Muon Detector workings

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1 Explanation

1.1 Correction to the cos^2 law

Normally the muon flux is defined as $I(\theta) = I_0 cos^2(\theta)$, however this law fails to account for the finite thickness of earth's atmosphere. To account for it, we use the following expression:

$$I(\theta) = I_0 \left(\frac{a}{-r|cos(\theta)| + \sqrt{r^2 + 2ar + (rcos(\theta))^2}} \right) \tag{1}$$

Where a is the height if earth's atmosphere, taken here to be $10km$ and r is the earth's radius, taken here to be $6400km$.

1.2 The zenith angle definition

Due to the fact that the area of the sky our detector looks at is a spherical rectangle (a rectangle projected on a sphere) is taking a big part of the surface of the unit sphere one integrates over and thus it also crosses the poles of the sphere, where one encounters a discontinuity, we make a change in the reference system. Instead of the detector rotating with respect to the sky, we have the sky rotate with respect to the detector, while the detector is fixed facing in direction of the equator (the normal vector of the detector stays on the x-y plane). On Figure 1 one can see the spherical coordinate system used for integration.

The zenith angle transformed will thus be:

$$\theta_{mod}(\theta_{zenith}, \theta, \phi) = arcsin(sin(\theta_{zenith} + \theta)cos(\phi/2)) \tag{2}$$

1.3 The area correction

The geometry of our detector needs also to be accounted for when constructing the model. This is because for a particle to be registered as a muon it must penetrate both scintillators. However the angles at which this can be accomplished are limited by the geometry of the detector itself as shown in Figure 2.

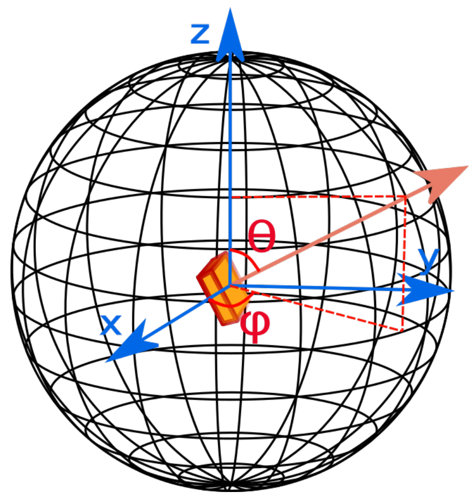

Figure 1: The spherical coordinate system.

Also the effective area which possible particles coming with a certain inclination angle with respect to the detector see, will also be a function of this inclination angle as illustrated in Figure 2.

The detector is basically a rectangular cuboid, thus it has a height- h, width- W and length- L. The area of the detector is defined by the $A = L\text{x}W$. After the correction for the area, it will be defined as $A' = L'.W'$, where we define L' and W' as:

$$L' = L - h.|tan(\theta)| \quad , where \quad \theta \in [-\alpha/2, \alpha/2] \tag{3}$$

$$W' = W - h.|tan(\phi)| \quad , where \quad \phi \in [-\beta/2, \beta/2] \tag{4}$$

Where the θ and ϕ are the zenith and azimuth angle respectively and they sweep values within the aperture angles for the length side - α and the width side - β, respectively. The illustrate the point for the aperture angles even better, refer to Figure 3

1.4 The final expression for the muon rate

To put it all together the expression used to model this detector (and can be used for any class of similar detectors) is:

$$\Phi = \int_{-\alpha/2}^{\alpha/2} \int_{-\beta/2}^{\beta/2} I(\theta_{mod}).A'(\theta, \phi).sin(\theta)\, d\theta\, d\phi \tag{5}$$

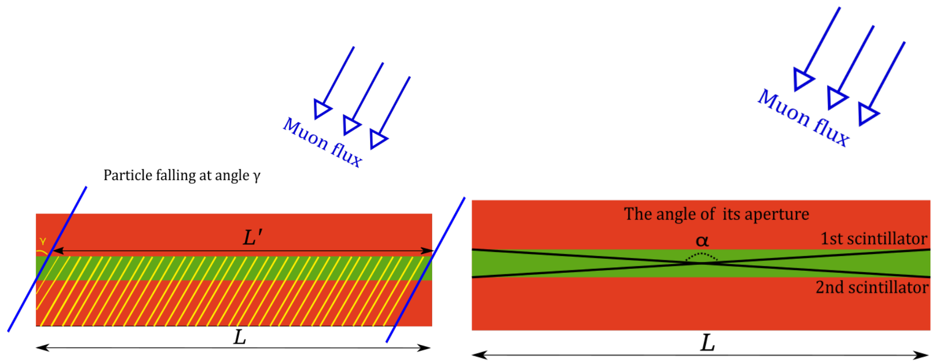

Figure 2: The first figure shows how depending on the angle the ray falls on the detector the effective detection area would be different (same consideration is to be done for the other side of the rectangular cuboid). The second figure defines an aperture angle α, all the particles which fall on the detector at angles steeper then $\alpha/2$ will not be registered.

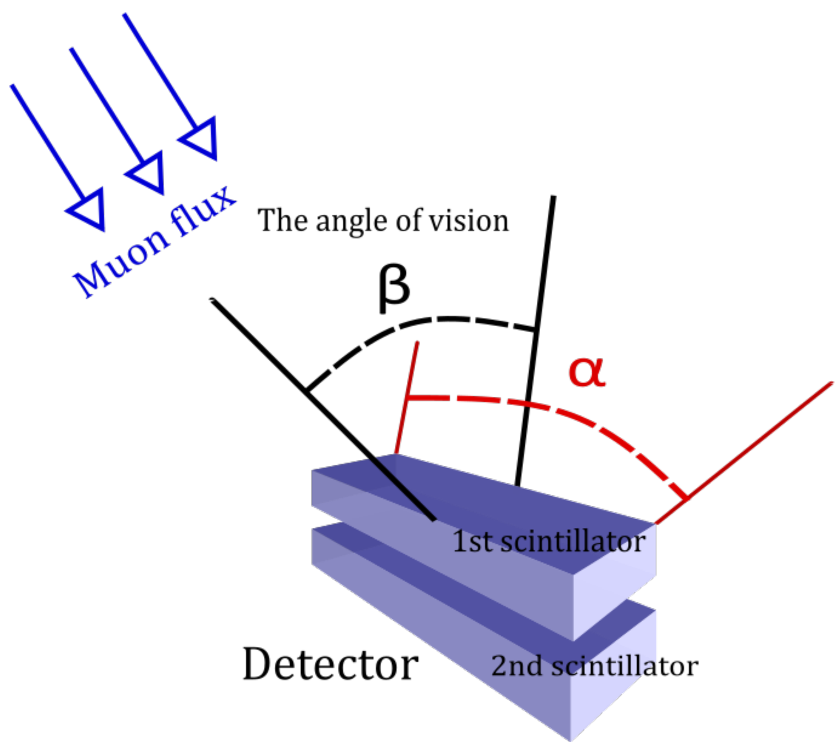

Figure 3: The aperture angles of the detector.

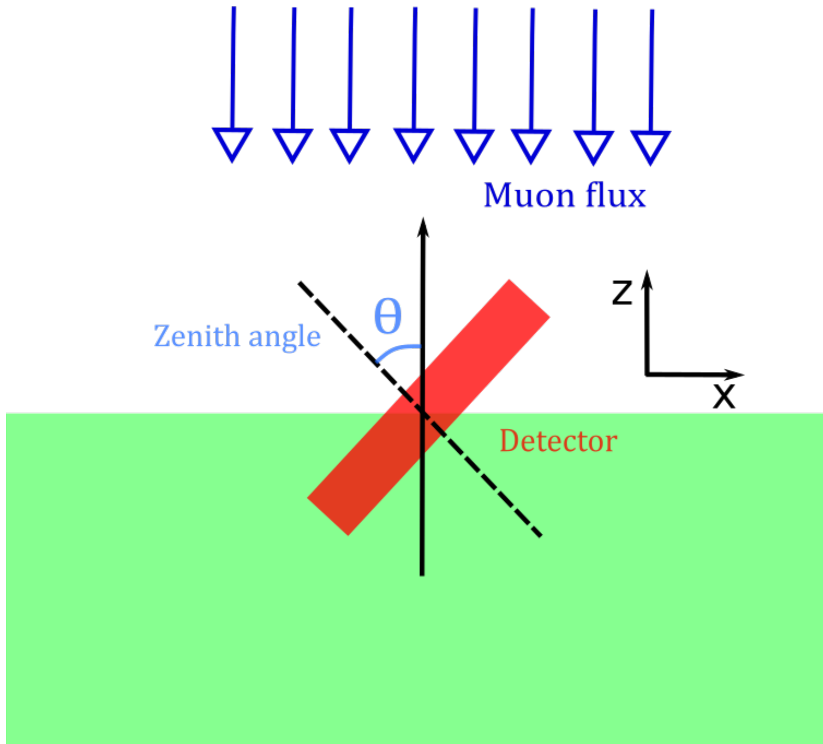

Figure 4: The zenith angle sweep .

Where we use the expressions from 1.1 to 1.3.

2 Example output

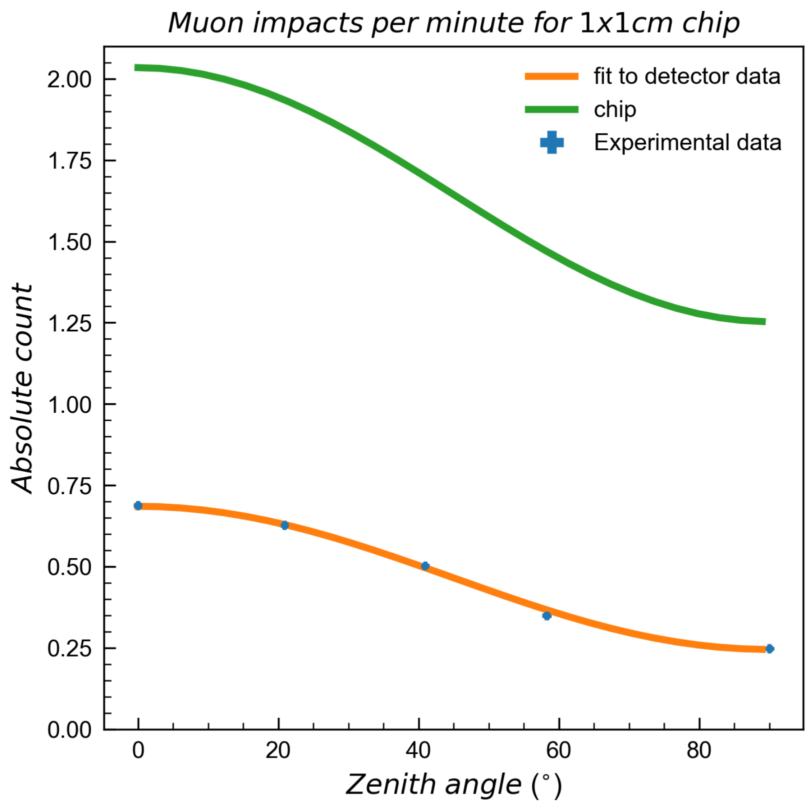

Figure 5: The fit for the detector data as a function of the zenith angle and one for an arbitrary rectangular solid body (like a chip for example).