1

$$\begin{cases} 10^{40} = (2 \cdot 5)^{40} = 2^{40} \cdot 5^{40} \\ 20^{30} = (2^2 \cdot 5)^{40} = 2^{60} \cdot 5^{30} \\ \gcd\left(10^{40}, 20^{30}\right) = 2^{40} \cdot 5^{30} \end{cases}$$

$$\begin{cases} \left|D\left(10^{40}\right)\right| = (40+1)(40+1) = 1681 \\ \left|D\left(20^{30}\right)\right| = (60+1)(30+1) = 1891 \\ \left|D\left(\gcd\left(10^{40}, 20^{30}\right)\right)\right| = (40+1)(30+1) = 1271 \end{cases}$$

$$\begin{aligned} & \left|D\left(10^{40} \vee 20^{30}\right)\right| \\ = & \left|D\left(10^{40}\right) \cup D\left(20^{30}\right)\right| \\ = & \left|D\left(10^{40}\right)\right| + \left|D\left(20^{30}\right)\right| - \left|D\left(10^{40}\right) \cap D\left(20^{30}\right)\right| \\ = & \left|D\left(10^{40}\right)\right| + \left|D\left(20^{30}\right)\right| - \left|D\left(10^{40} \wedge 20^{30}\right)\right| \\ = & \left|D\left(10^{40}\right)\right| + \left|D\left(20^{30}\right)\right| - \left|D\left(\gcd\left(10^{40}, 20^{30}\right)\right)\right| \\ = & 1681 + 1891 - 1271 \\ = & \boxed{2301} \end{aligned}$$

2

$$\begin{aligned} |S_{\neq}| &= |A_3' \cap B_5' \cap C_7'| \\ &= |S| - |A_3' \cup B_5' \cup C_7'| \\ &= |S| - (|A_3'| + |B_5'| + |C_7'| + |A_3' \cup B_5'| + |A_3' \cup B_5'|) \end{aligned}$$