This exam contains 8 pages (including this cover page) and 8 questions. The total number of points is 70.

This is a custom exam written by Trevor, from TrevTutor.com that covers Proofs, Functions, Relations, and Finite State Automata. The questions are based off the videos on the website, as well as questions in the free textbook *The Book of Proof.* Please use this midterm as a guide for figuring out what your weak points are with the material.

If you'd like to see more practice exams, as well as the video solution to this exam, check out TrevTutor.com.

Question	Points	Score
1	10	
2	5	
3	5	
4	10	
5	10	
6	10	
7	10	
8	10	
Total:	70	

1. (10 points) Prove the following using direct proofs, contrapositive proofs, or proof by contradiction.

   (a) (3 points) If $a|b$ and $a|c$, then $a|(b+c)$.

   (b) (3 points) If $n \in \mathbb{Z}$, then $5n^2 + 3n + 7$ is odd.

   (c) (4 points) Suppose $a, b \in \mathbb{Z}$. If both ab and $a+b$ are even, then both a and b are even.

2. (5 points) Using proof by contradiction, show that if A and B are sets, $A \cap (B - A) = \emptyset$.

3. (5 points) Prove the following statement about sets. You may use set laws if you wish.

$$(A \cup B) - C = (A - C) \cup (B - C)$$

4. (10 points) Prove the following using mathematical induction.

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

5. (10 points) Prove that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \dfrac{(x+1)^3}{(x-1)^3}$ is bijective.

6. (10 points) The following question relates to relations.

   (a) (6 points) Consider the relation $R = \{(a, b), (a, c), (c, c), (b, b), (c, b), (b, c)\}$ on the set $A = \{a, b, c\}$. Is R reflexive? Symmetric? Transitive? Explain why.

   (b) (4 points) Which of the following operators are equivalence classes. $\geq, =, <$. Explain your reasoning

7. (10 points) Prove the following statements.

    (a) (5 points) Consider a square whose side-length is 1 meter. Select any five points from inside this square. Prove that at least two of these points are within $\frac{\sqrt{2}}{2}$ meters of each other.

    (b) (5 points) Show that if 15 numbers are selected at random, at least 3 of them will have the same remainder when divided by 7.

8. (10 points) The following question relate to the the Euclidian Algorithm and Finite State Automata.

   (a) (4 points) Find $gcd(6939, 544)$.

   (b) (6 points) Construct a finite state automata using $\Sigma = \{a, b\}$ that accepts any string with two consecutive b's.