Homework 5 (計算方法設計，Design and Analysis of Algorithms)

註: 請在截止日期以前透過 eeclass 線上繳交作業，請注意不接受遲交。Please submit your assignment online through eeclass before the due date. Note that late submissions will not be accepted.

Due date: June 4, 2025

1. (25%) Let S be a (not necessarily sorted) sequence of n different integers. An integer x in S is called as *approximate median* of S if $|\{y \in S: y < x\}| \geq \frac{n}{4}$ and $|\{y \in S: y > x\}| \geq \frac{n}{4}$. Design an $O(n)$ time algorithm to find all the approximate medians of S.

2. (25%) For the following profit matrix, find an optimal allocation of resources to maximize the total profit for these four projects and four resources by using the dynamic programming method.

Project \ Resource	**1**	**2**	**3**	**4**
1	3	7	10	12
2	1	2	6	9
3	2	4	8	9
4	4	2	7	10

3. (25%) Find an optimal binary tree for $a_1, a_2, \cdots, a_6$ by using the dynamic programming method, if the identifiers $a_1, a_2, \cdots, a_6$ have probabilities 0.3, 0.2, 0.05, 0.2, 0.1, 0.15 respectively and all other identifiers have zero probability.

4. (25%) Suppose that there are n jobs, labeled as $1,2, \ldots, n$, in which each job i has a start time s_i, a finish time f_i and a positive weight w_i, where $1 \leq i \leq n$. Moreover, two jobs i and j are said to be compatible if their time intervals $[s_i, f_i]$ and $[s_j, f_j]$ do not overlap (i.e., $f_i \leq s_j$ or $f_j \leq s_i$). Please use the dynamic programming technique to design a polynomial-time algorithm for finding a maximum-weight subset of mutually compatible jobs (15%). Please also analyze the time complexity of your algorithm in big-*O* notation (10%).