Contents

Chapter 1

Overview of Nonlinear dynamics

1.1 Overview of Non-Linear Dynamics

As time flows, all the things in nature change, which states the general principal of nature that evolution is the key for growth, and evolution is the function of time which changes their physical state, which constitute dynamics. Therefore, dynamics deals with the change in bodies due to interplay of force in action. Evolution of dynamics has come a long way starting from common engineering and Newtonian dynamics which showed us that object in motion stays in motion even though accelerating force is removed, which explained us the motion of objects in space. Fundamentally, we have inertial motion that is acted upon by a force of attraction between their masses reaching across the intervening space. The dynamics is on the principle that inertia is inherent with matter. The basic concept of dynamics are inertial motion and the idea that a force can act at a distance across space. Newton combined these two ideas and turned orbital motions into perpetual falls. Depending upon whether the force acting on object is linear or non-linear there classifies two dynamic systems, linear dynamical system and non-linear dynamical system respectively.

The form of dynamical system is essential to determines the form of force acted upon the object. Here we are concentrating upon the non-linear dynamical system

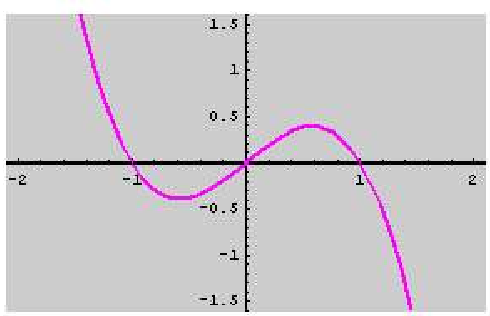


Figure 1.1: An-harmonic Oscillator-Force V(x) Vs displacement(x)

Figure 1.1 and Figure 1.2 depicts the form of the force F(x) and the corre-

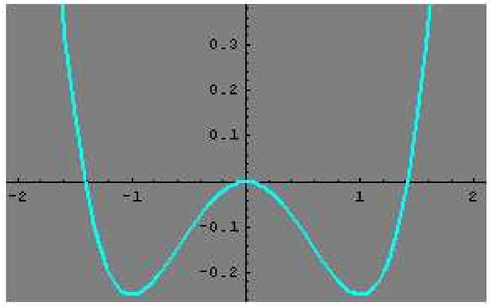


Figure 1.2: An-harmonic Oscillator-Force V(x) Vs displacement(x)

sponding potential function V (x) = 1 2 kx2 + 4 x 4 respectively for k = 1 and =1. Thus we can find that weather the equation set which describes the dynamics of motion is non-linear depending upon the force acting upon the it is linear or non-linear in nature . The point is that the properties of non-linear differential equation drastically differ that of linear differential equation hence exhibit difference in its characteristics and physical form. For nonlinear system of equation there are no such systematic analytic tools which are available hence there is very little that we know about the non-linear dynamical system though recently some unique general phenomena understanding these system have been discovered.

1.2 Chaos and Soliton

Chaos

Chaos theory, the field of study in mathematics and physics used to describe the behaviour of dynamical systems which are very sensitive to its initial state of the system i.e. small perturbation in initial condition yields significantly varying behaviour, also referred to as butterfly effect. This implies that long term prediction is impossible even the system is deterministic, also known as deterministic chaos. The dynamical system evolves its state with time that may exhibit dynamics. Some dynamical systems exhibit chaotic behaviour everywhere but in most cases the chaos exists only in a subset space, since a set of initial conditions may lead to same chaotic region e.g. a chaotic behaviour may take place on an attractor. An attractor is the state in dynamical system which a system attains after certain long period of time. Chaos can be controlled by using some stabilization algorithm based on small perturbation in some unstable orbits.

Chaotic optical communication systems have been very attractive field of research from last couple of decades. In this type of communication, the user message signal is transmitted using chaotic carrier signal and retrieved at receiver upon synchronization with transmitter. Chaos based communication is very hard to intercept, since it is very sensitive to the initial condition.

Chaos in optical fibre is generated by laser instability while using different schemes in semiconductor and fibre lasers. Optical chaos is observed in many cases of non-linear optical fibre system.

Soliton

Soliton is a nonlinear wave which has the following two properties:

(1) A localized wave propagates without change of its properties (shape, velocity etc.),

(2) Localized waves are stable against mutual collisions and retain their identities.

In optics the term soliton is used to refer to any optical field that does not change during propagation because of a delicate balance between nonlinear and linear effects in the medium.

There are two main kinds of solitons:

Spatial solitons: the nonlinear effect can balance the diffraction. The electromagnetic field can change the refractive index of the medium while propagating, thus creating a structure similar to a graded-index fibre. If the field is also a propagating mode of the guide it has created, then it will remain confined and it will propagate without changing its shape.

Temporal solitons: if the electromagnetic field is already spatially confined, it is possible to send pulses that will not change their shape because the non-linear effects will balance the dispersion. Those solitons were discovered first and they are often simply referred as ”solitons” in optics.

A solitary wave is a traveling wave consisting of a single peak or trough that propagates in isolation without change in size, shape or speed. The most canonical PDE that is used to describe solitary waves in shallow water is the Kortewegde Vries (KdV) equation,

$$\mathrm{U}_t + 6_{uux} + u_{xxx} = 0.$$

The term ut describes the time evolution of the wave propagating in one direction and the nonlinear term uux describes the steepening or narrowing of the wave. The last term uxxx is called the dispersion term and it describes the spreading of the wave. The equation was first formulated by Boussinesq in 1875 and was later rediscovered by Korteweg and de Vries in 1895. In addition to describing solitary waves, the KdV equation also describes ion-acoustic waves in plasma.

In this section we show the graphical representation to the solved non-linear PDE by Solomon Manukure and Timesha Booker for localized solitary wave solution (i.e., rapidly decreasing towards 0 as mod(x)—infinity) of the KdV equation which are commonly referred to as 1-soliton solutions of the KdV equation.

The 1-soliton and 2-soliton solutions of the KdV equation Fig1.3 depicts the 1 soliton solution, whereas Fig 1.4, Fig 1.5 Fig 1.6, Fig 1.7, Fig 1.8, Fig 1.9 illustrates the 2-soliton solution with two solitary waves undergoing a particle-like collision with each other.

In the 2-soliton interaction, the taller wave catches up to the shorter wave and eventually overtakes it, showing that taller waves travel faster than shorter waves. Thus, the speed of the solitary wave is dependent on the amplitude (height); an observation first made by John Scott Russell.

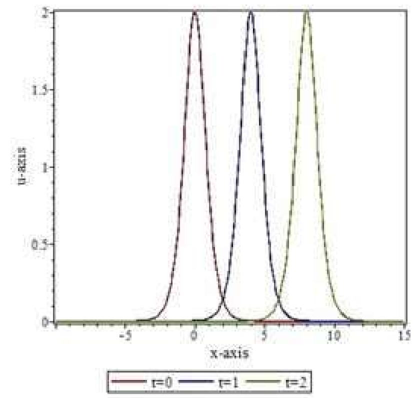


Figure 1.3: 1-soliton for different values of t

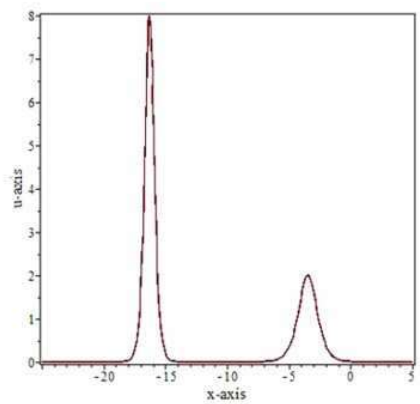


Figure 1.4: 2-soliton for t = -1

1.3 History of Soliton

Solitons or solitary waves date back to the early 1800s. They were first observed by John Scott Russell, a Scottish civil engineer and naval architect in the nineteenth century. After his initial discovery which he named the great wave of translation, he conducted extensive water tank experiments to find certain important properties of the solitary wave. Russell claimed that this solitary wave was of fundamental importance, but many scientists disagreed. Leading scientists such as George Airy and George G. Stokes attempted to give a theoretical explanation of the solitary wave, but were not successful. Airy argued wrongly

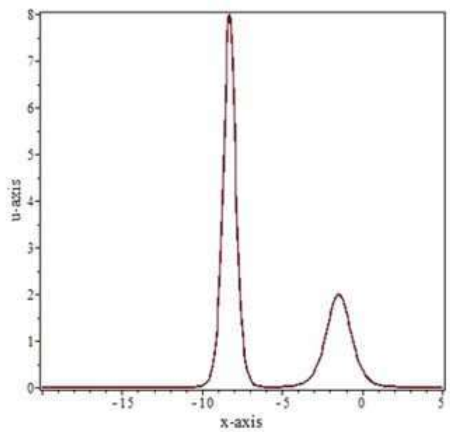


Figure 1.5: 2-soliton for t = -0.5

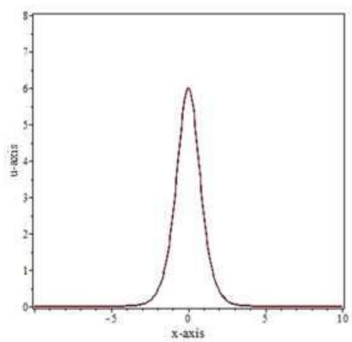


Figure 1.6: 2-soliton for t = 0.

that a solitary wave was a consequence of his linear shallow water theory while Stokes doubted the permanent form of solitary waves. The first theoretical description of the solitary wave was given by Boussinesq in 1871 and Rayleigh in 1876. However, the controversy surrounding solitary waves remained until the work of Korteweg and de Vries on the famous KdV equation in 1895.

The full significance of the solitary wave was at last established in 1965 by Zabusky and Kruskal who numerically analysed the KdV equation and observed a particle-like behaviour. In particular, they observed that solitary waves retained their shape and speed after collision. As a result, they coined the word solitons to describe solitary waves. The concept of soliton has been applied in

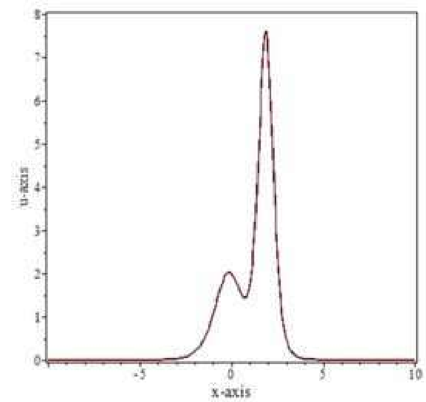


Figure 1.7: 2-soliton for t = 0.1

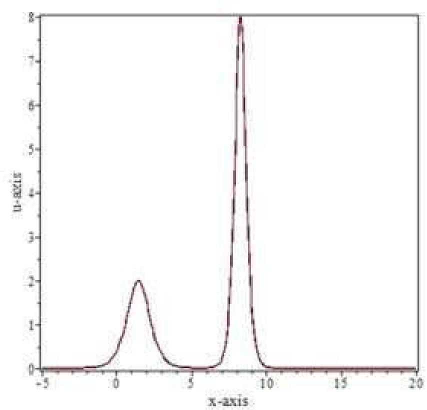


Figure 1.8: 2-soliton for t = 0.5

several areas in the applied sciences. Many dynamical systems that are known to be integrable have been found to possess soliton solutions through what is called the inverse scattering transform (IST) which, in some sense, is a non-linear analog of the Fourier transform for linear partial differential equations. The IST was developed by Gardner, Greene, Kruskal and Miura and is arguably considered the most important discovery in the field of mathematical physics in the 20th century. Solitons arise as solutions to certain nonlinear partial differential equations.

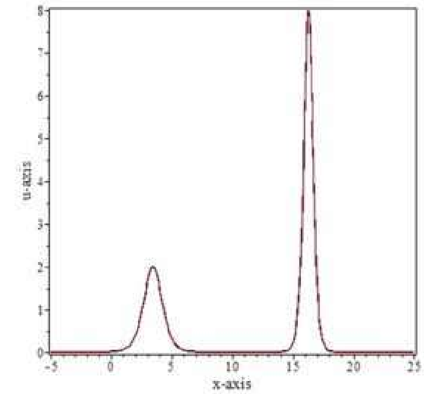


Figure 1.9: 2-soliton for t = 1

1.4 Classification of Soliton

As discussed before there exist two main forms of solitons

1.) Spatial Solitons

1974- Ashkin Bjorkholm

By this spatial soliton the non-linear effect can balance the diffraction.

Basically, spatial soliton is based on principle of kerr effect which introduces a self-phase modulation that changes the refractive index according to intensity.

Kerr effect is change in refractive index of material accordance with applied electric field The phase change can be expressed as product of phase constant and width of path the field covered

In optics the electric field is due to light itself and this is the reason for variation of refractive index which is proportional to local irradiance of light.

Change in refractive index is responsible for non-linear optical effects.

Kerr effect is observed in multi-mode fibre optics

2.) Temporal Solitons

These limits transmission bit rate in optical fibres is group velocity dispersion History of temporal soliton

In 1973 Akira Hasegama and Fred Tappert of ATT Bell Labs gave first suggestions that soliton exist in optical fibres due to the balance between self-phase modulation and anomalous dispersion

In 1973 Robin Bullogh made first mathematical report of soliton existence in optics

1.5 Role of soliton in optical fibre

Solitons have many applications in pure and applied mathematics, particularly in areas such as differential equations, Lie groups, Lie algebras, differential and algebraic geometry. Until the discovery of the inverse scattering transform (IST) for finding multi-soliton solutions, many nonlinear PDEs could not be solved analytically. However, the advent of solitons and the IST has led to the discovery or formulation of many other methods for solving nonlinear PDEs. Notable among them is the so-called Hirota bilinear method which has been widely used to find multi-soliton solutions and other exact solutions to nonlinear PDEs. The existence of soliton solutions to a nonlinear PDE can also lead to certain exact solutions. For example, it is known that lumps, which are rationally localized solutions, can be obtained from n-soliton solutions by computing long wave limits.

Equations that admit soliton solutions have profound mathematical structures and properties. One of the key properties of these equations is that they have an infinite number of conservation laws and associated symmetries which are strongly related to their integrability. Another property is the existence of a Hamiltonian or a bi-Hamiltonian structure which allows one to describe and analyse a system without having to solve the associated equations explicitly. The study of solitons and Hamiltonian theory has revolutionized research in mathematical physics leading to the emergence of new concepts and theories in many fields such as classical mechanics, quantum mechanics and Lie theory.

In fields such as fluid dynamics, plasma, nonlinear optics, astrophysics and molecular biology, soliton theory has been exploited to study many important practical problems. For example, in fibre optics, the concept of solitons has been remarkably employed in the transmission of digital signals over long distances. Apart from applications in communication, solitons also find applications in optical switches. Today, optical solitons remain one of the most active research topics in soliton theory due to its considerable potential applications in information and communications technology.

In biology, soliton theory has been applied to describe signal and energy propagation in bio-membranes, the nervous system and low frequency collective motion in proteins and DNA. Solitons also feature prominently in the study of plasmas, which consist of a large number of charged particles. For example, Dusty plasmas, which contain small charged dust particles, have been modelled using nonlinear oscillator chains that admit several types of solitary-wave solutions.

Aside from the KdV equation, solitons exist in many other (1+1)-dimensional nonlinear PDEs such as the Boussinesq equation, the nonlinear Schrdinger equation, the sineGordon equation and the CamassaHolm equation. There are also higher dimensional equations that possess soliton solutions. Notable examples include the KadomtsevPetviashvili equation, the HirotaSatsumaIto equation and the DaveyStewartson equation. Recently, many integrable equations have been modified or generalized to form new equations which are mostly nonintegrable, but have copious applications in nonlinear dynamics.

Chapter 2

Review of Fiber-Optics

2.1 Review of optical fibre.

Optical fibre is the technology which is associated with data transmission using light pulse in a close glass fibre using the basic principle of total internal reflection. The light pulse travelling in a long fibre is usually made of smooth glass or fibre. The most important reason to use a optical fibre in an Optical Fibre Communication System [OFCs] is that using these technique the signal travelling in optical fibre does not gets affected due to any hindrance such as electromagnetic interference or any static noise. The complete application of optical fibre is depended upon the principle of total internal reflection. The fibres are designed such that they facilitate the propagation of light along with the optical fibre depending on the requirement of power and distance of transmission. The general visualization of optical fibre is as shown in Fig:2.1

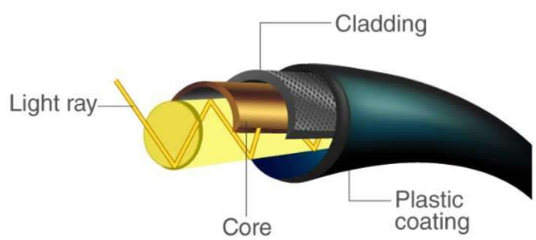


Figure 2.1: General visualization of Optical Fibre

The first instance of glass being drawn into the fibres dates back to 8th century but not until 1000 years later in 18th century the pair of French brothers named Chappe invented the first optical telegraph which was a tower outfitted with series of lights which was used as a communication system by operator to relay signal back and forth, after this the development in optical fibre and communication using optic fibre has sky rocketed in the field of communication.

In the 1840s, physicists Daniel Collodon and Jacques Babinet showed that light could be directed along jets of water for fountain displays. In 1854, John Tyndall, a British physicist, demonstrated that light could travel through a curved stream of water thereby proving that a light signal could be bent. He proved this by setting up a tank of water with a pipe that ran out of one side.

As water flowed from the pipe, he shone a light into the tank into the stream of water. As the water fell, an arc of light followed the water down.

In year 1880 almost 100 years later the French Chappe brothers, Alexander Graham Bell researched on the communication based on optical fibre communication which led to an invention which he patented as his first optical telephone which was named as photophone. Though his earlier invention of telephone was more successful commercially as it was more realistic back in the time.

71 years later in year 1951, Holger Moeller made an astounding invention and applied for a Danish patent on fibre-optic imaging in which he proposed cladding glass or plastic fibres with a transparent low-index material, but was denied because of Baird and Hansells patents. Three years later, Abraham Van Heel and Harold H. Hopkins presented imaging bundles in the British journal Nature at separate times. Van Heel later produced a cladded fibre system that greatly reduced signal interference and crosstalk between fibres.

Also in 1954, the maser was developed by Charles Townes and his colleagues at Columbia University. Maser stands for microwave amplification by stimulated emission of radiation.

The laser was introduced in 1958 as a efficient source of light. The concept was introduced by Charles Townes and Arthur Schawlow to show that masers could be made to operate in optical and infrared regions. Basically, light is reflected back and forth in an energized medium to generate amplified light as opposed to excited molecules of gas amplified to generate radio waves, as is the case with the maser. Laser stands for light amplification by stimulated emission of radiation.

In 1960, the first continuously operating helium-neon gas laser is invented and tested. That same year an operable laser was invented which used a synthetic pink ruby crystal as the medium and produced a pulse of light.

In 1961, Elias Snitzer of American Optical published a theoretical description of single mode fibres whose core would be so small it could carry light with only one wave-guide mode. Snitzer was able to demonstrate a laser directed through a thin glass fibre which was sufficient for medical applications, but for communication applications the light loss became too great.

Charles Kao and George Hockham, of Standard Communications Laboratories in England, published a paper in 1964 demonstrating, theoretically, that light loss in existing glass fibres could be decreased dramatically by removing impurities. In 1970, the goal of making single mode fibres with attenuation less then 20dB/km was reached by scientists at Corning Glass Works. This was achieved through doping silica glass with titanium. Also in 1970, Morton Panish and Izuo Hayashi of Bell Laboratories, along with a group from the Ioffe Physical Institute in Leningrad, demonstrated a semiconductor diode laser capable of emitting continuous waves at room temperature.

In 1973, Bell Laboratories developed a modified chemical vapor deposition process that heats chemical vapours and oxygen to form ultra-transparent glass that can be mass-produced into low-loss optical fibre. This process still remains the standard for fibre-optic cable manufacturing.

The first non-experimental fibre-optic link was installed by the Dorset (UK) police in 1975. Two years later, the first live telephone traffic through fibre optics occurs in Long Beach, California.

In the late 1970s and early 1980s, telephone companies began to use fibres extensively to rebuild their communications infrastructure. Sprint was founded on the first nationwide, 100 percent digital, fibre-optic network in the mid-1980s.

The erbium-doped fibre amplifier, which reduced the cost of long-distance fibre systems by eliminating the need for optical-electrical-optical repeaters, was invented in 1986 by David Payne of the University of Southampton and Emmanuel Desurvire at Bell Laboratories. Based on Desurvires optimized laser amplification technology, the first transatlantic telephone cable went into operation in 1988.

In 1991, Desurvire and Payne demonstrated optical amplifiers that were built into the fibre-optic cable itself. The all-optic system could carry 100 times more information than cable with electronic amplifiers. Also in 1991, photonic crystal fibre was developed. This fibre guides light by means of diffraction from a periodic structure rather than total internal reflection which allows power to be carried more efficiently then with conventional fibres therefore improving performance.

The first all-optic fibre cable, TPC-5, that uses optical amplifiers was laid across the Pacific Ocean in 1996. The following year the Fibre Optic Link Around the Globe (FLAG) became the longest single-cable network in the world and provided the infrastructure for the next generation of Internet application.

Today, a variety of industries including the medical, military, telecommunication, industrial, data storage, networking, and broadcast industries are able to apply and use fibre optic technology in a variety of applications.

2.2 Classification of Optical Fibre

The classification of optical fibre is done based on following

1.) Material used.

2.) Refractive index.

3.) Mode of propagation.

The classification based on Material used is: -

a.) Plastic Optical Fibres: - In this the core of optical fibre is made up of plastic i.e., polymethylmethacrylate

b.) Glass Fibres: - In this the core of optical fibre is made of extremely thin and fine glass fibres.

The classification based on refractive index is: -

a.) Step index fibres: - In this the core is surrounded by the cladding, which has a single and uniform index of refraction throughout the core.

b.) Graded index fibres: - In this the refractive index of the optical fibre decreases as the radial distance of the fibre axis increases.

The classification based on the mode of propagation is: -

a.) Single-mode fibre: - In Single-mode optical fibre there passes only one signal at a time thus this fibre is used for long distance data transmission for communication.

b.) Multi-mode fibre: - In Multi-mode optical fibre there passes multiple signals at a time through the core thus used for the short distance data transmission in communication.

The mode of propagation and refractive index in the core of optical fibre for communication is used to form four combination type of optic fibres such as: -

1.) Step index ingle mode fibre

2.) Graded index single mode fibre

3.) Step index multi-mode fibre

4.) Graded index multi-mode fibre

2.3 Non-Linear Wave in Optical Fibre

Fibre nonlinearities

The response of any dielectric material to the light becomes nonlinear for intense electromagnetic fields. Fundamentally, the origin of nonlinear response is related to the anharmonic motion of bound electrons under influence of an applied field.

Induced nonlinear polarization:

P=0[(1). E+ (2):EE+ (3) EEE+]

Here 0 is the vacuum permittivity and (j) (j = 1, 2,) is jth order susceptibility.

The linear susceptibility represents the dominant contribution to P. The second-order susceptibility is nonzero only for media that lack inversion symmetry at the molecular level. Since SiO2 is a symmetric molecule, it vanishes and optical fibres do not exhibit second-order nonlinear effects.

The lowest order nonlinear effects in optical fibre originate from third-order susceptibility, which is responsible for phenomena such as third harmonic generation, four-wave mixing (FWM) and nonlinear refraction.

Origin of Nonlinear Effects

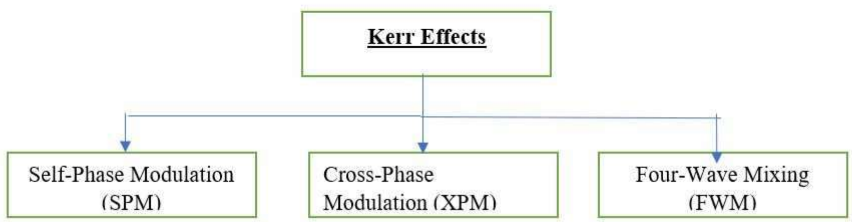
Fibre nonlinearities arise from the two basic mechanisms:

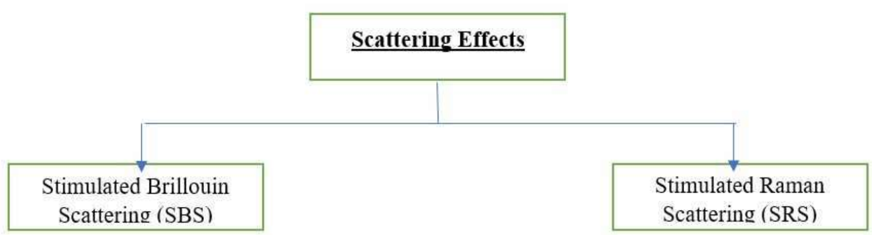
The first mechanism, most of the nonlinear effects in optical fibres originate from nonlinear refraction, a phenomenon that refers to the intensity dependence of refractive index of silica resulting from the contribution of third-order susceptibility. This mechanism gives rise to Kerr Effects.

The second mechanism for generating nonlinearities in fibre is the scattering phenomena.

These mechanisms give rise to SBS and SRS. These are inelastic scattering processes where frequency of scattered light is changed.

Classification of Non-linear Effects





Why fibre nonlinearity is important in order to design light wave system?

Nonlinearity may arise for the following reasons:

High transmitting power with small cross-section of fibre core

For long transmission distance

Many channels to enhance system capacity

2.4 Mathematical Model in Optical Fibre

A mathematical model is a description of system using mathematical concept and language.

The procedure of developing a mathematical model is termed as mathematical modelling

These are use to solve various situation problems in daily life, physics, etc.

Elements of mathematical model

Mathematical model can take many forms this includes dynamical system, differential equations and game theoretic models.

In physical sciences it contains most of the following elements: -

1. Governing equations: - It displays how the value of unknown or dependent variables change. When one or more known (independent) variables change.

2. Assumptions and constraints

Initial and Boundary conditions: - In dynamic systems a boundary value problem is differential equations together with set of additional constraints called boundary conditions

3. Mathematical models are of different types

a. Linear vs Non-linear: - If all operators in mathematical model provide linearity the resulting mathematical model is defined as linear otherwise non-linear

b. Static vs Dynamic: - A dynamic model represents for a time independent changes in state of system while static model calculate system in equilibrium.

Dynamic model is represented in differential equations.

4. Supplementary sub models: -

a. Defining equations: - In physics defining equations are equations that define new quantity in the terms of base quantities.

b. Constitutive equations: - In physics constitutive equations is relation between two physical quantities especially kinetic quantities related to kinematics. This are the combined with other equations governing the physical laws to physical problem e.g.: - fluid mechanics, etc

Mathematical Models in Design

Mathematical models are essential part for simulation and design control system. The use of mathematical model is the simplified representation of reality into mimic the relevant features of system being analysed it makes the real-life satiations or problems in mathematical explanations to a believable situation.

Why Mathematical models are important?

It can help people to understand and explore the meaning of equations or functional relationship

Mathematical modeling is the conversion of problems from an application zone into manageable mathematical formulations with a hypothetical and arithmetical analysis that provides perception, answers, and guidance useful for the creating application. Mathematical modeling is valuable in various applications; it gives precision and strategy for problem solution and enables a systematic understanding of the system modeled. It also allows better design, control of a system, and the efficient use of modern computing capabilities.

Knowing the ins and outs of mathematical modelling is a crucial step from theoretical mathematical training to application-oriented mathematical expertise; it also helps the students master the challenges of our modern technological culture.

Chapter 3

Modulation Instability of Optical nonlinear wave propagation in double core optical fiber

3.1 Introduction

The communication system is widely used in todays modern fast-growing world, so the requirement of high bandwidth and data communication too, major point is there should be minimum interference and noise production while the transmission and reception during data communication.

Basically the communication system can transmit the information from one place to another which is separated by greater distance thus here optical fibre communication comes in hand. This communication for large distance is covered by optical fibre which carries high frequency 100THz in infrared region. The optical fibre communication uses single mode as well as multi-mode communication to transmit information.

There are advantages of transmission through optical fibre over electrical wave or microwave transmission system. Among other systems it is smaller, light-weighted, capable to carry high bandwidth and less signal degradation occurs in OFCs. OFCs are used as they have no effect of electric and magnetic field interferences on them. The tool we are using is a double core optical fibre (directional coupler) this functions as optical switching, wavelength division multiplexing device and also as power divider.

Double core optical fibre (DFC) is a modern instrument used widely in OFCs. Basically, the DFC is an optical fibre which consists two identical optical fibre with a bimodal structure and it supports both symmetric and asymmetric transmission modes. In DFC when the power is launched in one core it is transferred into another core as in both modes the group velocity is different hence the input pulse gets broaden and splits after propagating certain wave distance. This distortion effect of pulse is called as intermodal dispersion (IMD). Intermodal dispersion causes distorted pulse hence pulse splitting occurs. In double core optical fibre power transfer mechanics is characterised by the coupling coefficient here the coupling coefficient is depended on the wavelength and as the wavelength varies it leads to IMD, hence IMD limits the bandwidth of Double core optical fibre. Furthermore, during one research study by Tan et it was showed that DFC is effective only in high power continuous wave (CW) fibre laser construction, also by one more research done by the research group of A Hidayat, A Listanti, E Latifah, H Wisodo, Nugroho A P and A Taufiq it was also showed that pulse splitting and pulse distortion due to intermodal dispersion could be avoided by increasing the Kerr nonlinearity of the fibre. If we induce a non-linearity in a fibre it will balance the effect of dispersion, so the non-linearity in an optical fibre can be increased by increasing the power. Thus, the power of the input pulse to balance out both the effects must be several times of one soliton power and hence the intermodal dispersion coefficient(R1) to balance out the pulse splitting and pulse distortion must be very less i.e.

$$R1 << 1$$

, and thus the coupling coefficient(R) must also be very strong i.e.

$$R >> 1$$

.

However, the further investigation on the effect of intermodal dispersion and nonlinearity by the coupling coefficient to minimize the pulse splitting and pulse distortion it is still unclear whether it is possible if the Double core optical fibre could be used in propagating multiple waves from a single core without inducing pulse splitting and pulse distortion is yet to done.

3.2 Theoretical model of equation

$$\frac{\partial A_1}{\partial z} + \frac{1}{vg}\frac{\partial A_1}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A_1}{\partial t^2} = ikA_2 + i\gamma(|A_1|^2 + \sigma|A_2|^2)A_1 \quad (3.1)$$

$$\frac{\partial A_2}{\partial z} + \frac{1}{vg}\frac{\partial A_2}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A_2}{\partial t^2} = ikA_1 + i\gamma(|A_2|^2 + \sigma|A_1|^2)A_2 \quad (3.2)$$

$$A_1 = (A_o + \delta A)e^{i(kz-\omega t)} \tag{3.3}$$

$$A_2 = (B_o + \delta B)e^{i(kz-\omega t)} \tag{3.4}$$

$$2vg[ik(A_o + \delta A) + \delta A_z] + 2[-i\omega(A_o + \delta A) + \delta A_t] + vgi\beta_2[\omega^2(-A_o + \delta A_t) + \delta A_{tt}] = ik(B_o + \delta B) + i\gamma[(A_o^3 + 2A_o^2\delta A + A_o^2\delta A^*) + \sigma(A_oB_o^2 + A_oB_o\delta B^* + A_oB_o\delta B + B_o^2\delta A)] \tag{3.5}$$

$$2vg[ik(B_o + \delta B) + \delta B_z] + 2[-i\omega(B_o + \delta B) + \delta B_t] + vgi\beta_2[\omega^2(-B_o + \delta B_t) + \delta B_{tt}] = ik(A_o + \delta A) + i\gamma[(B_o^3 + 2B_o^2\delta B + B_o^2\delta B^*) + \sigma(B_oA_o^2 + B_oA_o\delta A^* + B_oA_o\delta A + A_o^2\delta B)] \tag{3.6}$$

Pertarbel Terms

$$2vg[ik\delta A + \delta A_z] + 2[-i\omega(\delta A) + \delta A_t] + vgi\beta_2[\omega^2(\delta A_t) + \delta A_{tt}] = ik\delta B + i\gamma 2A_o^2\delta A + i\gamma A_o^2\delta A^* + i\gamma\sigma A_oB_o\delta B^* + i\gamma\sigma A_oB_o\delta B + i\gamma\sigma B_o^2\delta A \tag{3.7}$$

$$2vg[ik\delta B + \delta B_z] + 2[-i\omega(\delta B) + \delta B_t] + vgi\beta_2[\omega^2(\delta B_t) + \delta B_{tt}] = ik\delta A + i\gamma 2B_o^2\delta B + i\gamma B_o^2\delta B^* + i\gamma\sigma B_oA_o\delta A^* + i\gamma\sigma B_oA_o\delta A + i\gamma\sigma A_o^2\delta B \tag{3.8}$$

Consider

$$\delta A = A_1e^{(i(Qz-\Omega t)} + A_2^*e^{(-i(Qz-\Omega^* t))} \tag{3.9}$$

$$\delta B = B_1 e^{(i(Qz-\Omega t)} + B_2^* e^{(-i(Qz-\Omega^* t))} \tag{3.10}$$

Substitute δA and δB in Pertarbel Term Equation

Positive Terms of first Pertarbel equation is

$$\begin{aligned} 2vg[ikA_1 + A_1(iQ)] + 2[-i\omega A_1 + A_1(-i\Omega)] + vgi\beta_2[\omega^2 A_1(-i\Omega) - A_1\Omega^2] &= \\ ikB_1 + i\gamma 2A_o^2 A_1 + i\gamma A_o^2 A_2 + i\gamma\sigma A_o B_o B_2 + i\gamma\sigma A_o B_o B_1 + i\gamma\sigma B_o^2 A_1 & \end{aligned} \tag{3.11}$$

Negative Terms of first Pertarbel equation is

$$\begin{aligned} 2vg[ikA_2^* + A_2^*(-iQ)] + 2[-i\omega A_2^* + A_2^*(i\Omega^*)] + vgi\beta_2[\omega^2 A_2^*(i\Omega^*) + A_2^*\Omega^{*2}] &= \\ ikB_2^* + i\gamma 2A_o^2 A_2^* + i\gamma A_o^2 A_1^* + i\gamma\sigma A_o B_o B_1^* + i\gamma\sigma A_o B_o B_2^* + i\gamma\sigma B_o^2 A_2^* & \end{aligned} \tag{3.12}$$

Positive Terms of second Pertarbel equation is

$$\begin{aligned} 2vg[ikB_1 + B_1(iQ)] + 2[-i\omega B_1 + B_1(-i\Omega)] + vgi\beta_2[\omega^2 B_1(-i\Omega) - B_1\Omega^2] &= \\ ikA_1 + i\gamma 2B_o^2 B_1 + i\gamma B_o^2 B_2 + i\gamma\sigma B_o A_o A_2 + i\gamma\sigma B_o A_o A_1 + i\gamma\sigma A_o^2 B_1 & \end{aligned} \tag{3.13}$$

Negative Terms of second Pertarbel equation is

$$\begin{aligned} 2vg[ikB_2^* + B_2^*(-iQ)] + 2[-i\omega B_2^* + B_2^*(i\Omega^*)] + vgi\beta_2[\omega^2 B_2^*(i\Omega^*) + B_2^*\Omega^{*2}] &= \\ ikA_2^* + i\gamma 2B_o^2 B_2^* + i\gamma B_o^2 B_1^* + i\gamma\sigma B_o A_o A_1^* + i\gamma\sigma B_o A_o A_2^* + i\gamma\sigma A_o^2 B_2^* & \end{aligned} \tag{3.14}$$

Transferring RHS terms to LHS for positive 1^{st} Pertarbel Equation

$$A_1(2vgik + 2vg(iQ) - 2i\omega - 2i\Omega + vg\beta_2\omega^2\Omega - vgi\beta_2\Omega^2 - i\gamma 2A_o^2 - i\gamma\sigma B_o^2) - A_2 i\gamma A_o^2 - B_1(ik + i\gamma\sigma A_o B_o) - B_2 i\gamma\sigma A_o B_o B_2 \quad (3.15)$$

Transferring RHS terms to LHS for negative 1^{st} Pertarbel Equation

$$-i\gamma A_o^2 A_1^* + (2vgik - 2vgiQ - 2i\omega + 2i\Omega^* - vg\beta_2\omega^2(\Omega^*) + vgi\beta_2\Omega^{*2} - i\gamma 2A_o^2 - i\gamma\sigma B_o^2)A_2^* - i\gamma\sigma A_o B_o B_1^* - (ik + i\gamma\sigma A_o B_o)B_2^* \quad (3.16)$$

Transferring RHS terms to LHS for positive 2^{nd} Pertarbel Equation

$$(-ik - i\gamma\sigma B_o A_o)A_1 - i\gamma\sigma B_o A_o A_2 + (2vgik + 2vgiQ - 2i\omega - 2i\Omega + vg\beta_2\omega^2\Omega - vgi\beta_2\Omega^2 - i\gamma 2B_o^2 - i\gamma\sigma A_o^2)B1 - i\gamma\sigma B_o^2 B_2 \quad (3.17)$$

Transferring RHS terms to LHS for negative 2^{nd} Pertarbel Equation

$$-i\gamma\sigma B_o A_o A_1^* + (-ik - i\gamma\sigma B_o A_o)A_2^* - i\gamma B_o^2 B_1^* + (2vgik - 2vgiQ - 2i\omega + 2i\Omega^* - vg\beta_2\omega^2\Omega^* + vgi\beta_2\Omega^2 - i\gamma 2B_o^2 + i\gamma\sigma A_o^2)B_2^* \quad (3.18)$$

3.3 Modeling and simulations

$A_0 = B_0 = 0.001$

Gamma 0.1 / 0.5 / 1.5 / 2.5 / 3.5 / 4.5 / 8.5

$\sigma = 1.0$

V=1.1

$\beta 2 = 1.0$

Equation () expresses the stability and instability of optical soliton in fibre.

The stability of optical soliton in fibre is determined by system parameters $(\gamma, k, \Omega, \sigma v)$

therefore in this work we have analysed the role of XPM on stability of optical wave in fibre

Hence we have plotted the equation with the choice of parameters $\gamma, k, \Omega, \sigma, v$

In this figure we have plotted graph between petrabel wave number vs unpertable wave number

And one axis represented the petrable frequency.

In this plot blue colour represents the stable part against petrable wave number Q and red colour represents the unstable part which describes the growth of the wave

In fig 1a we have fixed gamma as 0.,5 and observed that two peaks of modes which are at the centre of the graph. The modes are representing the unstable wave the remaining part describes the stable part of the wave.

The maximum amplitude of mode has 1400 for this case. Then the gamma has increased from 1.5,2.5,3.5 and observed the evolution of modes in optical fibre as depicted in fig 1b, 1c, 1d So in this fig we observed that the unstable mode is increasing as well as the amplitude is increasing.

Further we increased gamma to 8.5 the and due to this the amp mode of has decreased to 1200.

Therefore, in this graphical illustration we observed that the XPM is affecting the evolution of optical soliton as well as stability of soliton in optical fibre

3.4 Result and Discussion

We studied the modulation instability in Cross-Phase-Modulation and observed that amplitude instability is depending upon the change in gamma value to which we conclude amplitude stability decreases with respect to gamma when gamma increases and vice-versa. For this problem we considered the model of non-linear Schrodinger equation with inclusion of Cross-Phase Modulation to obtain the desired modulation instability criterion.

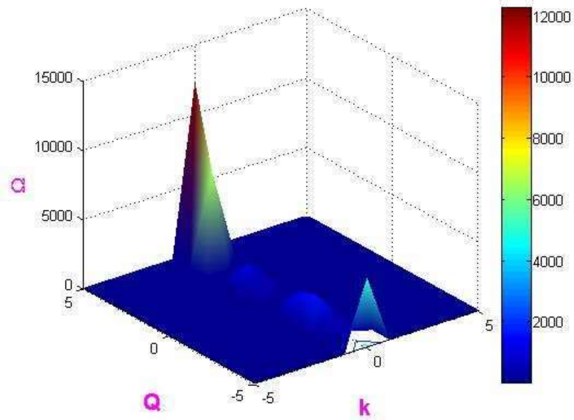


Figure 3.1: 1a figure $\gamma = 0.5$

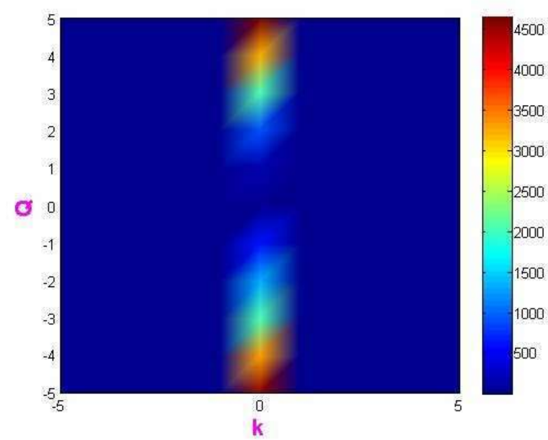


Figure 3.2: 1a figure $\gamma = 0.5[TopView]$

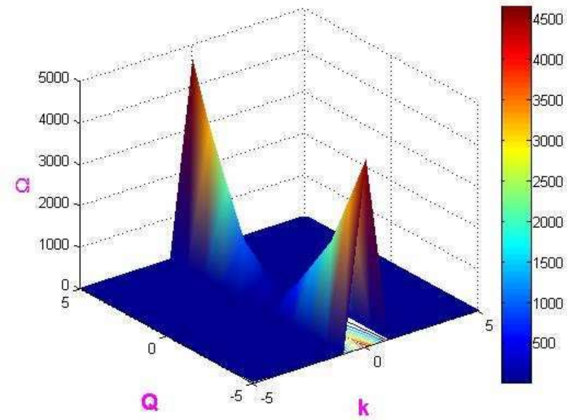


Figure 3.3: 1b figure $\gamma = 1.0$

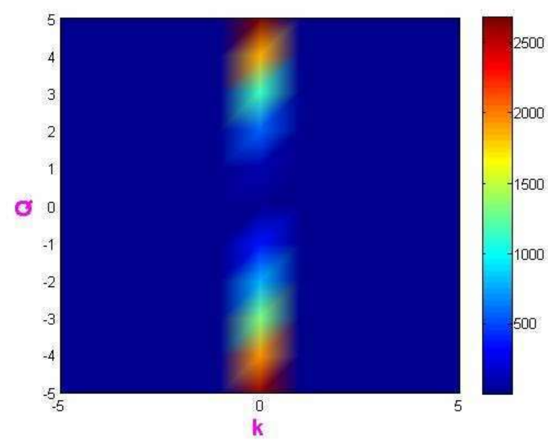


Figure 3.4: 1b figure $\gamma = 1.0[Topview]$

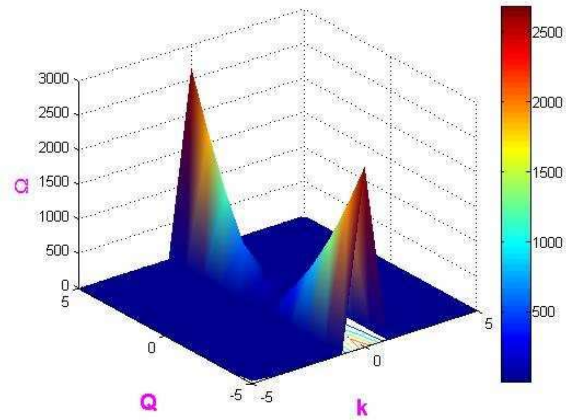


Figure 3.5: 1c figure $\gamma = 1.5$

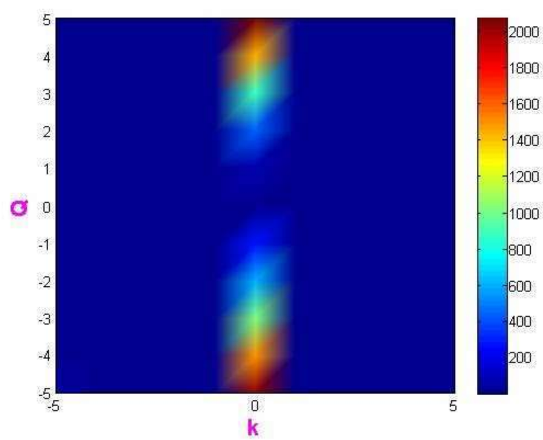


Figure 3.6: 1c figure $\gamma = 1.5[TopView]$

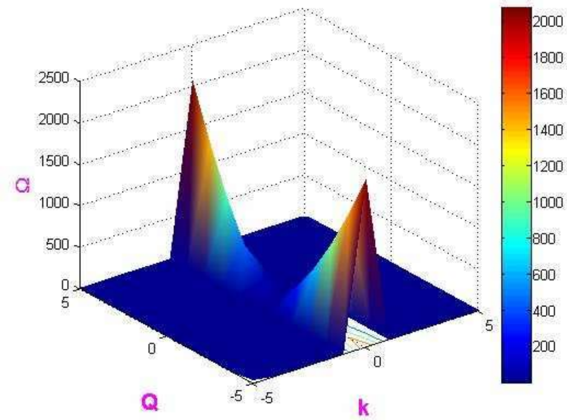


Figure 3.7: 1d figure $\gamma = 2.5$

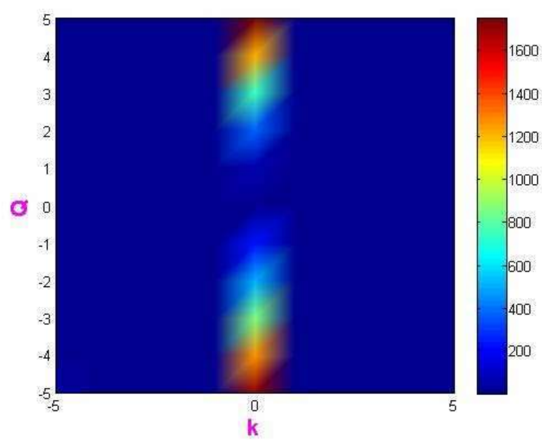


Figure 3.8: 1d figure $\gamma = 2.5[TopView]$

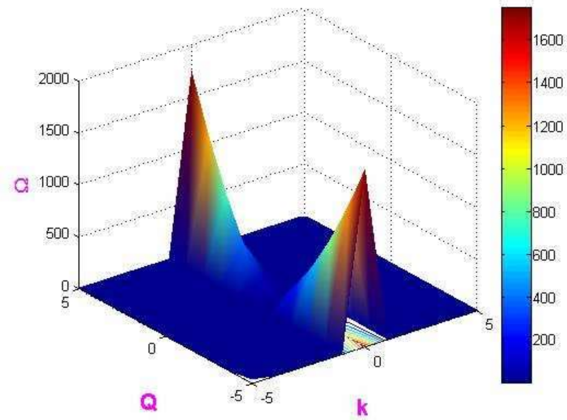


Figure 3.9: 1e figure $\gamma = 3.5$

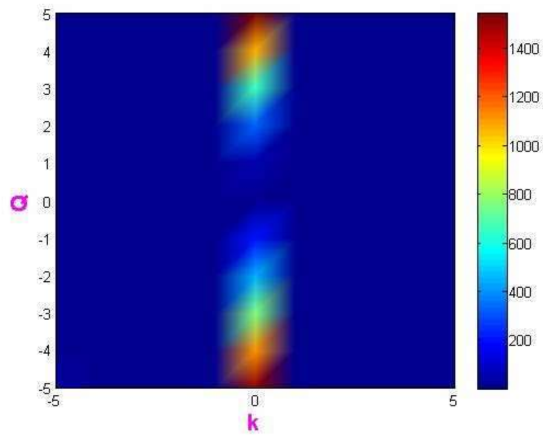


Figure 3.10: 1e figure $\gamma = 3.5[TopView]$

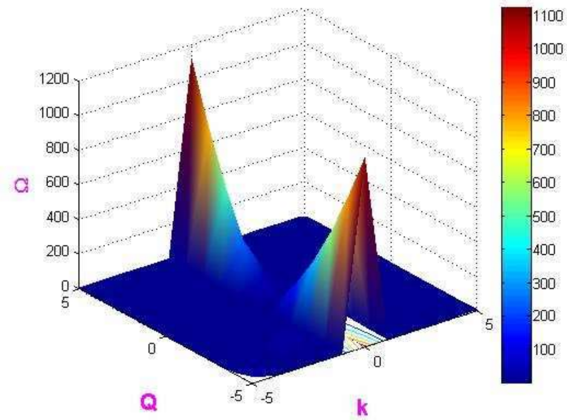


Figure 3.11: 1f figure $\gamma = 8.5$

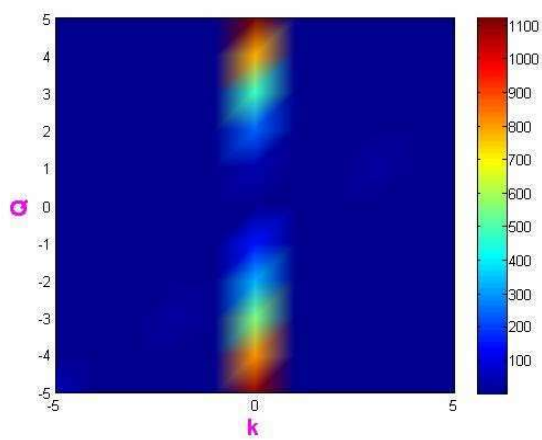


Figure 3.12: 1f figure $\gamma = 8.5[TopView]$

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