Name:		Centre/ Index Number:		Class:	

DUNMAN HIGH SCHOOL
Practice Paper
Year 6

MATHEMATICS (Higher 3) — 9820/01

Paper 1

13 October 2021

3 hours

Additional Materials: Answer Booklet
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name, centre number, index number and class on the question paper.

An answer booklet will be provided with this question paper. You should follow the instructions on the front cover of the booklet. If you need additional answer paper or graph paper ask the invigilator for a continuation booklet or answer paper or graph paper.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

For teachers' use:

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Total
Score									
Max Score	**10**	**12**	**15**	**14**	**10**	**14**	**12**	**13**	**100**

1 **Do not use a calculator in answering this question.**

By first using the substitution

$$u = x + \frac{1}{x},$$

find the exact value of

$$\int_{1}^{\frac{1}{2}(3+\sqrt{5})} \frac{x^2 - 1}{(x^2 - 1)\sqrt{(x^4 + x^2 + 1)}}\, dx.$$ [10]

1 The function f is such that

$$f(x+2) = af(x+1) - f(x),$$

for all real x and some constant a.

(i) In the case that f is a linear function, find all possibilities for f and a. [4]

(ii) In the case that $f(0) = 0$ and $|f(1)| = 1,$ use mathematical induction to prove that

$$f(n-1)f(n+1) + 1 = [f(n)]^2,$$

for all positive integers n. [6]

(iii) In the case that $a = 2,$ sketch one possibility for f which is **not** linear. [2]

3 Let s and G be the sum and the geometric mean, respectively, of the positive numbers $a_1, a_2, \ldots, a_n$.

(i) If a and b are two of the positive numbers $a_1, a_2, \ldots, a_n,$ such that $a \geq G \geq b,$ then prove that replacing a and b by G and $\frac{ab}{G},$ respectively, does not alter the geometric mean and does not increase the sum. [4]

(ii) Use part **(i)** and mathematical induction to prove that $s \geq nG.$ [6]

(iii) By applying the AM-GM inequality to the numbers $1+a_1, 1+a_2, \ldots, 1+a_n$ prove that

$$(1+a_1)(1+a_2)\ldots(1+a_n) \leq 1 + s + \frac{s^2}{2!} + \ldots + \frac{s^n}{n!}.$$ [5]

4 Let S be the set of 3-digit numbers each of whose digits are different and non-zero. For example, $123 \in S$, $312 \in S$, and $473 \in S$.

(i) How many numbers are there in S? [1]

(ii) Find the sum of all the numbers in S. [2]

(iii) How many numbers in S are

(a) divisible by 2, [2]

(b) divisible by 3, [4]

(c) **not** divisible by any of 2 or 3 or 5? [5]

5 A sequence $t_1, t_2, t_3, \ldots$ of positive numbers is such that $t_{n+2}^2 = t_n t_{n+1}$, for $n \geq 1$. It is given that the terms of this sequence converge to 1.

(i) Prove that $t_{n+1}\sqrt{t_n} = t_{n+2}\sqrt{t_{n+1}}$, for $n \geq 1$. [2]

(ii) Hence explain why $t_{n+1} = \dfrac{1}{\sqrt{t_n}}$, for $n \geq 1$. [3]

(iii) Given that $t_1 \neq 1$, find the sum to infinity of the series $\dfrac{\ln t_1}{\ln t_1} + \dfrac{\ln t_2}{\ln t_1} + \dfrac{\ln t_3}{\ln t_1} + \ldots$. [6]

6 **(a)** **(i)** Show that every positive integer $n \geq 28$ can be expressed as $n = 5x + 8y$, for non-negative integers x and y. [4]

(ii) Explain if n can always be uniquely expressed in the form of $5x + 8y$. [1]

(b) Let Q be the set of all primes in the form of $6k + 5$, for some integer k.

So $Q = \{5, 11, 17, 23, 29, 41, \ldots\}$.

(i) Prove that any positive number of the form $6k + 5$, for some integer k, is divisible by a prime in Q. [5]

(ii) Prove that there are infinitely many primes in Q. [4]

7 **(i)** Calculate the number of ways of distributing 6 identical balls into 4 distinct boxes, A, B, C and D, in each of the following cases.

(a) Boxes B and C each contain at least 1 ball. [2]

(b) Each box contains an even number of balls. [3]

(ii) A sandwich shop sells t types of sandwich. Consider the problem of calculating the number of distinct purchases of s sandwiches. Describe a distribution problem equivalent to this problem, and hence, or otherwise, find the number of distinct purchases. [3]

(iii) Find the number of distinct 9-letter arrangements which can be made with the letters of the word CAMBRIDGE such that no two vowels are adjacent. [4]

8 **(i)** Find the general solution of the differential equation $(x+1)\frac{\mathrm{d}y}{\mathrm{d}x}+2y=\mathrm{e}^{x}$. [4]

(ii) Let $\mathrm{a}(x)$, $\mathrm{b}(x)$ and $\mathrm{c}(x)$ be functions of x such that $\mathrm{a}''(x)-\mathrm{b}'(x)+\mathrm{c}(x)=0$. Find functions $\mathrm{f}(x)$ and $\mathrm{g}(x)$ such that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{f}(x)\frac{\mathrm{d}y}{\mathrm{d}x}+\mathrm{g}(x)y\right)=\mathrm{a}(x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\mathrm{b}(x)\frac{\mathrm{d}y}{\mathrm{d}x}+\mathrm{c}(x)y.$$

[Your answers must be given in terms of one or more of $\mathrm{a}(x)$, $\mathrm{b}(x)$ and $\mathrm{c}(x)$.] [3]

(iii) Hence use a substitution of the form

$$z=\mathrm{f}(x)\frac{\mathrm{d}y}{\mathrm{d}x}+\mathrm{g}(x)y$$

to find the general solution of the differential equation

$$(x^2+x)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+(4x+1)\frac{\mathrm{d}y}{\mathrm{d}x}+2y=(x+1)\mathrm{e}^{x}.$$ [6]