# Euler Method for reduced 3-body problem

$$\Delta x = \frac{\partial H}{\partial p_x} \cdot \Delta t = \left( p_x + y \right) \Delta t$$

$$\Delta y = \frac{\partial H}{\partial p_y} \cdot \Delta t = \left( p_y - x \right) \Delta t$$

$$\Delta p_x = -\frac{\partial H}{\partial x} \cdot \Delta t = \left( p_y - \frac{(1-\mu)(\mu + x)}{((\mu+x)^2 + y^2)^{3/2}} + \frac{(\mu)(1-\mu-x)}{((1-\mu-x)^2 + y^2)^{3/2}} \right) \cdot \Delta t$$

$$\Delta p_y = -\frac{\partial H}{\partial y} \cdot \Delta t = \left( -p_x - \frac{(1-\mu)y}{((\mu+x)^2+y^2)^{3/2}} - \frac{(\mu)y}{((1-\mu-x)^2+y^2)^{3/2}} \right) \cdot \Delta t$$

## Explicit

$$x_{i+1} = \left( p_{x,i} + y_i \right) \cdot \Delta t + x_i$$

$$y_{i+1} = \left( p_{y,i} - x_i \right) \cdot \Delta t + y_i$$

$$p_{x,i+1} = \left( p_{y,i} - \frac{(1-\mu)(\mu + x_i)}{((\mu+x_i)^2 + y_i^2)^{3/2}} + \frac{(\mu)(1-\mu-x_i)}{((1-\mu-x_i)^2 + y_i^2)^{3/2}} \right) \cdot \Delta t + p_{x,i}$$

$$p_{y,i+1} = \left( -p_{x,i} - \frac{(1-\mu)y_i}{((\mu+x_i)^2+y_i^2)^{3/2}} - \frac{(\mu)y_i}{((1-\mu-x_i)^2+y_i^2)^{3/2}} \right) \cdot \Delta t + p_{y,i}$$

## impicit

$$x_{i+1} = \left( p_{x,i+1} + y_{i+1} \right) \cdot \Delta t + x_i$$

$$y_{i+1} = \left( p_{y,i+1} - x_{i+1} \right) \Delta t + y_i$$

$$p_{x,i+1} = \left( p_{y,i+1} - \frac{(1-\mu)(\mu + x_{i+1})}{\left((\mu + x_{i+1})^2 + y_{i+1}^2\right)^{3/2}} + \frac{(\mu)(1-\mu-x_{i+1})}{\left((1-\mu-x_{i+1})^2 + y_{i+1}^2\right)^{3/2}} \right) \cdot \Delta t + p_{x,i}$$

$$p_{y,i+1} = \left( -p_{x,i+1} - \frac{(1-\mu)y_{i+1}}{\left((\mu + x_{i+1})^2 + y_{i+1}^2\right)^{3/2}} - \frac{(\mu)\, y_{i+1}}{\left((1-\mu-x_{i+1})^2 + y_{i+1}^2\right)^{3/2}} \right) \cdot \Delta t + p_{y,i}$$

# Symplectic

$$x_{i+1} = \left( \frac{p_{x,i+1}}{\cancel{\phantom{mm}}} + y_{i+1} \right) \Delta t + x_i$$

$$y_{i+1} = \left( \frac{p_{y,i+1}}{\cancel{\phantom{mm}}} - x_{i+1} \right) \Delta t + y_i$$

løs for $x_{i+1}, y_{i+1}$

$$p_{x,i+1} = \left( p_{y,i+1} - \frac{(1-\mu)(\mu + x_i)}{\left((\mu + x_i)^2 + y_i^2\right)^{3/2}} + \frac{\mu(1-\mu-x_i)}{\left((1-\mu-x_i)^2 + y_i^2\right)^{3/2}} \right) \Delta t + p_{x,i}$$

$$p_{y,i+1} = \left( -p_{x,i+1} - \frac{(1-\mu)\, y_i}{\left((\mu + x_i)^2 + y_i^2\right)^{3/2}} - \frac{\mu \cdot y_i}{\left((1-\mu-x_i)^2 + y_i^2\right)^{3/2}} \right) \Delta t + p_{y,i}$$

$$x_1 = \left( \frac{p_{x1}}{2} + y_1 \right) \cdot \Delta t + x_0 \quad (1)$$

$$y_1 = \left( \frac{p_{y1}}{2} + y_1 \right) \Delta t + y_0 \quad (2)$$

$(2) \rightsquigarrow (1)$ : $x_1 = \dfrac{p_{x_1}}{2}$