Vectors, Lines and Planes

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LaTeX

1 Vectors

1.1 Formal Definition

Vectors are members of a vector space $(V, S, +, \cdot)$.

1.2 Dot Product (Scalar Product)

$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\|\|\mathbf{v}\| \cos\theta$$

1.3 Cross Product (Vector Product)

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{bmatrix} u_1 v_2 - u_2 v_1 \\ u_3 v_1 - u_1 v_3 \\ u_2 v_3 - u_3 v_2 \end{bmatrix}$$

1.4 Length of Projections

1.4.1 u on v

$$|\mathbf{u} \cdot \hat{\mathbf{v}}| = \frac{|\mathbf{u} \cdot \mathbf{v}|}{\|\mathbf{v}\|} = \|\mathbf{u}\| \cos\theta$$

1.4.2 Tip of u on v

$$|\mathbf{u} \times \hat{\mathbf{v}}| = \frac{|\mathbf{u} \times \mathbf{v}|}{\|\mathbf{v}\|} = \frac{1}{\sqrt{v_1^2 + v_2^2 + v_3^3}} \left| \begin{bmatrix} u_1 v_2 - u_2 v_1 \\ u_3 v_1 - u_1 v_3 \\ u_2 v_3 - u_3 v_2 \end{bmatrix} \right|$$

$$= \sqrt{\frac{(u_1 v_2 - u_2 v_1)^2 + (u_3 v_1 - u_1 v_3)^2 + (u_2 v_3 - u_3 v_2)^2}{v_1^2 + v_2^2 + v_3^3}}$$