$$T : \mathbb{R}^n \to \mathbb{R}^m$$

The matrix A represents T, A is $m \times n$.

$$T(\mathbf{x}) = A\mathbf{x},\ \mathbf{x} \in \mathbb{R}^n$$

Also,

$$A\mathbf{x} = \mathbf{v},\ \mathbf{v} \in \mathbb{R}^m$$

$$\text{rank}(T) = \dim(\text{range}(T)) \leqslant m$$

$$S : \mathbb{R}^m \to \mathbb{R}^n$$

The matrix B represents S, B is $n \times m$.

$$S(\mathbf{y}) = B\mathbf{y},\ \mathbf{y} \in \mathbb{R}^m$$

Also,

$$B\mathbf{y} = \mathbf{u},\ \mathbf{u} \in \mathbb{R}^n$$

$$\text{rank}(S) = \dim(\text{range}(S)) \leqslant n$$

By rank-nullity theorem,

$$\begin{cases} \text{rank}(T) + \text{nullity}(T) = n \\ \text{rank}(S) + \text{nullity}(S) = m \end{cases} \implies \begin{cases} \text{rank}(T) = n \\ \text{rank}(S) = m \end{cases}$$

$$\begin{cases} m \geqslant \text{rank}(T) = n \\ n \geqslant \text{rank}(S) = m \end{cases} \quad \begin{cases} m \geqslant n \\ n \geqslant m \end{cases} \implies m = n \quad \square$$

$$\lim_{n\to\infty} A^n = A_\infty$$

$$\lim_{n\to\infty} A^n\mathbf{v} = A_\infty\mathbf{v} = \mathbf{v}_\infty$$

$$\begin{aligned} A_\infty\mathbf{v} &= \mathbf{v}_\infty \\ AA_\infty\mathbf{v} &= A\mathbf{v}_\infty \\ A_\infty\mathbf{v} &= A\mathbf{v}_\infty \\ AA_\infty\mathbf{v} &= AA\mathbf{v}_\infty \\ A_\infty\mathbf{v} &= A^2\mathbf{v}_\infty \\ &\vdots \\ A_\infty\mathbf{v} &= A_\infty\mathbf{v}_\infty \\ \mathbf{v} &= \mathbf{v}_\infty \\ A\mathbf{v} &= A\mathbf{v}_\infty \\ A\mathbf{v} &= \mathbf{v}_\infty \\ A\mathbf{v} &= \mathbf{v} \\ A\mathbf{v} &= 1\mathbf{v} \quad \square \end{aligned}$$