

Homework 2 (計算方法設計 · Design and Analysis of Algorithms)

註: 請在截止日期以前透過 eeclass 線上繳交作業 · 請注意不接受遲交。 Please submit your assignment online through eeclass before the due date. Note that late submissions will not be accepted.

Due date: April 4, 2025

1. (25%) Given an edge-weighted connected undirected graph $G = (V, E)$, where $n = |V|$ and $m = |E|$, a maximum spanning tree of G is a spanning tree with maximum weight. Please design a greedy algorithm to find a maximum spanning tree of G (10%) and analyze its time complexity (5%). Please also prove the correctness of your greedy algorithm (10%).
2. (25%) Prove that the following property is true. There is an optimal 2-way merge tree in which the two leaf nodes with minimum sizes are assigned to be brothers and their parent is an internal node of maximum distance from the root.
3. (25%) Design an algorithm that can compute the shortest path from a source node to each other node in an arbitrary directed, edge-weighted graph $G = (V, E)$ with negative cost edges, but no negative cycles (10%). Please also prove the correctness of your algorithm (10%) and analyze its time complexity (5%).
4. (25%) Given positive integers $P_1, P_2, \dots, P_n, W_1, W_2, \dots, W_n$ and M , the *knapsack problem* is to find X_1, X_2, \dots, X_n , where $0 \leq X_i \leq 1$, such that $\sum_{i=1}^n P_i X_i$ is maximized subject to $\sum_{i=1}^n W_i X_i \leq M$. Please design a greedy algorithm to find an optimal solution to the knapsack problem (10%), analyze its time complexity (5%) and prove its correctness (10%).