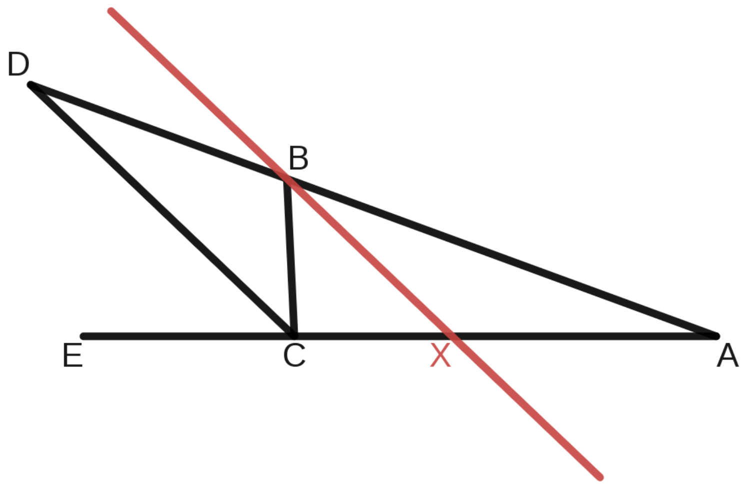
The Question

The *exterior angle bisector theorem* states that if $\frac{DB}{DA} = \frac{CB}{CA}$ holds for a point D on AB produced, then DC bisects $\angle ECB$, where E lies on AC produced. By constructing a suitable line parallel to DC or otherwise, prove this theorem.

Solution



By mid-point theorem, since $DC \parallel BX$, $\frac{DB}{DA} = \frac{CX}{CA}$. It is given that $\frac{DB}{DA} = \frac{CB}{CA}$. Hence, $\frac{CX}{CA} = \frac{DB}{DA} = \frac{CB}{CA} \iff CX = CB$. Since $\triangle CXB$ is isosceles, $\angle CXB = \angle BXC$. Let them be θ. By angular sum of $\triangle CXB$, $\angle BCX = \pi - 2\theta$. By corresponding angles, since $DC \parallel BX$, $\angle DCE = \theta$. By adjacent angles on a straight line, $\angle DCB = \pi - \theta - (\pi - 2\theta) = \theta = \angle DCE$. $\square$