COMBINATORICS
TUTORIAL 2: THE PRINCIPLE OF INCLUSION AND EXCLUSION

1 How many positive integers n are there such that n is a divisor of at least one of the numbers $10^{40},\ 20^{30}$? (Putnam 1983)

2 Let S be the set of 3-digit numbers abc such that $a, b, c \in \{1, 2, \ldots, 9\}$ and a, b, c are pairwise distinct (Thus, $489 \in S$ but $313 \notin S$ and $507 \notin S$). Find the number of members abc in S such that $a \neq 3$, $b \neq 5$ and $c \neq 7$.

3 In a shop, five different postcards are on sale. A customer wishes to send postcards to eight of his friends.

Use the principle of inclusion and exclusion to show that the number of ways in which he can send a postcard to each of his eight friends, buying at least one of each of the five types of cards, is

$$5^8 - \binom{5}{1}\cdot 4^8 + \binom{5}{2}\cdot 3^8 - \binom{5}{3}\cdot 2^8 + \binom{5}{4}\cdot 1^8 .$$

4 Each of ten ladies checks her hat and umbrella in a cloakroom and the attendant gives each lady back a hat and an umbrella at random. Show that the number of ways this can be done so that no lady gets back both of her possessions is

$$\sum_{r=0}^{10} (-1)^r \binom{10}{r} \{(10-r)!\}^2$$