Unit 4. Digital Audio

CS 3570

Shang-Hong Lai

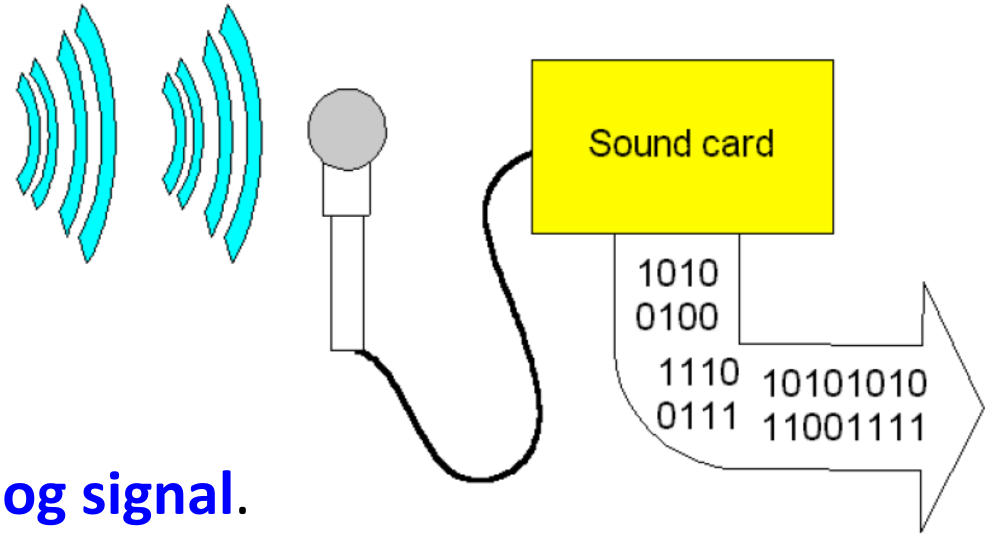
Content

- Audio signal sampling, digitization
- Nonlinear quantization
- Frequency Analysis
  - Discrete Fourier Transform(DFT)
  - Fast Fourier Transform(FFT)
- Dynamic processing
- Noise reduction
- Digital Audio Filter
  - FIR (finite-impulse response) filter
  - IIR (infinite-impulse response) filter
- Digital Audio Compression
  - MPEG-1 Audio Compression

Introduction

- Sound is a mechanical wave that is an oscillation of pressure transmitted through a solid, liquid, or gas.
- The perception of sound in any organism is limited to a certain range of frequencies (20Hz~20000Hz for humans).
- How do we process "sound"?
    - The changing air pressure caused by sound is translated into changing voltages.
    - The fluctuating pressure can be modeled as continuously changing numbers—a function where time is the input variable and amplitude (of air pressure or voltage) is the output.

Digitizing Sound



- Microphone:
  - Receives sound
  - Converts to **analog signal**.
- Computer like **discrete entities**

Need to convert Analog-to-Digital

Hardware (e.g. Soundcard)

Also known as Digital Sampling

Sample Rate and Bit Size

Bit Size — Quantization

- How do we store each sample value (Quantization)?
  - 8 Bit Value (0-255) or (-128~127)
  - 16 Bit Value (Integer) (0-65535) or (-32768~32767)

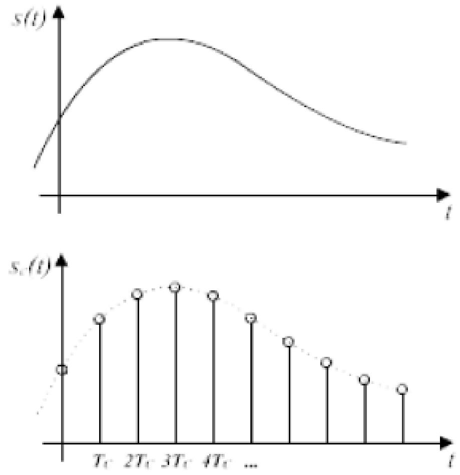
Sample Rate

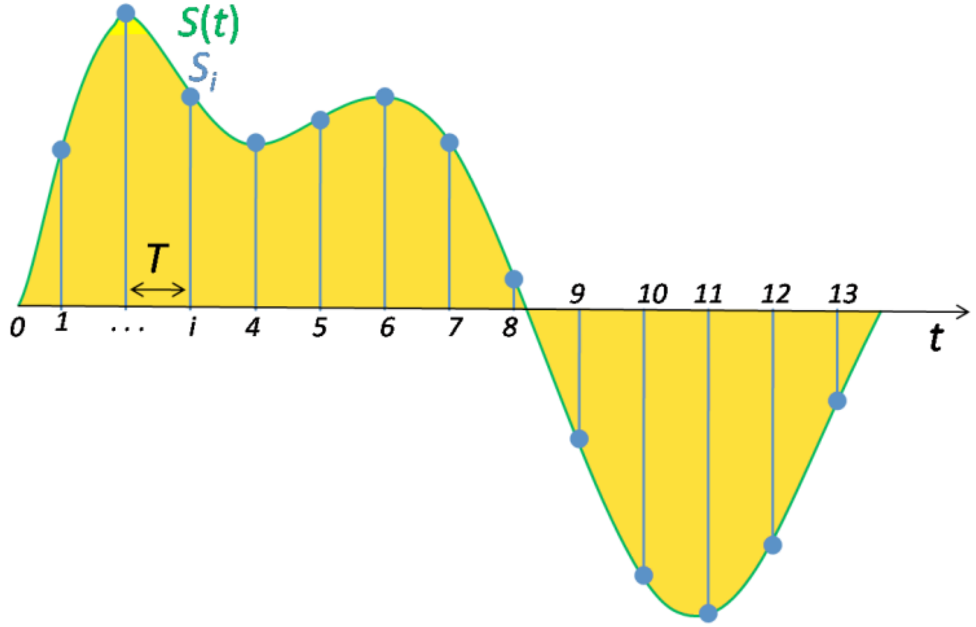
- How many Samples to take?
  - 8 KHz — Voice quality for phones and toys
  - 16 KHz — Commonly used for speech recognition
  - 44.1 KHz — CD Quality

Digital Sampling

Sampling process basically involves:

- Measuring the analog signal at regular discrete intervals
- Recording the values at these points





Nyquist Theorem

The Sampling Frequency is critical to the accurate reproduction of a digital version of an analog waveform

Nyquist's Sampling Theorem

The Sampling frequency for a signal must be **at least twice** the **highest frequency component** in the signal.

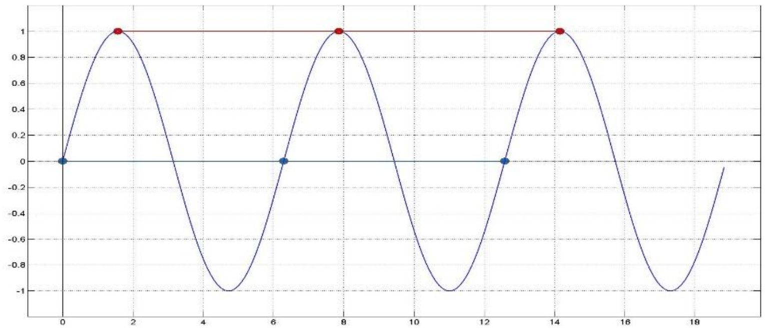
Given a sampling rate, the ***Nyquist frequency*** is the highest actual frequency component that can be sampled without aliasing.

Nyquist Theorem

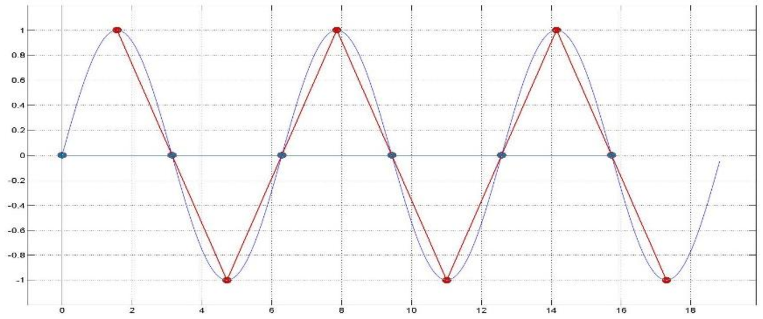
- Nyquist rate
  - Given an actual frequency to be sampled, the ***Nyquist rate*** is the lowest sampling rate that will permit accurate reconstruction of an analog digital signal.
- Ex:
  - If the highest frequency component is 10,000Hz, the Nyquist rate $f_{nr} = 2f_{max} =$ 20,000Hz

Sampling Rate vs Signal Frequency

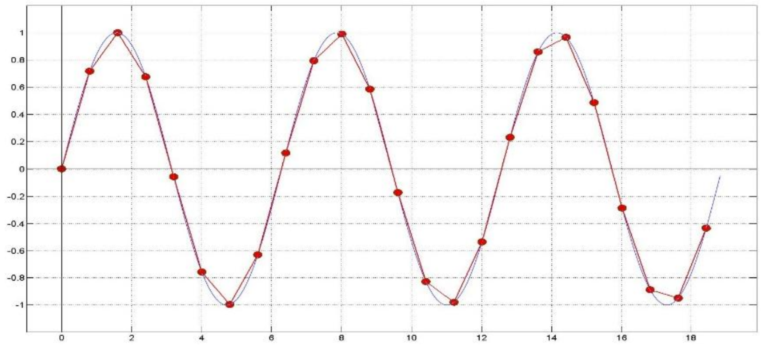
Sampling rate = Signal frequency



Sampling rate = Nyquist rate

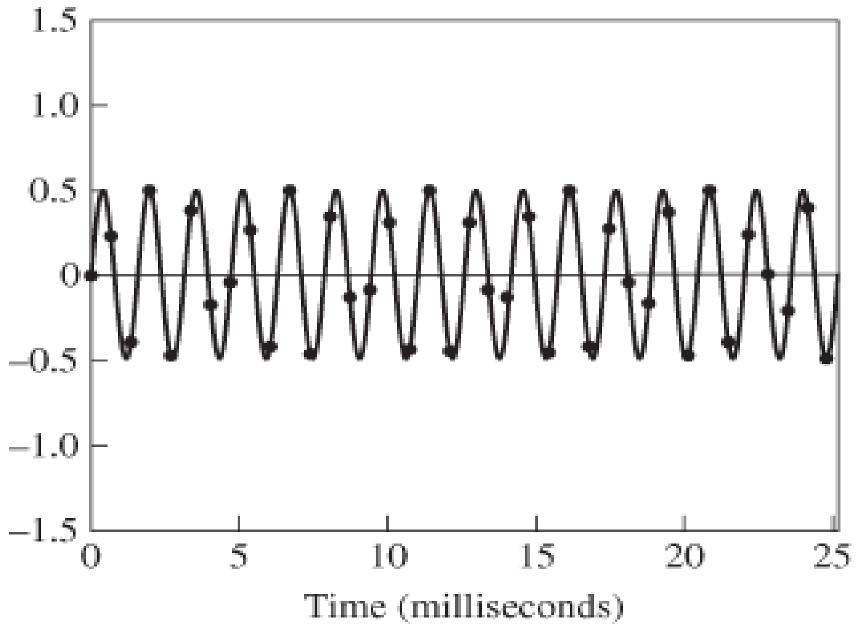


Sampling rate > Nyquist rate



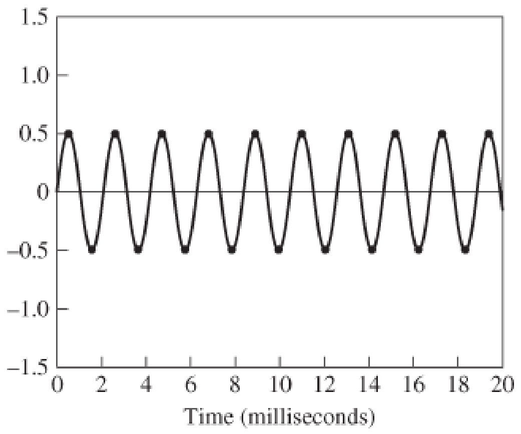
Sampling Rate and Aliasing

- In essence, the reason a too-low sampling rate results in aliasing is that there aren't enough sample points from which to accurately interpolate the sinusoidal form of the original wave.

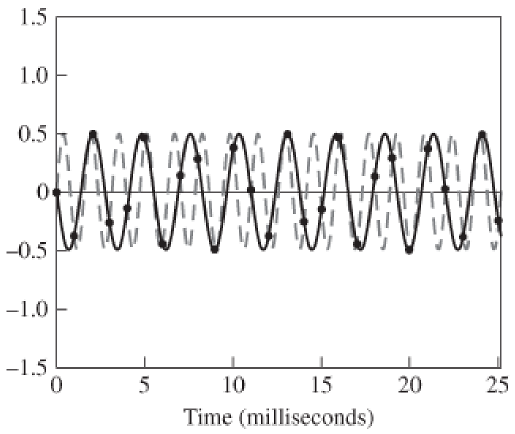


Samples taken more than twice per cycle will provide sufficient information to reproduce the wave with no aliasing

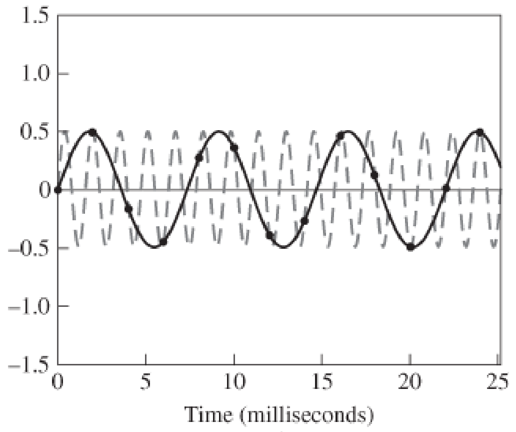
Sampling Rate and Aliasing



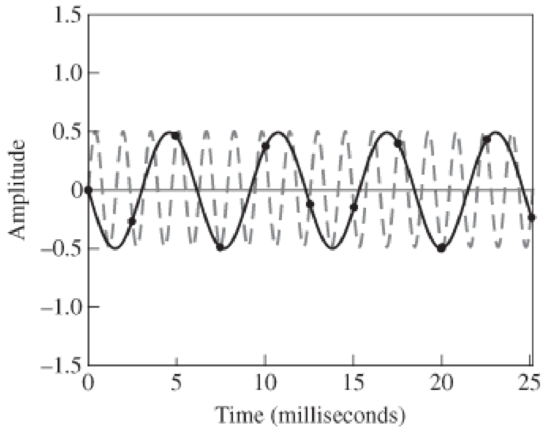
Samples taken exactly twice per cycle *can* be sufficient for digitizing the original with no aliasing



A 637 Hz wave sampled at 1000 Hz aliases to 363 Hz



A 637 Hz wave sampled at 500 Hz aliases to 137 Hz



A 637 Hz wave sampled at 400 Hz aliases to 163 Hz

Signal to Noise Ratio (SNR)

- The ratio of the power of the correct signal and the noise is called the *signal to noise ratio* (**SNR**) — a measure of the quality of the signal.
- The SNR is usually measured in decibels (**dB**), where 1 dB is a tenth of a **bel**. The SNR is defined as follows:

$$SNR = 10\log_{10}\frac{V_{signal}^2}{V_{noise}^2} = 20\log_{10}\frac{V_{signal}}{V_{noise}}$$

- (6.2)

The power in a signal is proportional to the square of the voltage. For example, if the signal voltage V_{signal} is 10 times the noise, then the SNR is

$$20 * \log_{10}(10) = 20 \text{ dB}.$$

Magnitude levels of common sounds in decibels

- The usual levels of sound we hear around us are described in terms of decibels, as a ratio to the quietest sound we are capable of hearing. Table 6.1 shows approximate levels for these sounds.

Threshold of hearing	0
Rustle of leaves	10
Very quiet room	20
Average room	40
Conversation	60
Busy street	70
Loud radio	80
Train through station	90
Riveter	100
Threshold of discomfort	120
Threshold of pain	140
Damage to ear drum	160

Signal to Quantization Noise Ratio (SQNR)

- Aside from any noise that may have been present in the original analog signal, there is also an additional error that results from quantization.
  - (a) If voltages are actually in 0 to 1 but we have only 8 bits to store values, then we force voltage values into 256 different values.
  - (b) This introduces a roundoff error. It is called **quantization noise** (or quantization error).
- The quality of the quantization is characterized by the Signal to Quantization Noise Ratio (**SQNR**).
- **Quantization noise**: the difference between the actual value of the analog signal, for the particular sampling time, and the nearest quantization interval value.
  - At most, this error can be as much as half of the interval.

SQNR

For a quantization accuracy of *N* bits per sample, the SQNR can be simply expressed:

$$SQNR = 20\log_{10}\frac{V_{signal}}{V_{quan\_noise}} = 20\log_{10}\frac{2^{N-1}}{\frac{1}{2}} \qquad (6.3)$$

$$= 20 ´\ N ´\ \log 2 = 6.02\,N\ (\mathrm{dB})$$

Notes:

(a) We map the maximum signal to $2^{N-1} - 1$ ($\simeq 2^{N-1}$) and the most negative signal to -2^{N-1}.

(b) Eq. (6.3) is the *Peak* signal-to-noise ratio, PSQNR: peak signal and peak noise.

(c) In other words, each bit adds about 6 dB of resolution, so 16 bits provide a maximum SQNR of 96 dB.

- 6.02N **is the worst case**. If the input signal is sinusoidal, the quantization error is statistically independent, and its magnitude is uniformly distributed between 0 and half of the interval, then it can be shown that the expression for the SQNR becomes:
    - SQNR = 6.02N + 1.76(dB) (6.4)

Linear and Non-linear Quantization

- **Linear format**: samples are typically stored as uniformly quantized values.
- **Non-uniform quantization**: set up more finely-spaced levels where humans hear with the most acuity.
  - Weber's Law stated formally says that equally perceived differences have values proportional to absolute levels:

$$\Delta\text{Response} \propto \Delta\text{Stimulus}/\text{Stimulus} \qquad (6.5)$$

  - Inserting a constant of proportionality k, we have a differential equation that states:

$$dr = k\,(1/s)\,ds \qquad (6.6)$$

with response r and stimulus s.

Non-linear Quantization

– Integrating, we arrive at a solution

$$r = k \ln(s) + C$$

• (6.7)

with constant of integration C.

Stated differently, the solution is

$$r = k \ln\left(s/s_0\right)$$

• (6.8)

s_0 = the lowest level of stimulus that causes a response (r = 0 when s = s_0).

Nonlinear quantization works by first transforming an analog signal from the raw s space into the theoretical r space, and then uniformly quantizing the resulting values.

Such a law for audio is called ***μ*-law** encoding, (or **u-law**). A very similar rule, called ***A*-law**, is used in telephony in Europe.

The equations for these very similar encodings are as follows:

Non-linear Quantization

- μ-law:

$$r = \frac{\operatorname{sgn}(s)}{\ln(1+\mu)} \ln\left\{1+\mu\left|\frac{s}{s_p}\right|\right\}, \qquad \left|\frac{s}{s_p}\right| \le 1 \tag{6.9}$$

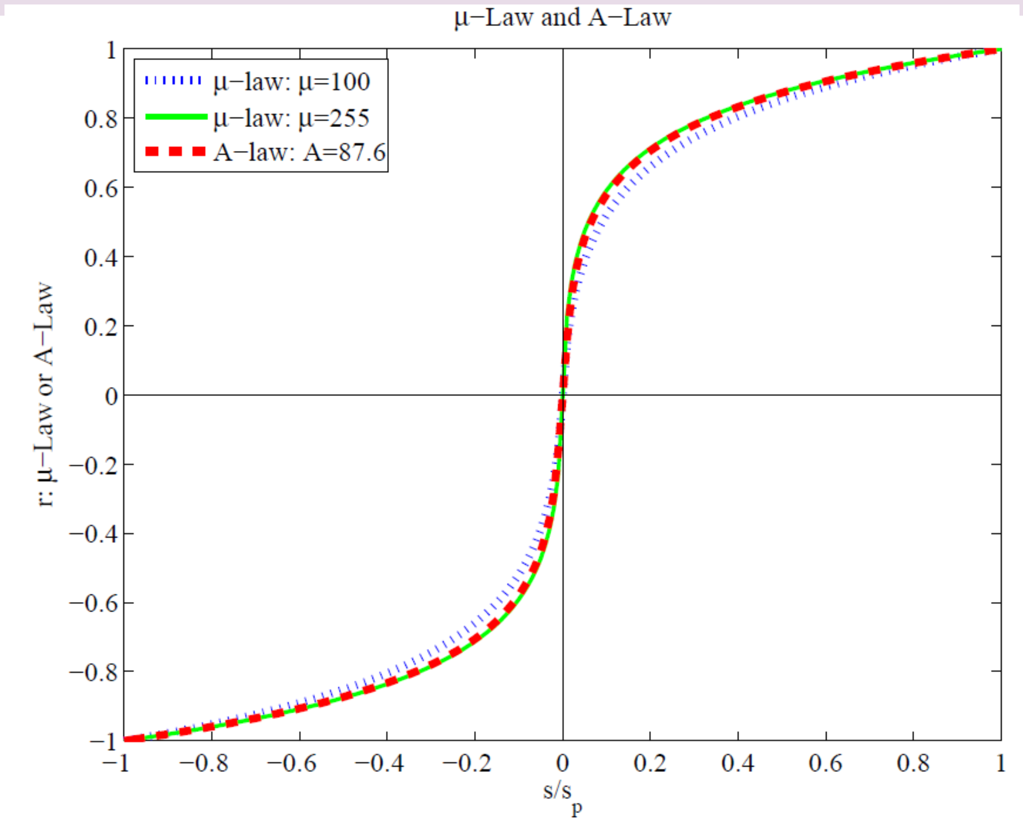
s_p is the peak signal value and s is the current signal value

- A-law:

$$r = \begin{cases} \frac{A}{1+\ln A}\left(\frac{s}{s_p}\right), & \left|\frac{s}{s_p}\right| \le \frac{1}{A} \\ \frac{\operatorname{sign}(s)}{1+\ln A}\left[1+\ln A\left|\frac{s}{s_p}\right|\right], & \frac{1}{A} \le \left|\frac{s}{s_p}\right| \le 1 \end{cases} \tag{6.10}$$

$$\text{where } \operatorname{sign}(s) = \begin{cases} 1 & \text{if } s > 0, \\ -1 & \text{otherwise} \end{cases}$$

Non-Linear Quantization



- **Fig. 6.6**: Nonlinear transform for audio signals.

- The parameter μ is set to μ = 100 or μ = 255; the parameter A for the A-law encoder is usually set to A = 87.6.
- The μ-law in audio is used to develop a nonuniform quantization rule for sound: uniform quantization of r gives finer resolution in s at the quiet end.

Bit Allocation

- In μ-law, we would like to put the available bits where the most perceptual acuity (sensitivity to small changes) is.
  1. Savings in bits can be gained by transmitting a smaller bit-depth for the signal.
  2. μ-law often starts with a bit-depth of 16 bits, but transmits using 8 bits.
  3. And then expands back to 16 bits at the receiver.

- Let $\mu = 255$.
- Now, we want s in $[-1, 1]$. The input is in -2^{15} to $(+2^{15} - 1)$, we divide by 2^{15} to normalize.
- Then the μ-law is applied to turn s into r.
- Now go down to 8-bit samples, using $\hat{r} = \text{sign}(s) * \text{floor}(128 * r)$.
- Now the 8-bit signal $\hat{r}$ is transmitted.
- At the receiver side, we normalize $\hat{r}$ by dividing by 2^7, and then apply the inverse μ-law function:

Bit Allocation

- At the receiver side, we normalize $\hat{r}$ by dividing by 2^7, and then apply the inverse μ-law function:

$$\hat{s} = \mathrm{sign}(s)\left(\frac{(\mu+1)^{|\hat{r}|} - 1}{\mu}\right)$$

- Finally, we expand back up to 16 bits:

$$\tilde{s} = ceil\left(2^{15} \times \hat{s}\right)$$

Companding puts the most accuracy at the quiet end near zero.

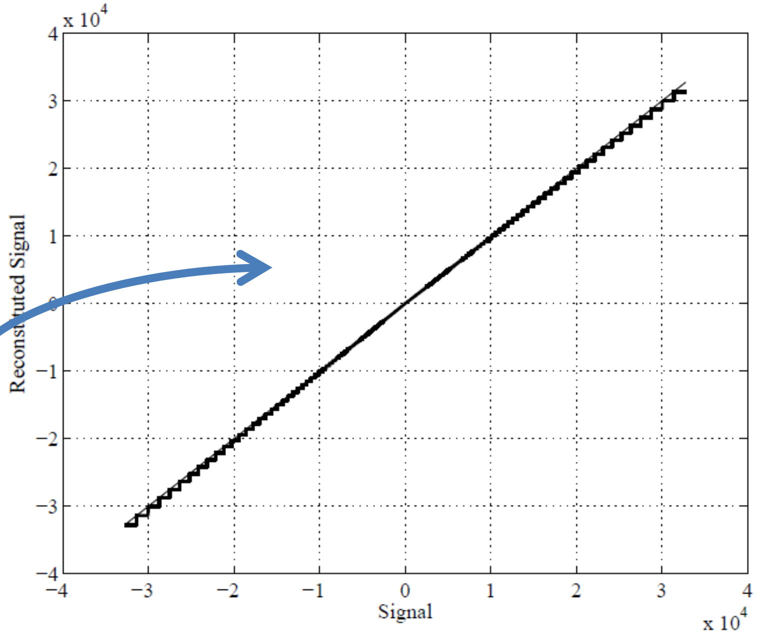
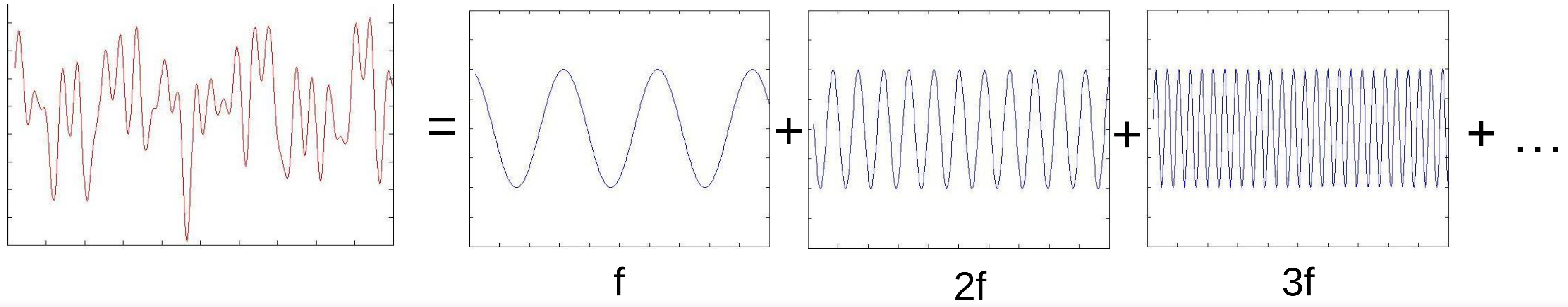


Fig. 6.7: Nonlinear quantization by companding.

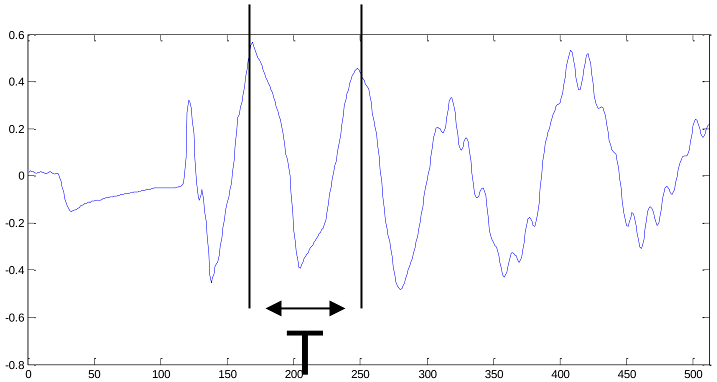
Frequency Analysis

- Time domain
  - Input: time (x-axis)
  - Output: amplitude(y-axis)
- A complex waveform is equal to an infinite sum of simple sinusoidal waves, beginning with a ***fundamental frequency*** and going through frequencies that are integer multiples of the fundamental frequency – ***harmonic frequencies***.

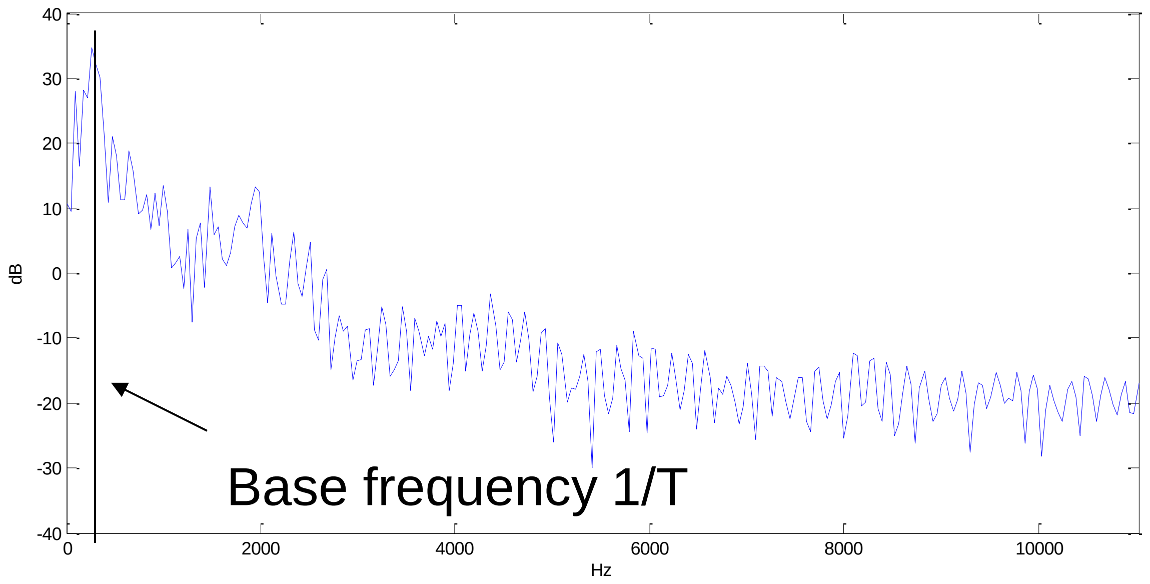


Frequency Domain Analysis

- **Fourier transform** can be used to decompose any signal into summation of sinusoidal waves.
- In Matlab, we can use **fft** (Fast Fourier Transform) for frequency domain analysis.



The time domain waveform



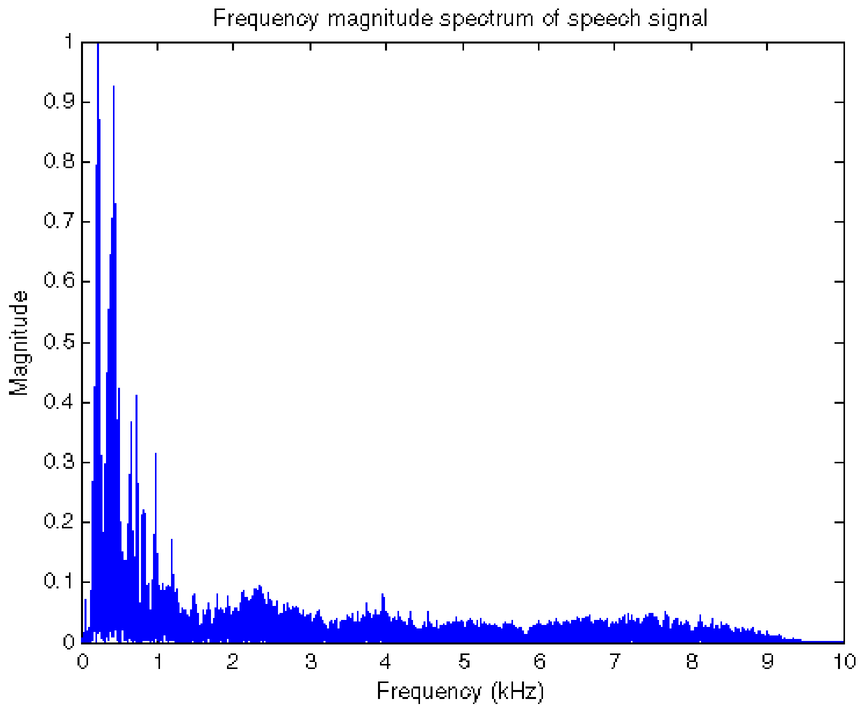
The frequency domain components.

Frequency Analysis

- Two views for frequency analysis
  - Frequency analysis view(spectrum analysis view)- common

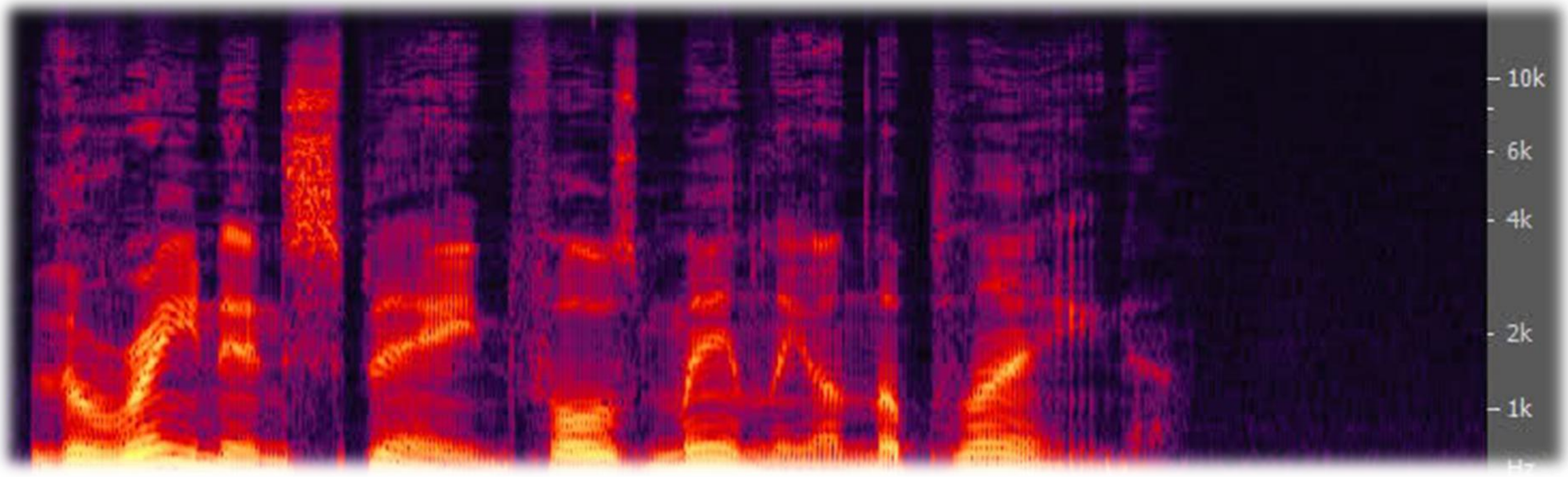
    x-axis: frequency

    y-axis: magnitude of the frequency component



Frequency Analysis

- Spectral view
  - x-axis: time, y-axis: frequency
  - color: magnitude of the frequency component
- Spectrogram



Discrete Fourier Transform

- The ***discrete Fourier transform*** (**DFT**) operates on an array of *N* audio samples, returning cosine and sine coefficients that represent the audio data in the frequency domain.
- DFT transforms a sequence of N complex numbers $\{f_k\} := f_0, f_1, \dots, f_{N-1}$ into another sequence of complex numbers, $\{F_n\} := F_0, F_1, \dots, F_{N-1}$

$$F_n = \frac{1}{N}\sum_{k=0}^{N-1} f_k \cos\left(\frac{2\pi nk}{N}\right) - i f_k \sin\left(\frac{2\pi nk}{N}\right) \qquad (4.8)$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} f_k e^{\frac{-i2\pi nk}{N}}$$

Discrete Fourier Transform

- Let f_k be a discrete integer function representing a digitized audio signal in the time domain, and F_n be a discrete, complex number function representing a digital audio signal in the frequency domain. Then the ***inverse discrete Fourier transform*** is defined by

$$\begin{aligned} \boldsymbol{f}_k &= \sum_{n=0}^{N-1} \left[\boldsymbol{a}_n \cos\left(\frac{2\pi nk}{N}\right) + i\boldsymbol{b}_n \sin\left(\frac{2\pi nk}{N}\right) \right] \\ &= \sum_{n=0}^{N-1} F_n e^{\frac{i2\pi nk}{N}} \end{aligned} \tag{4.7}$$

- Subscript k: signal value at time k

  Subscript n: nth frequency component

Discrete Fourier Transform

- The DC component, $\boldsymbol{a}_0$, is defined by $\boldsymbol{a}_0 = \frac{1}{N}\sum_{k=0}^{N-1} \boldsymbol{f}_k$, giving the average amplitude.
- The AC components are, for 1≤n≤N,

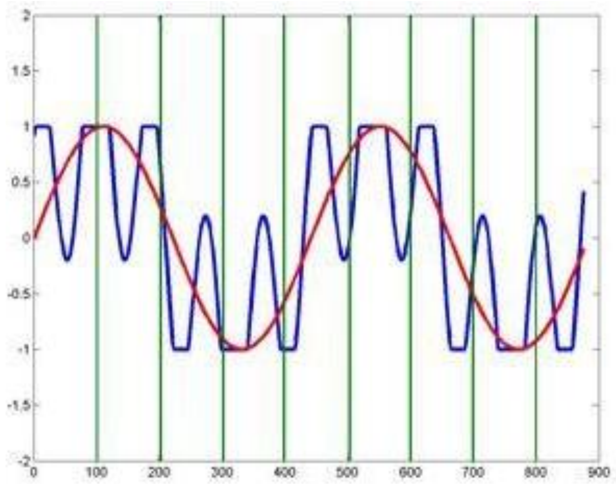
  $$\boldsymbol{a}_n = \frac{1}{N}\sum_{k=0}^{N-1} \boldsymbol{f}_k \cos(\frac{2\pi nk}{N})$$

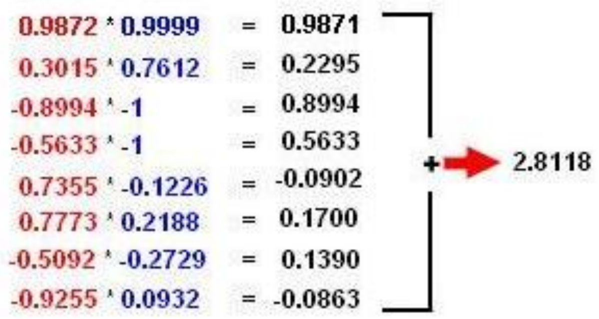
  $$\boldsymbol{b}_n = \frac{1}{N}\sum_{k=0}^{N-1} \boldsymbol{f}_k sin(\frac{2\pi nk}{N})$$

- Fundamental frequency $f = \frac{1}{N}$
- Fundamental angular frequency $\omega = 2\pi f = 2\pi/N$

How does DFT Work?

- Suppose the blue wave represents the complex audio and the red one is a sinusoidal wave of a certain frequency n.
- Green line means the sample points, N=8.
- If the sinusoidal wave fits the signal well, then **F**n is large, which means this frequency component takes a big ratio in the complex signal.





0.9872	* 0.9999	=	0.9871	
0.3015	* 0.7612	=	0.2295	
-0.8994	* -1	=	0.8994	
-0.5633	* -1	=	0.5633	
0.7355	* -0.1226	=	-0.0902	+ → 2.8118
0.7773	* 0.2188	=	0.1700	
-0.5092	* -0.2729	=	0.1390	
-0.9255	* 0.0932	=	-0.0863	

Phase Information in DFT

- In the equation for the inverse discrete Fourier transform given in (4.7, p31, the ***magnitude of the*** **n*th frequency component A*$_n$**, is given by

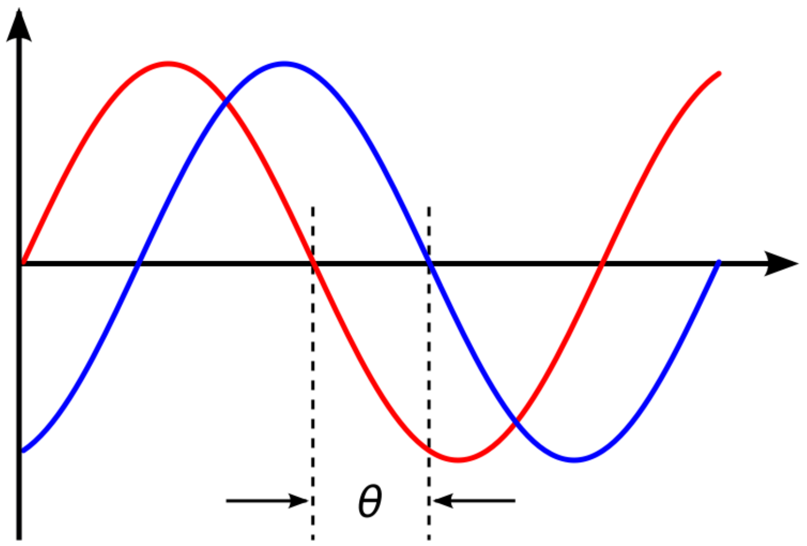
$$\boldsymbol{A}_n = |\boldsymbol{F}_n| = \sqrt{\boldsymbol{a}_n^2 + \boldsymbol{b}_n^2}\ ,\quad 0 \le n \le N-1$$

- The ***phase of the*** **n*th frequency component***, $\varnothing_n$, is given by
$$\varnothing_n = -tan^{-1}\left({}^{\boldsymbol{b}_n}/_{\boldsymbol{a}_n}\right), 0 \le n \le N-1$$

- The ***magnitude/phase form of the inverse DFT*** is given by
$$\boldsymbol{f}_k = \sum_{n=0}^{N-1} \boldsymbol{A}_n \cos(2\pi nk/N + \varnothing_n)$$

Phase Information

- Phase information in audio
  - Audio is a wave that continuously comes into your ears, so actually we don't detect the phase difference.
  - However, if two waves of the same frequency, one is phase shifted, come to you at the same time, then you will hear the destructive interference.



same frequency
different phase

Fast Fourier Transform(FFT)

- The usefulness of the discrete Fourier transform was extended greatly when a fast version was invented by Cooley and Tukey in 1965. This implementation, called the ***fast Fourier transform*** **(*FFT*)**, reduces the computational complexity from $O(N^2)$ to $O(N \log_2(N))$. N is the number of samples.

- The FFT is efficient because redundant or unnecessary computations are eliminated. For example, there’s no need to perform a multiplication with a term that contains sin(0) or cos(0).

FFT

- What would happen if the window size doesn’t fit an integer-multiple of the signal’s period?
- For example, Assume that the FFT is operating on 1024 samples of a 440 Hz wave sampled at 8000 samples per second. Then the window contains (1024/8000)*440=56.32 cycles => the end of the window would break the wave in the middle of a cycle.
- Due to this phenomenon(called ***spectral leakage***), the FFT may assume the original signal looks like Fig.4.21.

  The signal becomes discontinuous.

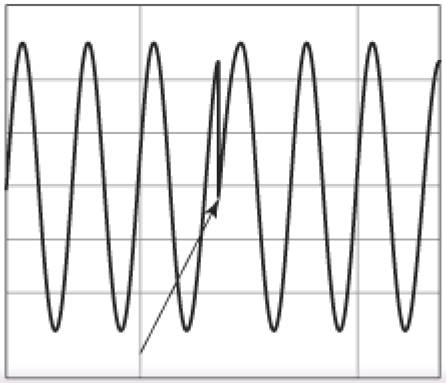
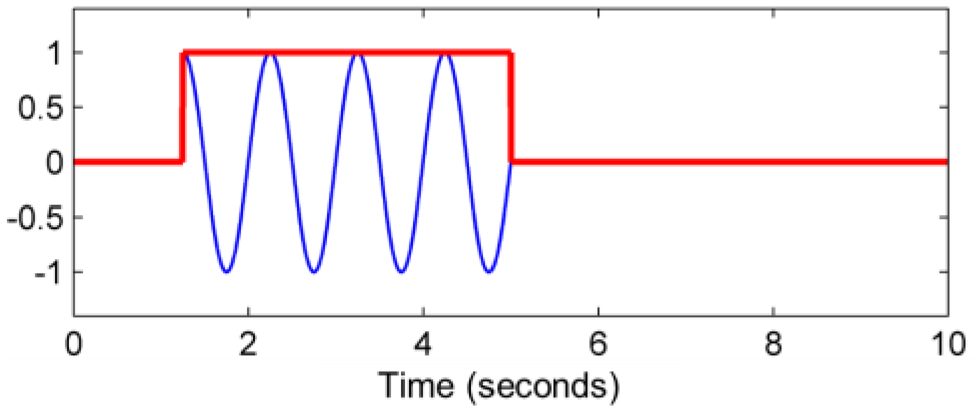
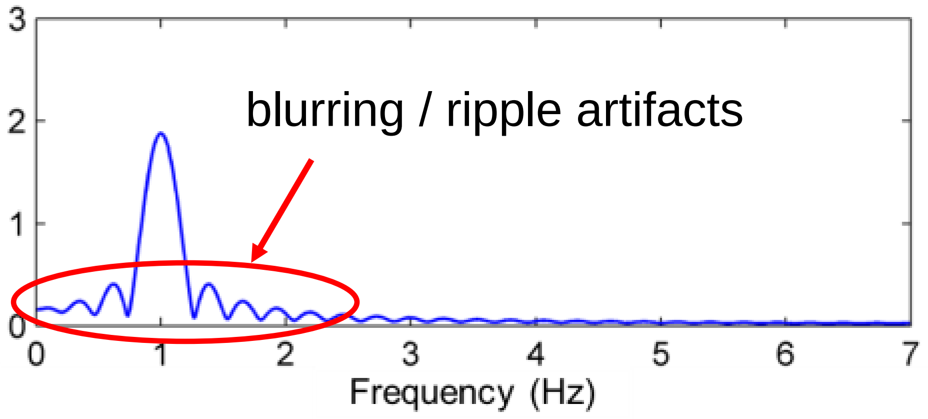


Fig.4.21

Spectral Leakage

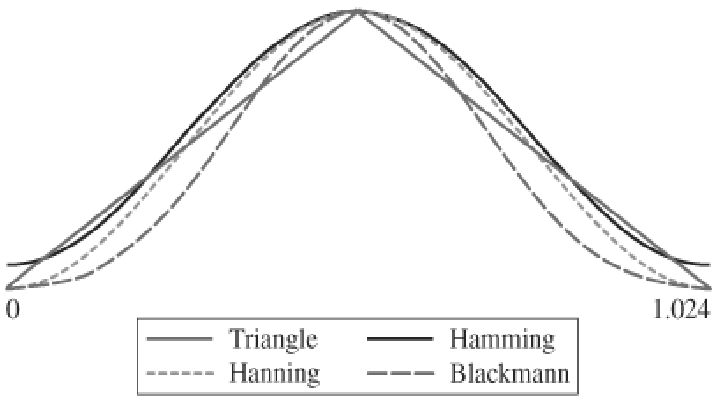
- Measure a signal of infinite length is impossible
- Perform Fourier transform on finite samples / time range (Windowing)
- Spectral leakage results in a relatively localized spreading of frequency components, with usually blurring / ripple artifacts





Windowing Function

- **Window function** - to reduce the amplitude of the sound wave at the beginning and end of the FFT window. If the amplitude of the wave is smaller at the beginning and end of the window, then the spurious frequencies will be smaller in magnitude as well.



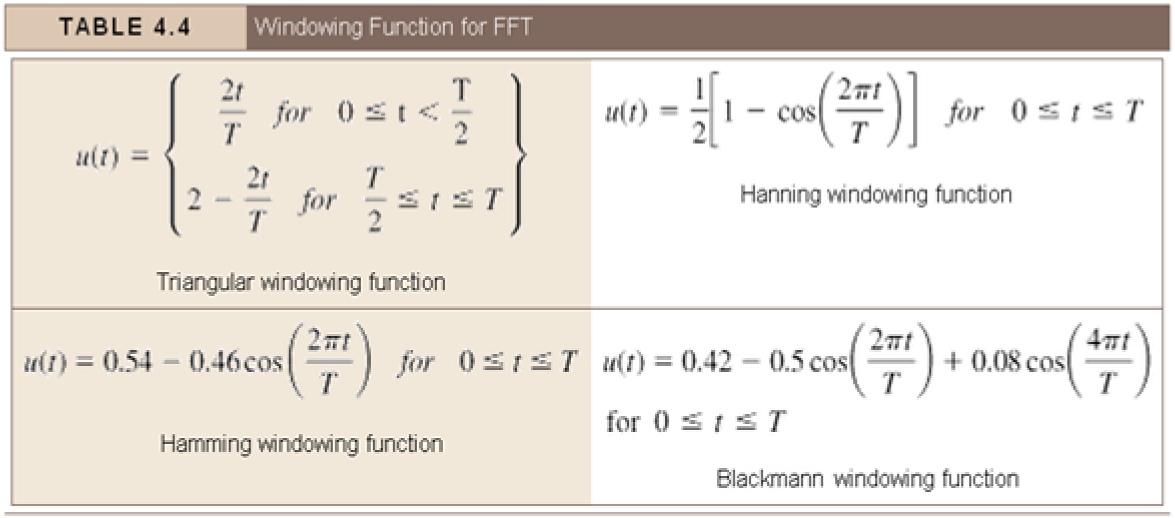


TABLE 4.4	Windowing Function for FFT
$u(t) = \begin{cases} \frac{2t}{T} & \text{for } 0 \le t < \frac{T}{2} \\ 2 - \frac{2t}{T} & \text{for } \frac{T}{2} \le t \le T \end{cases}$ Triangular windowing function	$u(t) = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi t}{T}\right)\right] \quad \text{for } 0 \le t \le T$ Hanning windowing function
$u(t) = 0.54 - 0.46\cos\left(\frac{2\pi t}{T}\right) \quad \text{for } 0 \le t \le T$ Hamming windowing function	$u(t) = 0.42 - 0.5\cos\left(\frac{2\pi t}{T}\right) + 0.08\cos\left(\frac{4\pi t}{T}\right)$ for $0 \le t \le T$ Blackmann windowing function

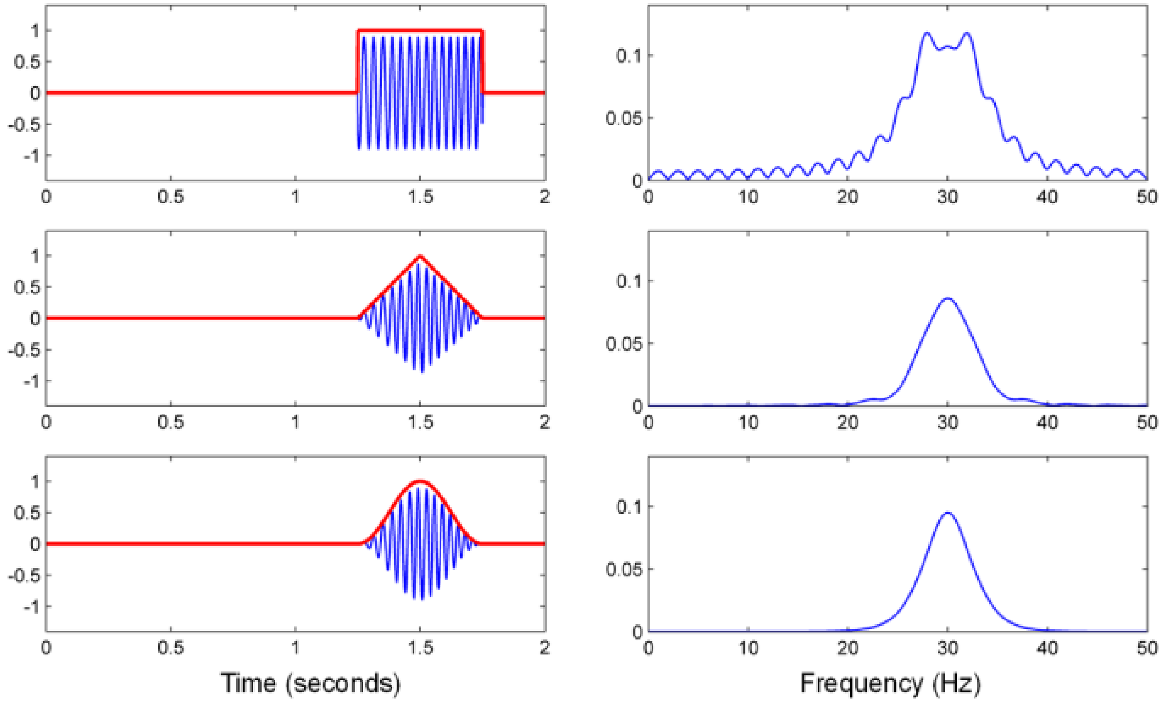
Windowing Function

- **Window function** - to reduce the amplitude of the sound wave at the beginning and end of the FFT window. If the amplitude of the wave is smaller at the beginning and end of the window, then the spurious frequencies will be smaller in magnitude as well.

Rectangular Window

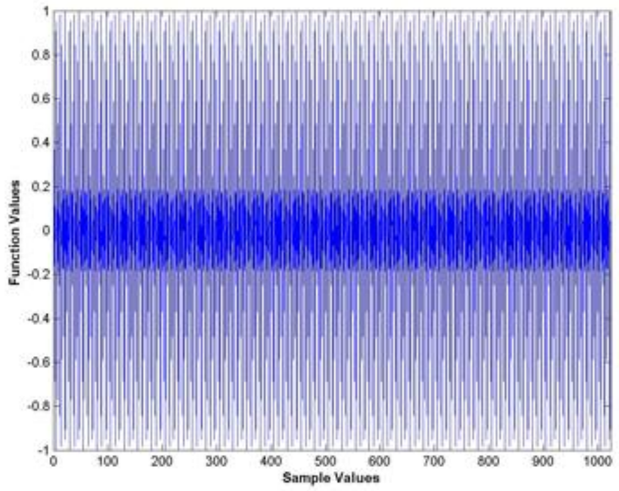
Triangular Window

Hann Window

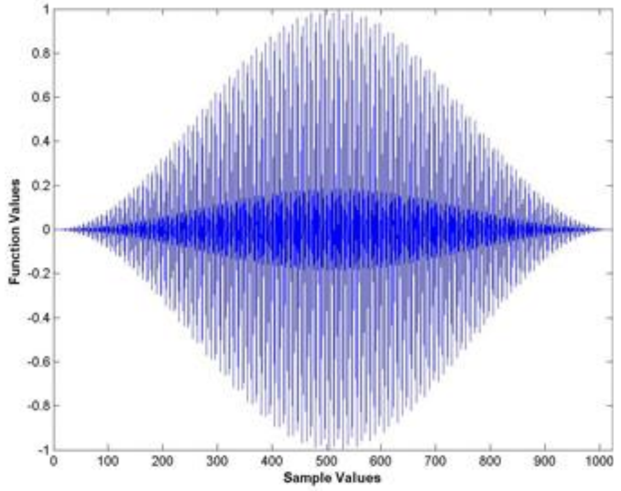


Window Function

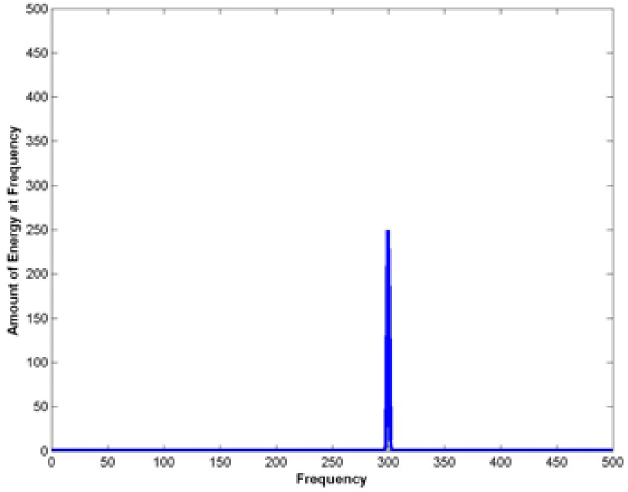
- The frequency components become more accurate, but the magnitudes also decrease. To counteract this, some other algorithms would be used.



Original wave



Applying window function



FFT result

Application of Frequency Analysis

- Translating an audio signal from time domain to frequency domain makes us observe distribution and range of its frequency more directly, then we can do audio processing and recognition, or design different filters.

- Speaker Recognition (Security system)
- Speech imitation (Tom Cat)
- Sound simulation (Animal sounds)

Musical Acoustics and Notation

- The range of human hearing is from about 20 Hz to about 20,000 Hz. As you get older, you lose your ability to hear high frequency sounds.
  - Test if you can hear the frequency you should be able to hear at your age
  - http://www.ultrasonic-ringtones.com/
- Octave equivalence
  - If the frequency of one note is 2^n times of the frequency of another, where *n* is an integer, the two notes sound “the same” to the human ear, except that the first is higher-pitched than the second.(n=1 => 高八度)

Decibels

- Decibels (*E*₀, *I*₀: threshold of human hearing)
  - Decibels-sound-pressure-level (dB_SPL)

$$dB\_SPL = 20\log_{10}\left(\frac{E}{E_0}\right), E_0 = 2\times 10^{-5}Pa$$

  - Decibels-sound-intensity-level (dB_SIL)

$$dB\_SIL = 10\log_{10}\left(\frac{I}{I_0}\right), I_0 = 10^{-12}W/m^2$$

- Decibels can be used to measure many things in physics, optics, electronics, and signal processing.
- A decibel is **not** an absolute unit of measurement.

DBFS

- In audio processing software, the amplitude is often shown in **dBFS. (*decibels-full-scale*)**

$$\mathrm{dBFS} = 20\log_{10}\left(\frac{\mathit{Sample\ Level}}{\mathit{Max\ Level}}\right)$$

- 0 dBFS represent the highest possible level of sound produced by audio. All other levels are expressed in **negative** numbers.
- dB vs dBFS
  - dB(decibels) are used to describe differences or changes in **sound level**
  - dBFS are used to describe signal levels in comparison with the **highest level your system** can handle.

Dynamic Range

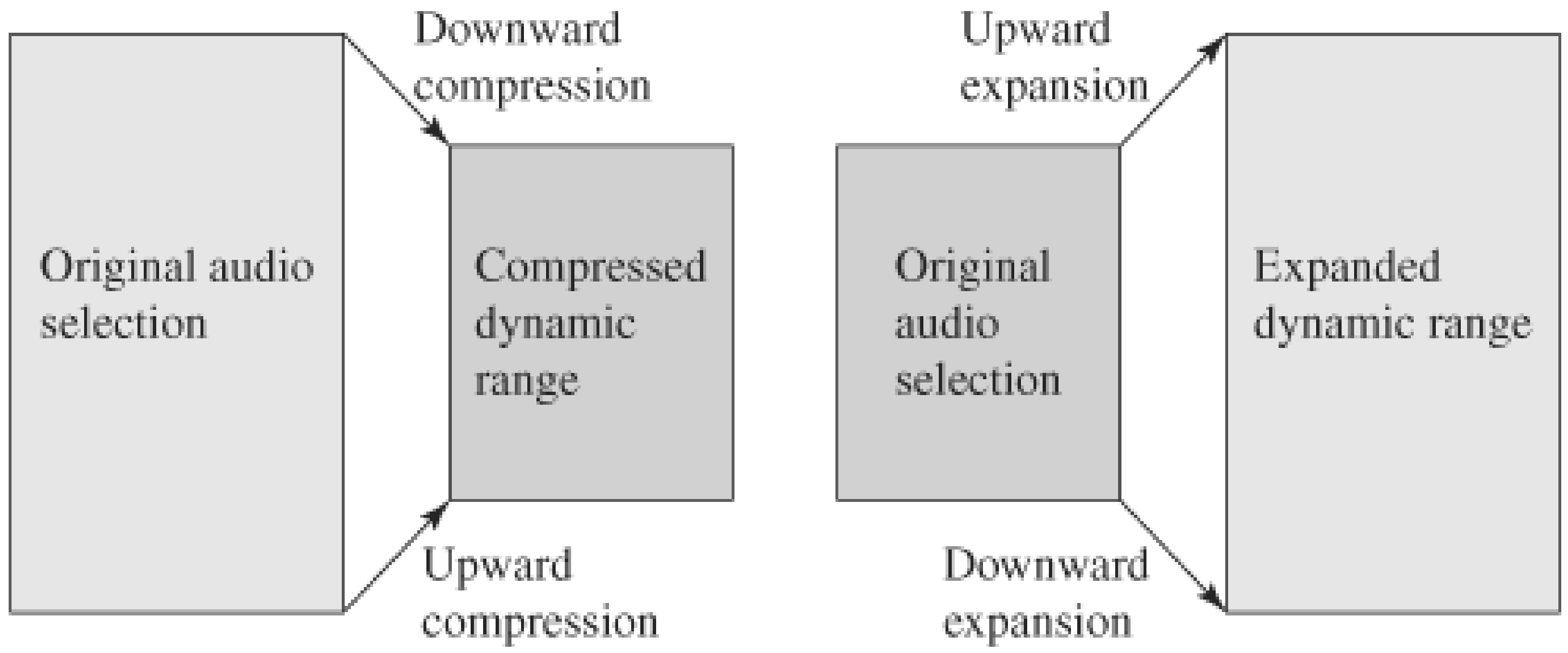
- Dynamic range is the ratio between the smallest nonzero value, which is 1, and the largest, which is 2^n. The *dynamic range of the audio file, d*, in decibels, is defined as $d = 20\log_{10} 2^n = 20n\,log_{10} 2 \approx 6n$
- The definition is identical to the definition of SQNR, and this is why you see the terms SQNR and dynamic range sometimes used interchangeably.
- Be careful not to interpret this as a 16-bit file allows louder amplitudes than an 8-bit file. Rather, dynamic range gives you a measure of the range of amplitudes that can be captured relative to the loss of fidelity compared to the original sound.

Dynamics Processing

- **Dynamics processing** is the process of adjusting the dynamic range of an audio selection, either to reduce or to increase.
- An increase in amplitude is called **gain** or **boost**. A decrease in amplitude is called **attenuation** or, informally, a **cut**.
- We introduce 4 digital dynamics processing tools here: **audio limiting**, **normalization**, **compression**, and **expansion**.

Compression and Expansion

- Types of dynamic range compression and expansion

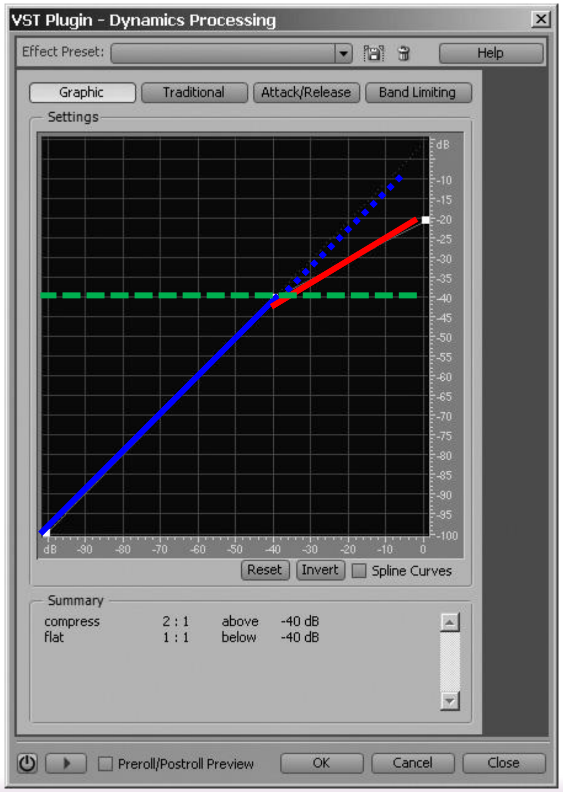


Compression and Expansion

- ***Downward compression*** lowers the amplitude of signals that are above a designated level, without changing the amplitude of signals below the designated level. It reduces the dynamic range.
- ***Upward compression*** raises the amplitude of signals that are below a designated level without altering the amplitude of signals above the designated level. It reduces the dynamic range.
- ***Upward expansion*** raises the amplitude of signals that are above a designated level, without changing the amplitude of signals below that level. It increases the dynamic range.
- ***Downward expansion*** lowers the amplitude of signals that are below a designated level without changing the amplitude of signals above this level. It increases the dynamic range.

Compression and Expansion - Examples

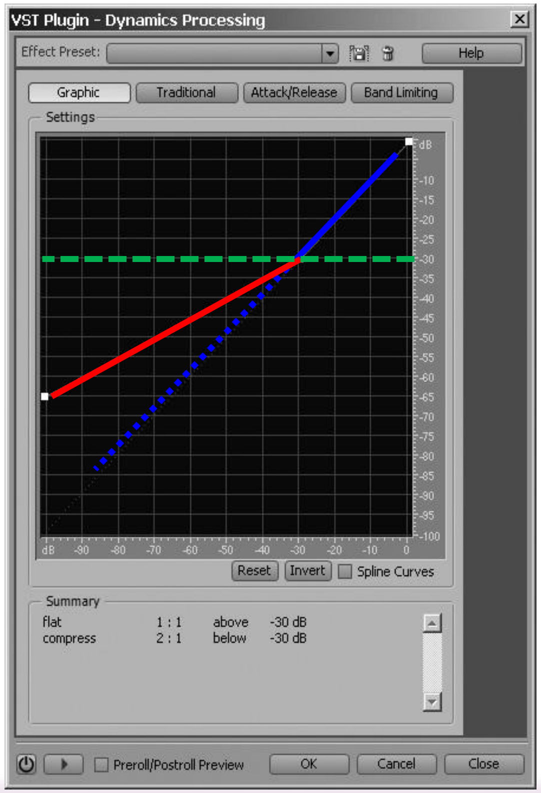
- Downward compression:
- Amplitudes above -40dB is lowered by a 2 : 1 ratio.



Summary

compress	2 : 1	above	-40 dB
flat	1 : 1	below	-40 dB

- Upward compression:
- Amplitudes below -30dB is compressed by a 2 : 1 ratio.



Summary

flat	1 : 1	above	-30 dB
compress	2 : 1	below	-30 dB

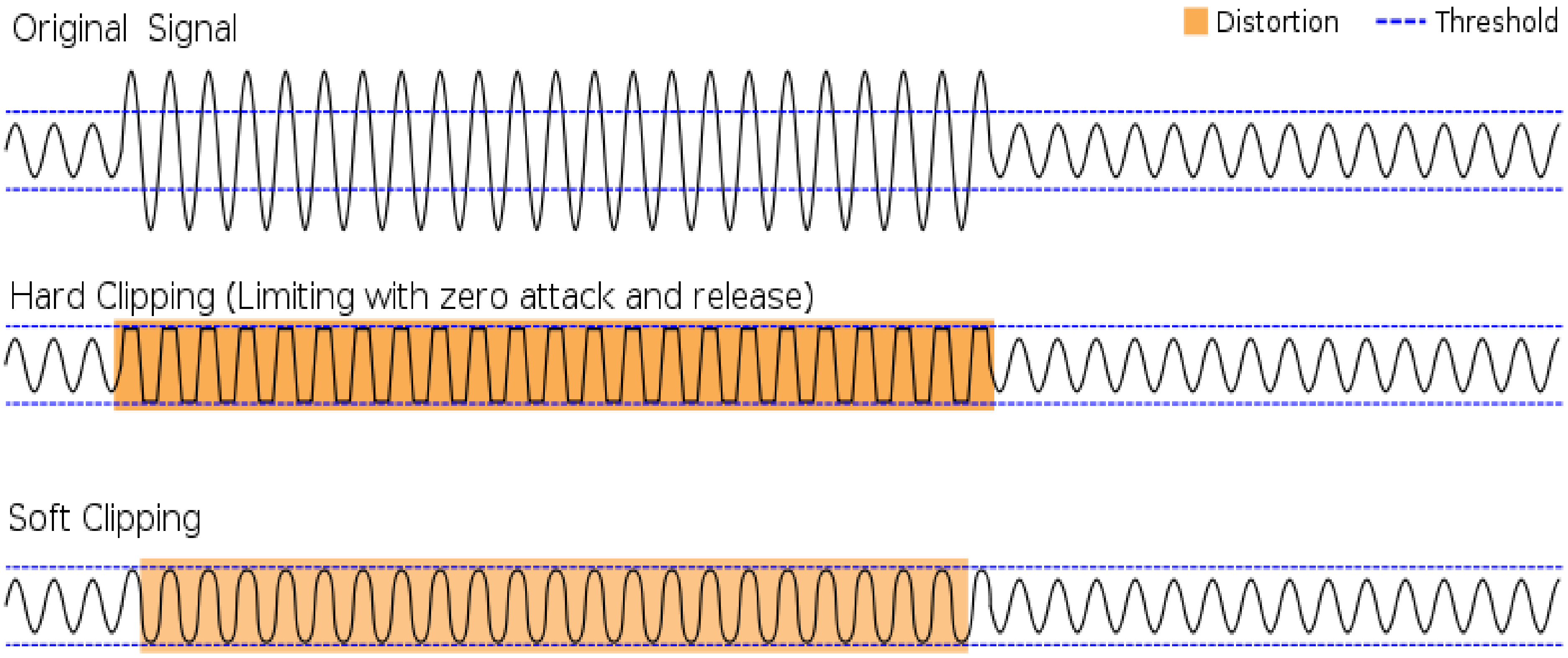
Limiting

- **Audio limiting** limits the amplitude of an audio signal to a designated level.

- Hard limiting (clipping)
  - cuts amplitudes of samples to a given maximum and/or minimum level.

- Soft limiting
  - audio signals above the designated amplitude are recorded at lower amplitude.

Limiting



http://en.wikipedia.org/wiki/File:Clipping_compared_to_limiting.svg

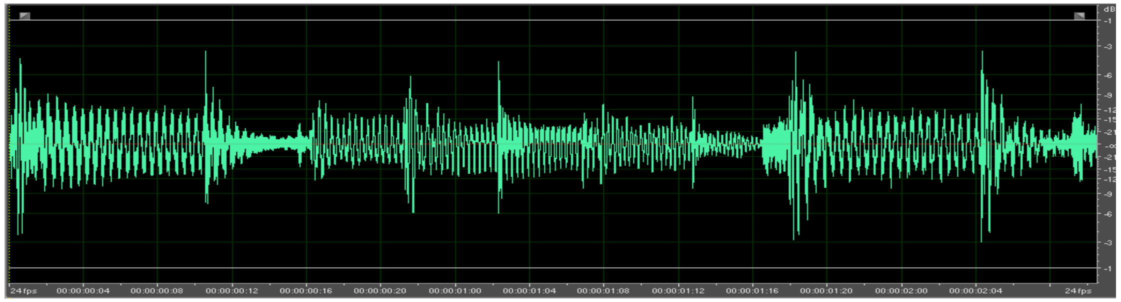
Normalization

- Often, normalization is used to increase the perceived loudness of a piece after the dynamic range of the piece has been compressed.

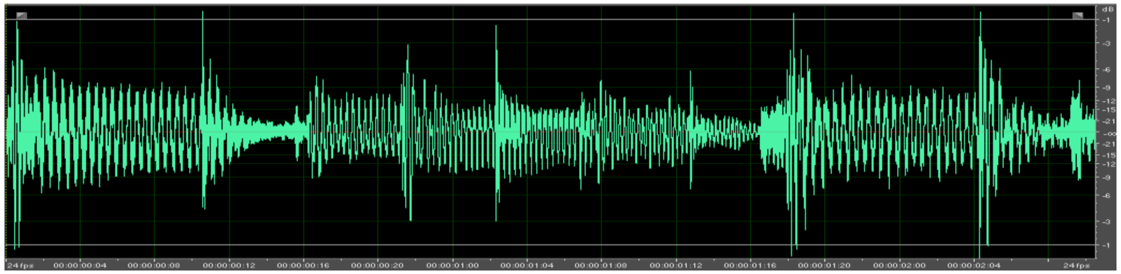
- Normalization steps:
  1. Find the highest amplitude sample in the audio selection.
  2. Determine the gain needed in the amplitude to raise the highest amplitude to maximum amplitude.
  3. Raise all samples in the selection by this amount.

Dynamics Processing - Example

- Bossa.wav - original

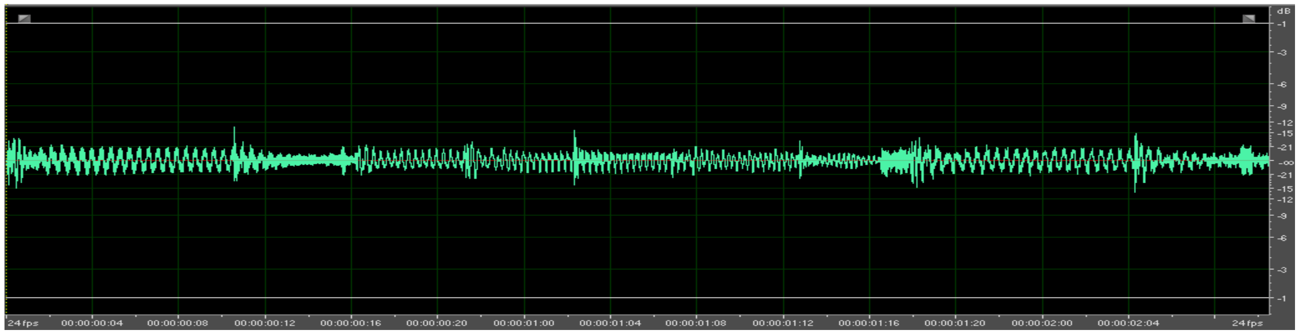


- Bossa.wav - normalized

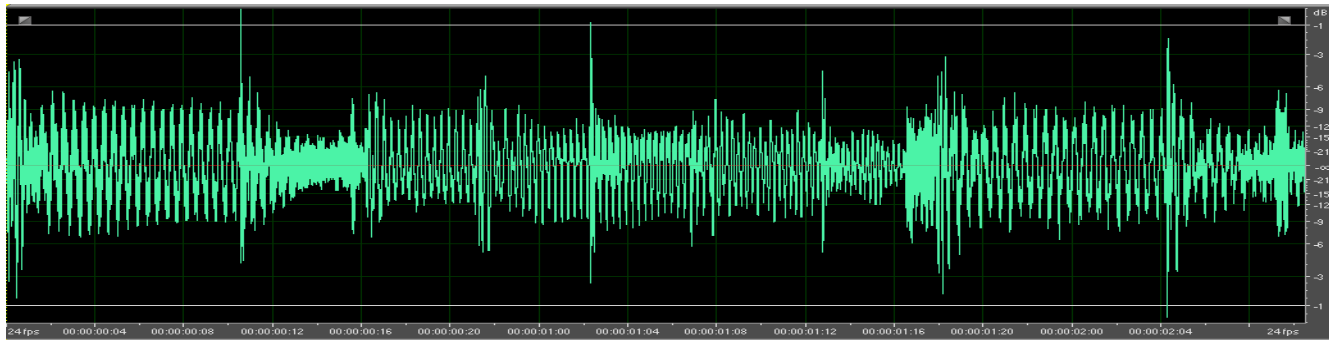


Dynamics Processing - Example

- Bossa.wav - compressed



- Bossa.wav – compressed + normalized

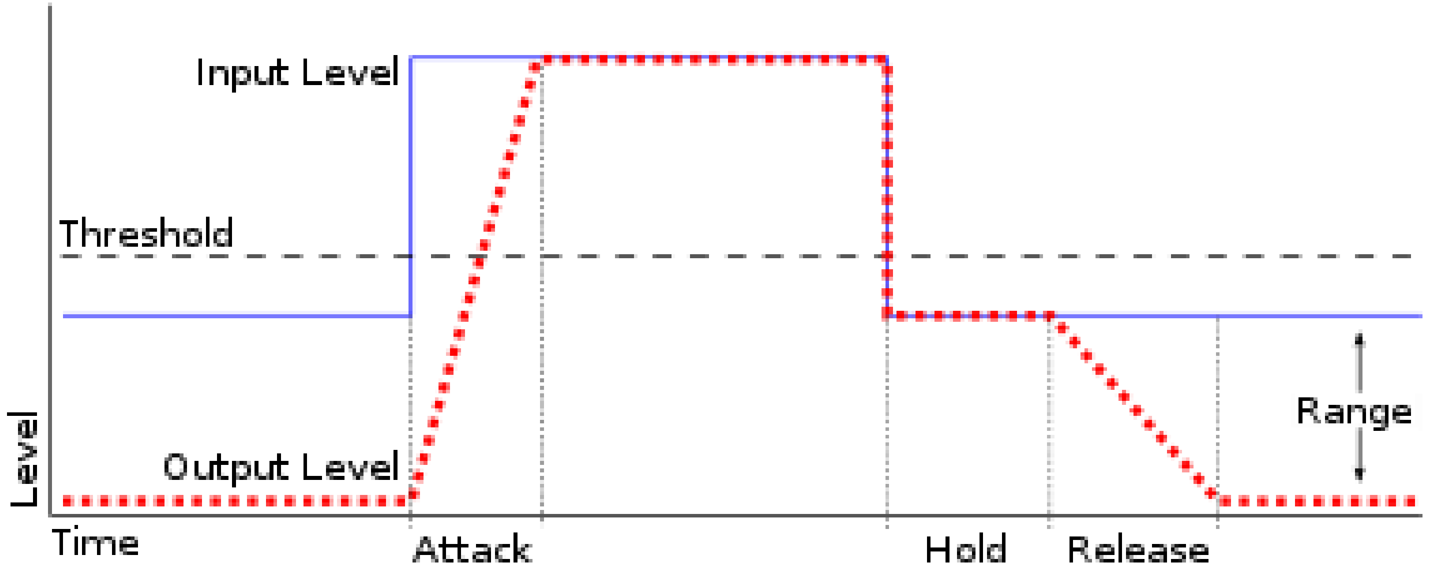


Audio Restoration

- Next, we introduce two basic types of audio restoration to alleviate the background noise that arises from the microphone, air, disk …etc.
  - Noise gating
  - Noise reduction

Noise Gating

- A **noise gate** allows a signal to pass through only when it is above a set threshold.
- It is used when the level of the signal is above the level of the noise. **It does not remove noise from the signal**. When the gate is open, both the signal and the noise will pass through.



http://en.wikipedia.org/wiki/Noise_gate

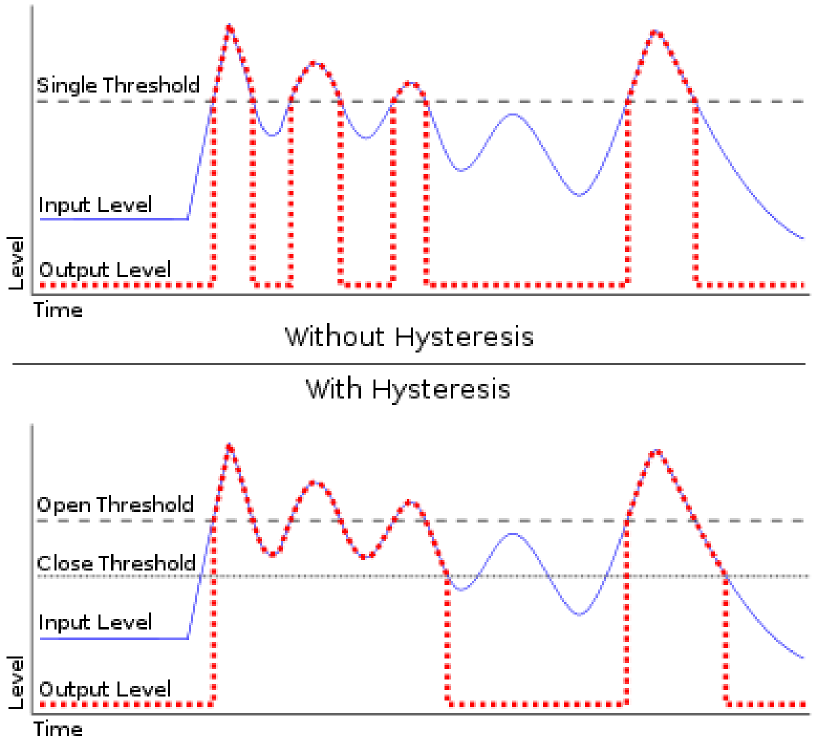
Noise Gating

- **Reduction Level**: the amplitude to which you want the below-threshold samples to be reduced.
- **Attack**: the attack time indicates how quickly you want the gate to open when the signal goes above the threshold, like fade-in.
- **Hold**: the amount of time the gate will stay open after the signal falls below the threshold.
- **Release**: The release time indicates how quickly you want the gate to close, like fade-out.

Noise Gating

- If the signal keeps moving back and forth around the threshold, the gate will open and close continuously, creating a kind of **chatter.**

- The **hysteresis** control indicates the difference between the value **n** that caused the gate to open and the value **m** that will cause it to close again. If *n* − *m* is large enough to contain the fluctuating signal, the noise gate won't cause chatter.



Noise Reduction

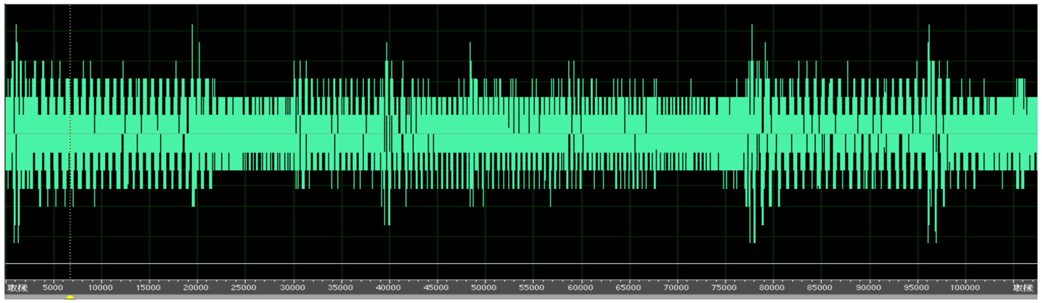
- Steps for noise reduction:
  1. Get a profile of the background noise. This can be done by selecting an area that should be silent, but that contains a hum or buzz.

  2. Determine the frequencies in the noise and their corresponding amplitude levels.

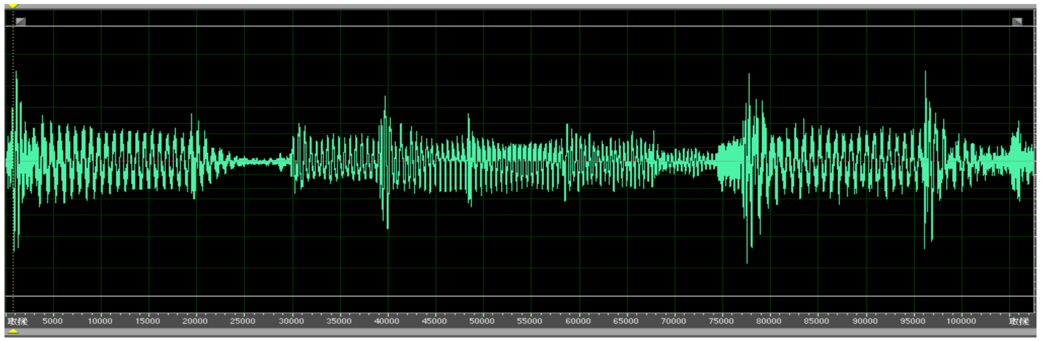
  3. The entire signal is processed in sections (FFT). The frequencies in each section are analyzed and compared to the profile, and if these sections contain frequency components similar to the noise, they can be eliminated below certain amplitudes.

Noise Reduction - Example

- Before noise reduction



- After noise reduction



Digital Audio Filter

- A digital audio filter is a linear system that changes the amplitude or phase of one or more frequency components of an audio signal.

- Types of digital audio filter
  - **FIR** (**finite-impulse response**) **filter**
  - **IIR** (**infinite-impulse response**) **filter**

FIR Filter

- Let *x*(*n*) be an audio signal of *N* samples for 0 ≤*n*≤ *N* – 1

  y(n) be the filtered signal

  h(n) be the convolution mask
- The FIR filter function is defined by

$$y(n) = h(0)x(n) + h(1)x(n-1) + \cdots + h(N)x(n-N)$$
$$= \sum_{k=0}^{N-1} h(k)x(n-k) = h(n) \otimes x(n)$$
$$where\ x(n-k) = 0\ if\ n-k < 0$$

- The number of the coefficients in h is the **order of the filter.**

IIR Filter

- The infinite form of the FIR filter function is defined by

$$y(n) = h(n) \otimes x(n) = \sum_{k=0}^{\infty} h(k)x(n-k)$$

$$where\ x(n-k) = 0\ if\ n-k < 0$$

- However, finding the values *h*(*n*) for an infinitely long mask is impossible. The equation can be transformed to a more manageable difference equation form.

IIR Filter

- The recursive form of IIR filter is

$$y(n) = h(n) \otimes x(n) = \sum_{k=0}^{N-1} a_k x(n-k) - \sum_{k=1}^{M} b_k y(n-k)$$

  a_k is the coefficient of the forward filter

  b_k is the coefficient of the feedback filter

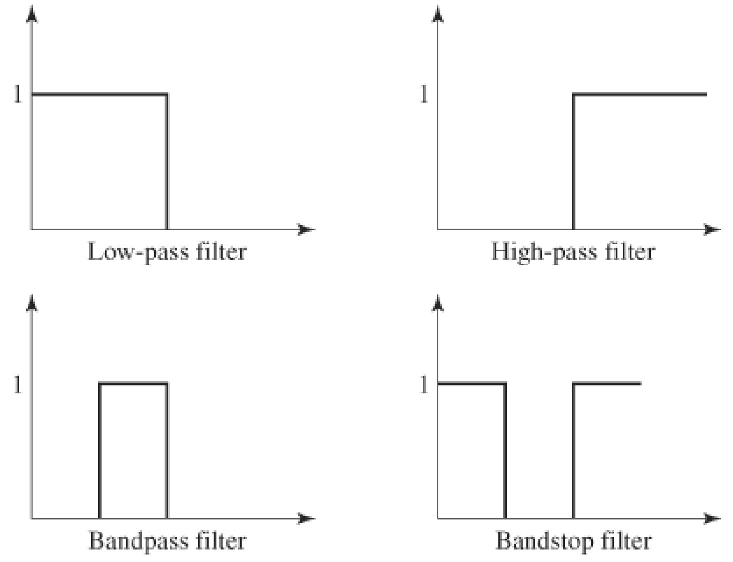
- y(n) depends on present and past input samples as well as on past outputs.

Impulse and Frequency Response

- The convolution mask **h(n)** for an FIR or IIR filter is sometimes referred to as the **impulse response**.
- The **frequency response**, **H(z)**, represents **h(n)** in frequency domain.
- A **frequency response graph** describes how a filter acts on an audio signal.

Filters in Audio Processing

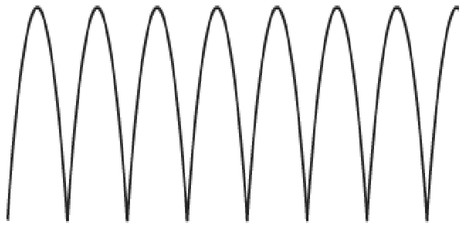
- **Band filters**
  - **low-pass filter**—retains only frequencies below a given level.
  - **high-pass filter**—retains only frequencies above a given level.
  - **bandpass filter**—retains only frequencies within a given band.
  - **bandstop filter**—eliminates all frequencies within a given band.



Horizontal axis: Frequency component
Vertical axis: Fraction of frequency component **retained** in filtered signal

Filters in Audio Processing

- **Comb filter —** add delayed versions of a wave to itself, resulting in phase cancellations that can be perceived as echo. Phase cancellations eliminate frequency components when two sine waves that are out-of-phase with each other are summed. Thus, the frequency response has the shape of a comb.


Frequency response of a comb filter

One-fold echo: a_k = [1, 0, 0, 0, ..., 0, 0.8] , **b_k** = [1]
(That is, 3199 zeros between 1 and 0.8.)
The output of the filter is: **y[n] = x[n] + 0.8*x[n-3200]**

Multiple-fold echo: a_k = [1], **b_k** = [1, 0, 0, 0, ..., 0, -0.8]
(That is, 3199 zeros between 1 and -0.8.)
The output of the filter is: **y[n] = x[n] + 0.8*y[n-3200]**

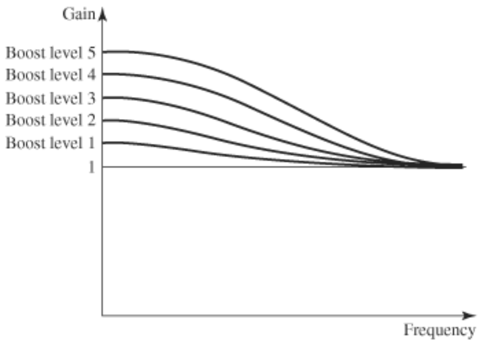
Filters in Audio Processing

- **Shelving filters —** shelving filters are similar to low- and high-pass filters except that they boost or cut frequencies up to a certain frequency.

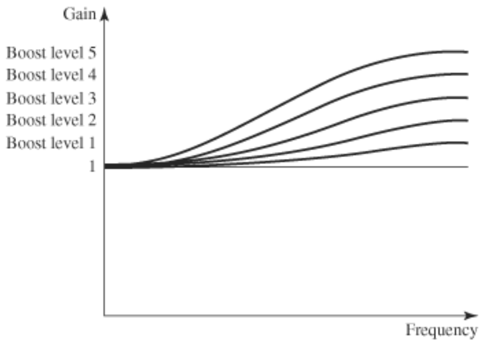
Low-shelf

High-shelf

Boosting

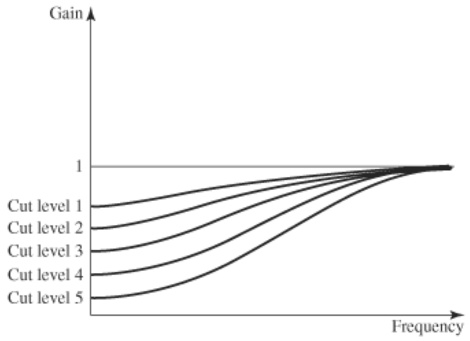


(a) Low-shelf filter for boosting low frequencies

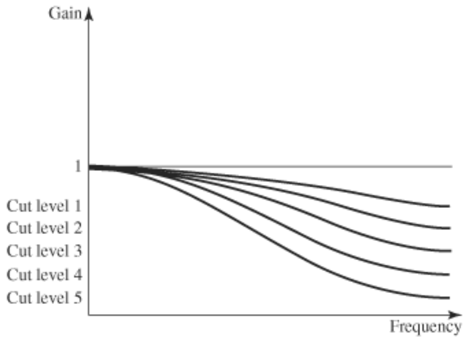


(b) High-shelf filter for boosting high frequencies

Cutting



(c) Low-shelf filter for cutting low frequencies

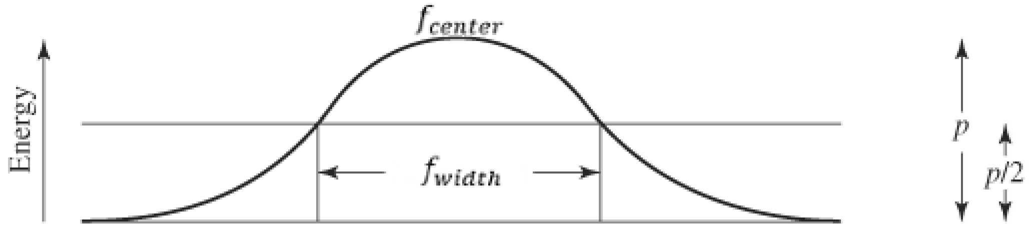


(d) High-shelf filter for cutting high frequencies

Filters in Audio Processing

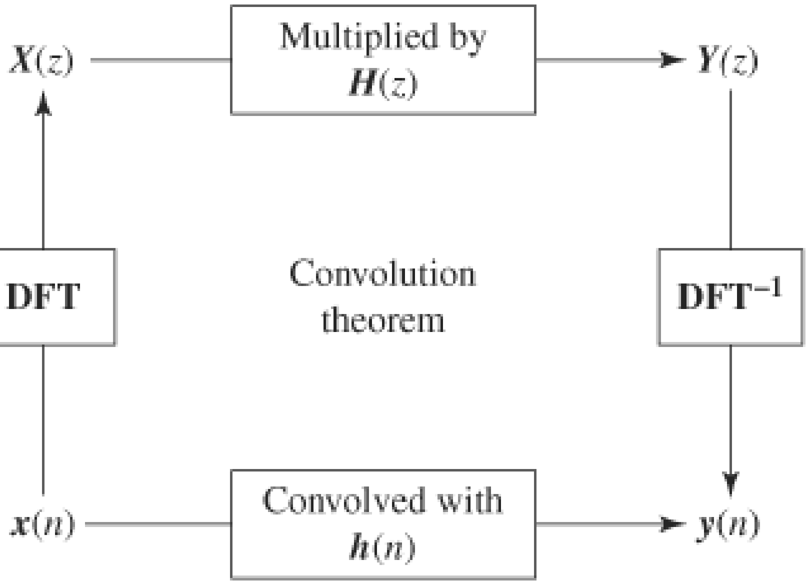
- **Peaking filter —** Ideally, for a bandpass filter, the unwanted frequencies would be filtered out entirely, but in reality this is not possible.
- The frequency response looks more like the bell curve. This type of filter is sometimes called a **peaking filter**.
- Given the graph of a peaking filter, let f_{width} be the width of the peak measured at the points ½ times the peak's height, and let f_{center} be the frequency at the geometric center of the peak, both in Hz. Then the Q-factor, Q, is defined as

$$Q = \frac{f_{center}}{f_{width}}$$



Relationship Between Convolution and Fourier Transform

- Let ***H***(*z*) be the discrete Fourier transform of a convolution filter ***h***(*n*), and let ***X***(*z*) be the discrete Fourier transform of a digital audio signal ***x***(*n*). Then $y(n) = h(n) \otimes x(n)$ is equivalent to the inverse discrete Fourier transform of ***Y***(*z*), where ***Y***(*z*) = ***H***(*z*)***X***(*z*).



Equivalent operations in time and frequency domains

FIR Filter Design – Terms

- The convolution mask **h**(n) is also called the **impulse response**, representing a filter in the **time domain**.
- Its counterpart in the frequency domain, the **frequency response H**(z), is also sometimes referred to as the **transfer function**.
- A frequency response graph can be used to show the desired frequency response of a filter you are designing.

FIR Filter Design – Ideal Filter

- An ideal low-pass frequency response graph, normalized.
- In the graph, angular frequency is on the horizontal axis. The vertical axis represents the fraction of each frequency component to be permitted in the filtered signal.
- The cutoff frequency ω_c must be less than π.(Nyquist theorem)

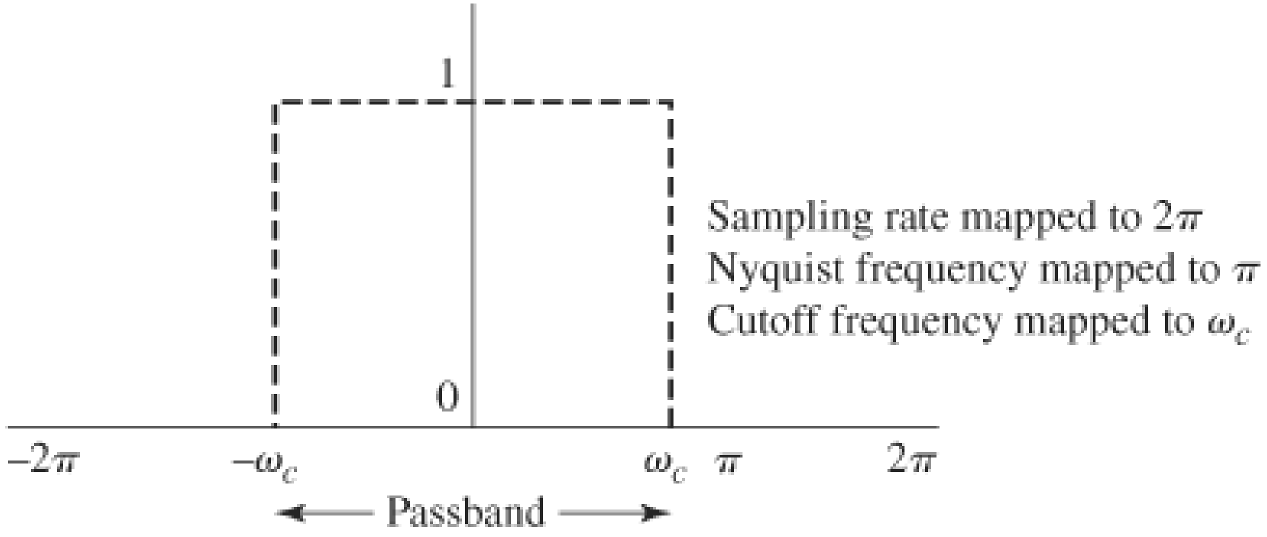
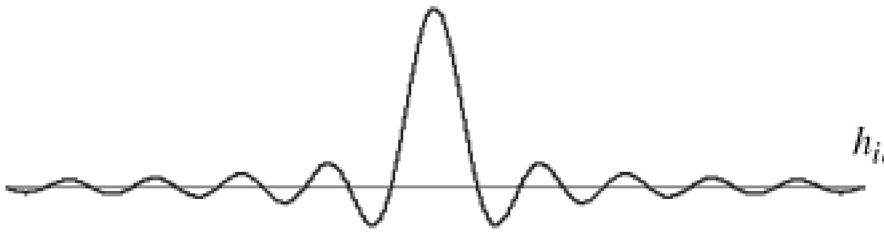

1
0
Sampling rate mapped to 2π
Nyquist frequency mapped to π
Cutoff frequency mapped to ω_c
-2π
$-\omega_c$
ω_c
π
2π
Passband

Figure 5.42

$$\omega_c = 2\pi \frac{f_c}{f_{sample}}$$

FIR Filter Design – Ideal Filter

- The frequency response graph shown in Figure 5.42 is an example of a **rectangle function.**
- If it is an idealized form of ***H***(*z*), then what would be the corresponding ideal impulse response, an idealized ***h***(*n*)?
- The inverse Fourier transform of a rectangle function in the frequency domain is a **sinc function** in the time domain.



$$h_{ideal}(n) = \frac{\sin(2\pi f_c n)}{\pi n} \quad \text{for } -\infty \le n \le \infty, n \ne 0$$
$$\text{and } 2f_c \text{ for } n = 0$$

FIR Filter Design – Ideal Filter

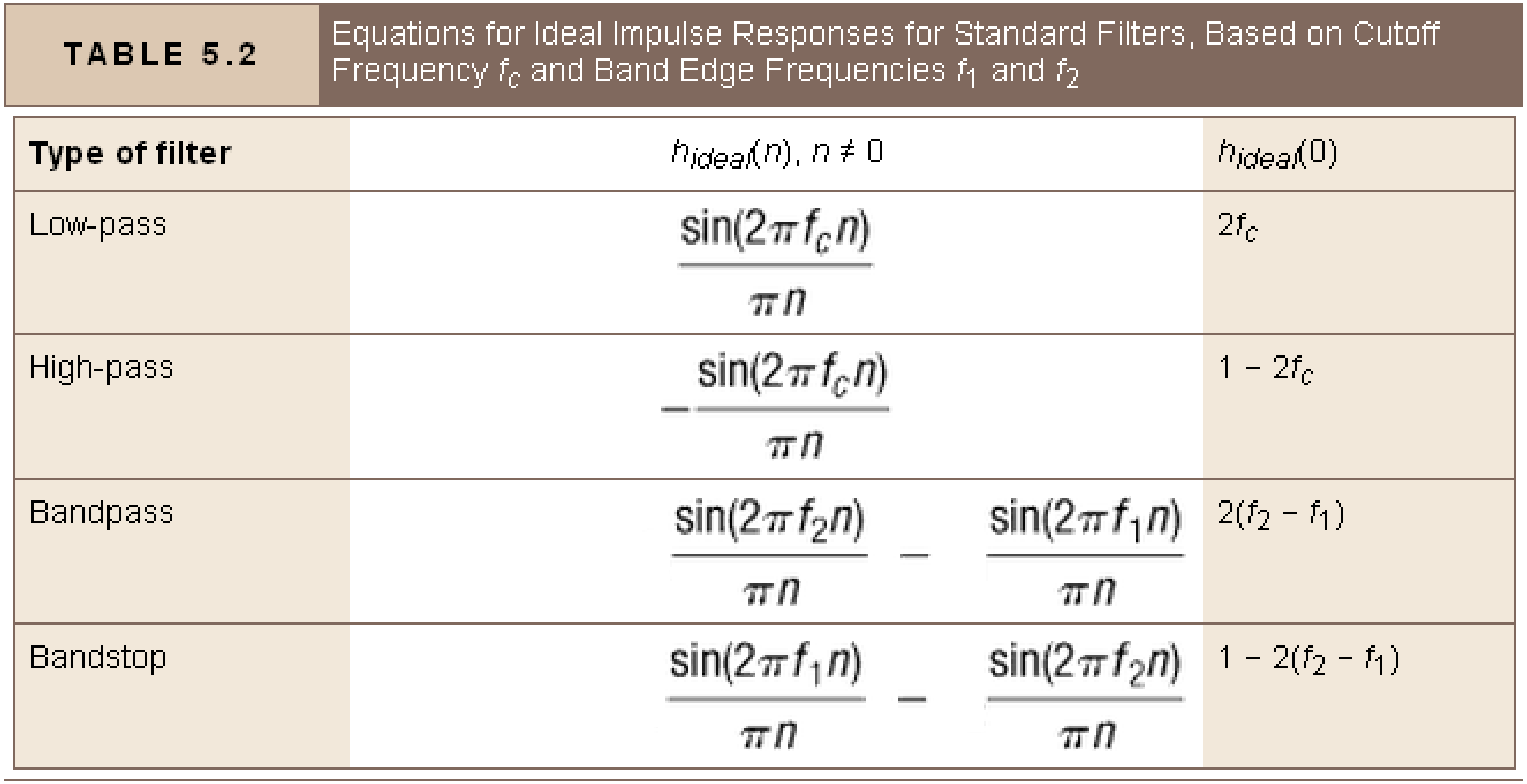


TABLE 5.2 Equations for Ideal Impulse Responses for Standard Filters, Based on Cutoff Frequency f_c and Band Edge Frequencies f_1 and f_2

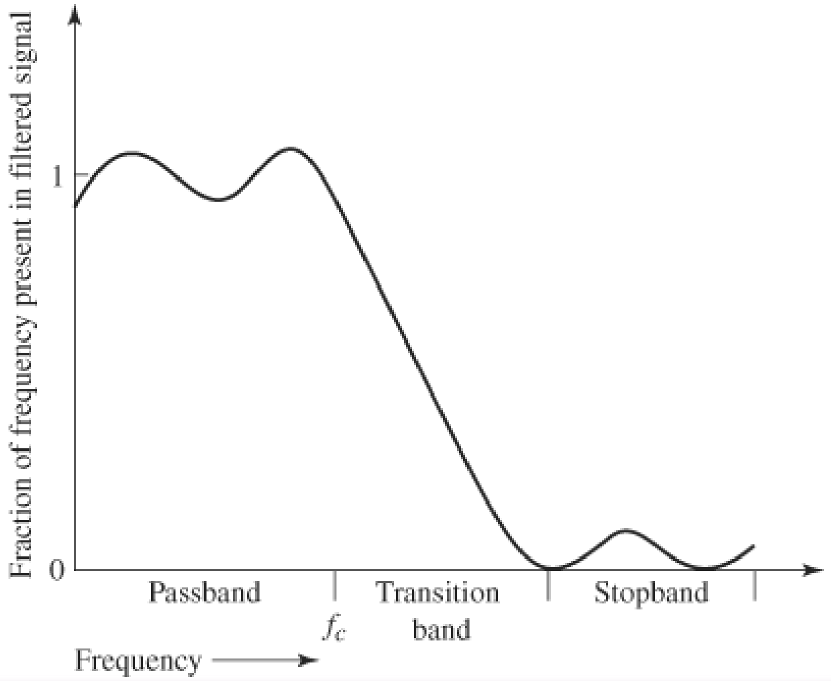
Type of filter	$h_{ideal}(n), n \neq 0$	$h_{ideal}(0)$
Low-pass	$\dfrac{\sin(2\pi f_c n)}{\pi n}$	$2f_c$
High-pass	$-\dfrac{\sin(2\pi f_c n)}{\pi n}$	$1 - 2f_c$
Bandpass	$\dfrac{\sin(2\pi f_2 n)}{\pi n} - \dfrac{\sin(2\pi f_1 n)}{\pi n}$	$2(f_2 - f_1)$
Bandstop	$\dfrac{\sin(2\pi f_1 n)}{\pi n} - \dfrac{\sin(2\pi f_2 n)}{\pi n}$	$1 - 2(f_2 - f_1)$

FIR Filter Design – Ideal to Real

- A sinc function goes on infinitely in the positive and negative directions … how can we implement an FIR filter?
- One way is to multiply the ideal impulse response by a **windowing function**. The purpose of the windowing function is to make the impulse response finite.
- However, making the impulse response finite results in a frequency response that is less ideal, containing "ripples".

FIR Filter Design - Realistic

- The **passband** corresponds to the frequencies the filter tries to retain. The **stopband** corresponds to the frequencies the filter attenuates or filters out.
- In a real filter, a **transition band** lies between passband and stopband, and the slope is not infinitely steep, as in an ideal filter.



Frequency response of a realistic low-pass filter

FIR Filter Design – Windowing Function

- Rectangular windowing function
  - 1
- Hanning windowing function
  - $w(n) = 0.5 - 0.5\cos(\frac{2\pi n}{N})$
- Hamming windowing function
  - $w(n) = 0.54 - 0.46\cos(\frac{2\pi n}{N})$
- Blackmann windowing function
  - $w(n) = 0.42 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos(\frac{4\pi n}{N-1})$

FIR Filter Design - Algorithm

```
/*Input: f_c, the cutoff frequency for the lowpass filter, in Hz
            f_samp, the sampling frequency of the audio signal to be filtered, in Hz
            N, the order of the filter; assume N is odd
  Output: a low-pass FIR filter in the form of an N-element array */

/*Normalize f_c and ω_c so that π is equal to the Nyquist angular frequency*/
f_c = f_c/f_samp
ω_c = 2*π*f_c
middle = N/2 /*Integer division, dropping remainder*/

/*Create the filter using the low-pass filter function from Table 5.2*/
/*Put a dummy value in for n = 0 to avoid a divide by 0 error)*/
for n = −N/2 to N/2
    if (n = 0) fltr(middle) = 1
    else fltr(n + middle) = sin(2*π*f_c*n)/(π*n)
fltr(middle) = 2*f_c

/*Multiply the elements of fltr by a windowing function chosen from Table 5.3. We use the Hanning window function */
for n = 0 to N-1
    fltr(n) = fltr(n) * (0.5 - 0.5*cos((2*π*n)/N))
```

Digital Audio Compression

- Time-Based Compression Methods
  - No need to transform the data into frequency domain.
  - A-law encoding, μ-law encoding, etc.
  - These methods are often considered conversion techniques rather than compression methods.
- The most effective audio compression methods require some information about the frequency spectrum of the audio signal. These compression methods are based on **psychoacoustical modeling** and **perceptual encoding**.

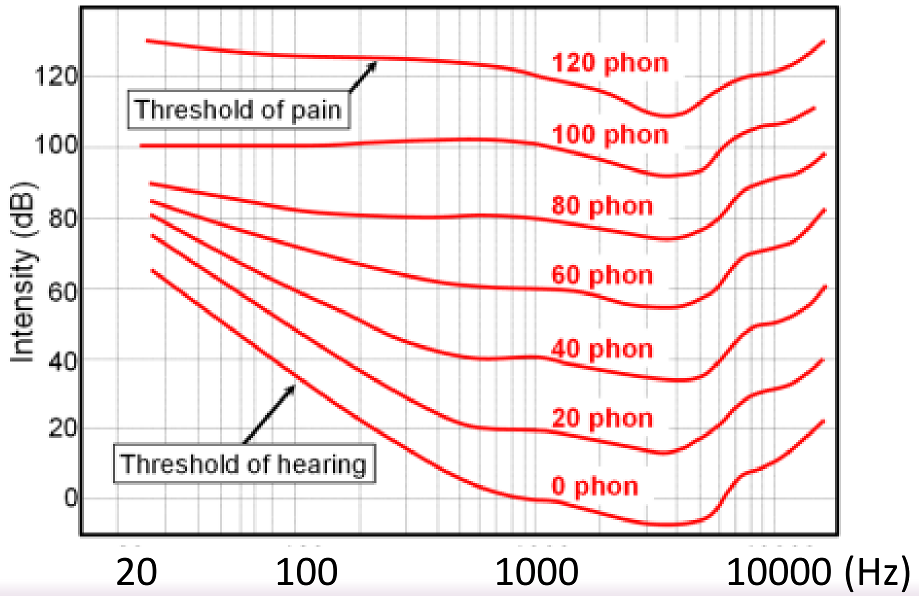
Psychoacoustics

- Psychoacoustics (心理聲學)
  - The study of subjective human perception of sounds.
  - The study of all the psychological interactions between humans and the world of sound.
- Psychoacoustical tests have shown that there is a great deal of nonlinearity in human sound perception.
- The octave from middle C (called C4) to C5 ranges from 261.63 Hz to 523.25 Hz, while C5 to C6 ranges from 523.25 to 1046.50 Hz. But the distance from C4 to C5 subjectively sounds the same as the distance from C5 to C6.

Psychoacoustics

- Humans hear best in the 1000 to 5000 Hz range, the frequency range of human speech.
- For a 100Hz and 1000Hz tone, the 100Hz tone needs larger amplitude to sound equally loud as the 1000Hz.
- Similarly, for a 10000Hz and 1000Hz tone, the 10000Hz tone needs larger amplitude to sound equally loud as the 1000Hz.

- The **phon** is a unit of perceived loudness level varying from frequencies
- 1 phon = 1 dB SPL at 1000 Hz
- 0 phon is the limit of perception, and inaudible sounds have negative phon levels

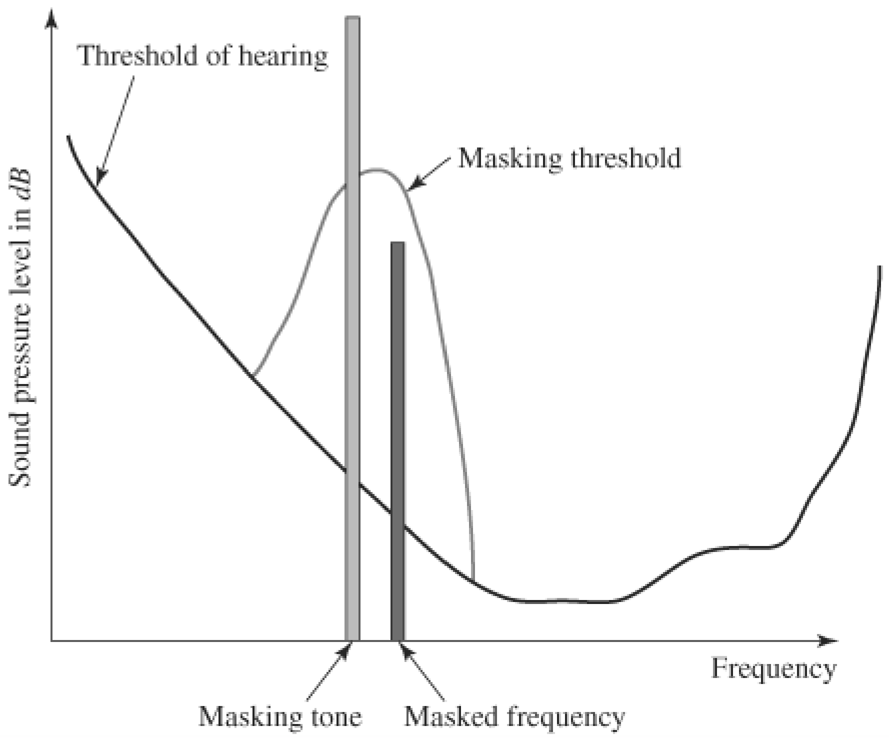


Psychoacoustics

- At low frequencies, we can distinguish between sound waves that are only a few Hz apart. At high frequencies, the frequencies must be a hundred or more Hz apart for us to hear the difference. The reason is that the ear is divided into frequency bands called ***critical bands***.
- Human ear perceive different frequencies by the response of different critical bands.
- Critical bands are narrower for low-frequency than for high-frequency sounds. Between 1 and 500 Hz, bands are about 100 Hz in width. The critical band at the highest audible frequency is over 4000 Hz wide.

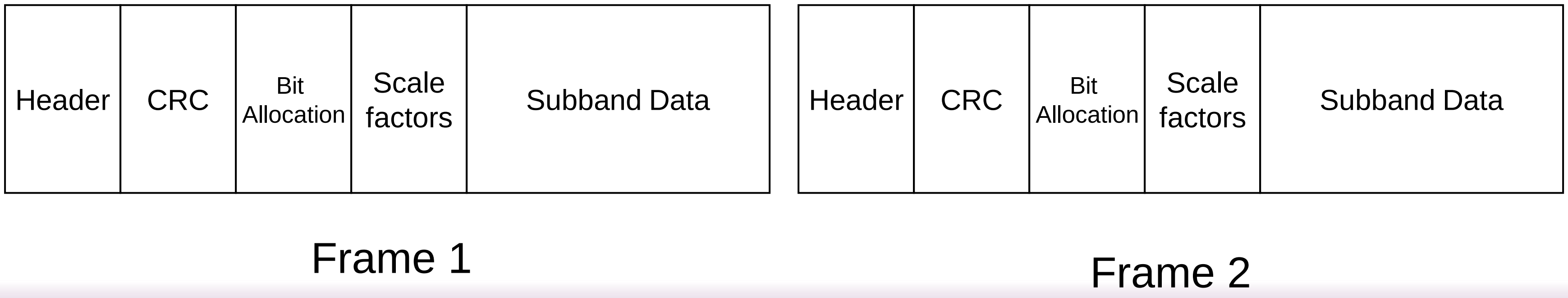
Psychoacoustics

- **Masking** - Within a small window of time, the loudest frequency sound can overpower the others in the critical band, making human unable to hear the other frequencies.



A Sketch of Compression

- A small window of time called a ***frame*** is moved across a sound file.
- In each frame, use different filters to divide the frame into bands of different frequencies.
- Calculate the masking curve for each band.
- Requantize the samples using fewer bits such that the quantization error is below the masking curve. That is, make sure the noise is inaudible.

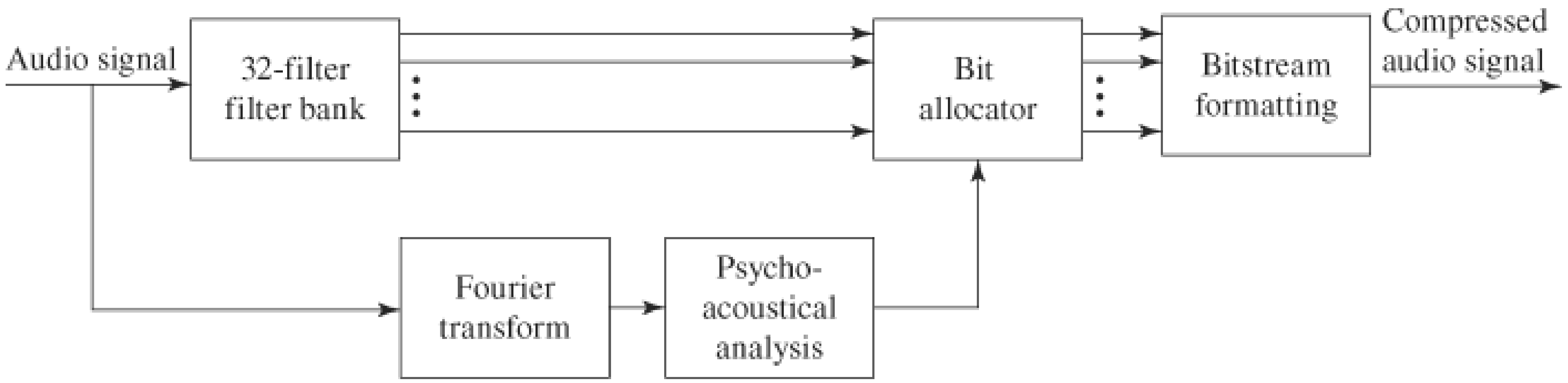


Overview of MPEG

- Acronym for the ***Motion Picture Experts Group,*** a family of compression algorithms for both digital audio and video.
- MPEG has been developed in a number of phases: MPEG-1, MPEG-2, MPEG-4, MPEG-7.
- MPEG-1 covers CD-quality audio suitable for video games.
- MPEG audio is also divided into three layers: Audio Layers I, II, and III.
- Each higher layer is more computationally complex, and generally more efficient at lower bitrates than the previous.
- The well-known MP3 audio file format is actually MPEG-1 Audio Layer III.

MPEG-1 Audio Compression

- Basic concepts of MPEG-1 compression



MPEG-1 Audio Compression - Step 1

- Divide the audio file into frames and analyze the psychoacoustical properties of each frame individually.
  - To analyze the masking phenomenon, we must look at a small piece of time because it happens when different frequencies are played at close to the same time.
  - Frames can contain 384, 576, or 1152 samples, depending on the MPEG phase and layer.
  - For the remainder of these steps, it is assumed that we're operating on an individual frame.

MPEG-1 Audio Compression - Step 2

- By applying a bank of filters, separate the signal into frequency bands.
  - The samples are divided into frequency bands for psychoacoustical analysis. Each filter removes all frequencies except for those in its designated band.
  - The use of filter banks is called **subband coding.** In MPEG-1, the number of filters is 32.
  - In MPEG-1 Layers I and II, the frequency bands created by the filter banks are uniform in size, which doesn’t match the width of the critical bands in human hearing. Layer III models critical bands more closely than layer I and II.

MPEG-1 Audio Compression - Step 3

- Perform a Fourier transform on the samples in each band in order to analyze the band’s frequency spectrum.
  - After the Fourier transform, we can know exactly how much of each frequency component occurs in each band.
  - From the frequency spectrum, a masking curve can be produced for each band.

MPEG-1 Audio Compression - Step 4

- Analyze the influence of tonal and nontonal elements in each band. (Tonal elements are simple sinusoidal components, such as frequencies related to melodic and harmonic music. Nontonal elements are transients like the strike of a drum or the clapping of hands.)
  - The longer the time window is in frequency analysis, the better the frequency resolution, but the worse the time resolution. This in turn can mean that transient signals may not be properly identified. Therefore, it is better to identify the transient first.

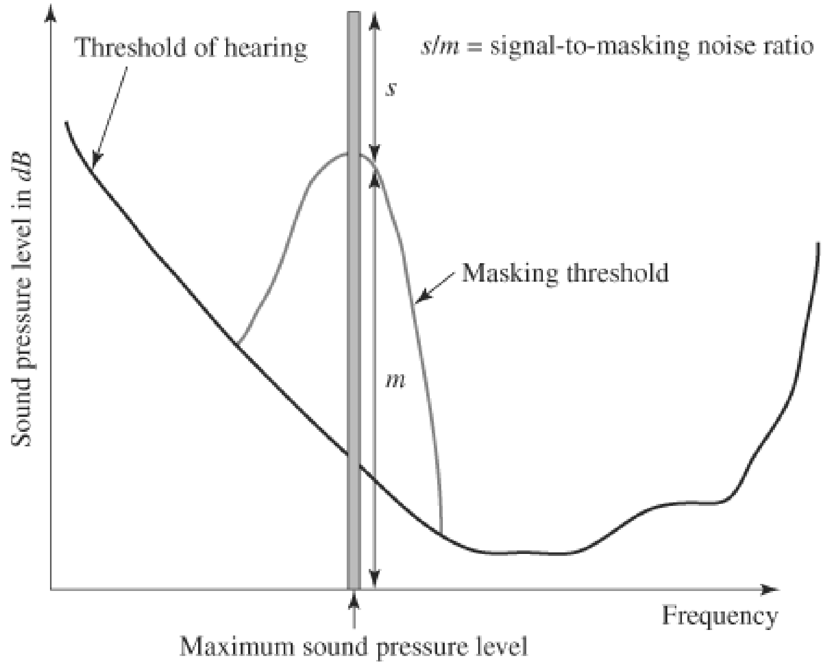
MPEG-1 Audio Compression - Step 5

- Determine how much each band's influence is likely to spread to neighboring frequency bands.
  - It isn't sufficient to deal with bands entirely in isolation from each other, since there can be a masking effect between bands.

MPEG-1 Audio Compression - Step 6

- Find the masking threshold and *signal-to-mask ratio* (*SMR*) for each band, and determine the bit depth of each band accordingly.

- SMR: the ratio between the peak sound pressure level and the masking threshold.


Threshold of hearing
s/m = signal-to-masking noise ratio
s
Masking threshold
m
Sound pressure level in dB
Frequency
Maximum sound pressure level

MPEG-1 Audio Compression - Step 6

- The masking phenomenon causes the noise floor within a band to be raised. When the noise floor is high relative to the maximum sound pressure level within a band, then fewer bits are needed.
- Fewer bits create more quantization noise, but it doesn’t matter if that quantization noise is below the masking threshold. The noise won’t be heard anyway.

MPEG-1 Audio Compression - Step 7

- Quantize the samples for the band with the appropriate number of bits, possibly following this with Huffman encoding.
  - MPEG-1 Layers 1 and 2 use linear quantization, while MP3 uses nonlinear.

MPEG-1 Audio Compression

ALGORITHM 5.2

```
algorithm MPEG-1_audio
/*Input: An audio file in the time domain
  Output: The same audio file, compressed*/
{
  Divide the audio file into frames
  For each frame {
    By applying a bank of filters, separate the signal into frequency bands.
    For each frequency band {
      Perform a Fourier transform to analyze the band's frequency spectrum
      Analyze the influence of tonal and nontonal elements (i.e., transients)
      Analyze how much the frequency band is influenced by neighboring bands
      Find the masking threshold and signal-to-mask ratio (SMR) for the band,
      and determine the bit depth in the band accordingly
      Quantize the samples from the band using the determined bit depth
      Apply Huffman encoding (optional)
    }
    Create a frame with a header and encoded samples from all bands
  }
}
```

Summary

- Audio signal sampling, digitization
- Nonlinear Quantization
- Frequency analysis
- Dynamic processing
- Noise reduction
- Digital audio filters
- FIR filter design
- Perceptual encoding
- MPEG-1 compression
- Lots of additional works on audio processing and analysis (speech recognition and synthesis, music recommendation)