MERIDIAN JUNIOR COLLEGE
2015 FASTEST FINGERS GC COMPETITION
PRELIMINARY ROUND

WRITE YOUR ANSWER IN THE BOX PROVIDED.

LEAVE ALL NON-EXACT ANSWERS TO 3 SIGNIFICANT FIGURES, UNLESS OTHERWISE STATED.

1. Evaluate $\dfrac{2\left(\sqrt[3]{\mathrm{e}}\right)+\left(\sqrt[6]{\mathrm{e}}\right)^2}{\sqrt[9]{\mathrm{e}^2}}$ to 3 significant figures.

2. A sequence is defined by

$u_1 = 1$ and $u_n = \dfrac{1}{5}u_{n-1} + n$, for $n \geq 2$

Find the least value of n such that $u_n > 40$.

3. Find the real root of the equation $\mathrm{e}x^3 - \pi x^2 + \left(\sin\dfrac{\pi}{3}\right)x - \ln 3 = 0$.

4. By solving for the system of equations,

$$5x^3 + 4y^2 - 3z = 139,$$
$$-2x^3 + 3y^2 + 4z = -26,$$
$$3x^3 - 5y^2 - 8z = 29,$$

find the exact value of $x + y + z$, given that $y \in \mathbb{R}^-$.

5. The curves C_1 and C_2 are defined as follows:

$C_1 : y = x + \dfrac{1}{x}$

$C_2 : (x-1)^2 + y^2 = k^2$

where k is a real positive value. Determine the value of k such that curves C_1 and C_2 intersect at exactly 1 point.

6. Find the point of intersection between the line l and plane π with equations as follows:

$l : \dfrac{-x-2}{3} = \dfrac{y+1}{2} = z - 1$

$\pi : x + y - 2z = 1$

7. The curve C is defined parametrically by $y = \ln t$ and $x = (1+t)^{\frac{2}{3}}$ where t > 0. Find the area enclosed by the curve C, the lines $y = 2$, $y = 3$ and the y- axis.

8. Given that $z_1 = -\frac{3}{\sqrt{5}} + \frac{1}{5}\mathrm{i}$ and $z_2 = \frac{1}{5} - \frac{3}{\sqrt{2}}\mathrm{i}$, find the argument of the complex conjugate of $\frac{(z_1)^2}{(z_2)^3}$. Give your answer in radians to 3 significant figures.
9. Find the number of ways in which the letters of the word CALCULATOR can be arranged if the arrangement cannot begin or end with the letter 'R'.
10. Given $X \sim \mathrm{Po}(\lambda)$ where $\lambda > 2$ and $\mathrm{P}(X = 3) = 0.17894$, find the value of λ.
11. In a company selling candies, on average, 1% of the candies are crushed. A test is conducted by randomly selecting 50 candies. To pass the test, there can only be at most 1 candy that is crushed. Find the probability that out of 10 tests, exactly 8 of them passed.
12. The random variable *X* has a normal distribution with mean μ and standard deviation σ. Given that $\mathrm{P}(X < 12) = 0.1$ and $\mathrm{P}(X > 34) = 0.2$, find the value of σ.
13. The random variable *X* has a normal distribution with mean μ and standard deviation σ. Given that $\mathrm{P}(|X - \mu| < 1) = 0.6$, find the value of σ.
14. A 2-tail *z*-test is conducted at α% significance level. If it is given that H_0 is rejected and the *z*-value (or z-statistic) obtained is 2.15, find the minimum value of α.
15.

x	44.5	42.3	33.5	28.2	18.5	13.9	15.7	10.3
y	22.5	25.0	28.0	30.5	38.0	40.5	42.5	48.0

Find the value of *b* of the least square regression line $\ln x = ay + b$.

ANSWERS

1. 3.35
2. 33
3. 1.18
4. 5
5. 2
6. $(4, -5, -1)$
7. 5.68
8. 2.15
9. 362880
10. 4.32

11. 0.170
12. 10.4
13. 1.19
14. 3.16
15. 5.16