

$$U \sim \text{U}(2f, 3f)$$

$$\text{pdf}_U(u) = \begin{cases} \frac{1}{f}, & 2f < u < 3f \\ 0, & \text{otherwise} \end{cases}$$

$$\text{cdf}_U(u) = \begin{cases} 0, & u \leq 2f \\ \frac{u}{f} - 2, & 2f < u < 3f \\ 1, & u \geq 3f \end{cases}$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$v = \left(\frac{1}{f} - \frac{1}{u} \right)^{-1}$$

$$V = \left(\frac{1}{f} - \frac{1}{U} \right)^{-1}$$

$$\begin{array}{ccccc} 2f & < & U & < & 3f \\ \frac{1}{3f} & < & \frac{1}{U} & < & \frac{1}{2f} \\ -\frac{1}{2f} & < & -\frac{1}{U} & < & -\frac{1}{3f} \\ \frac{1}{f} - \frac{1}{2f} & < & \frac{1}{f} - \frac{1}{U} & < & \frac{1}{f} - \frac{1}{3f} \\ \left(\frac{2}{3f} \right)^{-1} & < & \left(\frac{1}{f} - \frac{1}{U} \right)^{-1} & < & \left(\frac{1}{2f} \right)^{-1} \\ \frac{3}{2}f & < & V & < & 2f \end{array}$$

$$\begin{aligned}
\text{cdf}_V(v) &= P(V < v) \\
&= P\left(\left(\frac{1}{f} - \frac{1}{u}\right)^{-1} < v\right) \\
&= P\left(\frac{1}{f} - \frac{1}{U} > \frac{1}{v}\right) \\
&= P\left(\frac{1}{U} < \frac{1}{f} - \frac{1}{v}\right) \\
&= P\left(U > \left(\frac{1}{f} - \frac{1}{v}\right)^{-1}\right) \\
&= 1 - P\left(U < \left(\frac{v-f}{fv}\right)^{-1}\right) \\
&= 1 - \text{cdf}_U\left(\frac{fv}{v-f}\right) \\
&= \begin{cases} 1 - 0, & u \leq 2f \\ 1 - \left(\frac{\frac{fv}{v-f}}{f} - 2\right), & 2f < u < 3f \\ 1 - 1, & u \geq 3f \end{cases} \\
&= \begin{cases} 0, & v \leq \frac{3}{2}f \\ 2 - \frac{f}{v-f}, & \frac{3}{2}f < v < 2f \\ 1, & v \geq 2f \end{cases} \\
\text{pdf}_V(v) &= \begin{cases} \frac{f}{(v-f)^2}, & \frac{3}{2}f < v < 2f \\ 0, & \text{otherwise} \end{cases}
\end{aligned}$$

□

Range of V is the interval $\boxed{\left(\frac{3}{2}f, 2f\right)}$.

$$\begin{aligned}
E(V) &= \int_{-\infty}^{\infty} v \text{pdf}_V(v) dv \\
&= \int_{\frac{3}{2}f}^{2f} v \frac{f}{(v-f)^2} dv \\
&= \left(-v \frac{f}{v-f} + f \ln(v-f) \right) \Big|_{\frac{3}{2}f}^{2f} \\
&= -2f \frac{f}{2f-f} + f \ln(2f-f) + \frac{3}{2}f \frac{f}{\frac{3}{2}f-f} - f \ln\left(\frac{3}{2}f-f\right) \\
&= -2f \frac{f}{f} + f \ln(f) + \frac{3}{2}f \frac{f}{\frac{1}{2}f} - f \ln\left(\frac{1}{2}f\right) \\
&= -2f + f \ln(f) + 3f - f \ln(f) + f \ln(2) \\
&= \boxed{f \ln(2) + f}
\end{aligned}$$

Let m be the median of V .

$$\begin{aligned}
\int_{-\infty}^m \text{pdf}_V(v) dv &= \frac{1}{2} \\
\int_{\frac{3}{2}f}^m \frac{f}{(v-f)^2} dv &= \frac{1}{2} \\
-\frac{f}{v-f} \Big|_{\frac{3}{2}f}^m &= \frac{1}{2} \\
-\frac{f}{m-f} + \frac{f}{\frac{3}{2}f-f} &= \frac{1}{2} \\
-\frac{f}{m-f} + \frac{f}{\frac{1}{2}f} &= \frac{1}{2} \\
-\frac{f}{m-f} + 2 &= \frac{1}{2} \\
-\frac{f}{m-f} &= -\frac{3}{2} \\
2f &= 3m - 3f \\
3m &= 5f \\
m &= \boxed{\frac{5}{3}f}
\end{aligned}$$