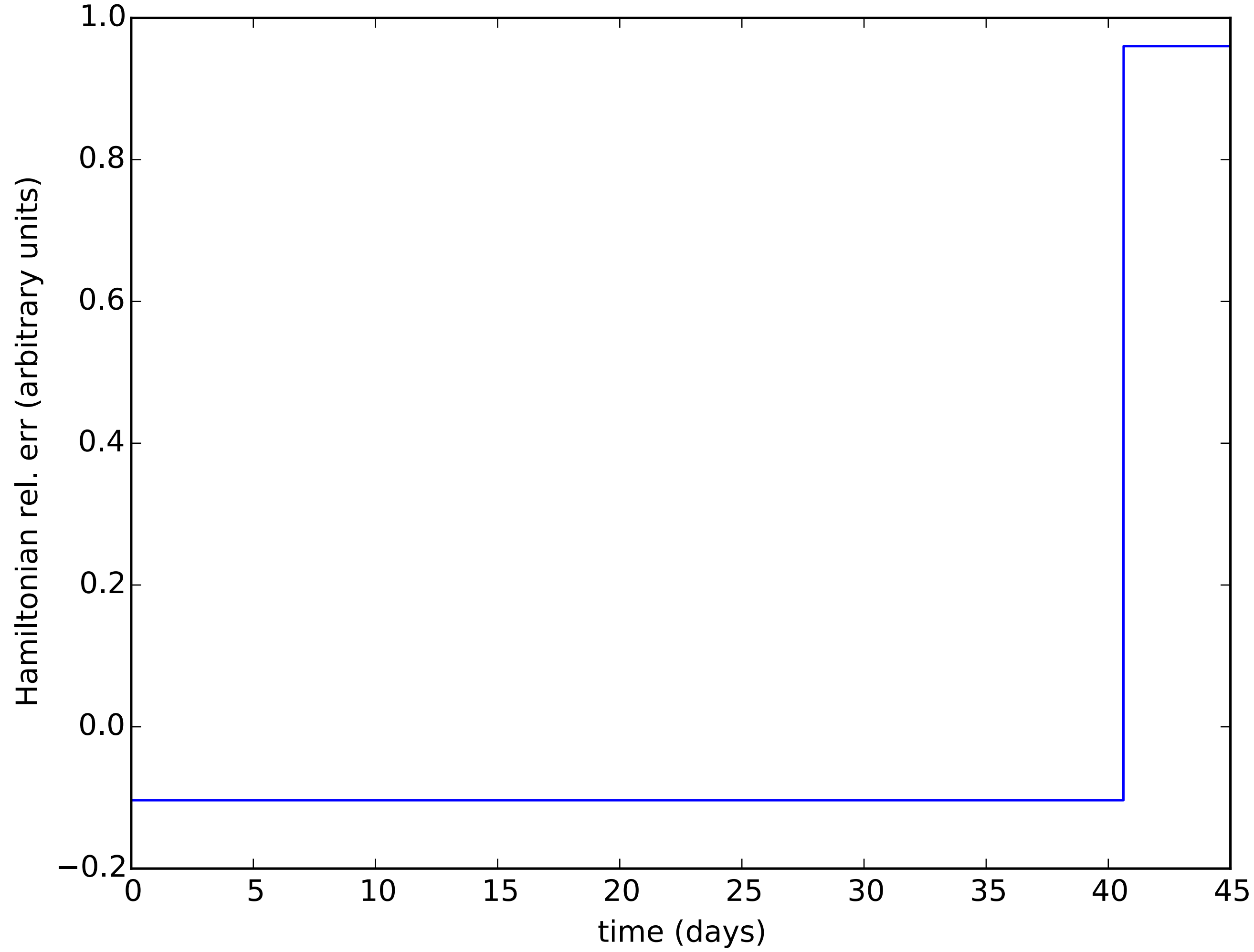


Numerical Method

Adaptive Störmer-Verlet (symplectic)



(x, y)

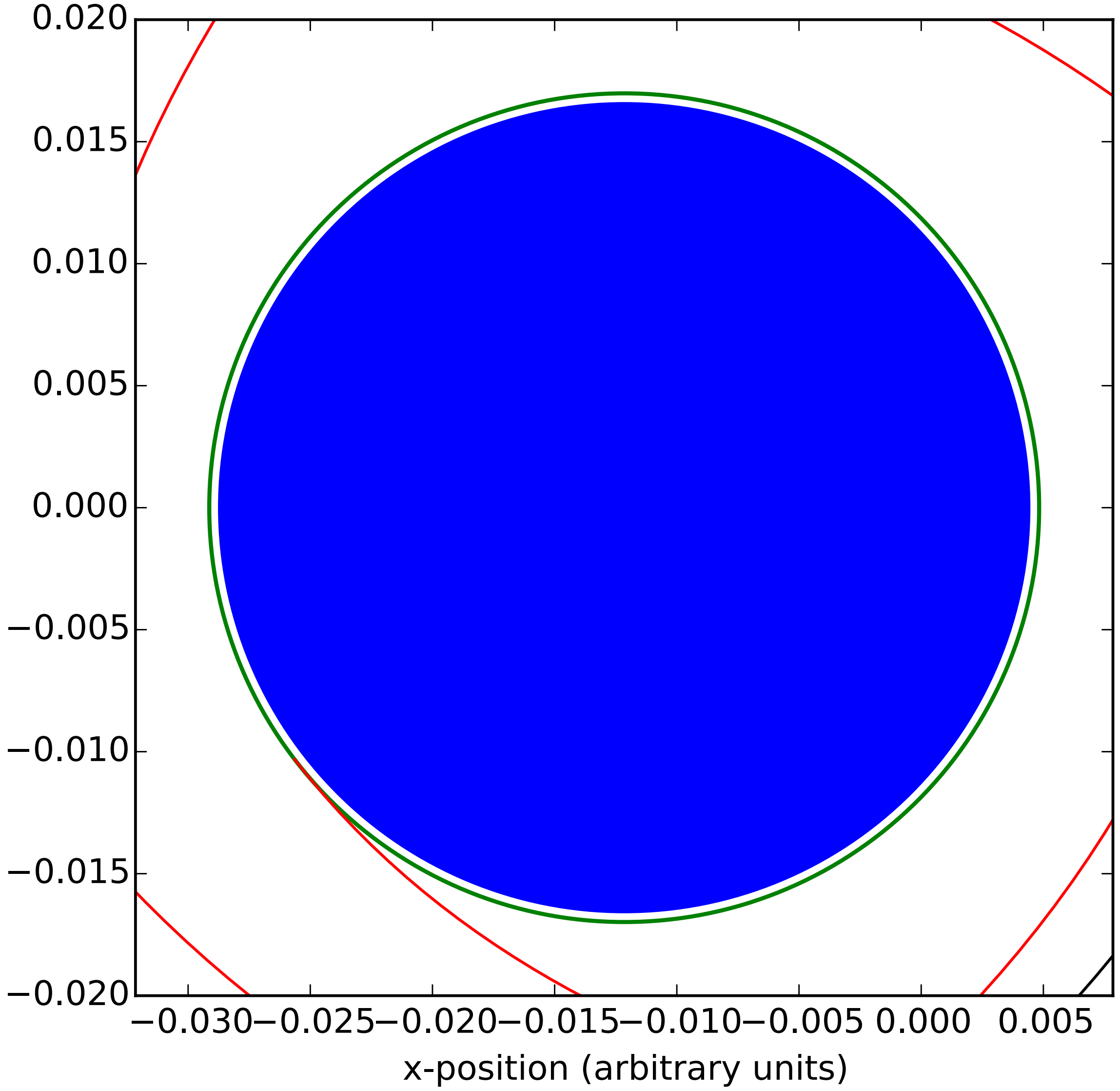


$\mathrm{Hamiltonian}$

Searching for Transfer Orbits

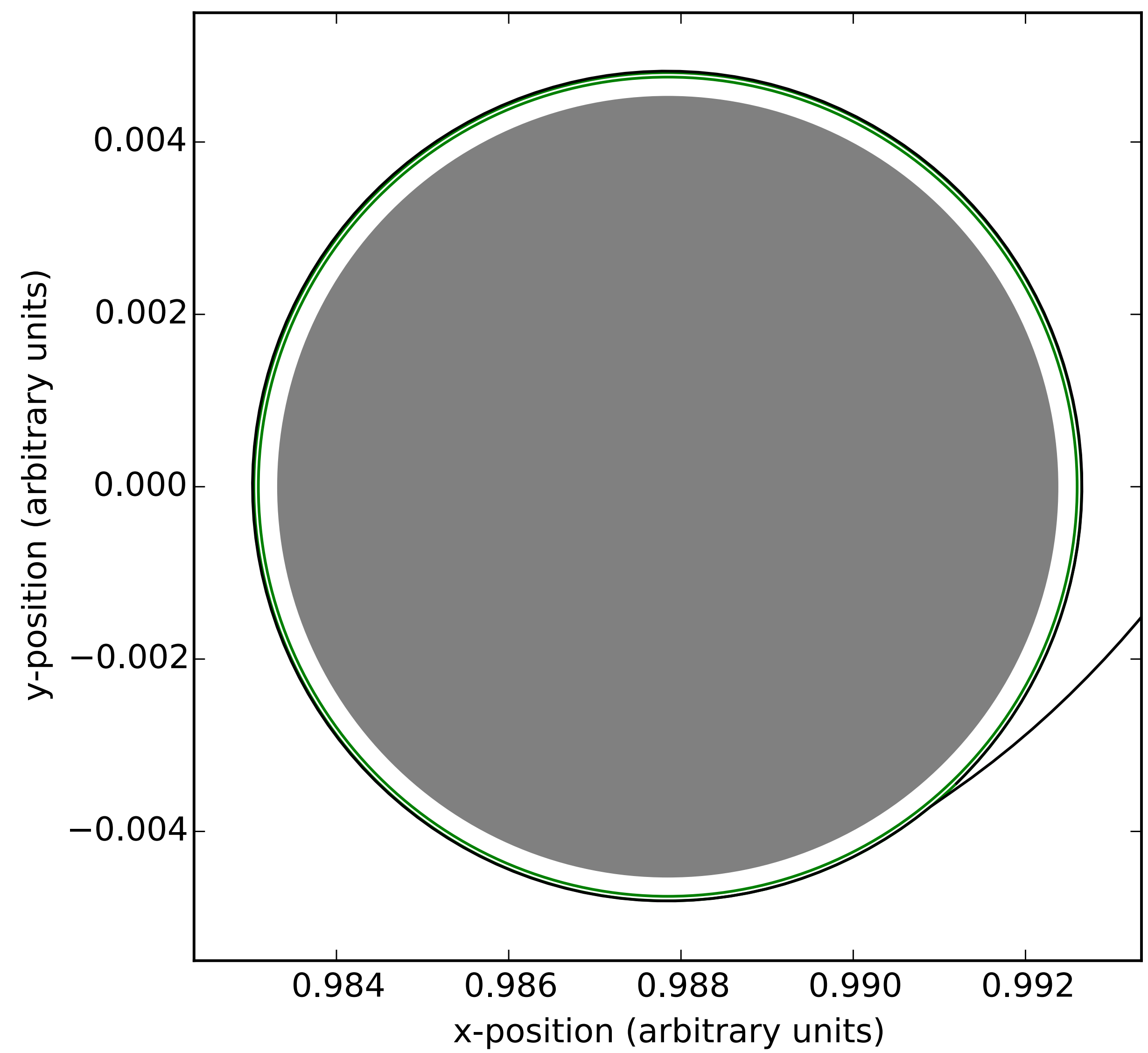
Entering Moon Orbit

Δv_{earth}



Exit from Earth orbit

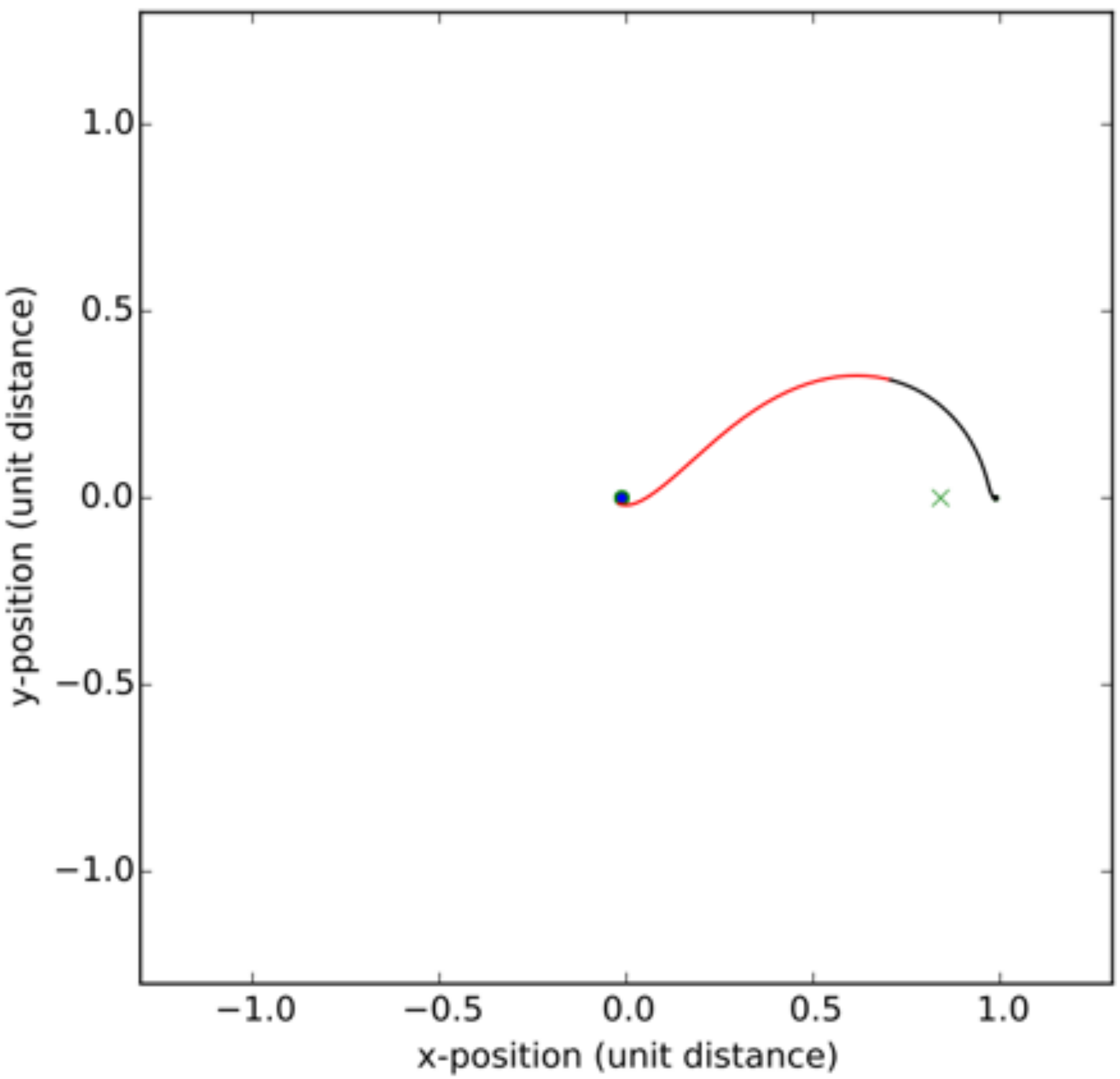
Δv_{moon}



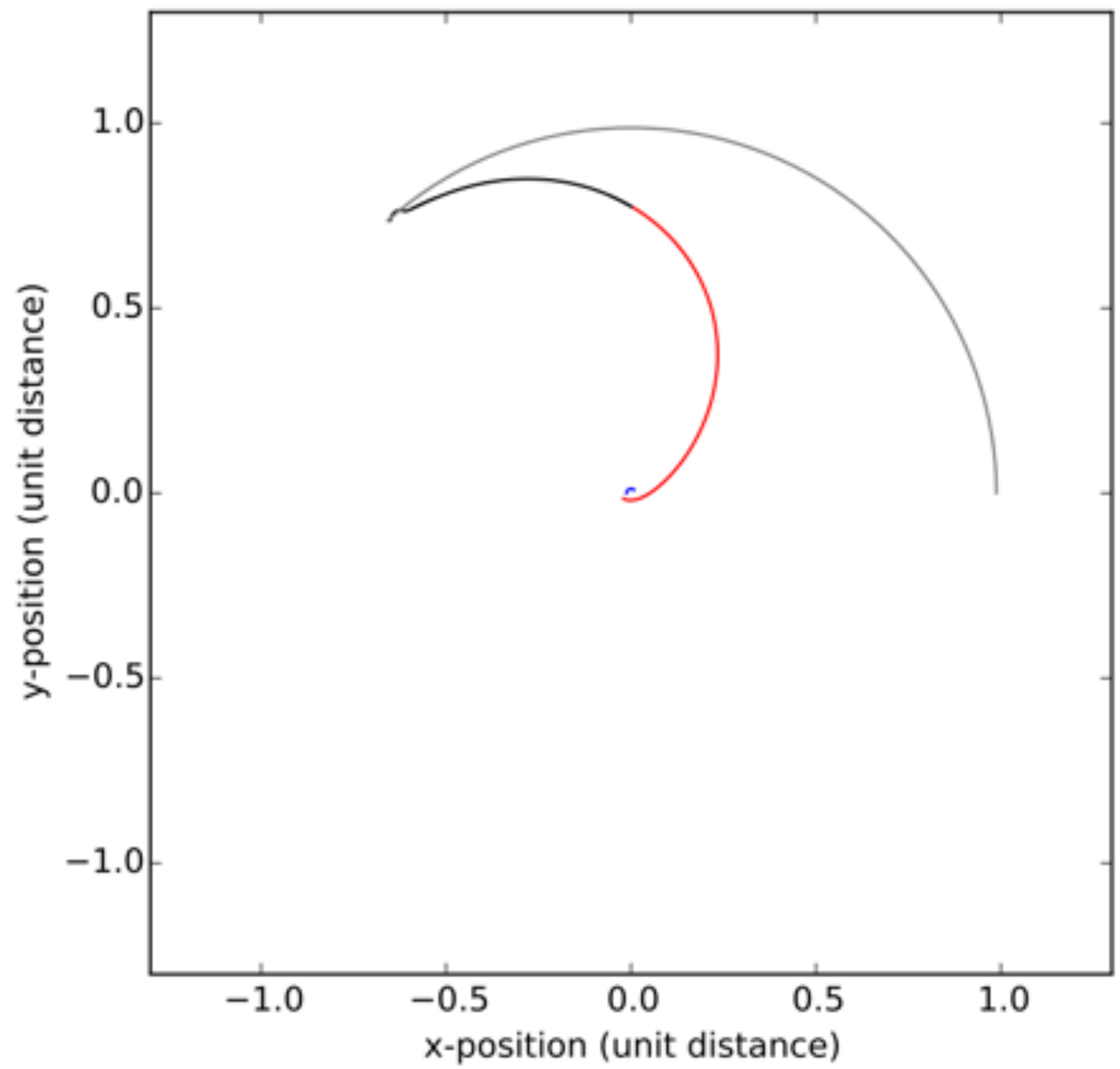
Entry to Moon orbit (100±10 km)

Searching for Transfer Orbits

Hohmann Transfer Orbit to the Moon



(x, y)



$(\mathcal{X}, \mathcal{Y})$

Δv_{total} = 3912 m/s

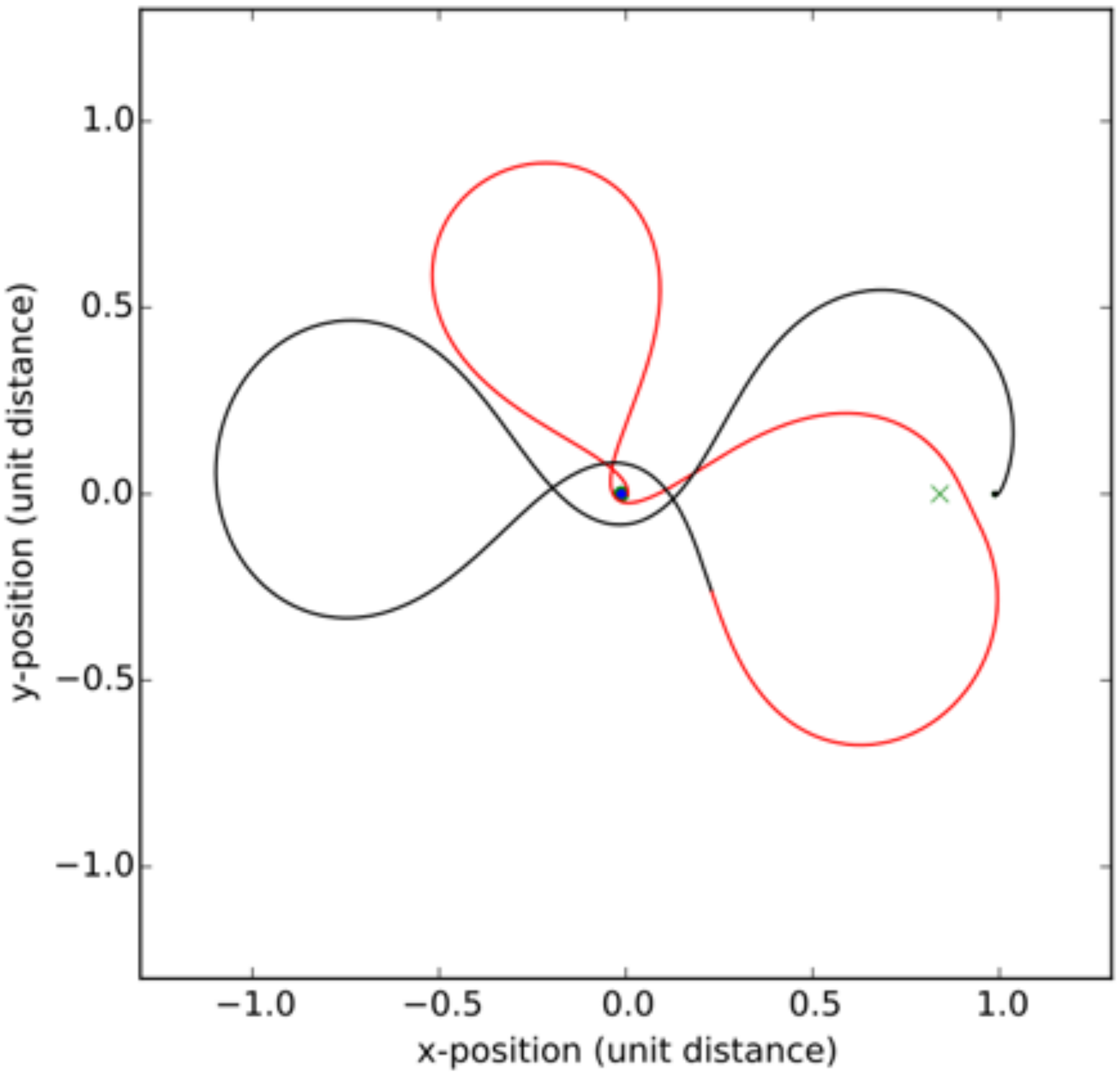
t_H = 4.3 days

From simple model:

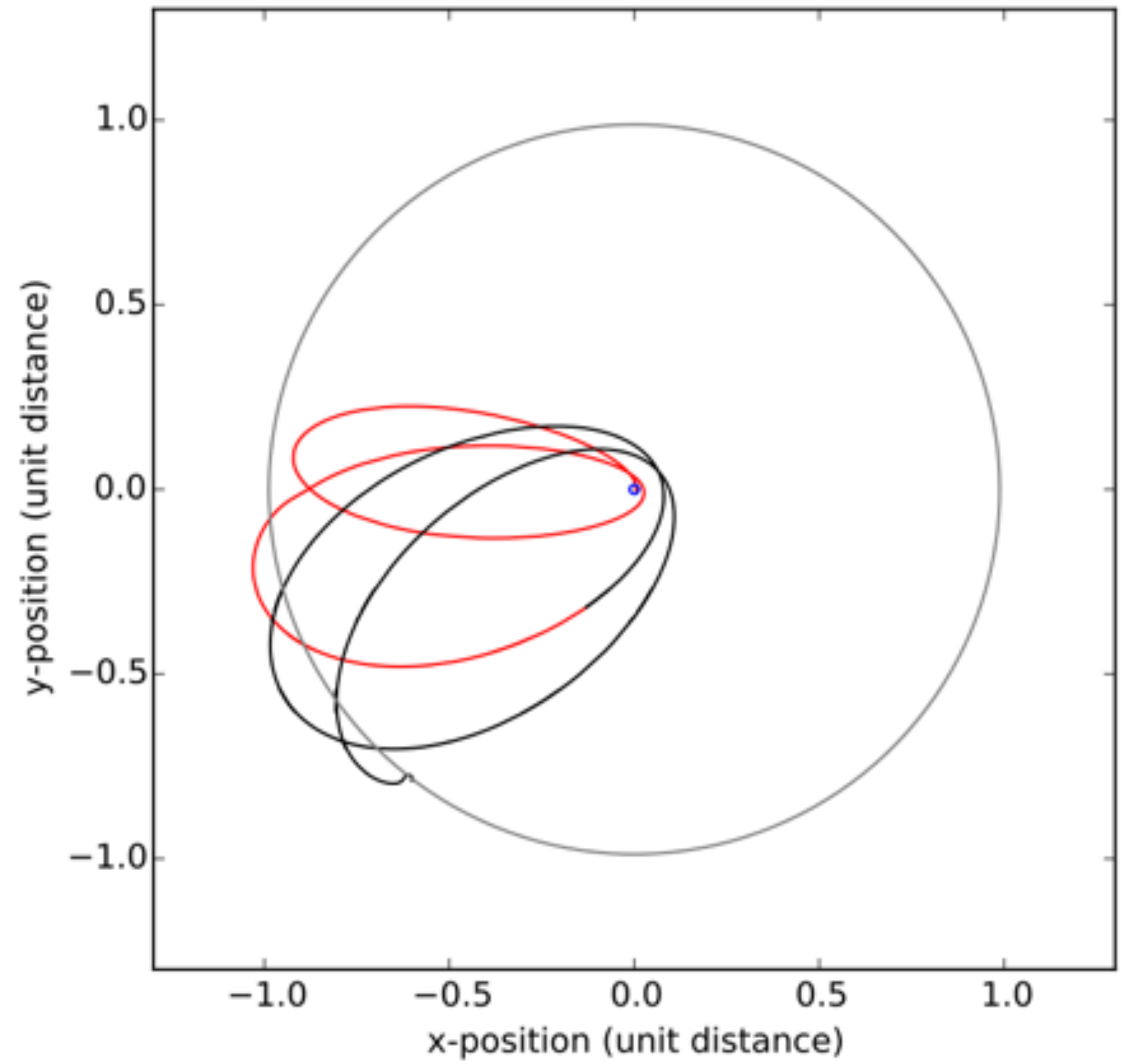
Δv_{Total} = 3946 m/s

Searching for Transfer Orbits

Low-Energy Transfer Orbit (short)



(x, y)



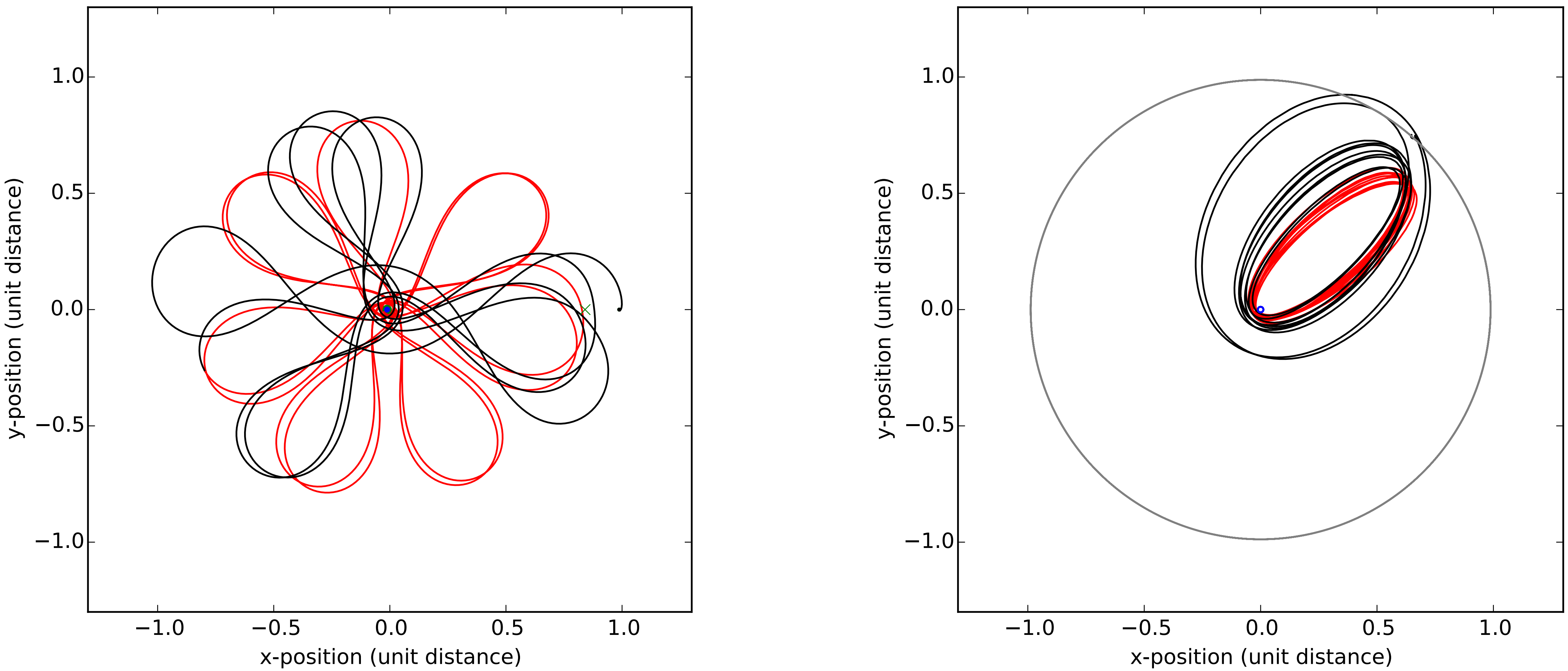
$(\mathcal{X}, \mathcal{Y})$

Δv_{total} = 3896 m/s

t_{short} = 41 days

Searching for Transfer Orbits

Low-Energy Transfer Orbit (long)



(x, y) $(\mathcal{X}, \mathcal{Y})$

Δv_{total} = 3795 m/s

t_{long} = 194 days

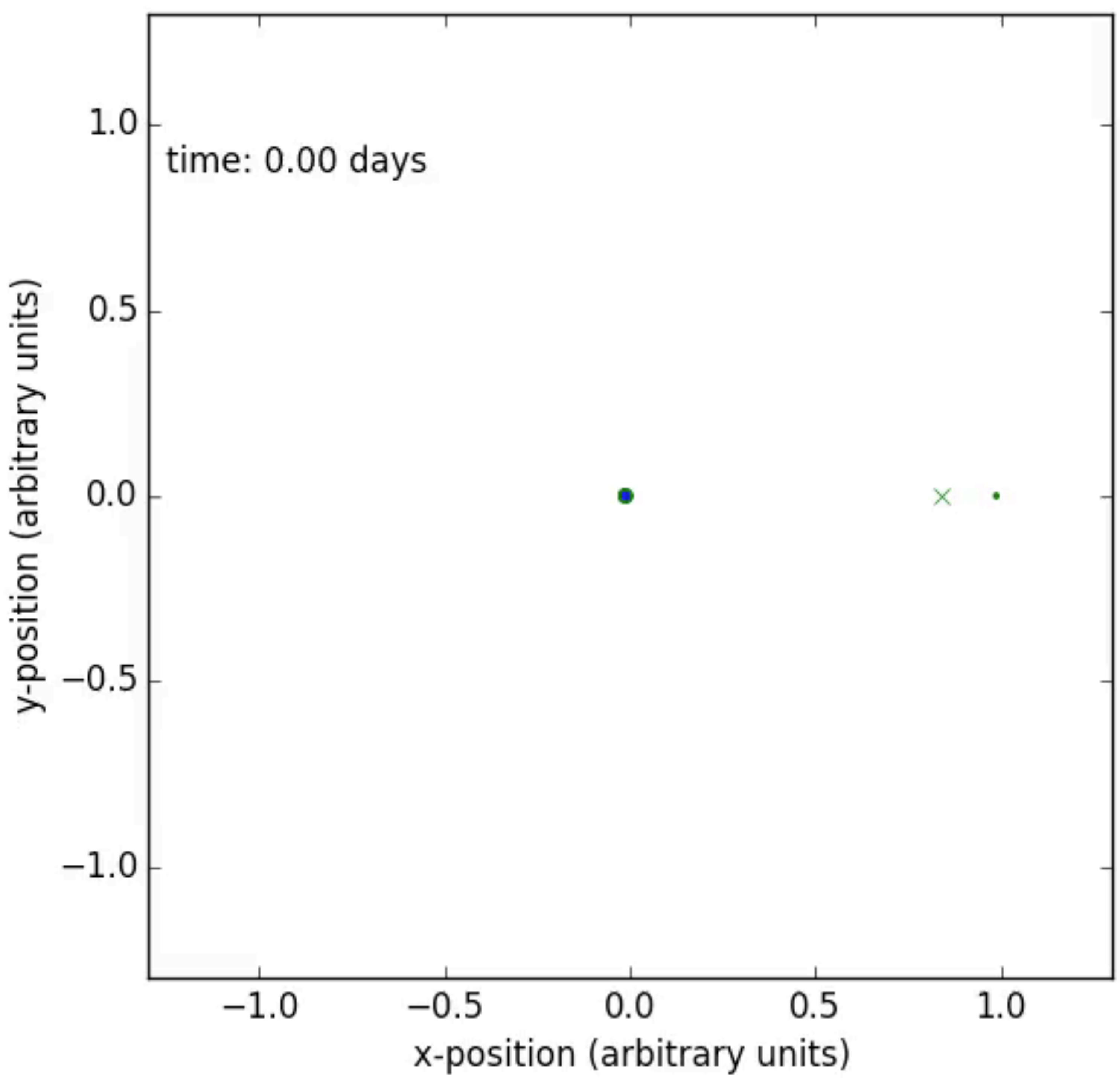
Results Summarized

All transfer orbits

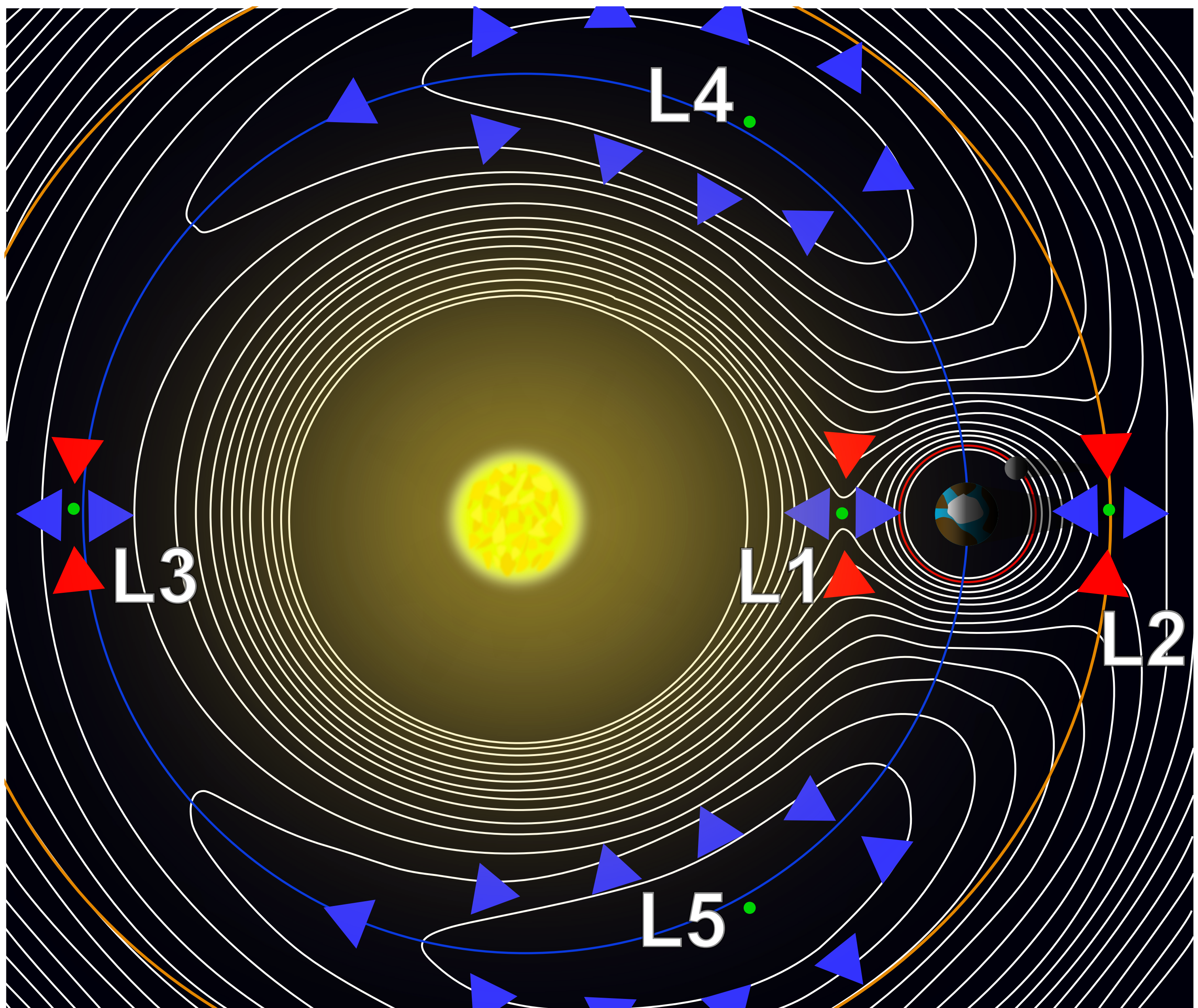
Trajectory	Flight time	Δv_{total} (km/s)	Δv_{earth} (km/s)	Δv_{moon} (km/s)
Minimum	N/A	3.721	3.099	0.622
Long LETO	194 days	3.795	3.091	0.704
Belbruno-Miller	3 months	3.838	3.187	0.651
Topputo	8 months	3.895	3.265	0.630
Short LETO	41 days	3.896	3.127	0.769
Hohmann - long (sim)	4.3 days	3.912	3.111	0.801
Hohmann - (model)	5.0 days	3.946	3.144	0.802
Hohmann - medium (sim)	3.00 days	4.015	3.136	0.880
Apollo (Hohman)	3.05 days	4.115	3.048	1.067
Hohmann - short (sim)	1.00 days	6.823	3.809	3.014

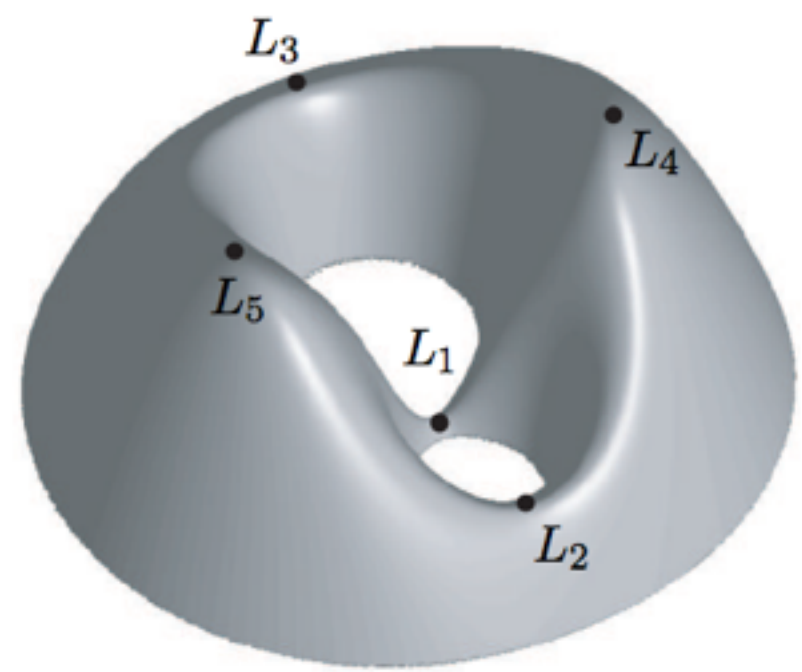
LETO Short Animation

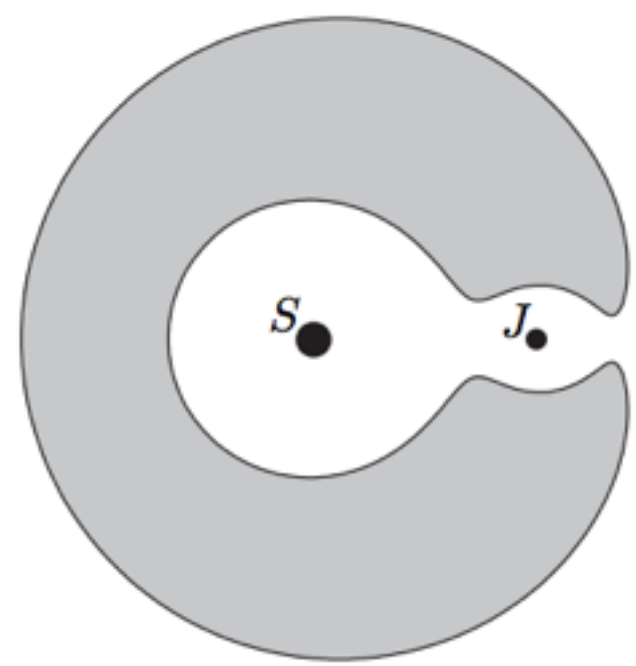
0.043484 days/frame, 60 FPS



Gravitational Potential and Lagrange Points







Wishlist

- Earth$\rightarrow L_1$ (forward) + $L_1\rightarrow$Moon (backward)
- 2D $\rightarrow$ 3D
- Include Sun's potential
- Higher-order integrator:
  4.-5.-order symplectic Runge-Kutta