MERIDIAN JUNIOR COLLEGE
2017 FASTEST FINGERS GC COMPETITION
PRELIMINARY ROUND

WRITE YOUR ANSWER IN THE BOX PROVIDED.

LEAVE ALL NON-EXACT ANSWERS TO 3 SIGNIFICANT FIGURES, UNLESS OTHERWISE STATED.

1. Evaluate the following

$$\ln 1 - \ln 2 + \ln 3 - \ln 4 + \ldots + \ln 19 - \ln 20$$

2. Find the sum of the first 30 terms of an Arithmetic Progression with first term 100 and common difference -1.5.

3. Evaluate $\displaystyle\sum_{r=0}^{10}\left((-1)^{r+1}2^r + \frac{2+r}{r!}\right)$.

4. Find the real root of the equation $(\pi x)^3 + \left(\frac{x}{\mathrm{e}}\right)^2 + x\ln\sqrt{13} = \tan 2$.

5. Find the largest value of k such that $\dfrac{2x+6}{3x-x^2} > (x-2)^3$ for all $x < k$ where $k \in \mathbb{R}^-$.

6. The curve C is defined by

$$x = \cos^{-1} t + \sin t,\ y = \left(\frac{1}{t} + \mathrm{e}^t\right)^{\frac{1}{3}}.$$

Find the gradient of C at $t = \dfrac{\sqrt{2}}{3}$.

7. A plane contains a point $A(3,2,2)$ and a line $l : \mathbf{r} = \begin{pmatrix}0\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}7\\2\\4\end{pmatrix}, \lambda \in \mathbb{R}$.

Find the value of a if the plane is parallel to $\begin{pmatrix}4\\a\\-2\end{pmatrix}$.

8. A curve has parametric equations $x = 3\sin\theta - 2$, $y = \cos\theta$, $-\pi < \theta \le \pi$. Find the area bounded by the curve. Give your answer to 3 significant figures.

9. Evaluate, to 4 decimal places, $\int_{\sqrt{2}-1}^{1} \frac{1}{(x+1)\sqrt{(x^2+2x+1)}}\,\mathrm{d}x$

10. Given that $z_1 = -\frac{1}{\sqrt{2}} + \frac{3}{4}\mathrm{i}$ and $z_2 = \frac{1}{2} - \frac{4}{\sqrt{3}}\mathrm{i}$, find the argument of the complex number

$$\left(\frac{(z_1)^2}{(z_2^*)^3}\right)^4 .$$

11. Evaluate ${}^5C_0 + {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 + \frac{{}^{10}\mathrm{P}_5}{{}^{10}\mathrm{C}_5} + \frac{{}^{12}\mathrm{P}_5}{{}^{12}\mathrm{C}_5} + \frac{{}^{15}\mathrm{P}_3}{{}^{15}\mathrm{C}_3}$.

12. The random variable X has the distribution $\mathrm{B}\left(64, \frac{5}{8}\right)$. Find $\mathrm{P}\left(X = 34 \text{ or } 35 \mid X \le \mathrm{E}(X)\right)$.

13. The random variable X has the distribution $\mathrm{N}(12, 0.3)$. The random variable Y has the distribution $\mathrm{N}(60, 2.5)$. Find the probability that the sum of 5 independent observations of X differs from a random observation of Y by at least 4.

14. A large number of students in a college have taken an examination. The time, x hours, taken by a student to prepare for the examination is noted for a sample of 200 students. The following results are obtained:

$$\sum x = 9670 \text{ and } \sum x^2 = 488400.$$

Determine the p-value for the test that the population mean time for a student to prepare for the examination is less than 50 hours.

15. A random sample of n pairs of values (s, t) is collected. The equation of the line of regression t on s is found to be $2t + 9s = 41$, and the equation of the line of regression s on t is $5s + 12t = 99$. Given $\sum t = 140$, find the value of $\sum s$.

16. The following observations of x and y have been reported.

x	170	161	120	102	95	84	53	42	13
y	2	30	35	65	73	80	82	120	210

Find the product moment correlation coefficient of $x + 20$ and $\frac{y}{100}$.

ANSWERS

1. -1.74 (3s.f.)
2. 2347.5
3. -675 (3s.f.)
4. -0.381 (3s.f.)
5. -0.173 (3s.f.)
6. 1.65 (3s.f.)
7. 2
8. 9.42 (3s.f.)
9. 0.2071 (4d.p.)
10. 2.32 (3s.f.)
11. 278
12. 0.137 (3s.f.)
13. 0.0455 (3s.f.)
14. 0.0113 (3s.f.)
15. 60
16. -0.916 (3s.f.)