Graphical Abstract

Topology Optimization strategies for continuous fiber-reinforced and functionally graded anisotropic composite structures: A brief review

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Highlights

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- Research highlight 1

- Research highlight 2

Topology Optimization strategies for continuous fiber-reinforced and functionally graded anisotropic composite structures: A brief review

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Abstract

Among all types of Additive Manufacturing (AM) technology, Continuous Fiber Fused Filament Fabrication (CF4) can fabricate high-performance composites compared to those manufactured with conventional technologies. AM provides the excellent advantage of a very high degree of reconfigurability, which is in high demand to support the immediate short-term manufacturing chain in medical, transportation, and other industrial applications. Additionally, the CF4 capability enables the fabrication of Continuous Fiber-Reinforced Composite (CFRC) materials and Functionally GRaded Anisotropic Composite (FGRC) structures. The current expedition in AM allows us to integrate Topology Optimization (TO) strategies to design realizable FRC and GFRC structures for a given performance. Various TO strategies for attaining lightweight and high-performance designs have been proposed in the literature, which exploits AM's design freedom. Therefore, the paper attempts to address works related to TO strategies employed to obtain optimal CFRC and FGRC structures. This review intends to overview, compare existing strategies, analyze their similarities and dissimilarities, and discuss challenges and future trends in this field.

Keywords:

1. Introduction

Additive Manufacturing. The cost-effective commercially available Additive Manufacturing (AM) or 3D Printing (3DP) technologies eliminate many limitations that previously plagued the manufacturing of highly tailored structural performance for multifunctional [] and multi-physics [] applications. Moreover, the short metamorphosis of AM technologies offers unique capabilities to realize the next-generation lightweight structure have brought great application potentials in several major industries such as aerospace [1, 2], automotive [] and medical[3]. First, AM's unique ability to fabricate a highly complex shape without a substantial increase in fabrication costs, along with the benefit s of reducing manufacturing preparation time, renders these technologies worthy investment for large-scale industries. Moreover, it offers the fabrication of lattice structures-a lightweight design comparative to the solid-filled parts. Thus, offer diversification of design to answers multifunctional material requirements in weight reduction, [4], their ability to dissipate energy [5, 6], heat [7] and vibration [8]. The printed polymer parts frequently consist of carbon nanotubes and short fibre to upgrade their mechanical performance. Still, it cannot outperform [9, 10], the mechanical strength offered by continuous fibre-reinforced composite laminate manufactured using conventional manufacturing tools. Hence, the shortcomings of 3D printed polymer composites aggravated the demand to develop Continuous Fiber Filament Fabrication (CF4) technology. CF4 technology provides a unique opportunity to reduce part distortion warping and support structures during printing, and fibre tension prevents nozzle clogging, a constant lookout with the polymer AM. Additionally, controlling the anisotropic properties of the fibre-reinforced composites can effectively distribute the loads throughout the laminate to maximize the structure's strength and stiffness. Recognizing these potentials of AM, returned a resurgent interest in utilizing the design strategies to exploit AM's performance-driven manufacturing technologies towards enhancing printed parts' overall functional performance. In contrast, to the geometric-driven or/and cost-driven manufacturing of components. The concept of performance-driven manufacturing is known as Design for Additive Manufacturing (DfAM) [11].

Topology optimization. Topology Optimization (TO), one of the DfAM methods, is an iterative design tool to optimize a quantifiable objective while being intended to sustains loads, constraints, and boundary conditions. Topology Optimization is frequently adopted to design structurally sound parts and has subsequentially surpassed optimization design tools such as shape and size optimization in isolation. The seminal work of Bendsøe and Kikuchi [12] introducing the concept of TO on the homogenization method; since then, TO has been developed rapidly. TO approaches can be summarized as follows: the homogenization method [12]; the Solid Isotropic Material with Penalization (SIMP) method [13, 14], the level set method [15, 16], the Evolutionary Structural Optimization (ESO) method [17]; Topology Derivatives [? ?] and Phase

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Field. The details of these approaches are discussed in the review papers [18, 19, 20] and some emerging TO methods for smooth boundary representation include the 'Metamorphic Development Method' (MDM) [21], and the 'Moving Morphable Method' (MMM) [22]. The general architecture of TO starts with the definition of maximizing or minimizing a single or multi-target-objective function to fulfil a set of constraints such as volume, displacement, or frequency [23]. Then, as part of an iterative process, design variables, Finite Element Analysis (FEA), sensitivity analysis, regularization, and optimization steps are repeated in this order until convergence is achieved [24].

Topology Optimization of fiber-reinforced composite materials. T CF4 technology allows fabrication of FRC material with the continuous spatial in-plane variation of fiber angle and fiber volume fraction, thus expanding design space as opposed to variable [25] and constant stiffness laminate [25]. Moreover, CF4 technology can achieve out-of-plane variation of fiber angle due to the fiber-reinforced composite's self-supporting characteristics. Numerous studies have shown that fiber orientation optimization can significantly tailor structural performance such as stress concentration[26], stiffness [27], load-bearing capacity, buckling load, and the natural frequency [28]. Therefore, the design of the FRC structures requires optimization methods that reflect design freedom offered by CF4 technologies, including constraints, to thoroughly exploit the anisotropic properties of FRC material [29]. The optimization concept applied to FRC materials allows finding the optimized material distribution, the optimized orientation of fiber paths, as well as optimized geometric contours of the laminate. Hence, the optimization method for FRC structures with continuous fiber parameterization schemes and optimization algorithms have notable influences on the quality of the solution. The article on the optimization of topology and fiber path orientation and thus, only related works are reviewed.

2. Paramterization schemes for fiber orientation

The parameterization scheme implements a numerical description of fiber orientation patterns and defines variables for the optimization. It should ensure spatial continuity of fiber angle so that CF4 technology can produce the structure, and it should also provide enough design freedom so that optimization algorithm can consider more candidate designs.

Continuous Parameterization

The straightforward parametrization of fiber orientation design uses the angle itself as the design variable. The design variable is then the continuous and independent parameter at design points, for instance, at centers of finite elements. Handling the continuous fiber orientation design presents difficulties due to a fourth-order transform tensor, which rotates the tensor to a given angle composed of multivalued sine and cosine functions, rendering a non-convex optimization problem. Thus, optimizing the fiber orientation is susceptible to the initial fiber configuration and produces difficulties obtaining the optimized solution. In addition, the lack of fiber continuity in a large region puts a question mark on the realizability of the attained optimized solution. As illustrated in [30], suboptimal solutions [25, 29] are the persistent outcome of a continuous fiber orientation design problem. Various modifications in parametrization schemes and different optimization algorithm methods have been considered to handle the sub-optimal design issue for the continuous fiber orientation problem — one brute-force way to avoid it by relaxing the design space. For instance, free material optimization (FMO)[31, 32] further relaxed the design space by independently parameterizing each stiffness tensor element as the design variable. However, interpretation of the obtained stiffness tensor and linking it to the feasible physical design makes this approach challenging. Xia and Shi [33]

An alternative is employing curvilinear parameterization schemes that define fiber paths as the graphs of analytical function, which guarantee continuity of fiber angle and have a small number of design variables [34, 35, 36]. Nevertheless, the restrictive design search space will limit the tailorability of the fiber path, thus deteriorates the optimization problem's stability [37] and quality of the optimized solution. Adjectently, the parameterization schemes can follow equidistant iso-contours of a level set function to represent curvilinear fiber paths [38, 39], thus naturally ensuring fiber continuity and being often parallel to the neighboring fiber paths. Furthermore, the optimization result becomes highly dependent on the initial configuration, and local solutions often appear [40].

Discrete Paramterization

The counter approach reduces the design search space to avoid multiple local minima issues where the optimized solution is highly sensitive to the initial fiber configuration. Thus, Stegmann and Lund [30, 41] stretched the design search space by choosing discrete fiber orientation candidates, define apriori, which parameterized the design space into the discrete material candidates. These transversely isotropic material models are defined for different fiber orientations for the same isotropic elasticity tensor and share some similarities with the multi-material optimization problem in [42, 43]. The suggested approach assigned weighting functions to different candidates and employed gradient-based optimization and penalization coefficient to force the weighting functions towards either zero or one to seek fiber convergence, i.e., one discrete material at each design point. This method is known as Discrete Material optimization (DMO). DMO laid the foundation for SFP (Shape Function and with Penalization) [44], BCP (Bi-value Coding Parameterization) [45] to perform discrete fiber orientation optimization . A comparison using various numerical examples on these methodologies on discrete fiber orientation optimization drawn in [46]. Contrarily to CFAO, it does not cover design problems for continuously varying orientation distributions. Thus, it permits a limited scope to fully exploit the potential of modern technology's continuously varying orientation in composites. [47, 27, 26]. Secondly, these methods fail to address the fiber convergence even against the significant penalization factor; hence, their benefit relies on an opti-

mization algorithm to circumvent impractical mixtures of fiber angles. Third, the discrete parameterization scheme should further minimize the number of design candidates for efficient optimization.

Kiyono et al. [46] proposed a continuation of the computational approach suggested by Yin and Ananthasuresh [43] by utilizing the normal distribution function to guarantee fiber convergence, low sensitivity to the initial fiber configuration, and the continuity of the fiber orientation Salas et al. [48] proposed a Self-Penalization Interpolation Model for Fiber Orientation (SPIMFO) based on convergent Talyor series for sine and cosine functions and optimized the dynamic design of laminated piezo composite actuators combining SPIMFO with SIMP method. Nomura et al. [49]. proposed a general methodology for the simultaneous design of structural topology and continuous and discrete material orientation with the isoparametric projection method. In that study, the Cartesian components of the orientation vector were chosen as the orientation design variables. The 2π ambiguity stems from the periodic nature of the angular representation is addressed to yields better control on manufacturability. However, the simultaneous design will significantly change the orientations with a change in topology, potentially causing LOSS.

Optimization Algorithm

Analytical approaches

Analytical methods determined early studies of optimal fibre orientation that dates back to the pioneering work of Pedersen on strain-based method [50, 51, 52] and followed by the stress-based method. In the strain-based method, analytically derived equations study the sensitivity of strain energy with respect to material orientation for orthotropic material based on the assumption that the strain energy is invariant on material orientation. A similar analytical deduction can be derived for the stress-based method that except constant stress field with respect to material orientation. These methods eliminate the need for iteration during the design process and significantly increase computational efficiency compared with mathematical programming. In addition, the stress-based method produces a slightly stiffer structure than the strain method because strong couplings exist among the orientational variables when the strain field is used. In both scenarios, optimization proceeds towards convergence. The material orientation tends to coincide with the major principal stress/strain fields for relatively 'weak' shear and some shear 'strong' types of orthotropic materials. However, the methods are highly dependent on the initial fibre configuration, and the solution of both approaches will fail when the local and repeated global minima (more than one solution has the same global minimum value) occurs [53]. Nevertheless, these methods form the basis for future research on material orientation optimization for fibre reinforced composite materials. These methods' shortcomings were due to the assumption of constant stress or strain fields that was removed by the energy-based method introduced by Luo and Gea [54, 55]. The proposed hybrid framework estimates the strain

fields' and stress fields' dependency on fibre orientation by introducing an approximate energy factor. Their research showed that an energy-based method produces numerically more optimal solutions to stress-based and strain-based methods. The method use inclusion model is used to analyze the variations of the strain and stress of one design cell due to the rotation of orthotropic material in that design cell. It assumes that the displacements are continuous at the interface of the design cell after the cell orientation changes. Thus, the strain of the design cell will not change because the cell is under the same displacement boundary condition. In contrast, the stress of the design cell will vary, suffer from stress discontinuity at the interface. Furthermore, the assumption to consider additional stress and strain fields for both the design cell and its surrounding resolves stress discontinuity caused at the interface. By introducing the concept of energy factor, Luo et al. evaluated additional stress and strain field.

Gradient-based approaches

[41] gradient based optimization of the buckling load of laminate composite structures considering fiber angle deisng variables. The optimization formualtion are based on either linear or geomterically nonlinear analysis and formulated as mathematical programming problems solved using gradient based techniques. Xia and Shi [56] First, a hierarchy of parameterizations are constructed. At each level of the hierarchy, one has a distinct parameterization for the fiber angle arrangement. From the top level to the bottom level of the hierarchy, the number and density of design points for the Shepard interpolation increase, thus the design freedom and the resolution of parameterization increase as well. Second, at each level of the hierarchy, an optimization problem is formulated, and these optimization problems are solved successively. After the optimization problem at a coarse level is solved, one goes to its neighboring finer level, using the solution of the former one to compute an initial design for the latter one. Again, the Shepard interpolation is used for the computation of initial design.

formulated cascadic multilevel optimization problem, where there results from coarse level serves as an initial design for finer level.

Heurstic-based approaches

Another outlook on approaching the LOSS problem is using an optimization solver better at global searching ability, such as the genetic algorithm [57] and simulated annealing algorithm [58]. The heuristic algorithm can find "global minimum" and allow handling a discrete variable, but this always sacrifices the computational cost. Sigmund [59] questions the usefulness of non-gradient approaches in TO, showing the inefficiency of non-gradient algorithm-based optimization problems with many design variables and constraints. As a result, gradient-based solver i.e. Method of Moving Asymptotes (MMA) [60] set the framework for the TO problem and has been pushed significantly to avoid the LOSS [61].

3. functionally graded anisotropic material

4. New articles

Shen et. al.[61] proposes normalized gradient by maximum algorithm to update material orientation of anisotropic material using gradient descent method. believe that the gradient descent method is well suited for optimal orientation of fiber due its simplicity and robustness. The first order derivative of compliance with respect to material orientation can be explicitly calculated in closed form. The only unknow coefficient in gradient descent method is the step length can guarantee a global descent direction which converge to a minimum, if chosen wisely. Strain-based method was developed based on the assumption that strain field is constant. Reuschel [62] developed Computer Aided Internal Optimization (CAIO) to align fiber orientation in principle stress direction so as to reduce the internal shear stress which was found to be weakness of FRCs. Brampton [38] combined Luo and Gea's work [63] with level-set method to gererate the optimized fiber orientation with continuous fiber path.

4.1. Formulation and optimization algorithm

The compliance is expressed in global form:

$$c = \mathbf{u}^T \mathbb{K} \mathbf{u} \tag{1}$$

Take partial derivative of the global equilibrium equation with respect to the material orientation of the ith element:

$$\frac{\partial c}{\partial \theta_i} = \mathbf{u}^T \frac{\partial \mathbb{K}}{\partial \theta_i} \mathbf{u} \tag{2}$$

Considering the quadrilateral element under plane stress conditon, the stiffness matrix formulation is given by

$$\mathbb{K}_i = \sum \omega(\eta, \zeta) \mathbb{B}_i(\eta, \zeta)^T \mathbf{T}_i^T \mathbf{D} \mathbf{T}_i \mathbb{B}_i(\eta, \zeta) |\mathbb{J}_i(\eta, \zeta)| \tag{3}$$

where four integration points are $\eta = \pm\frac{1}{\sqrt{3}}, \zeta = \pm\frac{1}{\sqrt{3}}$, $\mathbb{D}$ is the stress/strain matrix, $\mathbb{T}_i$ is the transformation matrix for the ith element and $\omega(\eta, \zeta)$ is the weight factor.

$$\mathbb{D} = \begin{bmatrix} \frac{E_{11}}{1-\nu_{12}\nu_{21}} & \frac{\nu_{12}E_{11}}{1-\nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{12}E_{11}}{1-\nu_{12}\nu_{21}} & \frac{E_{11}}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix}$$

$$\mathbb{T} = \begin{bmatrix} \cos^2\theta_i & \sin^2\theta_i & \sin\theta_i\cos\theta_i \\ \sin^2\theta_i & \cos^2\theta_i & -\sin\theta_i\cos\theta_i \\ -2\cos\theta_i\sin\theta_i & 2\cos\theta_i\sin\theta_i & \cos^2\theta_i - \sin^2\theta_i \end{bmatrix}$$

Taking partial derivative of stiffness matrix with respect to θ_i yields:

$$\frac{\partial \mathbb{K}_i}{\partial \theta_i} = \sum \mathbb{B}_i(\eta, \zeta)^T \left(\frac{\partial \mathbf{T}_i^T}{\partial \theta_i} \mathbf{D} \mathbf{T}_i + \mathbf{T}_i^T \mathbf{D} \frac{\partial \mathbb{T}_i}{\partial \theta_i} \right) \mathbb{B}_i(\eta, \zeta) |\mathbb{J}_i(\eta, \zeta)| \tag{4}$$

The 1st order partial derivative of the stiffness with respect to material orientation can be explicitly expressed as a function of transformation matrix and its 1st order derivative.

The 2nd order derivative of compliance is given as the following:

$$\frac{\partial^2 c}{\partial \theta_i \partial \theta_j} = \mathbf{u}^T \frac{\partial \mathbb{K}}{\partial \theta_j} \mathbb{K}^{-1} \frac{\partial \mathbb{K}}{\partial \theta_i} \mathbf{u} - \mathbf{u}^T \frac{\partial^2 \mathbb{K}}{\partial \theta_i \theta_j} \mathbf{u} + \mathbf{u}^T \frac{\partial \mathbb{K}}{\partial \theta_i} \mathbb{K}^{-1} \frac{\partial \mathbb{K}}{\partial \theta_j} \mathbf{u} \tag{5}$$

Desai et al. [64] : Gradient based numerical techniques were adopted for optimizing the topology and fiber orientation concurrently ([65][66]). The optimization approach proposed in [49] offer continuous and discrete sets of angles. It improves local minimum issue inherent to CFAO because th evolving design is independent of previous iteration design Zhou et al. [67] : Structural products with complex geometeries are usually assemblies of components with relative simple geometries This mainly because producing multiple components with simple geometeries are often less expensive than producing a single product with complex geometeries, even with th additional cost of assembly. A multi-components structural product is to design and optimize its overall geometery first, and then decompose it to refine part boundaries and joint configuration. Such practise, known as two-step approach, ir likely to yield suboptimal solution with respect to overall structural performance and/or manufacturing and assembly costs, since the optimal decomposition obtained in the second step is largely dependent on the optimal overall geomtery obtained in the first step. Acoording to the types of the prescibed design domain, topology optimization can be classified inot discrete (truss/beam) approaches, pioneered by Dorn[68], and continuu (pixel/voxel) approaches, pioneered by Bendsøe and Kikuchi [12]. Acoording to the types of optimization algorithm utilized, topology optimization can be classified into gradient methods and genetic algorithm. Recently, non-gradient methods received serious critique [59] from the topology optimization community regarding its applicability in continuum structure problems. It is due, mainly, to its lack of sensititives and the associated computational inefficiency. However, gradient methods are inapplicable to the problems that have non-differentiable objectives and constraints, for instance, as in case for the manufacturability objectives in the multi-components topology optimization.

A ground topology such that a node represent a structure element and, an edge represent a joint element. A given combination of structure element and joint can be interpreted as unique topology graph

Material density field

In a prescibed, fixed design domain Ω, a characterisitic function χ is defined to descirbe the material domain Ω_d to be optimized:

$$\chi(\mathbf{x}) = \begin{cases} 0, & \forall \mathbf{x} \in \Omega \backslash \Omega_d \\ 1, & \forall \mathbf{x} \in \Omega_d \end{cases} \tag{6}$$

where $\mathbf{x}$ stands for a design point in Ω and $\chi(\mathbf{x})$ is defined by a scalar function ϕ and the Heaviside function H such that:

$$\chi(\mathbf{x}) = H(\phi(\mathbf{x})) = \begin{cases} 0, & \forall \mathbf{x} \in \Omega \backslash \Omega_d \\ 1, & \forall \mathbf{x} \in \Omega_d \end{cases} \tag{7}$$

4

To eliminate checkerboard patterns therefore generating mesh-independent results, the Helmholtz PDE filter is introduced to regularize ϕ:

$$-R_\phi^2 \nabla^2 \tilde{\phi} + \tilde{\phi} = \phi \qquad (8)$$

where R_ϕ is the filter radius, and $\tilde{\phi}$ is the filtered field. Then the density field ρ can be defined by an additional smoothed Heaviside function $\tilde{H}$:

$$\rho = \tilde{H}(\tilde{\phi}) \qquad (9)$$

After the series of regularization from ϕ to ρ, the resulting density field is bounded between 0 and 1.

Material orientation vector field

The Cartesian representation of continuous angles proposed in [49], the original material orientation vector field $v^k = (\xi^k, \eta^k)$ bounded by a box constraint $v^k \in [-1, 1]^\Omega x [-1, 1]^\Omega$ if first regularized by Helmholtz PDE filter 8. Then, the (unbounded) $\tilde{v}^k$ is projected back to the original box constraint with smoothed Heaviside function $\tilde{H}$. The regularized orientation vector field $\bar{v}^k$ in a box domain is then projected to a circular domain through an isoparametric projection $\mathbf{N}^c$. The transformation from a box domain to a circular domain eliminates the need of the quadratic constraint for each design point, and ensures singularity-free numerical analysis.

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5

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