Complex Numbers

LaTeX

THEODORE

1 Definition of the Imaginary Unit

$$i^2 = -1$$

2 Rectangular Form

$$z = x + iy,\ x, y \in \mathbb{R}$$

2.1 Conjugate

$$z := x + iy \iff z^* = x - iy$$

3 Polar Form

$$z = re^{i\theta},\ r \in [0, \infty),\ \theta \in (-\pi, \pi]$$

3.1 Conjugate

$$z := re^{i\theta} \iff z^* = re^{-i\theta}$$

4 Equation Relating the Two Forms

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \arg(x + iy) = \begin{cases} -\pi + \tan^{-1}\frac{y}{x},\ x < 0, y < 0 \\ -\frac{\pi}{2},\ x = 0, y < 0 \\ \tan^{-1}\frac{y}{x},\ x > 0, y < 0 \\ \tan^{-1}\frac{y}{x},\ x > 0, y \geqslant 0 \\ \frac{\pi}{2},\ x = 0, y > 0 \\ \pi + \tan^{-1}\frac{y}{x},\ x < 0, y \geqslant 0 \end{cases}$$

5 Conjugate Root Theorem

If $p(z)$ is a polynomial in z whose coefficients are only real and $p(z) = 0$ has root z_1, then z_1^* is also a root.