# Topology Optimization strategies for continuous fiber-reinforced and functionally graded anisotropic composite structures: A brief review

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**Abstract**

Among all types of Additive Manufacturing (AM) technology,Continuous Fiber Fused Filament Fabrication (CF4) can fabricate high-performance composites compared to those manufactured with conventional technologies. AM provides the excellent advantage of a very high degree of reconfigurability, which is in high demand to support the immediate short-term manufacturing chain in medical, transportation, and other industrial applications. Additionally, the CF4 capability enables the fabrication of Continuous Fiber-Reinforced Composite (CFRC) materials and Functionally GRaded Anisotropic Composite (FGRC) structures. The current expedition in AM allows us to integrate Topology Optimization (TO) strategies to design realizable FRC and GFRC structures for a given performance. Various TO strategies for attaining lightweight and high-performance designs have been proposed in the literature, which exploits AM's design freedom. Therefore, the paper attempts to address works related to TO strategies employed to obtain optimal CFRC and FGRC structures. This review intends to overview, compare existing strategies, analyze their similarities and dissimilarities, and discuss challenges and future trends in this field.

*Keywords:*
topologoy optimization, anisotropic materials

## 1. Introduction

The cost-effective commercially available Additive Manufacturing (AM) or 3D Printing (3DP) technologies eliminate many limitations that previously plagued the manufacturing of highly tailored structural performance for multifunctional [1] and multi-physics [2] applications. Moreover, the short metamorphosis of AM technologies offers unique capabilities to realize the next-generation lightweight structure have brought great application potentials in several major industries such as aerospace [3, 4], automotive [5] and medical[6]. First, AM's unique ability to fabricate a highly complex shape without a substantial increase in fabrication costs, along with the benefit s of reducing manufacturing preparation time, renders these technologies worthy investment for large-scale industries. Moreover, it offers the fabrication of lattice structures-a lightweight design comparative to the solid-filled parts. Thus, offer diversification of design to answers multifunctional material requirements in weight reduction, [7], their ability to dissipate energy [8, 9], heat [10] and vibration [11]. The printed polymer parts frequently consist of carbon nanotubes and short fibre to upgrade their mechanical performance. Still, it cannot outperform [12, 13], the mechanical strength offered by continuous fibre-reinforced composite laminate manufactured using conventional manufacturing tools. Hence, the shortcomings of 3D printed polymer composites aggravated the demand to develop Continuous Fiber Filament Fabrication (CF4) technology. CF4 technology provides a unique opportunity to reduce part distortion warping and support structures during printing, and fibre tension prevents nozzle clogging, a constant lookout with the polymer AM. Additionally, controlling the anisotropic properties of the fibre-reinforced composites can effectively distribute the loads throughout the laminate to maximize the structure's strength and stiffness.

CF4 technology allows fabrication of FRC material with the continuous spatial in-plane variation of fiber angle and fiber volume fraction, thus expanding design space as opposed to variable [14] and constant stiffness laminate [14]. Moreover, CF4 technology can achieve out-of-plane variation of fiber angle due to the fiber-reinforced composite's self-supporting characteristics. Numerous studies have shown that fiber orientation optimization can significantly tailor structural performance such as stress concentration[15], stiffness [16], load-bearing capacity, buckling load, and the natural frequency [17]. Therefore, the design of the FRC structures requires optimization methods that reflect design freedom offered by CF4 technologies, including constraints, to thoroughly exploit the anisotropic properties of FRC material [18]. Recognizing these potentials of CF4, returned a resurgent interest in utilizing the design strategies to exploit AM's performance-driven manufacturing technologies towards enhancing printed parts' overall functional performance. In contrast, to the geometric-driven or/and cost-driven manufacturing of components. The concept of performance-driven manufacturing is known as Design for Additive Manufacturing (DfAM) [19].

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Topology Optimization (TopOpt), one of the DfAM methods, is an iterative design tool to optimize a quantifiable objective while being intended to sustains loads, constraints, and boundary conditions. Topology Optimization is frequently adopted to design structurally sound parts and has subsequentially surpassed optimization design tools such as shape and size optimization in isolation. The seminal work of Bendsøe and Kikuchi [20] introducing the concept of TopOpt on the homogenization method; since then, TopOpt has been developed rapidly. TopOpt approaches can be summarized as follows: the homogenization method [20]; the Solid Isotropic Material with Penalization (SIMP) method [21, 22], the level set method [23, 24], the Evolutionary Structural Optimization (ESO) method [25]; Topology Derivatives and Phase Field. The details of these approaches are discussed in the review papers [26, 27, 28] and some emerging TopOpt methods for smooth boundary representation include the 'Metamorphic Development Method' (MDM) [29], and the 'Moving Morphable Method' (MMM) [30]. The general architecture of TopOpt starts with the definition of maximizing or minimizing a single or multi-target-objective function to fulfil a set of constraints such as volume, displacement, or frequency [31]. Then, as part of an iterative process, design variables, Finite Element Analysis (FEA), sensitivity analysis, regularization, and optimization steps are repeated in this order until convergence is achieved [32].

The optimization concept applied to FRC materials allows finding the optimized material distribution, the optimized orientation of fiber paths, as well as optimized geometric contours of the laminate. Hence, the optimization method for FRC structures with continuous fiber parameterization schemes and optimization algorithms have notable influences on the quality of the solution. The article on the optimization of topology and fiber path orientation and thus, only related works are reviewed.

## 2. Topology Optimization for continuum structures

Topology Optimization optimizes material layout within a given design space $\Omega$ for a given set of loads $\{F^T, \Gamma_t\}$, boundary conditions $\Gamma_u$, and constraints to maximize the system's performance. It uses a finite element method (FEM) to evaluate the design performance, and the design is optimized using either gradient-based mathematical programming techniques or non-gradient-based algorithms.

### *2.1. Problem Statement*

A general form of topology optimization can be written as an optimization problem:

$$\begin{aligned} \min_{\rho} &: \Phi(\rho, \mathbf{U}(\rho)) \\ &:= \int_{\Omega} f(\rho, \mathbf{U}(\rho)) \\ \text{s.t.} &: G_i(\rho, \mathbf{U}(\rho)) \leq G_i^*, \quad i = 1, \ldots, Q \end{aligned} \tag{1}$$

An objective function $\Phi$ represents the quantity being minimized or maximized to maximize the system's performance—the density of the material in each location as a design

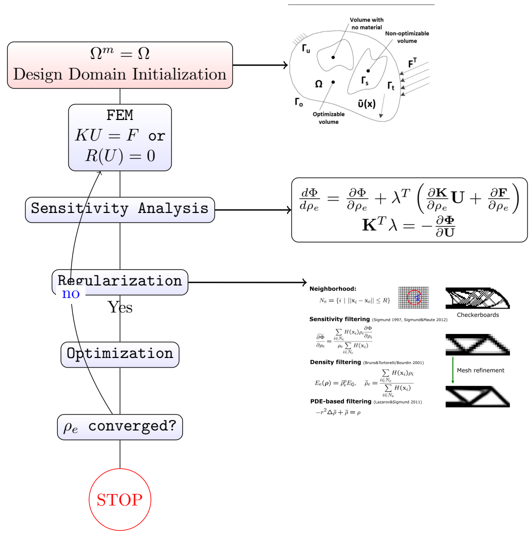


Figure 1: Density-based TopOpt framework

variable $\rho$ for the optimization problem. The constraints $G_i$ are characteristics that the solution must satisfy. The finite element method evaluates the field $\mathbf{U}$ that satisfies a linear or nonlinear state equation since these equations do not have a known analytical solution. Design space $\Omega$ defines the optimizable volume $V^*$ for the design to exist, and the volume in the design space that the optimizer cannot modify is considered non-optimizable volume $\Gamma_s$. A characterisitic function $\chi$ is defined to descirbe the material domain $\Omega_d$ to be optimized:

$$\chi(\mathbf{x}) = \begin{cases} 0, & \forall \mathbf{x} \in \Omega \backslash \Omega_d \\ 1, & \forall \mathbf{x} \in \Omega_d \end{cases} \tag{2}$$

where $\mathbf{x}$ stands for a design point in $\Omega$ and $\chi(\mathbf{x})$ is defined by a scalar function $\phi$ and the Heaviside function $H$ such that:

$$\chi(\mathbf{x}) = H(\phi(\mathbf{x})) = \begin{cases} 0, & \forall \mathbf{x} \in \Omega \backslash \Omega_d \\ 1, & \forall \mathbf{x} \in \Omega_d \end{cases} \tag{3}$$

To eliminate checkerboard patterns therefore generating mesh-independent results, the Helmholtz PDE filter [33] is introduced to regularize $\phi$:

$$-R_\phi^2 \nabla^2 \tilde{\phi} + \tilde{\phi} = \phi \tag{4}$$

where $R_\phi$ is the filter radius, and $\tilde{\phi}$ is the filtered field. Then the density field $\rho$ can be defined by an additional smoothed Heaviside function $\tilde{H}$:

$$\rho = \tilde{H}(\tilde{\phi}) \tag{5}$$

After the series of regularization from $\phi$ to $\rho$, the resulting density field is bounded between 0 and 1. The general architecture of the topology optimization framework is depicted in the schematic.

The following broader categorization of the implementation has been used to solve topology optimization problems.

### 2.2. Discrete Topology Optimization

The discrete variable optimization problem can be formulated as follows:

$$\begin{aligned}
\min_{\rho} &: \Phi(\rho, \mathbf{U}(\rho)) \\
\text{s.t.} &: \sum_{e=1}^{N} v_e\rho_e = \mathbf{v}^T\rho \leq V^* \\
&: g_i(\rho, \mathbf{U}(\rho)) \leq g_i^*, \quad i = 1, \ldots, M \\
&: \rho_e = \begin{cases} 0(\text{ void }) \\ 1(\text{ material }) \end{cases}, e = 1, \ldots, N \\
&: \mathbf{K}(\rho)\mathbf{U} = \mathbf{F}
\end{aligned} \tag{6}$$

Topology optimization problem is a binary problem representing the void and solid regions of the structure. The discrete TopOpt uses binary design variables, one for each of the finite elements in the mesh of the structural domain. The design variable 1 implies the finite element is filled with material, 0 indicates void. Consider an objective function $\Phi(\rho, \mathbf{U}(\rho))$, constrained by $g_i \leq g_i^*.i \in [1, M]$, with $\rho$ being the design variables, where $N$ number of finite elements, $\mathbf{K}$ and $\mathbf{U}$ is the assembled stiffness matrix and displacement vector corresponding to finite elements in the mesh.

The well-known discrete topology optimization method is the Bi-directional Evolutionary Structural Optimization (BESO). Interested readers find comprehensive reviews on the BESO methods in [34, 35]. Another outlook on approaching the discrete problem is using a genetic algorithm [36] that can find "global minimum" and allow handling a discrete variable, but this always sacrifices the computational cost. Furthermore, Sigmund [37] questions the usefulness of non-gradient approaches in Topology Optimization. Recently, Sivapuram et al. [38] combined the features of BESO and the sequential integer linear programming for discrete topology optimization.

### 2.3. Continuous Topology Optimization

The continuous variable optimization problem can be formulated as follows:

$$\begin{aligned}
\min_{\rho} &: \Phi(\rho, \mathbf{U}(\rho)) \\
\text{s.t.} &: \sum_{e=1}^{N} v_e\rho_e = \mathbf{v}^T\rho \leq V^* \\
&: g_i(\rho, \mathbf{U}(\rho)) \leq g_i^*, \quad i = 1, \ldots, M \\
&: 0 \leq \rho_{min} \leq \rho \leq 1 \\
&: \mathbf{K}(\rho)\mathbf{U} = \mathbf{F}
\end{aligned} \tag{7}$$

Density-based topology optimization is a broadly received idea in the topology optimization of continuum structures by using continuous density design variables that transform the binary variable optimization problem into a density distribution problem. The design variable can take any value from 0 to 1 such that $\rho \in [0, 1]$; thus, varying the material properties across the elements. Therefore, material properties are interpolated using a power function, where the intermediate properties are penalized, thus forcing the design back to the binary structure. This transformation enables the use of gradient-based information; unfortunately, the penalization parameter also introduces non-convexities, hence a high risk of falling into local minima. Furthermore, in SIMP, optimized solutions do not explicitly exhibit structural boundaries, which challenges solving problems where explicit boundary identification is essential, e.g., in design-dependent and multiphysics problems. Thus, topology optimization can be formulated in the nodal variables [39] that control an implicit function description of the shape to address the exact boundary identification problem.

## 3. Paramterization schemes for fiber orientation

The parameterization scheme implements a numerical description of fiber orientation patterns and defines variables for the optimization. It should ensure spatial continuity of fiber angle so that CF4 technology can produce the structure, and it should also provide enough design freedom so that optimization algorithm can consider more candidate designs.

### 3.1. Continuous parameterization

The continuous parametrization of fiber orientation design uses the angle itself as the design variable [40, 41]. The design variable is the continuous and independent parameter that provides flexibility in changing the orientation across the design points, with the relaxation of orientation design space. However, handling the continuous fiber orientation design presents difficulties due to a fourth-order transform tensor, which rotates to a given angle composed of multivalued sine and cosine functions, rendering a non-convex optimization problem. Furthermore, optimizing the fiber orientation is susceptible to the initial fiber configuration, thus causing difficulties obtaining the optimized solution. As illustrated in [42], suboptimal solutions [14, 18] are the persistent outcome of a continuous fiber orientation design problem—one brute-force way to avoid it by further relaxing the design space. For instance, free material optimization (FMO)[43, 44] parameterizing each stiffness tensor element independently as the design variable. Thus, securing the scheme from the complexity that stems from the design space's orientation design variable. However, as compensation, point-wise nonlinear constraints ensure the positive semi-definitiveness of the obtained stiffness tensor and link it to the feasible physical design, making this approach challenging. Nomura et al. [45] formulated orientation design variable as a tensor field to simplify the first tensor invariant constraint and remove nonlinear constraints successfully introduced due to the second tensor invariant. Still, as commented, the violation of these constraints is observed at the joint point of the structural members where the orientation shows the discontinuous distribution.

Reasonably arranging the fiber orientation is critical to effectively handling an anisotropic material, which is vital in designing next-generation lightweight composite structures. Frequently, fiber orientation optimization creates difficulty associated with local optima and discontinuous functions. To address this, gradient -free algorithms, such as the genetic algorithm particle swarm algorithm and simulated annealing algorithm [46], are more qualified because of their global searching ability [47, 48]. By allowing differentiable functions, mixed design variables, and discrete space, introduce a relaxed formulation that has the advantage of obtaining fewer local optima. The inefficiency of most gradient-free algorithms requiring numerous function evaluations is impractical for expensive finite element simulations; thus, adoption of gradient-based algorithms, i.e., Optimality Criteria Method (OCM), Method of Moving Asymptotes (MMA) [49], and Sequential Linear Programming (SLP) [50].

In particular, for orthotropic materials, early studies utilizing the analytical derived optimally criterion [51] for optimizing fiber orientation dates back to the pioneering work of Pedersen on strain-based method [52, 53, 54]. In Pedersen's work, strain energy density transformed into principal strain and concluded that material orientation axes along principal strain axes always give stationary energy density. However, Cheng [55] argued that the discussion is limited to a unit cell case where the orientation variable is separated from the design domain to obtain extreme strain energy. After that, a similar deduction using iterative optimality criteria [56, 57] formulated the stress-based method [58] by exercising invariant stress field for material orientation. Finally, Diaz and Bendsoe [59] extended the stress-based method for determining the optimal orientation optimization problem corresponding to multiple loads. Despite their similarity, the stress-based method produces a slightly stiffer structure than the strain method because strong couplings exist among the orientational variables when the strain field is used [55]. Conclusively, Gea and Luo [60] demonstrated that the fiber orientation coincides with the principal stress/strain fields for relatively 'weak' shear and some shear 'strong' types of orthotropic materials. Further, the methods are highly dependent on the initial fiber configuration, and both approaches will fail for shear 'strong' type materials due to repeated global minimum solutions. Nevertheless, these methods form the basis for future research on material orientation optimization for fiber-reinforced composite materials. These methods' shortcomings coerced to formulate the energy-based method introduced by Luo and Gea [61, 62]. The method uses an inclusion cell to estimates the strain fields' and stress fields' dependency on fiber orientation by introducing an approximate energy factor. Yet, the dependence of energy factors on the traction stress, material properties, and orientation of the inclusion cell and its surroundings make it challenging to formulate the framework for 3D and complex loading problems. Following the principles of the energy-based method, Yan et al. [63] proposed a hybrid stress-strain method by weighting the mean compliance's optimality condition in the stress and strain form. Numerical examples demonstrate their method on the shear weak and strong materials and extended for a 3D problem. The assumption on the elemental strain and stress field invariant to the neighboring elemental orientation is considered; however, it may restrict to solve 3D problems and may result in a suboptimal solution.

On the other hand, Shen et al. [64] questioned the lack of understanding about the orientation optimization algorithm to handle arbitrary constraints and loads. A step length scheme for orientation optimization is advised to achieve global descent by normalizing the gradient vector and introducing a parameter to control the magnitude of material orientation in each iteration. However, the verification lacks the effect of adding constraints in the orientation optimization problem on the update scheme, a critical factor for the optimality criteria method. Thus, a more generalized OCM for the topology optimization of transversely isotropic material is demanded from the perspective of scalability and multiloading situations. Recently, Kim et al. [65] interpreted the work of Patnaik et al. [66] on parametric optimization and proposed a generalized optimality criteria method for topology optimization problems. The approach eliminates the compulsion to satisfy the constraints during every optimization iteration but should be met upon convergence.

An alternative is employing curvilinear parameterization schemes that define fiber paths as the graphs of analytical function, which guarantee continuity of fiber angle and have a small number of design variables [67, 68, 69]. Nevertheless, the restrictive design search space will limit the tailorability of the fiber path, thus deteriorates the optimization problem's stability [70] and quality of the optimized solution. Adjectenly, the parameterization schemes can follow equidistant iso-contours of a level set function to represent curvilinear fiber paths [71, 72], thus naturally ensuring fiber continuity and being often parallel to the neighboring fiber paths. Furthermore, the optimization result becomes highly dependent on the initial configuration, and local solutions often appear [73].

### *3.2. Discrete paramterization*

The counter scheme reduces the orientation design space to avoid multiple local optima issues where the optimized solution is highly sensitive to the initial fiber configuration. Therefore, a discrete angle optimization formulation was solved using a genetic algorithm at the cost of a computational burden [74] [75] [76]. Lund's strategy [42] relaxes the combinatorial problem to a continuous optimization problem. By choosing discrete fiber orientation candidates, defined apriori, parametrize the orientation design space into the discrete material candidates. These transversely isotropic material models are defined for different fiber orientations for the same isotropic elasticity tensor. Thus the scheme share some similarities with the multi-material optimization problem in [77, 78]. The suggested scheme assigned weighting functions to different candidates and employed gradient-based optimization with penalization coefficient, forcing the weighting functions to seek a binary design and fiber convergence, i.e., one discrete material at each design point. This method is known as Discrete Material optimization (DMO). DMO laid the foundation for SFP (Shape Function and with Penalization) [79], BCP (Bi-value Coding parametrization) [80] to perform discrete fiber orientation optimization. A comparison between these methodologies utilizing various numerical examples is drawn in [81].

Contrarily to CFAO, it does not incorporate design problems for continuously varying orientation distributions. First, it is an imperative design consideration to circumvent stress constraint and degradation in the strength by order of magnitude lower than continuous fiber paths caused due to fiber discontinuity.

Consequently, it permits a limited scope to fully exploit the potential of modern technology's continuously varying orientation in composites [82, 16, 15]. Secondly, these methods fail to address the fiber convergence even against the significant penalization factor; hence, their benefit relies on an optimization algorithm to circumvent impractical mixtures of fiber angles. Third, the discrete parametrization scheme should further minimize the number of design candidates for efficient optimization. Concerning these drawbacks, Kiyono et al. [81] proposed a parametrizing scheme, which is a continuation of the computational approach suggested by Yin and Ananthasuresh [78]. Introducing a normal distribution function as a weighting function in their parametrizing scheme guarantees fiber convergence, low sensitivity to the initial fiber configuration, and the continuity of the fiber orientation. Another different work proposed a Self-Penalization Interpolation Model for Fiber Orientation (SPIMFO) based on convergent Talyor series for sine and cosine functions to optimized composite hyperelastic material [83] and the dynamic design of laminated piezo-composite actuators [84].

### *3.3. Hybrid paramterization*

Utilizing continuous and discrete methodology benefits is another alternative to fiber orientation optimization. The key idea in the following approaches is to fill the gaps by acknowledging the beneficial characteristics of both strategies to improve computational efficiency and/or reduce local optima and/or resolve and/or fiber continuity manufacturability issues. Therefore, an approach to reduce the risk of falling into local optimal without sacrificing the fiber continuity can use both discrete and continuous parametrization as suggested by Luo et al. [85]. Their work proposed a "coarse to fine" strategy, where the orientation design space is divided into discrete sub-intervals. After that, the CFAO searches for an optimized solution in a sub-interval, where the sub-interval selection problem is solved using the DMO approach. However, no criterion is defined to determine the number of sub-intervals required in advance.

Nevertheless, the proposed strategy provides flexibility to integrate alternatives that are suggested for DMO and CFAO approaches. Nomura et al. [86] studied the cartesian system for orientation design variables to improve initial design dependency and local optima issues encountered in the continuous parameterization approach. The parametrization scheme was further extended to yield an optimized discrete orientation design for a given discrete orientation set in their work. Moreover, the characteristics representing the orientation design variables into the vectorial form consider the $2\pi$ ambiguity, which occurs due to the periodic nature of the orientation design variable. Introducing vectorial design variable as a point-wise quadratic inequality constraint yields more interpolated elasticity tensor than the single variable polar representation.

Xia and Shi [87] develops a continuous global function by applying the shepherd interpolation method at scattered design points to represent the fiber orientation throughout the design domain. The benefit of the interpolation function is to ensure fiber continuity while considering reduced orientation design space in contrast with CFAO. Unfortunately, it suffers from the initial configuration and ends at the sub-optimal solution. In another work of Xia [88] applied multilevel optimization for fiber orientation optimization and verified its efficiency against the single-level optimization. Still, the optimization results in different fiber arrangements for different initial fiber orientations. As a result, the efficiency of the multilevel approach relies on the attained fiber orientation field at a coarse level since the optimization at the successive refined level starts from an initial design computed at its neighboring coarser level.

### *3.4. Feature-based parameterization*

The before-mentioned parameterization introduces low-level fiber material representations, such as pixel or voxel-based, thus representing the designs with variables proportional to the number of pixels or voxels in the design space. Moreover, these techniques render organic and free-form designs, which require sophisticated post-processing to distinguish fiber paths for realizability using AM technology. Therefore, to avail manufacturable solutions with designs with few variables, fiber material can be considered as a geometric feature with high-level parameters. High-level parameters refer to spatial dimensions associated with the feature's size, position, or orientation. Finally, feature-mapping techniques map these features onto a fixed mesh for analysis—an extensive review of feature-mapping methods by Wein et al. [89] details the components of feature-mapping techniques and discuss their implementation in structural optimization. Geometry projection [90] is a feature-mapping technique extended to represent the design via cylindrical bars reinforced with continuous fibers [91] and performs the analysis using a fixed finite element mesh. The interpolation of the material properties at the junction of multiple bars made of an anisotropic material is penalized as a convex combination of the penalized effective densities for each component. It demonstrates the method can easily integrate shape constraints on the structural form offered by CF4 technology. Thus, this work introduces the groundwork for using the geometry projection method for fiber-orientation optimization design problems.

## 4. Functionally graded anisotropic material

Functionally graded composites [92] are inhomogeneous materials, consisting of two or more materials, engineered to have a continuously varying spatial composition and structure. Recent studies [93, 94, 95] have shown that CF4 technology is ready to manufacture FRC structures with continuous yet spatially varying fiber paths and fiber volume fractions. Thus, if properly optimized, the spatial variation in FRC material properties may result in better performance than the fixed FRC material volume fraction. Therefore, a composite structure with FRC material in a fraction with voids and variable fiber density is termed a functionally graded anisotropic material. Furthermore, the before-mentioned gradation of FRC material brings considerably larger design freedom to design for additive manufacturing. Accordingly, Li et al. [96] considered a SIMP-based sequential topology optimization approach to design functionally graded fiber-reinforced anisotropic composites by considering fiber fraction along with material fraction in a given design space. A sequential process begins with designing an isotropic-material matrix with voids, inserting fiber selectively, then optimally orienting the fiber. However, the approach sacrifices the exploration of new topologies that might be optimal for anisotropic composite materials. Therefore, the following works investigated the simultaneous design of isotropic-material matrix topology, fiber material layout, and orientation.

Desai et al. [97] applied the topological derivative method for tailoring a spatially-varying fiber fraction. In addition, the dense arrangement of fibers is evenly spacing for the part's manufacturability while retaining their specific patterns. However, structural performance later in the simplification of the dense fiber arrangement is not evaluated, thus questioning the printed part's reliability. In their work, a differnet fiber orientation approach is achieved by computing anisotropic topological derivatives in the polar coordinate system. A possible solution to avoid postprocessing for dense fiber might be to consider the feature-based mapping methods with their technique.

The work, as mentioned earlier, implemented ”single-scale” approaches optimizing the distribution and orientation of FRC material. However, AM also provides an effective means to fabricate mono-scale structures and multiscale structures. Thus, spatially-varying material distribution and geometric patterns spanning at least two-scale or more scales hold a promising future for designing next-generation lightweight structures. On the other hand, the multiscale strategy for anisotropic material is challenging due to the following reason: length scale controls, models for fracture and damage criteria to capture actual anisotropic behavior, unique treatments at the boundaries' of the domain, etc. to be investigated through experiments or using appropriate numerical tools for estimating the actual performance of printed parts. Based on the author's knowledge, only a few works address the multiscale approach for fiber-reinforced composites. Hence, interested readers can refer to Wu et al. [98] review paper to understand the general framework for multiscale topology optimization. Kim et al. [99] adopted the homogenization method for designing spatially-varying fiber volume fraction and fiber orientation, and simultaneously, the SIMP designed the macrostructure composite topology. Finally, the de-homogenization procedure [100] applied on fiber microstructures obtained in the coarser mesh is visualized by projecting at a finer mesh. Various benchmark and multi-load structure problems are studied to conclude that locally varying FRC material further augments the global stiffness to the structure than the fixed fiber volume fraction or isotropic multi-material structure. In continuation of the Kim methodology, Jung [101] proposes a 3-D topology optimization approach for designing a functionally graded composite structure with spatially-varying fiber fractions and orientations. In conclusion, the multiscale framework has further enabled us to exploit the design freedom offered by the CF4 technology; however, no study to date fabricates and experimentally validates attained functionally graded composite design results.

## 5. Discussion

The discussion focuses on the suitability of given topology optimization for anisotropic material, given a pre-requisite understanding of the manufacturing process and its limitations. Therefore, the following discussion does not address composite manufacturing technology and its differences to adopt the particular topology optimization method.

The manufacturing design freedom extended by available composite manufacturing processes provides the flexibility to develop and integrate topology optimization approaches for designing anisotropic material orientations [102]. Therefore, the paradigm of performance-driven design focuses on investigating the suitability of topology optimization methods that can fully exploit the design freedom offered by manufacturing technology. Thus, the existing techniques for material orientation are broadcasted into four major classes, as stated previously.

Optimizing a prescribed set of alternative discrete angles, named the discrete orientation method, is often preferred in aerospace, automotive, and wind turbine industries for manufacturability reasons. DMO approach is favorable for composite laminate design [103] [104] [105] because a mixed-integer programming problem is formulated as a continuous problem that can be solved efficiently using gradient-based optimizers. As a result, substantial problems that might not be amenable to gradient-free methods can use DMO parameterization. An indirect approach is to apply lamination parameters as introduced by Tsai and Pagano [106]. However, despite these methods' popularity, they are limited to a prescribed set of alternative discrete angles, while the CF4 processes have higher freedom in orientation control can produce higher performing composites.

Therefore, the continuous orientation methods are most suitable for CF4 processes. Furthermore, these methods provide the highest freedom in shape and variable stiffness. Thus, the continuous orientation formulation directs material deposition path planning to ensure fiber trajectory curvature, fiber continuity, fiber fraction, and offset distance between adjacent fiber deposition, unlike discrete methods where fiber convergence and fiber continuity is difficult to attain. Papapetrou et al. [72] designed part's topology and material orientation simultaneously; the optimized results were post-processed using continuous fiber path planning to ensure realizability. A sequential scheme is proposed [107, 108] where fiber placement based on load transmission follows isotropic topology optimization contrary to Liu [109], who adopted concurrent fiber path planning and structural topology optimization. . The multi-axis material deposition technology using the robotic arm requires an extension of the topology optimization algorithm to envelop the 3D fiber orientation in contrast to in-plane printing. Schmidt et al. [110] introduced azimuth and elevation angles to extend the CFAO method for 3D fiber orientation. In addition, they emphasized the issues of nonconvexity of the compliance and sensitivity to the initial fiber orientation guess—a study by investigating the orientation parameter space to mitigate the issues [111]. Finally, the realizability of 3D printed composite is studied by Fedulov et al. [112], where they proposed a filtering technique for fast convergence.

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