In[12]:=
```
Quiet@Remove["`*"]
```

In[13]:=
```
SetOptions[SelectedNotebook[],
 PrintingStyleEnvironment → "Printout", ShowSyntaxStyles → True]
```

Setup

In[14]:=
```
(*Pariapsis: Point of least distance in elliptical orbit
  Apoapsis: Point of greatest distance in elliptical orbit*)
```

In[15]:=
```
μE = UnitConvert[G * PlanetData[Earth (planet), "Mass"], "km^3/s^2"]
```

Out[15]= $3.986 \times 10^5\ \text{km}^3/\text{s}^2$

In[16]:=
```
μM = UnitConvert[G * PlanetaryMoonData[Moon (planetary moon), "Mass"], "km^3/s^2"]
```

Out[16]= $4.90 \times 10^3\ \text{km}^3/\text{s}^2$

In[17]:=
```
(*Distance of Satellite's cicular orbit around Earth in Earth system*)
r1 = PlanetData[Earth (planet), "Radius"] + 160 km
```

Out[17]= 6527.4447 km

In[18]:=
```
(*Distance of Satellite's elliptical
 orbit around Earth at apoapsis in Earth system*)
r2 = PlanetaryMoonData[Moon (planetary moon), "AverageDistanceFromEarth"]
```

Out[18]= 3.850×10^5 km

In[19]:=
```
(*Distance of Satellite's circular orbit around Moon in Moon system*)
```

In[20]:=
```
r3 = PlanetaryMoonData[Moon (planetary moon), "Radius"] + 100 km
```

Out[20]= 1837.5 km

Hohmann transfer orbit to the Moon

Impulse A

In[21]:=
```
(*Speed in circular orbit, distance r1, at point A in Earth system*)
```

$$\text{vEarth} = \sqrt{\frac{\mu E}{r1}}$$

Out[21]= 7.814 km/s

In[22]:=
```
(*Speed in elliptical orbit, distance r1, at point A in Earth system*)
```

$$\text{vp} = \sqrt{\frac{\mu E}{r1} \frac{2\ r2}{r1 + r2}}$$

Out[22]= 10.96 km/s

In[23]:= (*Necessary Δv in point A, a speed-up of 3.14 km/s*)

$$\Delta vEarth = \sqrt{\frac{\mu E}{r1}} \left(\sqrt{\frac{2\ r2}{r1 + r2}} - 1\right) \text{ // N (*vp-vEarth*)}$$

Out[23]= 3.14423 km/s

Impulse B - neglecting circular motion around moon

In[24]:= (*Speed in elliptical orbit, distance r2, at point B in Earth system*)

$$va = \sqrt{\frac{\mu E}{r2} \frac{2\ r1}{r1 + r2}}$$

Out[24]= 0.1858 km/s

In[25]:= (*Speed in circular orbit, distance r2, at point B in Earth system (i.e. speed of the Moon in Earth system)*)

$$vNoMoon = \sqrt{\frac{\mu E}{r2}}$$

Out[25]= 1.017 km/s

In[26]:=

In[27]:= (*Change of speed needed to "follow along with the moon", a speed-up of 0.831 km/s*)

$$\Delta vNoMoon = \sqrt{\frac{\mu E}{r2}} \left(1 - \sqrt{\frac{2\ r1}{r1 + r2}}\right) \text{ // N (*v3NoMoon-v2B*)}$$

Out[27]= 0.831692 km/s

Moon circular orbit correction

In[28]:= (*However we can't follow the moon stationary along it's orbit since the Moon attracts the satellite and it would crash into the surface. We need to go into a circular orbit around the moon*)

In[29]:= (*Speed v3 in circular orbit, distance 100 km above the Moon surface, in Moon system*)

$$vMoon = \sqrt{\left(\frac{\mu M}{r3}\right)} \text{ // N}$$

Out[29]= 1.63342 km/s

In[30]:= (*We can choose to add or subtract this value to ΔvBNoMoon since we can go around in orbit both ways. Subtracting v3 from ΔvBNoMoon gives numerically the smallest Δv so that's what we'll do*)

In[31]:= (*Change in speed needed to go from geocentric elliptical orbit to circular Moon orbit, a speed-down of 0.802 km/s*)

ΔvMoon = ΔvNoMoon - vMoon

Out[31]= -0.801723 km/s

Total

In[32]:=
```
(*Regarding fuel/energy cost,
we're interested in in how much we need to change the speed,
irregardless of directio. Total Δv for
 the whole trip: impulse at A plus impulse at B*)
ΔvTotal = Abs[ΔvEarth] + Abs[ΔvMoon] // N
```

Out[32]= 3.94595 km/s

In[33]:=
```
(*Result if we had neglected the circular motion
 around the moon at the end. Close, but not identical*)
ΔvTotalNoMoon = Abs[ΔvMoon] + Abs[ΔvNoMoon]
```

Out[33]= 1.63342 km/s

Flight time

In[34]:= tHohmann = UnitConvert$\left[\frac{1}{2}\sqrt{\frac{4\,\pi\text{^}2 * \left(\frac{r1+r2}{2}\right)\text{^}3}{\mu E}}, \text{"Days"}\right]$

Out[34]= 4.99 days

Trying to minimize ΔvTotal by choosing appropriate moon oribt altitude

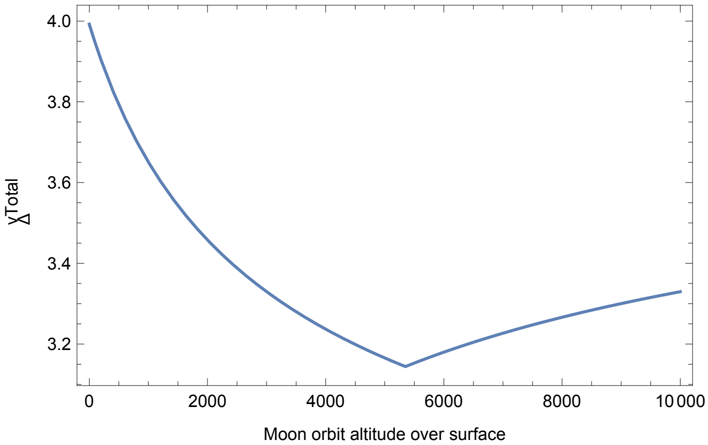
In[35]:= RMoon = PlanetaryMoonData[Moon (planetary moon), "Radius"]

Out[35]= 1737.5 km

Plot

In[36]:=
$$\text{Plot}\left[\text{Abs}\left[\sqrt{\frac{\mu E}{r1}}\left(\sqrt{\frac{2\,r2}{r1+r2}}-1\right)\right]+\text{Abs}\left[\sqrt{\frac{\mu E}{r2}}\left(1-\sqrt{\frac{2\,r1}{r1+r2}}\right)-\sqrt{\left(\frac{\mu M}{\text{RMoon}+\text{altitude}}\right)}\ /.\ \text{altitude}\to\text{Quantity}[x,\text{"Kilometers"}]\right],\{x,0,10^{\wedge}4\},\text{Frame}\to\text{True},\text{FrameLabel}\to\{\text{"Moon orbit altitude over surface"},\text{"}\Delta\text{vTotal"}\}\right]$$

Out[36]=



Moon orbit altitude that gives minimum ΔvTotal

In[37]:=
$$\text{Solve}\left[\sqrt{\frac{\mu E}{r2}}\left(1-\sqrt{\frac{2\,r1}{r1+r2}}\right)-\sqrt{\left(\frac{\mu M}{\text{RMoon}+\text{altitude}}\right)}==0,\ \text{altitude}\right]\ //\ N$$

Out[37]= {{altitude → 5350.04 km}}

Minimum ΔvTotal

In[38]:=
$$\Delta\text{vTotalMin}=\text{Abs}\left[\sqrt{\frac{\mu E}{r1}}\left(\sqrt{\frac{2\,r2}{r1+r2}}-1\right)\right]+\text{Abs}\left[\sqrt{\frac{\mu E}{r2}}\left(1-\sqrt{\frac{2\,r1}{r1+r2}}\right)-\sqrt{\left(\frac{\mu M}{\text{RMoon}+x}\right)}\ /.\ x\to 5350.039331517406060286 9\grave{}2.9098829878759\ \text{km}\right]\ //\ N$$

Out[38]= 3.14423 km/s

In[39]:= (*Yes but not practical*)

Moon hill sphere

In[40]:=
```
rHillMoon = N@(PlanetaryMoonData[Moon (planetary moon), "Mass"]/
    (3 * PlanetData[Earth (planet), "Mass"]))^(1/3)
```

Out[40]= 0.160053

In[41]:=
```
rHillMoon * PlanetaryMoonData[Moon (planetary moon), "AverageOrbitDistance"]
```

Out[41]= 61 618.7 km