ТВиМС
Д/З от 11.04

Киреев К.
8381

7а

$$F_\xi(x)=\begin{cases} 0, & x\le -2 \\ 1/8, & x\in(-2,-1] \\ 1/3, & x\in(-1,0] \\ 1/2, & x\in(0,2] \\ 1, & x>2 \end{cases}$$

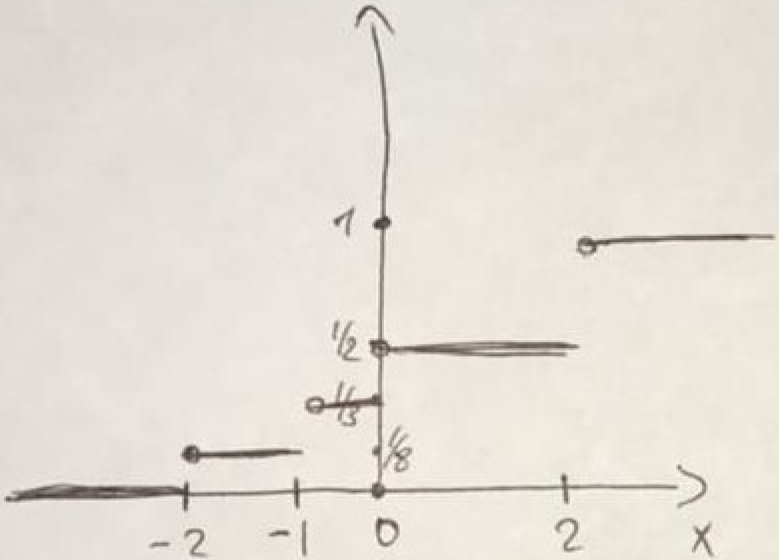
$\eta = G(\xi)$

$G(t)=t^4-t^2+2,\ t\in R$

$\eta=\xi^4-\xi^2+2$

$F_\eta(x)$ - ?

$F_\eta(x)=P(\eta<x)$

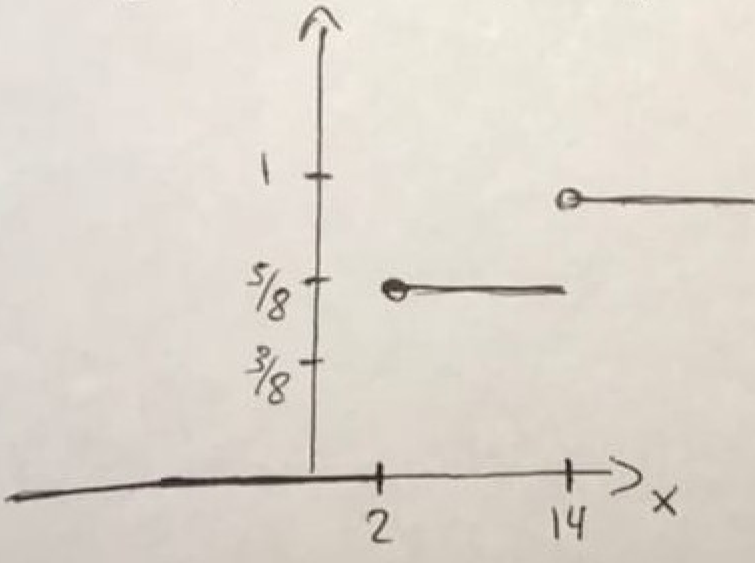


ξ	-2	-1	0	2	Σ
P	$1/8$	$5/24$	$1/6$	$1/2$	1

$\eta(-2)=14$ $\eta(0)=2$

$\eta(-1)=2$ $\eta(2)=14$

η	2	14
P	$3/8$	$5/8$



$$F_\eta(x)=\begin{cases} 0, & x\le 2 \\ 3/8, & x\in(2,14] \\ 1, & x>14 \end{cases}$$

7б

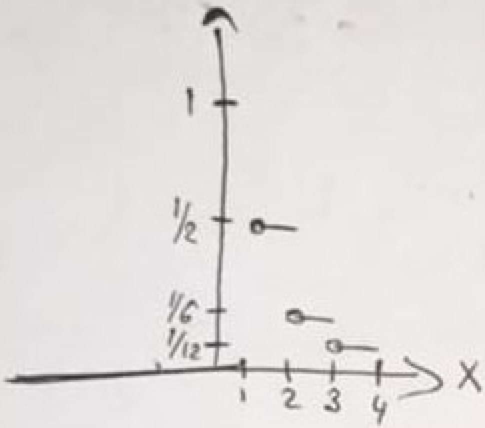
$$F_\xi(x)=\begin{cases}0, & x\le 1\\ 1-1/n, & x\in(n-1,n],\ n=1,2\ldots\end{cases}$$

$\eta=G(\xi);\ G(t)=\sin\pi t,\ t\in R$

$\eta=\sin\pi\xi;\ F_\eta(x)-?$

ξ	1	2	3	n
p	$1/2$	$1/6$	$1/12$	$1/n(n+1)$

$$1-\frac{1}{n}-\left(1-\frac{1}{n+1}\right)=\frac{1}{n+1}-\frac{1}{n}=\frac{1}{n(n+1)}$$



при $\forall\xi:\ \eta=\sin\pi\xi=0$

η	0
p	1

$$F_\eta(x)=\begin{cases}0, & x\le 0\\ 1, & x>0\end{cases}$$

8а

$$p_\xi(x)=\begin{cases}|x|, & x\in[-1,1]\\ 0, & x\notin[-1,1]\end{cases}$$

$G(t)=t^2-1,\ t\in R$

$F_\eta(x)=P(\eta<x)$

$supp(\xi)=[-1,1]$

$supp(\eta=G(\xi))=[-1,0]$

$$F_\eta(x)=P(\eta<x)=P(\xi^2-1<x)=P(\xi^2<x+1)=P(|\xi|<\sqrt{x+1})$$

$$F_\eta(x)=\int_{-\sqrt{x+1}}^{\sqrt{x+1}}|t|\,dt=x+1 \text{ для } x\in(-1,0]$$

$$F_\xi(x)=\begin{cases}0, & x\le -1\\ x+1, & x\in(-1,0]\\ 1, & x>0\end{cases}$$

$$p_\eta(x)=F'_\eta(x)\ \Rightarrow\ p_\eta(x)=\begin{cases}1, & x\in[-1,0]\\ 0, & x\notin[-1,0]\end{cases}$$

(8а) $p_\varepsilon(x) = \frac{1}{\sqrt{2\pi}} \exp(-x^2/2),\ x \in R$ $\quad G(t) = at + b,\ t \in R,\ a, b - \text{const}$

непрер., монот.

$p_\eta(x) = p_\varepsilon(G^{-1}(x))\,|(G^{-1})'(x)|$

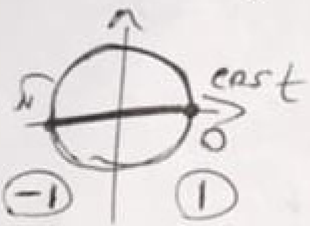
$G^{-1}(x) = \frac{x-b}{a}$

$(G^{-1})'(x) = 1/a$ $\quad \Rightarrow p_\eta(x) = p_\varepsilon\left(\frac{x-a}{b}\right) \cdot \left|\frac{1}{a}\right| = \frac{1}{|a|\sqrt{2\pi}} \cdot \exp\left(\frac{-\left(\frac{x-b}{a}\right)^2}{2}\right)$

(8б)

$p_\varepsilon(x) = \begin{cases} 1/2 \sin x, & x \in [0, \pi] \\ 0, & x \notin [0, \pi] \end{cases}$ $\quad G(t) = \cos t,\ t \in R$

Носитель распределения совпадает с интервалом $[0, \pi]$.



$F_\eta(x) = 0,\ x \le -1$

$F_\eta(x) = 1,\ x > 1$

$F_\eta(x) = ?,\ x \in (-1, 1]$

функция распределения

тобр. $\cos t$: $[-1, 1]$

$F_\eta(x) = P(\eta < x) = P(\cos(\varepsilon) < x) = P(\varepsilon < \arccos x) =$

$= \frac{1}{2} \int_{\arccos x}^{\pi} \sin t\, dt = x + 1/2$

$F_\eta(x) = \begin{cases} 0, & x \le -1 \\ x + 1/2, & x \in (-1, 1] \\ 1, & x > 1 \end{cases} \Rightarrow p_\eta(x) = \begin{cases} 0, & x \notin [-1, 1] \\ 1/2, & x \in [-1, 1] \end{cases}$

$p_\eta(x) = F'_\eta(x)$

82 $p_\varepsilon(x) = \frac{1}{2}\exp(-|x|),\ x\in R \qquad G(t) = t^2,\ t\in R$

Носитель распред. η совп. с $[0,\infty)$ т.к. квадрат

$$F_\eta(x) = P(\eta < x) = P(\varepsilon^2 < x) = P(|\varepsilon| < \sqrt{x}) = P(-\sqrt{x} < \varepsilon < \sqrt{x}) =$$

$$= \int_{-\sqrt{x}}^{\sqrt{x}} \frac{1}{2}\exp(-|t|)\,dt = 2\cdot\frac{1}{2}\int_0^{\sqrt{x}} \exp(-|t|)\,dt = 1 - \exp(-\sqrt{x})$$

$$F_\eta(x) = \begin{cases} 0, & x \le 0 \\ 1 - \exp(-\sqrt{x}), & x > 0 \end{cases}$$

$$p_\eta(x) = F'_\eta(x) \Rightarrow p_\eta(x) = \begin{cases} 0, & x \le 0 \\ \dfrac{\exp(-\sqrt{x})}{2\sqrt{x}}, & x > 0 \end{cases}$$