



MINISTRY OF EDUCATION, SINGAPORE  
in collaboration with  
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION  
General Certificate of Education Advanced Level  
Higher 2



## FURTHER MATHEMATICS

9649/02

Paper 2

October/November 2020

3 hours

Additional Materials: List of Formulae (MF26)

### READ THESE INSTRUCTIONS FIRST

An answer booklet and a graph paper booklet will be provided with this question paper. You should follow the instructions on the front cover of both booklets. If you need additional answer paper or graph paper ask the invigilator for a continuation booklet or graph paper booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of 6 printed pages and 2 blank pages.



Singapore Examinations and Assessment Board



Cambridge Assessment  
International Education

**Section A: Pure Mathematics [50 marks]**

- 1 Let  $S$  be the solution space of the following system of equations.

$$\begin{aligned}x_1 - x_2 + 2x_3 - x_4 + 5x_5 &= 0 \\x_1 + 3x_2 - 2x_3 + x_4 - x_5 &= 0 \\x_1 + x_3 + 3x_4 &= 0 \\x_2 - x_3 + x_4 - 2x_5 &= 0\end{aligned}$$

- (i) Find a basis for  $S$ . [5]
- (ii) State the dimension of  $S$ . [1]
- 2 (i) By considering the 7th roots of unity, express  $z^6 + z^5 + z^4 + z^3 + z^2 + z + 1$  as the product of three quadratic factors with real coefficients. Give each coefficient, where appropriate, in an exact trigonometric form. [3]

- (ii) Deduce that

$$\cos\left(\frac{1}{7}\pi\right) - \cos\left(\frac{2}{7}\pi\right) + \cos\left(\frac{3}{7}\pi\right) = \frac{1}{2}$$

and determine the exact value of

$$\cos\left(\frac{1}{7}\pi\right) \cos\left(\frac{2}{7}\pi\right) \cos\left(\frac{3}{7}\pi\right).$$

Full working must be shown.

[5]

- 3 The plane transformation  $T$ , from  $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ , is defined by

$$T : \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \mathbf{A} \begin{pmatrix} x \\ y \end{pmatrix}, \text{ where } \mathbf{A} = \begin{pmatrix} -1 & 7 \\ 2 & -6 \end{pmatrix}.$$

- (i) Find
- (a) the image of the point with coordinates  $(6, 1)$ , [1]
- (b) the coordinates of the point whose image point has coordinates  $(5, 14)$ , [2]
- (c) the image of the line  $y = 1$ . [3]
- (ii) The unit square  $OABC$ , where  $O = (0, 0)$ ,  $A = (1, 0)$ ,  $B = (1, 1)$  and  $C = (0, 1)$ , is transformed under  $T$  to the parallelogram  $OA'B'C'$ . Determine the area of  $OA'B'C'$ . [3]
- 4 The normal to the rectangular hyperbola  $xy = c^2$  at the point  $P\left(cp, \frac{c}{p}\right)$ ,  $p > 0$ , meets the curve again at the point  $Q$ .
- (i) Determine the coordinates of  $Q$ . [7]
- (ii) Prove that  $(PQ)^2 = 3(OP)^2 + (OQ)^2$ . [5]

- 5 The curve  $C$  is defined parametrically by  $x = 4t^3 + 9$ ,  $y = 24t\sqrt{t}$  for  $0 \leq t \leq a$ , where  $a$  is a positive constant.

When  $C$  is rotated about the  $x$ -axis, the surface area generated is  $X(a)$ .

When  $C$  is rotated about the  $y$ -axis, the surface area generated is  $Y(a)$ .

- (i) Show that  $X(a) = k\pi((a^3 + 9)^{1.5} - 27)$  for some integer  $k$  to be determined. [7]
- (ii) Find an expression, in terms of  $a$ , for  $Y(a)$ . [5]
- (iii) Find, correct to 4 decimal places, all values of  $a$  for which  $X(a) = Y(a)$  and justify that there are no others. [3]

### Section B: Probability and Statistics [50 marks]

- 6 Cheng sells insurance policies by phone. He knows from past experience that 17% of the calls he makes will be successful in selling a policy.

Cheng denotes by  $X$  the number of calls he makes each day up to and including his first successful call.

- (i) State two conditions under which  $X$  can be well modelled using a geometric distribution. [2]

You are now given that  $X$  follows a geometric distribution.

- (ii) Find the probability that the fifth call Cheng makes one day is the first to be successful. [1]
- (iii) Find how many calls Cheng must make in order to be 95% certain that at least one will be successful. [3]
- (iv) Write down the mean,  $a$ , and standard deviation,  $b$ , of  $X$ .

Hence find  $P(X > a + b)$ . [3]

- 7 An internet service provider (ISP) monitors the download speeds available to its domestic customers. Any occasion on which the speed drops below 5 megabits per second (Mbps) for a minute or more is referred to as an 'outage'. The ISP models the times between outages by the random variable  $T$  with cumulative distribution function

$$F(t) = 1 - e^{-kt}, \text{ for suitable } k \text{ and } t \geq 0.$$

In this distribution,  $T$  is measured in days.

- (i) Obtain the probability density function for  $T$ , name the distribution and state its mean and variance. [3]

- (ii) Show that  $P(2 \leq T < 3) = e^{-2k} - e^{-3k}$ . [1]

The ISP claims in its technical literature that the times between outages follow the given distribution with  $k = 0.5$ .

- (iii) What does this claim imply about the mean time between outages? [1]

A group of customers wish to test the accuracy of the claim. They obtain a random sample of 100 time intervals between outages. The data are summarised as follows.

Time between outages (days)	$0 \leq t < 1$	$1 \leq t < 2$	$2 \leq t < 3$	$3 \leq t < 4$	$4 \leq t < 5$	$5 \leq t$
Frequency	56	19	12	6	3	4

- (iv) Carry out a chi-squared goodness-of-fit test for the ISP's model.

Discuss what the test indicates about the ISP's claim. You should refer to the  $p$ -value for your test and to the two largest contributions to the test statistic. [9]

- 8 AutoLot is a company which sells used cars. The company has recently employed a new salesperson, Grace, and part of her job is to set the prices at which cars should be advertised. In order to investigate whether the average price that Grace sets is correct, AutoLot chooses a random sample of the cars that are currently for sale and have already been priced. Grace is asked to set a price for each of them. She is not told what the current price is.

The table below shows the prices (in thousands of dollars) at which the cars are on sale and the prices that Grace sets.

Car	A	B	C	D	E	F	G	H	I
Current price	21.0	12.2	17.5	15.3	16.5	23.5	34.4	47.5	85.0
Grace's price	19.9	12.4	17.0	15.2	16.8	22.8	32.1	42.0	79.0
Current price – Grace's price	1.1	–0.2	0.5	0.1	–0.3	0.7	2.3	5.5	6.0

- (i) Explain why it might not be appropriate to carry out a test based on the  $t$ -distribution using these data. [1]
- (ii) Carry out a Wilcoxon test, at the 5% level of significance using these data, and state the conclusions AutoLot should reach. [6]

One of AutoLot's managers comments that the same conclusion would be reached by carrying out a sign test.

- (iii) Check whether the manager is correct. [3]
- (iv) Discuss briefly the relative merits of the sign test and the Wilcoxon test. [2]

- 9 An American newspaper commissioned a survey of doctors' salaries in order to investigate whether the mean salary is higher for males than for females. A random sample of 200 doctors were asked to state their annual salary and their gender. The results were as follows.

	Sample size	Mean (\$)	Standard deviation (\$)
Males	107	265 500	28 400
Females	93	254 000	26 700

- (i) Carry out a suitable hypothesis test. State the  $p$ -value for the test and explain what it indicates. [5]
- (ii) On examining the individual salaries for the two groups of doctors, it is clear that they are not normally distributed. Explain what implications, if any, this has for the test carried out in part (i). [2]

- 10** A survey carried out in 2006 in the UK asked 2112 adults about their views on evolution. 1014 of the adults said that they accepted the theory of evolution.

(i) Find a 95% confidence interval for  $p_{2006}$ , the proportion of UK adults in 2006 who accepted the theory of evolution. [3]

A scientific paper published in 2012 stated that a survey had shown that 69% of UK adults accepted the theory of evolution. The paper also stated that the 99% confidence interval was  $69\% \pm 4.7\%$ .

(ii) Determine the sample size in the 2012 paper. [2]

A researcher in 2018 wanted to investigate the proportion,  $p_{2018}$ , of UK adults in 2018 who accepted the theory of evolution. She wanted to produce a 95% confidence interval of width at most 6%.

(iii) Determine the smallest sample size that will give a confidence interval of the required width whatever the value of  $p_{2018}$ . [3]