

Let $x = R_X$ and $y = R_Y$.

$$I_1 = \frac{\varepsilon}{\frac{xy}{x+y} + x}$$

$$I_2 = \frac{\varepsilon}{\frac{x}{2} + y}$$

$$\begin{aligned}\frac{I_1}{I_2} &= \frac{\frac{\frac{x}{2} + y}{\frac{xy}{x+y} + x}}{\frac{x}{2} + y} \\ &= \frac{(x+y)x + 2(x+y)y}{2xy + 2(x+y)x} \\ &= \frac{x^2 + xy + 2xy + 2y^2}{2xy + 2x^2 + 2xy} \\ &= \frac{x^2 + 3xy + 2y^2}{2x^2 + 4xy} \\ &= \frac{(x+2y)(x+y)}{2x(x+2y)} \\ &= \frac{x+y}{2x} \\ &= \frac{y}{2x} + \frac{1}{2}\end{aligned}$$

$$\boxed{\frac{I_1}{I_2} = \frac{R_Y}{2R_X} + \frac{1}{2}}$$