Introduction to dynamical system modelling

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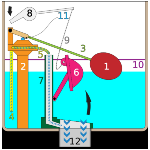
Module 06-23836: Computational Modelling with MATLAB

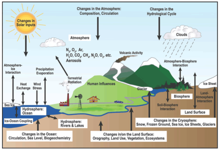
Outline of Topics

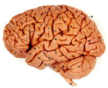
What is a system?

A System: consisting of interconnected components, built or evolved with a desired purpose.
Examples:

- Toilet tank.







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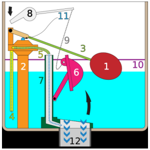

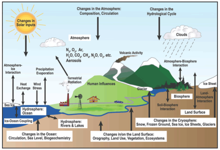

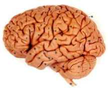

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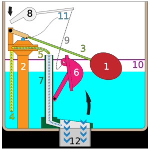

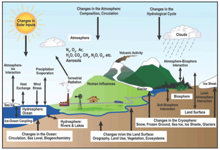

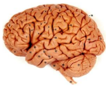

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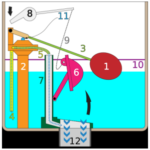

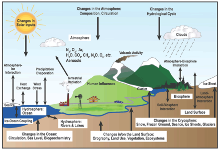

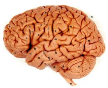

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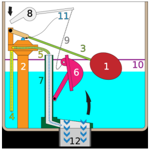

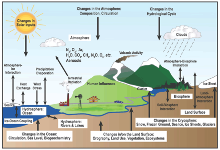

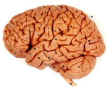

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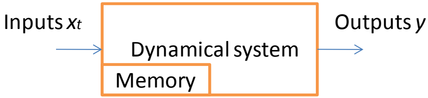
- **Dynamical system**: history $+$ current inputs $\Longrightarrow$ outputs.



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- Mathematically described with differential or difference equations.

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- Biological systems are complex: emergence.
- Reductionism might not work well
- System point of view on biological systems is new - **Systems Biology**.

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- Can be hybrid: Mathematical models + Computational models.

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Modelling methods

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- Other methods: data driven methods.

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- Einstein's razor' (Better!):
    - *The model should be made as simple as possible, but no simpler.*

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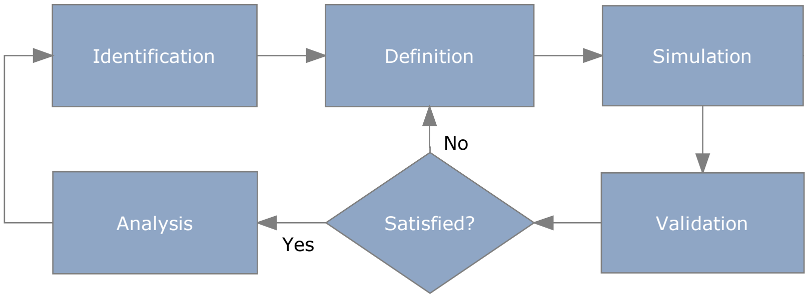
- Model building may feel like muddling through.

Modelling process



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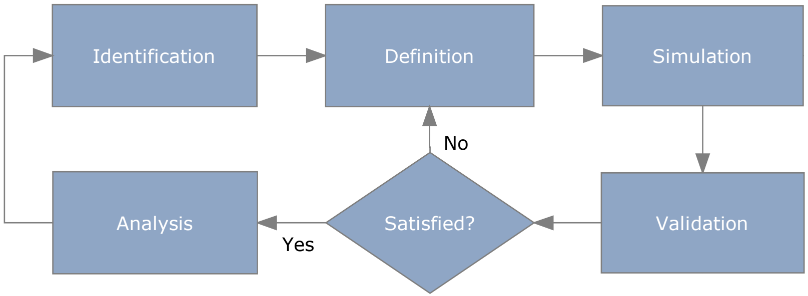

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- You might need to simplify your assumptions.

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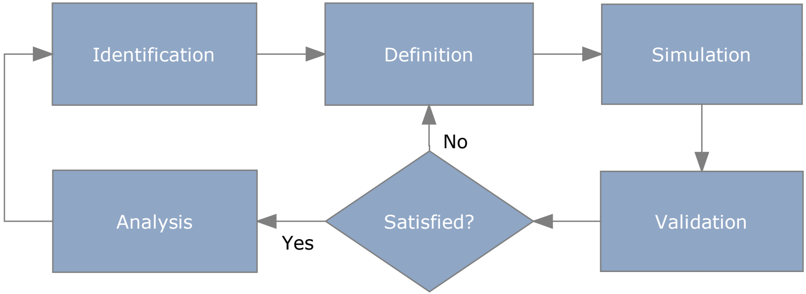

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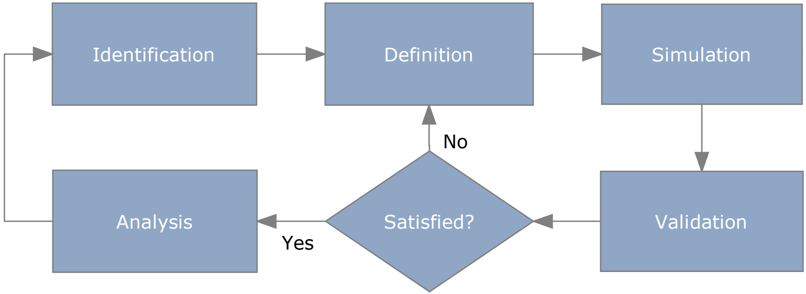

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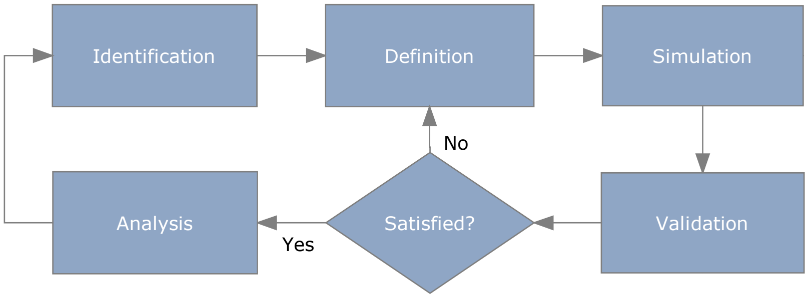

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- If the analysis proposes new questions, we need to go back to identification.

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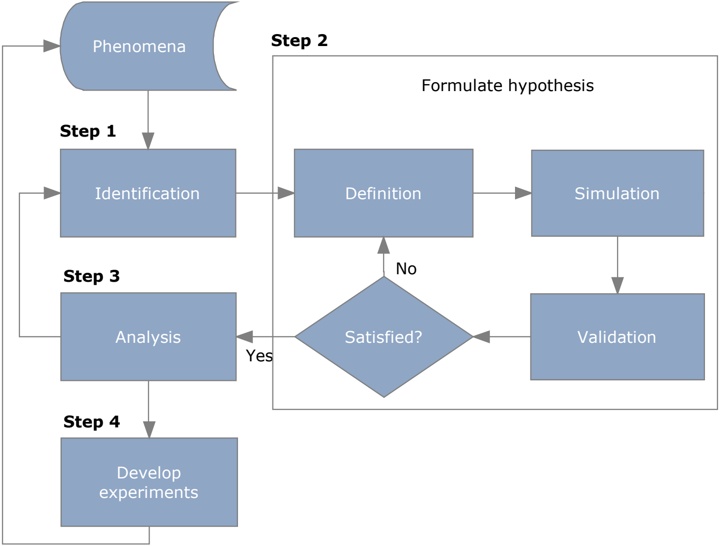
5. Repeat steps 3&4 until there are no discrepancies between hypothesis and phenomena.


Phenomena
Step 2
Formulate hypothesis
Step 1
Identification
Definition
Simulation
No
Step 3
Analysis
Satisfied?
Validation
Yes
Step 4
Develop
experiments

Too abstract?

After learning MATLAB next week, in week 4, I will follow the process to model predator-prey intreactions using Ordinary Differential Equations.