德明政府中学
Dunman High School

February 2021

Year 6 DM **50 minutes**

H2 Further Mathematics Test 4 **Total marks: 26**

1. There are **2** printed pages. Answer **all** the questions.
2. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
3. You are expected to use an approved graphing calculator.
4. You are reminded of the need for clear presentation in your answers.
5. At the end of the test, attach the **question paper to the back of your answer script**.
6. You may refer to the MF 26 formula list.

1 The continuous random variable X is uniformly distributed on the interval $[1,\ 2]$.

(i) State $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [2]

The random variable Y is defined by $Y = \dfrac{1}{X}$.

(ii) Show that $\mathrm{Var}(X) = \frac{1}{2} - (\ln 2)^2$. [3]

(iii) Estimate the probability that the mean of 50 independent observations of X is more than two times the mean of 100 observations of Y, stating clearly any distributions that you use. [3]

2 Country A, made up of many small towns, is about to hold an election. Let p be the proportion of voters who will vote for Candidate X.

(i) From a random sample of 1500 voters from a particular town, a media company obtained a 96% confidence interval of p as $(0.879,\ 0.912)$, and claimed that Candidate X had high chances of winning the election with high votes. Explain why the media company's claim is incorrect. [1]

A random sample of 1100 voters around the country was interviewed just after they had voted, and 678 of them said they had voted for Candidate X.

(ii) Using the data above, construct a 96% confidence interval for p. [3]

(iii) Explain what you understand by a 96% confidence interval in this context. [1]

3 Brand A electric light bulbs have lifetimes with mean 1200 hours. It is suspected that a particular batch of these bulbs is below standard in that the mean lifetime is less than 1200 hours. To test this suspicion, a random sample of 25 bulbs were taken from the batch and their lifetimes, x hours, were measured and summarised as follows:

$$\sum(x-1200) = -664, \quad \sum(x-1200)^2 = 558204.$$

(a) Test at 15% significance level whether the batch of electric light bulbs is below standard. State any assumptions that you used in your working. [6]

The manufacturer of Brand D batteries claimed that their batteries have mean lifetimes longer than Brand E batteries produced by a competitor. Batteries of both brands were randomly sampled and their lifetimes measured, to test the manufacturer's claim at 3% significance level. The results were as follows:

	Sample Size	Sample mean (hours)	Sample variance (hours^2)
Brand D	79	108	240
Brand E	71	100	220

(b) Assuming that Brand D and E batteries have the same population variance, find the critical region of the test. Explain if the manufacturer's claim is correct. [6]

(c) Explain how the test in part **(b)** would change if both brands of batteries do not have the same population variance. [1]