1

$$\max(m,n)\left\lceil \frac{\max(m,n)}{\min(m,n)} \right\rceil$$

2

When sorted from top to bottom, left to right, the order of the tiles are only changed when a vertical move is made. However, the change of the number of out-of-order pair is alway even. Since the original number of out-of-order pairs is 0, the number of out-of-order pairs will always stay even. Since the number of out-of-order pairs is odd on the target arrangement, it is impossible to arrange the tiles from the left arrangement to the right arrangement.

4

$$\min\left(\sum_{i=k}^{k+1} |u_i + u_{i-1}|\right) = (k+1-k) - (k-(k-1)) = 1 - 1 = \boxed{0}$$

$$\begin{aligned}
&\max\left(\sum_{i=k}^{k+1} |u_i + u_{i-1}|\right) \\
&= \max(u_{k+1} - u_k + u_k - u_{k-1}) \\
&= \max(2u_k - u_{k+1} - u_{k-1}) \\
&= 2n - 1 - 2 \\
&= \boxed{2n-3}
\end{aligned}$$

6

$$x\lfloor x\lfloor x\lfloor x\rfloor\rfloor\rfloor = 50$$

$$x\lfloor x\lfloor x\lfloor x\rfloor\rfloor\rfloor \approx x^4$$

$$x^4 = 50 \implies x = \pm\sqrt[4]{50} \approx \pm 2.66 \implies x\lfloor x\lfloor x\lfloor x\rfloor\rfloor\rfloor = 50 \implies x \in (-3,-2) \cup (2,3)$$

If $x > 0$, $x \in (2,3)$.

$$\begin{aligned} x &< 3 \\ \lfloor x\rfloor &\leqslant 2 \\ x\lfloor x\rfloor &< 6 \\ \lfloor x\lfloor x\rfloor\rfloor &\leqslant 5 \\ x\lfloor x\lfloor x\rfloor\rfloor &< 15 \\ \lfloor x\lfloor x\lfloor x\rfloor\rfloor\rfloor &\leqslant 14 \\ x\lfloor x\lfloor x\lfloor x\rfloor\rfloor\rfloor &< 42 \\ x\lfloor x\lfloor x\lfloor x\rfloor\rfloor\rfloor &\neq 50 \end{aligned}$$

If $x < 0$, $x \in (-3,-2)$.

$$x\lfloor x\lfloor x\lfloor x\rfloor\rfloor\rfloor = 50 \implies \lfloor x\lfloor x\lfloor x\rfloor\rfloor\rfloor = m \in \mathbb{Z}^-$$

$$xm = 50 \implies x = \frac{50}{m}$$

$$\begin{array}{ccccc} -3 & < & x & < & -2 \\ -3 & < & \frac{50}{m} & < & -2 \\ -\frac{1}{2} & < & \frac{m}{50} & < & -\frac{1}{3} \\ -25 & < & m & < & -\frac{50}{3} \\ -24 & \leqslant & m & \leqslant & -17 \end{array}$$

$$x = \frac{50}{m} \in \left\{-\frac{50}{24}, \ldots, -\frac{50}{17}\right\} \implies \boxed{x = -\frac{50}{19}}$$