(a)

$$2z^3 - 3z^2 + kz + 26 = 0,\ k \in \mathbb{R}$$

By the conjugate root theorem, since all the conefficient of z are real, if $z = 1 + ai$ is a root to the equation, $z = 1 - ai$ must also be a root to the equation. Also, any odd degree polynomial has at least one real root. Thus, the original expression in z can be factor as:

$$\begin{aligned}
&A(z - (1 + ai))(z - (1 - ai))(z - c) \\
&= A((z - 1) - ai)((z - 1) + ai)(z - c) \\
&= A\left((z - 1)^2 - (ai)^2\right)(z - c) \\
&= A\left(z^2 - 2z + 1 + a^2\right)(z - c) \\
&= Az^3 - 2Az^2 + A\left(1 + a^2\right)z - Acz^2 + 2Acz - A\left(1 + a^2\right)c \\
&= Az^3 - A(2 + c)z^2 + A\left(1 + a^2 + 2c\right)z - A\left(1 + a^2\right)c
\end{aligned}$$

Since this expression is identical to the original,

$$Az^3 - A(2 + c)z^2 + A\left(1 + a^2 + 2c\right)z - A\left(1 + a^2\right)c \equiv 2z^3 - 3z^2 + kz + 26$$

By comparing coefficients,

$$\begin{cases} A &= 2 \\ A(2 + c) &= 3 \\ A\left(1 + a^2 + 2c\right) &= k \\ A\left(1 + a^2\right)c &= -26 \end{cases} \implies \begin{cases} 2(2 + c) &= 3 \\ 2\left(1 + a^2 + 2c\right) &= k \\ 2\left(1 + a^2\right)c &= -26 \end{cases}$$

$$\begin{cases} c &= -\frac{1}{2} \\ 2\left(1 + a^2 + 2c\right) &= k \\ 2\left(1 + a^2\right)c &= -26 \end{cases} \implies \begin{cases} 2\left(1 + a^2 + 2\left(-\frac{1}{2}\right)\right) &= k \\ 2\left(1 + a^2\right)\left(-\frac{1}{2}\right) &= -26 \end{cases}$$

$$\begin{cases} 2a^2 &= k \\ a^2 &= 25 \end{cases} \implies \boxed{\begin{cases} k &= 50 \\ a &= 5 \end{cases}}$$

(b)

(i)

$$\begin{aligned}
(x + iy)^2 &= 15 + 8i \\
x^2 + 2ixy - y^2 &= 15 + 8i \\
\left(x^2 - y^2\right) + (2xy)i &= 15 + 8i
\end{aligned}$$

Since $x, y \in \mathbb{R}$, by comparing the coefficients of the real and imaginary parts,

$$\begin{cases} x^2 - y^2 &= 15 \\ 2xy &= 8 \end{cases}$$

$$\begin{aligned} y = \frac{4}{x} \implies x^2 - \left(\frac{4}{x}\right)^2 &= 15 \\ x^2 - \frac{16}{x^2} &= 15 \\ x^4 - 16 &= 15x^2 \\ x^4 - 15x^2 - 16 &= 0 \\ \left(x^2+1\right)\left(x^2-16\right) &= 0 \\ x^2 - 16 &= 0 \\ x^2 &= 16 \\ x &= \pm 4 \\ y &= \frac{4}{\pm 4} \\ y &= \pm 1 \end{aligned}$$

Possible values: $(x, y) = \boxed{(-4, -1)}$ OR $\boxed{(4, 1)}$.

(ii)

$$\begin{aligned} z^2 - (2+7i)z &= 15 - 5i \\ z^2 - (2+7i)z - (15-5i) &= 0 \end{aligned}$$

$$\begin{aligned} z &= \frac{-(-(2+7i)) \pm \sqrt{(-(2+7i))^2 - 4(1)(-(15-5i))}}{2(1)} \\ &= \frac{(2+7i) \pm \sqrt{(2+7i)^2 + 4(15-5i)}}{2} \\ &= \frac{1}{2}\left(2 + 7i \pm \sqrt{4 + 28i - 49 + 60 - 20i}\right) \\ &= \frac{1}{2}\left(2 + 7i \pm \sqrt{15 + 8i}\right) \\ &= \frac{1}{2}\left(2 + 7i \pm \sqrt{4^2 + 2(4)(i) + i^2}\right) \\ &= \frac{1}{2}\left(2 + 7i \pm \sqrt{(4+i)^2}\right) \\ &= \frac{1}{2}(2 + 7i \pm (4+i)) \end{aligned}$$

$$
\begin{aligned}
z_2 &= \frac{1}{2}(2 + 7i - (4 + i)) \\
&= \frac{1}{2}(-2 + 6i) \\
&= -1 + 3i \\
&= \sqrt{(-1)^2 + 3^2} e^{\pi - \tan^{-1} 3} \\
&= \boxed{\sqrt{10} e^{\pi - \tan^{-1} 3}}
\end{aligned}
$$

$$
\begin{aligned}
z_1 &= \frac{1}{2}(2 + 7i) + (4 + i)) \\
&= \frac{1}{2}(6 + 8i) \\
&= 3 + 4i
\end{aligned}
$$

$$
\begin{aligned}
\arg z_1^2 z_2^* &= 2 \arg z_1 - \arg z_2 \\
&= 2 \tan^{-1} \frac{4}{3} - \left(\pi - \tan^{-1} 3\right) \\
&= \boxed{2 \tan^{-1} \frac{4}{3} + \tan^{-1} 3 - \pi}
\end{aligned}
$$