SEC 1 MATH REVISION NOTES

Your Ultimate Guide To Ace Sec 1 Math

JIMMY LING

Published by Jimmy Maths

Website: www.jimmymaths.com

While every care has been taken to compile this book, errors may still arise in the course of compilation and production. If you notice any error, kindly email to jimmyling@jimmymaths.com so that we can review it.

About the Author

Jimmy Ling

- Graduated from NUS with degree in Mathematics
- Director and Math tutor of Grade Solution Learning Centre
- Online Math Coach at Jimmy Maths
- More than 10 years of teaching experience

Preface

Hi there, thank you for purchasing this book. You have made a right choice in your path to succeed in Secondary School Math.

This book contains all the key concepts which you must know for Sec 1 Math.

This Revision Notes is broken down into different chapters, according to the latest MOE syllabus.

Each chapter is further broken down into different concepts and each concept is explained with examples.

Read and understand the concepts. Understand the examples behind the concepts to make sure that you know how to apply them next time.

Practice is essential. After understanding the concepts, you need to put in sufficient practice to master them. You can check out the practice questions given in our Sec 1 Math Online Course.

As long as you understand the concepts and put in enough practice, you will Ace your Sec 1 Math exams.

Important

For first time users of this book, you need to be familiar with the concepts and strategies used by Jimmy Maths in order to achieve maximum benefits.

If you are not familiar yet, you are encouraged to visit the resources in the next page to learn the concepts.

Online Courses and Tuition Classes

Sec 1 Math Online Course

Find out more by clicking below:

Sec 1 Math Online Course

- Learn from recorded videos
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If you have any questions, you can email us at jimmyling@jimmymaths.com

Group Tuition

Together with a team of tutors, we conduct Math tuition at our tuition centre, Grade Solution Learning Centre. Let us coach your child face-to-face in understanding and mastering Math concepts. In our centre, we will impart problem solving skills and motivate your child, on top of academic contents. Looking forward to see your child in our classes one day!

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Chapter 1: Factors and Multiples

Prime Numbers

Prime number is a natural number greater than 1 that has no positive divisors other than 1 and itself.

Examples: 2, 3, 5, 7, 11, 13, 17, 19, 23….

Prime Factorisation

Process of breaking up a composite number into its prime factors.

Example

$$\begin{array}{r|l}
2 & 1172 \\ \hline
2 & 588 \\ \hline
2 & 294 \\ \hline
3 & 147 \\ \hline
7 & 49 \\ \hline
7 & 7 \\ \hline
 & 1
\end{array}$$

$$\begin{aligned} 1176 &= 2\times2\times2\times3\times7\times7 \\ &= 2^3\times3\times7^2 \text{ (Index Notation)} \end{aligned}$$

Highest Common Factor (HCF)

The Highest Common Factor (H.C.F) of two (or more) numbers is the largest number that divides evenly into all the numbers.

Example: The common factors or of 12 and 18 are 1, 2, 3 and 6. The largest common factor is 6, so this is the H.C.F. of 12 and 18.

Lowest Common Multiple (L.C.M)

The Lowest Common Multiple (LCM) of two (or more) numbers is the smallest number that is divisible by all the numbers.

Example: The L.C.M of 12 and 18 is 36.

Finding H.C.F in Index Form

Compare and take the **Lower Power**

Example

Find the H.C.F of $2\times3^2\times5$ and $2^2\times3^3\times7\times13$.

$$
\begin{array}{l}
2\times3^2\times5 \\
2^2\times3^3\quad\times7\times13 \\
\hline
\end{array}
$$

$HCF = 2\times3^2$

Finding H.C.F using Prime Factorisation

Example

Find the highest common factor of 168 and 324.

2	168, 324
2	84, 162
3	42, 81
	14, 27

$HCF = 2^2\times3$

$\quad = 12$

You stop when there is no common factor between the 2 numbers.

Finding LCM in Index Form

Compare and take the **Higher Power**

$2 \times 3^2 \times 5^2 \times 11$

$2^3 \times 3 \times 5^3$

$LCM = 2^3 \times 3^2 \times 5^3 \times 11$

Finding LCM using Prime Factorisation

Find the LCM of 48 and 90.

2	48, 90
3	24, 45
	8, 15

You stop when there is no common factor between the 2 numbers.

$$\begin{aligned} LCM &= 2 \times 3 \times 8 \times 15 \\ &= 720 \end{aligned}$$

How about 3 numbers?

Find the LCM of 30, 36 and 40.

2	30, 36, 40
2	15, 18, 20
3	15, 9, 10
5	5, 3, 10
	1, 3, 2

$$\begin{aligned} LCM &= 2^2 \times 3 \times 5 \times 1 \times 3 \times 2 \\ &= 360 \end{aligned}$$

You stop when there is no common factor between any 2 numbers.

Word Problems

Example 1

Mrs. Ong packs 48 highlighters, 60 paper clips and 108 pencils equally into some bags.
a. Find the largest number of bags Mrs. Ong packs.
b. Calculate the number of highlighters, paper clips and pencils there are in each bag.

2	48, 60, 108
2	24, 30, 54
3	12, 15, 27
	4, 5, 9

a. Largest number of bags = $2 \times 2 \times 3 = 12$ bag
b. Highlighters → $48 \div 12 = 4$
Paper Clips → $60 \div 12 = 5$
Pencils → $108 \div 12 = 9$

Example 2

Joe, Roy and Shawn are running on a circular track. They take 48 seconds, 56 seconds and 60 seconds to complete a lap respectively. They begin together at the starting point at 0900. At what time do they next meet together?

2	48, 56, 60
2	24, 28, 30
2	12, 14, 15
3	6, 7, 15
	2, 7, 5

Duration to meet = L.C.M $= 2^3 \times 3 \times 2 \times 7 \times 5$
$= 1680$ seconds
$= 28$ minutes

They next meet at 9.28 a.m.

Square Root

$4 = 2^2$

Divide the power by 2

$\sqrt{4} = 2$

$9 = 3^2$

Divide the power by 2

$\sqrt{9} = 3$

$36 = 2^2 \times 3^2$

Divide the power by 2

$\sqrt{36} = 2 \times 3$
$= 6$

4, 9 and 36 are called perfect squares
→ All the powers of the factors are divisible by 2

Example

a. Express 676 as a product of prime factor.
b. Find $\sqrt{676}$

2	676
2	338
13	169
13	13
	1

Remember to check your answer with your calculator.

a. $676 = 2^2 \times 13^2$

b. $\sqrt{676} = \sqrt{2^2 \times 13^2}$
$= 2 \times 13$
$= 26$

Cube Root

$8 = 2^3$

Divide the power by 3

$\sqrt[3]{8} = 2$

$27 = 3^3$

Divide the power by 3

$\sqrt[3]{27} = 3$

$216 = 2^3 \times 3^3$

Divide the power by 3

$$\sqrt[3]{216} = 2 \times 3$$
$$= 6$$

8, 27 and 216 are called perfect cubes
→ All the powers of the factors are divisible by 3

Example

a. Express 216 as a product of prime factor.
b. Find $\sqrt[3]{216}$

2	216
2	108
2	54
3	27
3	9
3	3
	1

Remember to check your answer with your calculator.

a. $216 = 2^3 \times 3^3$

b. $\sqrt[3]{216} = \sqrt[3]{2^3 \times 3^3}$
$= 2 \times 3$
$= 6$

Perfect Square

$4 = 2^2$ $\qquad$ $\sqrt{4} = 2$

$9 = 3^2$ $\qquad$ $\sqrt{9} = 3$

$16 = 2^4$ $\qquad$ $\sqrt{16} = 2^2$

$25 = 5^2$ $\qquad$ $\sqrt{25} = 5$

Perfect Squares → 4, 9, 16, 25...
→ All the powers of the factors are divisible by 2

Example 1 - Making into a Perfect Square

Given $a = 2\times 3^2 \times 5^3$, find the smallest integer n such
That $a\times n$ is a perfect square.

$a = 2\times 3^2 \times 5^3$

$\times 2 \qquad \times 5 \quad \rightarrow \quad n = 2\times 5$

$\qquad\qquad\qquad\qquad\qquad = 10$

$2^2 \times 3^2 \times 5^4$

Example 2 - Making into a Perfect Square

Find the smallest value of n such that $252n$ is a perfect square.

2	252
2	126
3	63
3	21
7	7
	1

Double check with calculator!

$252 = 2^2 \times 3^2 \times 7$

$\times 7 \quad \rightarrow \quad n = 7$

$2^2 \times 3^2 \times 7^2$

$252\times 7 = 1764$

$\sqrt{1764} = 42$

Perfect Cube

$8 = 2^3$ $\quad\quad$ $\sqrt[3]{8} = 2$

$27 = 3^3$ $\quad\quad$ $\sqrt[3]{27} = 3$

$64 = 2^6$ $\quad\quad$ $\sqrt[3]{64} = 2^2$

Perfect Cube → 8, 27, 64...
→ All the powers of the factors are divisible by 3

Example 1 - Making into a Perfect Cube
Given $a = 2 \times 3^2 \times 5^3$, find the smallest integer n such that $a \times n$ is a perfect cube.

$a = 2 \times 3^2 \times 5^3$

$\times 2^2 \quad \times 3$

$2^3 \times 3^3 \times 5^3$

$n = 2^2 \times 3$
$= 12$

Example 2 - Making into a Perfect Cube
Find the smallest value of n such that $252n$ is a perfect cube.

2	252
2	126
3	63
3	21
7	7
	1

Double check with calculator!

$252 = 2^2 \times 3^2 \times 7$

$\times 2 \quad \times 3 \quad \times 7^2$

$2^3 \times 3^3 \times 7^3$

$n = 2 \times 3 \times 7^2$
$= 294$

$252 \times 294 = 74088$
$\sqrt[3]{74088} = 42$

Chapter 2: Real Numbers

Type of Numbers

Integers → Numbers without Decimals

Positive Integers → 1, 2, 3 , 4, 5…..
Negative Integers → -1, -2, -3, -4, -5…..

Rational Numbers → Numbers which can be expressed as Fractions

All integer are rational

Rational → 1, 2, 3, -1, -2, -3…

All fractions are rational

$\frac{1}{2}, \frac{2}{3}, -\frac{3}{4}, \frac{22}{7}$

Irrational → $\pi, \sqrt{2}, \sqrt[3]{5}, \frac{10}{\sqrt{3}}$

Example 1

Given the following numbers:

$$2\pi,\ \sqrt{3},\ -\frac{5}{3},\ \sqrt[3]{8},\ 1.412$$

Write down the irrational numbers

$2\pi = 6.283185...$ Irrational

$\sqrt{3} = 1.73205...$ Irrational

$-\frac{5}{3}$ Rational

$\sqrt[3]{8} = 2 = \frac{2}{1}$ Rational

$1.412 = 1\frac{412}{1000}$ Rational

Ans: $2\pi,\ \sqrt{3}$

Decimals

Terminating → 0.2, 0.25

Recurring → $0.\dot{3} = 0.3333333...$

$0.21\dot{3} = 0.213333333...$

$0.\dot{2}1\dot{3} = 0.213213213...$

All Terminating and Recurring Decimals are Rational

$$0.2 = \frac{1}{5}$$

$$0.\dot{3} = 0.3333333... = \frac{1}{3}$$

$$0.1\dot{6} = 0.16666... = \frac{1}{6}$$

Example 1
Arrange the following decimals in ascending order:

$0.213,\ 0.21\dot{3},\ 0.21,\ 0.\dot{2}1\dot{3},\ 0.22$

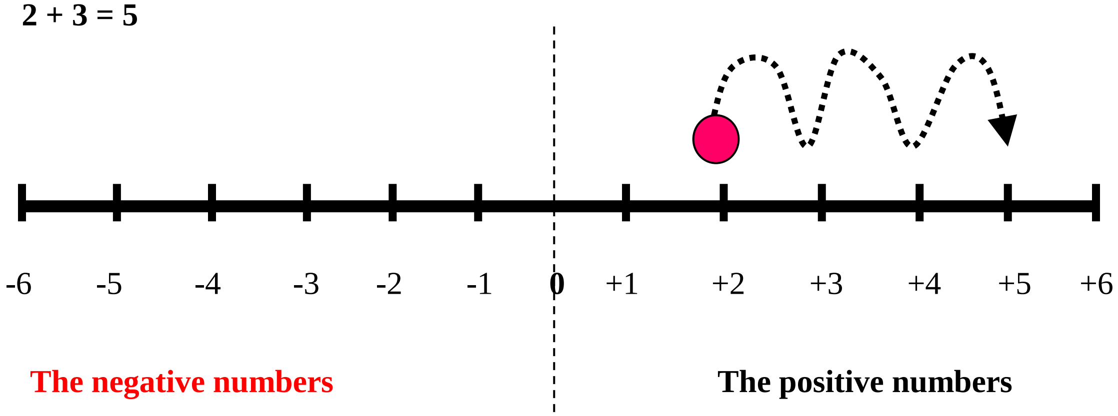
$0.21\dot{3} = 0.21333...$

$0.\dot{2}1\dot{3} = 0.213213213...$

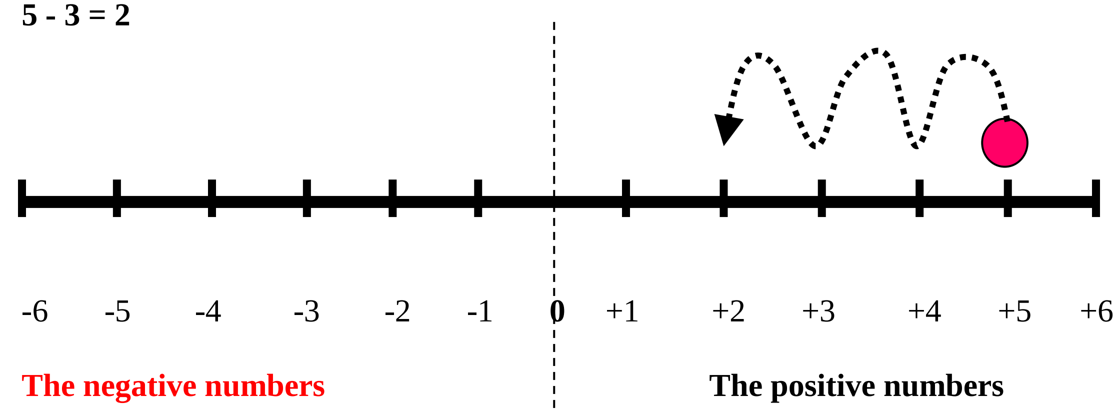
Ans: $0.21,\ 0.213,\ 0.\dot{2}1\dot{3},\ 0.21\dot{3},\ 0.22$

Number Line

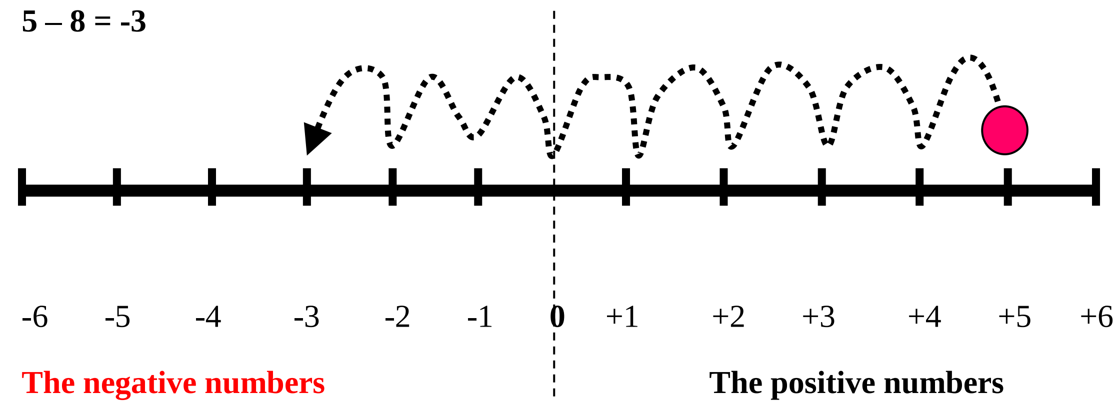
2 + 3 = 5



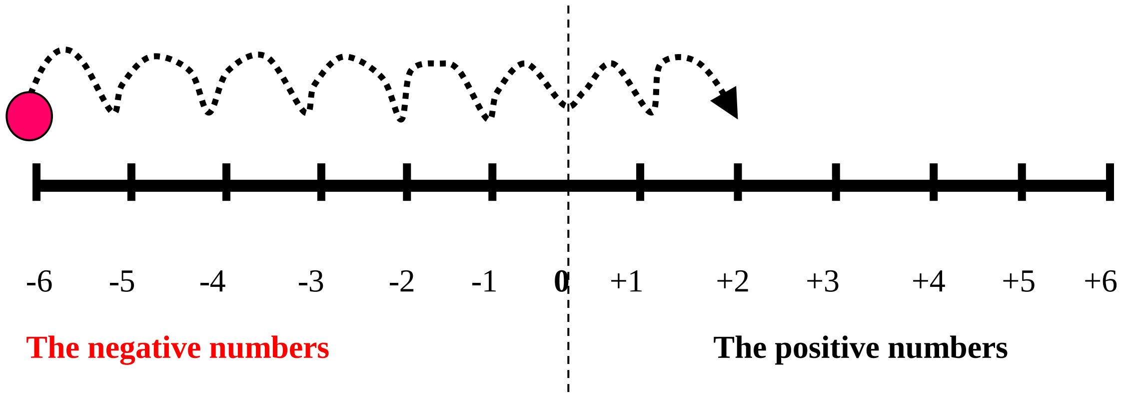
5 - 3 = 2



5 – 8 = -3



-6 + 8 = 2



Examples

$4-8=-4$ $\qquad$ $-2-5=-7$

$3-5=-2$ $\qquad$ $-6-5=-11$

$3-12=-9$ $\qquad$ $-4-8=-12$

Try Yourself

$3-8=-5$

$7-12=-5$

$-2-12=-14$

$-4+8=4$

$-3-6=-9$

$-10+7=-3$

Times and Divide for Negative Number

The 4 Rules of Multiplication

1. Plus × Plus = Plus

   $2 \times 1 = 2$

2. Plus × Minus = Minus

   $2 \times -1 = -2$

3. Minus × Plus = Minus

   $-2 \times 1 = -2$

4. Minus × Minus = Plus

   $-2 \times -1 = 2$

Example 1 - Multiplication

1. $2 \times -3 = -6$
2. $-3(-3) = -3 \times -3 = 9$
3. $2 + (-1) = 2 - 1 = 1$
4. $3 - (-2) = 3 + 2 = 5$
5. $5 + (-8) = 5 - 8 = -3$

Example 2 – Division

1. $12 \div -3 = -4$
2. $-8 \div 4 = -2$
3. $-27 \div -3 = 9$
4. $12 \div (-6 + 3) = 12 \div -3 = -4$
5. $-5 + 4 \div (-2) = -5 + (-2)$

   $= -5 - 2$

   $= -7$

Order of Operations

(BODMAS rule)

1. **B**rackets
2. **O**rder (Power **O**f)
3. **D**ivision and **M**ultiplication, left to right
4. **A**ddition and **S**ubtraction, left to right

Example 1

$$2\times(3-4)^3+8\div2^2=2\times(-1)^3+8\div2^2$$

$$=2\times(-1)+8\div4$$

$$=-2+8\div4$$

$$=-2+2$$

$$=0$$

Example 2

$$\left\{-5-[7+(-2)^2]\right\}\times(-2)=\left[-5-(7+4)\right]\times(-2)$$

$$=(-5-11)\times(-2)$$

$$=-16\times(-2)$$

$$=32$$

Fractions

Example 1

$$3\frac{2}{5}-1\frac{3}{4}\div\left(-\frac{7}{12}\right)=\frac{17}{5}-\frac{7}{4}\div\left(-\frac{7}{12}\right)$$

$$=\frac{17}{5}-\frac{7}{4}\times\left(-\frac{12}{7}\right)$$

$$=\frac{17}{5}-\frac{\cancel{7}}{\cancel{4}}\times\left(-\frac{\cancel{12}^{3}}{\cancel{7}}\right)$$

$$=\frac{17}{5}-\left(-\frac{3}{1}\right)$$

$$=\frac{17}{5}+3$$

$$=6\frac{2}{5}$$

Example 2

$$\left[\left(\frac{5}{6}-\frac{1}{4}\right)\div1\frac{1}{3}\right]\times\left(-\frac{2}{3}\right)=\left[\left(\frac{10}{12}-\frac{3}{12}\right)\div\frac{4}{3}\right]\times\left(-\frac{2}{3}\right)$$

$$=\left[\frac{7}{12}\times\frac{3}{4}\right]\times\left(-\frac{2}{3}\right)$$

$$=\left[\frac{7}{\cancel{12}^{4}}\times\frac{\cancel{3}}{4}\right]\times\left(-\frac{2}{3}\right)$$

$$=\frac{7}{16}\times\left(-\frac{2}{3}\right)$$

$$=\frac{7}{\cancel{16}^{8}}\times\left(-\frac{\cancel{2}}{3}\right)=-\frac{7}{24}$$

Chapter 3: Estimation

Rounding Off

Decimal Place

$24.16 \approx 24.2$ (1 d.p)

Nearest 10, 100 etc

$7894 \approx 7890$ (Nearest Tens)

≈ 7900 (Nearest Hundreds)

Nearest unit

$789g = 0.789kg$

$\approx 1\, kg$(Nearest Kg)

Significant Figures

Significant figures show the accuracy in measurements.

The higher the significant figures, the accurate the answer is.

For example, the weight of a book is 864 g vs 864.3 g, which measurement is more accurate?

The answer is 864.3 g as it contains more significant figures.

<u>Rules of Significant Figures</u>

1.23 has 3 significant figures (All the digits give us information on how accurate the measurement is).

0.123 has 3 significant figures (The first zero only tell us the size, and not the accuracy of the measurement).

0.1230 has 4 significant figures (The last zero tells us that this number has been rounded off, so it is significant).

0.10023 has 5 significant figures (All the zeros between the significant figures are significant).

10.0 has 3 significant figures (The zero between the significant figures is significant too).

10 has either 2 significant figures or 1 significant figure (Depending on the question)

For example, $10.2 \approx 10$ (2 sf)

Or $10.2 \approx 10$ (1 sf)

Example 1

Round off each of the following to 3 significant figures.

$24.16 \approx 24.2$

$0.02584 \approx 0.0258$

$201.84 \approx 202$

$0.09995 \approx 0.100$

$$0.0509\dot{8} = 0.0509888...$$
$$\approx 0.0510$$

$12450 \approx 12500$ **Be careful! Many students write as 125 (Wrong!!)**

Example 2

The number 799899 corrected to n significant figures is 800000. Write down the possible values of n.

$799899 \approx 800000$ (1 sf) ✓

$799899 \approx 800000$ (2 sf) ✓

$799899 \approx 800000$ (3 sf) ✓

$799899 \approx 799900$ (4 sf) ✗

Ans: 1, 2, 3

Estimation

Example 1

By rounding each number to 1 significant figure, estimate the value of

$$\frac{212 \times 5.21}{79.8 - 28.4}$$

$$\frac{212 \times 5.21}{79.8 - 28.4} \approx \frac{200 \times 5}{80 - 30}$$

$$= \frac{1000}{80 - 30}$$

$$= \frac{1000}{50}$$

$$= 20$$

Example 2

Estimate the following to 1 significant figure, showing the estimates in your working clearly.

$$\frac{2.23\times\sqrt{402}}{1.95^2} \approx \frac{2.2\times\sqrt{400}}{2.0^2}$$

$$= \frac{2.2\times 20}{4}$$

$$= \frac{44}{4}$$

$$= 11$$

$$\approx 10 \text{ (1 sf)}$$

Chapter 4: Algebraic Simplification

Remember The 4 Rules of Multiplication!

1. Plus × Plus = Plus
2. Plus × Minus = Minus
3. Minus × Plus = Minus
4. Minus × Minus = Plus

Substitution

Example

Given that $x = 5$ and $y = -2$, evaluate $\frac{x}{2x+y} - \frac{y}{x}$.

$$\frac{x}{2x+y} - \frac{y}{x}$$
$$= \frac{5}{2(5)+(-2)} - \frac{-2}{5}$$
$$= \frac{5}{10-2} + \frac{2}{5}$$
$$= \frac{5}{8} + \frac{2}{5}$$
$$= \frac{41}{40}$$

Algebra Simplification for Times and Divide

Examples

1. $3a \times 5a \ = 15a^2$

2. $27a \div 9b \times 3c$

$$= 27a \times \frac{1}{9b} \times 3c$$
$$= \frac{9ac}{b}$$

Algebraic Expansion

Example

Expand and Simplify $2(a+b)-3(2a-b)$

$$2(a+b)-3(2a-b)$$
$$=2a+2b-6a+3b$$
$$=2a-6a+2b+3b$$
$$=-4a+5b$$

Algebraic Fraction

Example

Express $\dfrac{x-2}{4}+\dfrac{2x-2}{6}$ as a single fraction in its simplest form.

$$\frac{x-2}{4}+\frac{2x-2}{6}$$
$$=\frac{3(x-2)}{12}+\frac{2(2x-2)}{12}$$
$$=\frac{3x-6+4x-4}{12}$$
$$=\frac{7x-10}{12}$$

Algebraic Factorisation

Identify the H.C.F of the terms and put outside the bracket.

Examples

1. $12xy-36yz$
$=12y(x-3z)$

2. $4x-8x^2$
$=4x(1-2x)$

Chapter 5: Algebraic Equations

Solving Linear Equations

Example

Solve for *x* in the following equation:

$$3(4x-1)=7(2x-5)$$

Expand by opening the brackets first

$$\begin{aligned}&3(4x-1)=7(2x-5)\\&12x-3=14x-35\\&12x-14x=-35+3\\&-2x=-32\\&x=\frac{-32}{-2}\\&x=16\end{aligned}$$

Solving Equations Involving Algebraic Fractions

Example

Solve for *x* in the following equation:

$$\frac{2p+1}{6}-\frac{6-5p}{5}=\frac{12p-15}{10}$$

Multiply each term by 30 (The LCM of 6, 5 and 10)

$$\begin{aligned}&\left(\frac{2p+1}{6}\right)\times30-\left(\frac{6-5p}{5}\right)\times30=\left(\frac{12p-15}{10}\right)\times30\\&5(2p+1)-6(6-5p)=3(12p-15)\\&10p+5-36+30p=36p-45\\&10p+30p-36p=-45-5+36\\&4p=-14\\&p=\frac{7}{2}\end{aligned}$$

Solving Equations Involving Word Problems

Example 1

When a number is added to two-fifths of itself, the result is 35. What is the number?

Let *x* be the number.

$$x+\frac{2}{5}x=35$$

$$\frac{7}{5}x=35$$

$$\begin{aligned} x &= 35\div\frac{7}{5} \\ &= 35\times\frac{5}{7} \\ &= 25 \end{aligned}$$

Example 2

The sum of 3 consecutive even numbers is 42. Find the smallest number.

Let the 3 numbers be *x, x + 2, x + 4*.

$$x+x+2+x+4=42$$

$$3x+6=42$$

$$3x=42-6$$

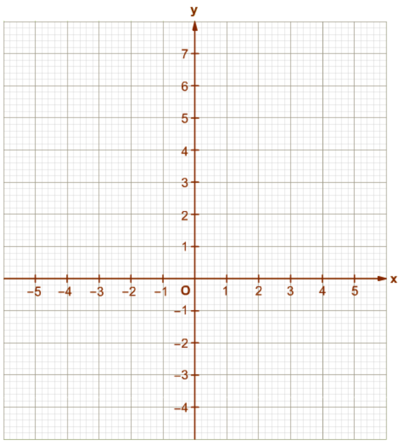
$$3x=36$$

$$\begin{aligned} x &= 36\div 3 \\ &= 12 \end{aligned}$$

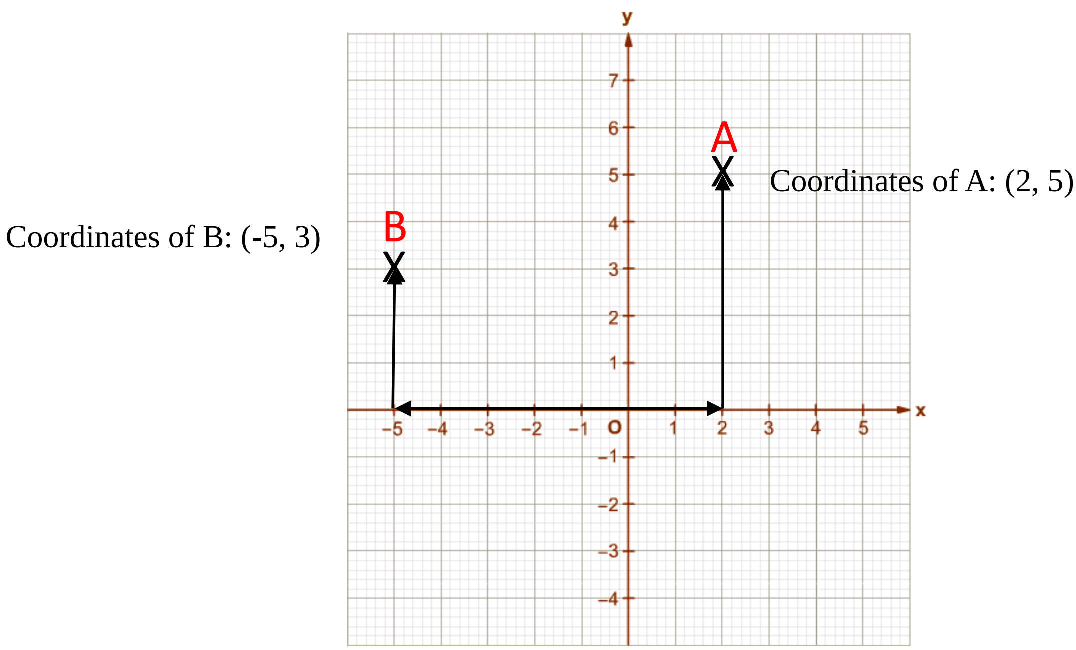
Chapter 6: Coordinates and Linear Functions

Coordinates

Cartesian Plane

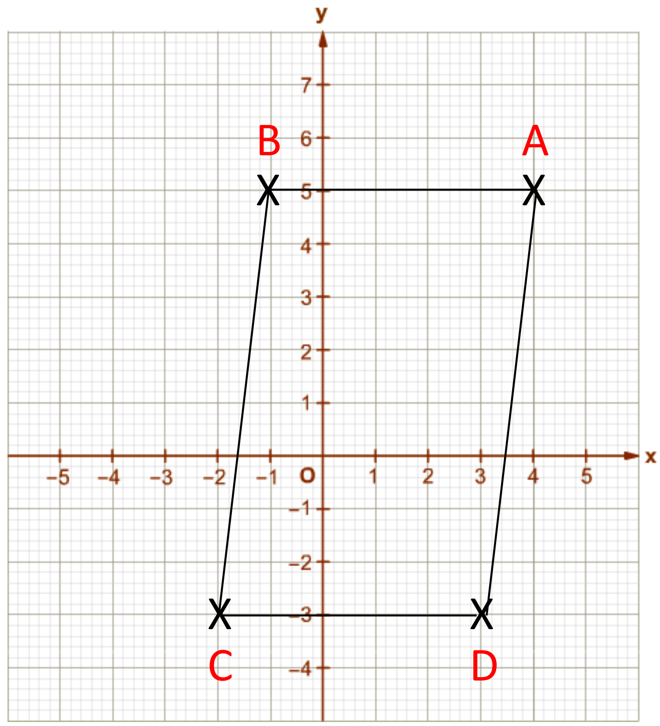


- The position of a point on the Cartesian plane
- Expressed in the form of an ordered pair (x, y)
- *x is the x-coordinate* and y, is the y-coordinate.



Example 1

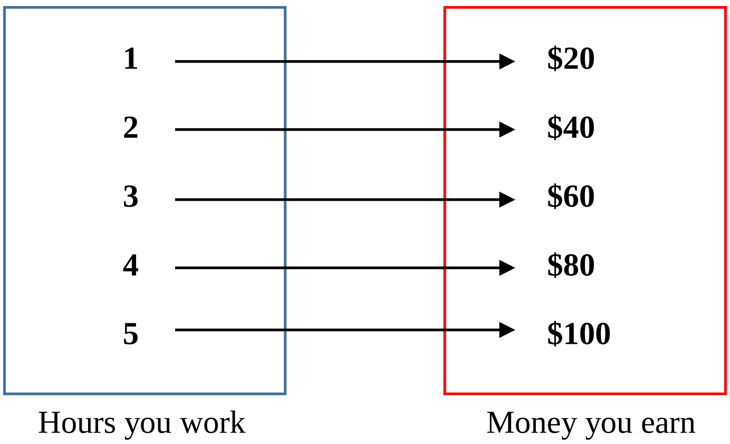
Plot and label the points A (4, 5), B (-1, 5), C (-2, -3).
Plot and write down the coordinates of D such that ABCD is a parallelogram.



<u>What is a function?</u>

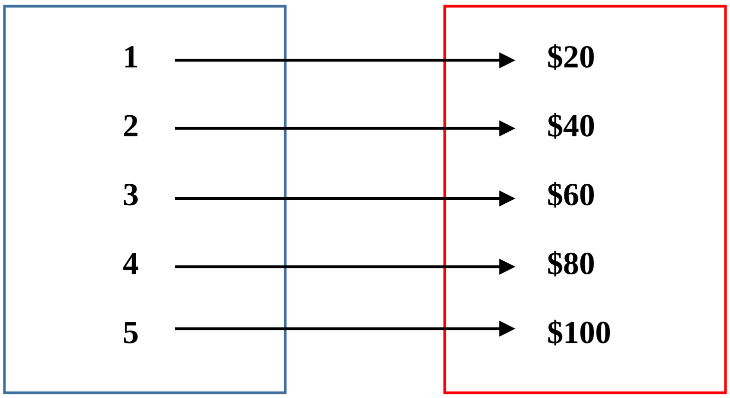
A rule that gives one output per input.

Rule: $20 per hour

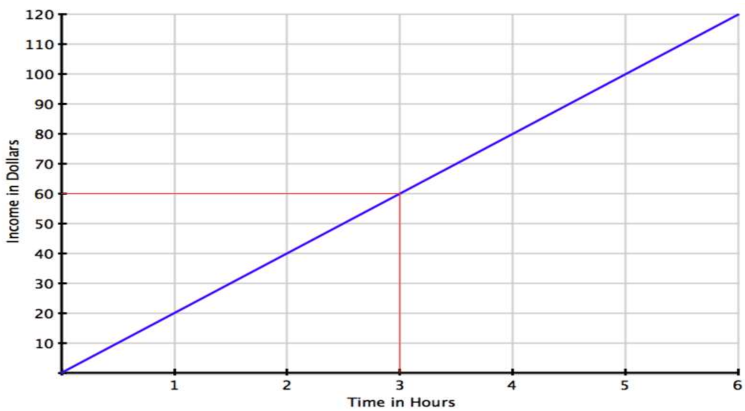


Linear Function

Straight Line Graph



Hours you work Money you earn



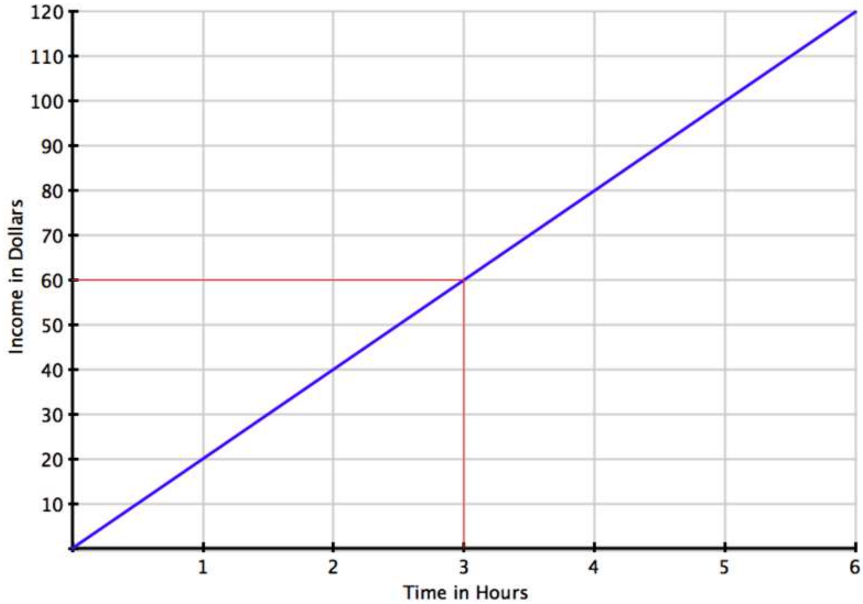
It is a linear function.

Equation of linear function

The rule that defines all the points lying on the line.

x (Time in hours)	1	2	3	4	5
y (Income in dollars)	20	40	60	80	100

Equation: $y = 20x$

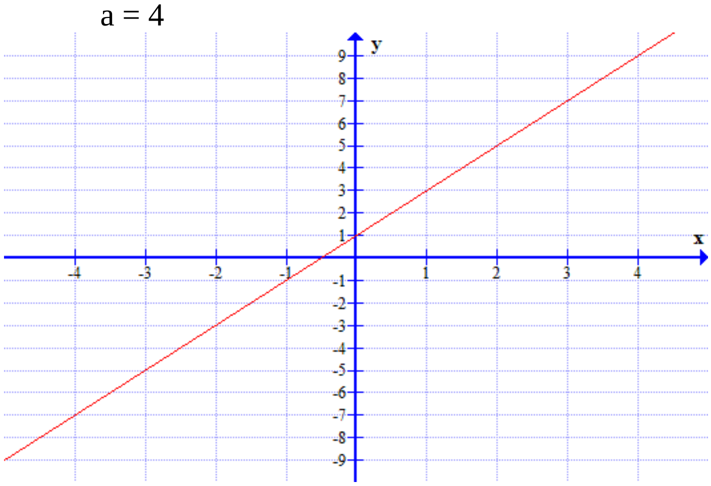


Example 1
Given that the equation of a line is $y = 2x + 1$

a. Does (3, 8) lies on the line?
b. Given that (a, 9) lies on the line, find the value of a?

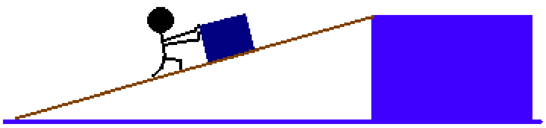
a. Sub $x = 3$,
$y = 2(3) + 1 = 7$
Since $7 \neq 8$, (3, 8) does not lie on the line.

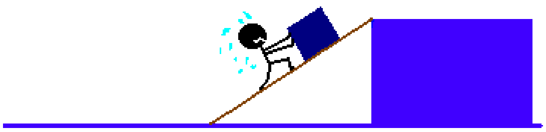
b. Sub $x = a, y = 9$,
$9 = 2a + 1$
$9 - 1 = 2a$
$2a = 8$
$a = 4$



Gradient
Determines the slope of a line

This line has a lower gradient.

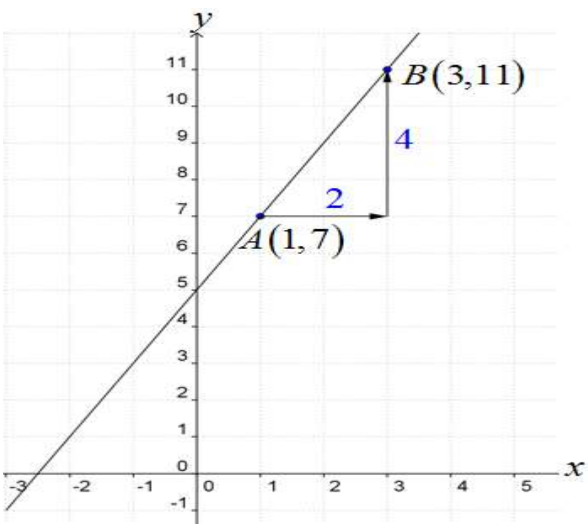




This line has a higher gradient.

Calculation of Gradient

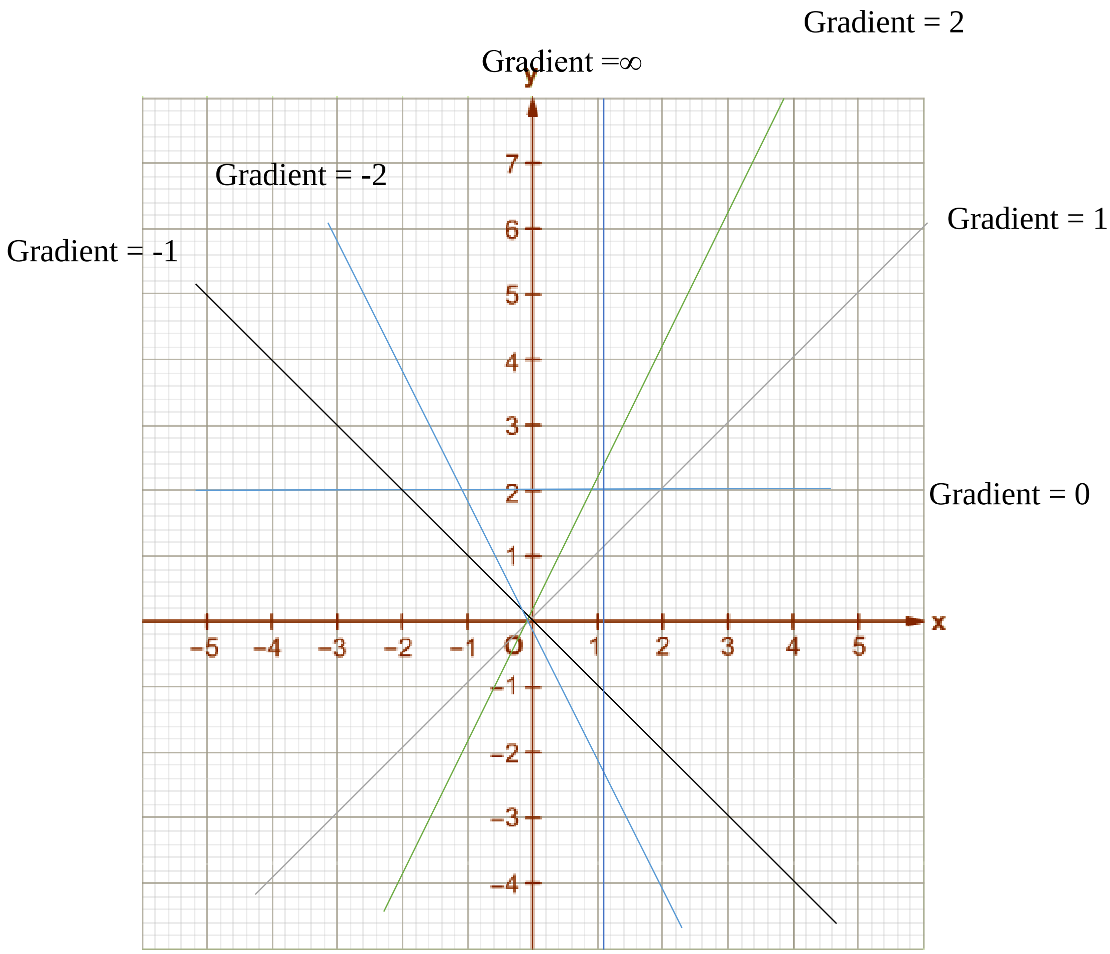
$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Change in y}}{\text{Change in x}}$$



Change in y = 11 – 7 = 4

Change in x = 3 – 1 = 2
Gradient = 4 ÷ 2 = 2

Different Gradients



Example 1

Calculate the gradient of a line that passes through A (-2, 5) and B (3, -8)

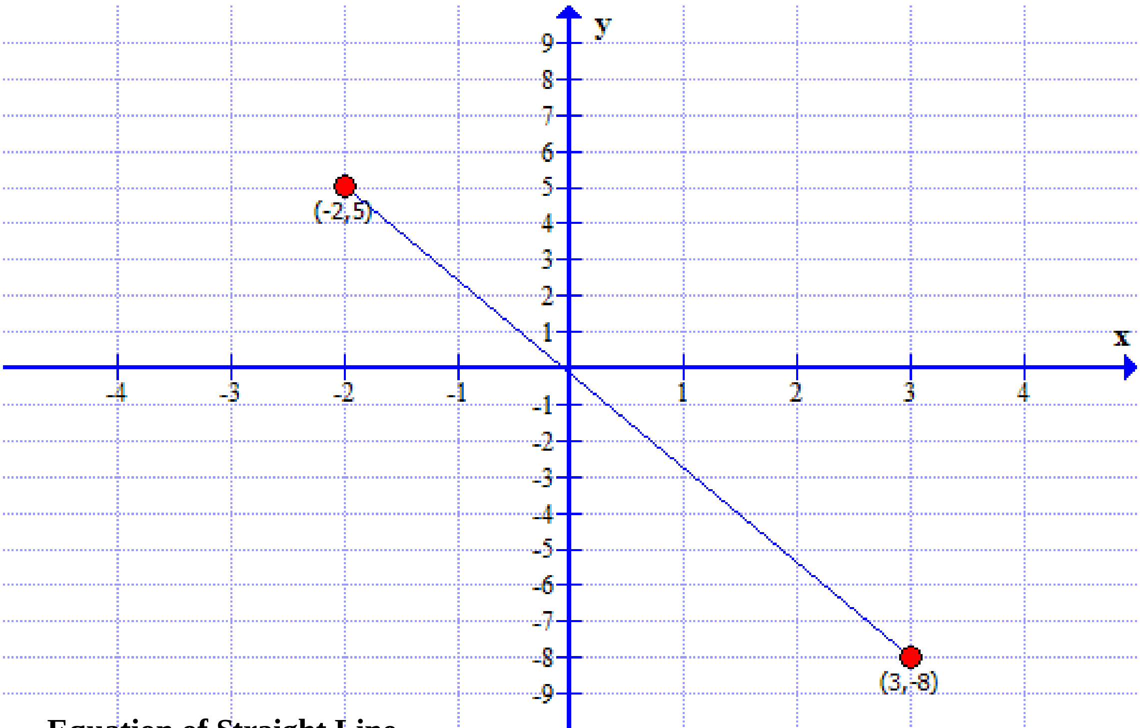
$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-8-5}{3-(-2)}$$

$$= \frac{-13}{5}$$

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5-(-8)}{-2-3}$$
$$= \frac{-13}{5}$$

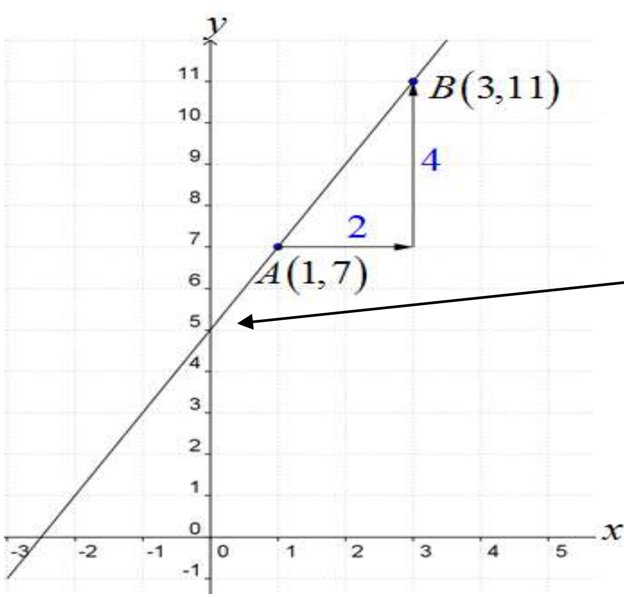


Equation of Straight Line

$y = mx + c$

m = gradient

c = y-intercept (The point when the graph cuts the y-axis)



m = gradient = 4 ÷ 2 = 2

c = y-intercept = 5

: $y = 2x + 5$

Example 2

Find the gradients and the y-intercepts of the following lines:

a. $2y = 3x + 4$

b. $3x + 4y = 1$

y = mx + c

a. $2y = 3x + 4$

$y = \frac{3}{2}x + 2$

Gradient = $\frac{3}{2}$

y-intercept = 2

b. $3x + 4y = 1$

$4y = -3x + 1$

$y = -\frac{3}{4}x + \frac{1}{4}$

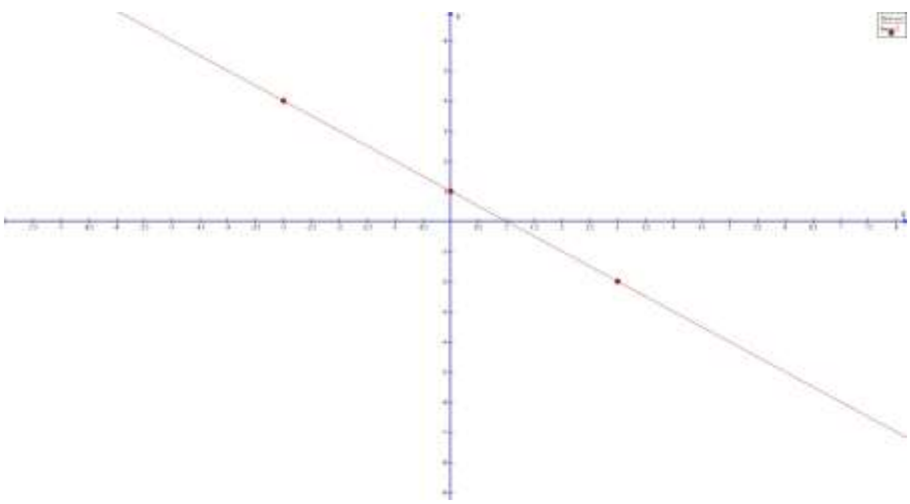
Gradient = $-\frac{3}{4}$

y-intercept = $\frac{1}{4}$

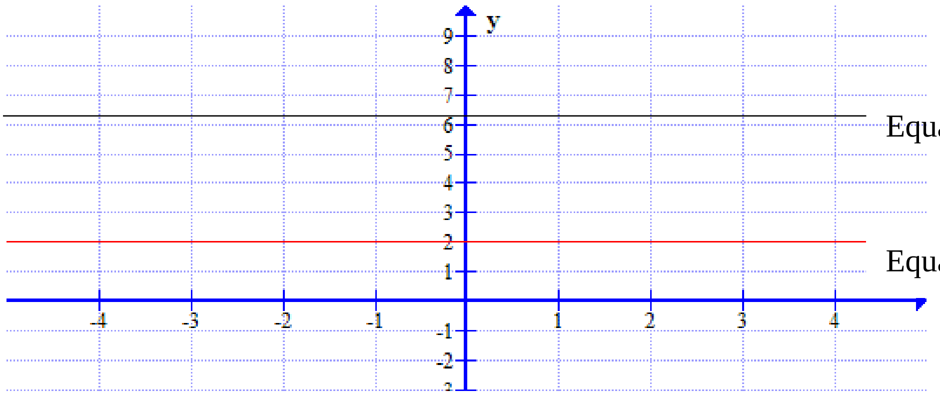
Drawing of linear function

Complete the table below and draw the graph of $y = -x + 1$

x	-3	0	3
y	4	1	-2



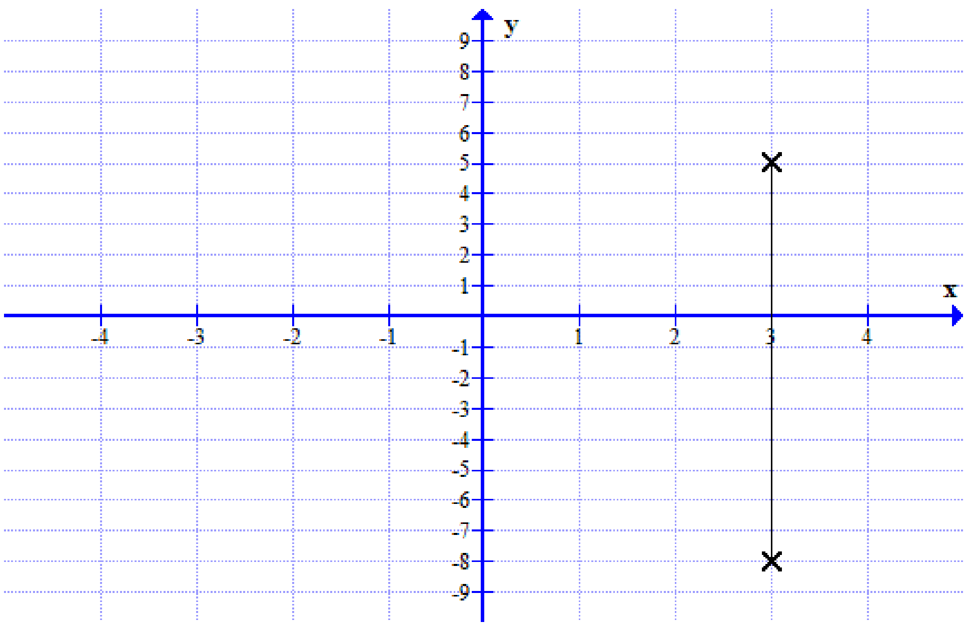
Graphs of Horizontal and Vertical Lines



Equation: y = 6

Equation: y = 2

Equation of a horizontal line is always: $y = a$



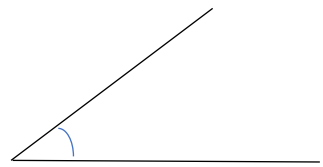
Equation: $x = 3$

Equation of a vertical line is always: $x = a$

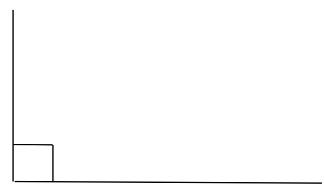
Chapter 7: Geometry (Angles)

1) There are different types of angles.

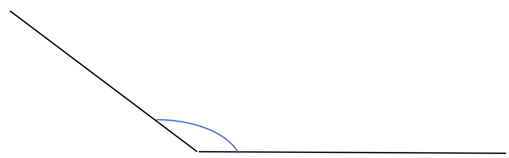
a) An **acute** angle is less than 90°.



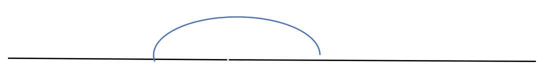
b) A **right angle** is equal to 90°.



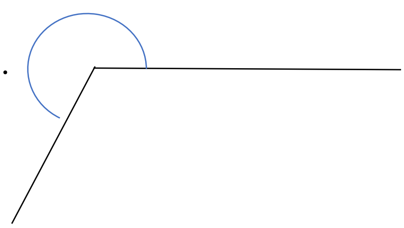
c) An **obtuse** angle is between 90° and 180°.



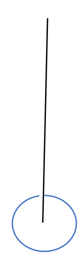
d) A **straight line** is equal to 180°.



e) A **reflex angle** is larger than 180° but less than 360°.



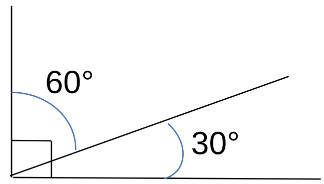
f) A **complete turn** is 360°.



2) Two angles are said to be **complementary angles** if their sum is 90°.

For example,

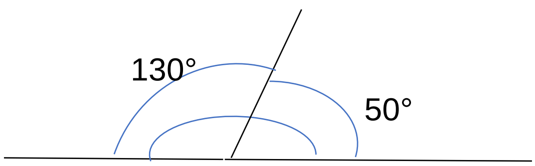
30° and 60° are complementary angles as 30° + 60° = 90°.



3) Two angles are called **supplementary angles** if their sum is 180°.

For example,

130° and 50° are supplementary angles because 130° + 50° = 180°.

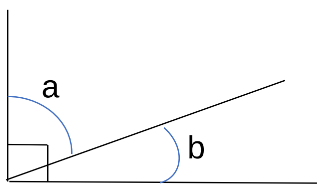


4) Two angles are called **adjacent angles** if

   a) they share the common vertex and a common arm, and
   b) they lie on the opposite sides of the common arm.

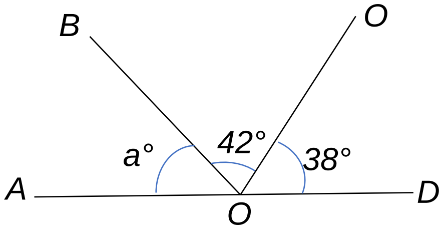
   For example,

   a and b are adjacent angles.



Example 1

Calculate the **unknown angle *a*** in the figure below.



Solution

Observe that ∠AOB, ∠BOC and ∠COD are **adjacent angles** on a straight line.

Therefore, we can form an equation comprising the information above

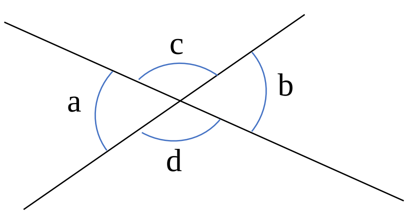
$a° + 42° + 38° = 180°$ (***Sum of angles on a line***)

$a° + 80° = 180°$

$a° = 180° - 80°$

$a° = 100°$

5) **Vertically opposite angles** are those on the opposite sides of two intersecting lines.



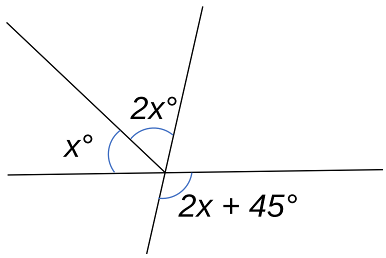
The figure shows two pairs of vertically opposite angles.

i) a and b; and
ii) c and d.

Vertically opposite angles are always equal in size.

Example 2

What is the **value of x** in the figure below?



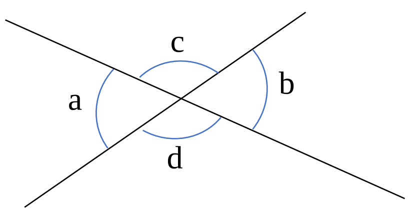
<u>Solution</u>

$x° + 2x° = 2x° + 45°$ ***(vertically opposite angles)***

$x° + 2x° - 2x° = 45°$

$x = 45°$

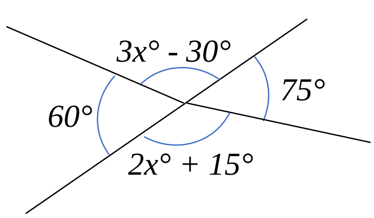
6) In the diagram shown below, a, b, c and d are called **angles at a point**.



a + b + c + d = 360° as they make up a complete turn.

Example 3

In the figure below, not drawn to scale, find the value of *x*.



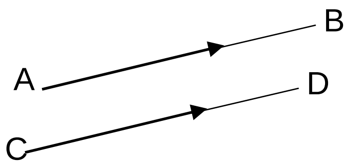
$3x° - 30° + 60° + 2x° + 15° + 75° = 360°$ ***(angles at a point)***

$$5x° + 120° = 360°$$

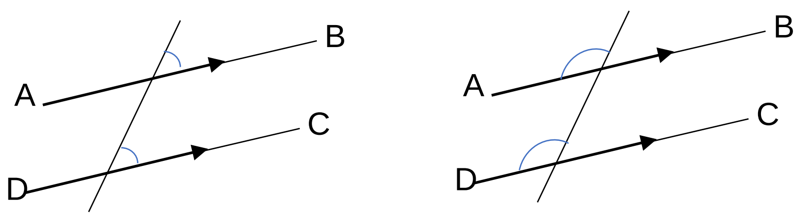
$$5x° = 360° - 120°$$

$$x = 48°$$

7) **Parallel lines** are the straight lines that never intersect. In geometry, we use a pair of arrows to indicate parallel lines. The diagram below shows parallel lines AB and CD. Their relationship can be written as: AB // CD.

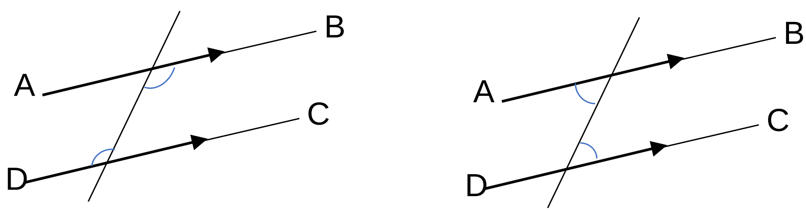


8) Two angles are said to be **corresponding angles** if they have the same relative positive at the each intersection where a line cuts across a pair of parallel lines.



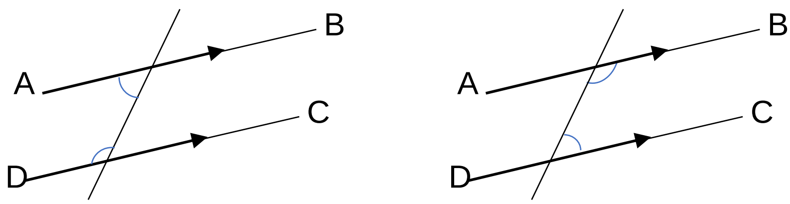
The corresponding angles are equal to each other.

9) Two angles are called **alternate angles** if they lie on the different side of the line cutting across a pair of parallel lines.



The alternate angles are equal to each other.

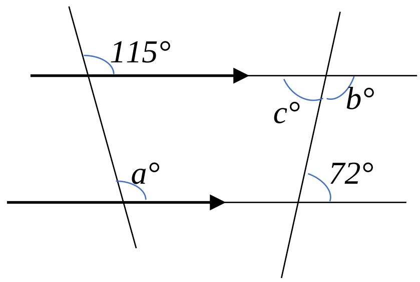
10) Two angles are known as **interior angles** if they lie on the same side of the line that lie on the different side of the line cutting across a pair of parallel lines.



The interior angles are supplementary.

Example 4

Calculate the unknown angles in the figure below.



<u>Solution</u>

a° = 115° ***(corresponding angles)***

b° + 72° = 180° ***(interior angles)***

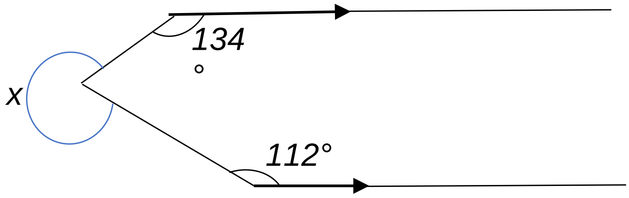
b° = 180° – 72°

b° = 108°

c° = 72° ***(alternate angles)***

Example 5

Calculate the unknown angle in the figure below.



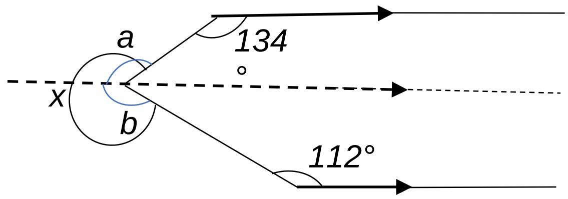
Solution

Observing the figure, there are two line segments between the two parallel lines.

In this case, we can draw the third parallel line at the point of intersection of the two line segments.

After drawing the dotted line, we can then apply the angle properties to deduce the value of *x*.

$a = 134°$ *(alternate angles)*



$b = 112°$ *(alternate angles)*

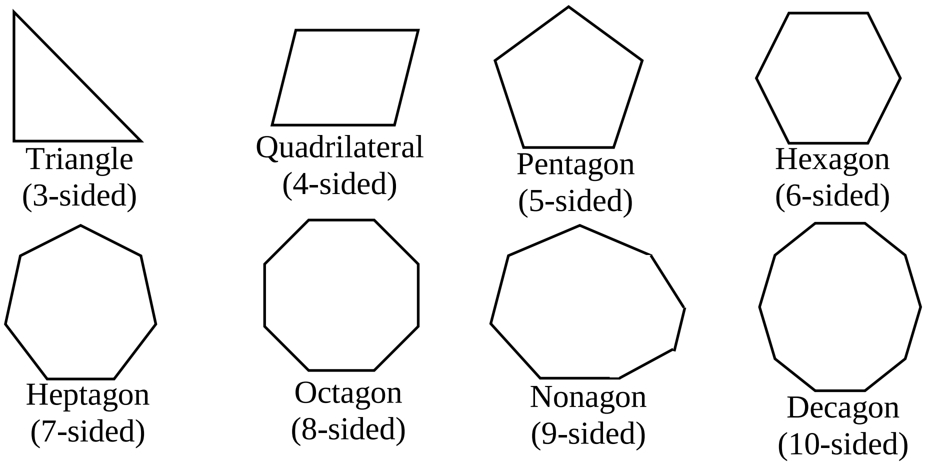
$x = a + b$

$x = 134° + 112°$

$x = 246°$

Chapter 8: Polygons

1) A **polygon** is a closed plane figure with three or more straight lines. Polygons are named according to the number of sides they have.

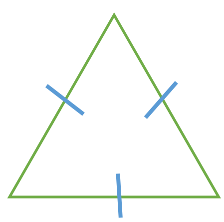


Note: Polygon with n sides is called an *n*-gon.

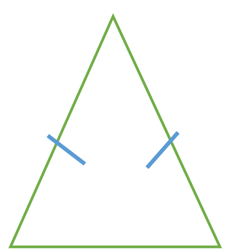
2) A **regular polygon** is a polygon with all its sides and angles equal.

3) Triangles can be classified according to the number of equal sides they have.

a) An **equilateral triangle** is a triangle with 3 equal sides. Each of its angles is 60°.

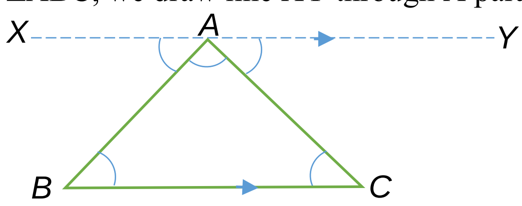


b) An **isosceles triangle** is a triangle with 2 equal sides. Its base angles are equal.



4) The **sum of the interior angles** of a triangle is always 180°.

Considering ΔABC, we draw line XY through A parallel to BC.



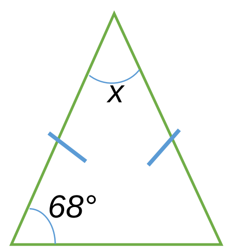
$\angle XAB = \angle ABC$ and $\angle YAC = \angle ACB$ (***alternate*** $\angle$***s, XY//BC***)

$\angle XAB + \angle BAC + \angle YAC = 180°$ (***Sum of*** $\angle$***s on a straight line***)

$\therefore \angle ABC + \angle BAC + \angle ACB = 180°$

Example 1

Find the unknown angle marked *x* in the figure.



Solution

The figure shows an isosceles triangle. The two base angles of the isosceles triangle are equal in size.

$\therefore\ x° + 68° + 68° = 180°$ (***Sum of*** $\angle$***s in a*** Δ)

$x° + 136° = 180°$

$x° = 180° - 136°$

$x° = 44°$

Example 2

The angles of a triangle are $(2x-45)°$, $(x+15)°$ and $\frac{3}{2}x°$. Find the value of *x*.

Solution

$(2x-45)° + (x+15)° + \frac{3}{2}x° = 180°$ (***Sum of*** $\angle$***s in a*** Δ)

$3x° - 30° + \frac{3}{2}x° = 180°$

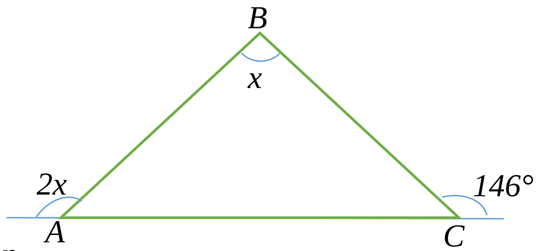
$$3x° + \frac{3}{2}x° = 180° + 30°$$

$$\frac{9}{2}x° = 210°$$

$$x = 46\frac{2}{3}$$

Example 3

Find the value of x in the figure below.



Solution

$\angle ACB = 180° – 146°$ ***(adjacent ∠s on a straight line)***
$= 34°$

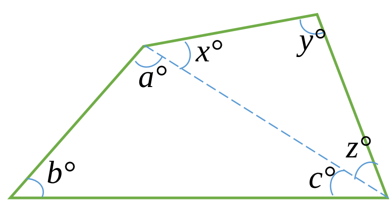
$2x° = x° + \angle ACB$ ***(exterior ∠s = sum of interior opposite ∠s)***
$2x° – x° = \angle ACB$
$x° = \angle ACB$
$x = 34$

5) A **quadrilateral** is a 4-sided polygon. Any quadrilateral can be divided into 2 triangles along its diagonal. Since the angles of a triangle add up to 180°, the angle sum of a quadrilateral is 180° × 2 = 360°.

$a° + b° + c° = 180°$ ***(Sum of ∠s in a Δ)***

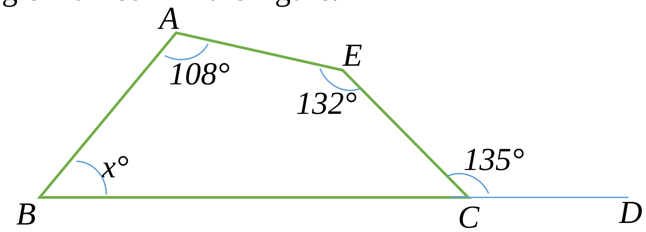
$x° + y° + z° = 180°$ ***(Sum of ∠s in a Δ)***

$a° + b° + c° + x° + y° + z° = 360°$



Example 4

Calculate the unknown angle marked *x* in the figure.



Solution

$\angle BCE = 180° - 135°$ (***adjacent ∠s on a straight line***)

$= 45°$

$\angle BCE + x° + 108° + 132° = 360°$

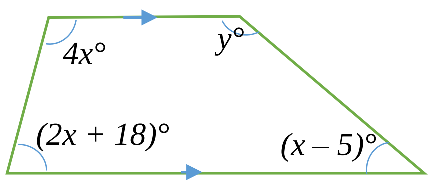
$x° + 285° = 360°$

$x° = 75°$

6) A **trapezium** is a quadrilateral with exactly one pair of parallel opposite sides.

Example 5

Find the unknown angles marked *x* and *y* in the figure below.



Solution

$(4x)° + (2x+18)° = 180°$ (***interior ∠s***)

$(6x+18)° = 180°$

$6x° = 162°$

$x° = 27°$

$(y)° + (x-5)° = 180°$ (***interior ∠s***)

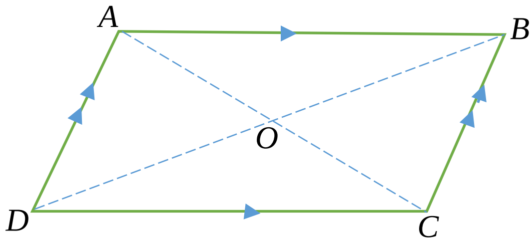
$(y)° + (27-5)° = 180°$

$y° + 22° = 180°$

$y° = 180° - 22°$

$y° = 158°$

7) A **parallelogram** is a trapezium with two pairs of parallel opposite sides.



Note that

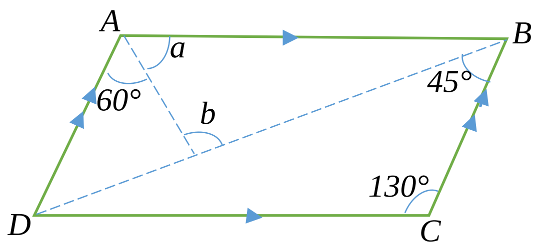
a) The opposite sides of a parallelogram are equal are parallel.
AB = DC, AD = BC and AB // DC, AD // BC

b) The opposite angles of a parallelogram are equal.
∠BAD = ∠DCB and ∠ADC = ∠ABC

c) The diagonals of a parallelogram bisect each other.
AO = OC and DO = OB

Example 6

ABCD is a parallelogram. Calculate the unknown angles marked *a* and *b*.



<u>Solution</u>

$a° + 60° = 130°$ **(opposite ∠s of parallelogram are equal)**

$a° = 130° – 60°$

$a° = 70°$

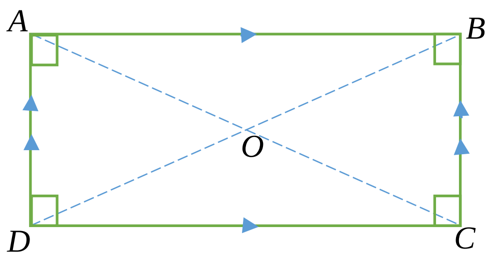
$\angle ADB = 45°$ **(alternate ∠s, AD // BC)**

$b° = 60° + \angle ADB$ **(exterior ∠s = sum of interior opposite ∠s)**

$b° = 60° + 45°$

$b° = 105°$

8) A **rectangle** is a parallelogram with four right angles.



Note that

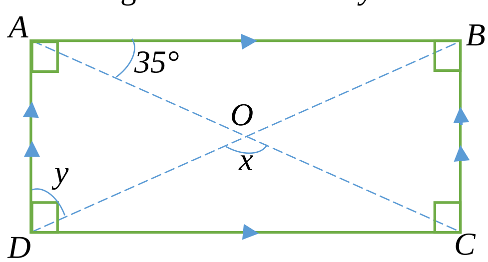
a) ∠DAB = ∠ABC = ∠BCD = ∠CDA = 90°

b) If we divide the rectangles along the 2 diagonals, AC and BC, there will be 4 isosceles triangles.
ΔAOD, ΔAOB, ΔBOC and ΔCOD

c) The diagonals of a rectangle are equal in length. AC = BD
∴AO = OC = BO = OD as the diagonals also bisect each other.

Example 7

ABCD is a rectangle. Find the unknown angles marked *x* and *y*.



<u>Solution</u>

∠ADC is a right angle.

$$\angle ADO + 35° = 90°$$

$$\angle ADO = 90° – 35°$$

$$\angle ADO = 55°$$

Since the diagonals AC and BD bisect each other, ΔAOD is an isosceles Δ.

$$y° = \angle ADO$$

$$y° = 55°$$

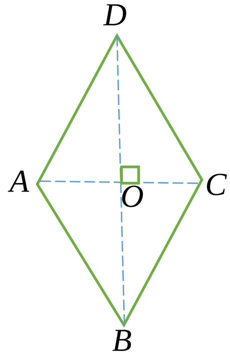
∠AOB is an exterior angle of ΔOAD.

$$x° = \angle AOD + y°$$

$x° = 55° + 55°$

$x° = 110°$

9) A **rhombus** is a parallelogram with all its 4 sides equal in length.



Note that

a) Its four sides are equal in length. AB = BC = CD = DA
b) The opposite sides are parallel. AB // DC and BC // AD
c) The opposite angles are equal. ∠DAB = ∠DCB and ∠ABC = ∠ADC
d) The diagonals of a rhombus bisect each other at right angles.
AO = OC, BO = OD and ∠AOD = ∠COD = ∠BOA = ∠BOC = 90°
e) The diagonals of a rhombus bisect the opposite angles. Since the opposite angles are equal in size,
∠OAB = ∠OAD = ∠DCA = ∠ACB and ∠ADB = ∠CDB = ∠ABD = ∠DBC

Example 8

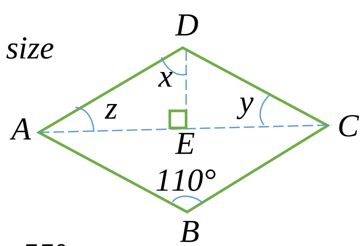
ABCD is a rhombus. Find the unknown angles marked *x, y* and *z.*

Solution

The opposite angles of a rhombus are equal in size

$\angle ADC = 110°$

Since DE ⊥ AC, DE is part of diagonal DB.



$x° = \frac{1}{2}\angle ADC = 55°$

$z° + x° + 90° = 180°$ (***Sum of ∠s in a*** Δ)

$z° + 55° + 90° = 180°$

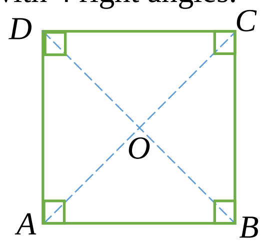
$z° = 180° – 55° – 90°$

$z° = 35°$

The diagonal AC bisects the opposite angles which are equal.

$y° = z° = 35°$

10) A **square** is a rhombus with 4 right angles.

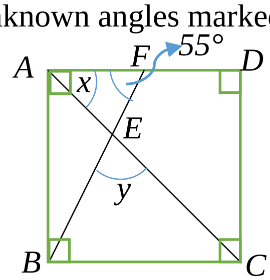


Note that

a) It has 4 equal sides. AB = BC = CD = DA
b) Its opposite sides are parallel. AB // DC and BC // AD
c) It has 4 right angles. ∠DAB = ∠ABC = ∠BCD = ∠CDA = 90°
d) Its diagonals are equal in length and bisect each other at right angles.
AO = OC = BO = OD and ∠AOD = ∠AOB = ∠BOC = ∠COD = 90°
e) The diagonals of a square bisect the opposite angles. Since all its 4 angles are 90°, ∠OAD = ∠OAB = ∠DCA = ∠ACB = ∠CDB = ∠ABD = ∠CBD = 45°
f) If we divide the square along its diagonals, we will get four congruent, isosceles right-angled triangles. They are ΔAOB, ΔBOC, ΔCOD and ΔAOD.

Example 9

ABCD is a square. Find the unknown angles marked *x* and *y*.



Solution

The diagonals of a square bisect the opposite angles which are all right angles.

$x° = 45°$

$\angle AEF = 180° – x° – 55°$ (**Sum of ∠s in a** Δ)

$\angle AEF = 180° – 45° – 55°$

$\angle AEF = 80°$

$y° = \angle AEF$ ***(vertically opposite ∠s)***

$y° = 80°$

11) A **regular polygon** is a polygon with equal angles and equal sides. Its n interior angles add up to $(n – 2) \times 180°$.

Example

Each interior angle of a regular polygon is 165.6°. How many sides does the polygons have?

Solution

Form an equation,

Each interior angle of a regular polygon $= \dfrac{(n-2)\times 180°}{n}$

$$\frac{(n-2)\times 180°}{n} = 165.6°$$

$$(n – 2) \times 180° = 165.6° \times n$$

$$180° \times n – 165.6° \times n = 360°$$

$$14.4° \times n = 360°$$

$$n = 25$$

∴ The polygon has 25 sides.

12) The sum of the exterior angles of an n-sided polygon is always 360°.

Example

Find the exterior angle of a 12-sided regular polygon.

Solution

Let x the size of each exterior angle.

$12 \times x = 360°$

$x° = 30°$

Example

The ratio of an interior angle to an exterior angle of a regular n-sided polygon is 8 : 1. How many sides does the polygon have?

Solution

Let 8a and a be the interior angle and the exterior angle at the same vertex respectively.

exterior angle + interior angle = 180° **(adjacent ∠s on a straight line)**

$$a + 8a = 180°$$

$$a = 20°$$

$$\text{Sum of exterior angles} = 360°$$

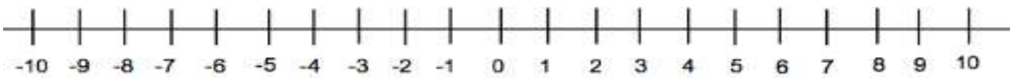
$$20°(n) = 360°$$

$$n = 18$$

∴ *The polygon has 18 sides.*

Chapter 9: Inequalities

Number Line



1) a is more than b

$a > b$

2) a is lesser than b

$a < b$

3) a is more than or equal to b

$a \geq b$

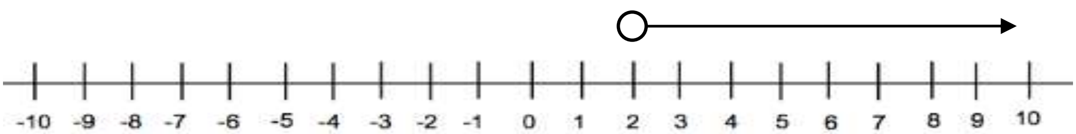
4) a is lesser than or equal to b

$a \leq b$

Examples

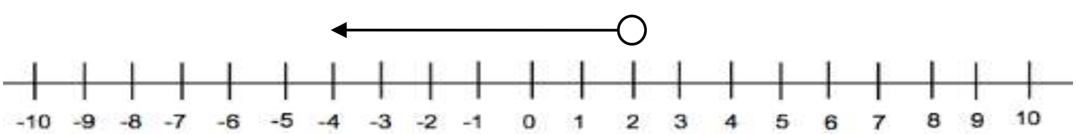
1) x is more than 2

$x > 2$



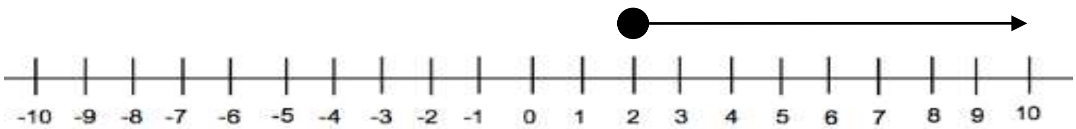
2) x is lesser than 2

$x < 2$



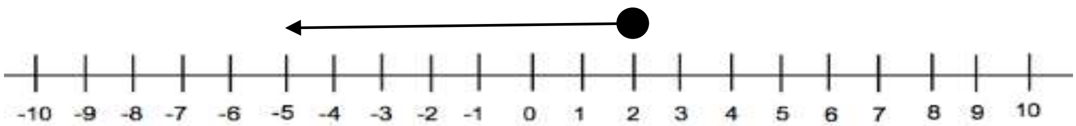
3) x is more than or equal to 2

$x \geq 2$



4) x is lesser than or equal to 2

$x \leq 2$

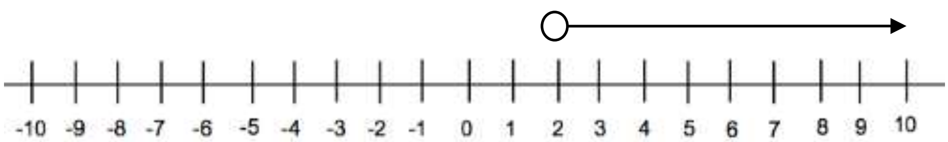


<u>Maximum and Minimum</u>

If x is an integer and $x > 2$,

What is the max value of x?
There is no max!

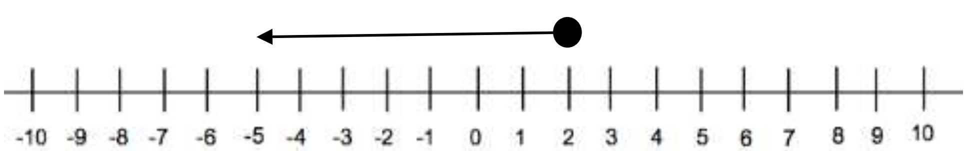
What is the min value of x?
3



If x is an integer and $x \leq 2$,

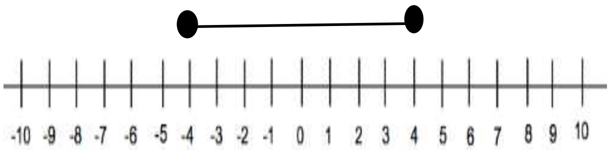
What is the max value of x?
2

What is the min value of x?
There is no min!

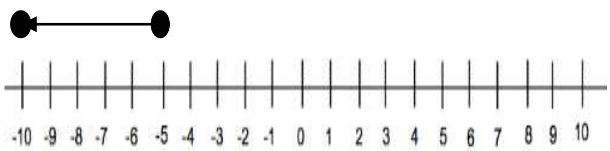


Example 1

Given that $-4 \le x \le 4$ and $-10 \le y \le -5$, find the smallest value of $x^2 + y^2$.



$$(-4)^2 + (-10)^2$$
$$= 16 + 100$$
$$= 116$$



$$(0)^2 + (-5)^2$$
$$= 0 + 25$$
$$= 25$$

<u>Solving Inequalities</u>

How do we solve inequalities?
It is almost the same as solving equations!

$x+1>2$	$x-1>2$	$2x>2$	$\frac{x}{2}>2$
$x>2-1$	$x>2+1$	$x>2\div 2$	$x>2\times 2$
$x>1$	$x>3$	$x>1$	$x>4$

Be Careful!

$$-x>2$$
$$x>2\div -1$$
$$x>-2$$

You have to switch the inequality!

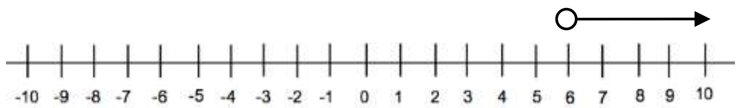
$$-x>2$$
$$x<2\div -1$$
$$x<-2$$

Example 1

Solve the following inequality and represent your answers on a number line.

$$\frac{2x}{3} - 1 > 3$$

$$\frac{2x}{3} > 3 + 1$$



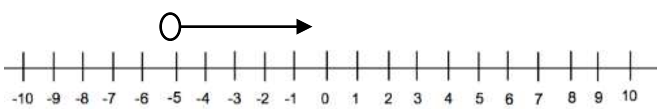
$$2x > 4 \times 3$$

$$x > 12 \div 2$$

$$x > 6$$

Example 2

$$\frac{1}{2}(x+3) < x+4$$
$$x+3 < 2(x+4)$$
$$x+3 < 2x+8$$
$$x-2x < 8-3$$
$$-x < 5$$
$$x > -5$$



Inequalities Word Problems

Common Terms You Should Know

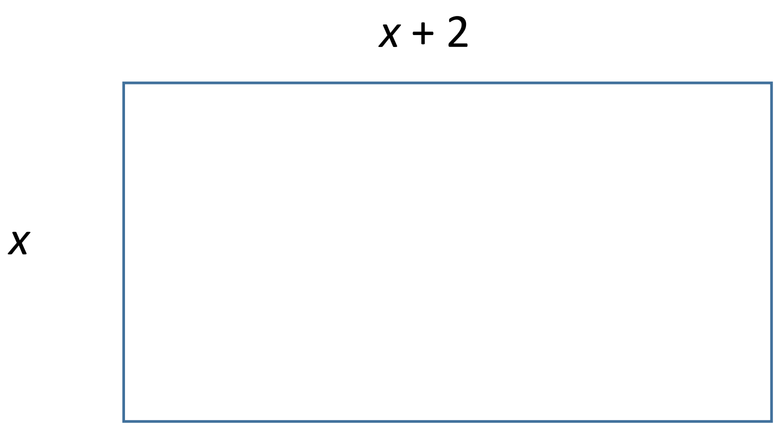
At least 30
$x \geq 30$
At most 30
$x \leq 30$
Cannot be more than 30
$x \leq 30$
Cannot be lesser than 30
$x \geq 30$
Budget of $30
$x \leq \$30$

Example 1

The breadth of a rectangle is given as x cm. Its length is 2 cm longer than its breadth. If the perimeter of this rectangle is at most 60 cm,
(a) form an inequality in x,
(b) solve the inequality,
(c) hence, find the largest possible area of the rectangle.

(a) Perimeter = $2(x + 2) + 2x$
$= 4x + 4$

$4x + 4 \leq 60$



(b) $4x + 4 \leq 60$
$4x \leq 60 - 4$
$x \leq 56 \div 4$
$x \leq 14$

(c) Largest breath = 14 cm
Largest length = 16 cm
Largest area = $14 \times 16 = 224$ cm^2

Chapter 10: Patterns

Patterns with Same Difference

Important Formula: Nth Term = a + (n – 1) d

a → 1st term, d → common difference

Example: 1, 3, 5, 7, 9

n	
1	1
2	3 = 1 + 1 × 2
3	5 = 1 + 2 × 2
4	7 = 1 + 3 × 2
5	9 = 1 + 4 × 2
n	Nth term = a + (n – 1) d

a → first term

d → difference

$$\begin{aligned} \text{nth term} &= 1+(n-1)2 \\ &= 1+2n-2 \\ &= 2n-1 \end{aligned}$$

$$\begin{aligned} \text{50th term} &= 2(50)-1 \\ &= 99 \end{aligned}$$

Example 1

A certain number sequence is given as:

3, 7, 11, 15, …….. , ……….

(a) Write down the next two terms of the sequence.
(b) Write down an expression ,in terms of n, for the nth term of the sequence.
(c) Find the 45th term.

(a) 19, 23

(b) $\text{nth term} = 3+(n-1)4$
$= 3+4n-4$
$= 4n-1$

(c) $\text{45th term} = 4(45)-1$
$= 179$

<u>Patterns with Square Numbers</u>

Example: 1, 4, 9, 16, 25…

n	
1	$1 = 1^2$
2	$4 = 2^2$
3	$9 = 3^2$
4	$16 = 4^2$
5	$25 = 5^2$
n	Nth term = n^2

Example 1

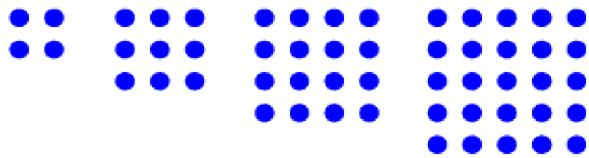


Figure n:	1	2	3	4
No of dots:	4	9	16	25

n	**No of dots**
1	$4 = (1 + 1)^2$
2	$9 = (2 + 1)^2$
3	$16 = (3 + 1)^2$
4	$25 = (4 + 1)^2$

a. Find the number of dots in the nth figure.

nth term = $(n+1)^2$

b. How many dots does figure 10 have?

$$\begin{aligned} \text{10th term} &= (10+1)^2 \\ &= 121 \end{aligned}$$

c. Which figure has 225 dots?

$$\begin{aligned} (n+1)^2 &= 225 \\ n+1 &= \sqrt{225} \\ n+1 &= 15 \\ n &= 14 \end{aligned}$$

Sum of Consecutive Numbers

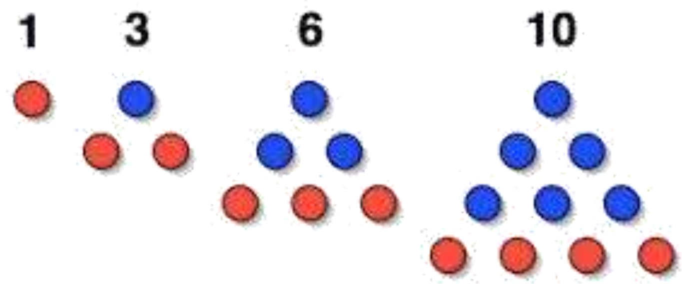
Example: 1, 3, 6, 10, 15

n	
1	1
2	3 = 1 + 2
3	6 = 1 + 2 + 3
4	10 = 1 + 2 + 3 + 4
5	15 = 1 + 2 + 3 + 4 + 5
n	Nth term = 1 + 2 + 3 + … + n

$$
\begin{aligned}
\text{nth term} &= 1+2+3+\ldots+(n-1)+n \\
&= (n+1)\times\frac{n}{2} \\
&= \frac{n(n+1)}{2}
\end{aligned}
$$

Example 1

Figure: 1 2 3 4



n	Total dots
1	1
2	3 = 1 + 2
3	6 = 1 + 2 + 3
4	10 = 1 + 2 + 3 + 4
5	15 = 1 + 2 + 3 + 4 + 5
n	Nth term = 1 + 2 + 3 + … + n

a. Find the number of dots in the nth figure.

$$\text{nth term} = \frac{n(n+1)}{2}$$

b. How many dots does figure 10 have?

$$\text{10th term} = \frac{10(10+1)}{2}$$
$$= 55$$

c. If a figure has 18 red dots, how many blue dots does it have?

18 red dots → 18^{th} figure

$$\text{Total dots} = \frac{18(18+1)}{2}$$
$$=171$$

$$\text{Blue dots} = 171-18$$
$$=153$$

Chapter 11: Ratio and Proportion

Ratio Involving Rational Number

What is a ratio?

- A ratio is a comparison of 2 quantities in the **same units**

Example:

3 m : 20 cm

$$15:1=\frac{15}{1}$$

= 300 cm : 20 cm

= 15 : 1

Ratio in Simplest Form (Integers)

Example 1

$$\frac{1}{2}:\frac{1}{3}=3:2$$

Multiply both sides of the ratio by the common multiple of 2 and 3 to remove the fractions.

Example 2

$$1\frac{1}{2}:2\frac{1}{3}=\frac{3}{2}:\frac{7}{3}$$

$$=9:14$$

Example 3

$$0.22:0.5=22:50$$

$$=11:25$$

Multiply both sides of the ratio by 100 to remove the decimal points

Equivalent Ratios

Example 1

$$\frac{1}{2}:3=\frac{2}{3}x:5$$
$$\frac{1}{2}\div 3\times 5=\frac{2}{3}x$$
$$\frac{5}{6}=\frac{2}{3}x$$
$$x=\frac{5}{6}\div\frac{2}{3}=\frac{5}{4}$$

Combining Ratios (Repeated Identity Concept)

Example 1

If a : b = 2 : 3 and b : c = 4 : 5 , what is a : b : c?

a : b	b : c
=2 : 3	=4 : 5
=8 : 12	=12 : 15

a : b : c

=8 :12 : 15

Increase in Ratio

- When a quantity is increased in the ratio a : b, you multiply the quantity by $\frac{a}{b}$

Example

Increase 6 in the ratio 5 : 3

$$6\times\frac{5}{3}=10$$

Example 1

Mrs Tan distributed some sweets to 5 children and discovered that she had some more sweets left. She increased the number of sweets per child in the ratio 5 : 3. As a result each child gets 4 more sweets and Mrs Tan had no more sweets left after that. How many sweets did Mrs Tan have?

$5-3=2$
2 units = 4 sweets
5 units = 10 sweets
$10 \times 5 = 50$ sweets

Decrease in Ratio

- When a quantity is decreased in the ratio a : b, you multiply the quantity by $\frac{a}{b}$

Example 1

Decrease 6 in the ratio 2 : 3

$$6 \times \frac{2}{3} = 4$$

Example 2

Mr Tan was paying $800 for his monthly rent. After one year, he managed to decrease his rent by $50 after discussing with his landlord. Find the decrease in the ratio of his monthly rent.

$800-50=750$
For decrease in ratio, the smaller number is in front of the ratio.

$$\frac{750}{800} = \frac{15}{16}$$

15:16

Chapter 12: Percentage

Discount

A discount is a reduction in the price.

Discount = Marked Price – Selling Price

$$\text{Percentage Discount} = \frac{\text{Discount}}{\text{Marked Price}} \times 100\%$$

During the Great Singapore Sale, all the items were on a 10% discount. A shirt has a marked price of $20. Find the selling price.

$$100\% - 10\% = 90\%$$
$$90\% \times \$20 = 0.9 \times \$20$$
$$= \$18$$

GST

GST (Goods and Services Tax) is an increase on the price. In Singapore, the GST is 7%.

$$\text{Price after GST} = (100\% + GST) \times \text{Selling Price}$$

A price of a watch was $150 before 7% GST. There was also a 20% discount on the watch. How much do you need to pay if you want to buy the watch?

$$80\% \times \$150 = \$120$$
$$107\% \times \$120 = \$128.40$$

$$107\% \times \$150 = \$160.50$$
$$80\% \times \$160.50 = \$128.40 \text{ (Same!)}$$

Conversion

To convert a fraction or decimal to percentage, multiply it by 100%.

$$\frac{1}{4}=\frac{1}{4}\times100\%$$
$$=25\%$$

$$1\frac{1}{2}=\frac{3}{2}\times100\%$$
$$=150\%$$

$$0.8=0.8\times100\%$$
$$=80\%$$

$$1.2=1.2\times100\%$$
$$=120\%$$

To convert a percentage into a fraction or decimal,

1) Drop the percentage
2) Divide by 100

$$25\%=\frac{25}{100}$$
$$=\frac{1}{4}$$
$$=0.25$$

$$140\%=\frac{140}{100}$$
$$=1\frac{40}{100}$$
$$=1\frac{2}{5}$$
$$=1.4$$

Finding Percentage of a Number

To find percentage of a number,

1) Write the percentage as a fraction
2) Multiply the fraction by the number

$$25\%\text{ of }80=\frac{25}{100}\times80$$
$$=\frac{1}{4}\times80$$
$$=20$$

$$125\%\text{ of }80=\frac{125}{100}\times80$$
$$=\frac{5}{4}\times80$$
$$=100$$

Express one Quantity as a Percentage of Another

To express one quantity as a percentage of another,

1) Express as a fraction
2) Multiply the fraction by 100%

Express 15 cm as a percentage of 0.8 m

0.8 m = 80 cm

$$\frac{15}{80} \times 100\% = 18.75\%$$

Reverse Percentage

25% of x is equal to 30, what is x?

1) Express 25% as a fraction.
2) Divide 30 by the fraction.

$$\frac{25}{100} x = 30$$

$$\begin{aligned} x &= 30 \div \frac{25}{100} \\ &= 30 \div \frac{1}{4} \\ &= 30 \times 4 \\ &= 120 \end{aligned}$$

Percentage Increase and Decrease

$$\text{Percentage Increase} = \frac{Increase}{Original} \times 100\%$$

$$\text{Percentage Decrease} = \frac{Decrease}{Original} \times 100\%$$

The price of a bag increased from \$28 to \$32. Find the percentage increase?

$$32 - 28 = 4$$

$$\frac{4}{28} \times 100\% = 14\frac{2}{7}\%$$

Chapter 13: Rate and Speed

Rate

Definition of Rate

- Measures how fast something changes
- Rate of Change of Distance = Speed (m/s)
- Rate of Change of Volume = Water Rate (litres per min)
- Printing Rate (Example: 100 pages/min)
- Working Rate (Example: Clean 1 house in 3 days)

Example 1

Mary can clean 2 tables per minute. How many tables can she clean in 1 hour?

1 minute → 2 tables

1 hour → $2 \times 60 = 120$ tables

Example 2

Printer A can print a certain number of pages in 10 hours. Printer B can print the same number of pages in 4 hours. Both printers started printing at the same time for 6 hours. How many pages can Printer B print if Printer A printed 3000 pages in the 6 hours?

Rate of printer A → $3000 \div 6 = 500$ pages / hour

Number of pages printed by A in 10 hours → $500 \times 10 = 5000$

Rate of printer B → $5000 \div 4 = 1250$ pages / hour

Number of pages printed by B in 6 hours → $1250 \times 6 = 7500$ pages

Mixed Rates – Example 1

Marcus takes 2 hours to clean a house on his own. Edmund takes 3 hours to clean a house on his own. If they work together, how long will they take to clean a house? Leave your answer in hours and mins.

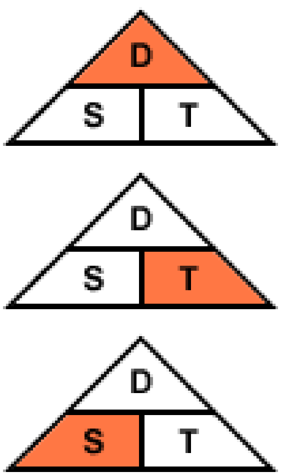
$$\text{Marcus' rate} \;\rightarrow\; \frac{1}{2}$$

$$\text{Edmund's rate} \;\rightarrow\; \frac{1}{3}$$

$$\text{Marcus + Edmund rate} \;\rightarrow\; \frac{1}{2}+\frac{1}{3}=\frac{5}{6}$$

$$\text{Time Taken} \;\rightarrow\; 1\div\frac{5}{6}=1.2 \text{ hours} = 1 \text{ hour } 12 \text{ mins}$$

<u>Speed</u>



Distance = Speed x Time

$$\text{Time}=\frac{\text{Distance}}{\text{Speed}}$$

$$\text{Speed}=\frac{\text{Distance}}{\text{Time}}$$

- Average Speed is Total Distance ÷ Total Time

Example 1

A driver drove at a speed of 80 km/h for 45 min. Find the total distance he had covered.

Check the units first

45 mins = 45/60 = ¾ h

Distance = Speed × Time

$80 \times 3/4 = 60$ km (Ans)

Example 2

A train travelled 52 km in 30 minutes. Find its average speed in km/h.

Check the units first

30 mins = 30/60 = ½ h

Speed = Distance ÷ Time

$52 \div \frac{1}{2} = 52 \times 2 = 104$ km/h (Ans)

Example 3

Express 120 km/h in m/s.

Recall: 1 km = 1000 m

1 hour = 3600 sec

120 km/h = 120 000 m/h

= 120 000 ÷ 3600 m/s

= 13 m/s (Ans)

Example 4

Usain Bolt ran 100 m in 9.58 seconds. Calculate his speed in km/h.

Speed = Distance ÷ Time

100 ÷ 9.58 = 10.44 m/s

10.44 m/s = 0.01044 km/s

= 0.01044 × 3600 km/h

= 37.58 km/h (Ans)

Average Speed

- Average Speed is Total Distance ÷ Total Time

Example 1

Mr Tan travelled on an expressway for 2h at 60 km/h. He then travelled another 3h at 80 km/h. Find the average speed for the whole journey.

D = 60 x 2 = 120km

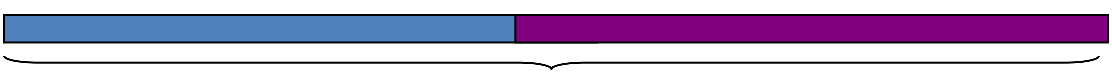
S = 80km/h

T = 3h

D = 80 x 3 = 240km

S = 60km/h

T = 2h



Total Distance = 120 + 240 = 360 km

Total Time = 5h

Average Speed = 360 ÷ 5 = <u>72 km/h</u>

Example 2

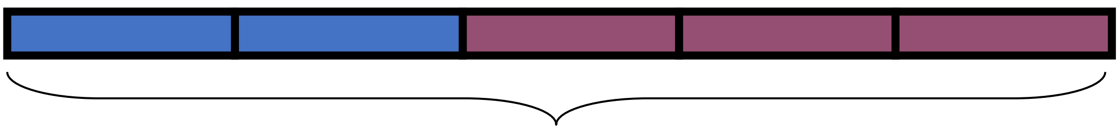
Mary took 1.5 h to cover 2/5 of a journey. She covered the remaining 120 km in 2.5 h. Find the average speed for the whole journey.

D = 120 ÷ 3 x 2 = 80 km

T = 1.5 h

D = 120 km

T = 2.5 h



Total Distance = 200 km

Total Time = 4h

Average Speed = 200 ÷ 4 = <u>50 km/h</u>

Chapter 14: Mensuration

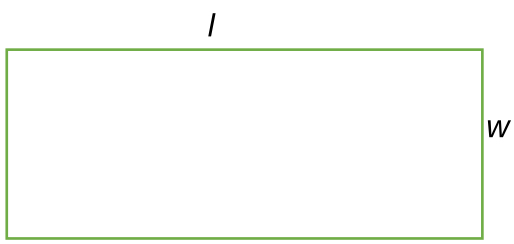
Perimeter

1. We use **millimetres** (mm), **centimetres** (cm), **metres** (m) and **kilometres** (km) as units for measuring lengths or distances.

   The units are related as follows:
   1km = 1000m; 1m = 100cm; 1cm = 10mm

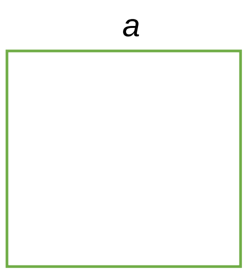
2. Perimeter of a closed figure is the total distance along its boundary. It can be found by adding the lengths of all its sides.

3. Perimeter of a **rectangle** = 2 × (length + width)



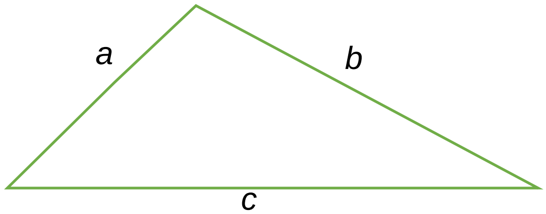
Perimeter of rectangle = 2 (l + w)

4. Perimeter of a **square** = 4 × length



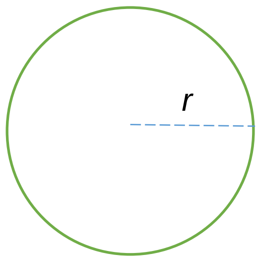
Perimeter of square = 4a

5. Perimeter of a **triangle** = sum of all three sides



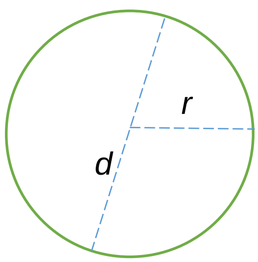
Perimeter of triangle = $a + b + c$

6. Perimeter of a **circle** is referred to as the circumference. The **circumference**, *C*, of a circle of radius, r, can be obtained by using this formula, circumference = $2 \times \pi \times r$ where $\pi = 3.14$ or $\frac{22}{7}$.



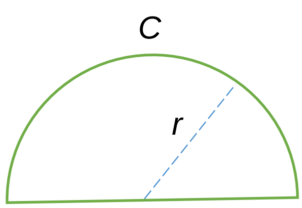
Circumference of circle = $2\pi r$

Note: The diameter of a circle, d, is twice the length of the radius. Therefore, we can also find the circumference using the formula, circumference = $\pi \times d$ where $\pi = 3.14$ or $\frac{22}{7}$.



Circumference of circle = πd

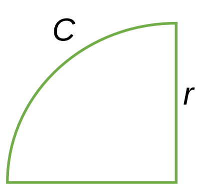
7. A **semicircle** is half a whole circle.



(Curved line)

Circumference of semicircle = πr or $\frac{1}{2}\pi d$

8. A **quadrant** is a quarter of a whole circle. Circumference of a quadrant = $\frac{1}{4} \times \pi \times d$ where $\pi = 3.14$ or $\frac{22}{7}$.



(Curved line)
Circumference of quadrant = $\frac{1}{4}\pi d$ or

Example 1

Find the diameter of the semicircle with perimeter 36cm. Give your answers correct to 3 significant figures. (Take $\pi = 3.14$)

Solution

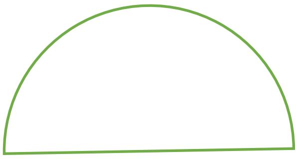
Let the diameter of the semicircle be d cm

$$\frac{1}{2}\pi d + d = 36$$

$$d\left(\frac{1}{2}\pi + 1\right) = 36$$

$$d = \frac{36}{\left(\frac{1}{2}\pi + 1\right)}$$

$$d = 14.0 \text{ cm (3s.f)}$$



<u>Area</u>

9. Area of a closed figure is the amount of space it covers. It is measured in square units.

10. Note:

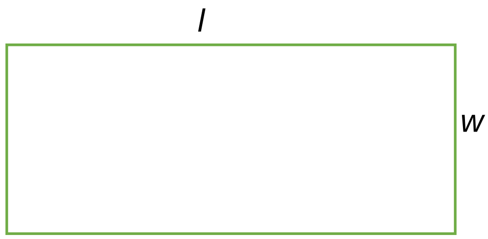
a) $1 \text{ cm}^2 = 1 \text{ cm} \times 1 \text{ cm}$
$= 10 \text{ mm} \times 10 \text{ mm}$
$= 100 \text{ mm}^2$

b) $1 \text{ m}^2 = 1 \text{ m} \times 1 \text{ m}$
$= 100 \text{ cm} \times 100 \text{ cm}$
$= 10\ 000 \text{ cm}^2$

c) $1 \text{ km}^2 = 1 \text{ km} \times 1 \text{ km}$
$= 1\ 000 \text{ m} \times 1\ 000 \text{ m}$
$= 1\ 000\ 000 \text{ m}^2$

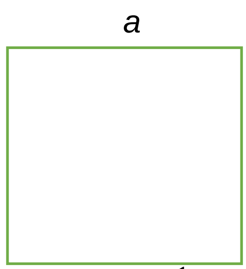
d) Hectare (ha) is a unit for measuring large land areas such as farms.
$1 \text{ ha} = 10\ 000 \text{ m}^2$

11. Area of a **rectangle** = length × width



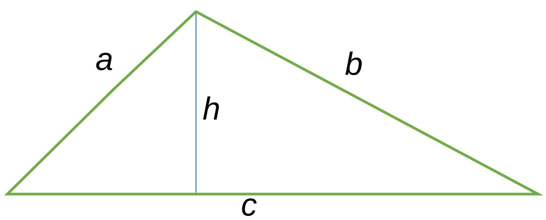
Area of rectangle = $l \times w$

12. Area of a **square** = length × length



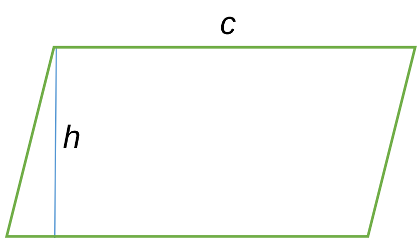
Area of square = a^2

13. Area of a **triangle** = $\frac{1}{2}$ × base × height



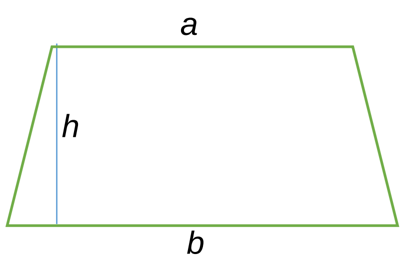
Area of triangle = $\frac{1}{2} \times c \times h$

14. Area of a **parallelogram** = base × height



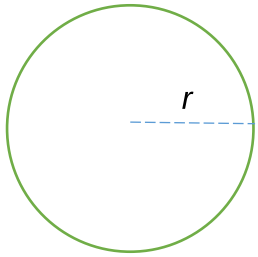
Area of parallelogram = $c \times h$

15. Area of a **trapezium** = $\frac{1}{2}$ × (sum of parallel sides) × height



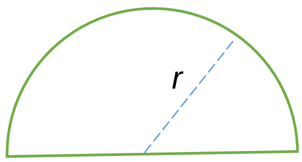
Area of trapezium = $\frac{1}{2} \times (a + b) \times h$

16. Area of a **circle** = $\pi \times r^2$, where $\pi = 3.14$ or $\frac{22}{7}$.



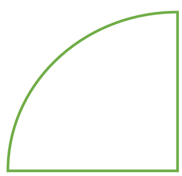
Area of circle = $\pi \times r^2$

17. Area of a **semicircle** = $\frac{1}{2} \times \pi \times r^2$, where $\pi = 3.14$ or $\frac{22}{7}$.



Area of semicircle = = $\frac{1}{2} \times \pi \times r^2$

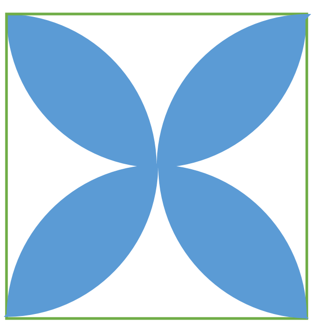
18. Area of a **quadrant** = $\frac{1}{4} \times \pi \times r^2$, where $\pi = 3.14$ or $\frac{22}{7}$.



Area of quadrant = $\frac{1}{4} \times \pi \times r^2$

Example 2

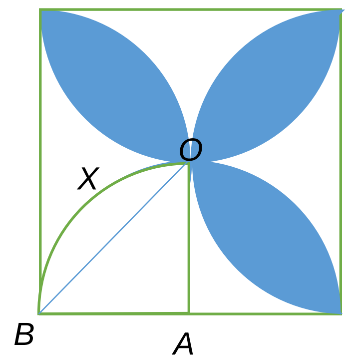
The figure below is made up of 4 semicircles enclosed in a square of edge 10cm. What is the area of the shaded region? (Take $\pi = 3.14$)



Solution

We can solve part of the problem by looking at the quadrant ABO.

AB = 5 cm, AO = 5cm



Area of $\Delta ABO = \frac{1}{2} \times AB \times AO$

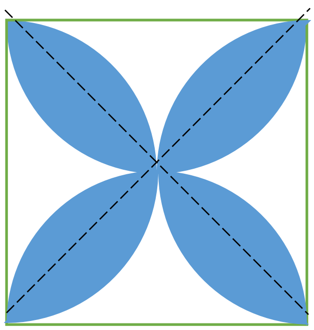
$= \frac{1}{2} \times 5 \times 5$

$= 12.5 \text{ cm}^2$

Area of quadrant ABO = $\frac{1}{4} \times \pi \times 5 \times 5$

$= \frac{1}{4} \times 3.14 \times 5 \times 5$

$= 19.625 \text{ cm}^2$ (5s.f)



Area of BXO = $19.625 \text{ cm}^2 - 12.5 \text{ cm}^2$

$= 7.125 \text{ cm}^2$

Area of shaded area in the figure = $7.125 \text{ cm}^2 \times 8$

$= 57 \text{ cm}^2$

Volume

Volume of an object is the amount of space it occupies. We use cubic metre (m^3), cubic centimetre (cm^3) and millimetre (mm^3) as units for measuring volumes.

19. Note:

a) $1 \text{ cm}^3 = 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm}$

$= 10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm}$

$= 1000 \text{ mm}^3$

b) $1 \text{ m}^3 = 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m}$

$= 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm}$

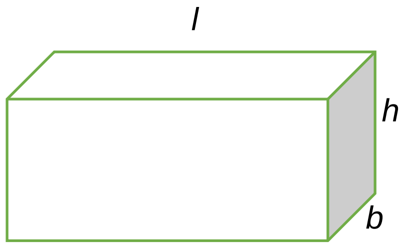
$= 1\,000\,000 \text{ cm}^3$

c) $1 \text{ km}^3 = 1 \text{ km} \times 1 \text{ km} \times 1 \text{ km}$

$= 1\,000 \text{ m} \times 1\,000 \text{ m} \times 1\,000 \text{ m}$

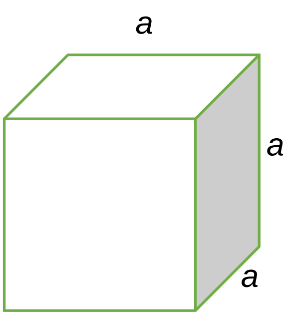
$= 1\,000\,000\,000 \text{ m}^3$

20. Volume of a **cuboid** = length × breadth × height



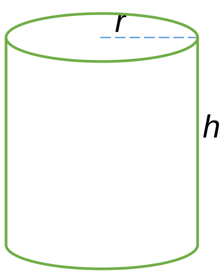
Volume of cuboid = $l \times b \times h$

21. Volume of a **cube** = length × length × length



Volume of cube = a^3

22. A cylinder is a special prism with a circular cross-section. Hence, its volume can be found by multiplying the area of the circular base by its height.

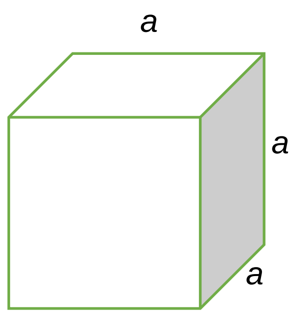


Volume of cylinder = $\pi r^2 h$

Surface Area

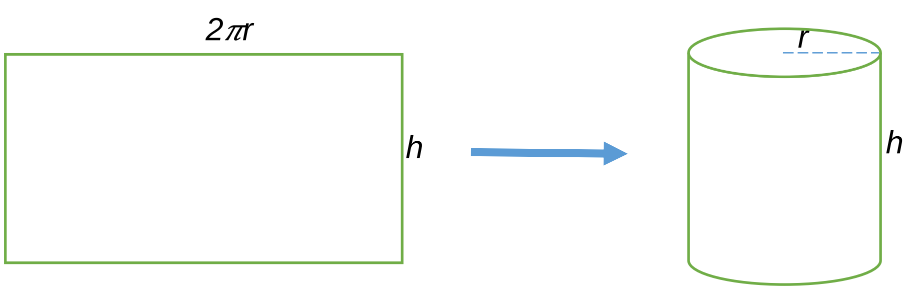
23. The surface area of a solid is the sum of the areas of all its faces. It is measured in square units.

24. A cube has 6 equal faces. Therefore, its surface area is obtained by multiplying the area of one face by 6.



Surface area of cube
$= 6 \times a \times a$
$= 6a^2$

25. The lateral surface of a cylinder is also called the curved surface. If we rolled up a piece of rectangular sheet, the rectangular sheet will become the curved surface of the cylinder.



Note:

a) Length of rectangle = circumference of the circle

$= 2\pi r$

b) Breadth of rectangle = height of the cylinder

$= h$

c) Area of rectangle = curved surface area of the cylinder

$= 2\pi rh$

Hence, total surface area of a solid cylinder = area of curved surface + 2 × area of base

$= 2\pi rh + 2\pi r^2$

26. The capacity of a container is the volume of liquid that can fit inside the container. The standard units often used for capacity are ***litre*** *(l)*, ***millilitre*** *(ml)* and ***kilolitre*** *(kl)*.

Note:

a) $1\ ml = 1\ \text{cm}^3$

b) $1\ l = 1000\ ml$
$= 1000\ \text{cm}^3$

c) $1\ kl = 1000\ l$
$= 1\ \text{m}^3$

Mass and Density

27. The density of a substance is the mass of one unit volume of the substance. It is calculated using the formula:

$$Density = \frac{Mass}{Volume}$$

28. We normally use kg/m^3 and g/cm^3 as units for measuring density. If $1\ \text{cm}^3$ of a substance has a mass of 250 g, we say its density is $250\ \text{g/cm}^3$. Similarly, if the density of a metal is $1400\ \text{kg/m}^3$, it means that $1\ \text{m}^3$ of the metal has a mass of 1400 kg.

Example 3

Find the density of a cylindrical solid with radius 24 cm and height 20 cm if it has a mass of 3 kg. Give your answer in g/cm3 correct to 2 decimal places. (Take $\pi = 3.14$)

Solution

Volume of solid $= \pi \times 24 \times 24 \times 20$
$= 36\ 172.8\ \text{cm}^3$

Mass of the solid = 3000g

Density of the solid = $\dfrac{3000}{36172.8}\ g / cm^3$
$= 0.08\ \text{g/cm}^3$ (2dp)

Chapter 15: Data Handling

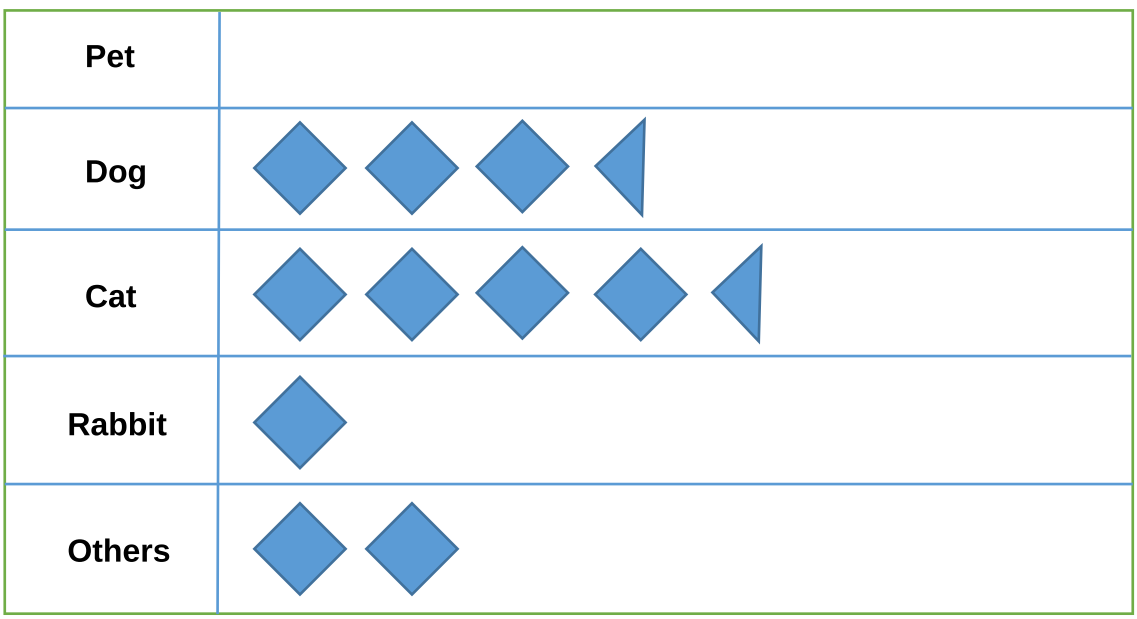
1. Statistics is a science of collecting, organising, interpreting and analysing data in order to assist the user to make decisions.

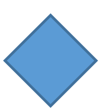
2. A survey on the pets owned by a group of 44 children was conducted and its result were tallied as shown below.

Pet Name	Tally	Frequency
Dog	卌 卌 ////	14
Cat	卌 卌 卌 ///	18
Rabbit	////	4
Others	卌 ///	8

In the table, the number of times each pet appears is called as **frequency**.

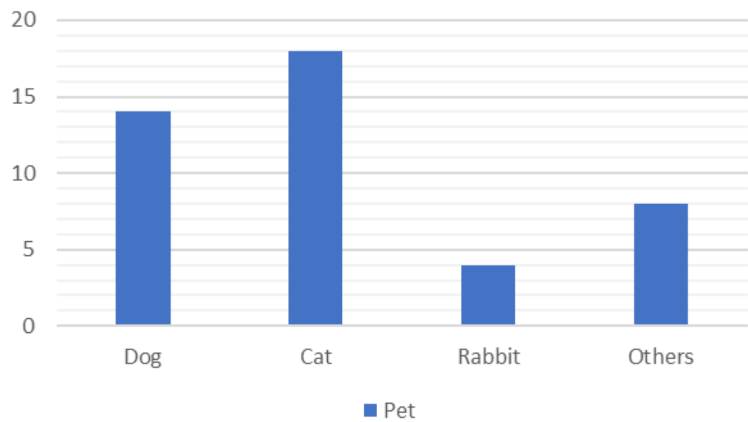
We can present the table of data using a **pictogram** shown below.



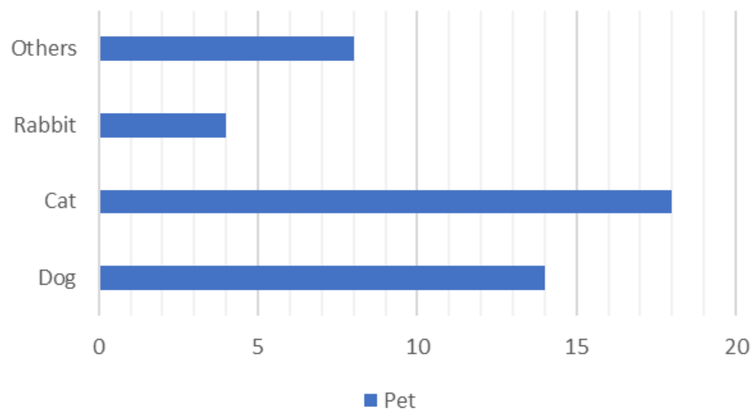
Key: Each  represents 4 animals.

3. A **pictogram** is statistical diagram that uses pictures to represent data. It gives a quick comparison of different categories. However, it is not suitable for large quantities of data.
Note: When drawing a pictogram, you must remember to include a key to explain what the individual symbol represents.

4. The information about the pets owned by 48 children can be represented by a horizontal or a vertical **bar graph**.



Vertical bar graph



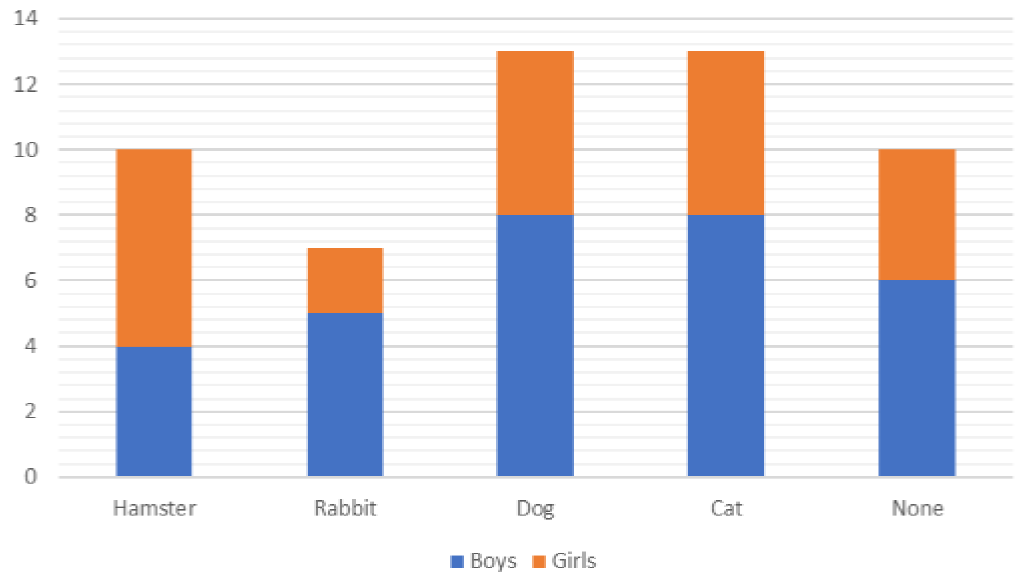
Horizontal bar graph

Observe that

a) A bar is made up of bars of the same width.
b) A bar graph has two axes. One axis shows the frequencies while the other shows the categories.
c) The height of a bar varies to represent the corresponding frequency.
d) There is a space between each bar of the same width to distinguish clearly between the categories.

Example 1

A survey is conducted on a group of children about the pets they own. The results are shown in a bar graph below.



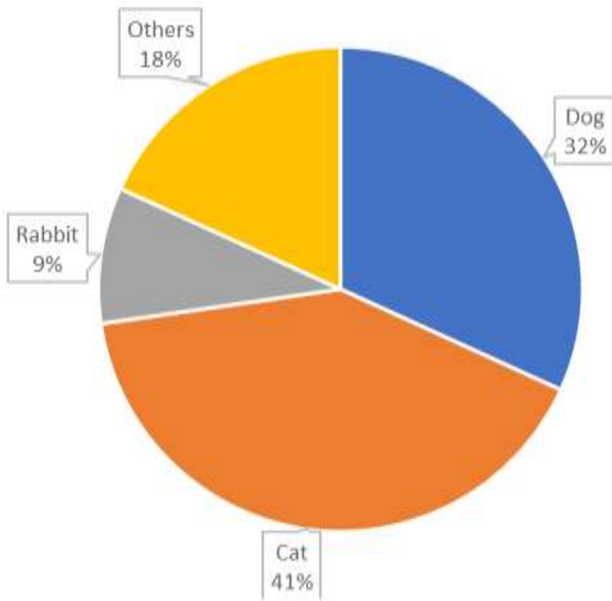
a) How many children own a cat?
b) How many boys do not have a pet?
c) Are rabbits more popular with girls or boys?
d) What is the least popular pet with boys?

Solutions

a) 13 children
b) 6 boys
c) The bar representing the rabbits owned by the girls is 2 whereas the bar representing the rabbits owned by the boys is 5. Hence, **rabbits are more popular with the boys than the girls.**
d) Hamster is the least popular pet with the boys.

5. A **pie chart** uses sectors of a circle to represent relative quantities. It compares the proportions of a whole instead of the actual numerical values. However, it is not suitable to display too many categories. Also, we need to calculate the angle size of each sector before drawing a pie chart.

6. The following pie chart shows the number of pets owned by 44 children.



Observe that
a) Each sector corresponds to the percentage of pets for that category.
b) The angle of each sector is proportional to the number of pets for that category.

7. The angle size of each sector can be calculated as follows:

- The angle of the sector representing dogs $= \frac{14}{44} \times 360° \approx 115°$
- The angle of the sector representing cats $= \frac{18}{44} \times 360° \approx 147°$
- The angle of the sector representing rabbits $= \frac{4}{44} \times 360° \approx 33°$
- The angle of the sector representing others $= \frac{8}{44} \times 360° \approx 65°$

Example 2

Arthur spends $2664 per month. The way in which he spends is shown in the table below.

Item	Spending ($)
Food	629
Housing	1110
Clothes	333
Utilities	111
Others	481

a) Draw a pie chart to show how he spends his money.
b) What percentage of his expenditure is on housing?

Solutions

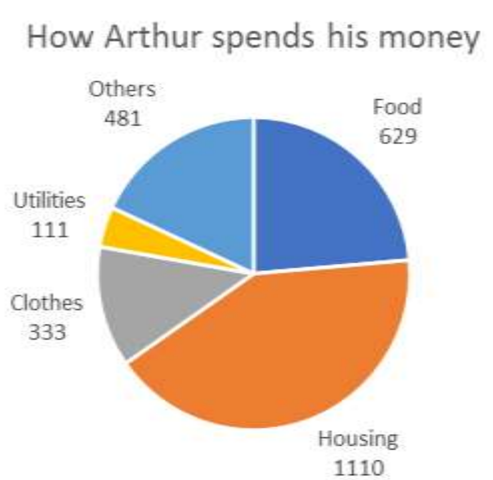
a) The angle of the sector representing food $= \frac{629}{2664} \times 360° = 85°$

The angle of the sector representing housing $= \frac{1110}{2664} \times 360° = 150°$

The angle of the sector representing clothes $= \frac{333}{2664} \times 360° = 45°$

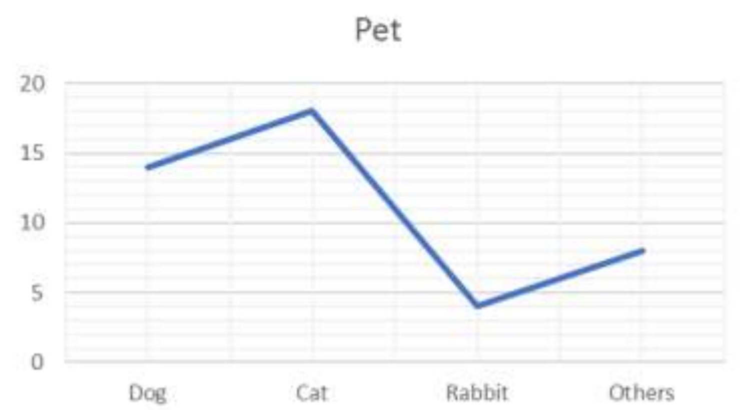
The angle of the sector representing utilities $= \frac{111}{2664} \times 360° = 15°$

The angle of the sector representing others $= \frac{481}{2664} \times 360° = 65°$

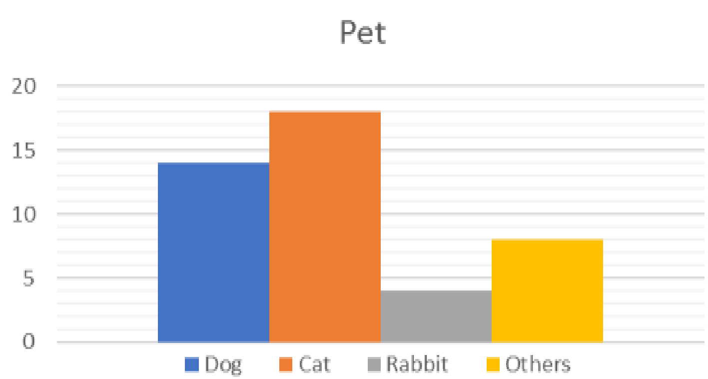


b) Percentage of his expenditure on housings = $\frac{1110}{2664} \times 100\% = 41\frac{2}{3}\%$
$\approx 41.7\%$ (1dp)

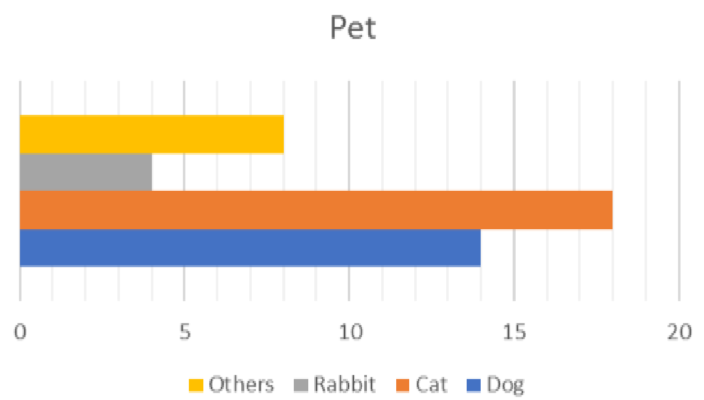
8. A **line graph** is drawn by plotting the frequencies at various points and then joining the points with line segments. This is particularly useful when we wish to show trends over time.



9. A **histogram** is a vertical bar graph with no space in between the bars. The area of each bar is proportional to the frequency it represents.



Vertical Histogram



Horizontal Histogram

In Summary

A **bar graph** is used to **compare quantities.**

A **pie chart** is used to **compare proportions.**

A **line chart** is used to **show trends.**

A **histogram** is similar to bar graph except it has **no gaps between the bars** and **the area is proportional to the frequency.**

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