

Real Numbers

1 Real Numbers

The real numbers consist of all the numbers on the real number line.

$$\mathbb{R} = (-\infty, \infty)$$

1.1 Example

Which of the following are real numbers?

$$-69420, -2.345, 0, 0.0001, \sqrt{2}, \pi, 100, 1234567890$$

2 Types of Real Numbers

There are two main types of real numbers, rational and irrational numbers.

- Rational Numbers $\mathbb{Q} = \{\frac{p}{q}, p, q \in \mathbb{Z}\}$
 - Integers $\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \dots\}$
 - * Positive Integers $\mathbb{Z}^+ = \{1, 2, 3, 4, 5, \dots\}$
 - * Negative Integers $\mathbb{Z}^- = \{-1, -2, -3, -4, -5, \dots\}$
 - Decimals
 - * Terminating $\{0.6, -0.69, 4.2, \dots\}$
 - * Recurring $\{0.\dot{6}, -0.\dot{6}9, 0.\dot{6}0\dot{9}, 4.\dot{2}, \dots\}$
- Irrational Numbers $\{\pi, e, \sqrt{2}, \sqrt[3]{5}, \dots\}$

Fact: both the sum and the product of a rational number and an irrational number are irrational.

2.1 Example 1

Which of the following are

rational numbers?

irrational numbers?

$$2\pi, 3e, \sqrt{3}, \sqrt{4}, -\frac{6}{5}, \sqrt{8}, \sqrt[3]{8}, 420.69$$

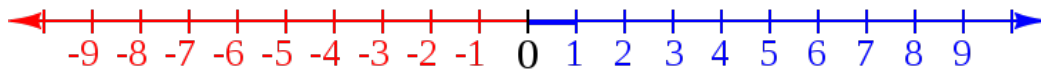
2.2 Example 2

Arrange the following in ascending order.

$$1, 0.21\overline{3}, 0.21\dot{3}, -0.22, 0.\dot{2}1\dot{3}, 0.2\dot{1}\dot{3}, 0$$

3 Number Line

The number line is used to give an visual representation of the real numbers and their operations.



3.1 Example

Represent the following equations on the number line.

$$3 + 5 = 8$$

$$3 - 5 = -2$$

$$-3 + 5 = 2$$

$$-3 - 5 = -8$$

4 Multiplication of Negative Numbers

The multiplication involving negative numbers is summarised below.

\times	1	-1
1	1	-1
-1	-1	1

Fact: $-a = (-1) \times a$.

4.1 Example

Compute the following.

$$5 \times 7, (-5) \times 7, 5 \times (-7), (-5) \times (-7)$$

5 Order of Operations

By convention, the order of operations is as follows.

1. Operations in the inner-most bracket.
2. Powers.
3. Multiplications and divisions, from left to right.
4. Additions and subtractions, from left to right.

5.1 Example

Compute the following.

$$2 \times (3 - 4)^3 + 8 \div 2^2$$

$$(-5 - (7 + (-2)^2)) \times (-2)$$

$$\left(\left(\frac{5}{6} - \frac{1}{4} \right) \div \frac{4}{3} \right) \times \left(-\frac{2}{3} \right)^2$$