1

(a)

$$((\neg s \land l) \lor s) \not\Longrightarrow (s \implies \neg l)$$

(b)

2

$$(a \mid b \iff b = na) \land (b \mid c \iff c = mb)$$
$$c = nb = n(ma) = (nm)a \iff c \mid a \quad \square$$

3

$$x \in \mathbb{Z}$$

Suppose $2 \mid x^2 - 1$.

$$2 \mid x^2 - 1$$
$$x^2 - 1 = 2a$$
$$(x-1)(x+1) = 2a$$

For $(x-1)(x+1)$ to be even, $(x-1) \lor (x+1)$ must be even. Suppose $x - 1 = 2n$.

$$x - 1 = 2n$$
$$x + 1 = 2n + 2$$
$$x + 1 = 2(n+1)$$
$$(x-1)(x+1) = (2n)(2(n+1))$$
$$x^2 - 1 = 4\left(n^2 + 1\right)$$

WLOG, same thing happens when $(x+1)$ is even or $(x-1) \land (x+1)$ are even. $\square$

4

(i)

Case 1: $a = k$, $k \in \mathbb{R}^+$.

$$a^2 = aa = kk = k^2 > 0$$

Case 2: $a = -k$, $k \in \mathbb{R}^+$.

$$a^2 = aa = (-k)(-k) = k^2 > 0$$

Case 3: $a = 0$.

$$a^2 = (0)(0) = 0$$

Thus, $\forall a \in \mathbb{R}$, $a^2 \geqslant 0$. $\square$

(ii)

$$\begin{aligned}
(a-b)^2 &\geqslant 0 \\
a^2 - 2ab + b^2 &\geqslant 0 \\
-2ab &\geqslant -\left(a^2 + b^2\right) \\
ab &\leqslant \frac{1}{2}\left(a^2 + b^2\right) \quad \square
\end{aligned}$$

(iii)

$$\begin{aligned}
(ca - c^{-1}b)^2 &\geqslant 0 \\
c^2a^2 - 2ab + c^{-2}b^2 &\geqslant 0 \\
-2ab &\geqslant -\left(c^2a^2 + c^{-2}b^2\right) \\
ab &\leqslant \frac{1}{2}\left(c^2a^2 + c^{-2}b^2\right) \quad \square
\end{aligned}$$

5

(i)

$$\begin{aligned}
(\alpha y - \beta x)^2 &\geqslant 0 \\
\alpha^2 y^2 - 2\alpha\beta xy + \beta^2 x^2 &\geqslant 0 \\
\frac{\alpha^2}{x^2} + \frac{\beta^2}{y^2} &\geqslant \frac{2\alpha\beta}{xy}
\end{aligned}$$