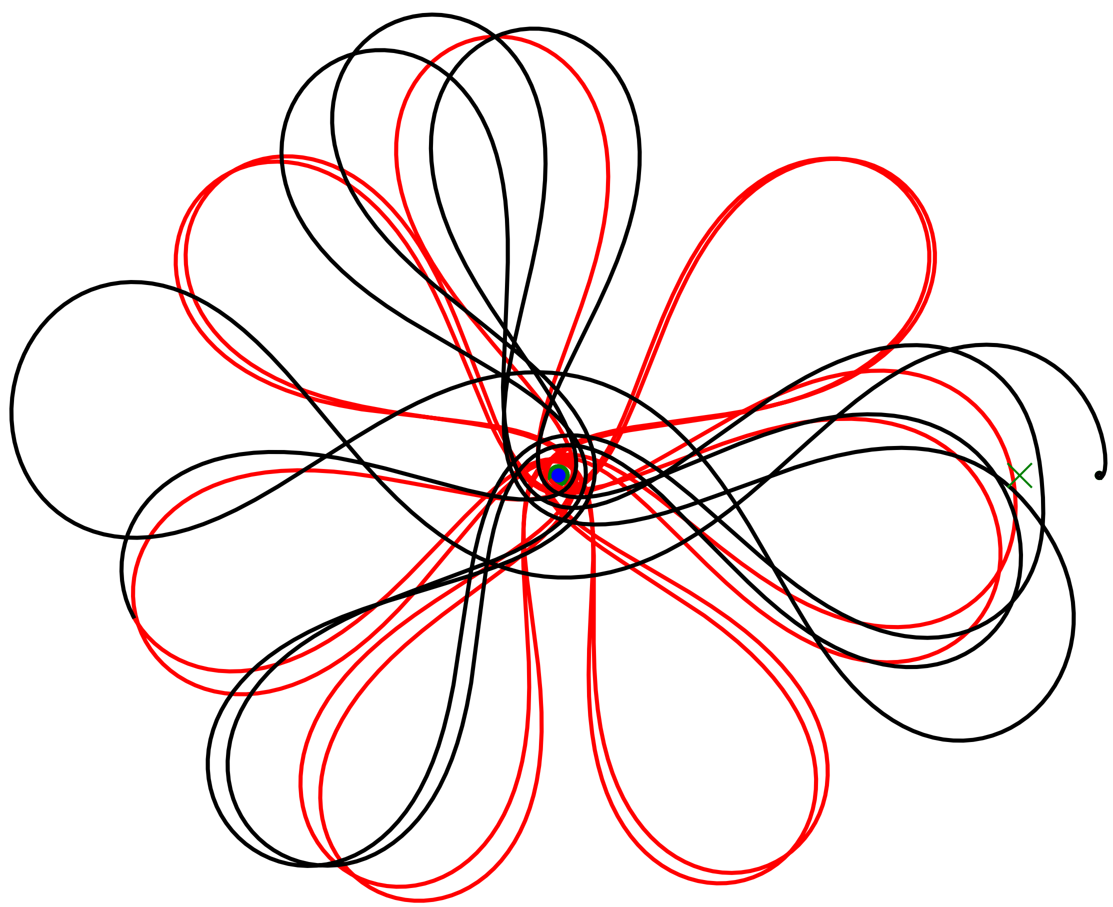
Low-Energy Transfer Orbits



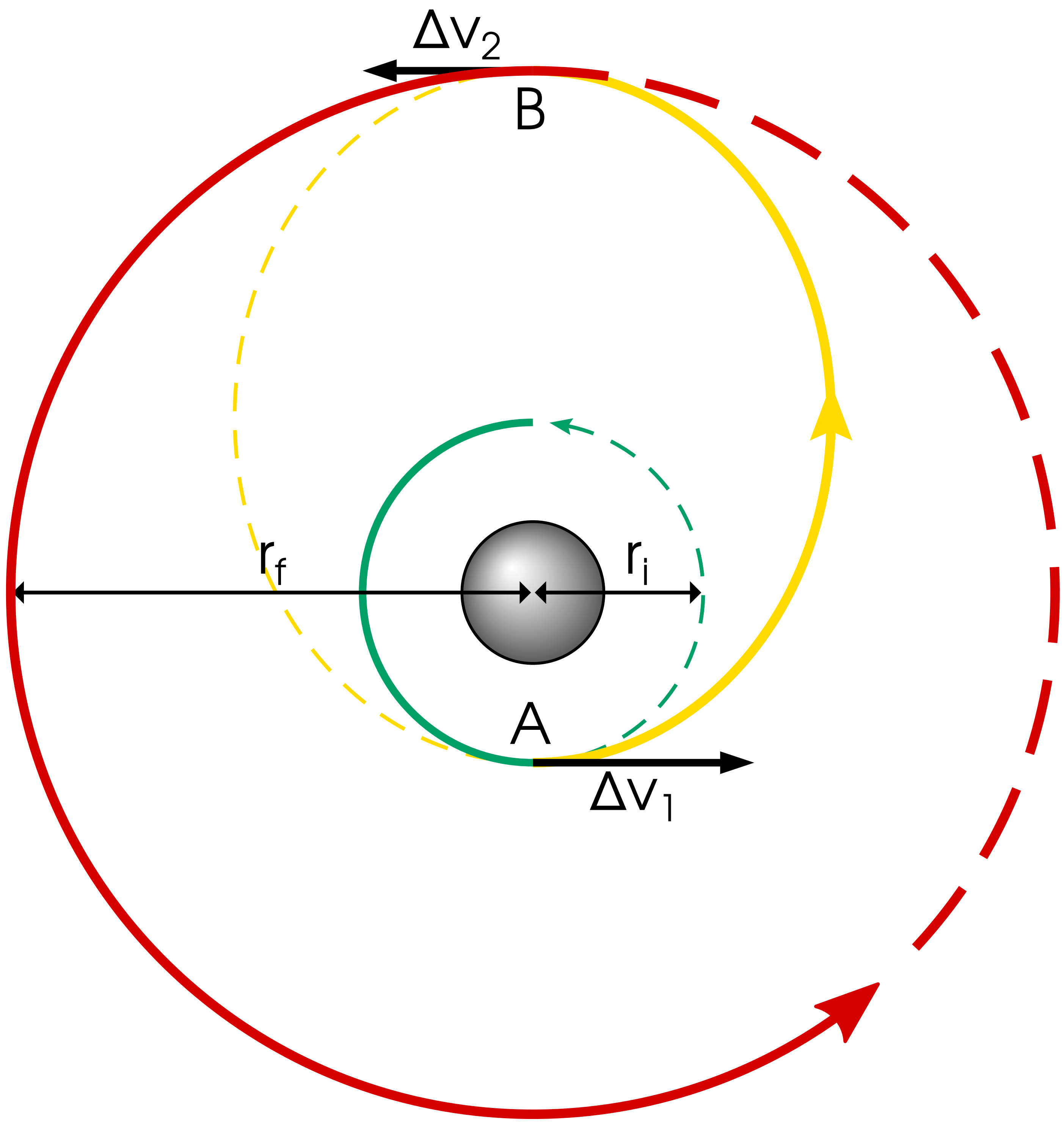
A Theoretical and Numerical Study

Gandalf Saxe, DTU

What are transfer orbits?

A way of getting from A to B in space

Hohmann



Why Are Transfer Orbits Interesting?

NASA manned spacecraft: Orion
Manned Mars Mission: 2030's

(NASA)

"The first crewed mission — called EM-2 — is now scheduled for April 2023; the flight was originally scheduled for August 2021"

- The Verge, September 16th

Mars One manned spacecraft: Dragon
Manned Mars Mission: ~2026

(SpaceX)

"Elon Musk argues that we must put a million people on Mars if we are to ensure that humanity has a future"

- Interview with aeon.co, 30 September 2014

Why Are Transfer Orbits Interesting?

Answer: Fuel efficiency

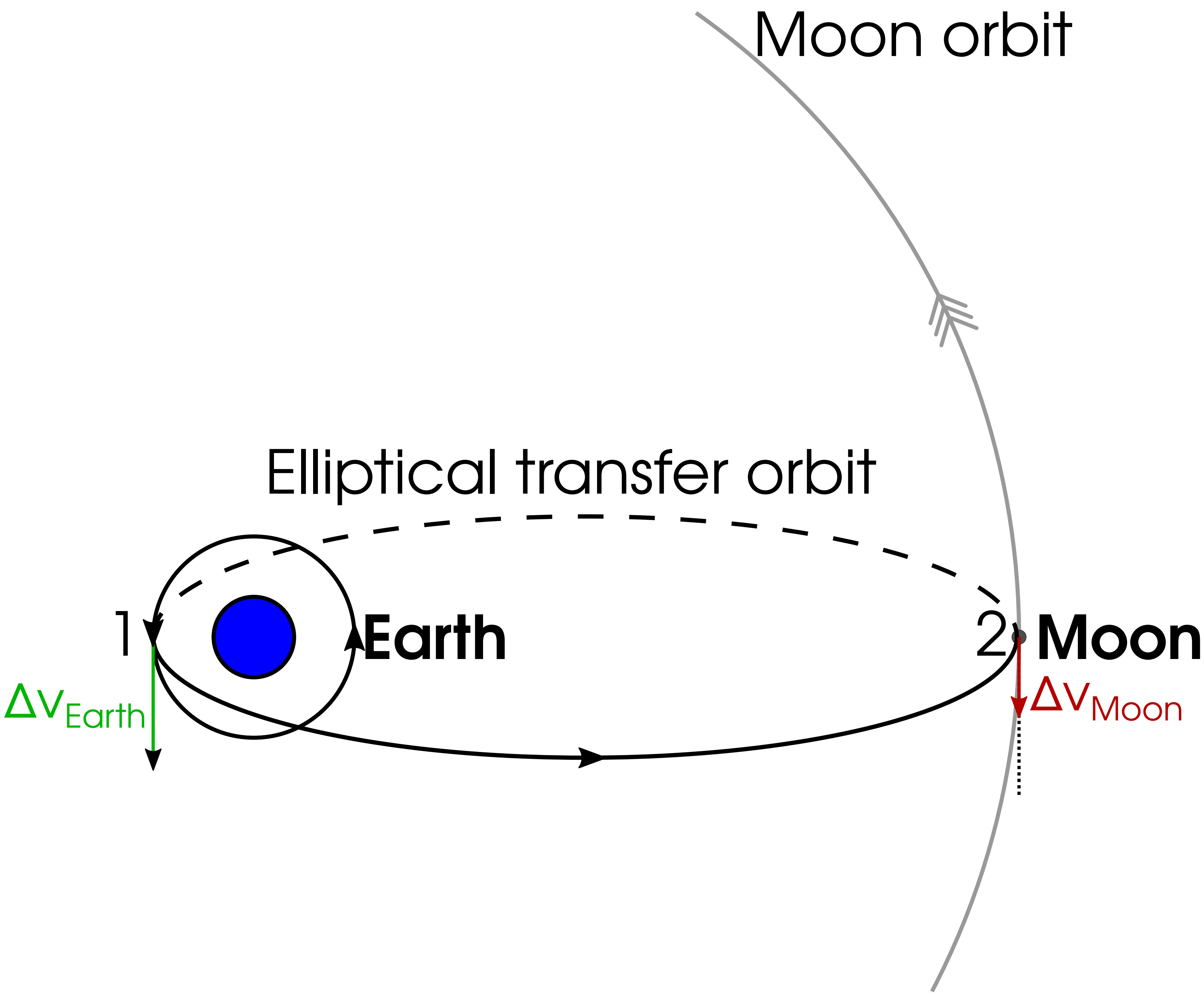
Cost of low Moon orbit: ~ \$100,000 per kg
Cost of moon landing: ~ \$1,000,000 per kg

Low-energy transfer orbit can double the payload to Moon!

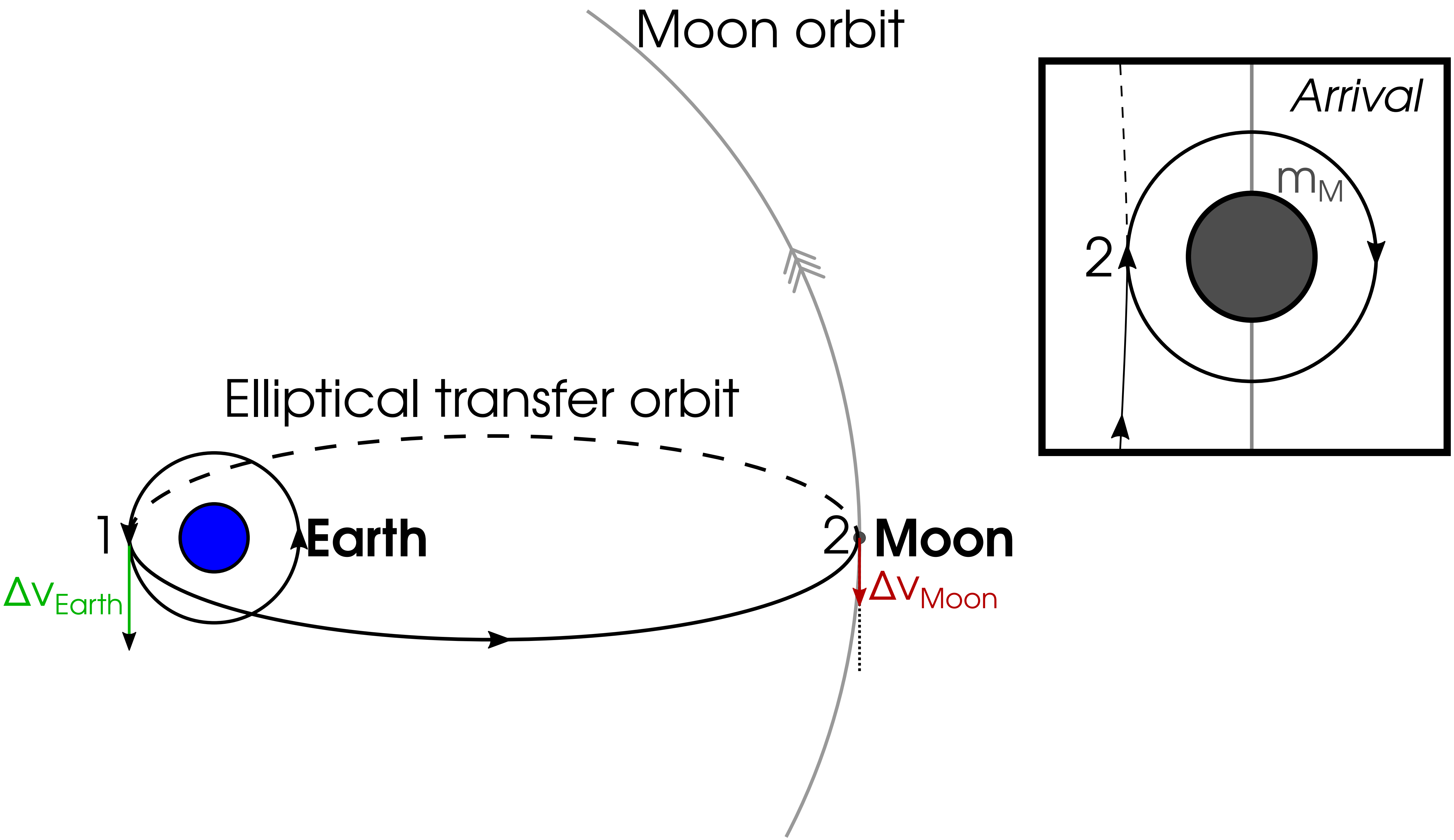
Sources:

- https://www.astrobotic.com/lunar-delivery
- Jacob Akira Okada. Painting the Way to the Moon. 2015.
- Edward a Belbruno and John P Carrico. “Calculation of Weak Stability Boundary Ballistic Lunar Transfer Trajectories”. In: Astrodynamics specialist conference. Denver, Colerado, 2000. doi: doi:10.2514/6.2000-4142.

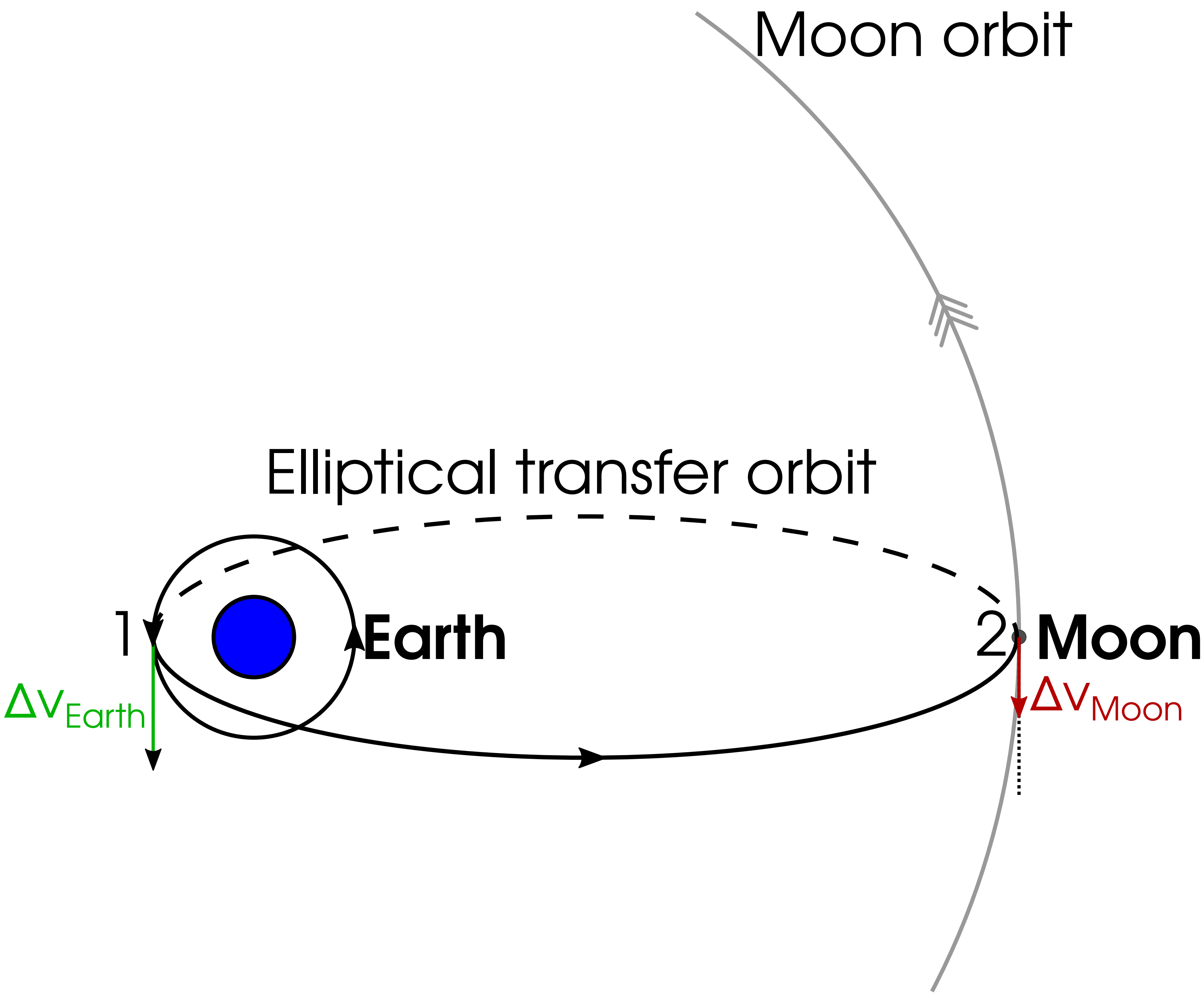
Hohmann Transfer Orbit to the Moon

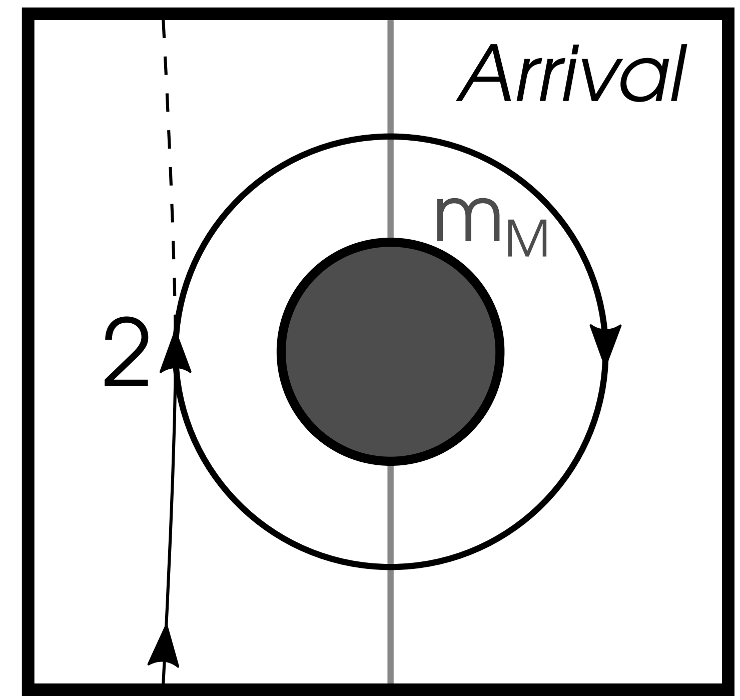


Hohmann Transfer Orbit to the Moon



Hohmann Transfer Orbit to the Moon





$t_H = 5.0\ \mathrm{days}$

$\Delta v_{\mathrm{Earth}} = 3144$ m/s

$\Delta v_{\mathrm{Moon}} = 802$ m/s

$\Delta v_{\mathrm{Total}} = 3946$ m/s

It's all about low Δv

$t_H = 5.0$ days

Δv_{Earth} = 3144 m/s

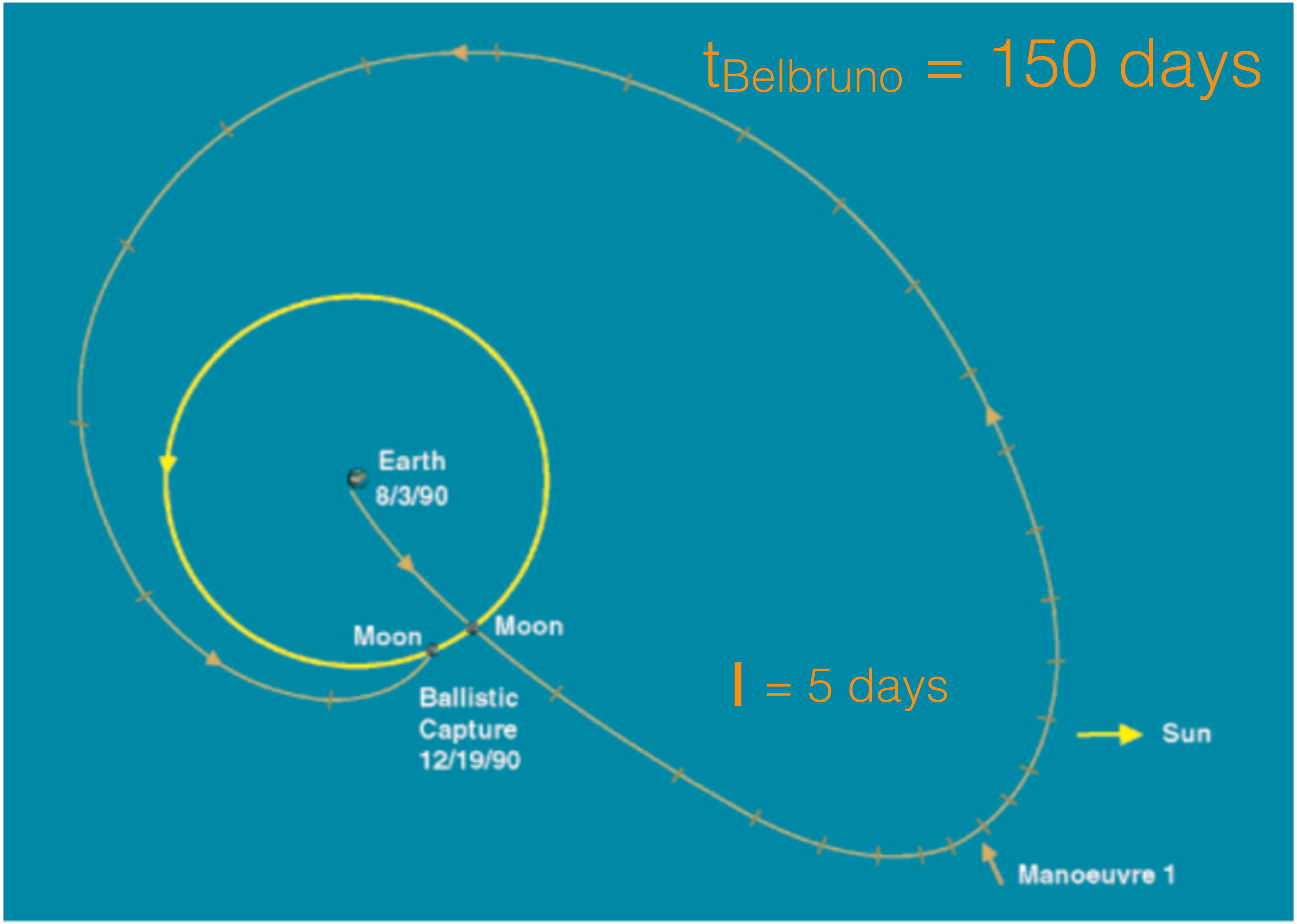
Δv_{Moon} = 802 m/s

Δv_{Total} = 3946 m/s

Find transfer orbits with Δv low as possible!

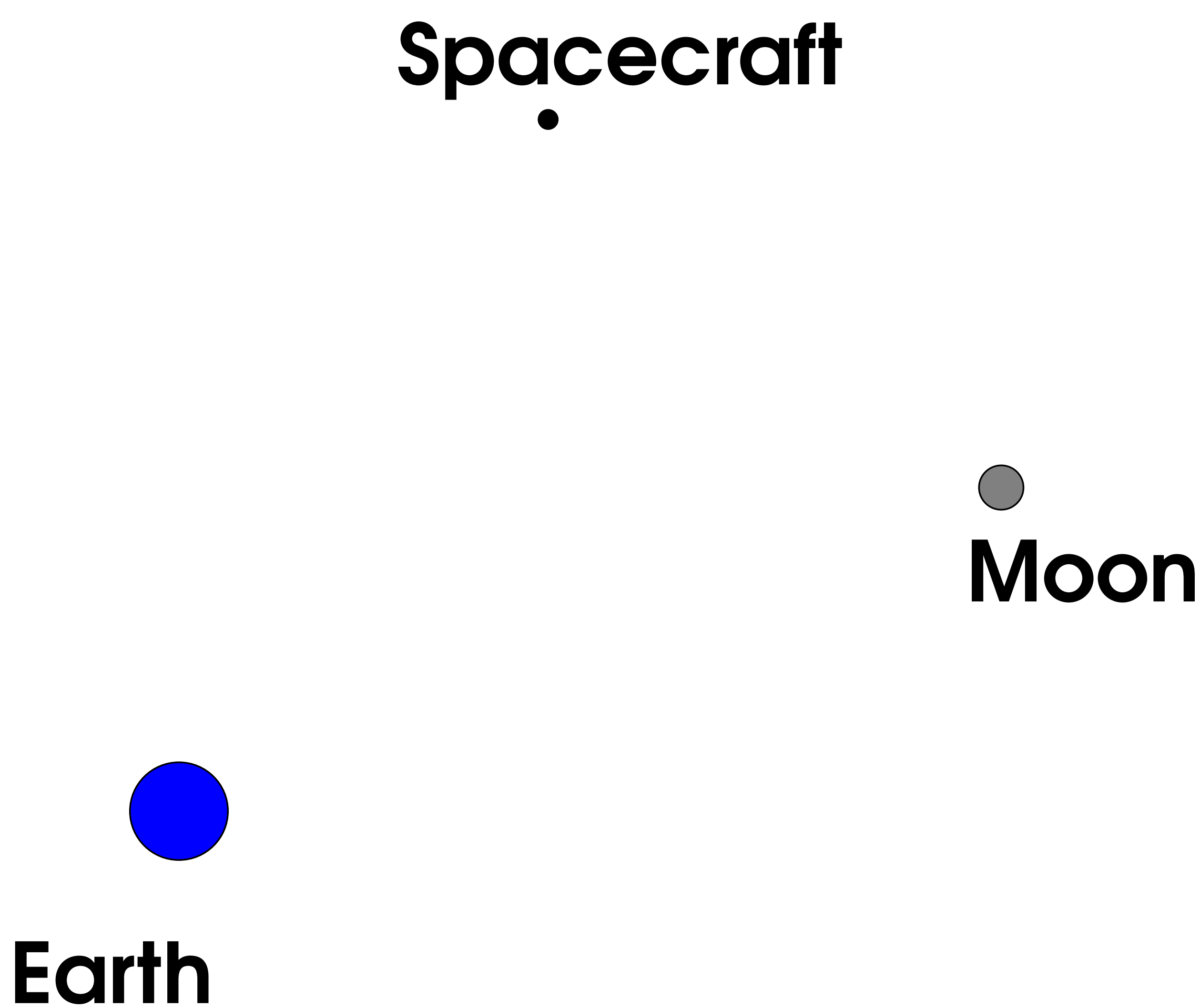
Δv_{Moon} can be reduced up to ~25%

Low Energy Transfer Orbits

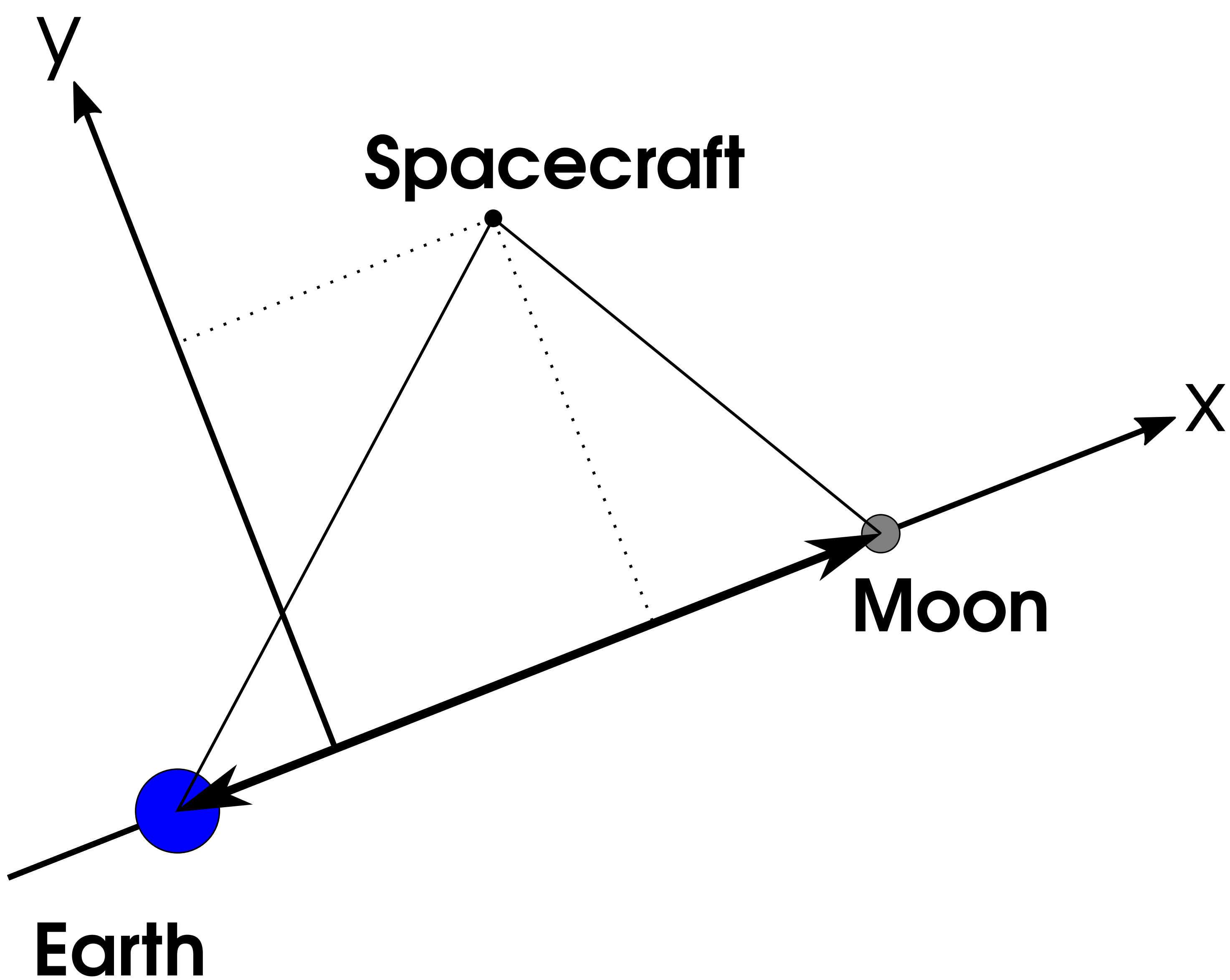


Hiten: Japanese Spacecraft, 1990

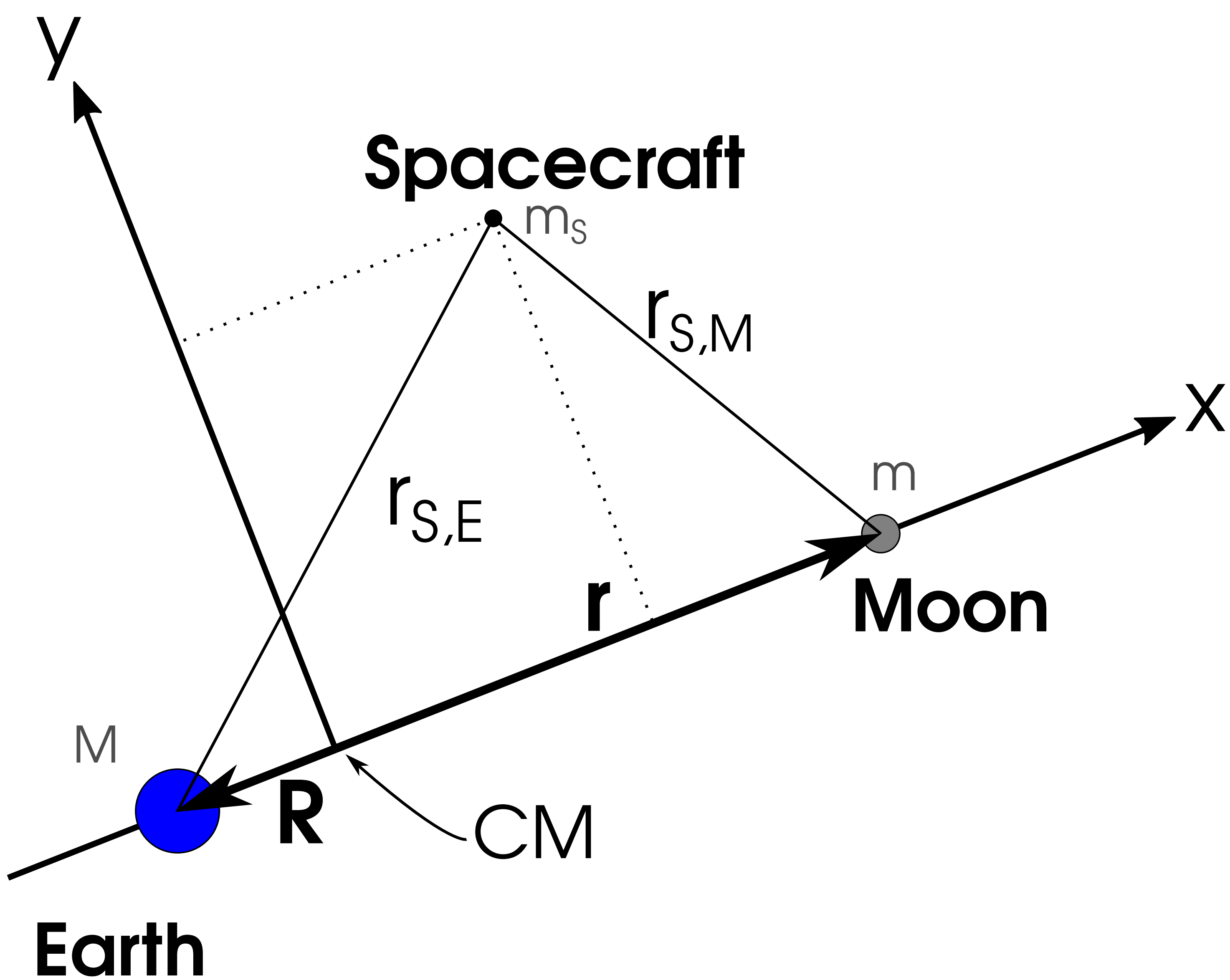
Restricted 3-body problem



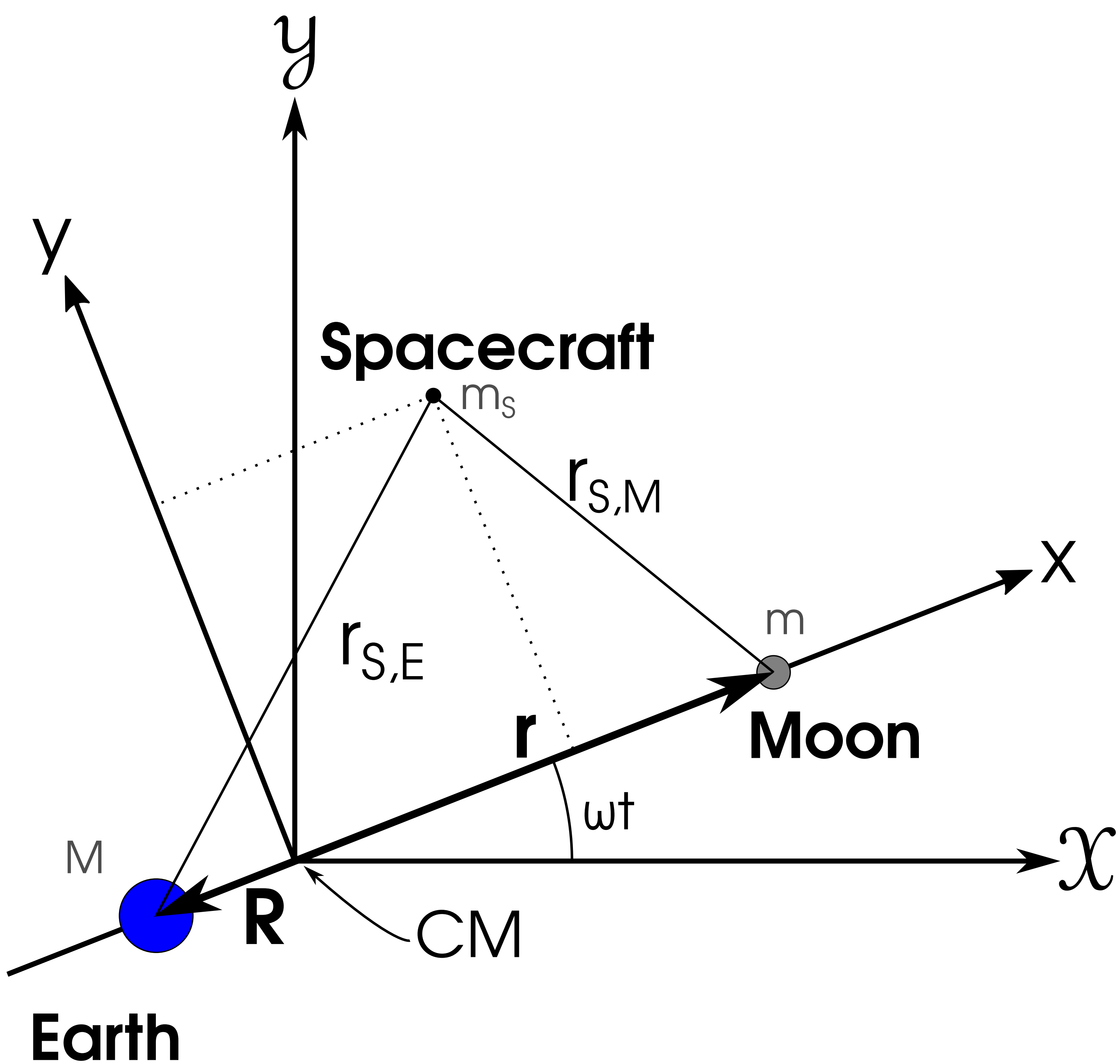
Restricted 3-body problem



Restricted 3-body problem



Restricted 3-body problem



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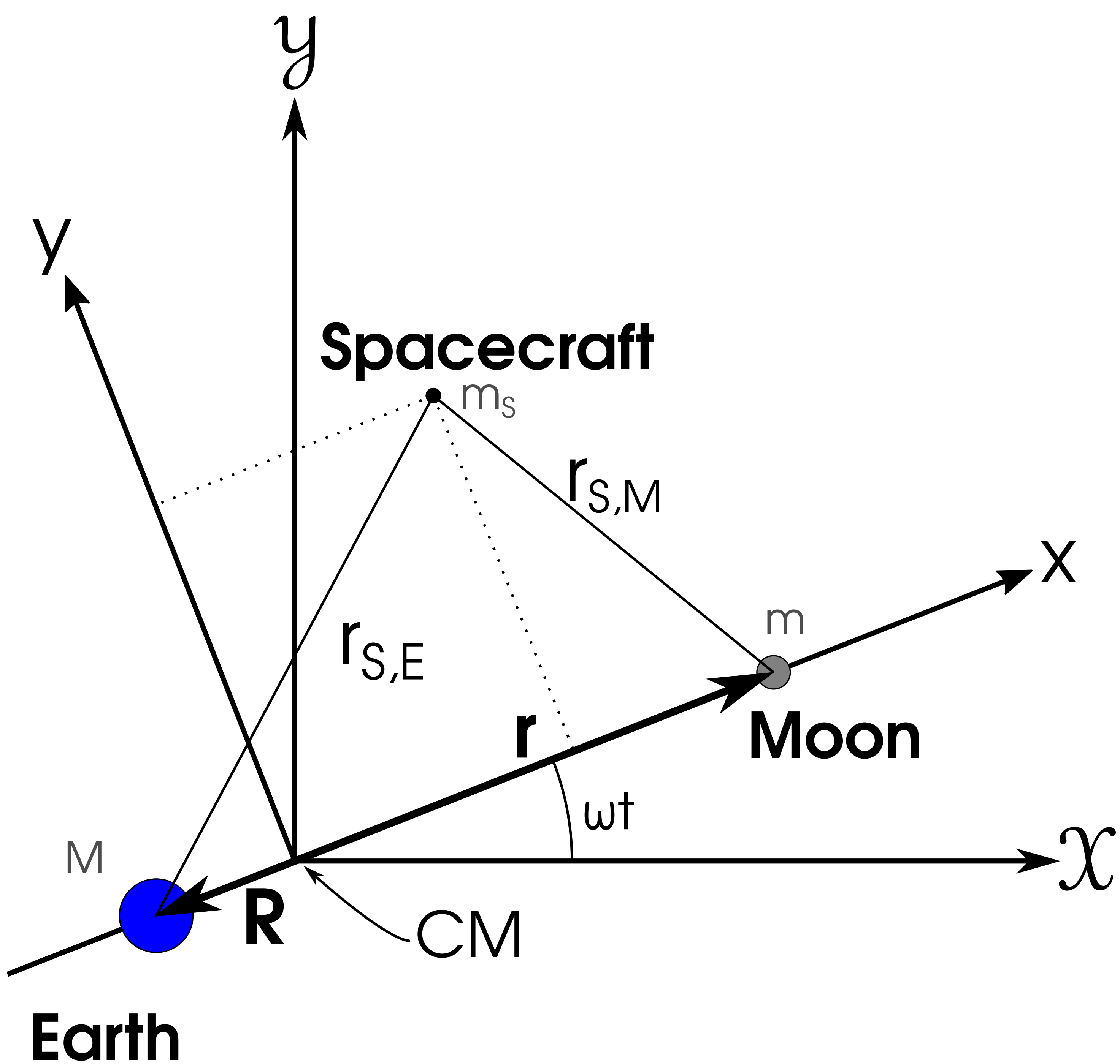
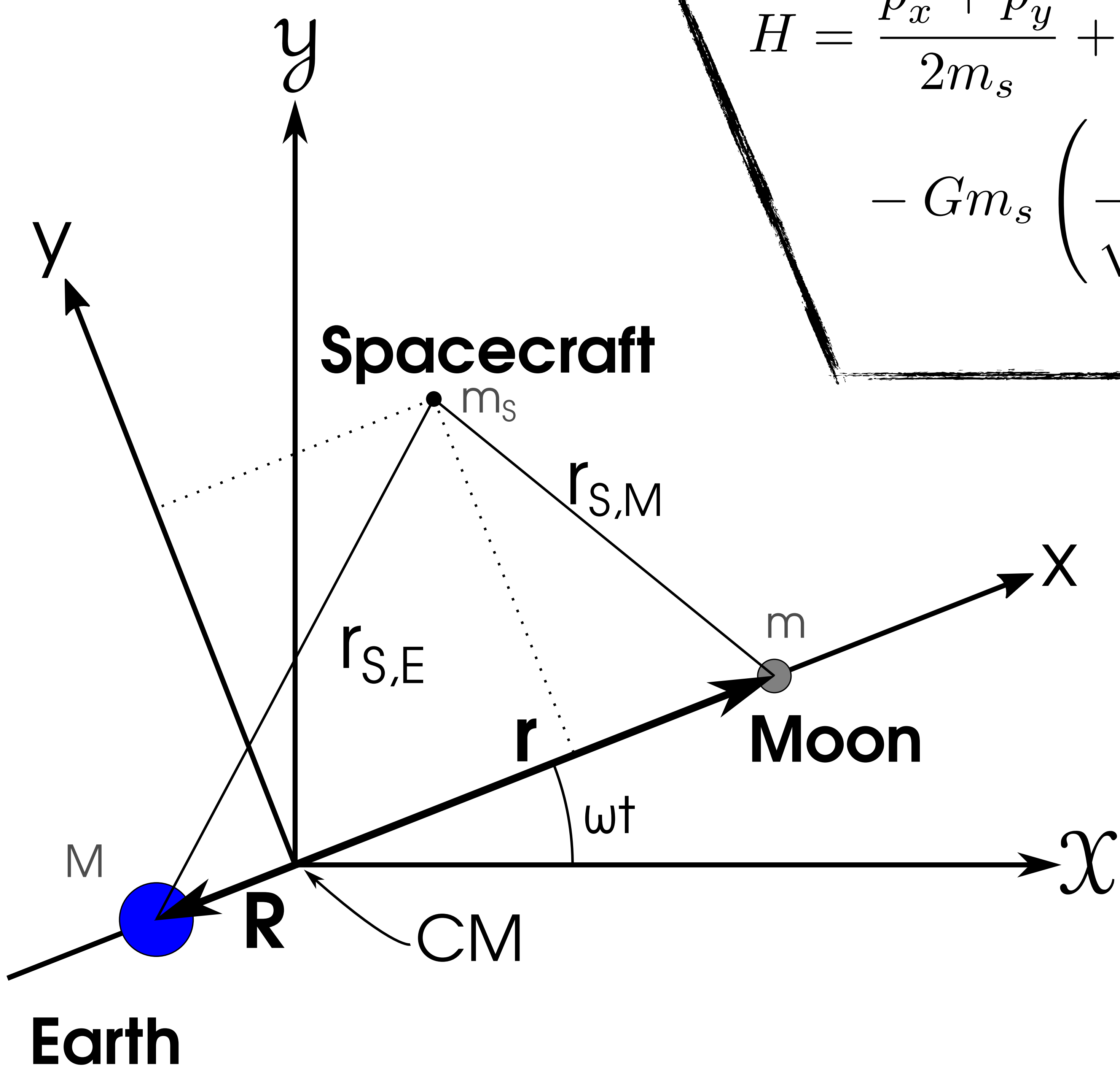


Assumptions:

- 2D
- 2-body

Restricted 3-body problem

$$H = \frac{p_x^2 + p_y^2}{2m_s} + p_x \omega y - p_y \omega x$$
$$- G m_s \left(\frac{M}{\sqrt{(x+R)^2 + y^2}} + \frac{m}{\sqrt{(x-r)^2 + y^2}} \right)$$



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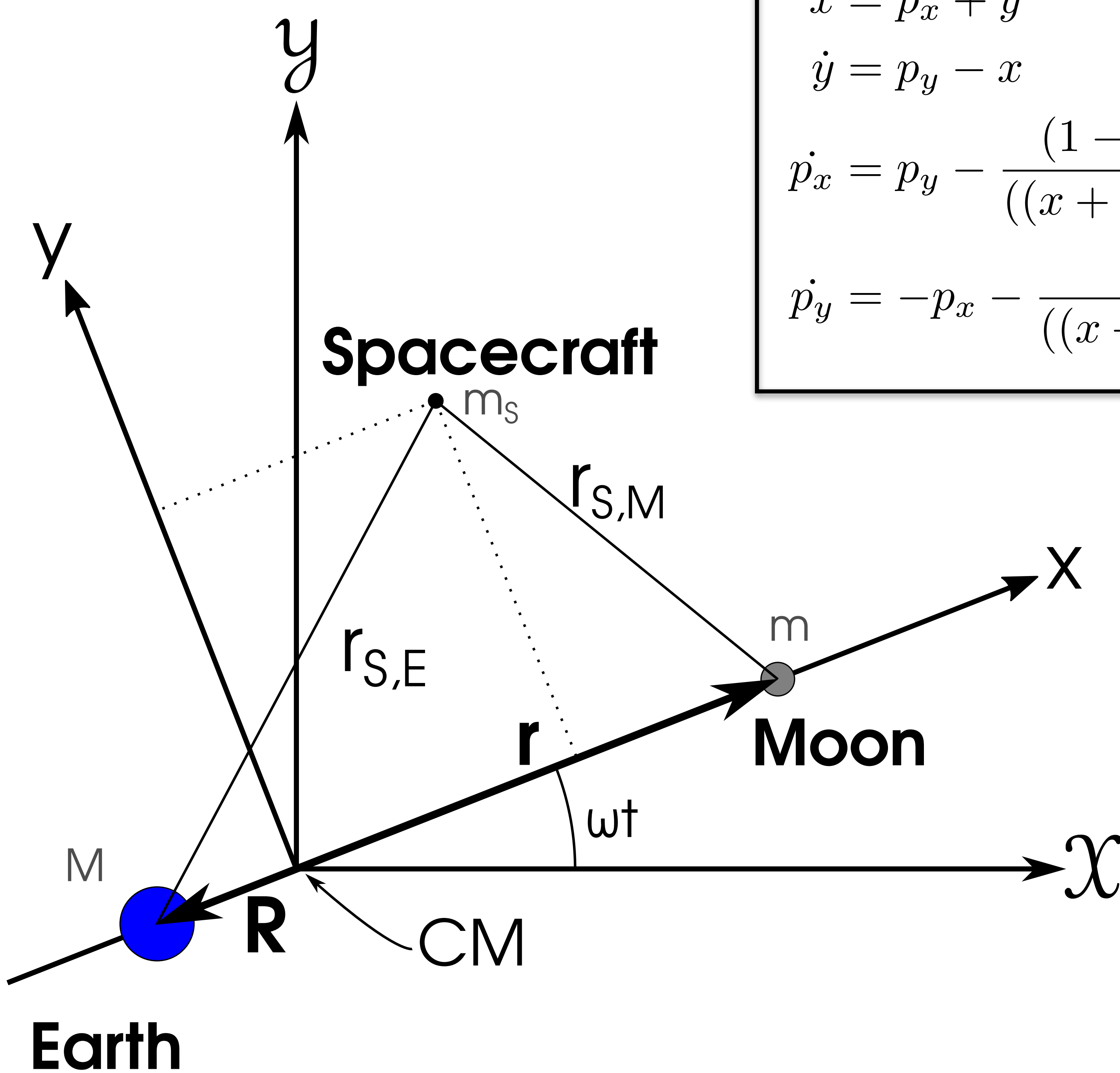


Equations of Motion

$$\dot{x} = p_x + y$$

$$\dot{y} = p_y - x$$

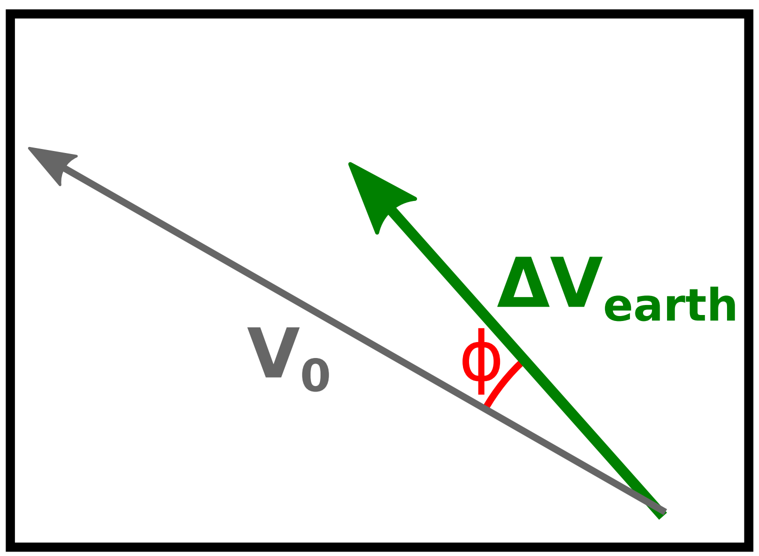
$$\dot{p_x} = p_y - \frac{(1-k)(x+k)}{((x+k)^2+y^2)^{3/2}} - \frac{k(x-1-k)}{((x-1-k)^2+y^2)^{3/2}}$$

$$\dot{p_y} = -p_x - \frac{(1-k)y}{((x+k)^2+y^2)^{3/2}} - \frac{ky}{((x-1-k)^2+y^2)^{3/2}}$$

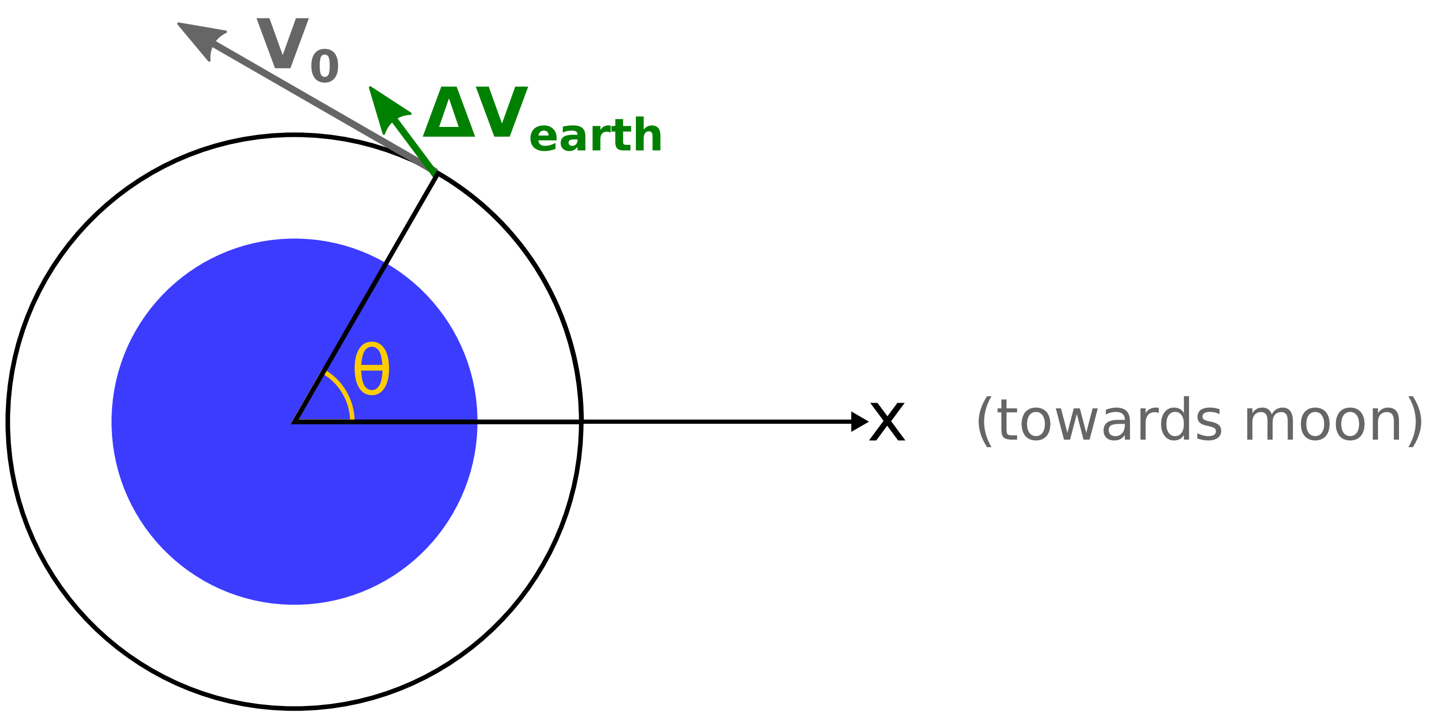
Assumptions:

- 2D
- 2-body

Searching for Transfer Orbits

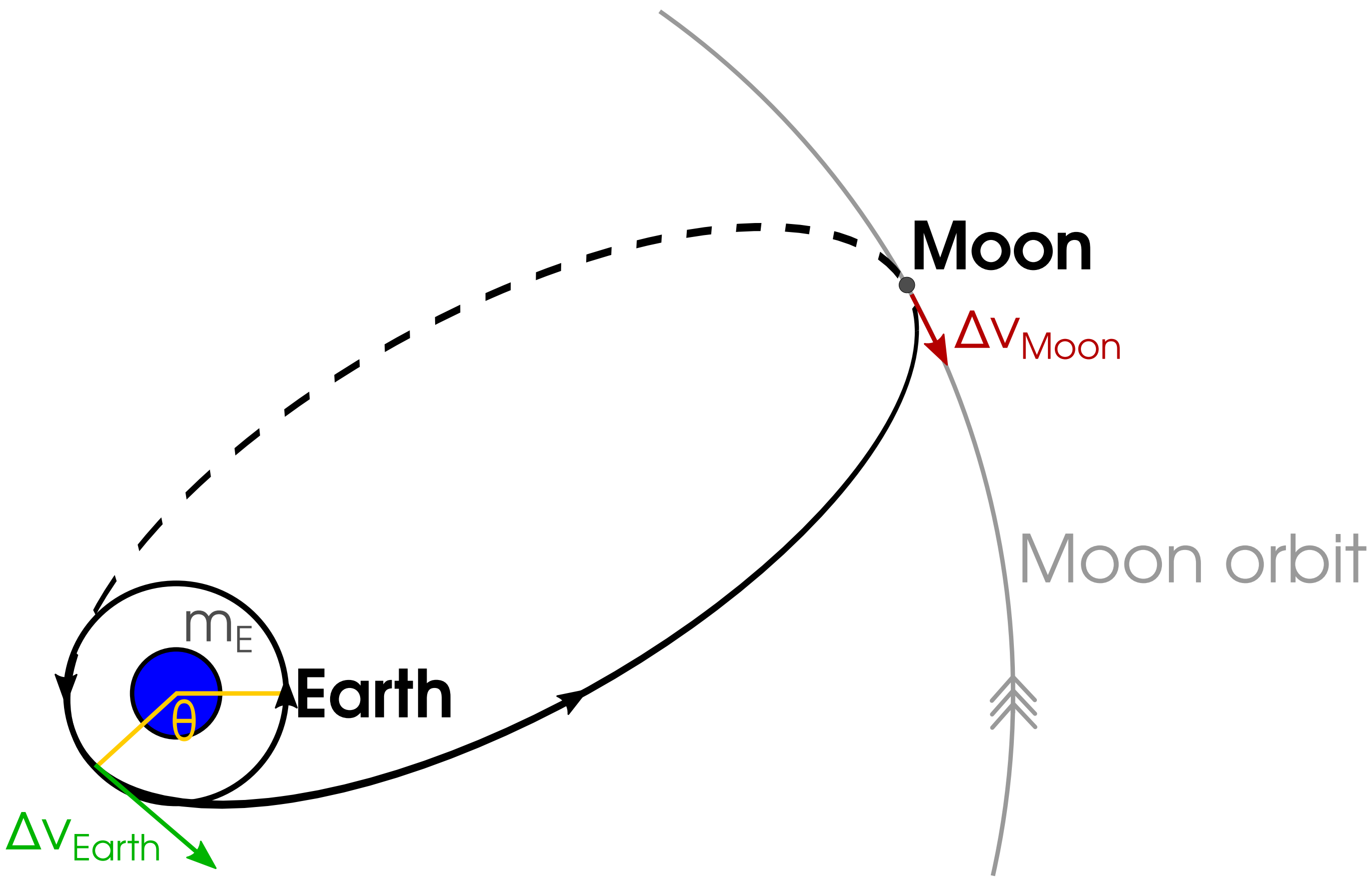


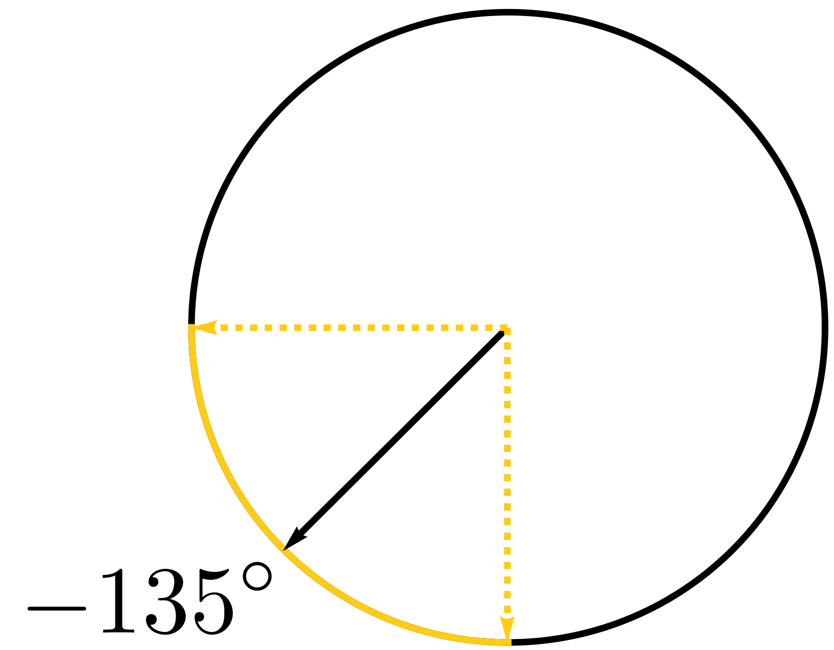
1. θ: Position in orbit
2. Δv_{earth}: Velocity change
3. ϕ: Angle to velocity vector



Searching for Transfer Orbits

Hohmann Transfer Orbit to the Moon





ϕ: 0° constant

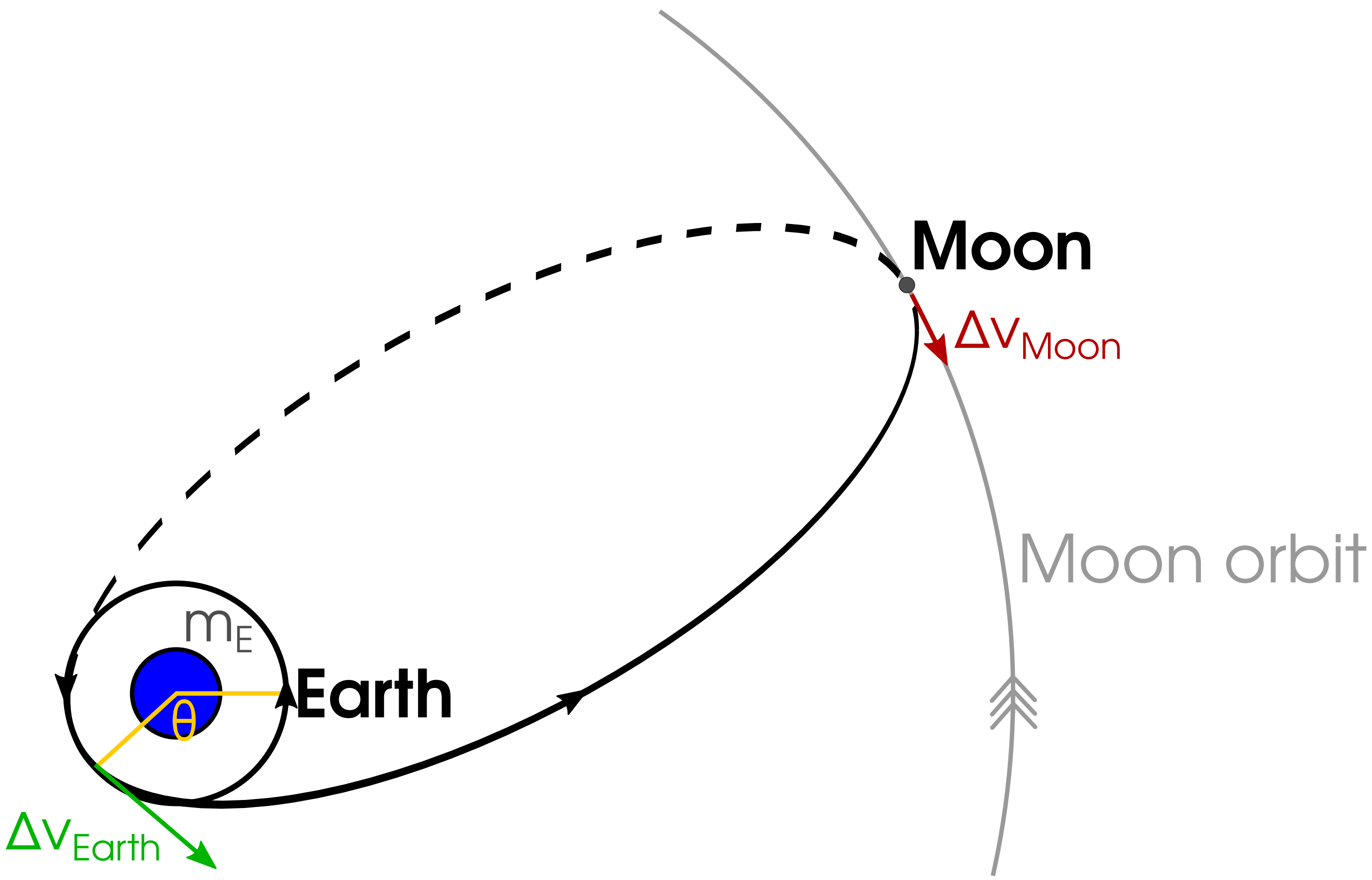
First search

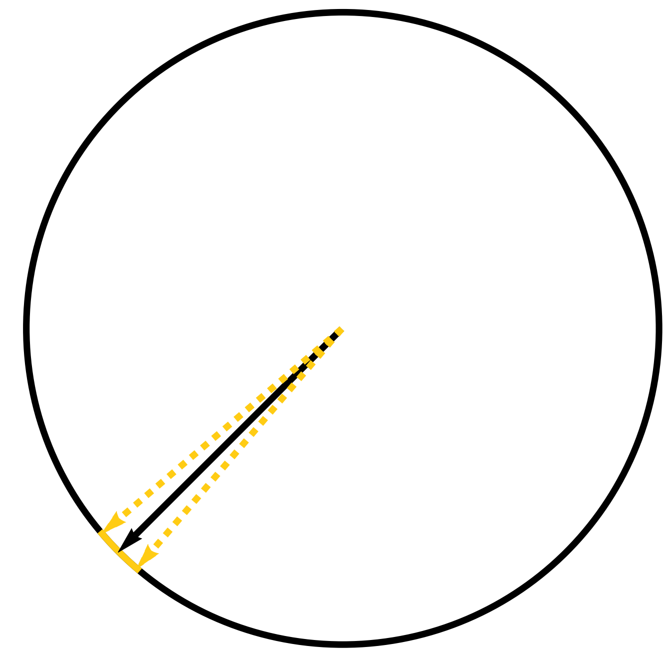
---

100 positions · 200 velocities = 20,000 simulations

Searching for Transfer Orbits

Hohmann Transfer Orbit to the Moon





First search

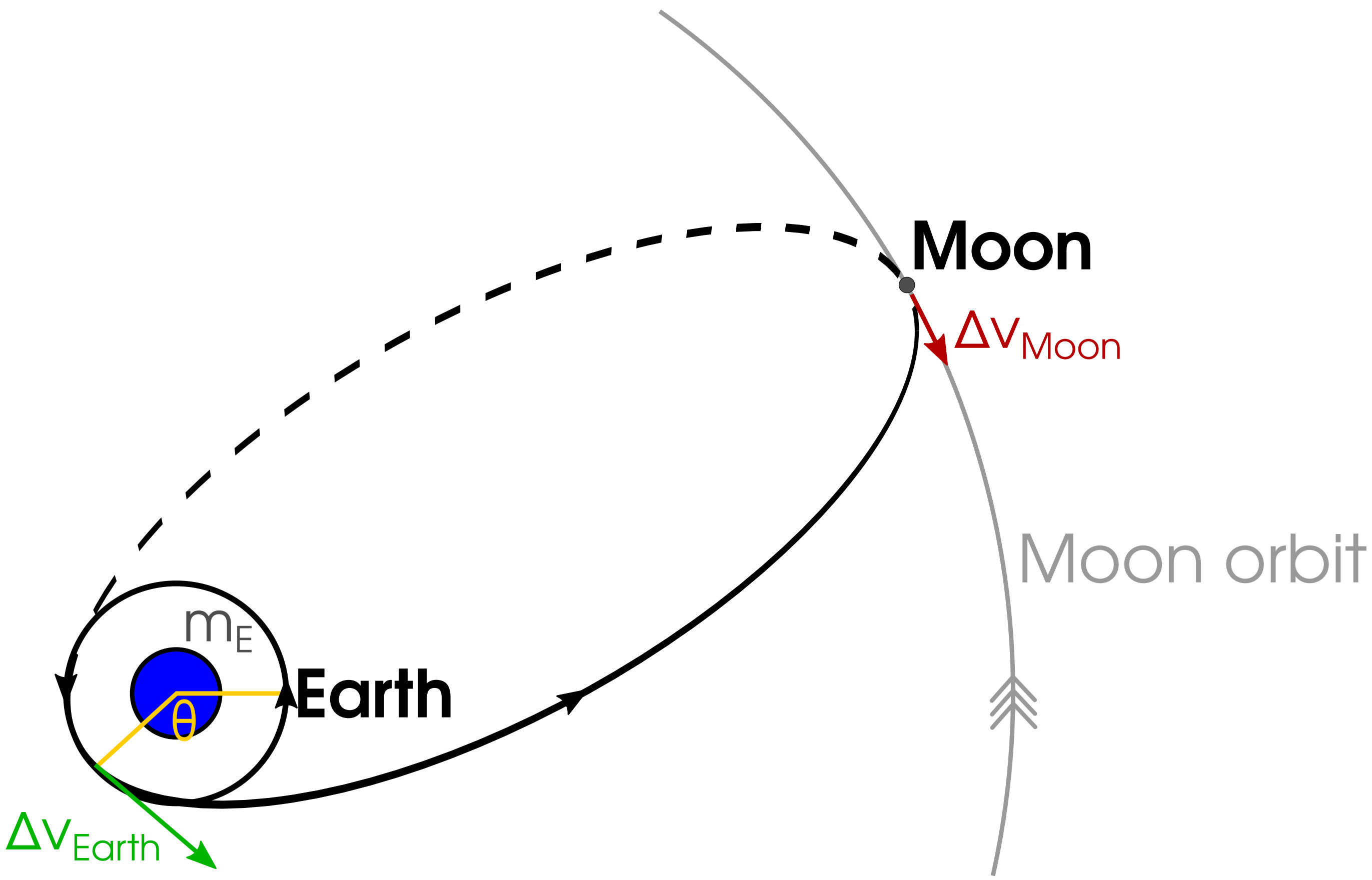
100 positions · 200 velocities = 20,000 simulations

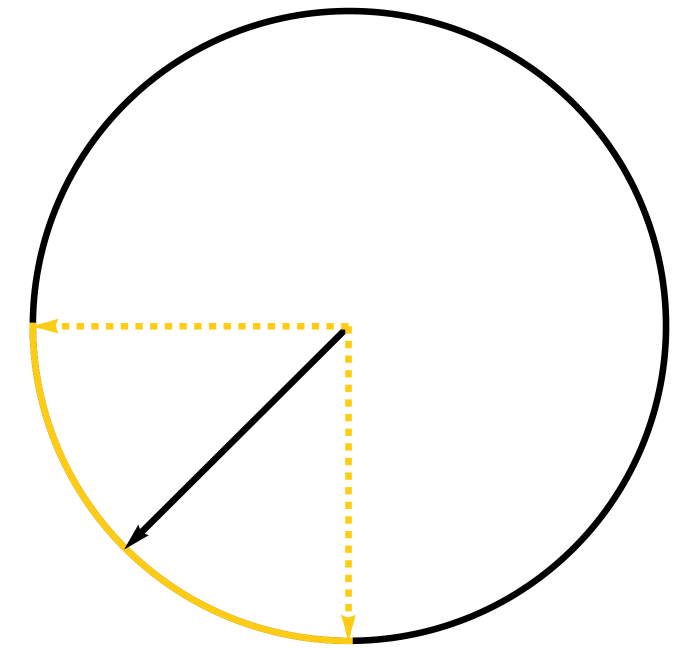
Refinement

15 positions · 15 velocities · 15 angles = 3375 simulations

Searching for Transfer Orbits

Hohmann Transfer Orbit to the Moon





First search

---

100 positions · 200 velocities = 20,000 simulations

Refinement

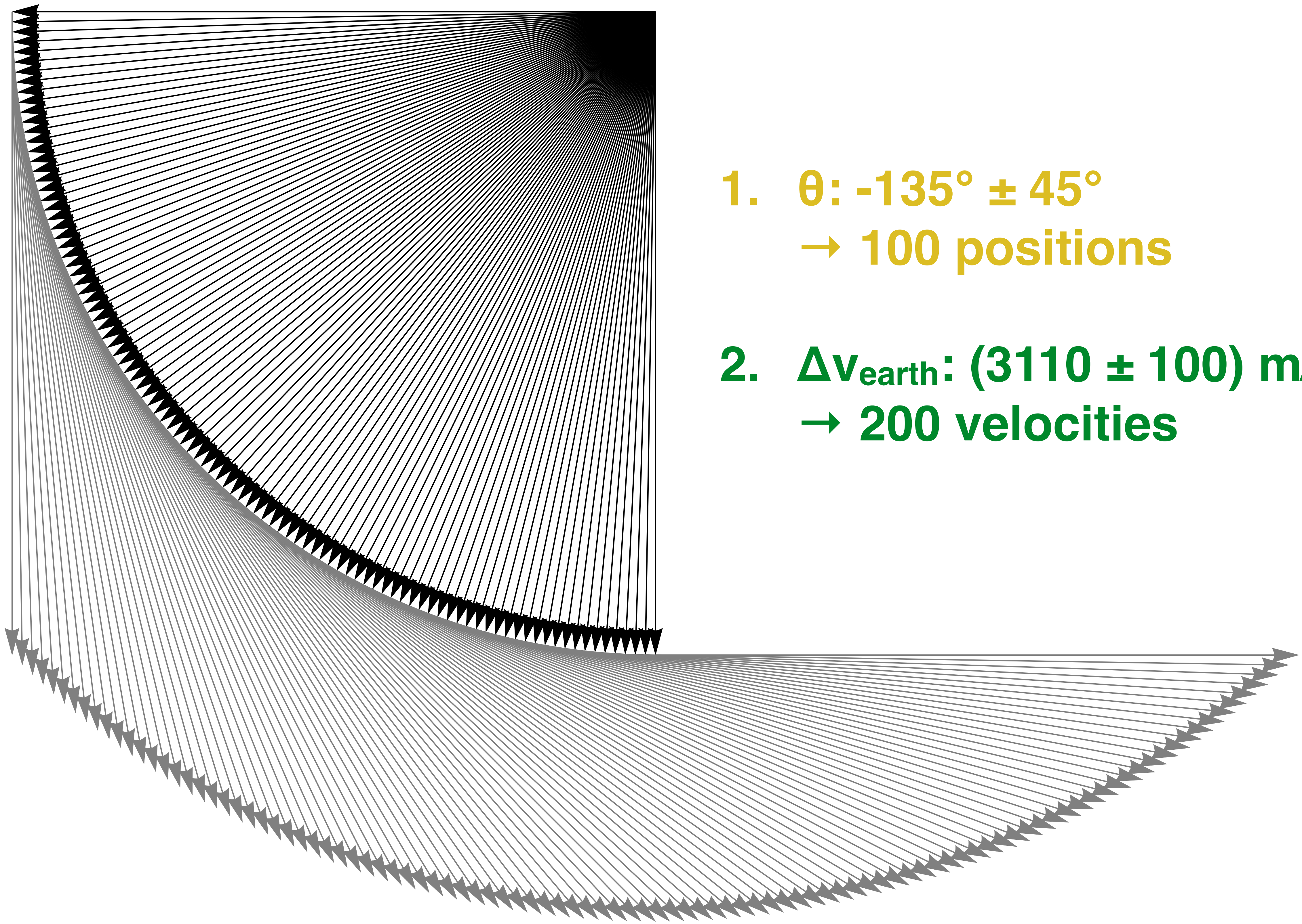
---

15 positions · 15 velocities · 15 angles = 3375 simulations

Total: 20,000 + 3375 = 23,375 simulations

Searching for Transfer Orbits

Hohmann Transfer Orbit to the Moon

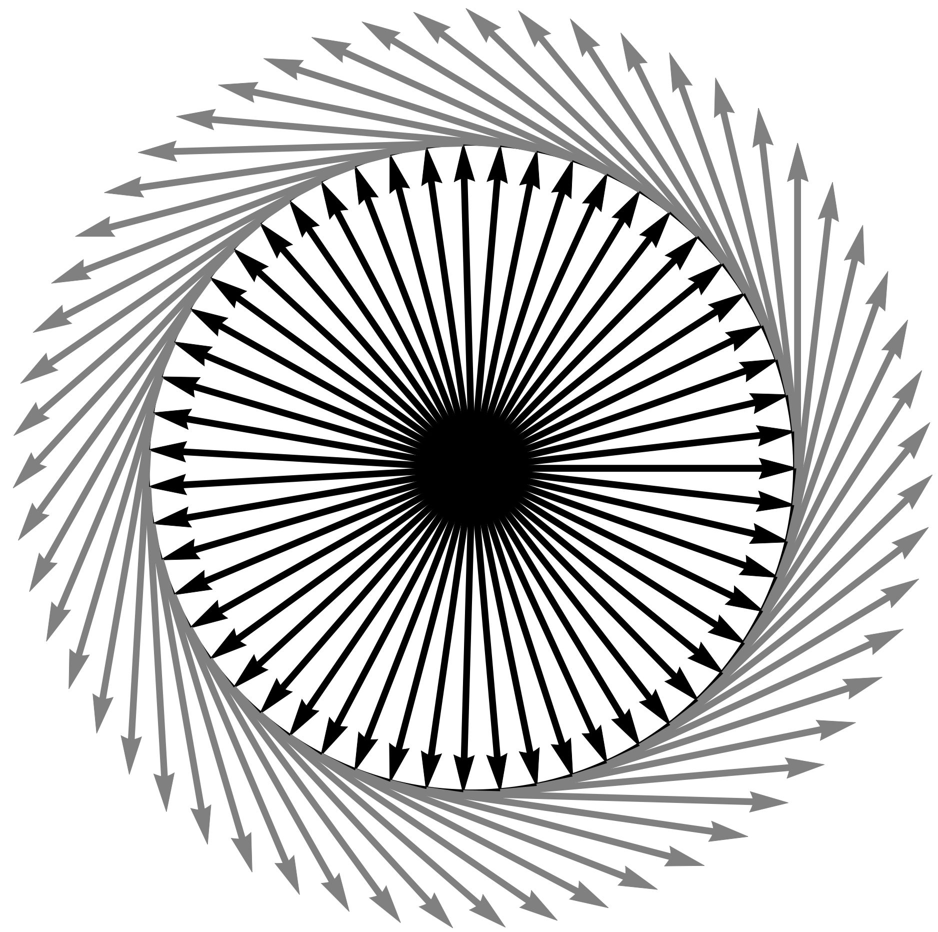


1. **θ: -135° ± 45°**
   → 100 positions

2. **Δv_{earth}: (3110 ± 100) m/s**
   → 200 velocities

Searching for Transfer Orbits

Low Energy Transfer Orbit to the Moon



First search + 7 refinements

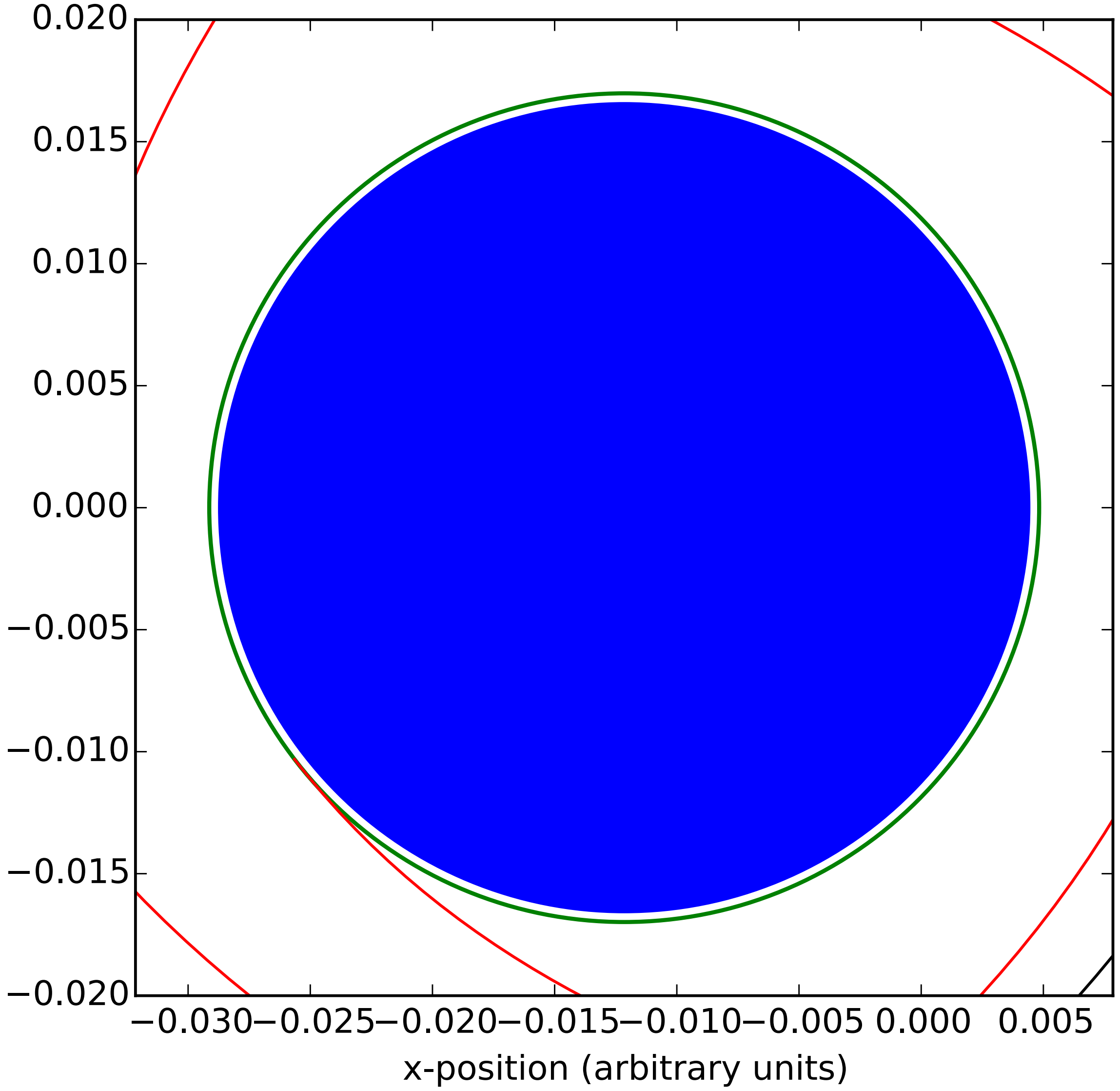
1. **θ: 0 ± 180° → 55 positions**
2. **Δv_{earth}: (3120 ± 100) m/s → 55 velocities**
3. **ϕ: 0° ± 1.8° → 55 angles**

TOTAL $55 \cdot 55 \cdot 55 \cdot 8$ = **1,331,000**

Searching for Transfer Orbits

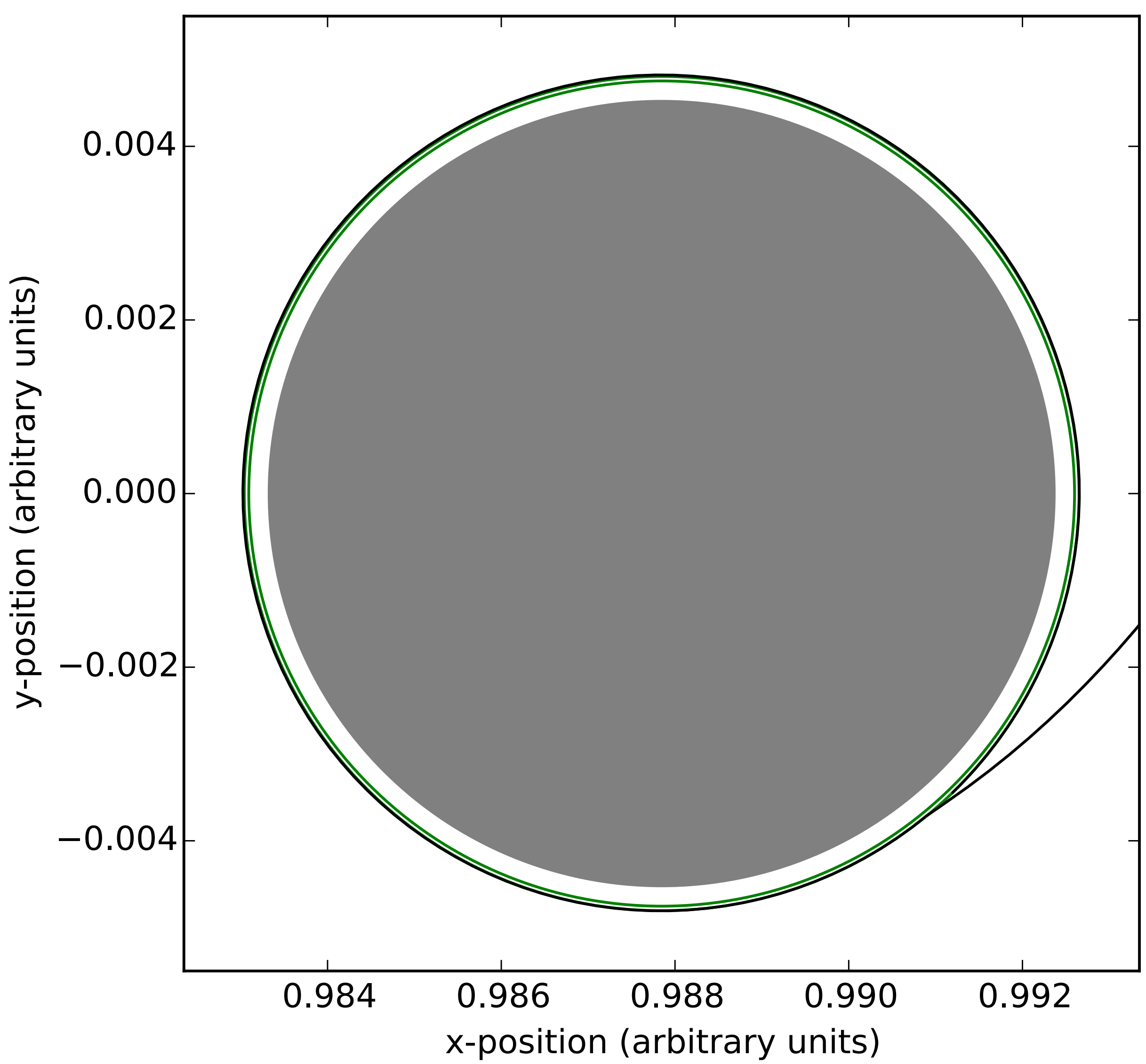
Entering Moon Orbit

Δv$_{earth}$



Exit from Earth orbit

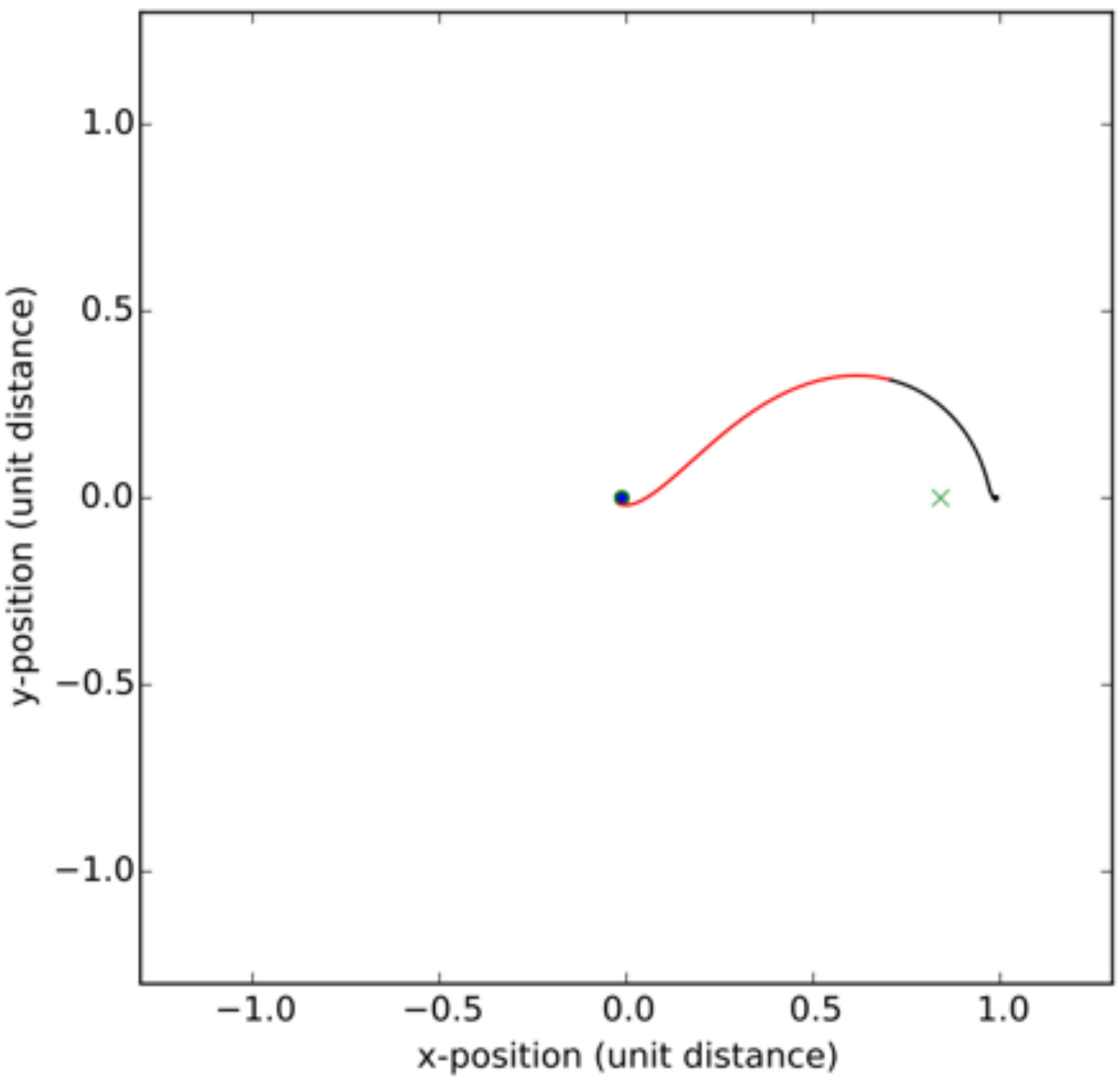
Δv$_{moon}$



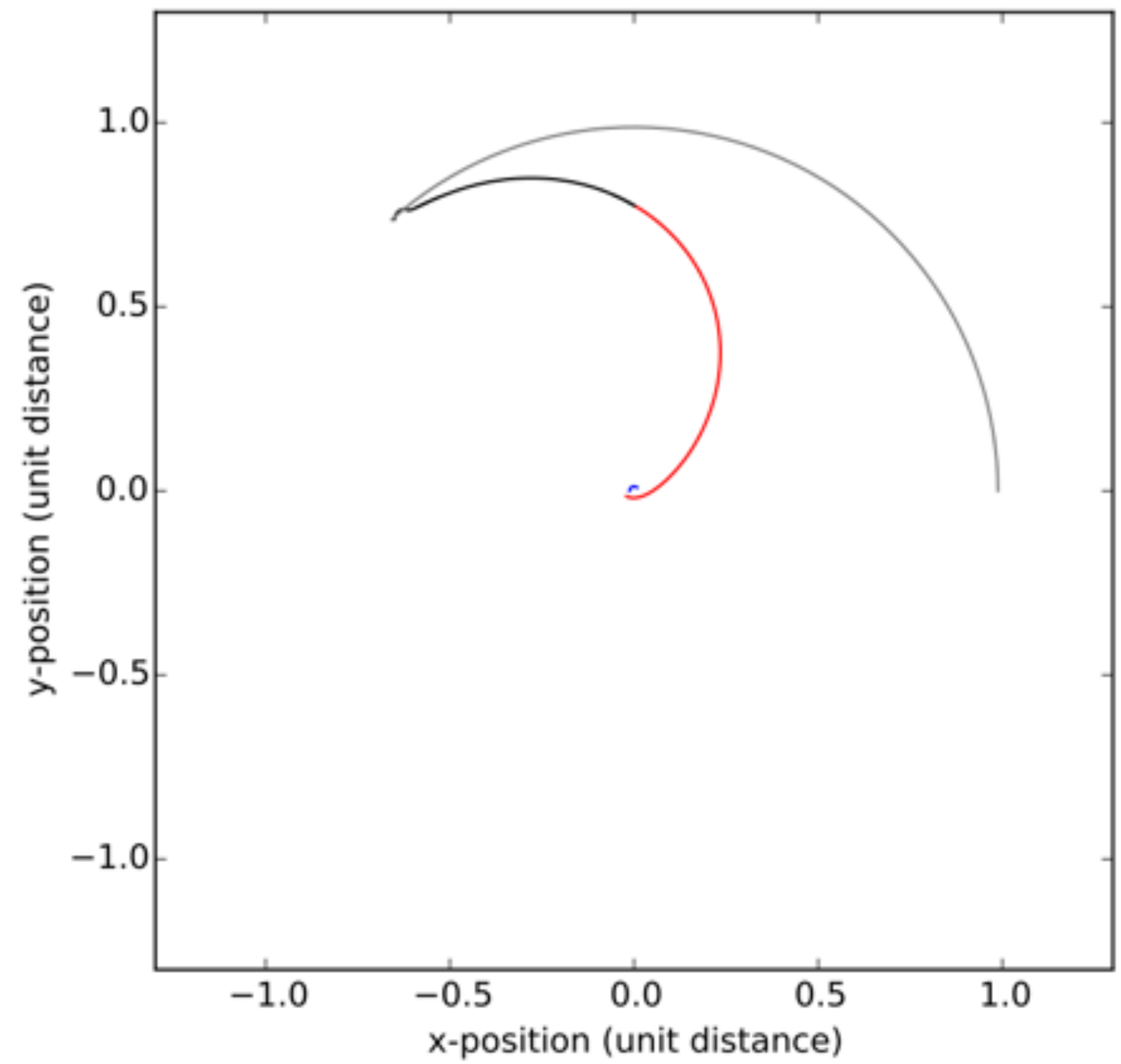
Entry to Moon orbit (100±10 km)

Searching for Transfer Orbits

Hohmann Transfer Orbit to the Moon



(x, y)



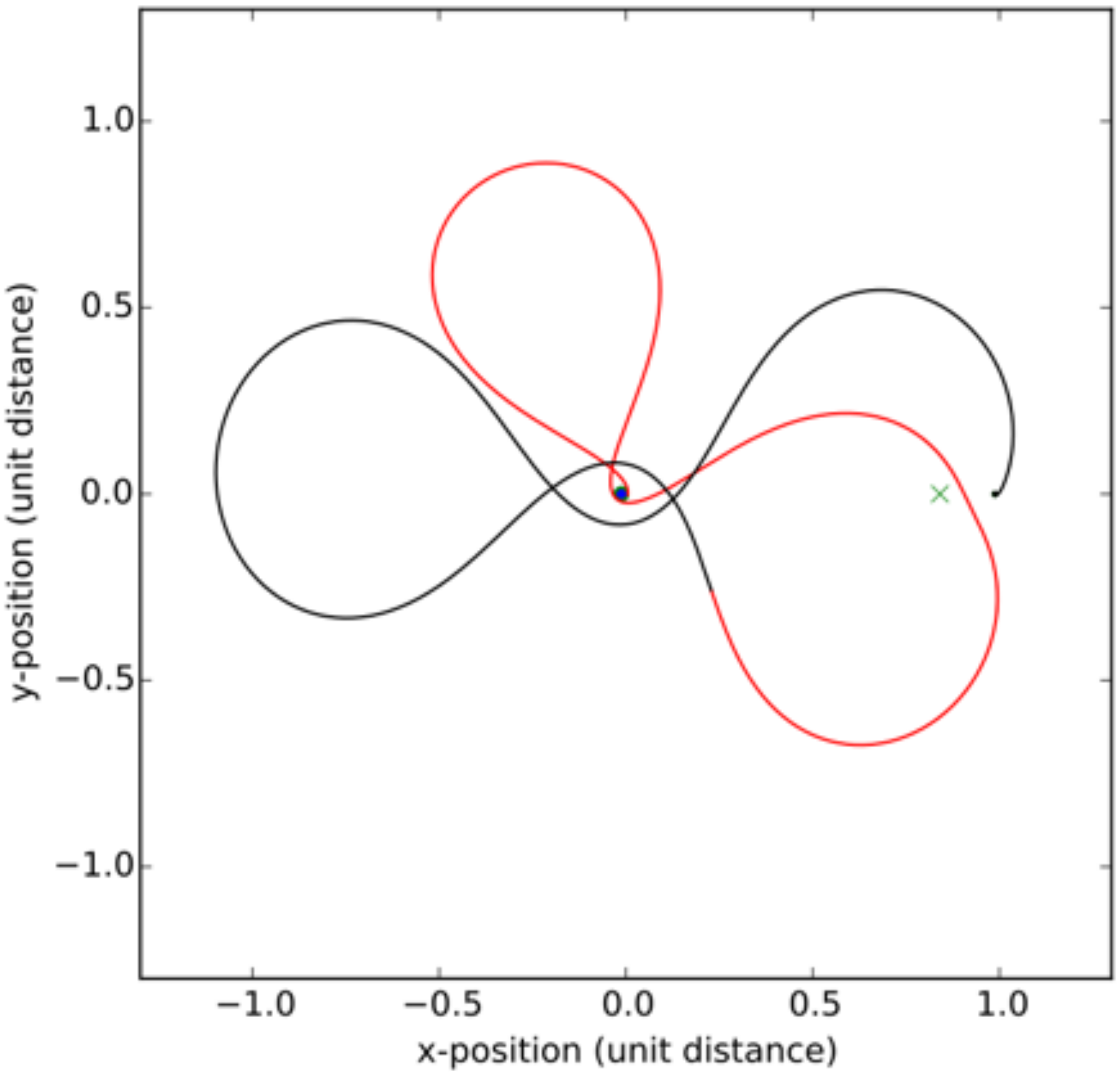
$(\mathcal{X}, \mathcal{Y})$

Δv_{total} = 3912 m/s

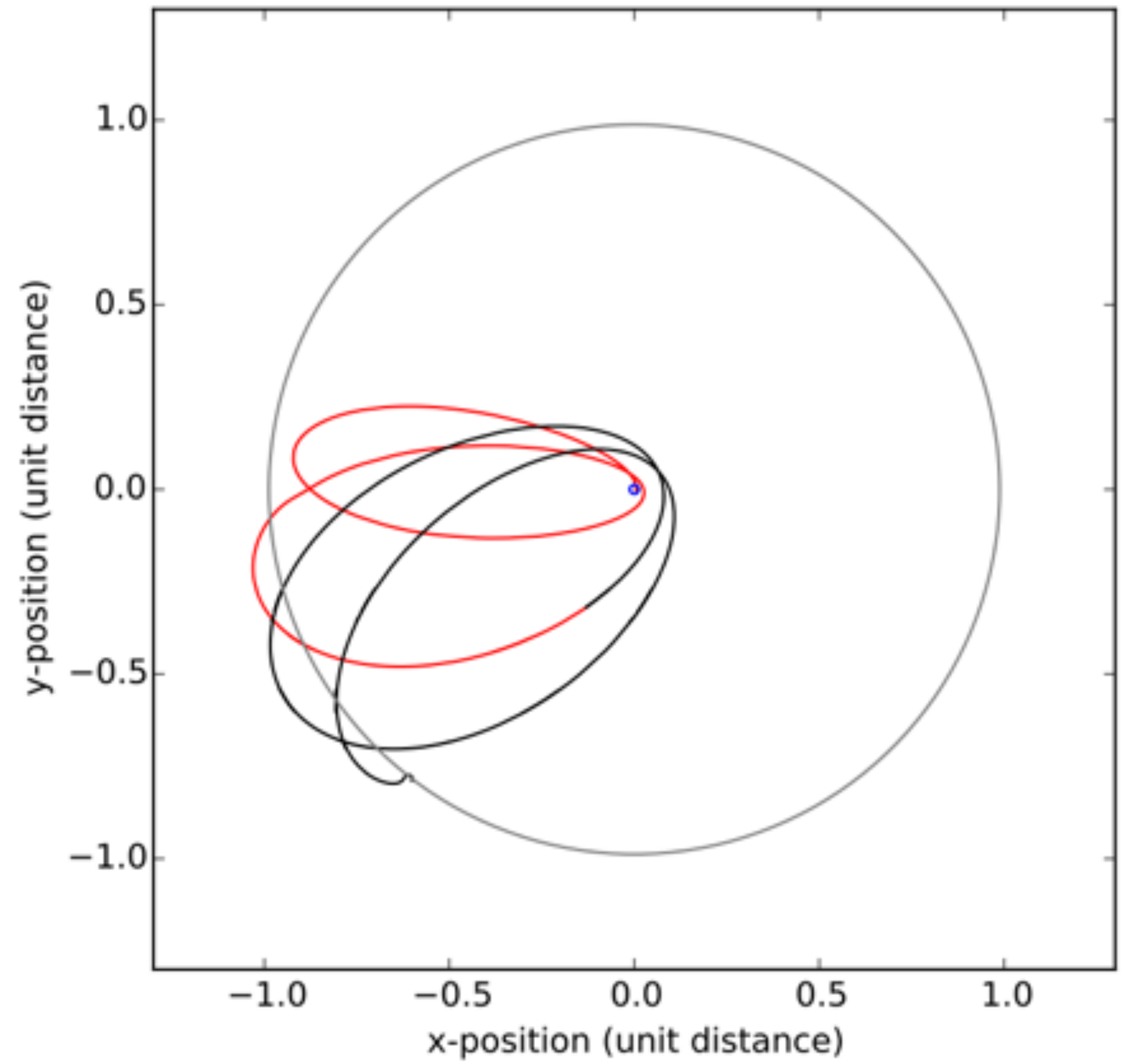
t_H = 4.3 days

Searching for Transfer Orbits

Low-Energy Transfer Orbit (short)



(x, y)



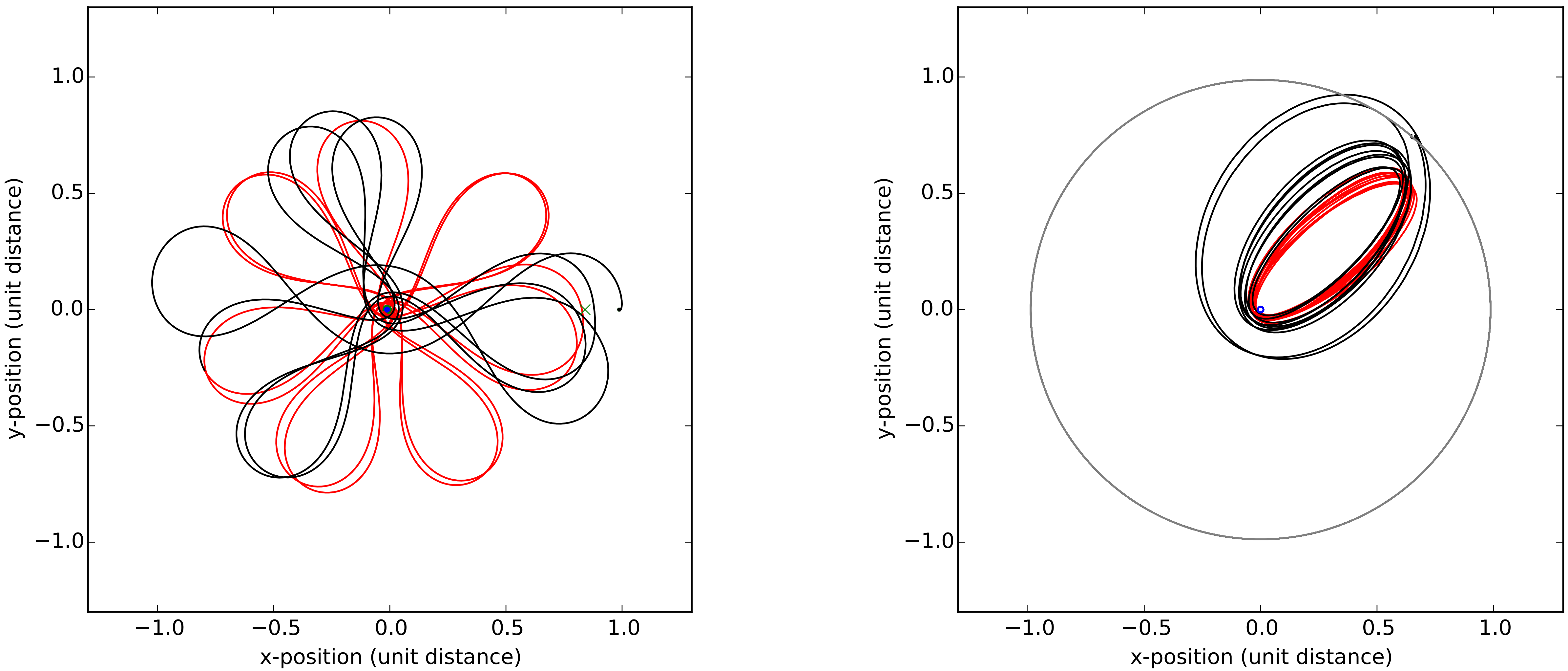
$(\mathcal{X}, \mathcal{Y})$

$\Delta v_{total} = 3896$ m/s

$t_{short} = 41$ days

Searching for Transfer Orbits

Low-Energy Transfer Orbit (long)



(x, y) $(\mathcal{X}, \mathcal{Y})$

Δv_{total} = 3795 m/s

t_{long} = 194 days

Results Summarized

All transfer orbits

Trajectory	Flight time	Δv_{total} (km/s)	Δv_{earth} (km/s)	Δv_{moon} (km/s)
Minimum	N/A	3.721	3.099	0.622
Long LETO	194 days	3.795	3.091	0.704
Belbruno-Miller	3 months	3.838	3.187	0.651
Topputo	8 months	3.895	3.265	0.630
Short LETO	41 days	3.896	3.127	0.769
Hohmann - long (sim)	4.3 days	3.912	3.111	0.801
Hohmann - (model)	5.0 days	3.946	3.144	0.802
Hohmann - medium (sim)	3.00 days	4.015	3.136	0.880
Apollo (Hohman)	3.05 days	4.115	3.048	1.067
Hohmann - short (sim)	1.00 days	6.823	3.809	3.014

Conclusion

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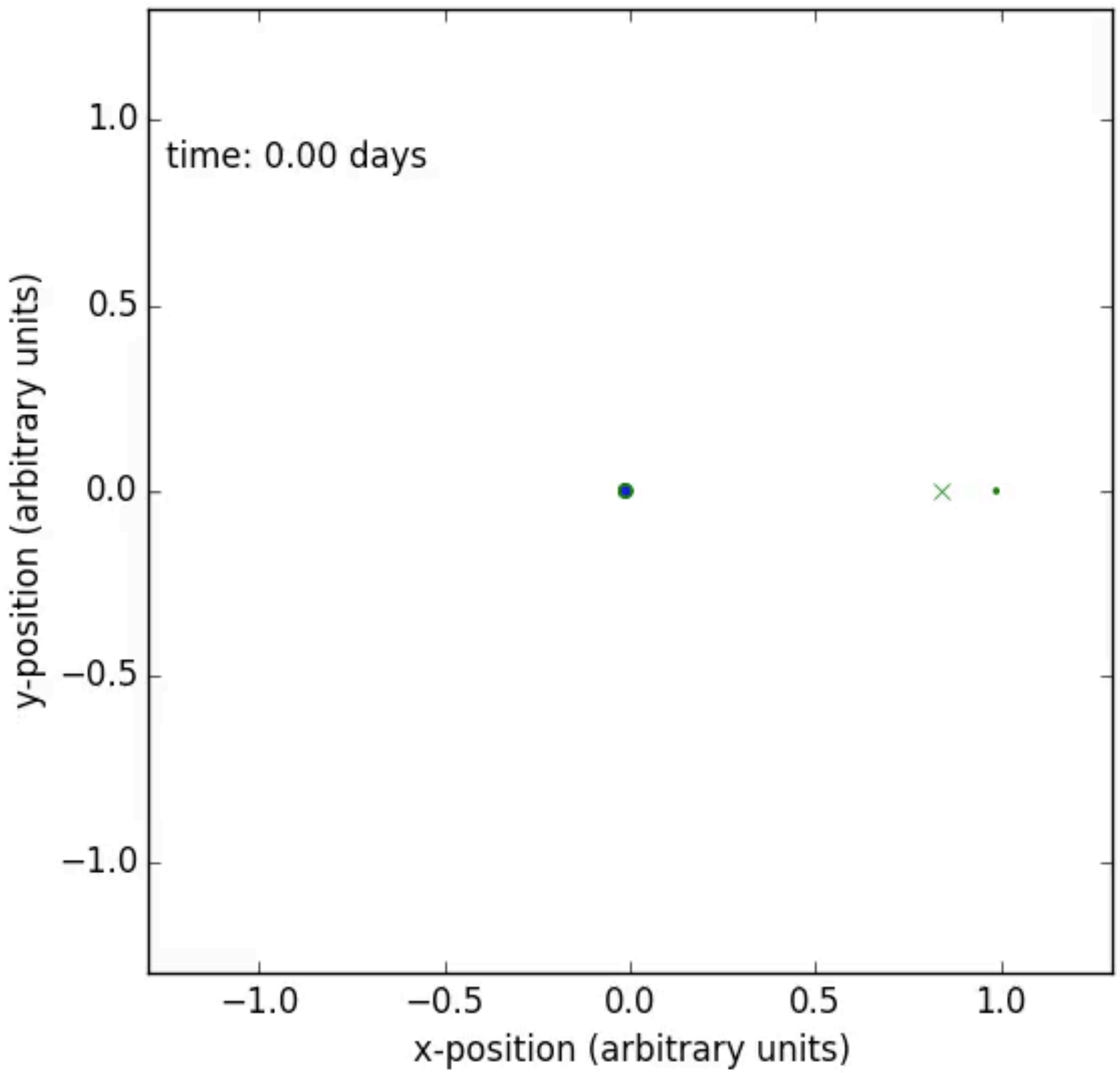
- I have used this to estimate a low Δv for a potential Moon mission, and can easily be extended to Mars missions.

(My advisor at DTU, Poul G. Hjorth, is so excited about the results that we will be writing an article together next January for publication)

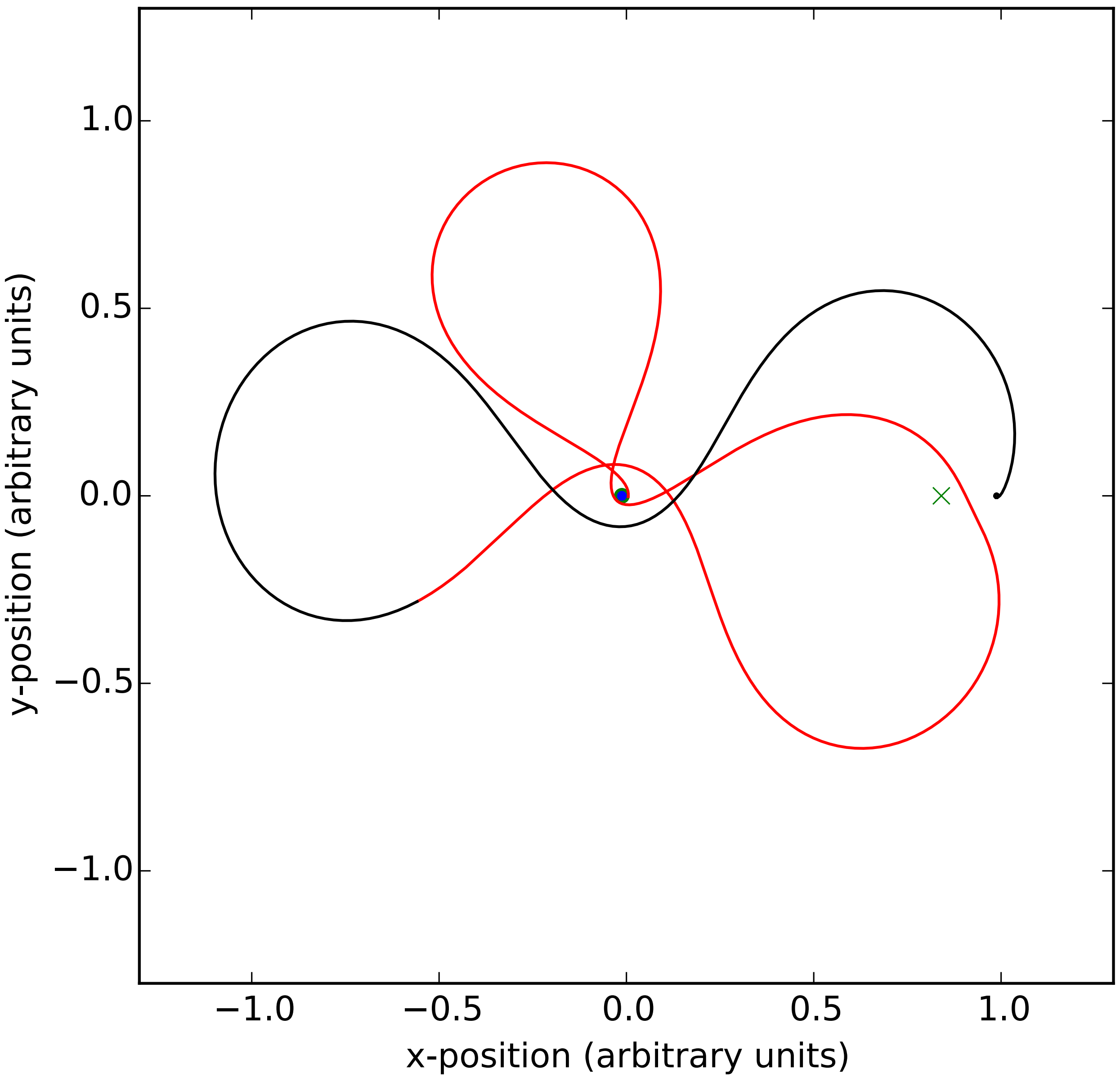
LETO Short Animation

0.043484 days/frame, 60 FPS

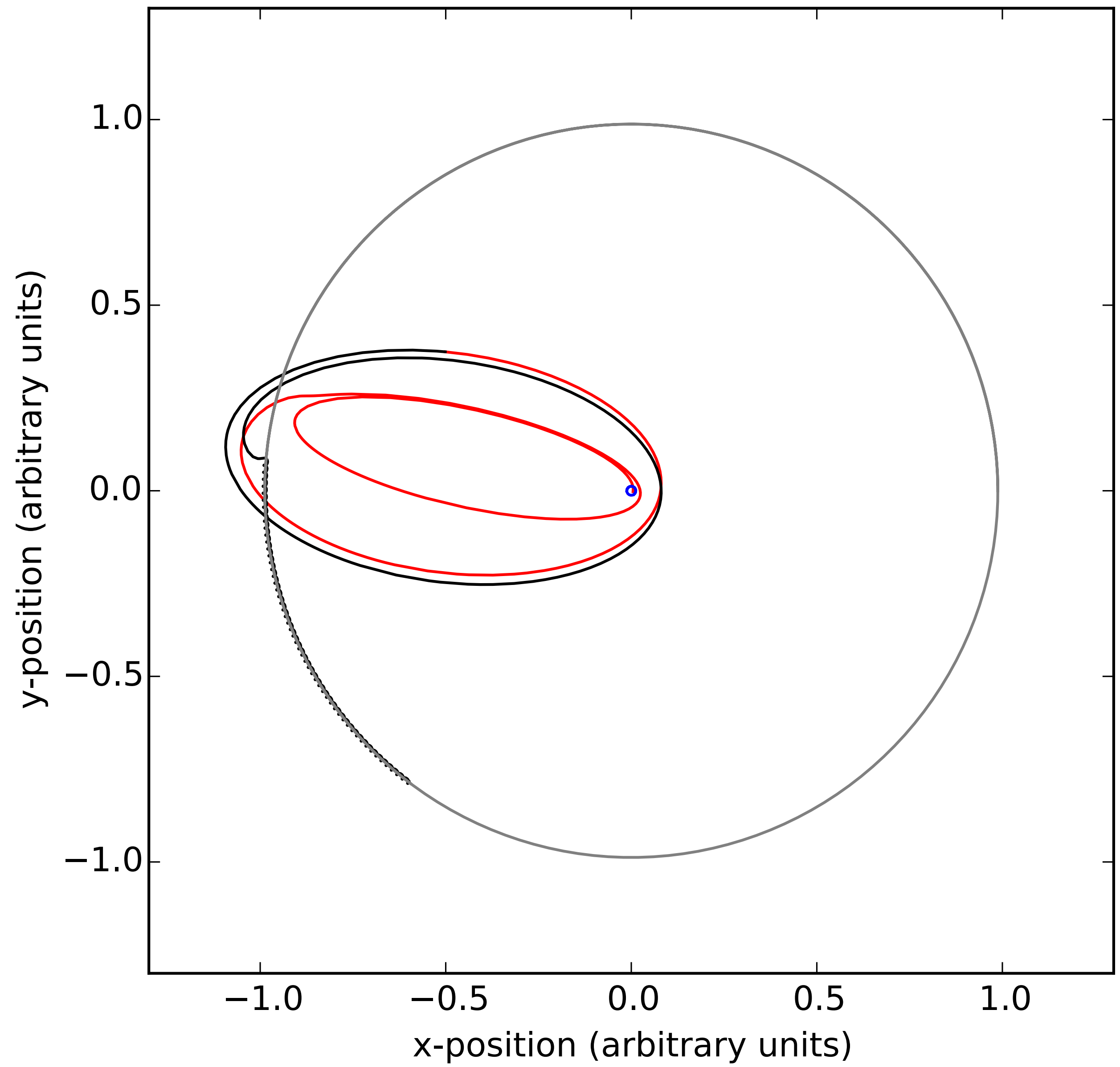


Numerical Method

Adaptive Störmer-Verlet (symplectic)



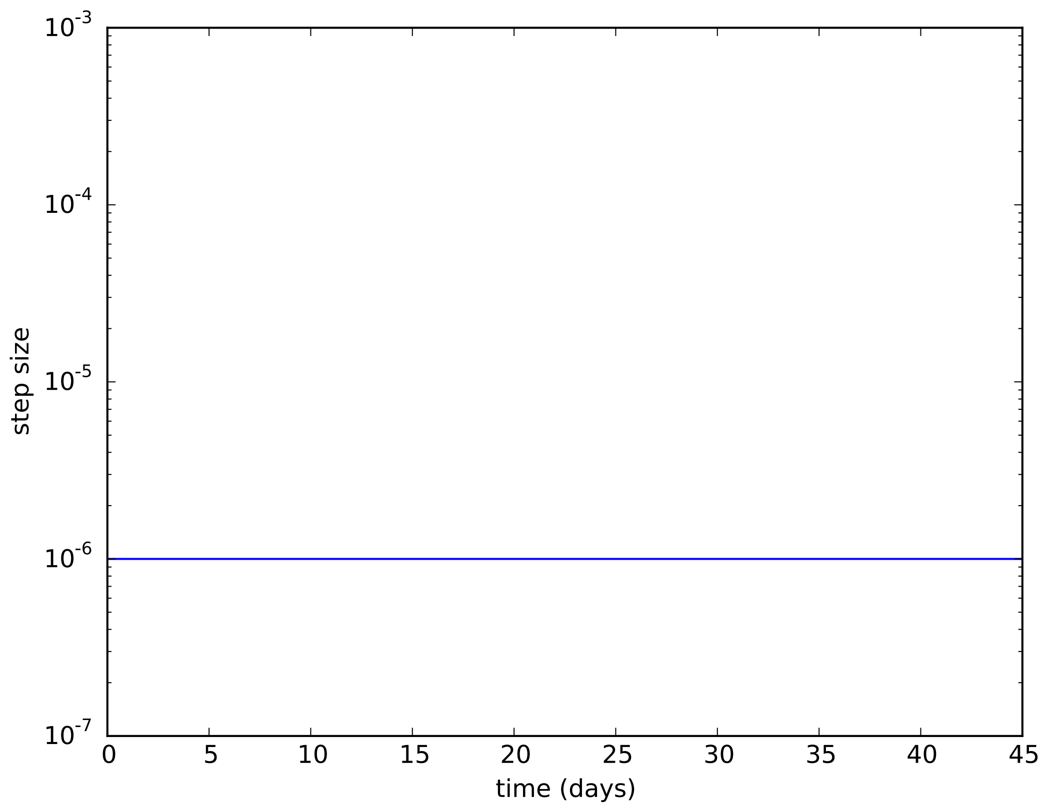
(x, y)

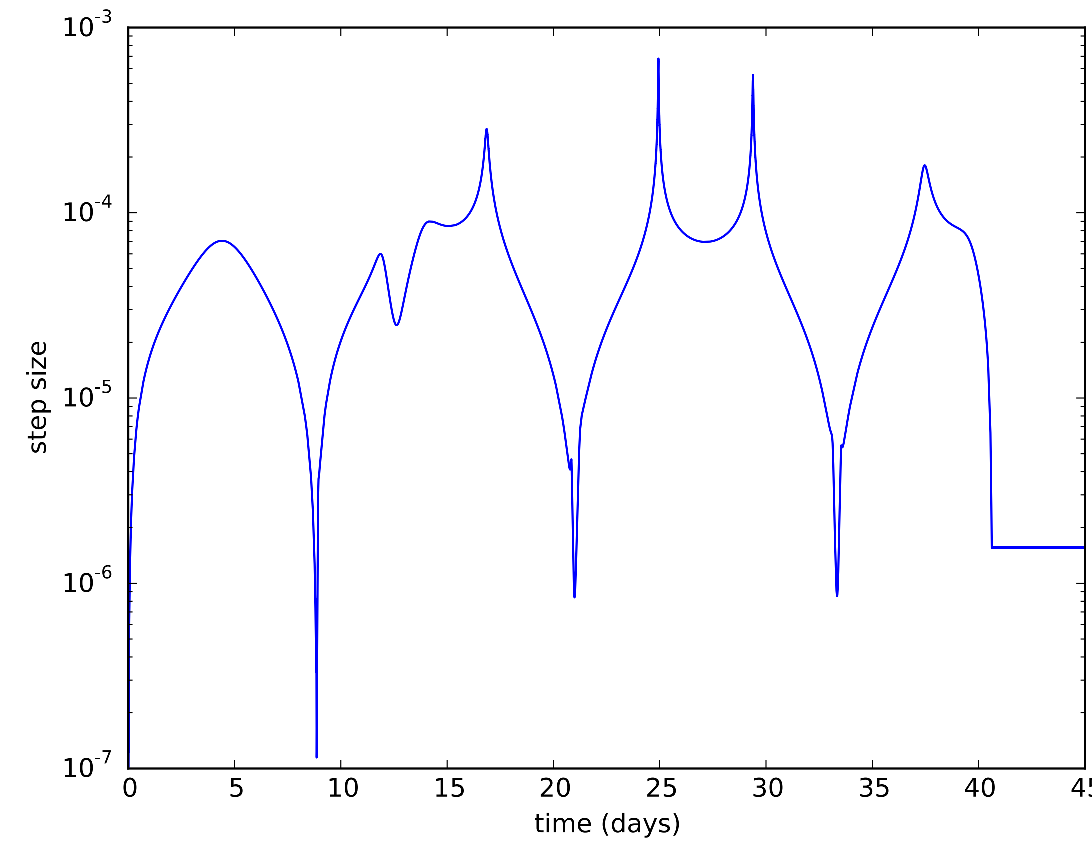


$(\mathcal{X}, \mathcal{Y})$

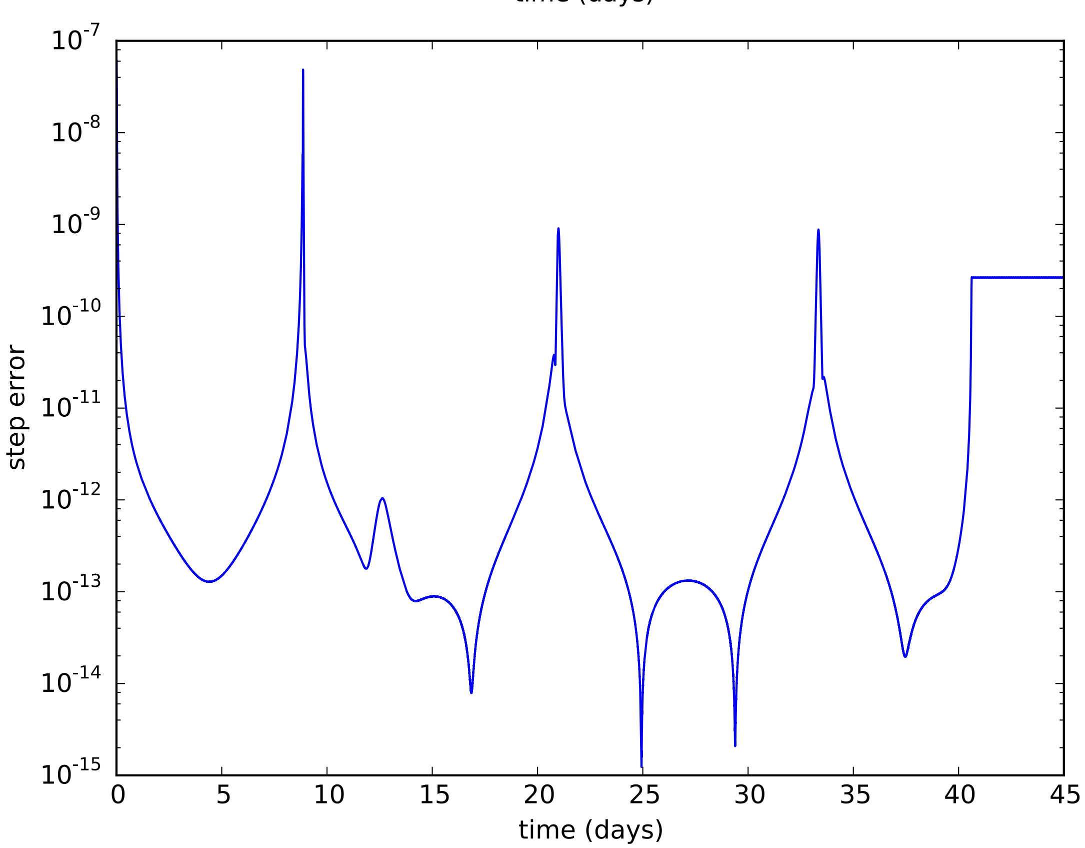
Numerical Method

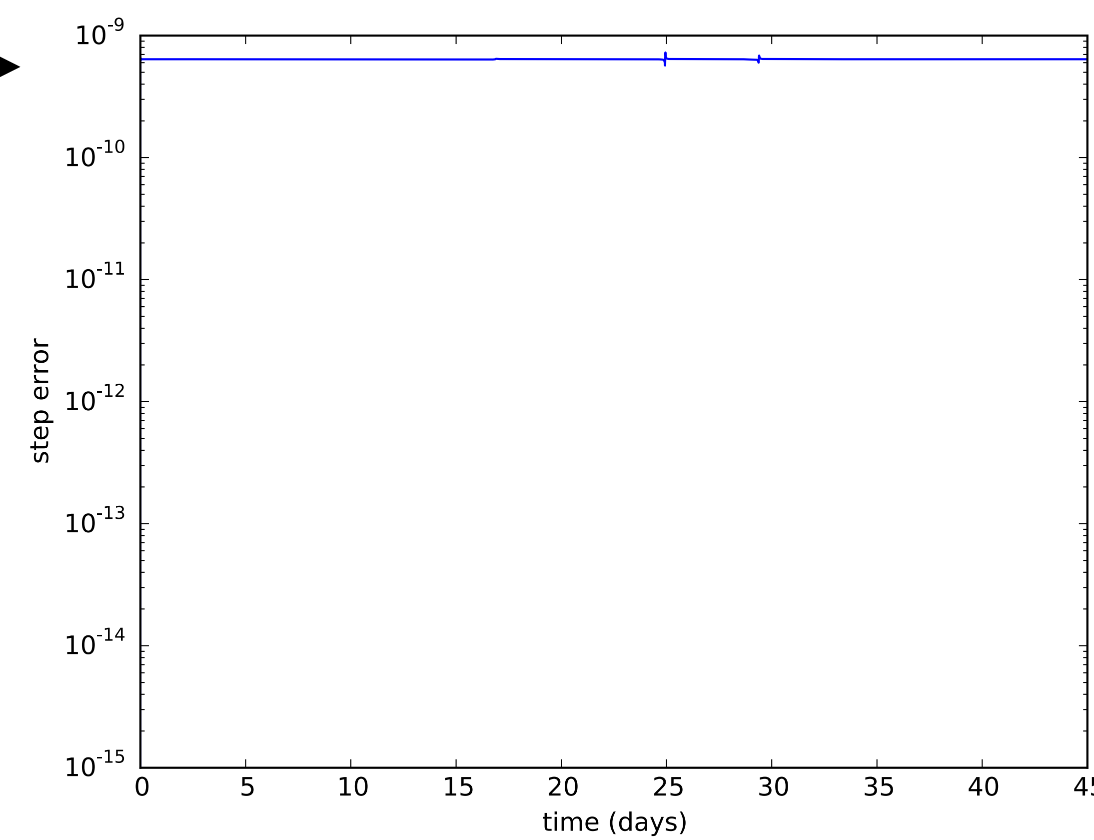
Adaptive Störmer-Verlet (symplectic)





Adaptive





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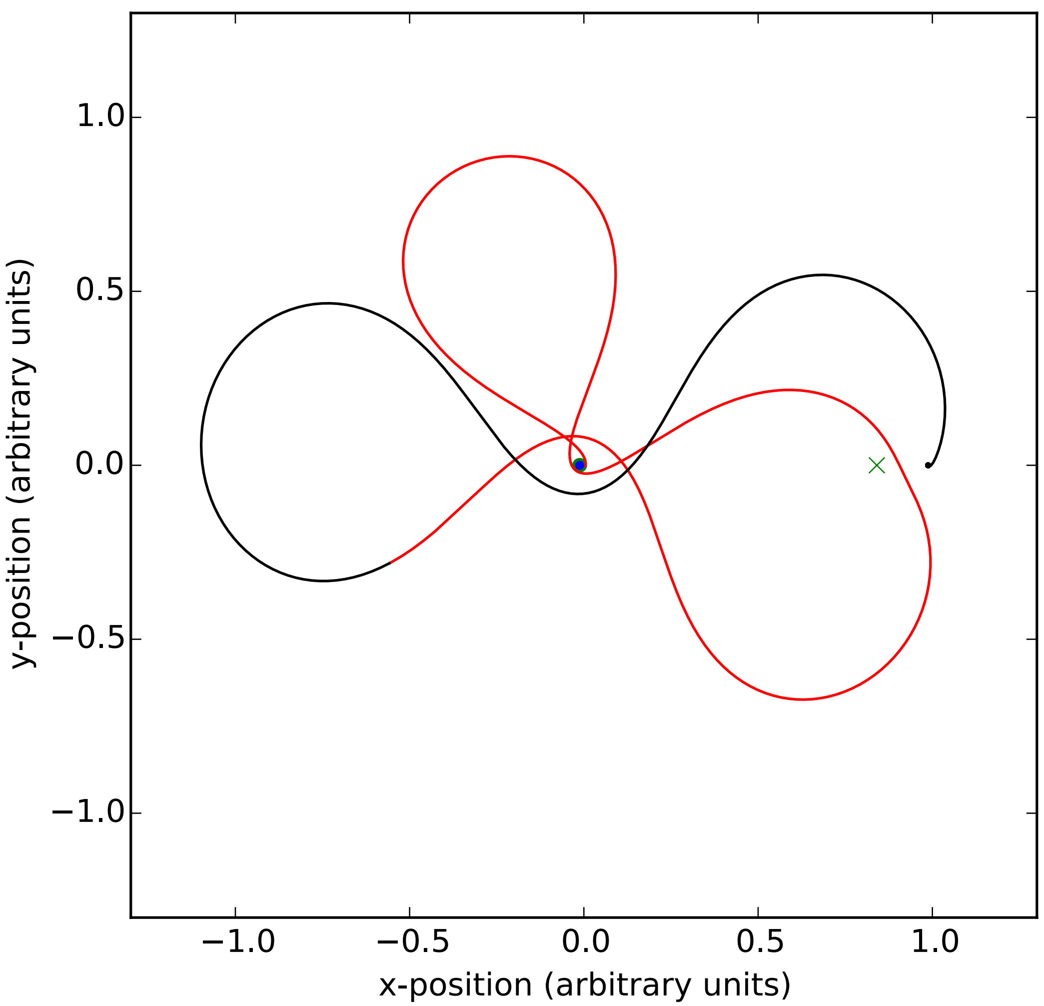
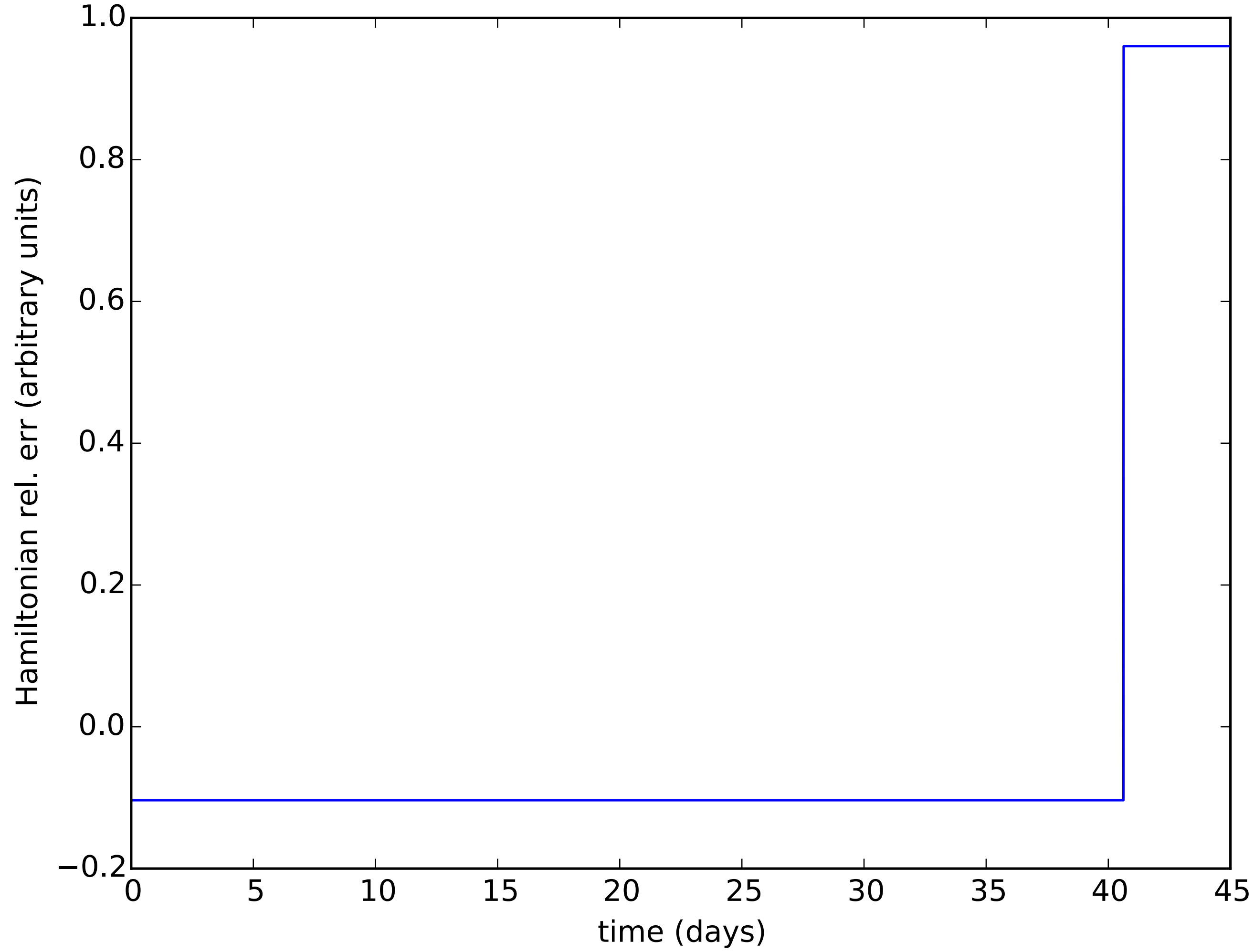
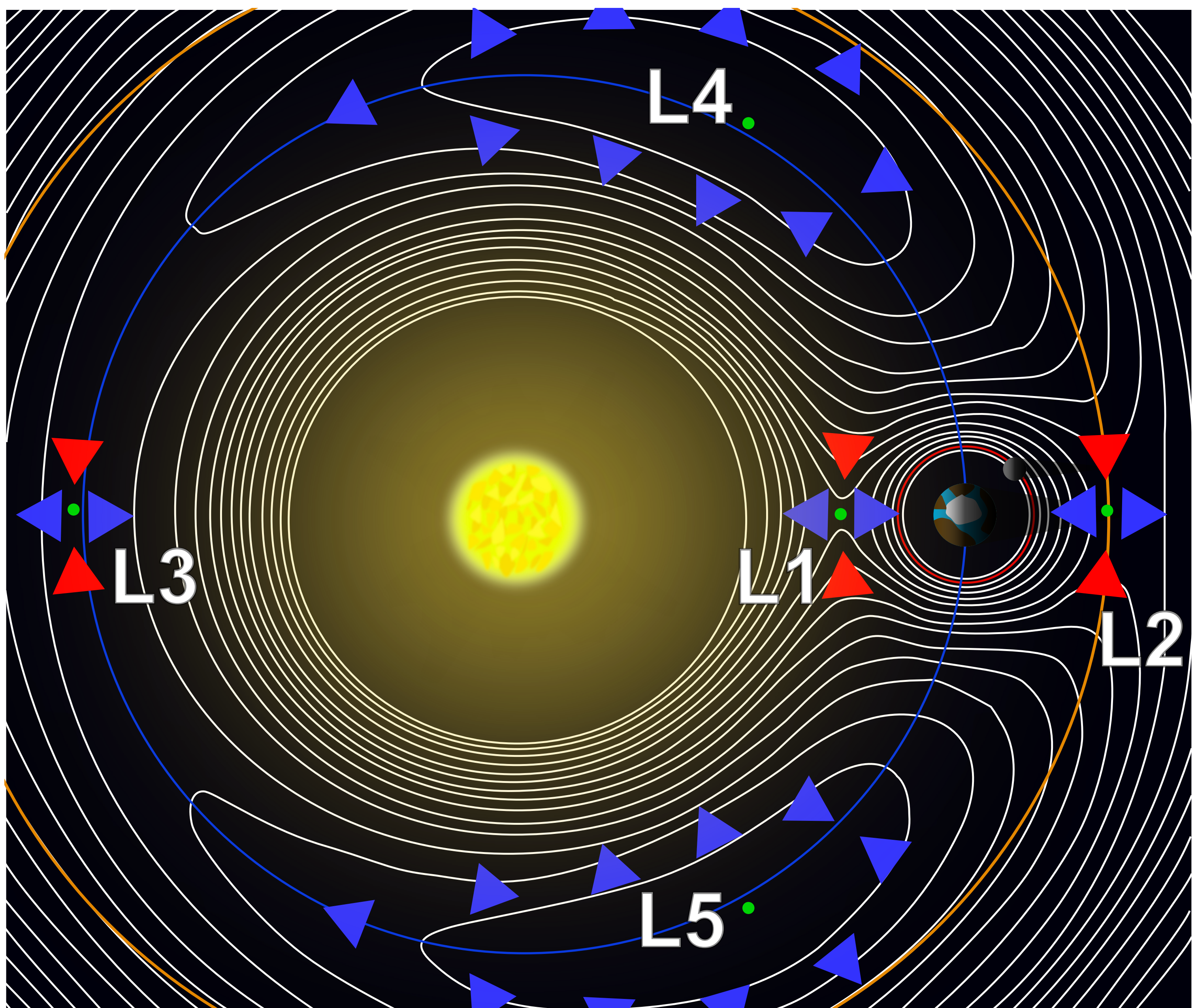


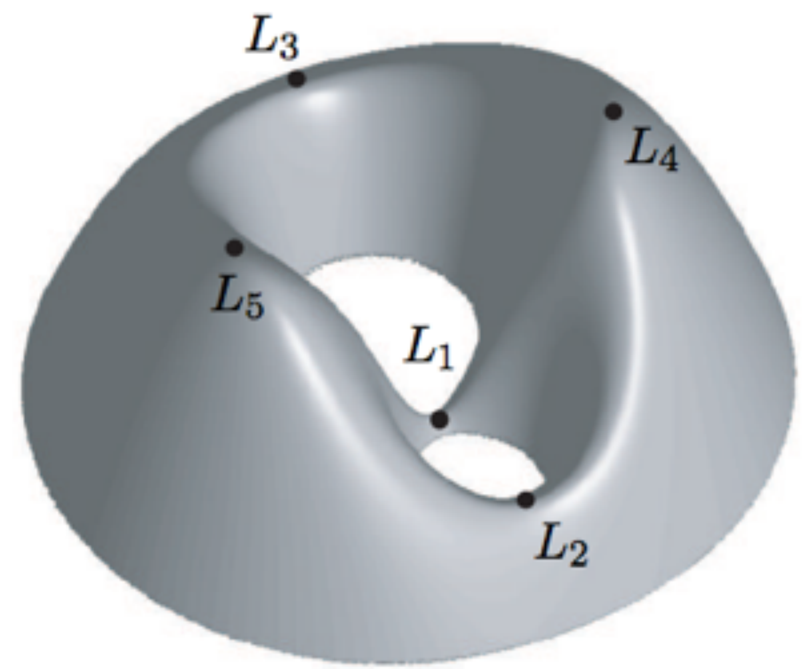
$$(x, y)$$

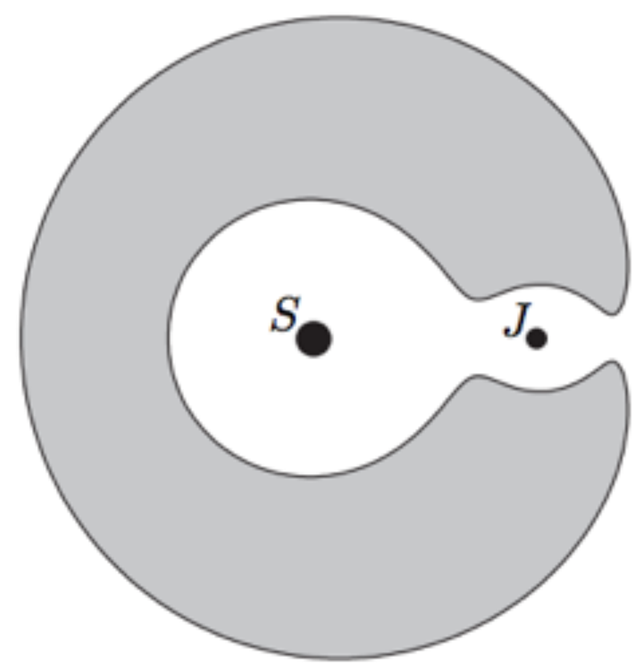


$$\text{Hamiltonian}$$

Gravitational Potential and Lagrange Points







Wishlist

- Earth$\rightarrow L_1$ (forward) + $L_1 \rightarrow$Moon (backward)
- 2D → 3D
- Include Sun’s potential
- Higher-order integrator:
  4.-5.-order symplectic Runge-Kutta