1

Let the mean mass of the contents of a bag be μ grams.

$$\begin{cases} H_0 : \mu = 10000 \\ H_a : \mu \neq 10000 \end{cases}$$

$$\bar{x} = \frac{\sum (x - 10000)}{80} + 10000 \approx 9968.6$$

$$s^2 = \frac{1}{79} \left(\sum (x - 10000)^2 - \frac{\left(\sum (x - 10000)\right)^2}{80} \right) \approx 24448$$

Under H_0, the test statistic $\bar{X} \sim N\left(\mu, \frac{s^2}{n}\right)$ approximately.

$$p\text{-value} = 2P\left(\bar{X} \leqslant \bar{x}\right) \approx 0.072463 \leqslant 0.10$$

Since p-value $\leqslant 10\%$, H_0 is rejected in favour of H_a and a conclusion is made that there is sufficient evidence at a 10% significance level that the mean mass of a bag differs from 10 kg.

'at the 10 % significance level' means that there is a 10% probability when the null hypothesis of the mean mass of the contents of a bag does not differ from 10 kg is rejected when in fact the mean mass is 10 kg.

'p-value' is the probability at which the test statistic $\bar{X}$ is more or equally extreme as the sample statistic $\bar{x}$.

2

Let the children in Ms Patricia's school sleep an average of μ hours.

$$\begin{cases} H_0 : \mu = \mu_0 = 6.5 \\ H_a : \mu < \mu_0 = 6.5 \end{cases}$$

$$\bar{x} = \frac{\sum x}{n} = 6.325$$

$$s^2 = \frac{n}{n-1}\left(\frac{\sum x^2}{n} - \bar{x}^2\right) \approx 0.12214$$

Under H_0, the test statistic $T = \frac{\bar{X}-\mu_0}{S\sqrt{\frac{1}{n}}} \sim t(n-1)$.

$$p\text{-value} = P\left(T \leqslant \frac{\bar{x}-\mu_0}{s\sqrt{\frac{1}{n}}}\right) \approx 0.099807 > 0.08$$

Since p-value $> 8\%$, H_0 is not rejected and a conclusion is made that there is insufficient evidence at a 8% significance level that the children in Ms Patricia's school sleep an average of fewer than 6.5 hours each night.

$$T \sim t(14)$$

$$s^2 = \frac{15}{14}(0.849) = \frac{2547}{2800}$$

$$p\text{-value} \leqslant 0.08$$

$$P\left(T \leqslant \frac{\bar{x}-6.5}{\frac{2547}{2800}\sqrt{\frac{1}{15}}}\right) \leqslant 0.08$$

$$\frac{\bar{x}-6.5}{\frac{2547}{2800}\sqrt{\frac{1}{15}}} \leqslant a \approx -1.4839$$

$$\bar{x} \leqslant \frac{2547a}{2800\sqrt{15}} + 6.5 \approx 6.1515$$

$$\bar{x} \in [0, 6.15]$$