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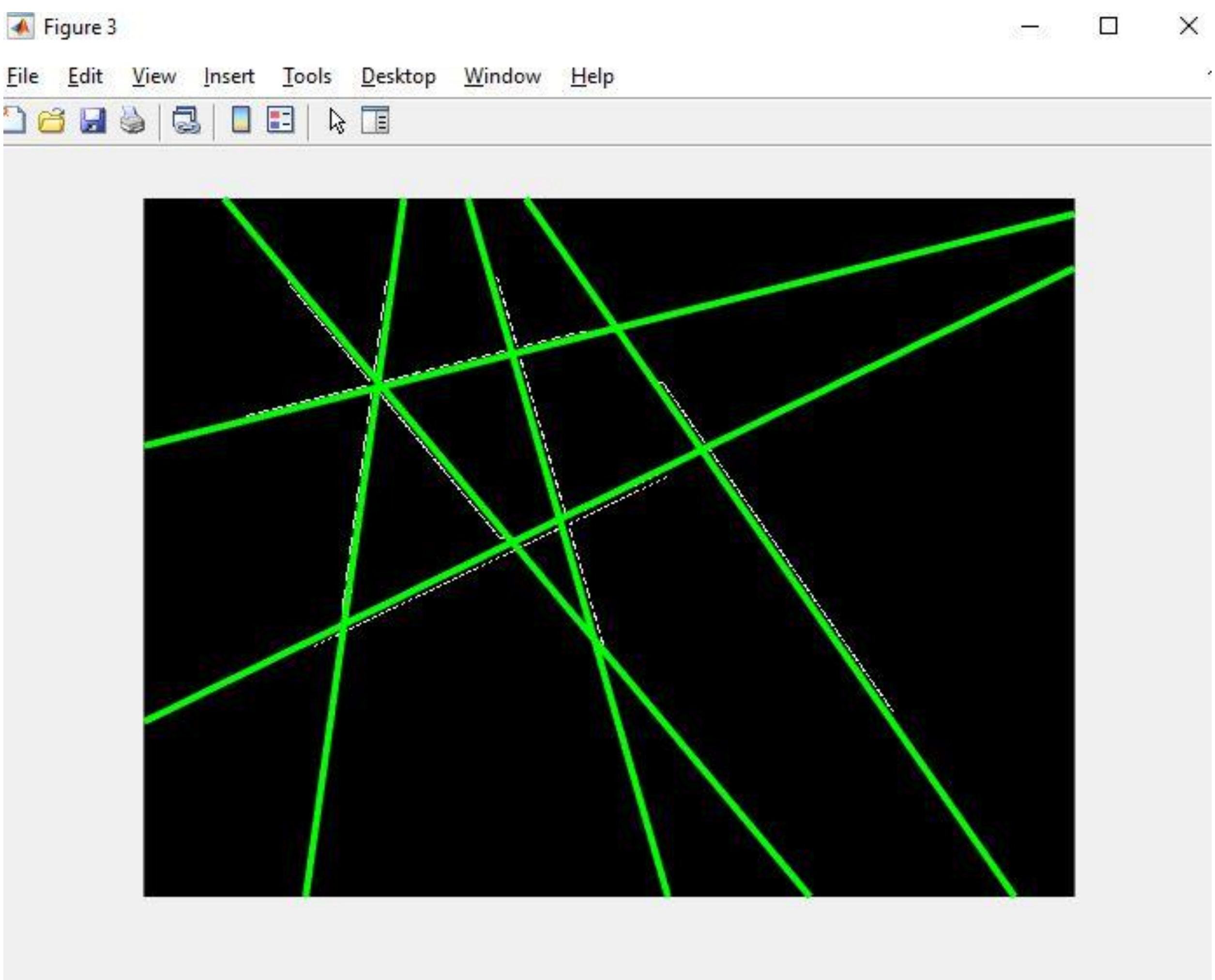
CSE 6367

Assignment 2

Solution 1:



Solution 2:

Solution 3:

Let R be n-dimensional rotation matrix.

We know that rotation can be represented as orthogonal matrices.

Therefore, for R acting on R^n,

1. $R^T = R^{-1}$ …Rotation is an orthogonal matrix.
2. $\det(RR^T) = \det(In)$ …Taking determinants of both sides.
3. $\det(R)\det(R^T) = 1$ …From (3)
4. $\det(R)^2 = 1$ …As $\det(R) = \det(R^{-1})$
5. $(\det(R))^2 - 1 = 0$
6. $(\det(R) + 1)(\det(R) - 1) = 0$
7. $\det(R) + 1 = 0$ or $\det(R) - 1 = 0$
8. $\det(R) = 1$ or $\det(R) = -1$

Solution 4:

(a) Let R1 and R2 be two rotation matrices on the plane. Prove or disprove the following: R1R2 = R2R1.

Let R1 = [0, -1;
1, 0];

R2 = [-1, 0;
0, -1];

Multiplying them, we get:

R1R2 = [0, 1;
-1, 0];

R2R1 = [0, 1;
-1, 0];

Hence proved that R1R2 = R2R1.

Explanation: Matrices commute if they preserve each other's ***eigenspaces***.
In a plane, no matter what, the eigenvectors of a rotation matrix are [i,1] and [−i,1]. So, since all such matrices have the same eigenvectors, they will always commute.

(b) Let R1 and R2 be two rotation matrices in 3D space. Prove or disprove the following: R1R2=R2R1.

Let R1 = [0, -1, 0;
1, 0, 0;
0, 0, 1];

R2 = [0, 0, 1;
1, 0, 1;
-1, 0, 0];

Multiplying them, we get:

R1R2 = [0, -1, 0;
0, 0, 1;
-1, 0, 0];

R2R1 = [0, 0, 1;
1, 0, 0;
0, 1, 0];

Hence, disproved that R1R2 ≠ R2R1.

Explanation: In *three* dimensions, there's always one real eigenvalue for a rotation matrix, so that eigenvalue has a real eigenvector associated with it: the axis of rotation.
This eigenvector doesn't share values with the rest of the eigenvectors for the rotation matrix. So, the axis is an eigenspace of dimension 1, so **rotations with different axes can't share eigenvectors**, so they cannot commute.