Topology Optimization strategies for continuous fiber-reinforced and functionally graded anisotropic composite structures: A brief review

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Abstract

Topology Optimization (TO) recently gained importance due to the development of Additive Manufacturing (AM) processes able to produce components with good mechanical properties. Among all types of additive manufacturing technologies, continuous fibre fused filament fabrication (CF4) can fabricate high-performance composites compared to those manufactured with conventional technologies. AM provides the excellent advantage of a very high degree of reconfigurability, which is in high demand to support the immediate short-term manufacturing chain in medical, transportation, and other industrial applications. Additionally, CF4 enables the fabrication of continuous fibre-reinforced composite (CFRC) materials and functionally graded anisotropic composite (FGRC) structures. The current version of AM allows us to integrate topology optimization strategies to design realizable fibre-reinforced composite (FRC) and FGRC structures for a given performance. Various TO strategies for attaining lightweight and high-performance designs have been proposed in the literature, which exploits the design freedom of AM. Therefore, this paper attempts to address works related to strategies employed to obtain optimal CFRC and FGRC structures. This paper intends to review, compare existing strategies, analyse their similarities and dissimilarities, and discuss challenges and future trends in this field.

Keywords:
topologoy optimization, fiber-reinforced composites, additive manufacturing, functionally graded material

1. Introduction

2. Introduction

Cost-effective commercially avaiable additive manufacturing (AM) or 3D Printing (3DP) technologies eliminate many limitations that previously plagued the manufacturing of highly tailored structural performance for multi-functional [1] and multi-physics [2] applications. Moreover, AM offers unique capabilities to realize the next-generation lightweight structure have brought great application potentials in several major industries such as aerospace [3, 4], automotive [5] and medical[6]. First, AM techniques have the unique ability to fabricate highly complex shapes without a substantial increase in fabrication costs; also, the benefit of reducing manufacturing preparation time renders these technologies viable for large-scale industries. Moreover, it offers lattice structures, which are lightweight compared to solid parts. Thus, AM offers weight reduction [7] and the ability to dissipate energy [8, 9], heat [10] and vibrations [11]. Printed polymer parts frequently consist of carbon nanotubes and short fibre to upgrade their mechanical performance. Still, printed parts cannot outperform [12, 13], the mechanical strength offered by continuous fibre-reinforced composite laminate manufactured using conventional manufacturing tools. Hence, the shortcomings of 3D printed polymer composites support the development of continuous fibre filament fabrication (CF4). CF4 provides a unique opportunity to reduce part distortion warping and support structures during printing, and fibre tension prevents nozzle clogging, a constant challenge with polymer AM techniques. Additionally, controlling the anisotropic properties of FRCs can effectively distribute the loads throughout the laminate to maximize the strength and stiffness of the fabricated structures.

CF4 technology allows fabrication of FRC material with the continuous spatial in-plane variation of fiber angle and fiber volume fraction, thus expanding the design space compared to that for variable [14] and constant stiffness laminate [15]. Moreover, CF4 technology can achieve out-of-plane variation of fiber angle due to the fiber-reinforced composite's self-supporting characteristics of FRCs. Numerous studies have shown that fiber orientation optimization can significantly tailor structural performance such as stress concentration[16], stiffness [17], buckling load, and the natural frequency [18]. Therefore, the design of the FRC structures requires optimization methods that reflect design freedom offered by CF4 technologies, including constraints, to thoroughly exploit the anisotropic properties of FRC material [19]. These advantages of CF4 have resulted in an interest in utilizing design strategies to enhance the overall functional performance of printed parts. This is in contrast to the geometric-driven or/and cost-driven manufacturing of components. The concept of performance-driven manufacturing is known as Design for Additive Manufacturing (DfAM) [20].

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Topology optimization (TO), one of the DfAM methods, is an iterative design tool to optimize a quantifiable objective while being able to sustain loads, constraints, and boundary conditions. TO is frequently adopted to design structurally sound parts and has subsequentially surpassed design tools, such as shape and size optimization, in isolation. The seminal work of Bendsøe and Kikuchi [21] introduced the concept of TO for the homogenization method; since then, TO has developed rapidly. TO approaches can be summarized as follows: the homogenization method [21], the Solid Isotropic Material with Penalization (SIMP) method [22, 23], the level set method [24, 25], the Evolutionary Structural Optimization (ESO) method [26], and Phase Field [27]. The details of these approaches are discussed in the review papers [28, 29, 30] and some emerging TO methods for smooth boundary representation include the 'Metamorphic Development Method' (MDM) [31], and the 'Moving Morphable Method' (MMM) [32]. The general architecture of TO starts with the definition of maximizing or minimizing a single or multi-target-objective function to fulfil a set of constraints such as volume, displacement, or frequency [33]. Then, as part of an iterative process, design variables, finite element methods (FEMs), sensitivity analysis, regularization, and optimization steps are repeated in this order until convergence is achieved [34].

The optimization concept applied to FRC materials enables the optimal material distribution, optimized orientation of fibre paths, and optimized geometric contours of a laminate to be found. Hence, the optimization method for FRC structures with continuous fibre parameterization schemes and algorithms have notable influences on the quality of the solution. The article is a brief review of topology and fibre path orientation optimization of FRCs, and thus, only related works are reviewed.

3. Topology Optimization for continuum structures

TO optimizes material layout within a given design space Ω for a given set of loads $\{F^T, \Gamma_t\}$, boundary conditions Γ_u, and constraints to maximize the system's performance. It uses FEMs to evaluate the design performance, and the design is optimized using either gradient-based mathematical programming techniques or non-gradient-based algorithms.

3.1. Problem Statement

A general form of topology optimization can be written as an optimization problem:

$$\begin{aligned} \min_{\rho} &: \Phi(\rho, \mathbf{U}(\rho)) \\ &:= \int_{\Omega} f(\rho, \mathbf{U}(\rho)) \\ \text{s.t.} &: G_i(\rho, \mathbf{U}(\rho)) \leq G_i^*, \quad i = 1, \ldots, Q \end{aligned} \tag{1}$$

An objective function Φ represents the quantity being minimized or maximized to to maximize the performance of a system. The density of the material in each location as a design variable ρ for the optimization problem. The constraints G_i are

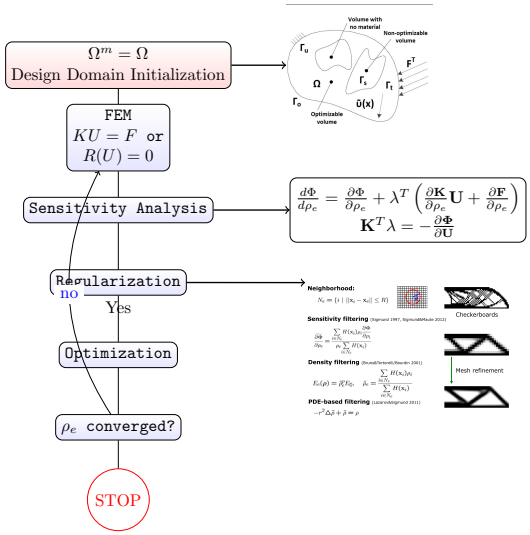

Figure 1: Density-based TO framework

characteristics that the solution must satisfy. The FEM evaluates the field $\mathbf{U}$ that satisfies a linear or nonlinear state equation, since these equations do not have a known analytical solution. The design space Ω defines the optimizable volume V^* for the design to exist, and the volume in the design space that the optimizer cannot modify is considered non-optimizable volume Γ_s. A characterisitic function χ is defined to descirbe the material domain Ω_d to be optimized:

$$\chi(\mathbf{x}) = \begin{cases} 0, & \forall \mathbf{x} \in \Omega \backslash \Omega_d \\ 1, & \forall \mathbf{x} \in \Omega_d \end{cases} \tag{2}$$

where $\mathbf{x}$ stands for a design point in Ω and $\chi(\mathbf{x})$ is defined by a scalar function ϕ and the Heaviside function H such that:

$$\chi(\mathbf{x}) = H(\phi(\mathbf{x})) = \begin{cases} 0, & \forall \mathbf{x} \in \Omega \backslash \Omega_d \\ 1, & \forall \mathbf{x} \in \Omega_d \end{cases} \tag{3}$$

To eliminate checkerboard patterns therefore generating mesh-independent results, the Helmholtz PDE filter [35] is introduced to regularize ϕ:

$$-R_{\phi}^2 \nabla^2 \tilde{\phi} + \tilde{\phi} = \phi \tag{4}$$

where R_{ϕ} is the filter radius, and $\tilde{\phi}$ is the filtered field. Then the density field ρ can be defined by an additional smoothed Heaviside function $\tilde{H}$:

$$\rho = \tilde{H}(\tilde{\phi}) \tag{5}$$

After the series of regularization from ϕ to ρ, the resulting density field is bounded between 0 and 1. The general architecture of the topology optimization framework is depicted in the schematic.

The following broader categorization of the implementation has been used to solve topology optimization problems.

3.2. Discrete Topology Optimization

The discrete variable optimization problem can be formulated as follows:

$$\begin{aligned} \min_{\rho} &: \Phi(\rho, \mathbf{U}(\rho)) \\ \text{s.t.} &: \sum_{e=1}^{N} v_e \rho_e = \mathbf{v}^T \rho \leq V^* \\ &: g_i(\rho, \mathbf{U}(\rho)) \leq g_i^*, \quad i = 1, \ldots, M \\ &: \rho_e = \begin{cases} 0(\text{ void }) \\ 1(\text{ material }) \end{cases}, e = 1, \ldots, N \\ &: \mathbf{K}(\rho)\mathbf{U} = \mathbf{F} \end{aligned} \tag{6}$$

The TO problem is a binary problem representing the void and solid regions of the structure. The discrete TO uses binary design variables, one for each of the finite elements in the mesh of the structural domain. The design variable 1 implies the finite element is filled with material, 0 indicates void. For example, consider an objective function $\Phi(\rho, \mathbf{U}(\rho))$, constrained by $g_i \leq g_i^*.i \in [1, M]$, with ρ being the design variables, where N number of finite elements, $\mathbf{K}$ and $\mathbf{U}$ is the assembled stiffness matrix and displacement vector corresponding to finite elements in the mesh.

The well-known discrete topology optimization method is the Bi-directional Evolutionary Structural Optimization (BESO). Interested readers find comprehensive reviews on the BESO methods in [36, 37]. Another outlook on approaching the discrete problem is using a genetic algorithm [38] that can find "global minimum" and allow handling a discrete variable, but this always sacrifices the computational cost. Furthermore, Sigmund [39] questions the usefulness of non-gradient approaches in TO. Recently, Sivapuram et al. [40] combined the features of BESO and the sequential integer linear programming for discrete topology optimization.

3.3. Continuous Topology Optimization

The continuous variable optimization problem can be formulated as follows:

$$\begin{aligned} \min_{\rho} &: \Phi(\rho, \mathbf{U}(\rho)) \\ \text{s.t.} &: \sum_{e=1}^{N} v_e \rho_e = \mathbf{v}^T \rho \leq V^* \\ &: g_i(\rho, \mathbf{U}(\rho)) \leq g_i^*, \quad i = 1, \ldots, M \\ &: 0 \leq \rho_{min} \leq \rho \leq 1 \\ &: \mathbf{K}(\rho)\mathbf{U} = \mathbf{F} \end{aligned} \tag{7}$$

Density-based topology optimization is a broadly received idea in the TO of continuum structures by using continuous density design variables that transform the binary variable optimization problem into a density distribution problem. The design variable can take any value from 0 to 1 such that $\rho \in [0, 1]$; thus, the material properties across the elements are varied. Therefore, material properties are interpolated using a power function, where the intermediate properties are penalized, thus forcing the design back to the binary structure.

This transformation enables the use of gradient-based information; unfortunately, the penalization parameter also introduces nonconvexities; hence, there is a high risk of falling into local minima. Furthermore, in SIMP, optimized solutions do not explicitly exhibit structural boundaries, which provides a challenge when solving problems where explicit boundary identification is essential, e.g., in design-dependent and multiphysics problems. Thus, TO can be formulated in the nodal variables [41] that control an implicit function description of the shape to address the exact boundary identification problem.

4. A brief note on optimization algorithms in TO

Reasonably arranging the fiber orientation is critical to effectively handling an anisotropic material, which is vital for designing next-generation lightweight composite structures. Frequently, fiber orientation optimization creates a difficulty associated with local optima and discontinuous functions. Thus, to address this, gradient-free algorithms [42] are more qualified because of their global searching ability [43, 44]. Furthermore, by allowing differentiable functions, mixed design variables, and discrete space, introduce a relaxed formulation that has the advantage of obtaining fewer local optima. However, the inefficiency of most gradient-free algorithms requires numerous function evaluations, which is impractical for expensive finite element simulations. Hence, adoption of gradient-based algorithms, i.e., Optimality Criteria Method (OCM), Method of Moving Asymptotes (MMA) [45], and Sequential Linear Programming (SLP) [46] become a reasonable choice for the TO problems.

The OCM is derived using the Lagrange function, which is composed of objective and constraint functions that satisfy the Karush-Kuhn-Tucker (KKT) condition for an optimal solution. The OCM procedure has double loops, where the inner loop updates the design variable, and the outer loop updates the Lagrange multiplier based on the KKT condition. However, the method cannot handle multiple constraints because the coupling of the Lagrange multiplier and the design variables requires solving a nonlinear equation. Therefore, Shen et al. [47] questioned the lack of understanding about the orientation optimization algorithm to handle arbitrary constraints and loads in the OCM. A step length scheme for orientation optimization is advised to achieve global descent by normalizing the gradient vector and introducing a parameter to control the magnitude of material orientation in each iteration. However, the verification lacks the effect of adding constraints in the orientation optimization problem on the update scheme, a critical factor for the OCM. Thus, a more generalized OCM for the topology optimization of an anisotropic material is demanded from scalability and multiloading situations. Recently, Kim et al. [48] interpreted the work of Patnaik et al. [49] on parametric optimization and proposed a generalized optimality criteria method for topology optimization problems. The approach eliminates the compulsion to satisfy the constraints during every optimization iteration but should be met upon convergence.

On the other hand, SLP and MMA are general-purpose optimization strategies supporting various engineering multi-

objective and multi-constraints nonlinear problems (NLP). In these first-order methods, the gradient information about a design point approximates the constraint and objective functions. In particular, for MMA, a hybrid form of the linear and reciprocal approximation [50] has the advantage of being convex, which introduced the term convex linearization (CONLIN) [51] for approximating the optimization problem. Svanberg introduces a convex approximation variation that stabilizes and speeds up the convergence of process optimization by controlling moving asymptotes while the approximation remains convex and first-order. Furthermore, because the subproblem is separable and convex, a dual approach or a primal-dual interior-point method can efficiently solve the NLP. However, the reciprocal approximation in MMA might eliminate the linearity of approximation; therefore, SLP can serve as a suitable candidate unless convexity is not needed [52].

5. Paramterization schemes for fiber orientation

The parameterization scheme implements a numerical description of fiber orientation patterns and defines variables for the optimization. It should ensure spatial continuity of fiber angles so that CF4 technology can produce the structure. It should also provide enough design freedom so that the optimization algorithm can consider more candidate designs. This section discusses various parameterizations schemes used in the literature to optimize fiber-reinforced composite structures.

5.1. Continuous parameterization

The continuous parametrization of fiber orientation (CFO) design uses the angle itself as the design variable [53, 54]. The design variable is the continuous and independent parameter that provides flexibility in changing the orientation across the design points, relaxing orientation design space. However, handling a continuous fiber orientation design presents difficulties due to a fourth-order transform tensor that rotates to a given angle composed of multivalued sine and cosine functions, resulting in a non-convex optimization problem. Furthermore, optimizing the fiber orientation is susceptible to the initial fiber configuration, thus causing difficulties obtaining the optimized solution. As illustrated in [55], suboptimal solutions are the persistent outcome of a continuous fiber orientation design problem. One brute-force way to avoid it is by further relaxing the design space. For instance, free material optimization (FMO)[56, 57] parameterizing each stiffness tensor element independently as the design variable. This secures the scheme from the complexity of the orientation design variable of the design space. However, as compensation, point-wise nonlinear constraints ensure the positive semi-definitiveness of the obtained stiffness tensor and link it to the feasible physical design, making this approach challenging. Nomura et al. [58] formulated orientation design variable as a tensor field to simplify the first tensor invariant constraint and remove nonlinear constraints successfully introduced due to the second tensor invariant. Still, as commented, the violation of these constraints is observed at the joint point of the structural members where the orientation shows the discontinuous distribution.

For anisotropic materials, early studies utilizing the analytically derived optimally criterion [59] for optimizing fiber orientation dates back to the pioneering work of Pedersen on strain-based method [60, 61, 62]. In that work, strain energy density was transformed into principal strain, and it was concluded that material orientation axes that lie along principal strain axes always give stationary energy density. However, Cheng [63] argued that the discussion is limited to a unit cell case where the orientation variable is separated from the design domain to obtain extreme strain energy. After that, a similar deduction using iterative optimality criteria [64, 65] formulated the stress-based method [66] by exercising an invariant stress field for material orientation. Finally, Diaz and Bendsoe [67] extended the stress-based method to determine the optimal orientation optimization problem corresponding to multiple loads. Despite their similarity, the stress-based method produces a slightly stiffer structure than the strain method because strong couplings exist among the orientational variables when the strain field is used [63]. Conclusively, Gea and Luo [68] demonstrated that the fiber orientation coincides with the principal stress/strain fields for relatively weak shear and some strong shear types of anisotropic materials.

Further, the methods are highly dependent on the initial fiber configuration, and both approaches will fail for shear 'strong' type materials due to repeated global minimum solutions. Nevertheless, these methods form the basis for future research on material orientation optimization for FRC materials. The shortcomings of these methods encouraged the formulation of the energy-based method introduced by Luo and Gea [69, 70]. This method uses an inclusion cell to estimate the dependency of the strain fields and stress fields on the fiber orientation by introducing an approximate energy factor. Yet, the dependence of energy factors on the traction stress, material properties, and direction of the inclusion cell and its surroundings make it challenging to formulate the framework for 3D and complex loading problems. Following the principles of the energy-based method, Yan et al. [71] proposed a hybrid stress-strain method by weighting the optimality condition of the mean compliance in the stress and strain form. Numerical examples demonstrate their method on weak and strong shear materials and extension to 3D problems. The assumption regarding the elemental strain and stress field invariant to the neighboring elemental orientation is considered; however, it may restrict the solution of 3D problems and result in a suboptimal solution.

An alternative is employing curvilinear parameterization schemes that define fiber paths as the graphs of analytical function, which guarantee continuity of fiber angle and have a small number of design variables [72, 73, 74]. Nevertheless, the restrictive design search space will limit the tailoring of the fiber path, thus deteriorating the stability of the optimization problem [14] and quality of the optimized solution. Also, the parameterization schemes can follow equidistant iso-contours of a level set function to represent curvilinear fiber paths [75, 76], naturally ensuring fiber continuity and being often parallel to the neighboring fiber paths. Furthermore, the optimization result becomes highly dependent on the initial configuration, and local solutions often appear [77].

5.2. Discrete paramterization

The counter scheme reduces the orientation design space to avoid multiple local optima issues, where the optimized solution is highly sensitive to the initial fiber configuration. Therefore, a discrete orientation optimization formulation was solved using a genetic algorithm at the cost of a computational burden [78] [79] [80]. Lund [55] relaxes the combinatorial problem to a continuous optimization problem. By choosing discrete fiber orientation candidates, which are defined a priori, the orientation design space is parametrized into discrete material candidates. These transversely isotropic material models are defined for different fiber orientations for the same isotropic elasticity tensor. Thus the scheme share some similarities with the multi-material optimization problem in [81, 82]. The suggested scheme assigned weighting functions to different candidates and employed gradient-based optimization with penalization coefficient, forcing the weighting functions to seek a binary design and fiber convergence, i.e., one discrete material at each design point. This method is known as Discrete Material optimization (DMO). DMO laid the foundation for Shape function and with penalization (SFP) [83], bi-value coding parametrization (BCP) [84] to perform discrete fiber orientation optimization. A comparison for these methodologies that use various numerical examples is contained in [85].

DMO does not incorporate design problems for continuously varying orientation distributions. First, it is an imperative design consideration to circumvent stress constraints and a degradation in the strength by an order of magnitude compared to that for continuous fibre paths due to fibre discontinuity. Consequently, it permits a limited scope to fully exploit the potential of modern technology's continuously varying orientation in composites [86, 17, 16]. Secondly, these methods fail to address the fiber convergence even against the significant penalization factor; hence, their benefit relies on an optimization algorithm to circumvent impractical mixtures of fiber orientations. Third, the discrete parametrization schemes should further minimize the number of design candidates for efficient optimization. Concerning these drawbacks, Kiyono et al. [85] proposed a parametrizing scheme that continues the computational approach suggested by Yin and Ananthasuresh [82]. Introducing a normal distribution function as a weighting function in their parametrizing scheme guarantees fiber convergence, a low sensitivity to the initial fiber configuration, and continuity of the fiber orientation. Another different work proposed a self-penalization interpolation model for fiber orientation (SPIMFO) based on convergent Talyor series for sine and cosine functions to optimized composite hyperelastic material [87] and the dynamic design of laminated piezo-composite actuators [88].

5.3. Hybrid paramterization

Utilizing continuous and discrete methodology benefits is another alternative to fiber orientation optimization. The key idea in the following approaches is to fill the gaps by acknowledging the beneficial characteristics of both strategies to improve computational efficiency and/or reduce local optima and/or resolve fiber continuity and/or manufacturability issues. Therefore, an approach to reduce the risk of falling into local optimal without sacrificing the fiber continuity can use both discrete and continuous parametrization as suggested by Luo et al. [89]. Their work proposed a coarse-to-fine strategy, where the orientation design space is divided into discrete sub-intervals. After that, the CFO searches for an optimized solution in a sub-interval, where the sub-interval selection problem is solved using the DMO approach. However, no criterion is defined to determine the number of sub-intervals required in advance.

Nevertheless, the proposed strategy provides flexibility to integrate alternatives that are suggested for DMO and CFO approaches. Nomura et al. [90] studied the cartesian system for orientation design variables to improve initial design dependency and local optima issues encountered in the continuous parameterization approach. The parametrization scheme was further extended to yield an optimized discrete orientation design for a given discrete orientation set in their work. Moreover, the characteristics representing the orientation design variables into the vectorial form consider the 2π ambiguity, which occurs due to the periodic nature of the orientation design variable. Introducing vectorial design variable as a point-wise quadratic inequality constraint yields more interpolated elasticity tensor than the single variable polar representation. However, the optimization algorithm treats continuous and discrete problems as two different problems; thus, investigating the coupled optimization framework might be an outlook for consideration.

Xia and Shi [91] develops a continuous global function by applying the shepherd interpolation method at scattered design points to represent the fiber orientation throughout the design domain. The benefit of the interpolation function is that it ensure fiber continuity while considering a reduced orientation design space in contrast with CFO. Unfortunately, it suffers from the initial configuration and ends at the sub-optimal solution. Another work of Xia [92] applied multilevel optimization for fiber orientation optimization and verified its efficiency against the single-level optimization. Still, the optimization results in different fiber arrangements for different initial fiber orientations. As a result, the efficiency of the multilevel approach relies on the attained fiber orientation field at a coarse level since the optimization at the successive refined level starts from an initial design computed at its neighboring coarser level.

5.4. Feature-based parameterization

The parameterization, as mentioned earlier, introduces low-level fiber material representations, such as pixel or voxel-based, thus representing the designs with variables proportional to the number of pixels or voxels in the design space. Moreover, these techniques render organic and free-form designs, which require sophisticated postprocessing to distinguish fiber paths for the use of CF4. Therefore, to avail manufacturable solutions with designs with few variables, fiber material can be considered as a geometric feature with high-level parameters. High-level parameters refer to spatial dimensions associated with the size, position, or orientation of a feature. Finally, feature-mapping techniques map these features onto a fixed mesh for analysis; an extensive review of feature-mapping methods by

Wein et al. [93] details the components of feature-mapping techniques and discusses their implementation in structural optimization. Geometry projection [94] is a feature-mapping technique extended to represent the design via cylindrical bars reinforced with continuous fibers [95] and performs the analysis using a fixed finite element mesh. The interpolation of the material properties at the junction of multiple bars made of an anisotropic material is penalized as a convex combination of the penalized effective densities for each component. It demonstrates the method can easily integrate shape constraints on the structural form offered by CF4. Thus, this work introduces the groundwork for using the geometry projection method for fiber-orientation optimization design problems.

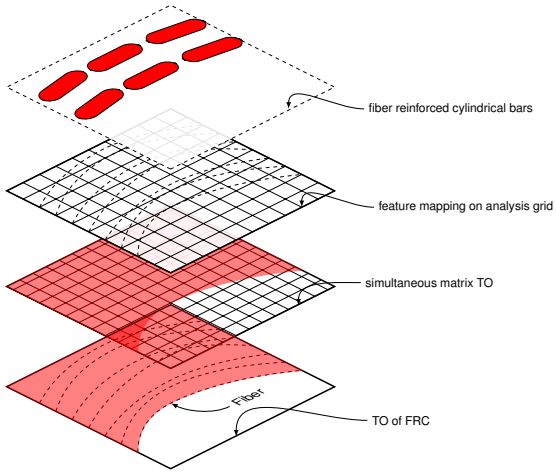

Figure 2: Schematics for hybrid geometry projection method for FRC

6. Functionally graded anisotropic material

Functionally graded composites [96] are inhomogeneous materials that consisting of two or more materials and are engineered to continuously vary the spatial composition and structure. Recent studies [97, 98, 99] have shown that CF4 is ready to manufacture FRC structures with continuous yet spatially varying fibre paths and fibre volume fractions. Thus, if properly optimized, the spatial variation in FRC material properties may result in better performance than that for a fixed FRC material volume fraction. Therefore, a composite structure comprising a FRC material with voids and a variable fibre density is termed a functionally graded anisotropic material (FGRC). Furthermore, the aforementioned gradation of a FRC material provides a considerably larger design freedom to CF4. Accordingly, Li et al. [100] considered a SIMP-based sequential TO approach to design FGRC by considering fiber fraction along with material fraction in a given design space. A sequential process begins with designing an isotropic-material matrix with voids, inserting fiber selectively, and then, optimally orienting the fibers. However, this approach sacrifices the exploration of new topologies that might be optimal for FGRC. Therefore, the following works investigated the simultaneous design of isotropic material matrix topology, fiber material layout, and orientation.

Desai et al. [101] applied the topological derivative method to tailor a spatially varying fiber fraction. In addition, the dense arrangement of fibers was evenly spacing for the manufacturability of the part while retaining their specific patterns. However, the structural performance as a result of the simplification of the dense fiber arrangement was not evaluated, thus throwing into question the reliability of the printed part. In their work, a different fiber orientation approach was achieved by computing anisotropic topological derivatives in the polar coordinate system. A possible solution to avoid postprocessing for dense fiber arrangements with their technique might be to consider feature-based mapping methods.

The work, as mentioned earlier, implemented single scale approaches to optimize the distribution and orientation of the FRC material. However, CF4 also provides an effective means to fabricate mono-scale structures and multiscale structures. Thus, spatially varying material distributions and geometric patterns spanning at least two or more scales hold a promising future for designing next-generation lightweight structures. On the other hand, the multiscale strategy for anisotropic materials is challenging due to the following reasons: length scale controls, ability to produce models for fracture and damage criteria to capture actual anisotropic behaviour, and unique treatments at the boundaries of the domain, for example. These factors must be investigated through experiments or the use of appropriate numerical tools to estimate the actual performance of printed parts. Only a few works address the multiscale approach for FRCs based on the knowledge of the authors. Hence, interested readers can refer to Wu et al. [102] review paper to understand the general framework for multiscale TO. Kim et al. [103] adopted the homogenization method for designing spatially varying fiber volume fractions and fiber orientations, and simultaneously, the SIMP was used to design the macrostructure composite topology. Finally, the de-homogenization procedure [104] applied to fiber microstructures obtained in the coarser mesh was visualized by projecting at a finer mesh. Various benchmark and multi-load structure problems have been studied and concluded that locally varying FRC materials augment the global stiffness to the structure more than a fixed fiber volume fraction or isotropic multi-material structure. In continuation of the Kim methodology, Jung [105] proposes a 3D TO approach for designing a FGRC with spatially-varying fiber fractions and orientations. In conclusion, the multiscale framework has further enabled us to exploit the design freedom offered by CF4; however, no study to date fabricates and experimentally validates the results of a FGRC.

7. Discussion and future trends

The discussion focuses on the suitability of given TO for anisotropic materials given a pre-requisite understanding of the manufacturing process and its limitations. Therefore, the following discussion does not address composite manufacturing and its differences to adopt the particular TO method.

The manufacturing design freedom extended by available composite manufacturing processes provides the flexibility to develop and integrate TO approaches for designing anisotropic

material orientations [106]. Therefore, the paradigm of performance-driven design focuses on investigating the suitability of TO methods that can fully exploit the design freedom offered by manufacturing technologies. Thus, the existing techniques for material orientation are categorized into four major classes, as stated previously.

The optimization of a prescribed set of alternative discrete angles, which is referred to as the DMO, is often preferred in the aerospace, automotive, and wind turbine industries for manufacturability reasons. The DMO approach is favourable for composite laminate designs [107] [108] [109] because a mixed-integer programming problem is formulated as a continuous problem that can be solved efficiently using gradient-based optimizers. As a result, substantial problems that might not be amenable to gradient-free methods can use DMO parameterization. An indirect approach is to apply lamination parameters, as introduced by Tsai and Pagano [110]. However, despite these methods' popularity, they are limited to a prescribed set of alternative discrete angles, while the CF4 processes have higher freedom in orientation control can produce higher performing composites.

Therefore, the continuous orientation methods are most suitable for CF4 processes. Furthermore, these methods provide the highest freedom in terms of shape and variable stiffness. Thus, the continuous orientation formulation directs the material deposition path planning to ensure the fiber trajectory curvature, fiber continuity, fiber fraction, and offset distance between adjacent fibers, unlike discrete methods where the fiber convergence and fiber continuity are challenging to attain. Papapetrou et al. [76] designed the topology and material orientation in parts simultaneously; the optimized results were post-processed using continuous fiber path planning to ensure realizability. A sequential scheme was proposed [111, 112] where the fiber placement that was based on load transmission follows isotropic TO; this is contrary to Liu [113], who adopted concurrent fiber path planning and structural TO. The multi-axis material deposition technology using a robotic arm requires an extension of the TO algorithm to envelop the 3D fiber orientation, in contrast to in-plane printing. Schmidt et al. [114] introduced azimuth and elevation angles to extend the CFO method for 3D fiber orientation. In addition, they emphasized the issues of nonconvexity of the compliance and sensitivity to the initial fiber orientations by investigating the orientation parameter space to mitigate the problems[115]. Finally, the realizability of 3D printed composite is studied by Fedulov et al. [116], where they proposed a filtering technique for fast convergence.

Utilizing TO methods for exploring the CF4, generally speaking, heighten the composite manufacturing cost, especially when committing these technologies for large-scale structure parts. Therefore, understanding the trade-off among commercial aspects, i.e., realizability, practicability, and structural design, requires assimilating the benefits of the discrete, continuous, and multi-component methodology. Thus, a hybrid parameterization scheme optimizes the structural topology and material orientation, including multi-component optimization (MTO) that decomposes product geometry while guaranteeing manufacturing constraints that might significantly impact the quality and cost of the end product. Initially, a genetic algorithm was used to solve MTO [117], and then recently, a gradient-based optimization algorithm was used by Zhou et al.[118]. Zhou et al. [119] further extended their work for structures made of multiple composite components with tailored material orientations, without a prescribed set of alternative discrete angles. Therefore, this method can produce regions fabricated separately and joined with either continuous or discrete material orientation methods.

Feature-based parametrization follows the ideology of ready-to-manufacturability with a necessary restriction on the spatial distribution of the fiber orientations. It envelops commercial aspects for the realizability of composite parts by introducing CAD-based features to ease the manufacturing process with the potential for layerwise design. Moreover, it further simplifies the design space by reducing the design variables considerably. It is noted that published work only considered stiffness-driven design. However, it is also critical to consider failure modes for composite parts manifested using the layerwise AM process. Incorporating these failure criteria renders markedly different designs that raise the method's relevance in fabricating FRC structures.

8. Closure

This work presents a brief review of TO methods for FRC structures applied to CF4. First, the study underlines the single-scale TO approaches that simultaneously or sequentially design fiber orientation and structural's topology. It further reports the usefulness of multiscale TO for realizing FRC and extending it for FGRC structures. Moreover, it highlights emerging TO methodologies such as feature-mapping and multi-component are suitable choices to attain ready-to-manufacture design and to introduce manufacturing constraints with ease.

The TO methods can attain continuous fiber orientation by incorporating continuous or discrete fiber angle or both as a design variable in the numerical optimization scheme. However, continuous strategies are susceptible to the initial fiber configuration due to the problem's nonconvexity and restrictive design space. On the other hand, discrete strategy solves the problem's nonconvexity using the multi-material method but increases the number of design variables. Therefore, a hybrid implementation that couples both strategies serve as a promising strategy to avoid earlier mentioned issues, but an efficient algorithm still needs to be addressed. In addition, the recent use of the geometry projection method for anisotropic materials is new attention in the fiber orientation optimization methods.

Overall, continuous parameterization schemes are considered for spatially varying fiber orientation and/or fiber volume fraction. In addition, these methods are amenable to CF4 and multi-axis material deposition technology. However, realizing part using continuous strategies may require path planning techniques to account for manufacturing constraints, whereas discrete strategies clearly illustrate the fiber paths in the design domain; hence parts can avoid further postprocessing. Therefore, DMO is widely chosen in the automotive and aerospace industries due to manufacturability ease. On the other hand,

feature-mapping methods provide ready-to-manufacture parts; however, it still requires validation for complex problems, such as bucking stability and eigenvalue analysis. Hence, the choice of the TO methods for designing complex FRC parts applied to AM requires various considerations; thus, this review highlights the different aspects of TO methods used for FRC structures to laid an essential foundation for the researchers entering the field of the TO of FRC material.

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