No of common divisors of a and b is the number of divisors of $\gcd(a, b)$.
There are 2^{n-1} ways to right n as a sum of positive integers.

$$
\begin{aligned}
|A \cup B \cup C| &= |A \cup (B \cup C)| \\
&= |A| + |B \cup C| - |A \cap (B \cup C)| \\
&= |A| + |B \cup C| - |(A \cap B) \cup (B \cap C)| \\
&= |A| + |B| + |C| - |(A \cap B) \cup (B \cap C)|
\end{aligned}
$$