MERIDIAN JUNIOR COLLEGE
2016 FASTEST FINGERS GC COMPETITION
PRELIMINARY ROUND

WRITE YOUR ANSWER IN THE BOX PROVIDED.

LEAVE ALL NON-EXACT ANSWERS TO 3 SIGNIFICANT FIGURES, UNLESS OTHERWISE STATED.

1. Evaluate $\ln\frac{1}{2}+\ln\frac{2}{3}+\ln\frac{3}{4}+...+\ln\frac{9999}{10000}$.

2. Evaluate $5(9)+8(13)+11(17)+14(21)+...+119(161)$

3. The sequence $u_0,u_1,u_2,u_3,...$ is defined by the recurrence relation $u_{n+1}=\sqrt{2u_n}+\mathrm{e}^n,\ n\geq 0$ and $u_0=7$. Find $u(10)$.

4. Given the following system of equations,

$$6\mathrm{e}^x+3\mathrm{e}^y+2\mathrm{e}^z=21$$
$$4\mathrm{e}^x+6\mathrm{e}^y-2\mathrm{e}^z=-10$$
$$2\mathrm{e}^x+9\mathrm{e}^y+3\mathrm{e}^z=31\ ,$$

find the value of $x+y+z$.

5. The equations of planes π_1 and π_2 are given as follows:

$$\pi_1:\mathbf{r}\bullet\begin{pmatrix}1\\2\\4\end{pmatrix}=6$$
$$\pi_2:\mathbf{r}\bullet\begin{pmatrix}-2\\3\\-1\end{pmatrix}=2$$

A line l with direction vector **d** is parallel to plane π_1 and lies in plane π_2. Given that $\begin{pmatrix}x\\y\\z\end{pmatrix}$ is a unit vector that is parallel to **d**, find the value of $|x+y+z|$.

6. A curve has equation $y=\ln|x|,\ x\neq 0$. Find the area bounded by the curve and the lines $y=-1$ and $y=1$.

7. Find the modulus of z if $\left(\frac{1+\mathrm{i}}{21-13\mathrm{i}}\right)z=\left(\frac{2-3\mathrm{i}}{-8+5\mathrm{i}}\right)$.
8. Given that $1+\mathrm{i}$ is a root of the equation $kz^3-3z^2+2z+2=0$, find the value of the real number k.
9. Find the number of 3-letter code words that can be formed from the letters in "FFGCC".
10. A multiple choice test consists of 10 questions, each with four possible answers, of which only one is correct. If a student randomly chooses the answer to each question and answers each question sequentially such that all questions are attempted, find the probability that the last question is the fifth correctly answered question in the test.
11. The time taken for students to travel to school follows a normal distribution with mean 95 minutes and standard deviation 21 minutes.
    Three students are randomly selected. Find the probability that exactly two students take more than 120 minutes each to travel to school, while the time taken for the remaining student to travel to school is between 80 and 112 minutes.

12. An experiment involves throwing 3 fair dice, and a success is recorded when all 3 dice show distinct even numbers. The experiment is repeated 100 times. Using a suitable approximation, find the probability that the total number of successes is more than 5.
13. A survey yields the following data:

x	18	19	21	24	25
frequency	19	24	15	20	18

14. A two-tailed Z-test is performed to determine if the population mean is 22. Find the p-value.

x	0.1	0.2	0.5	0.8	0.9	1.3	1.5
y	15.8	18.2	20.4	25	36.2	32.4	38.1

    One of the data points appears to be incorrect. By removing this data point, calculate the product moment correlation coefficient between x and y.
15. A weighing machine used to measure a person's mass is monitored. The following table gives the actual mass, x, and the machine's reading, y, in kg.

x	49	55	64	72	83	89	98	104
y	47	59	64	68	85	89	94	108

    Using a suitable regression line, calculate an estimate for the percentage error in the machine's reading if the actual mass of a person is 75 kg.

ANSWERS

1. –9.21
2. 264810
3. 8181
4. 0.981
5. 0.816
6. 4.70
7. 6.67
8. 2
9. 18
10. 0.0292
11. 0.0227
12. 0.0632
13. 0.0109
14. 0.990
15. 0.0199 / 0.0200 (using 7 s.f. values) / 0.0196 (using 6 s.f. values) / 0.0236 (using 5 s.f. values)