$$y - \bar{y} = b(x - \bar{x}) \iff y = (\bar{y} - b\bar{x}) + bx \equiv y = a + bx$$

$$\implies a = \bar{y} - b\bar{x}$$

$$\begin{aligned}
v_i &= y_i - (a + bx_i) \\
&= y_i - (\bar{y} - b\bar{x} + bx_i) \\
&= y_i - \bar{y} + b(\bar{x} - x_i) \\
\sum v_i^2 &= \sum (y_i - \bar{y} + b(\bar{x} - x_i))^2 \\
\left(\sum v_i^2\right)' &= \sum 2(y_i - \bar{y} + b(\bar{x} - x_i))(\bar{x} - x_i) \\
\left(\sum v_i^2\right)' &= 0 \\
\sum 2(y_i - \bar{y} + b(\bar{x} - x_i))(\bar{x} - x_i) &= 0 \\
\sum (y_i - \bar{y})(\bar{x} - x_i) + b\sum (\bar{x} - x_i)^2 &= 0 \\
b\sum (x_i - \bar{x})^2 &= \sum (y_i - \bar{y})(x_i - \bar{x}) \\
b &= \boxed{\frac{\sum (y_i - \bar{y})(x_i - \bar{x})}{\sum (x_i - \bar{x})^2}}
\end{aligned}$$