1

An unbiased estimate for p, is:

$$p \approx p_s = \frac{\sum x}{n} = \frac{900}{1000} = 0.9$$

Let $X \sim B(1, p)$.

$$\mathrm{E}(X) = p \approx p_s,\ \mathrm{Var}(X) = p(1-p) \approx p_s(1-p_s)$$

It is clear that the sample proportion $P_s = \bar{X}$. By the Central Limit Theorem, since $n = 1000$ is large, $P_s \sim N\left(p_s, \frac{p_s(1-p_s)}{n}\right)$ approximately.

It is know that the $100(1-\alpha)\%$ confidence interval for p is given by:

$$p_s \pm z_c\sqrt{\frac{p_s(1-p_s)}{n}},\ P(|Z| < z_c) = 1 - \alpha$$

Thus, a 99.5% confidence interval for p, is:

$$0.9 \pm z_c\sqrt{\frac{0.9(0.1)}{1000}},\ P(|Z| < z_c) = 99.5\%$$

$$\begin{aligned} P(|Z| < z_c) &= 99.5\% \\ P(Z < z_c) &= 0.9975 \\ z_c &\approx 2.8070 \end{aligned}$$

Therefore, a 99.5% confidence interval for $p \approx \boxed{(0.873, 0.927)}$.

When a large number of samples of 1000-letter time-of-arrival recordings are collected and each of their respective 99.5% confidence intervals are calculated, approximately 99.5% of these confidence intervals contains p.

$$\begin{aligned} P(|Z| < z_c') &= 99.9\% \\ P(Z < z_c') &= 0.9995 \\ z_c' &\approx 3.2905 \end{aligned}$$

$$\begin{aligned} z_c'\sqrt{\frac{p_s q_s}{n'}} &< 0.005 \\ n' &> \left(\frac{z_c'}{0.005}\right)^2 (0.9)(0.1) \approx 38979.24 \\ \min(n') &= \boxed{38980} \end{aligned}$$

2

(a)

(i)

$$\mu \approx \bar{x} = \frac{\sum(x-18)}{n} + 18 = \frac{388}{60} + 18 = \frac{367}{15} \approx \boxed{24.467}$$

$$\sigma^2 \approx s^2 = \frac{1}{n-1}\left(\sum(x-18)^2 - \frac{\left(\sum(x-18)\right)^2}{n}\right) = \frac{1}{59}\left(2550 - \frac{388^2}{60}\right) = \frac{614}{885} \approx \boxed{0.694}$$

(ii)

$$\begin{aligned}
P(|Z| < z_c) &= 94\% \\
P(Z < z_c) &= 0.97 \\
z_c &\approx 1.8808
\end{aligned}$$

$$\begin{aligned}
z_c \frac{\sigma}{\sqrt{n}} &= \frac{0.18}{2} \\
z_c \frac{s}{\sqrt{n}} &\approx 0.09 \\
n &= \left(\frac{z_c}{0.09}\right)^2 \frac{614}{885} \\
&\approx \boxed{303}
\end{aligned}$$

(iii)

$$\begin{aligned}
z_c' \frac{\sigma}{\sqrt{n}} &> \frac{0.18}{2} \\
z_c' \frac{\sigma}{\sqrt{n}} &> z_c \frac{\sigma}{\sqrt{n}} \\
z_c' &> z_c \\
z_c' &> 1.8808 \\
P(Z < z_c') &> 0.97 \\
P(|Z| < z_c') &> 94\%
\end{aligned}$$

(b)

An unbiased estimate for p, the passing rate, is:

$$p \approx p_s = 60\% = 0.6$$

Let $X \sim B(1, p)$.

$$\mathrm{E}(X) = p \approx p_s,\ \mathrm{Var}(X) = p(1-p) \approx p_s(1-p_s)$$

It is clear that the sample passing rate $P_s = \bar{X}$. By the Central Limit Theorem, since $n = 50$ is large, $P_s \sim N\left(p_s, \frac{p_s(1-p_s)}{n}\right)$ approximately.

It is know that the $100(1-\alpha)\%$ confidence interval for p is given by:

$$p_s \pm z_c \sqrt{\frac{p_s(1-p_s)}{n}},\ P(|Z| < z_c) = 1 - \alpha$$

Thus, a 95% confidence interval for p, is:

$$0.6 \pm z_c \sqrt{\frac{0.6(0.4)}{50}},\ P(|Z| < z_c) = 95\%$$

$$\begin{aligned}
P(|Z| < z_c) &= 95\% \\
P(Z < z_c) &= 0.975 \\
z_c &\approx 1.9600
\end{aligned}$$

Therefore, a 95% confidence interval for $p \approx \boxed{(0.464, 0.736)}$.

$$a = \boxed{46.4},\ b = \boxed{73.6}$$

This statement is false. One can think of an extreme case where classes only have a passing rate of 0% or 100%, while still satisfying the condition where a random sample of 50 scripts having a passing rate of 60%, leading to the confidence interval concerned. This means that in reality, the percentage of classes having a passing rate between a% and b% can be any number in the interval $[0, 100]$.