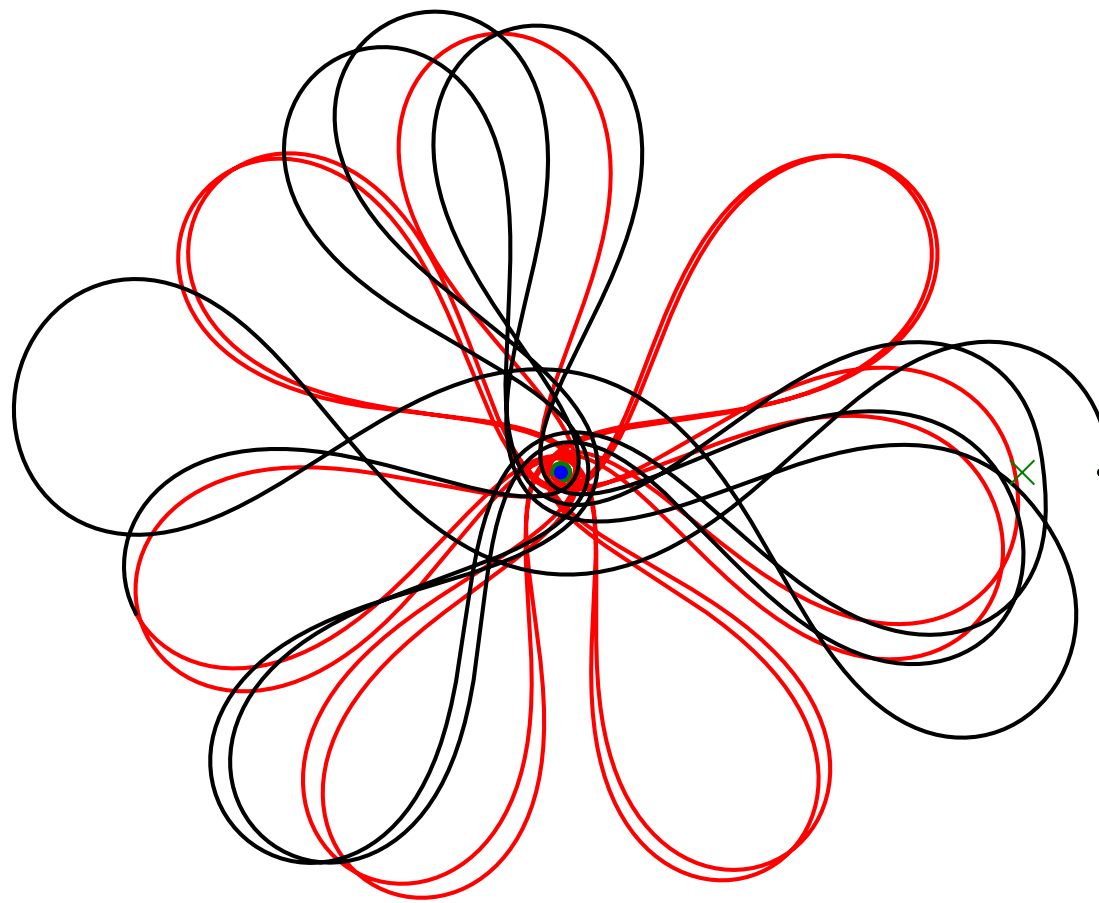


Low-Energy Transfer Orbits



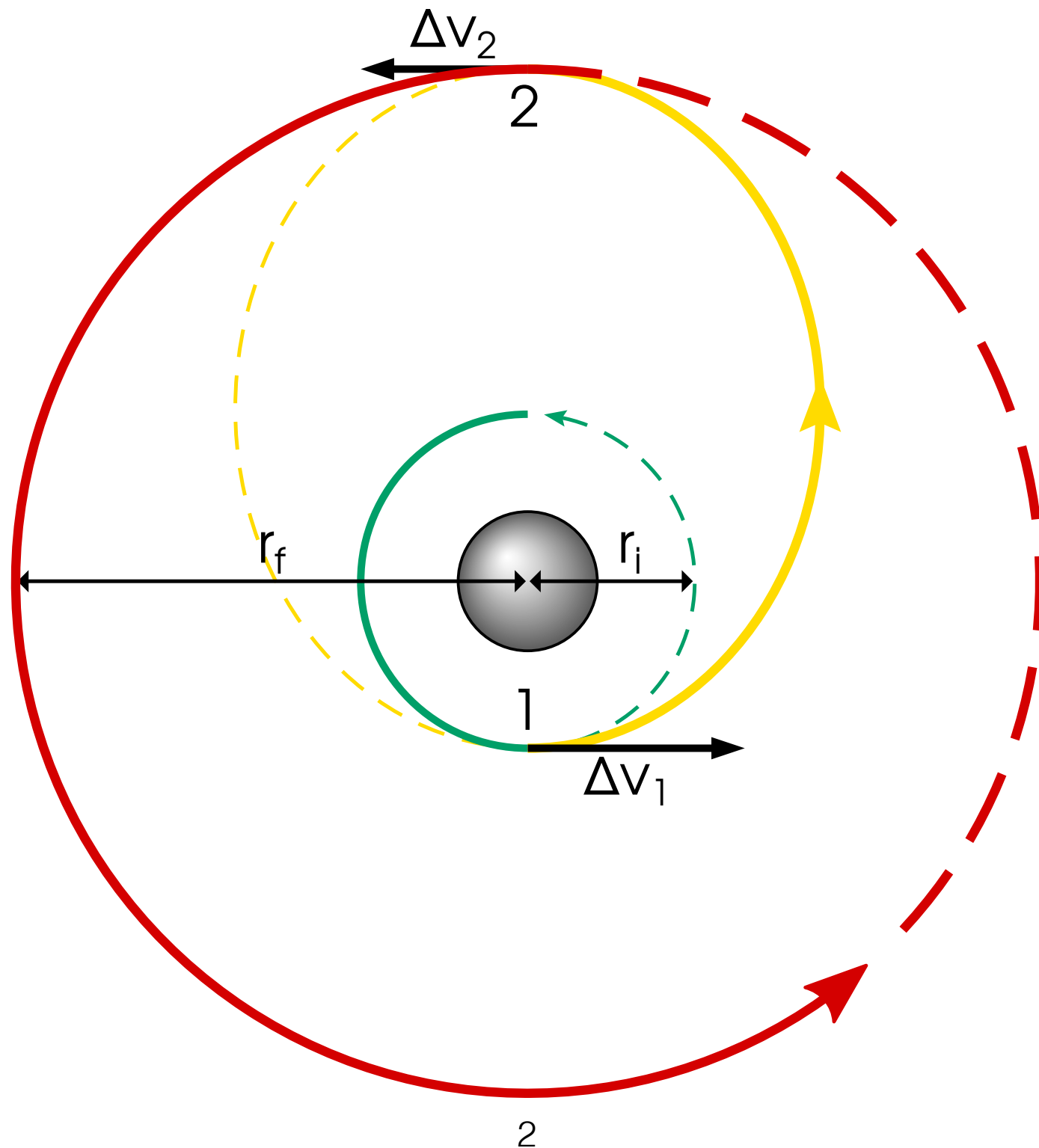
A Theoretical and Numerical Study

Gandalf Saxe

What are transfer orbits?

A way of getting from A to B in space

Hohmann



Why Are Transfer Orbits Interesting?

NASA manned spacecraft: Orion

Manned Mars Mission: 2030's



“The first crewed mission — called EM-2 — is now scheduled for April 2023; the flight was originally scheduled for August 2021”

- *The Verge*, September 16th



Why Are Transfer Orbits Interesting?

Mars One manned spacecraft: Dragon
Manned Mars Mission: ~2026

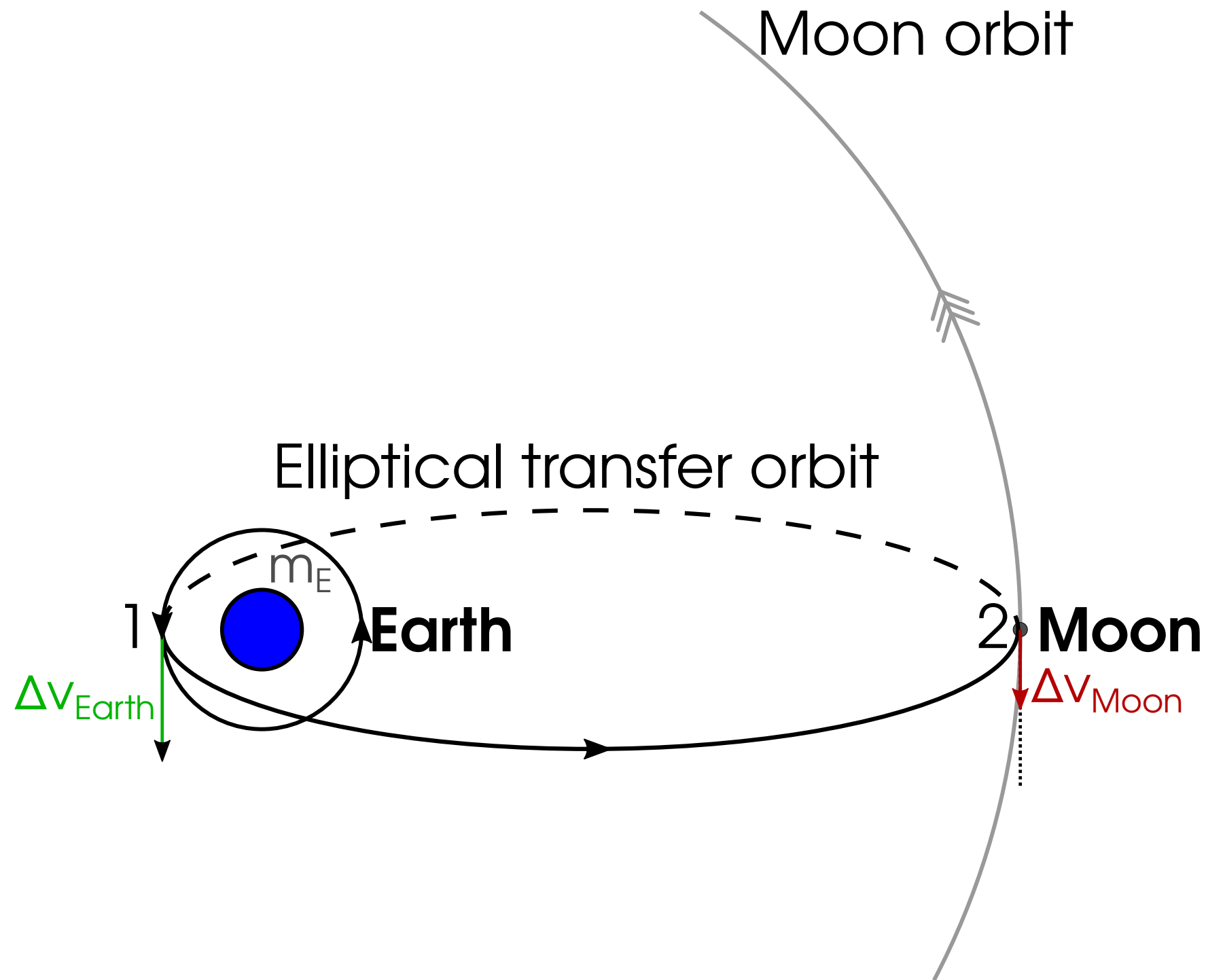


“Elon Musk argues that we must put a million people on Mars if we are to ensure that humanity has a future”

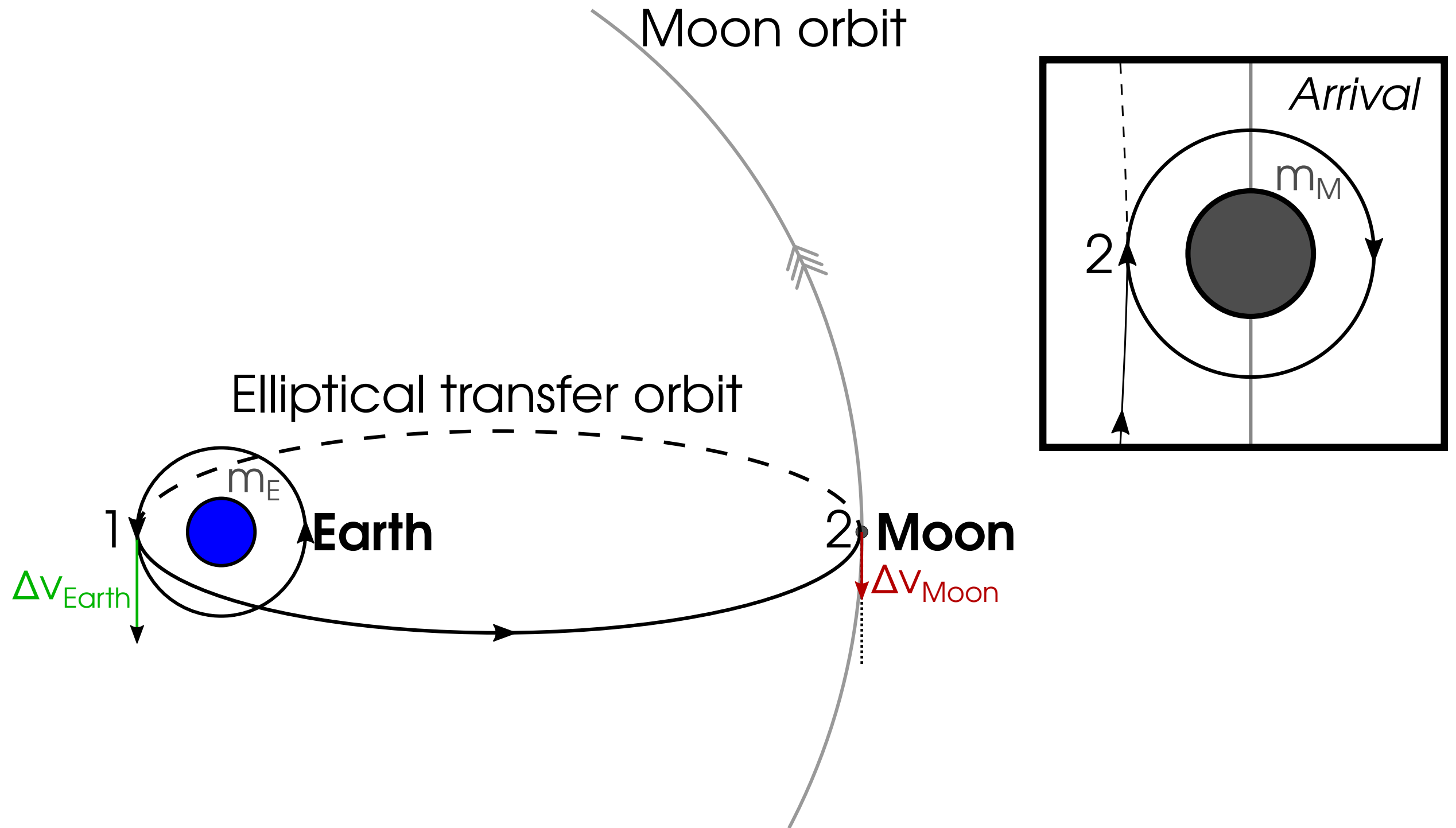
- Interview with aeon.co, 30 September 2014



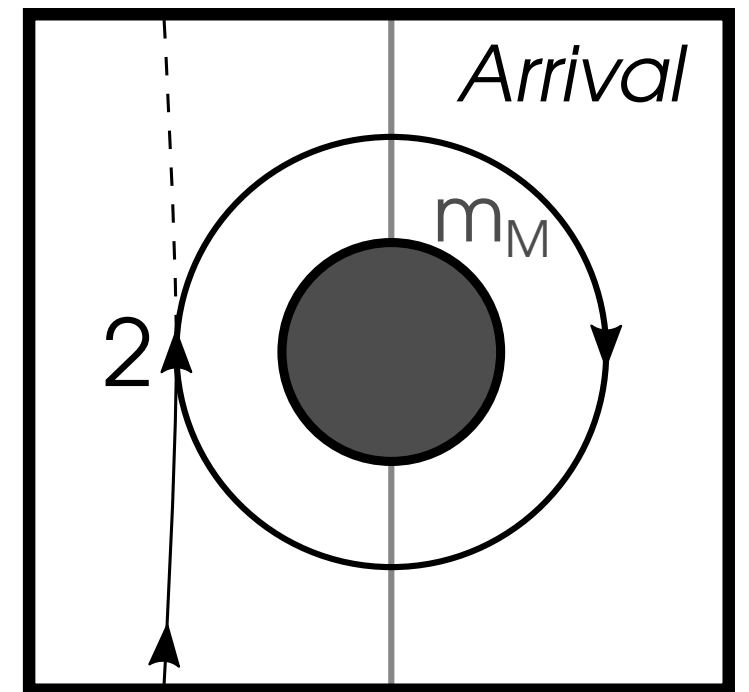
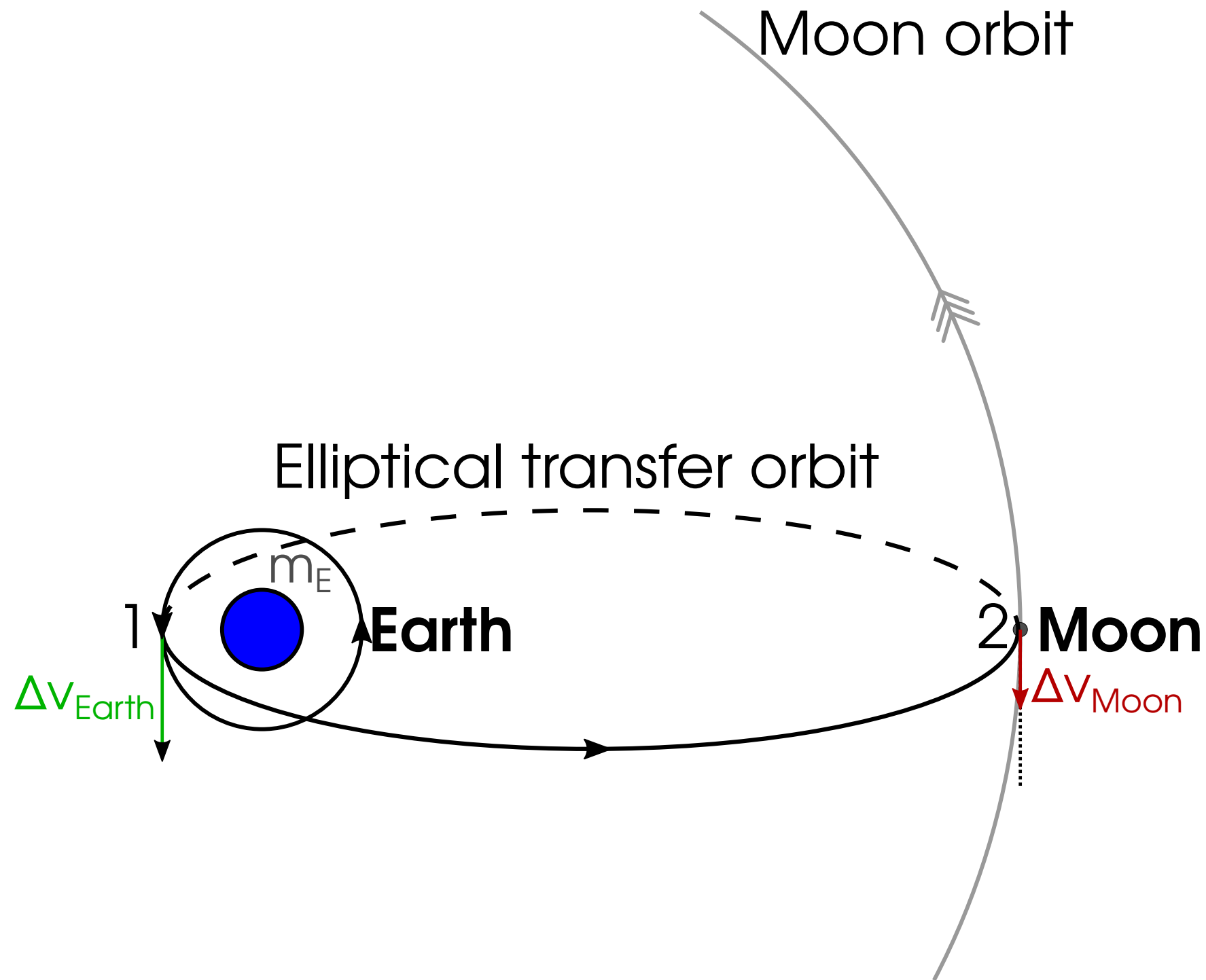
Hohmann Transfer Orbit to the Moon



Hohmann Transfer Orbit to the Moon



Hohmann Transfer Orbit to the Moon



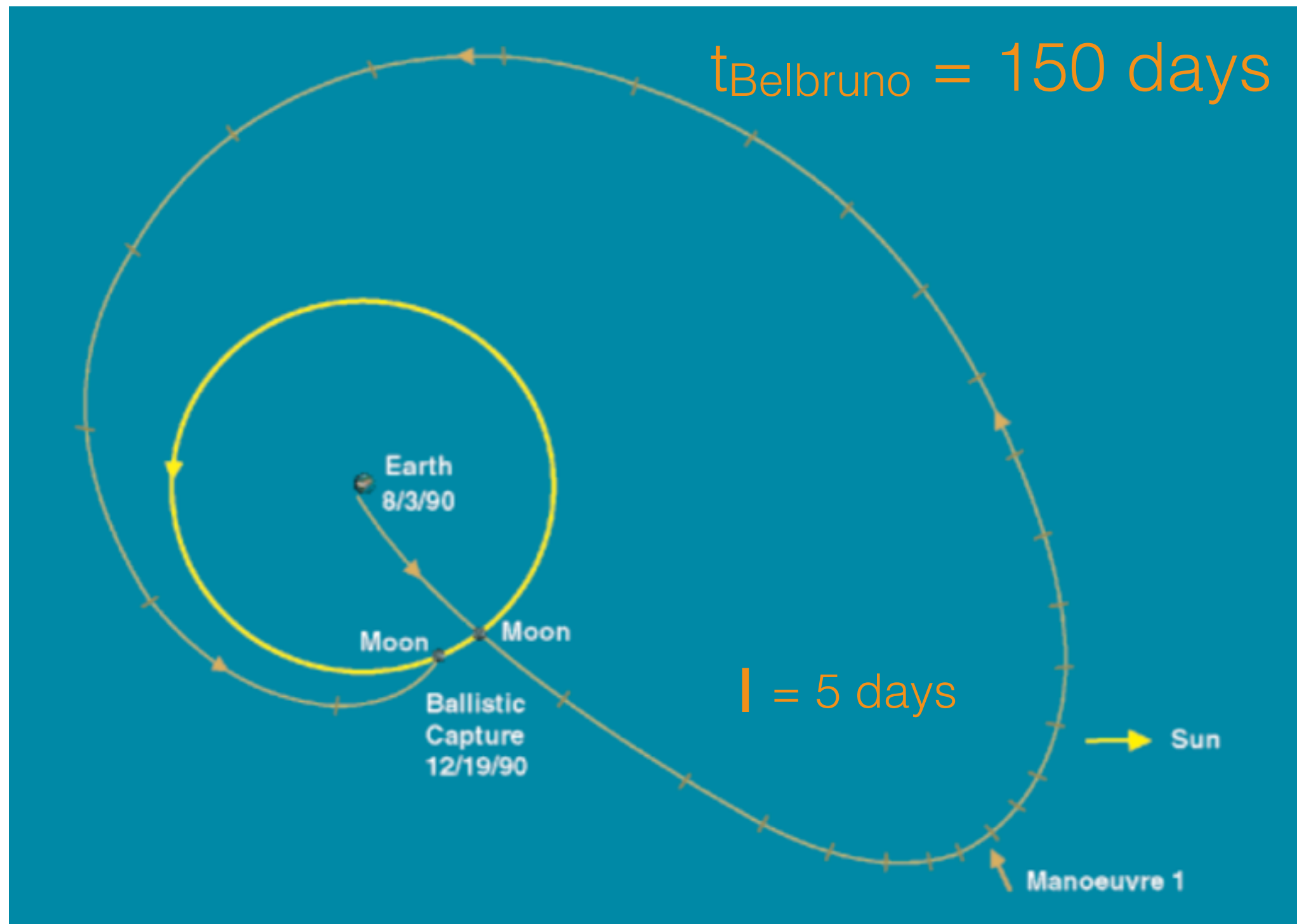
$$t_H = 5.0 \text{ days}$$

$$\Delta V_{\text{Earth}} = 3144 \text{ m/s}$$

$$\Delta V_{\text{Moon}} = 802 \text{ m/s}$$

$$\Delta V_{\text{Total}} = 3946 \text{ m/s}$$


Low Energy Transfer Orbits



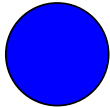
Hiten: Japanese Spacecraft, 1990

Restricted 3-body problem

Spacecraft

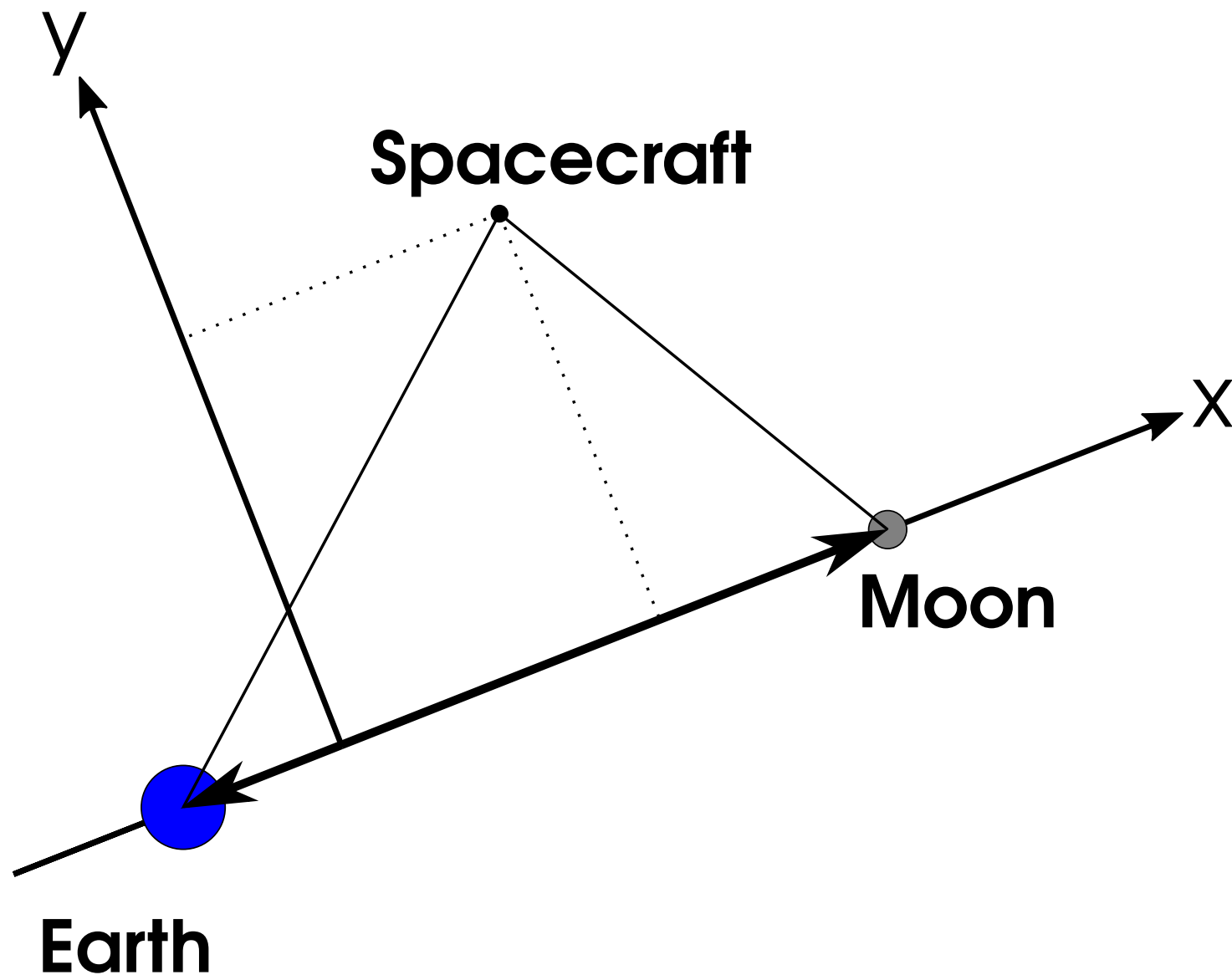


Moon

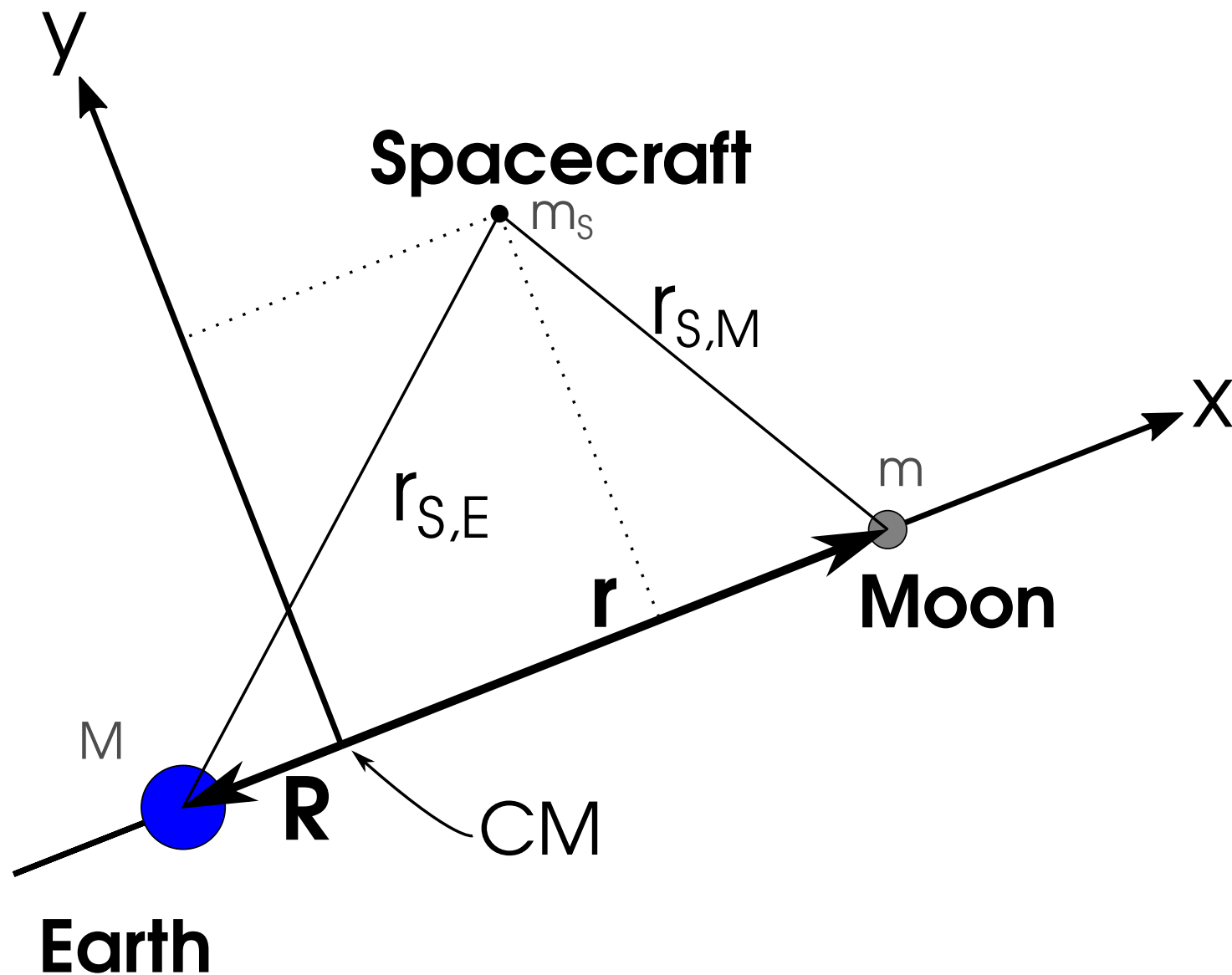


Earth

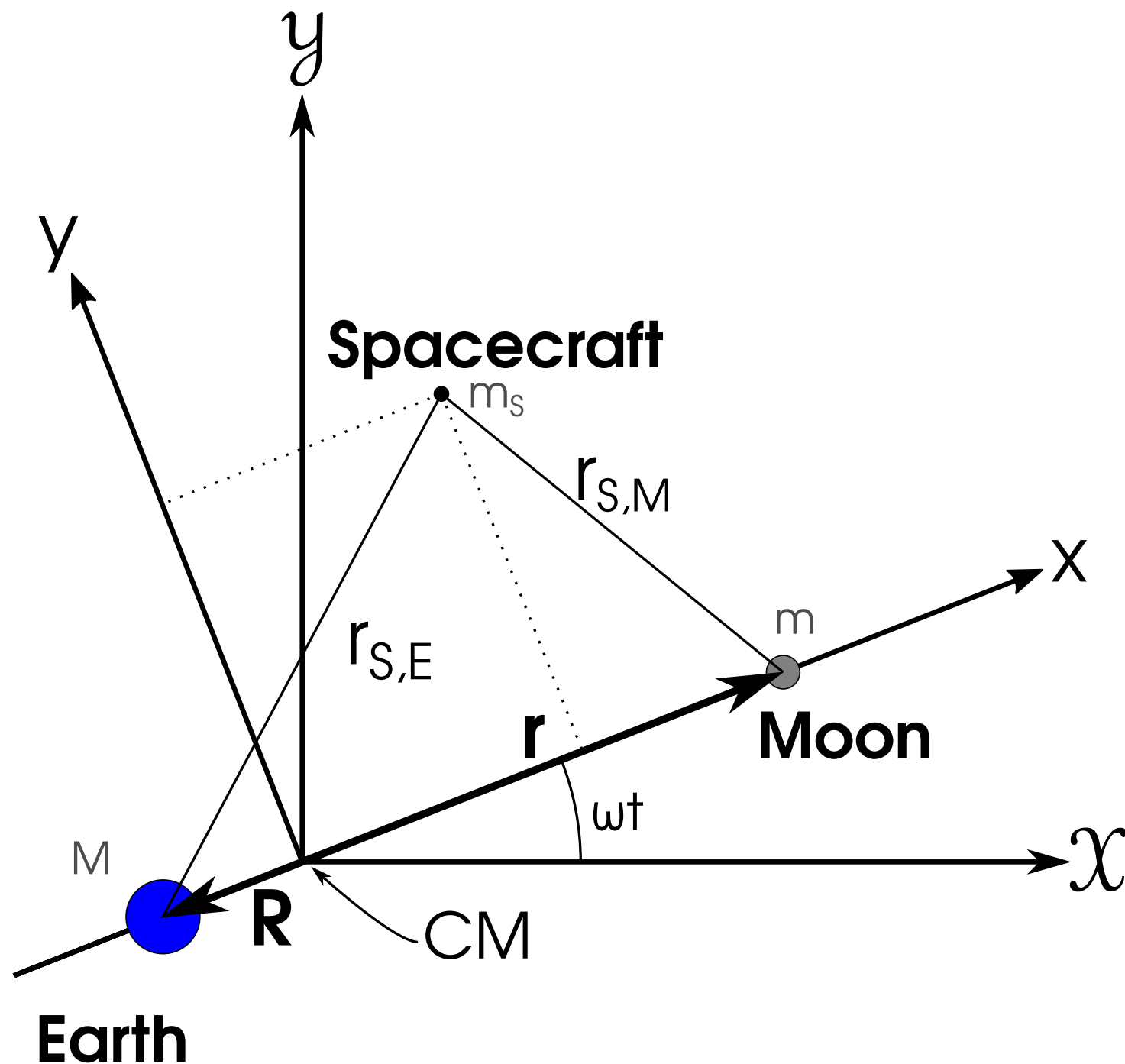
Restricted 3-body problem



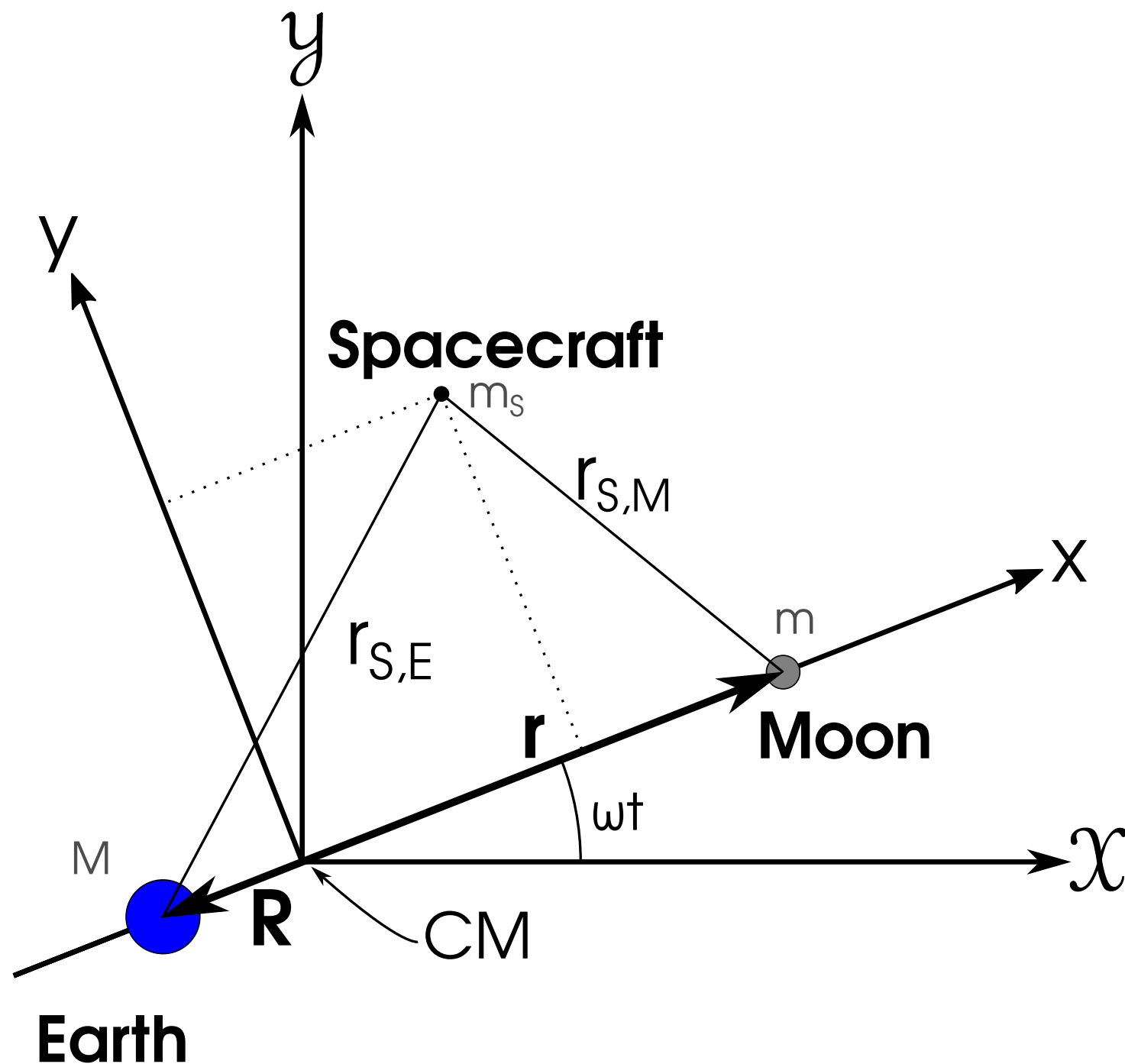
Restricted 3-body problem



Restricted 3-body problem



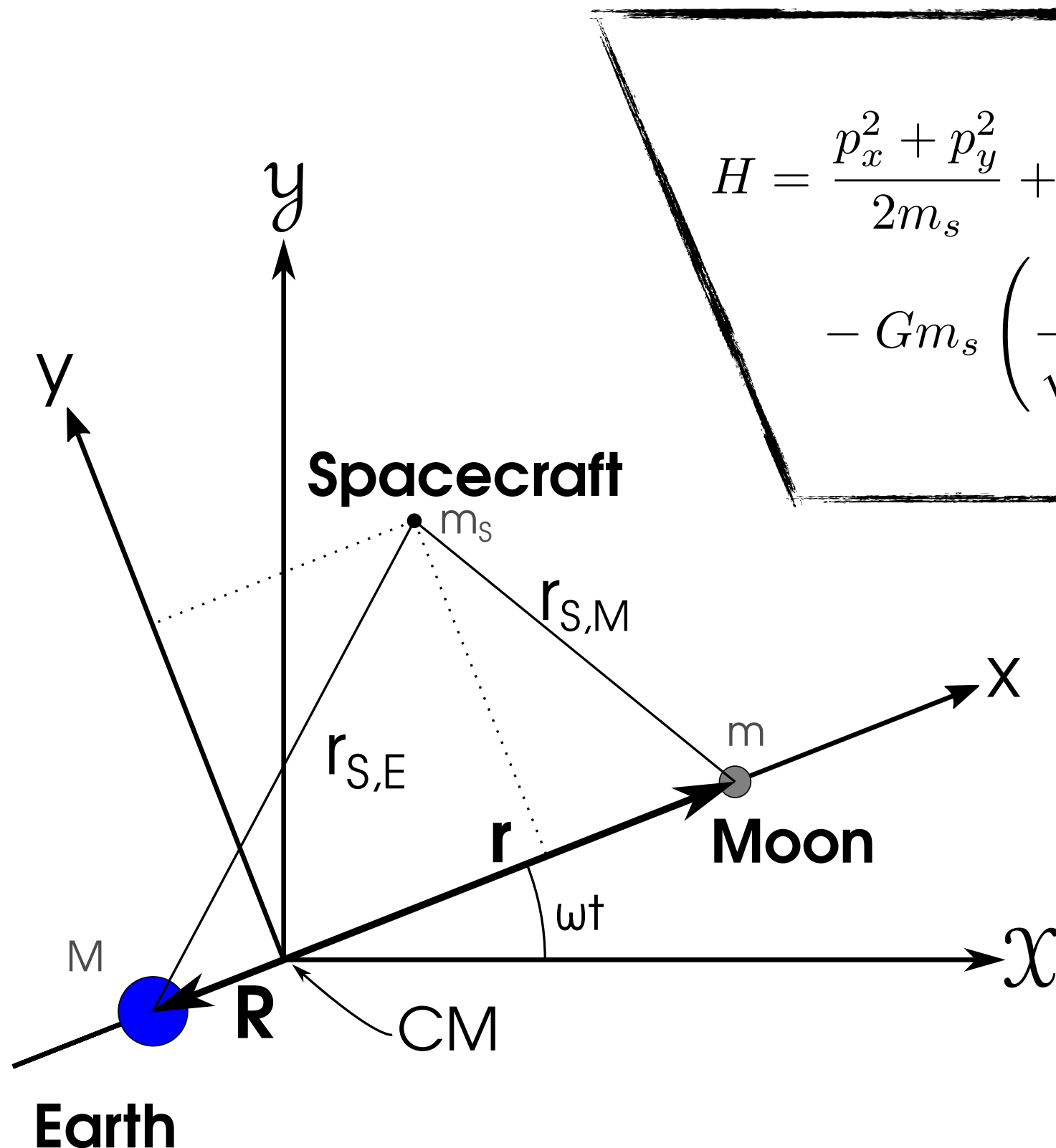
Restricted 3-body problem



Assumptions:

- 2D
- 2-body

Restricted 3-body problem



$$H = \frac{p_x^2 + p_y^2}{2m_s} + p_x\omega y - p_y\omega x$$

$$- Gm_s \left(\frac{M}{\sqrt{(x+R)^2 + y^2}} + \frac{m}{\sqrt{(x-r)^2 + y^2}} \right)$$

Assumptions:

- 2D
- 2-body

Restricted 3-body problem

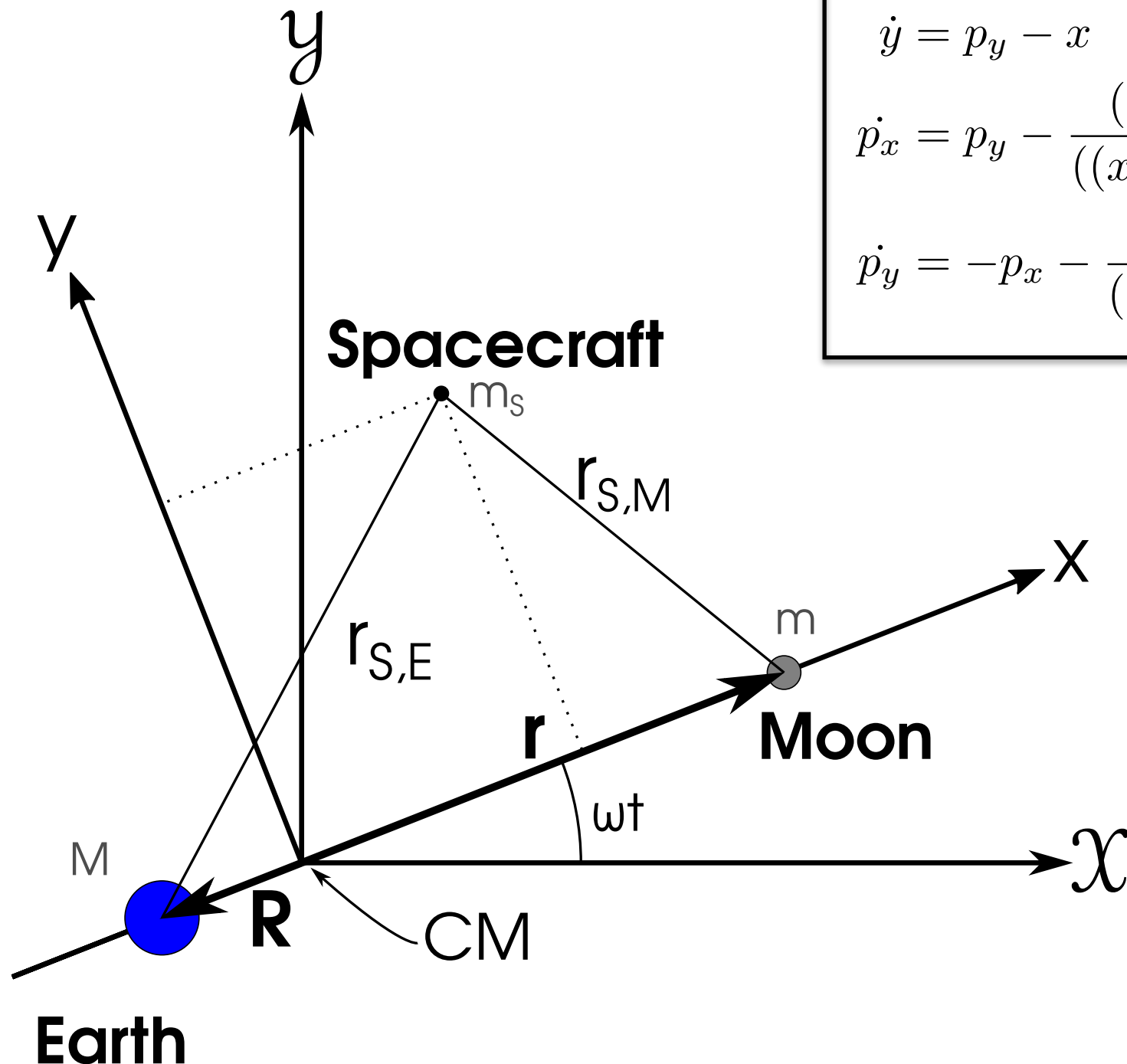
Equations of Motion

$$\dot{x} = p_x + y$$

$$\dot{y} = p_y - x$$

$$\dot{p}_x = p_y - \frac{(1-k)(x+k)}{((x+k)^2 + y^2)^{3/2}} - \frac{k(x-1-k)}{((x-1-k)^2 + y^2)^{3/2}}$$

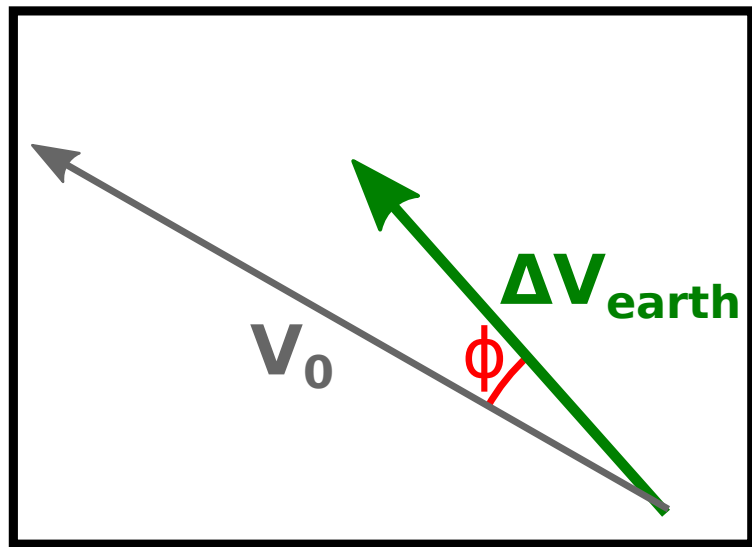
$$\dot{p}_y = -p_x - \frac{(1-k)y}{((x+k)^2 + y^2)^{3/2}} - \frac{ky}{((x-1-k)^2 + y^2)^{3/2}}$$



Assumptions:

- 2D
- 2-body

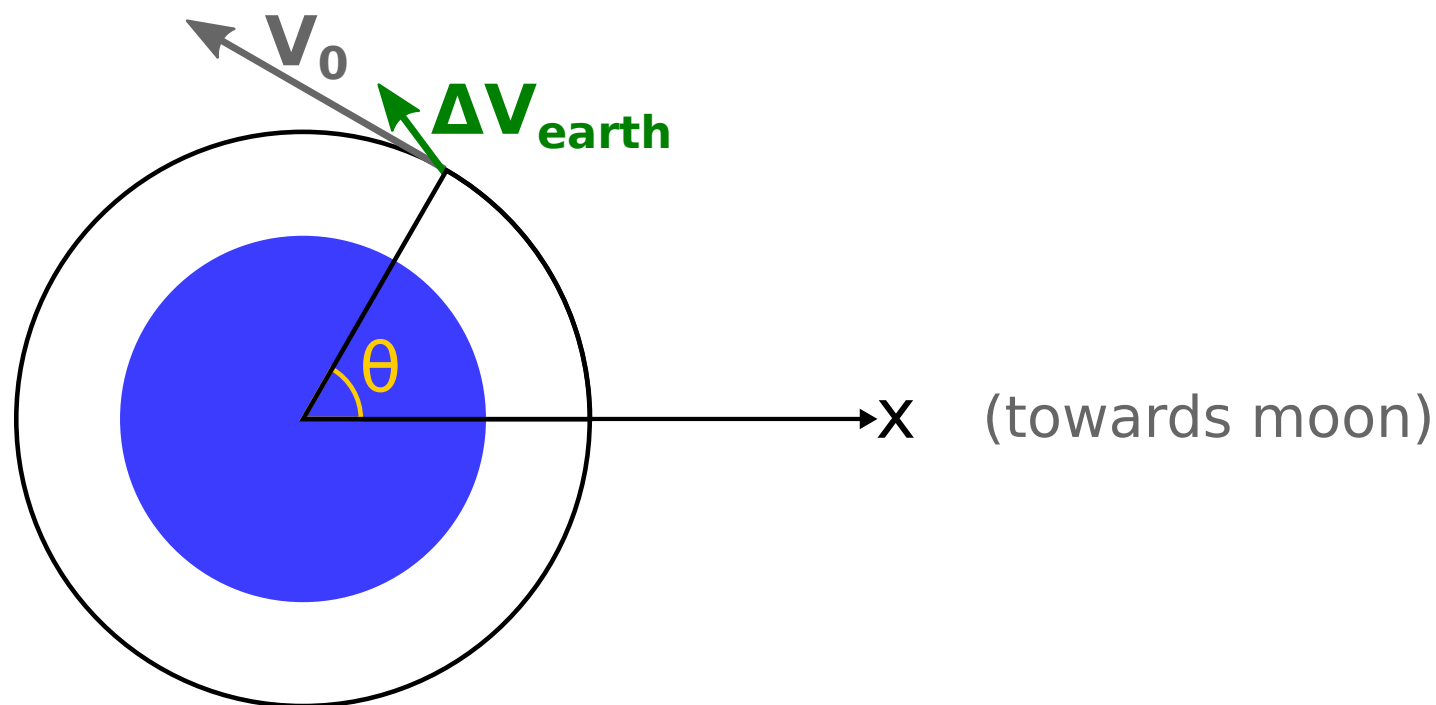
Searching for Transfer Orbits



1. θ : Position in orbit

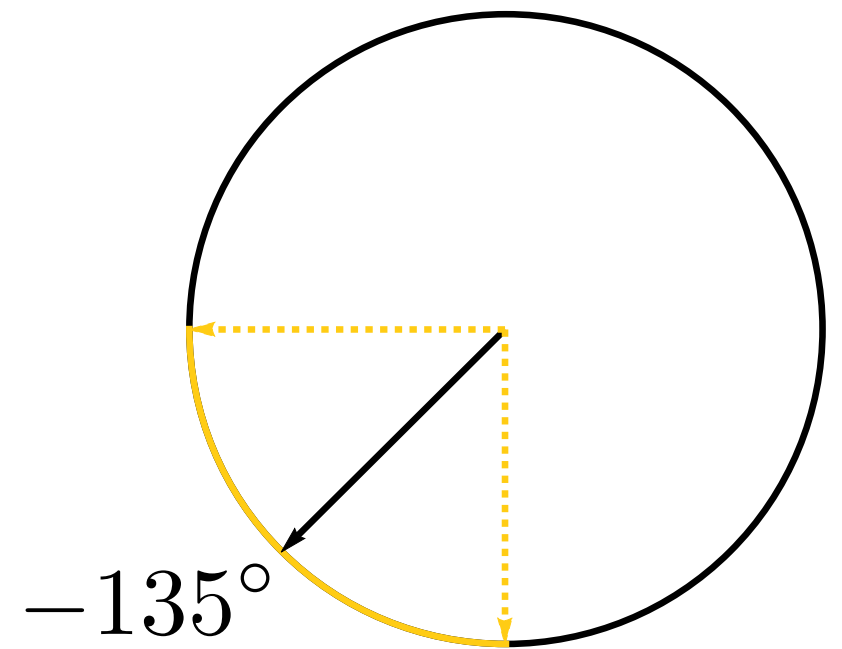
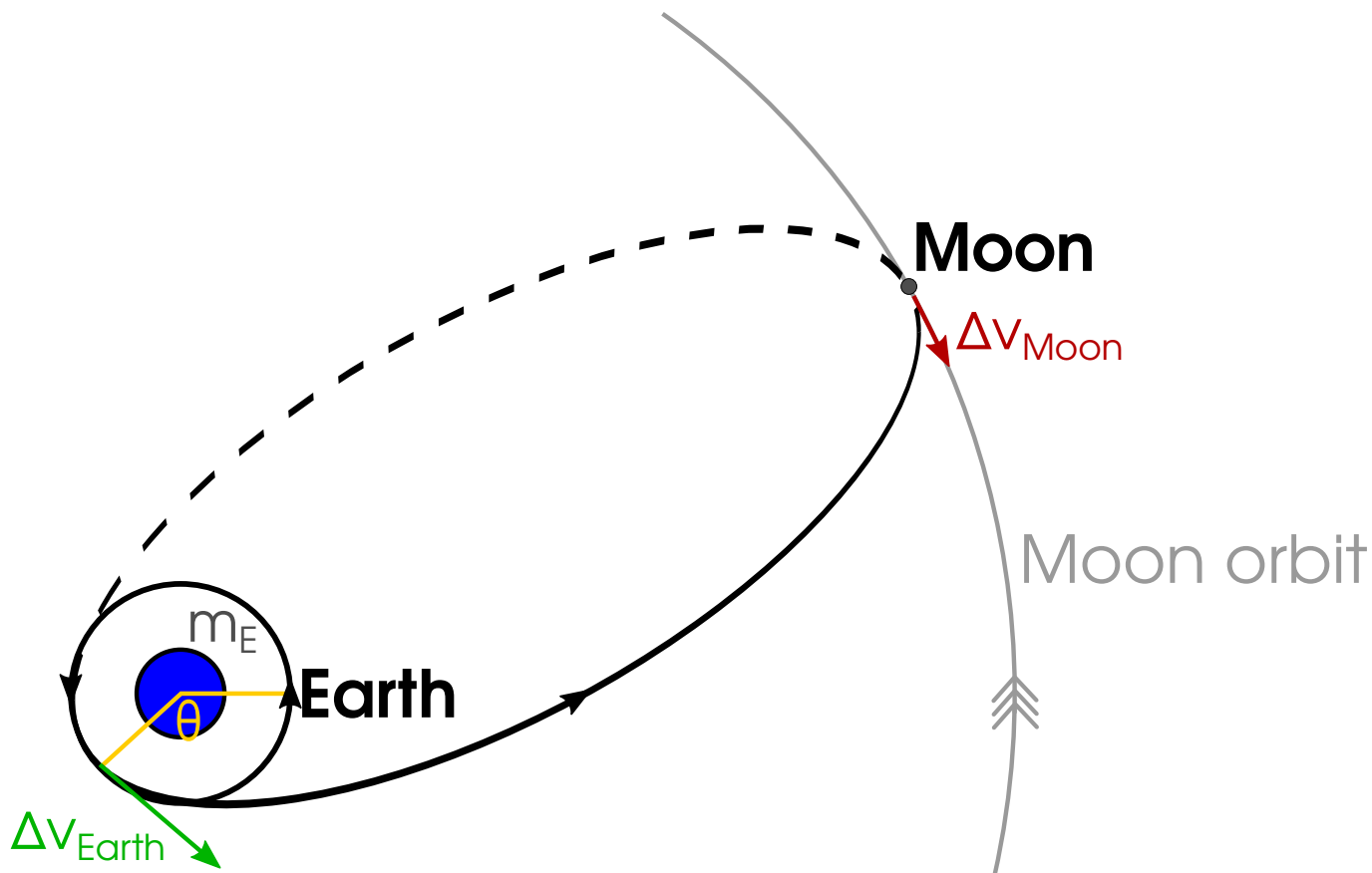
2. Δv_{earth} : Velocity change

3. ϕ : Angle to velocity vector



Searching for Transfer Orbits

Hohmann Transfer Orbit to the Moon



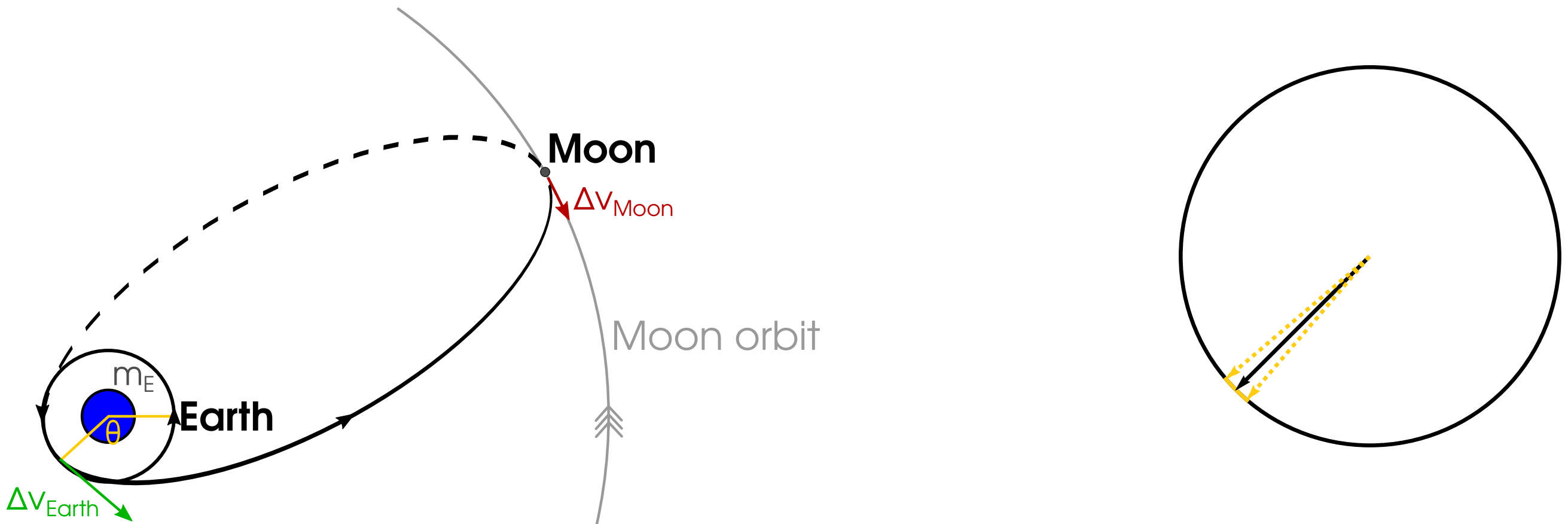
$\phi: 0^\circ$ constant

First search

100 positions · 200 velocities = 20,000 simulations

Searching for Transfer Orbits

Hohmann Transfer Orbit to the Moon



First search

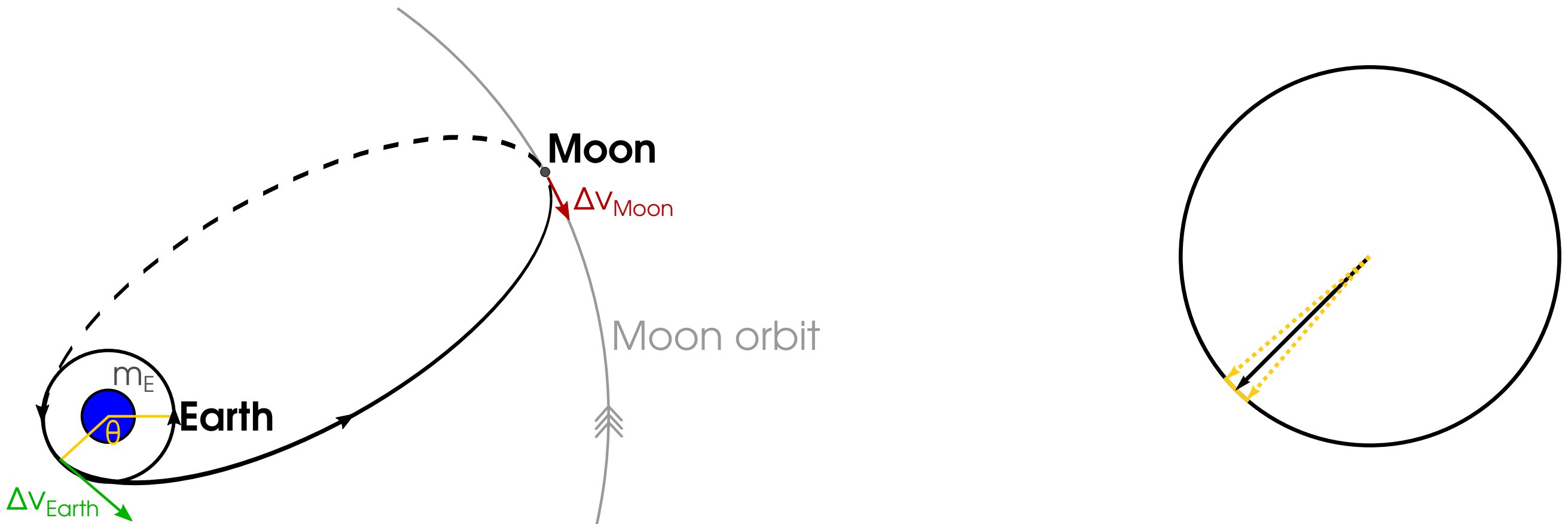
100 positions · 200 velocities = 20,000 simulations

Refinement

15 positions · 15 velocities · 15 angles = 3375 simulations

Searching for Transfer Orbits

Hohmann Transfer Orbit to the Moon



First search

100 positions · 200 velocities = 20,000 simulations

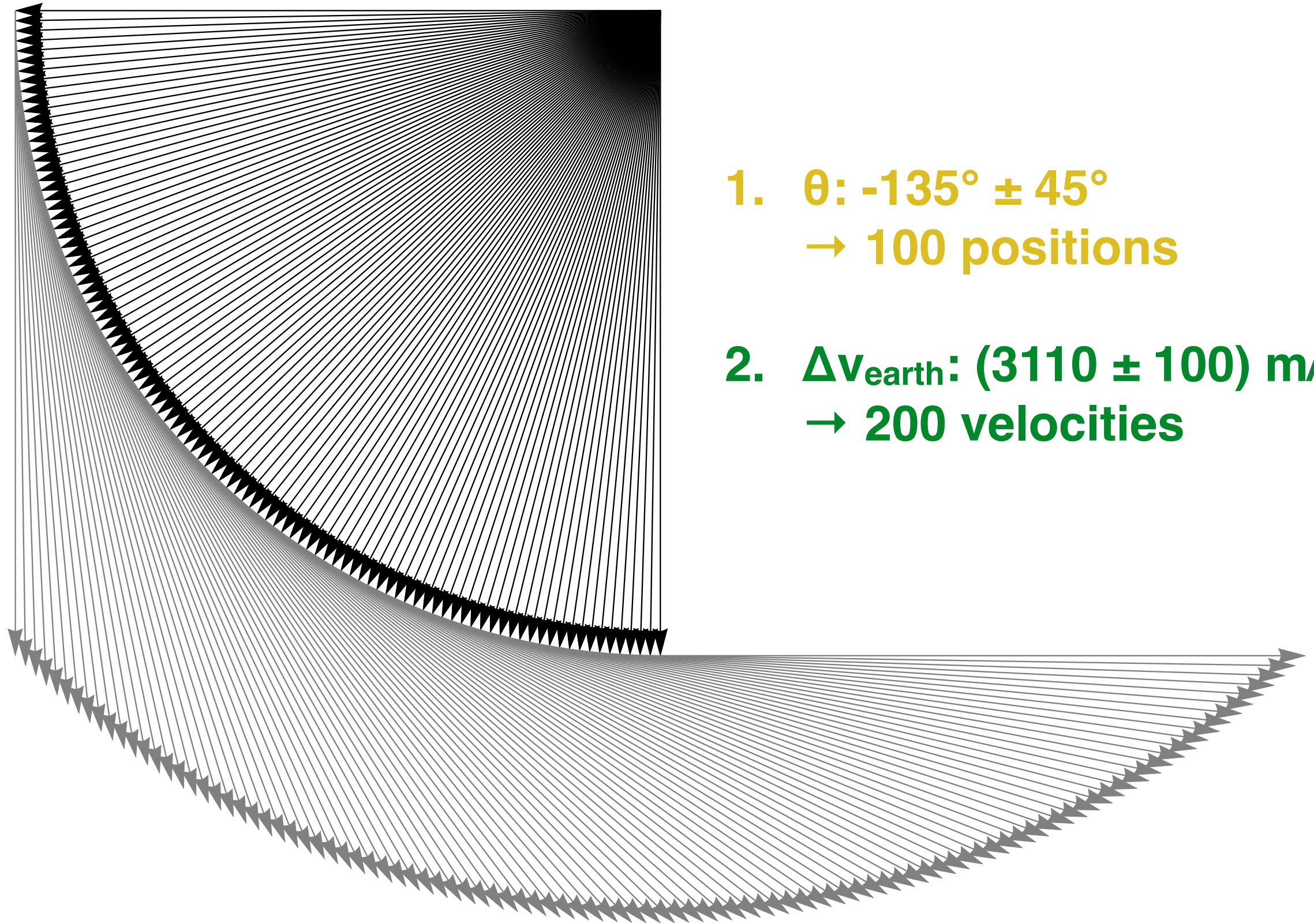
Refinement

15 positions · 15 velocities · 15 angles = 3375 simulations

Total: 20,000 + 3375 = 23,375 simulations

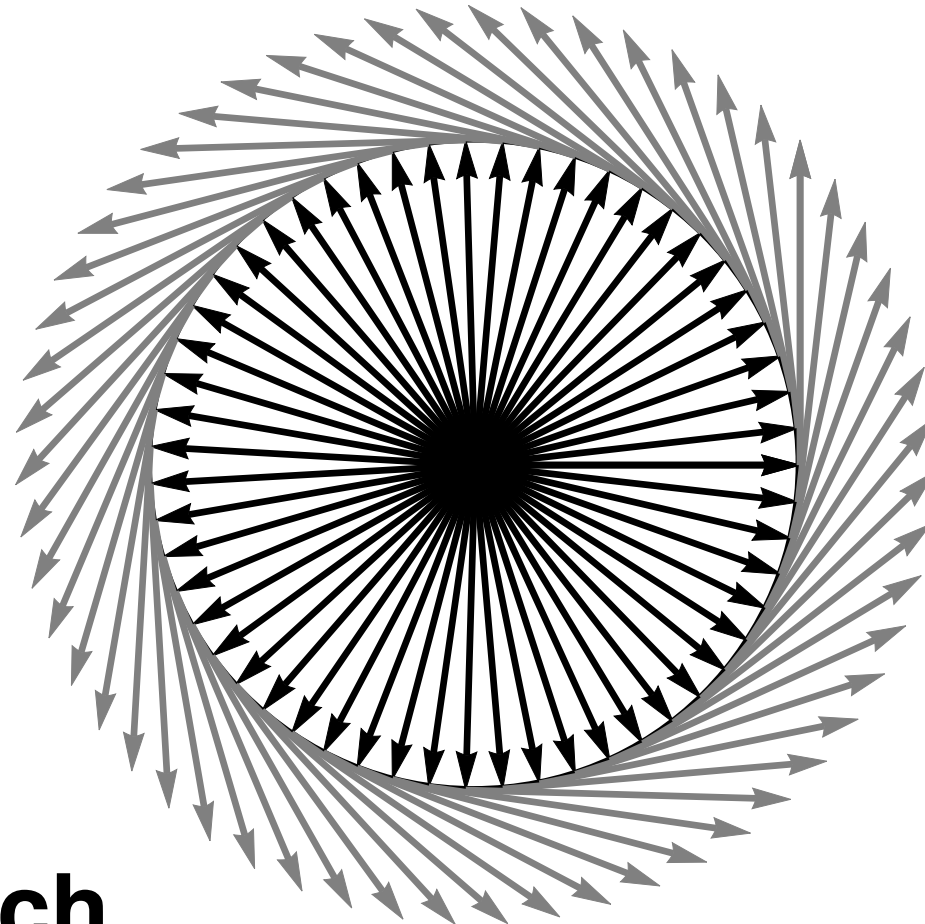
Searching for Transfer Orbits

Hohmann Transfer Orbit to the Moon



Searching for Transfer Orbits

Low Energy Transfer Orbit to the Moon



First search

1. $\theta: 0 \pm 180^\circ \rightarrow 55$ positions

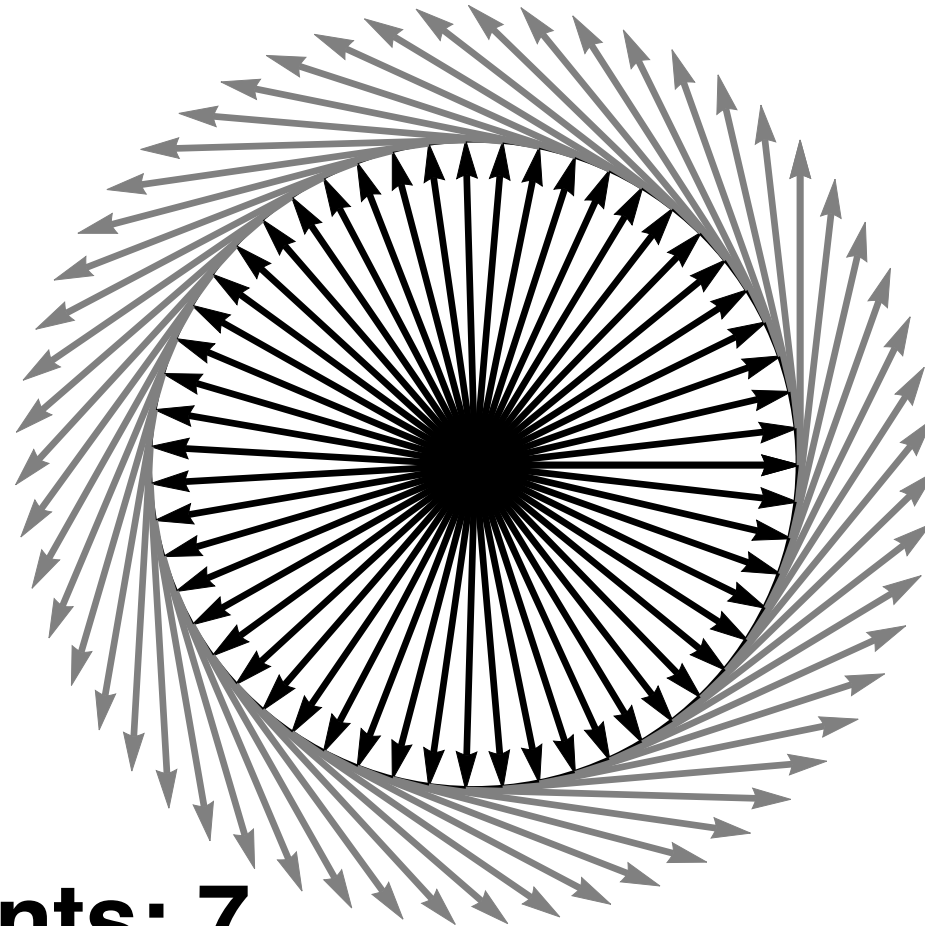
2. $\Delta v_{\text{earth}}: (3120 \pm 100) \text{ m/s} \rightarrow 55$ velocities

3. $\phi: 0^\circ \pm 1.8^\circ \rightarrow 55$ angles

SUBTOTAL $55 \cdot 55 \cdot 55 = 166,375$

Searching for Transfer Orbits

Low Energy Transfer Orbit to the Moon



Refinements: 7

1. θ : best $\pm \pi/10$ \rightarrow 55 positions

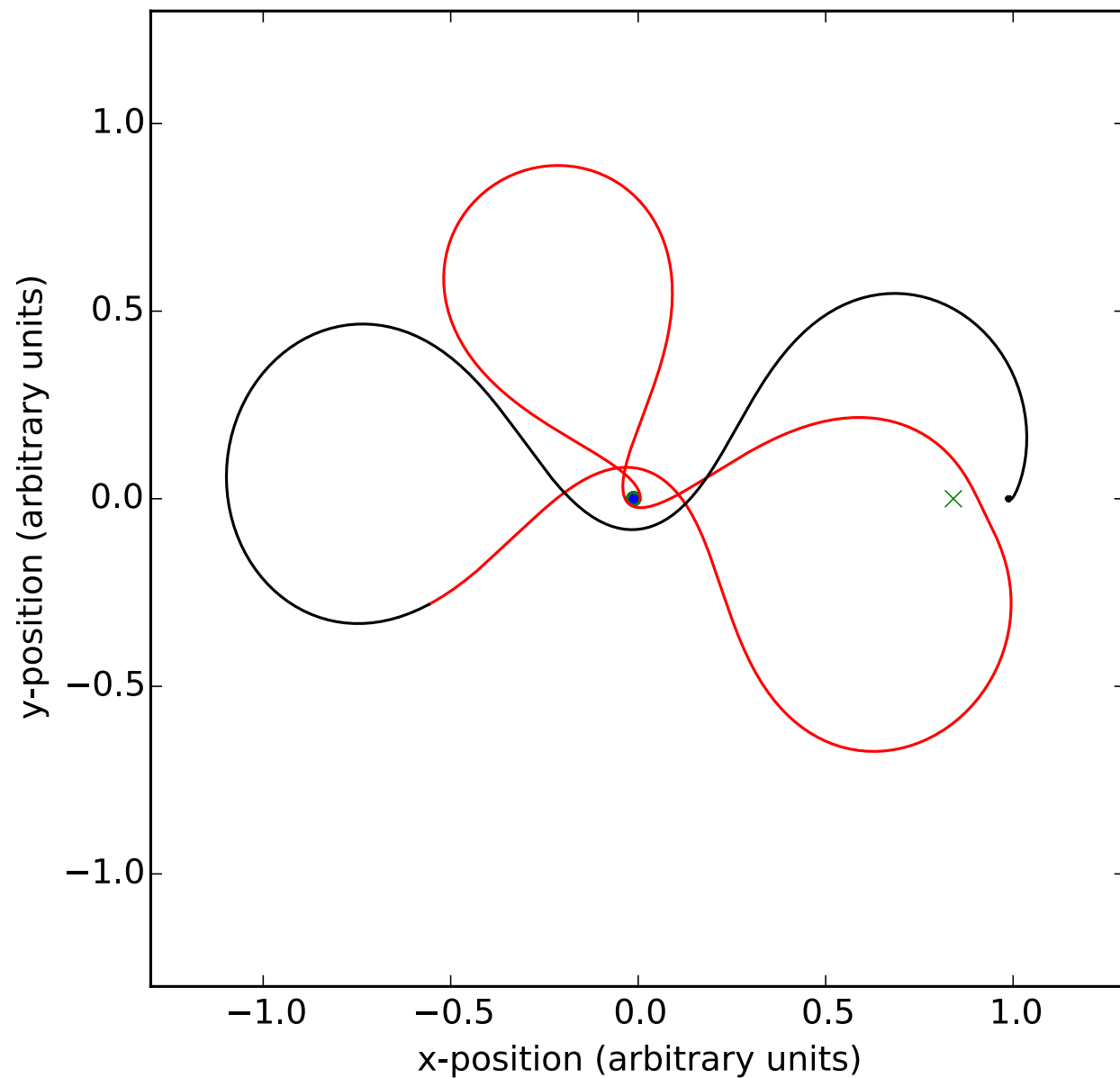
2. Δv_{earth} : (best $\pm 100/10$) m/s \rightarrow 55 velocities

3. ϕ : best $\pm 1.8^\circ/10$ \rightarrow 55 angles

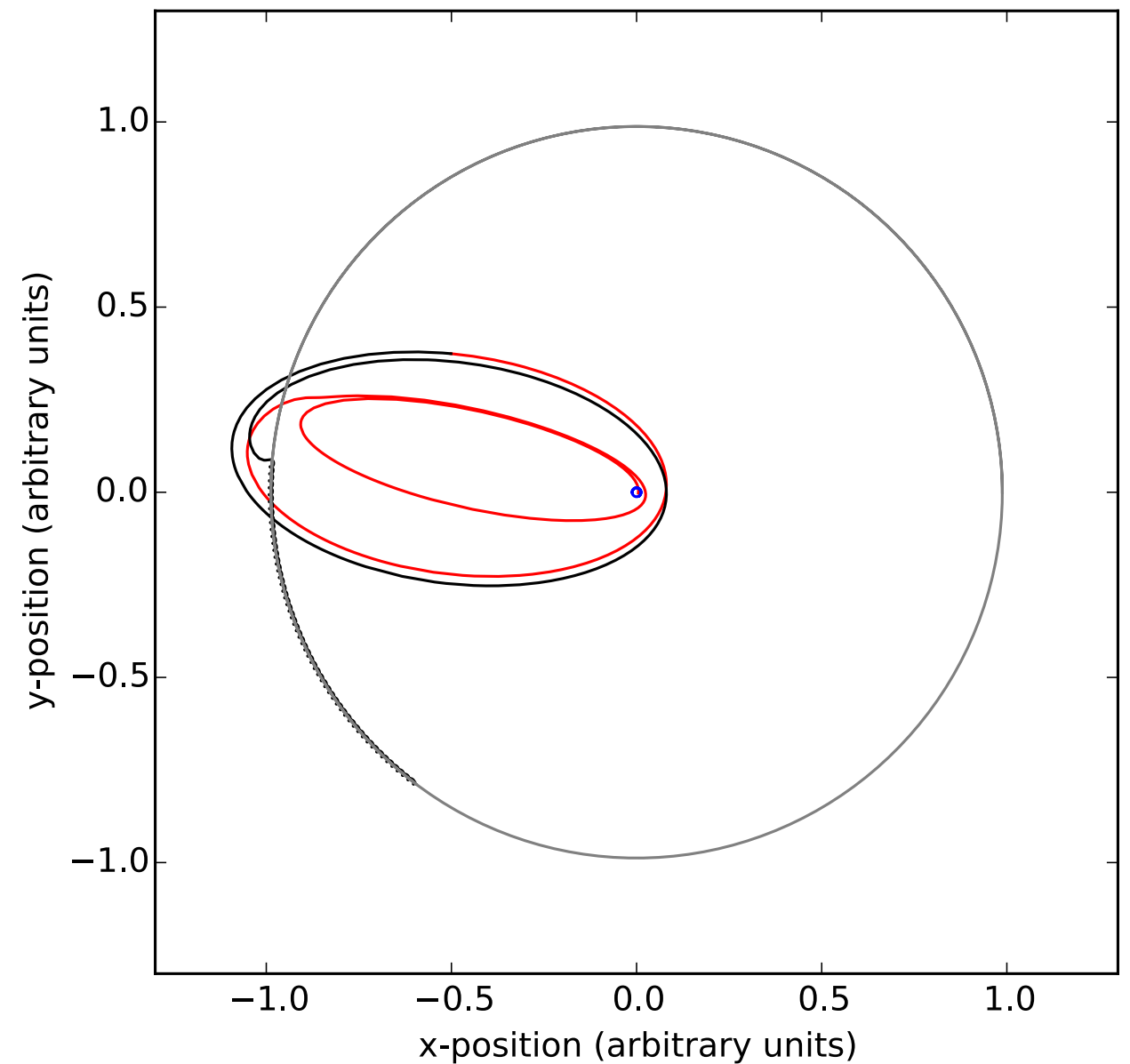
TOTAL $55 \cdot 55 \cdot 55 \cdot 8 = 1,331,000$

Numerical Method

Adaptive Störmer-Verlet (symplectic)



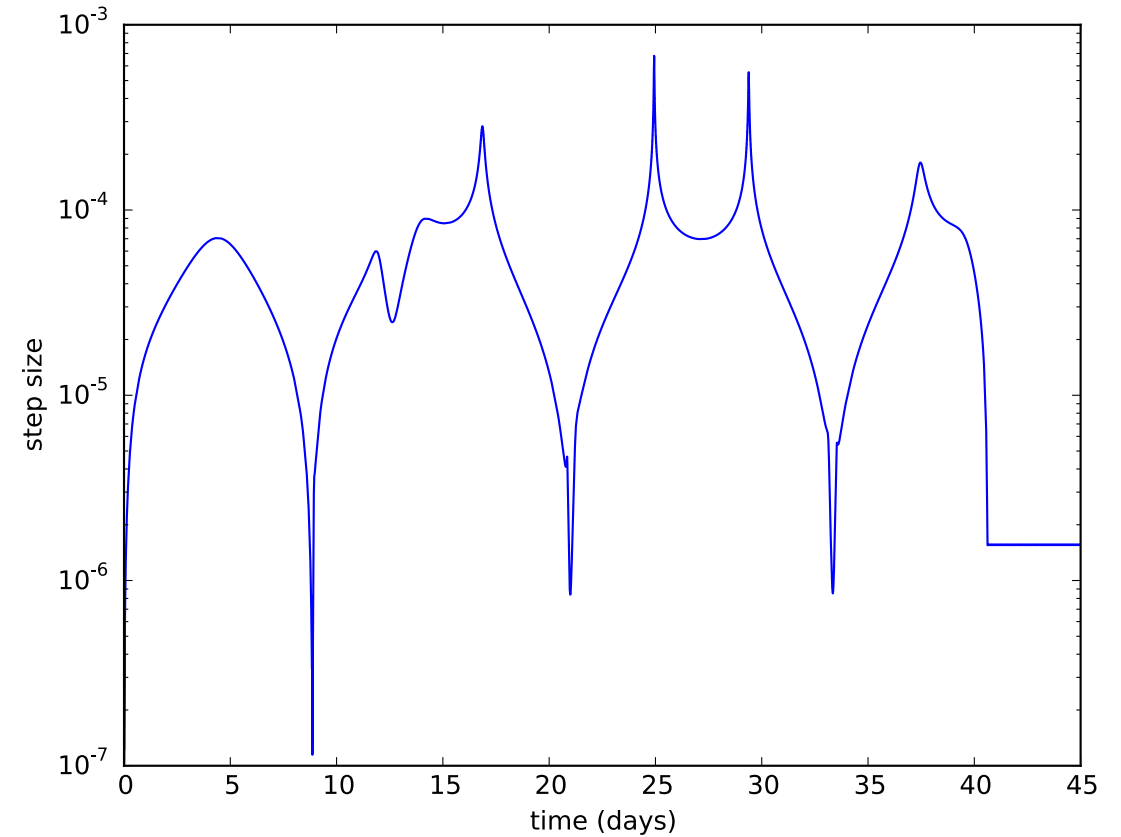
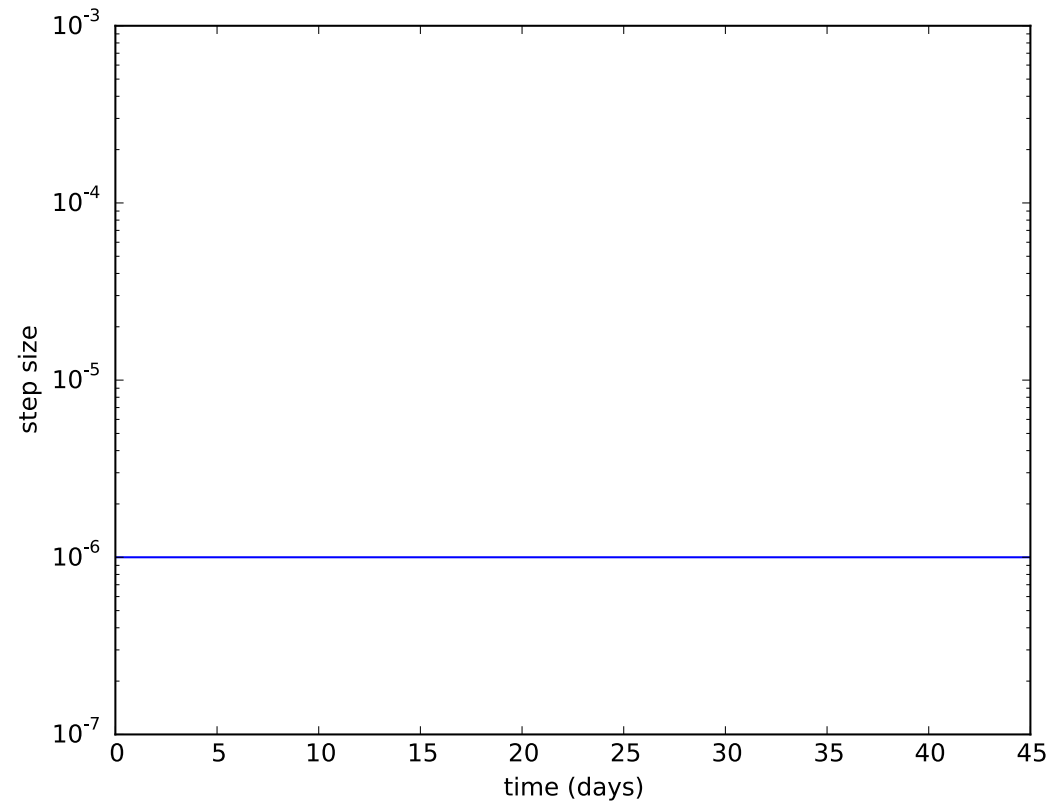
(x, y)



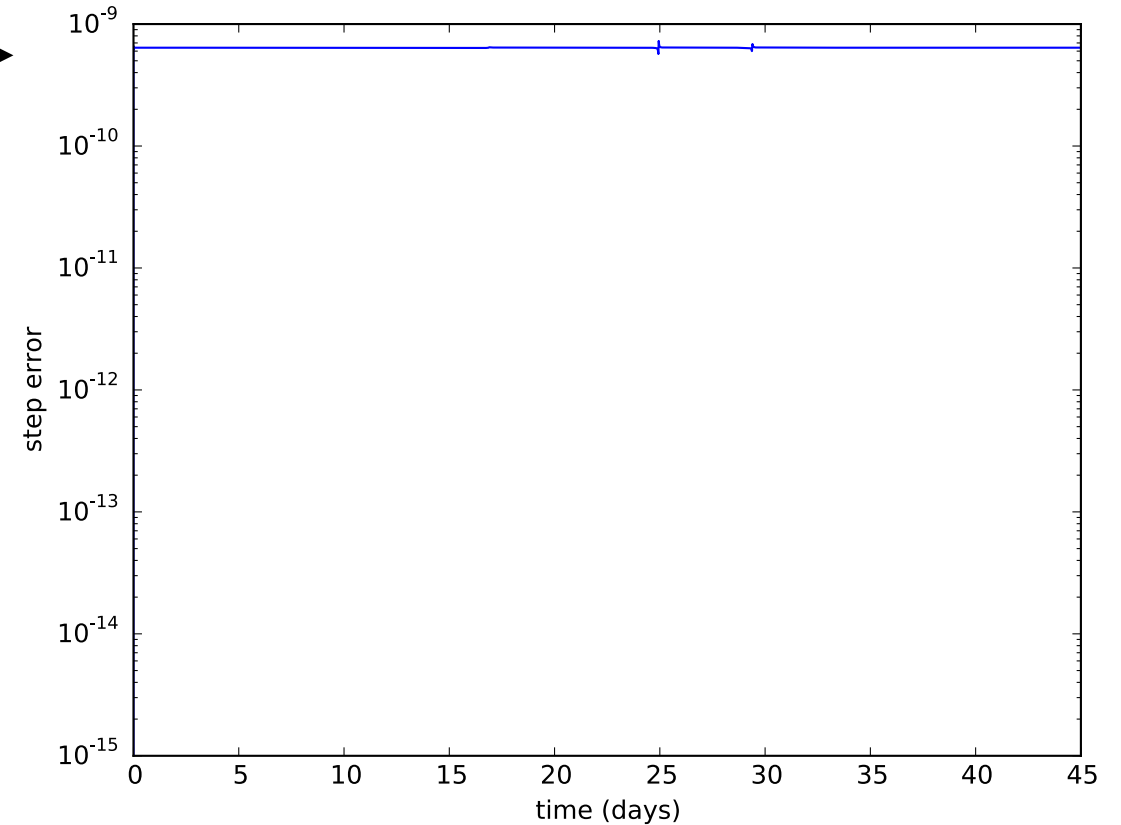
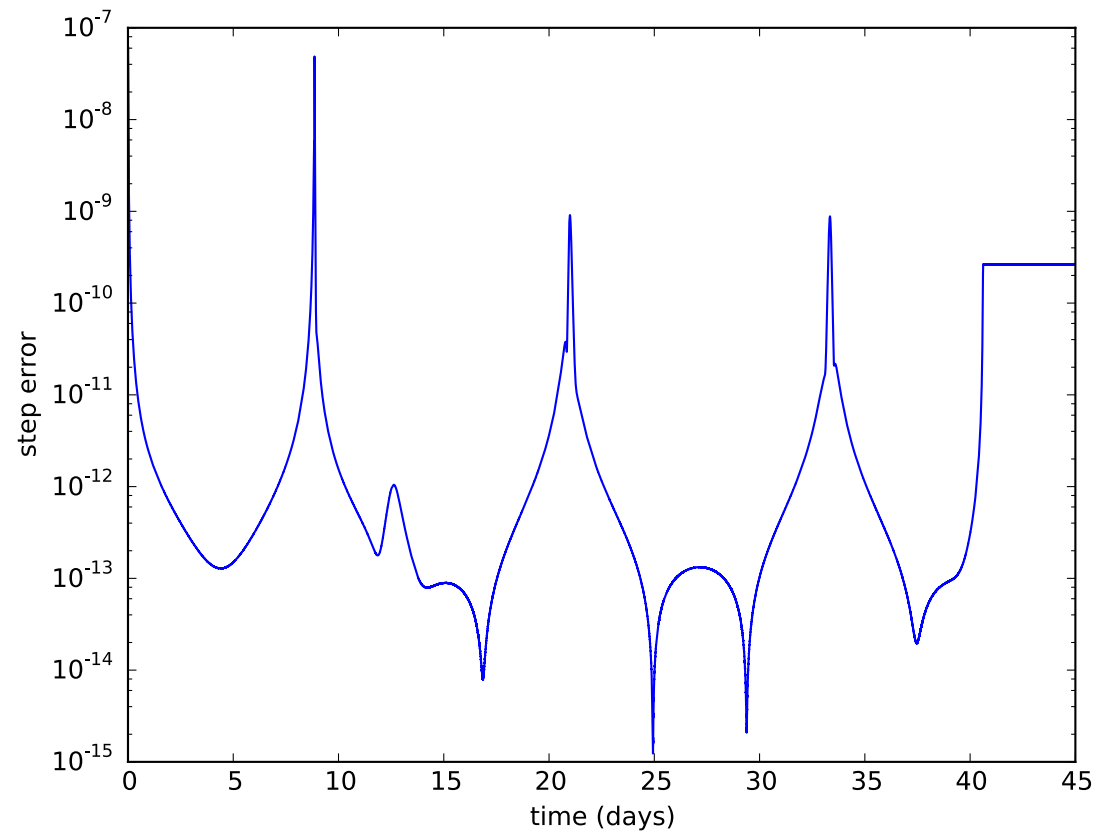
(x, y)

Numerical Method

Adaptive Störmer-Verlet (symplectic)

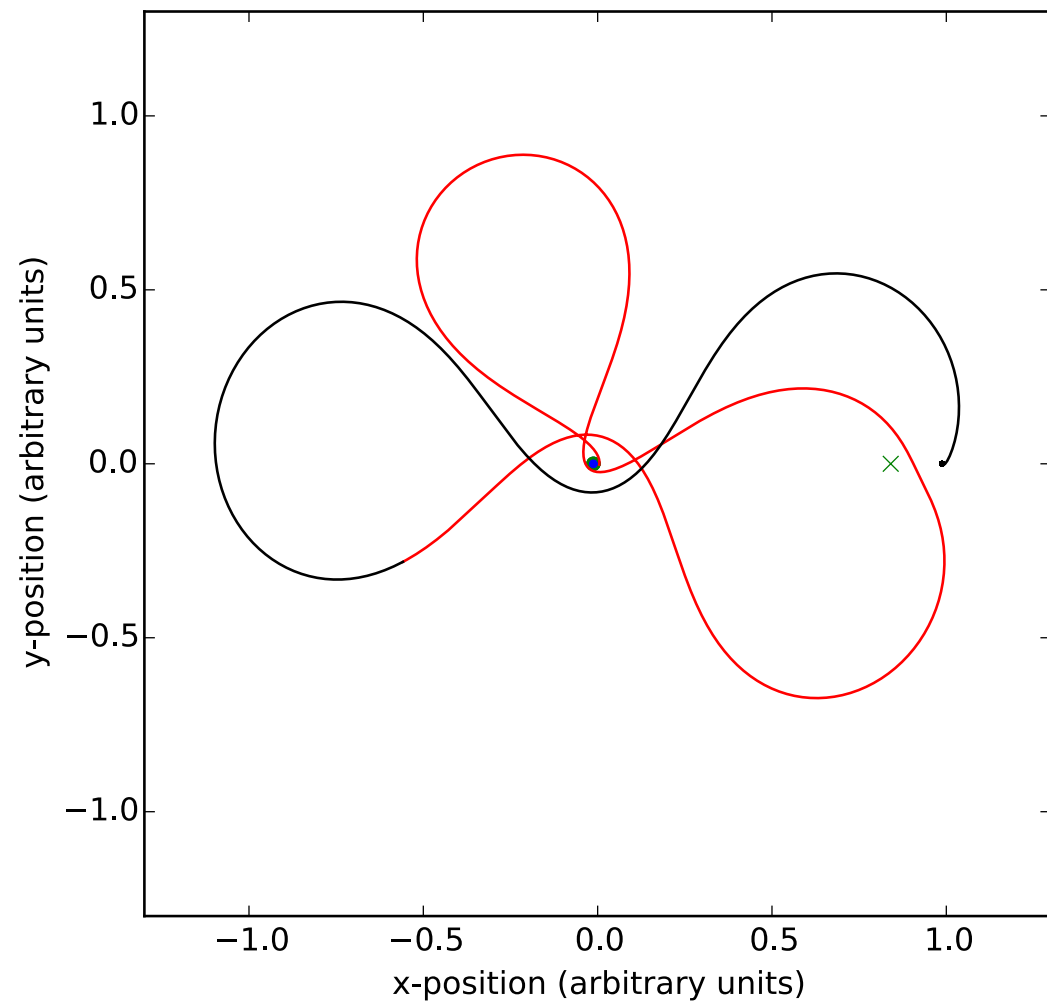


Adaptive

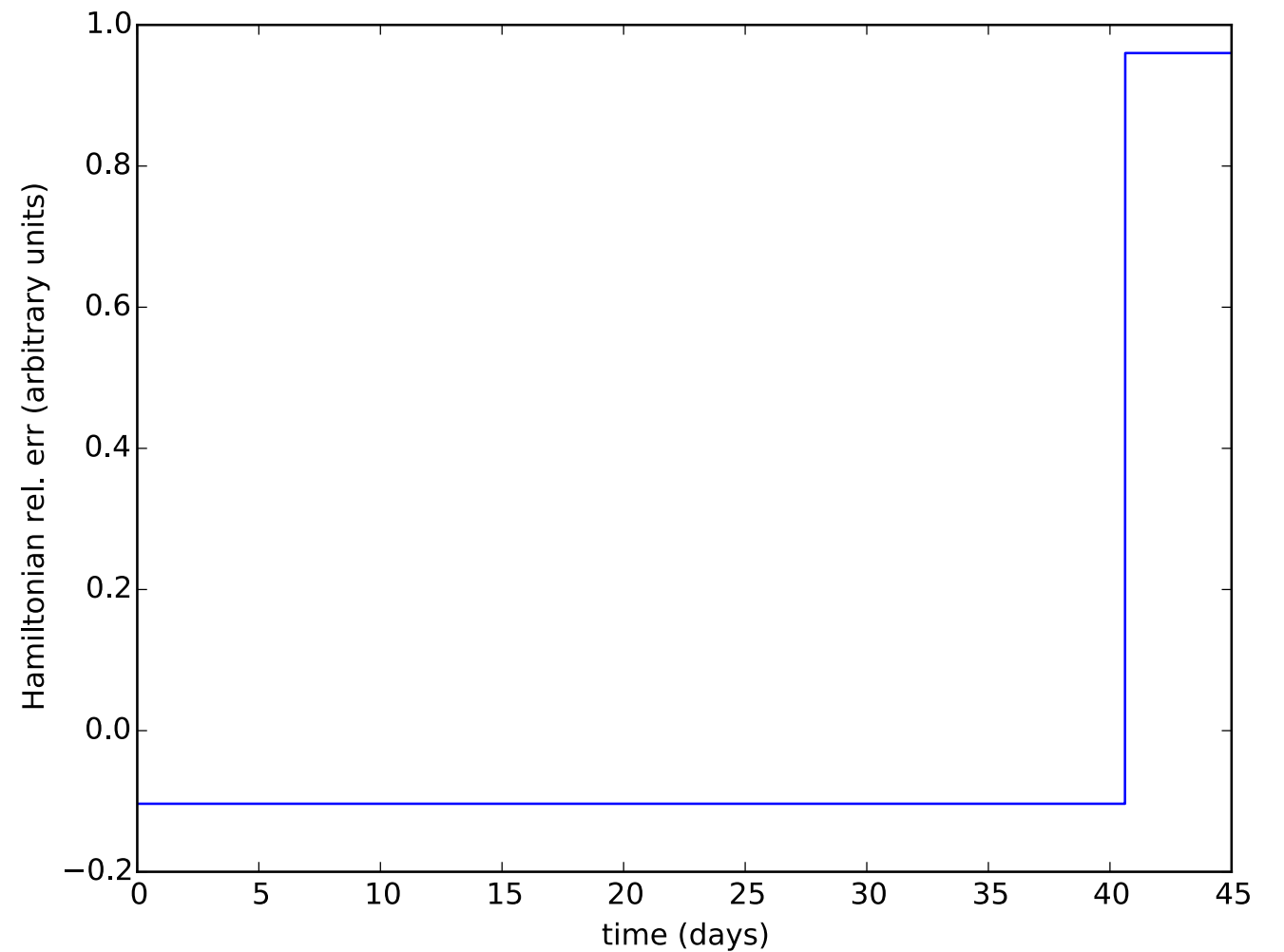


Numerical Method

Adaptive Störmer-Verlet (symplectic)



(x, y)

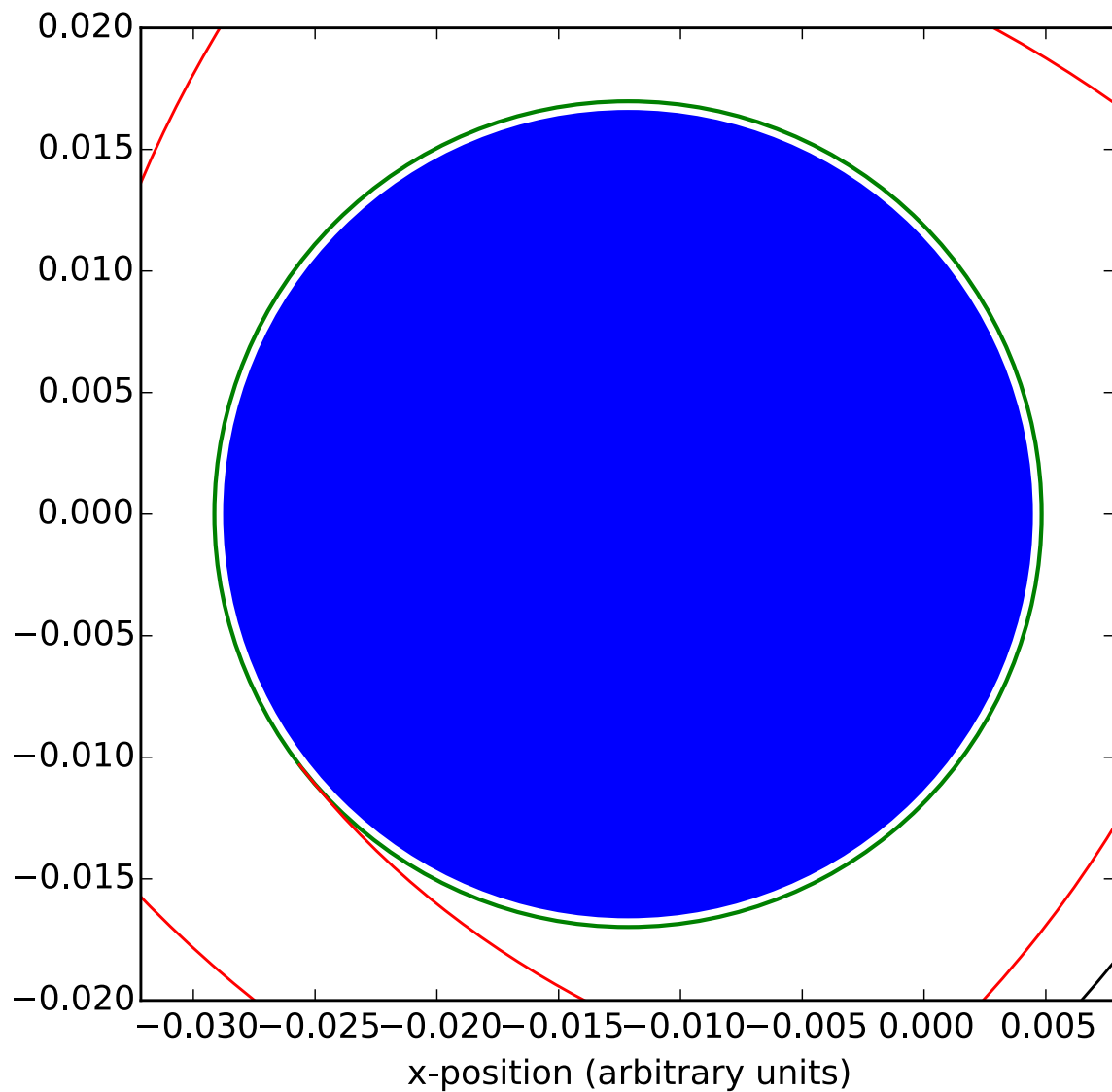


Hamiltonian

Searching for Transfer Orbits

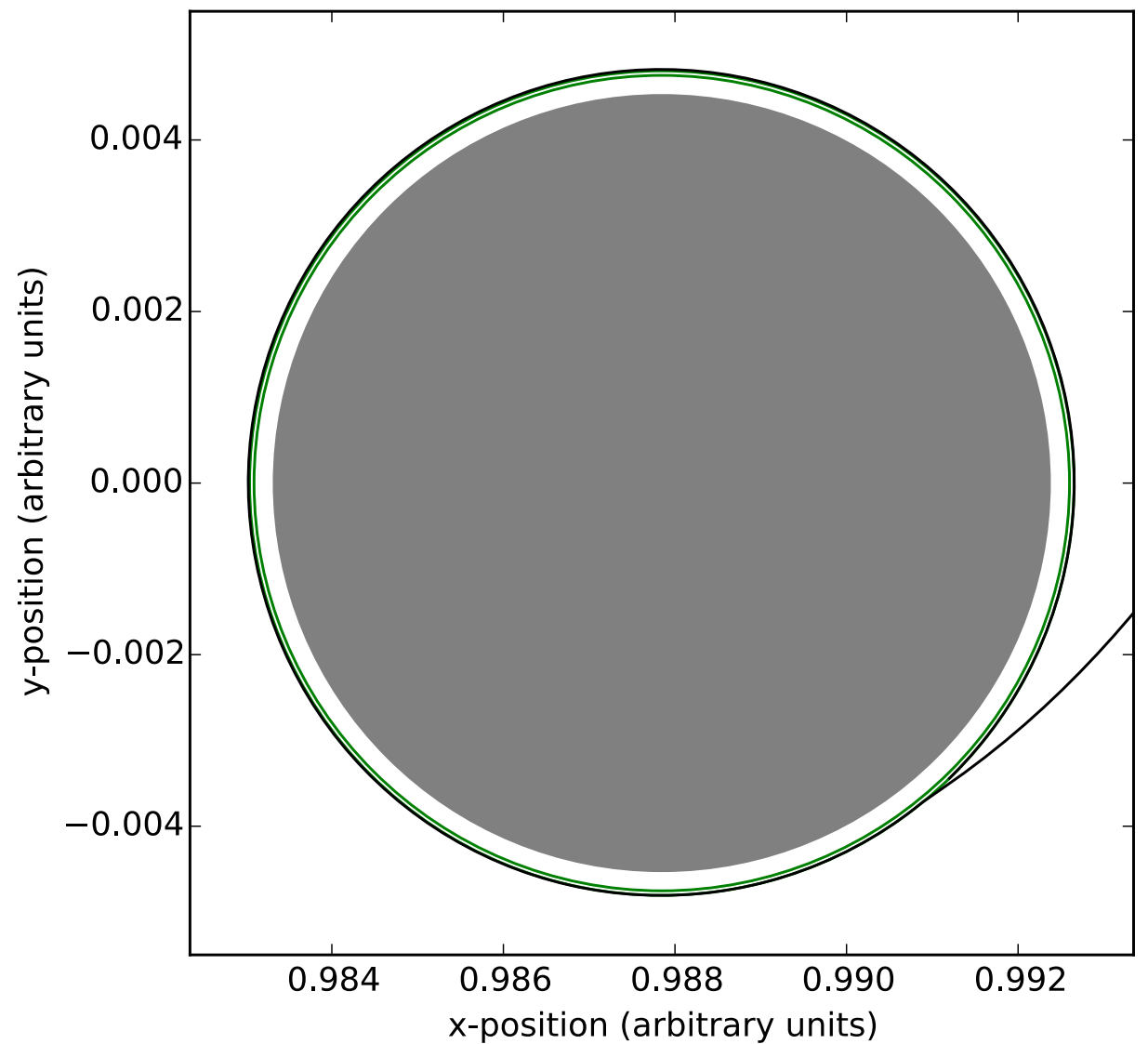
Entering Moon Orbit

ΔV_{earth}



Exit from Earth orbit

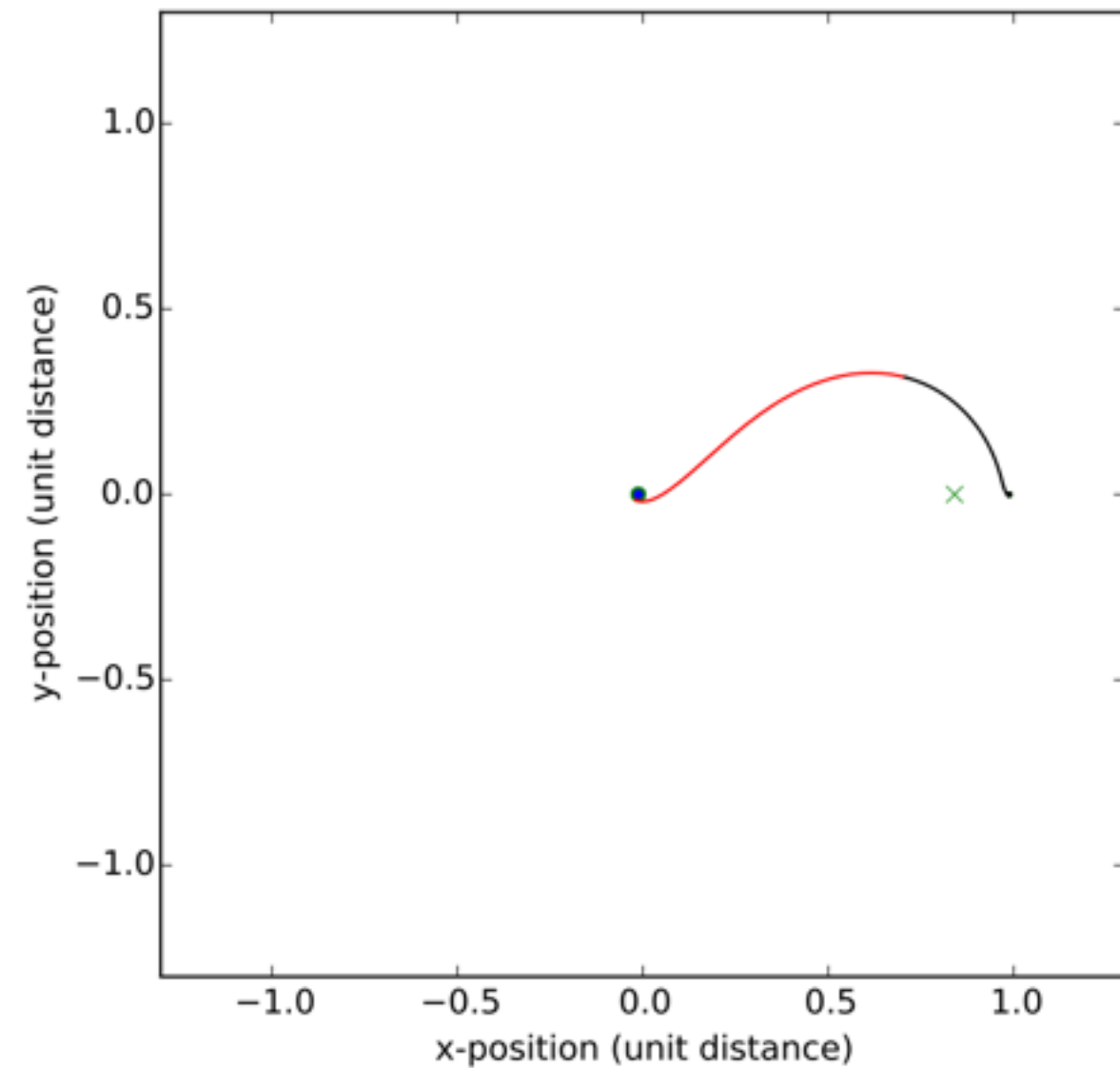
ΔV_{moon}



Entry to Moon orbit (100 ± 10 km)

Searching for Transfer Orbits

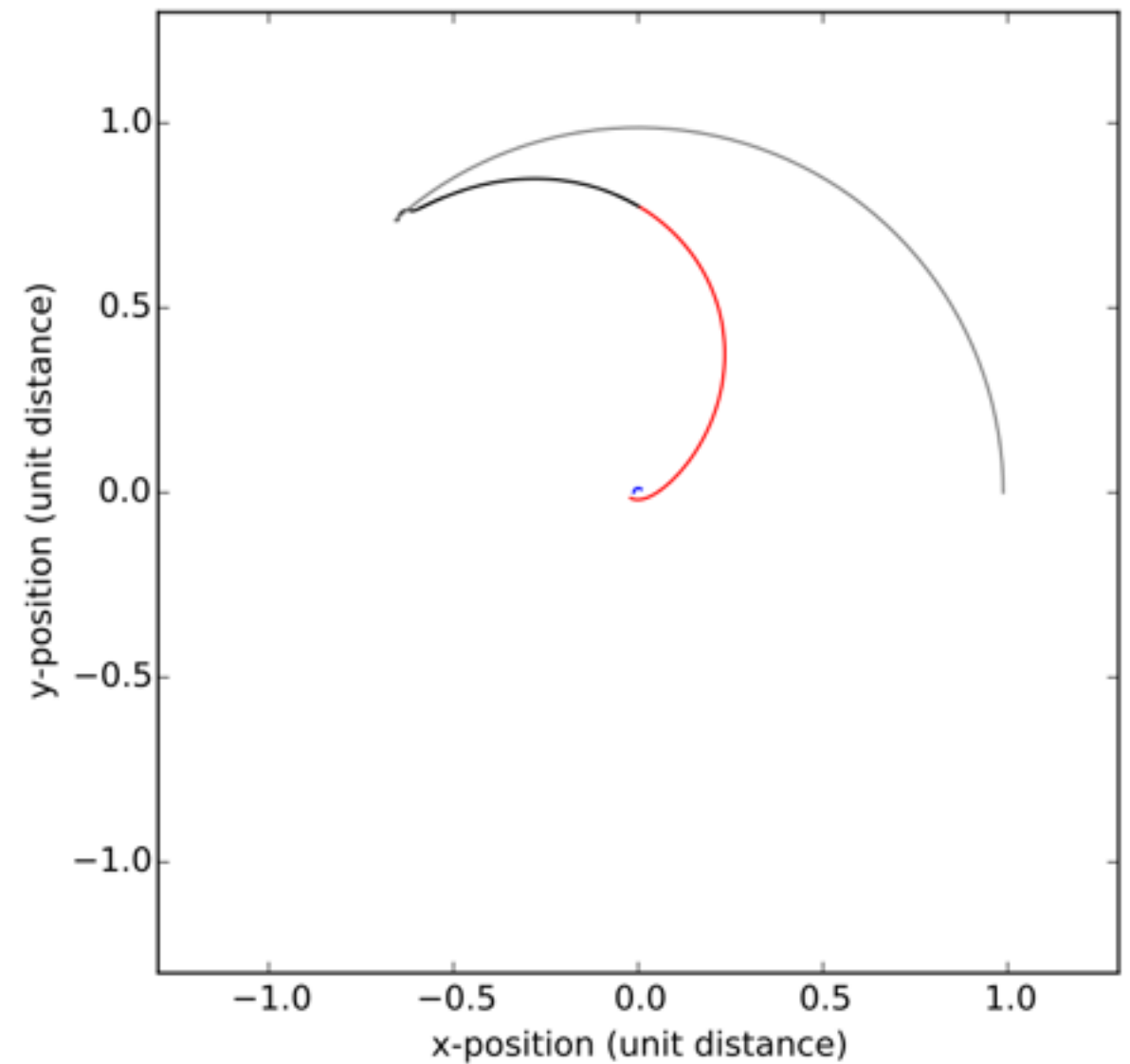
Hohmann Transfer Orbit to the Moon



(x, y)

$$\Delta v_{\text{total}} = 3912 \text{ m/s}$$

$$t_H = 4.3 \text{ days}$$



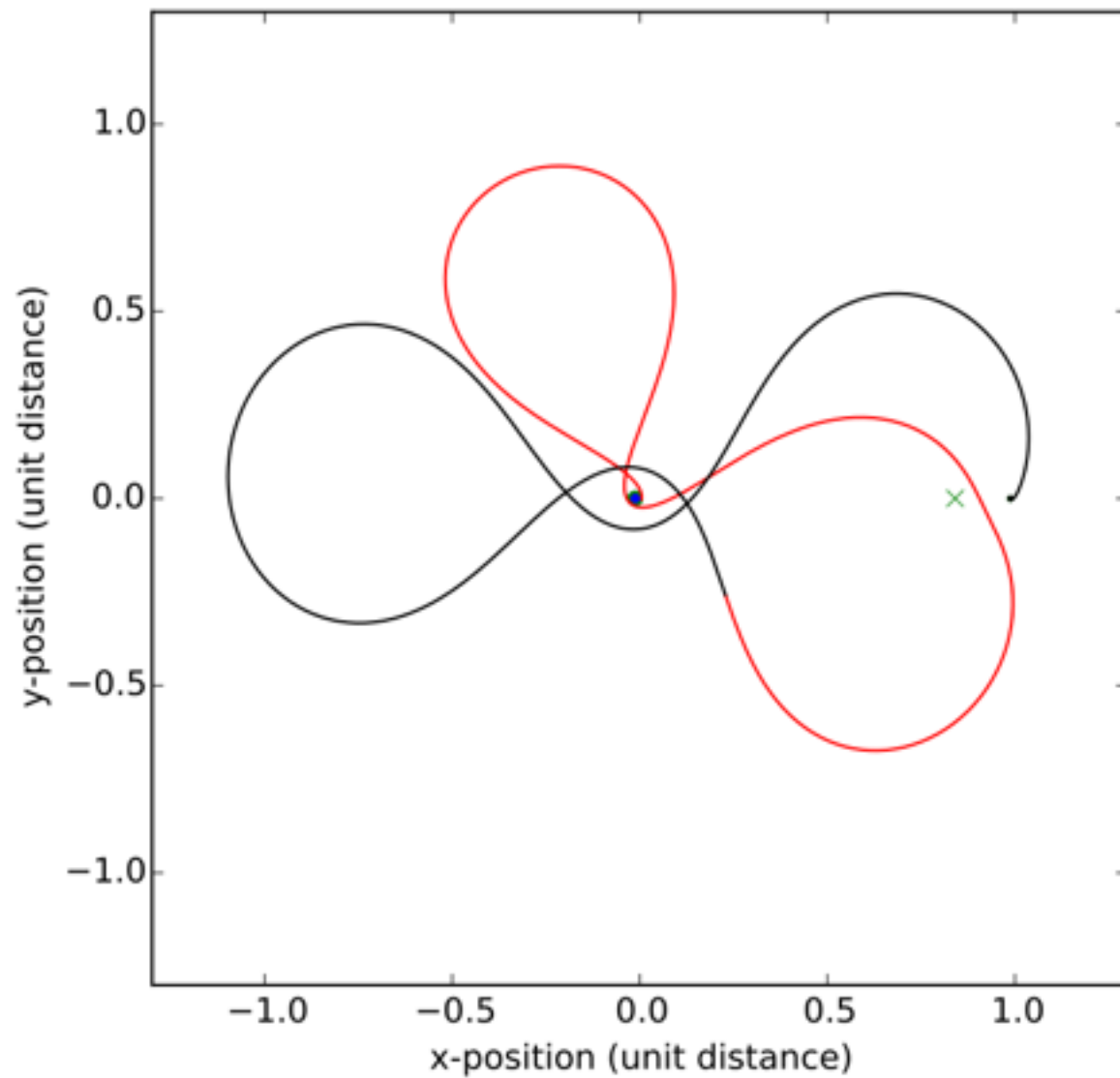
(x, y)

From simple model:

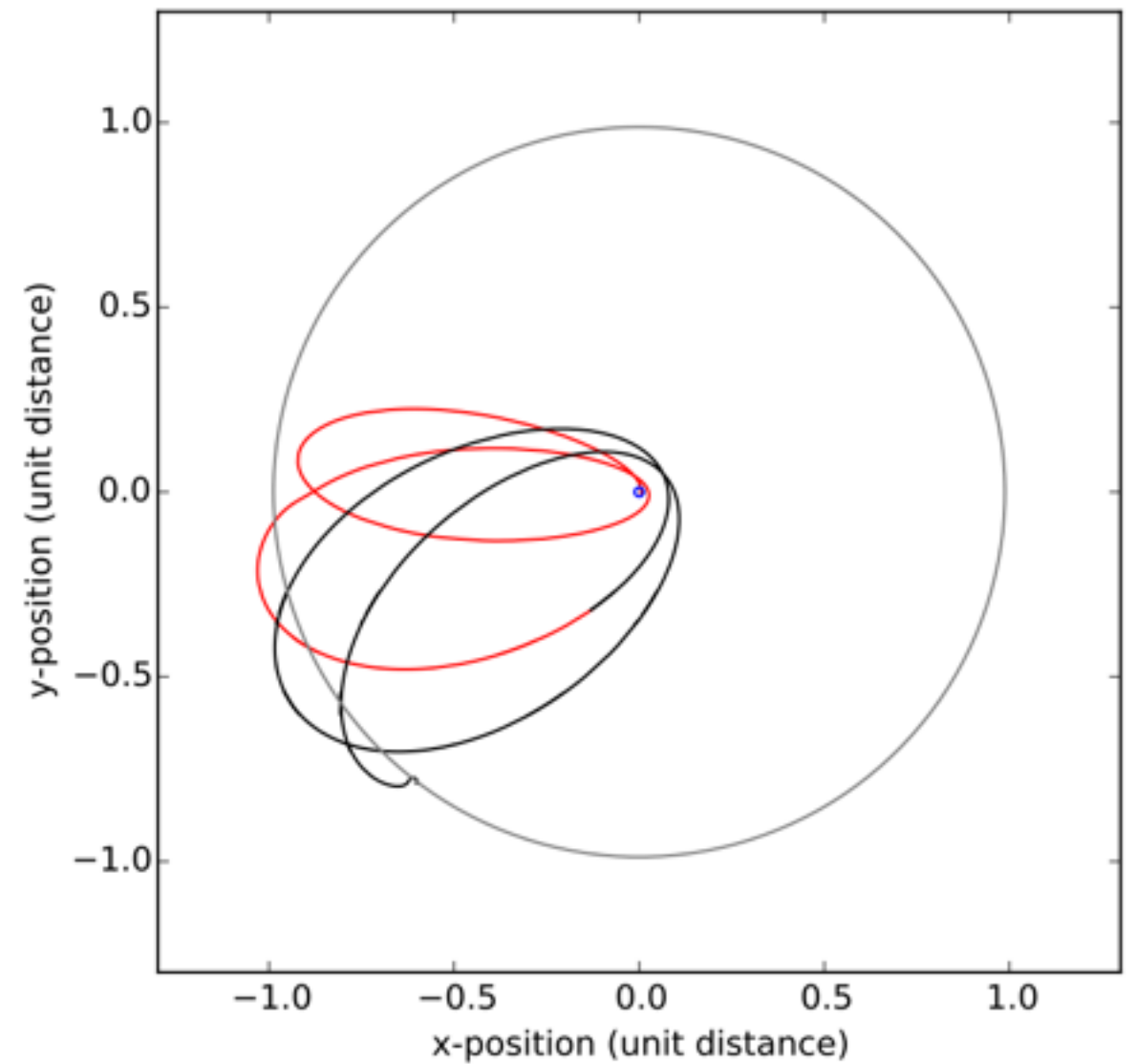
$$\Delta v_{\text{Total}} = 3946 \text{ m/s}$$

Searching for Transfer Orbits

Low-Energy Transfer Orbit (short)



(x, y)



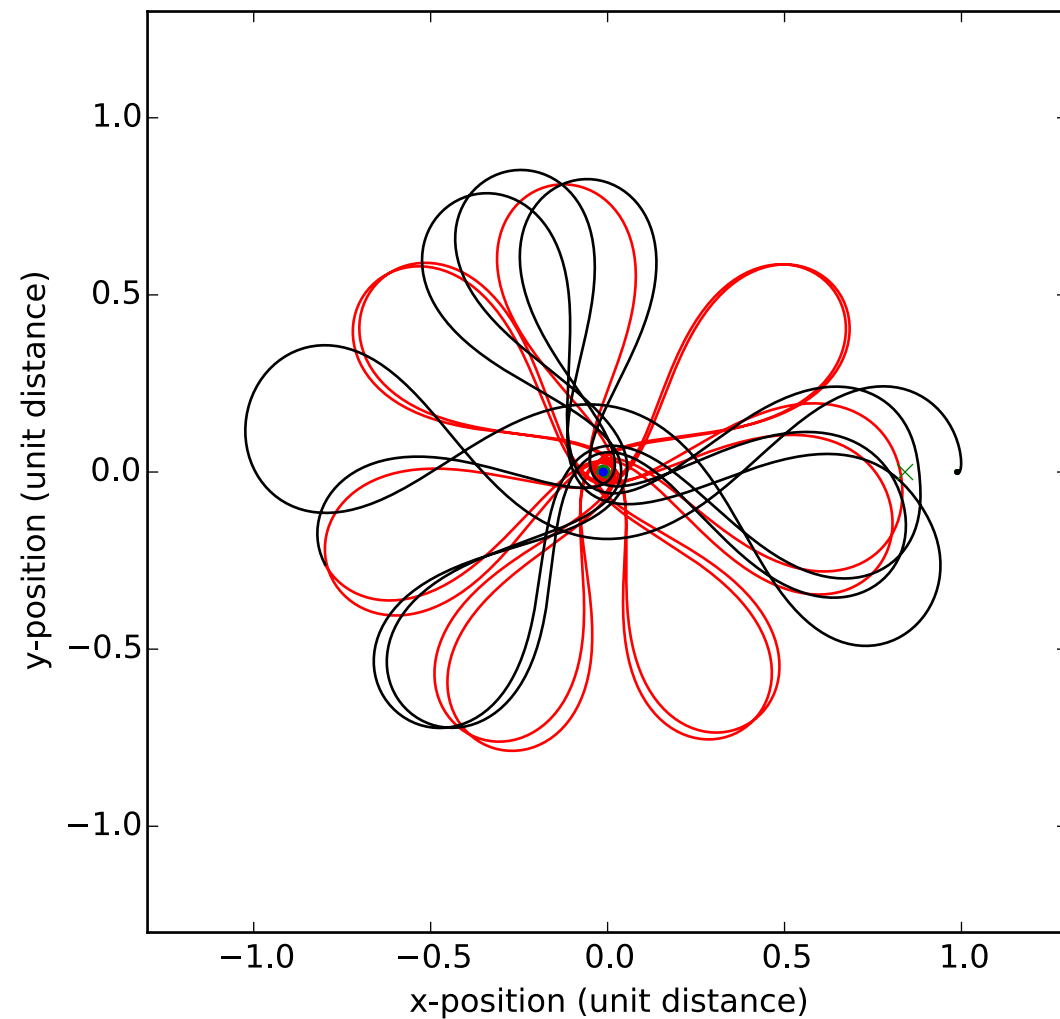
(x, y)

$$\Delta v_{\text{total}} = 3896 \text{ m/s}$$

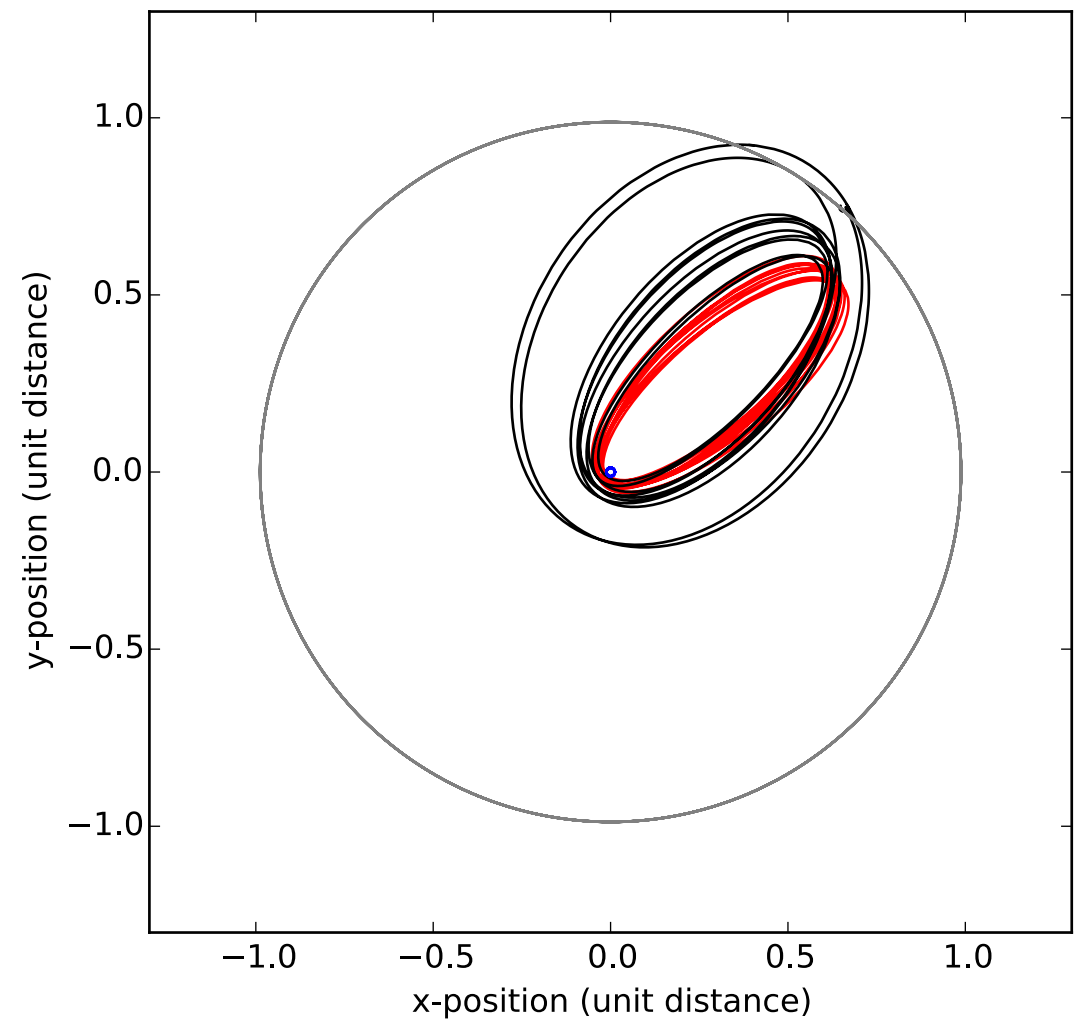
$$t_{\text{short}} = 41 \text{ days}$$

Searching for Transfer Orbits

Hohmann Transfer Orbit to the Moon



(x, y)



(x, y)

$$\Delta v_{\text{total}} = 3795 \text{ m/s}$$

$$t_{\text{long}} = 194 \text{ days}$$

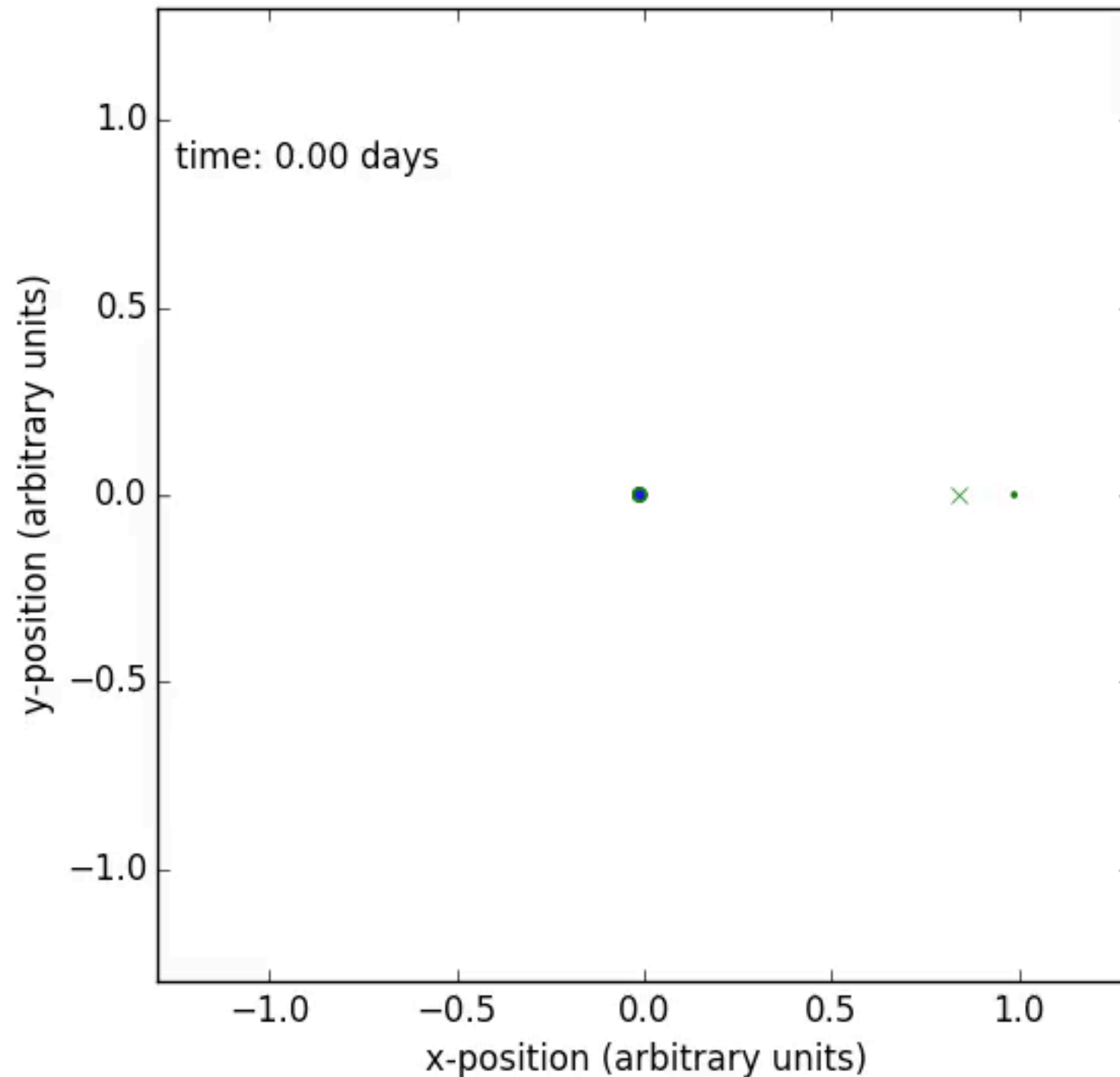
Results Summarized

All transfer orbits

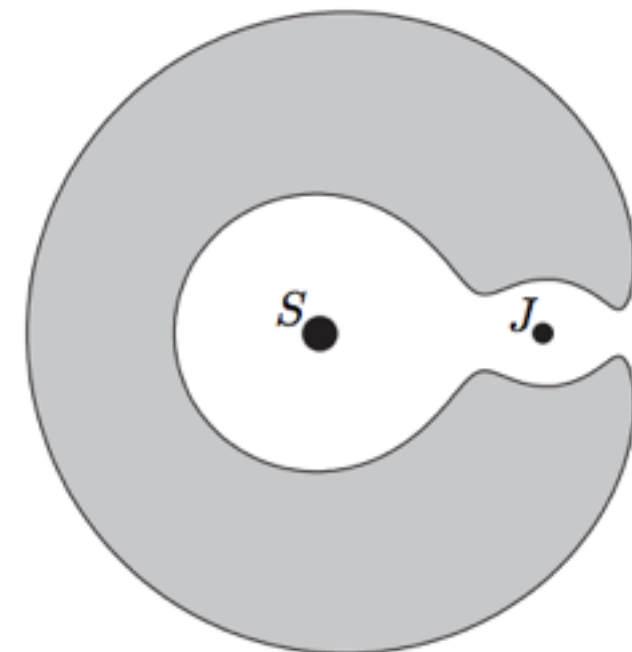
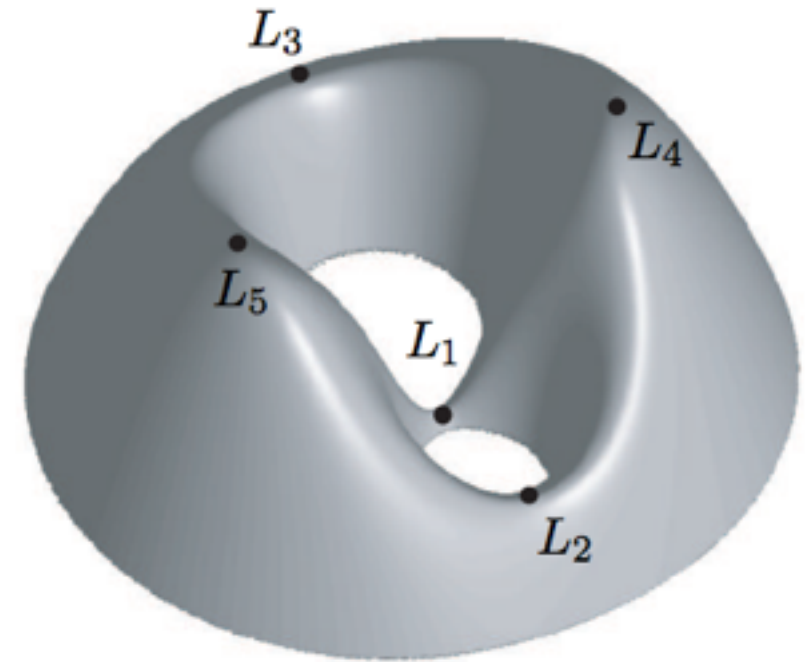
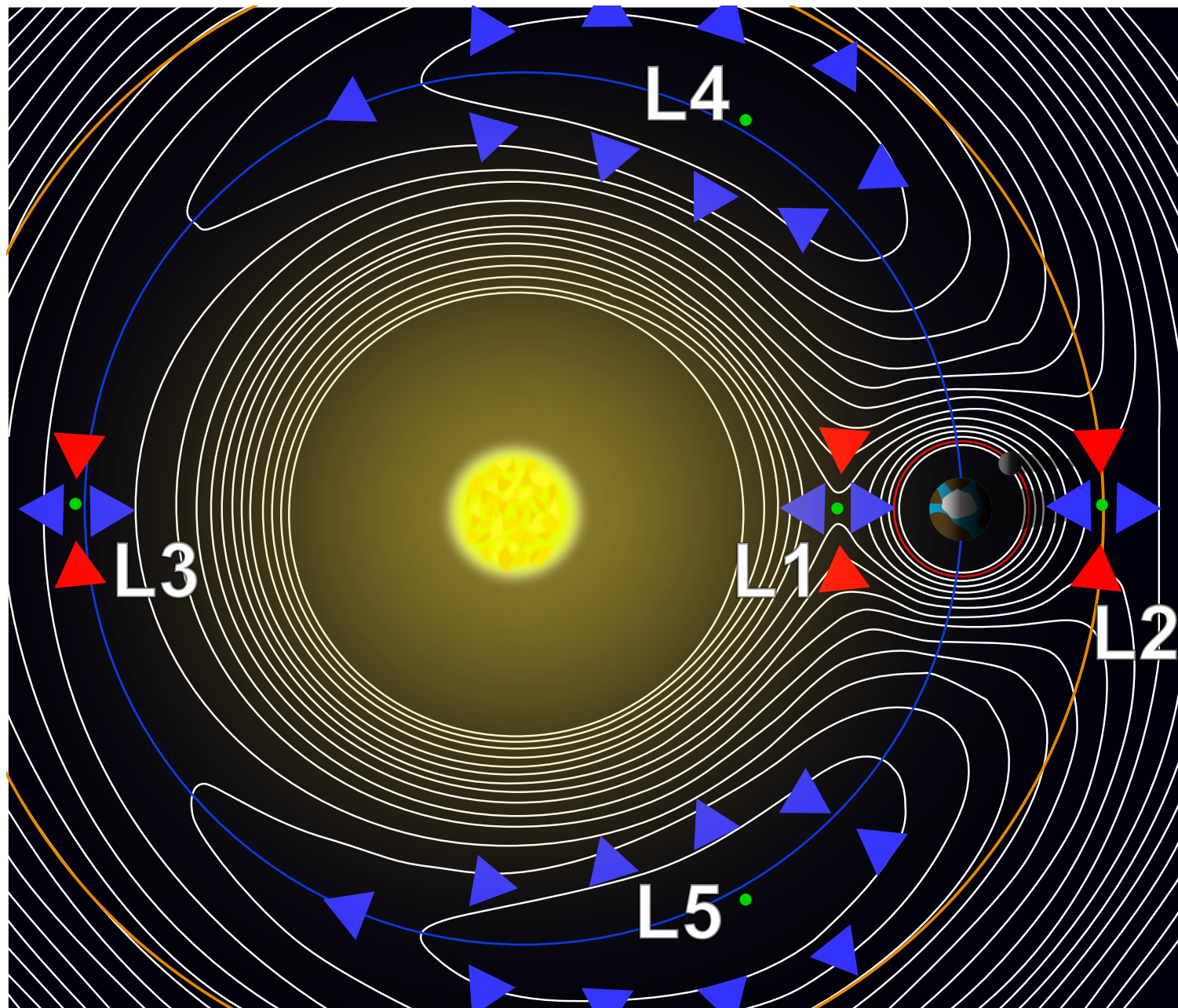
Trajectory	Flight time	Δv_{total} (km/s)	Δv_{earth} (km/s)	Δv_{moon} (km/s)
Minimum	N/A	3.721	3.099	0.622
Long LETO	194 days	3.795	3.091	0.704
Belbruno-Miller	3 months	3.838	3.187	0.651
Topputo	8 months	3.895	3.265	0.630
Short LETO	41 days	3.896	3.127	0.769
Hohmann - long (sim)	4.3 days	3.912	3.111	0.801
Hohmann - (model)	5.0 days	3.946	3.144	0.802
Hohmann - medium (sim)	3.00 days	4.015	3.136	0.880
Apollo (Hohman)	3.05 days	4.115	3.048	1.067
Hohmann - short (sim)	1.00 days	6.823	3.809	3.014

LETO Short Animation

0.043484 days/frame, 60 FPS



Gravitational Potential and Lagrange Points



Wishlist

- Earth \rightarrow L₁ (forward) + L₁ \rightarrow Moon (backward)
- 2D \rightarrow 3D
- Include Sun's potential
- Higher-order integrator:
4.-5.-order symplectic Runge-Kutta