Assignment $\mathbb{B}g$

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$$\begin{cases} y = y(x) \\ \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{5}(\tan x - xy) \end{cases}$$

(i)

$$\begin{cases} 0 < h < \frac{\pi}{2} \\ h \approx 0 \\ y(0) = 0 \end{cases}$$

(a)

$$\begin{aligned}
&y_0 = y(0) = 0 \\
&y_1 = y_0 + \Delta x \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(x,y)=(x_0,y_0)} = 0 + \frac{h}{2}\frac{1}{5}(\tan 0 - 0 \cdot 0) = 0 \\
&y_2 = y_1 + \Delta x \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(x,y)=(x_1,y_1)} = 0 + \frac{h}{2}\frac{1}{5}\left(\tan\frac{h}{2} - \frac{h}{2} \cdot 0\right) = \frac{h}{10}\tan\frac{h}{2}
\end{aligned}$$

$$y(h) \approx \boxed{\frac{h}{10}\tan\frac{h}{2}}$$

(b)

$$\begin{aligned}
y_0 &= y(0) = 0 \\
y_1 &= y_0 + \Delta x \frac{\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(x,y)=(x_1,y_1)} \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{(x,y)=(x_0,y_0)}}{2} \\
&= 0 + h\frac{\frac{1}{5}(\tan 0 - 0 \cdot 0) + \frac{1}{5}(\tan h - h \cdot 0)}{2} = \frac{h}{10}\tan h
\end{aligned}$$

$$y(h) \approx \boxed{\frac{h}{10}\tan h}$$

(ii)

$$\begin{aligned}
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{1}{5}(\tan x - xy) \\
\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{x}{5}y &= \frac{1}{5}\tan x \\
e^{0.1x^2}\left(\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{x}{5}y\right) &= e^{0.1x^2}\left(\frac{1}{5}\tan x\right) \\
e^{0.1x^2}\frac{\mathrm{d}y}{\mathrm{d}x} + 0.2xe^{0.1x^2}y &= 0.2e^{0.1x^2}\tan x \\
\frac{\mathrm{d}}{\mathrm{d}x}e^{0.1x^2}y &= 0.2e^{0.1x^2}\tan x \\
e^{0.1x^2}y(x) &= \int 0.2e^{0.1x^2}\tan x\,\mathrm{d}x \\
e^{0.1h^2}y(h) &= \int 0.2e^{0.1h^2}\tan h\,\mathrm{d}h \\
e^{0.1h^2}y(h) &= \int_0^h 0.2e^{0.1x^2}\tan x\,\mathrm{d}x \quad \square
\end{aligned}$$

$$\begin{aligned}
\int_0^h 0.2e^{0.1x^2}\tan x\,\mathrm{d}x &\approx \int_0^h 0.2e^{0.1x^2}x\,\mathrm{d}x \\
&= \int_0^h 0.2xe^{0.1x^2}\,\mathrm{d}x \\
&= \left. e^{0.1x^2}\right|_0^h \\
&= e^{0.1h^2} - e^0 \\
&= e^{\frac{h^2}{10}} - 1
\end{aligned}$$

$$y(h) \approx \boxed{e^{\frac{h^2}{10}} - 1}$$