NCERT

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Question - 10th CBSE

1) The centre of a circle is at (2,0). If one end if a diameter is at (6,0), then the other end is at :

   1. $(0, 0)$
   2. $(4, 0)$
   3. $(-2, 0)$
   4. $(-6, 0)$

Solution

$$\mathbf{O} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{1}$$

$$\mathbf{A} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \tag{2}$$

$$\mathbf{O} = \frac{(\mathbf{A} + \mathbf{B})}{2} \tag{3}$$

$$\mathbf{B} = 2\mathbf{O} - \mathbf{A} \tag{4}$$

$$= 2\begin{pmatrix} 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{5}$$

$\implies$ Option (C) is true

2) AD is a median of ΔABC with vertices $A\,(5,-6), B\,(6,4)$ and $C\,(0,0)$. Length AD is equal to:

1. $\sqrt{68}$
2. $2\sqrt{15}$
3. $\sqrt{101}$
4. 10

Solution

As Midpoint of **BC** is given by **D**

$$\mathbf{D} = \frac{\mathbf{B} + \mathbf{C}}{2} \tag{6}$$

$$= \frac{1}{2}\begin{pmatrix} 6 \\ 4 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{7}$$

Distance between **AD** is given by it's norm.

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix} 5 \\ -6 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix} \tag{8}$$

$$\|\mathbf{A} - \mathbf{D}\| \triangleq \sqrt{(\mathbf{A} - \mathbf{D})^\top (\mathbf{A} - \mathbf{D})} \tag{9}$$

$$= \sqrt{\begin{pmatrix} 2 & -8 \end{pmatrix} \begin{pmatrix} 2 \\ -8 \end{pmatrix}} = \sqrt{2^2 + 8^2} = \sqrt{68} \tag{10}$$

$\implies$ Option (A) is true.

3) If the distance between the points $(3, -5)$ and $(x, -5)$ is 15 units, then the values of x are

1. $12, -18$
2. $-12, 18$
3. $18, 5$
4. $-9, -12$

Solution

Let A and B denote the points and the distance between them be denoted by norm of **A**-**B**.

$$\mathbf{A} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} \tag{11}$$

$$\mathbf{B} = \begin{pmatrix} x \\ -5 \end{pmatrix} \tag{12}$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 3 - x \\ -5 - (-5) \end{pmatrix} = \begin{pmatrix} 3 - x \\ 0 \end{pmatrix} \tag{13}$$

Solution

$$d = ||\mathbf{A} - \mathbf{B}|| \triangleq \sqrt{(A-B)^\top (A-B)} \tag{14}$$

$$= \sqrt{\begin{pmatrix} 3-x & 0 \end{pmatrix} \begin{pmatrix} 3-x \\ 0 \end{pmatrix}} = \sqrt{(3-x)^2} \tag{15}$$

$$15 = \pm (3-x) \tag{16}$$

$$\implies x = -12, 18 \tag{17}$$

$\implies$ Option (B) is true.

4) Solve the following system of linear equations algebraically:
$2x + 5y = -4; 4x - 3y = 5$

Solution

The above system of equations can be written as:

$$\begin{pmatrix} 2 & 5 \\ 4 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 \\ 5 \end{pmatrix} \tag{18}$$

Writing the augmented matrix for using Gauss elimination

$$\left(\begin{array}{cc|c} 2 & 5 & -4 \\ 4 & -3 & 5 \end{array}\right) \xleftarrow{R_2 \to R_2 - 2R_1} \begin{pmatrix} 2 & 5 & -4 \\ 0 & -13 & 13 \end{pmatrix} \xleftarrow{R_1 \to \frac{13}{5} R_1 + R_2} \begin{pmatrix} \frac{26}{5} & 0 & \frac{13}{5} \\ 0 & -13 & 13 \end{pmatrix} \tag{19}$$

$$\implies \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -1 \end{pmatrix} \tag{20}$$

Question - 10th CBSE

5) The sum of the digits of a 2-digit number is 14. The number obtained by interchanging its digits exceeds the given number by 18. Find the number.

Solution

Let the digits of the number be x_1(tens) and x_2(units).Given

$$x_1 + x_2 = 14 \tag{21}$$
$$10x_2 + x_1 = 18 + 10x_1 + x_2 \tag{22}$$
$$\implies x_1 - x_2 = -2 \tag{23}$$

Solving the equations (21),(23) in their matrix forms

$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 14 \\ -2 \end{pmatrix} \tag{24}$$
$$\tag{25}$$

Let:

$$\mathbf{A} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \tag{26}$$
$$AA^\top = \mathbf{I} \tag{27}$$

Solution

A is an orthogonal matrix.

$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 14 \\ -2 \end{pmatrix} \tag{28}$$

$$2\mathbf{I}\mathbf{x} = \begin{pmatrix} 12 \\ 16 \end{pmatrix} \tag{29}$$

$$\implies \mathbf{x} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} \tag{30}$$

6) Find the ratio in which the point $\left(\frac{8}{5}, y\right)$ divides the line segment joining the points $(1, 2)$ and $(2, 3)$.Also, find the value of y.

Solution

Let the points be denoted by **A**,**B** and **C** respectively.

$$A = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{31}$$

$$B = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{32}$$

$$C = \begin{pmatrix} \frac{8}{5} \\ y \end{pmatrix} \tag{33}$$

For Collinearity:

$$\text{rank} \begin{pmatrix} 1 & 1 & 1 \\ A & B & C \end{pmatrix} = 2 \tag{34}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 8/5 \\ 2 & 3 & y \end{pmatrix} \leftrightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & (2-1) & (\frac{8}{5}-1) \\ 0 & (3-2) & (y-3) \end{pmatrix} \leftrightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & \frac{3}{5} \\ 0 & 1 & y-3 \end{pmatrix} \tag{35}$$

Solution

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & \frac{3}{5} \\ 0 & 1 & y-3 \end{pmatrix} \xleftrightarrow{R_3 \to R_3 - R_2} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & \frac{3}{5} \\ 0 & 0 & y-\frac{18}{5} \end{pmatrix} \tag{36}$$

$$\implies y = \frac{18}{5} \tag{37}$$

7) $ABCD$ is a rectangle formed by the points $A(-1,-1)$,$B(-1,6)$,$C(3,6)$ and $D(3,-1)$. P,Q,R and S are mid-points of sides AB,BC,CD and DA respectively. Show that the diagonal of the quadrilateral $PQRS$ bisect each other.

Solution

$$A = \begin{pmatrix} -1 \\ -1 \end{pmatrix}, B = \begin{pmatrix} -1 \\ 6 \end{pmatrix}, C = \begin{pmatrix} 3 \\ 6 \end{pmatrix}, D = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \tag{38}$$

$$\mathbf{P} = \frac{\mathbf{A} + \mathbf{B}}{2} \tag{39}$$

$$\mathbf{Q} = \frac{\mathbf{B} + \mathbf{C}}{2} \tag{40}$$

$$\mathbf{R} = \frac{\mathbf{C} + \mathbf{D}}{2} \tag{41}$$

$$\mathbf{S} = \frac{\mathbf{D} + \mathbf{A}}{2} \tag{42}$$

Solution

Let $\mathbf{O}_1$ and $\mathbf{O}_2$ be the midpoints of PR and QS respectively

$$\mathbf{O}_1 = \frac{\mathbf{P}+\mathbf{R}}{2} = \frac{\mathbf{A}+\mathbf{B}+\mathbf{C}+\mathbf{D}}{4} \tag{43}$$

$$\mathbf{O}_2 = \frac{\mathbf{Q}+\mathbf{S}}{2} = \frac{\mathbf{A}+\mathbf{B}+\mathbf{C}+\mathbf{D}}{4} \tag{44}$$

Since the midpoints of the diagonals coincide, the diagonals bisect each other.

8) The sum of first and eight terms of an A.P is 32 and their product is 60. Find the first term and common difference of the A.P. Hence, also find the sum of its first 20 terms.

Solution

Let the first and eighth terms be $x(0)$ and $x(7)$ respectively,given:

$$x(0) + x(7) = 32 \tag{45}$$

$$x(0) = 32 - x(7) \tag{46}$$

$$x(0)\,x(7) = 60 \tag{47}$$

From (46) and (47)

$$x(7)\,(32 - x(7)) = 60 \tag{48}$$

The roots are $(30, 2)$, therefore ,if $x(7) = 30$ then $x(0) = 2$ and if $x(7) = 2$ then $x_0 = 30$
Now

$$x(n) = (x(0) + nd)\,u_{(n)} \tag{49}$$

Where d is the common difference of the A.P and u_n is the unit step function.
$(u_{(n)} = 0 \forall n < 0, u_{(n)} = 1 \forall n \geq 0)$

$$\implies x(7) = (x(0) + 7d) \tag{50}$$
$$\implies 7d = \pm 28 \implies d = \pm 4 \tag{51}$$

Therefore the A.P is $2, 6, 10...$ or $30, 26, 22....$
Considering the former for calculations and taking Z-Transform of (49) for sum.
Since

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{52}$$

Let $y(n)$ denote the sum, let:

$$y(n) = x(n) * h(n) \tag{53}$$
$$= \sum_{k=-\infty}^{\infty} x(k)h(n-k) \tag{54}$$

Solution

Replace $h(n)$ with $u_{(n)}$.

$$y(n) = \sum_{k=0}^{n} x(k) u_{(n-k)} \tag{55}$$

$$= x(0)u_{(n)} + x(1)u_{(n-1)} + ...x(n)u_{(0)} \tag{56}$$

This denotes the sum of terms $x(0), x(1)....x(n)$ i.e. first $n+1$ terms.
From (52)

$$u_{(n)} \xrightarrow{\mathcal{Z}} \frac{1}{(1-z^{-1})} \tag{57}$$

$$nu_{(n)} \xrightarrow{\mathcal{Z}} \frac{z^{-1}}{(1-z^{-1})^2} \tag{58}$$

$$\implies X(z) = \frac{2}{(1-z^{-1})} + \frac{4z^{-1}}{(1-z^{-1})^2}, \quad |z| > |1| \tag{59}$$

Solution

Now as convolution in the time domain corresponds to multiplication in the frequency domain and (59) and (57).

$$Y(z) = X(z) * H(z) \tag{60}$$

$$= \left(\frac{2}{(1-z^{-1})} + \frac{4z^{-1}}{(1-z^{-1})^2}\right)\left(\frac{1}{(1-z^{-1})}\right), \quad |z| > |1| \tag{61}$$

Using normal inversion for inverse Z-transform:

$$Y(z) = \frac{2}{(1-z^{-1})^2} + \frac{4z^{-1}}{(1-z^{-1})^3}, \quad |z| > |1| \tag{62}$$

$$= \frac{8z^{-1}}{1-z^{-1}} + \frac{10z^{-2}}{(1-z^{-1})^2} + \frac{4z^{-3}}{(1-z^{-1})^3} + 2 \tag{63}$$

Solution

For proceeding forwards here are some important generalizations.
Shifting property

$$x(n-k) \leftrightarrow z^{-k}X(z) \tag{64}$$

Differentiation property

$$nx(n) \leftrightarrow -zX'(z) \tag{65}$$

From (57) and (64)

$$u_{(n-1)} \xrightarrow{\mathcal{Z}} \frac{z^{-1}}{1-z^{-1}} \tag{66}$$

From (57) and (65)

$$nu_{(n)} \xrightarrow{\mathcal{Z}} -z\frac{d}{dz}\left(\frac{1}{1-z^{-1}}\right) \tag{67}$$

$$nu_{(n)} \xrightarrow{\mathcal{Z}} \frac{z^{-1}}{(1-z^{-1})^2} \tag{68}$$

From (64)

$$(n-1)u_{(n-1)} \xrightarrow{\mathcal{Z}} z^{-1}\frac{z^{-1}}{(1-z^{-1})^2} \tag{69}$$

$$(n-1)u_{(n-1)} \xrightarrow{\mathcal{Z}} \frac{z^{-2}}{(1-z^{-1})^2} \tag{70}$$

Now, using (65) and writing the corresponding L.H.S

$$(n)(n-1)u_{(n-1)} \xrightarrow{\mathcal{Z}} \frac{2z^{-2}}{(1-z^{-1})^3} \tag{71}$$

Using (64)

$$\frac{(n-1)(n-2)u_{(n-2)}}{2} \xrightarrow{\mathcal{Z}} \frac{z^{-3}}{(1-z^{-1})^3} \tag{72}$$

Solution

The inverse-Z of a constant will be $\delta(n)$, so it is ruled out. Plugging these values in (63) we get

$$y(n) = 8u_{(n-1)} + 10(n-1)u_{(n-1)} + 4\frac{(n-1)(n-2)u_{(n-2)}}{2} + 2\delta(n) \quad (73)$$

Putting $n = 19$

$$y(19) = 2(19+1)^2 = 800 \quad (74)$$

Solution

Using contour integration for inverse Z-transform

$$y(19) = \frac{1}{2\pi j} \oint_C Y(z)\, z^{18}\, dz \tag{75}$$

$$= \frac{1}{2\pi j} \oint_C \left(2z^{20}\left(z-1\right)^{-2} + 4z^{20}\left(z-1\right)^{-3}\right)\, dz \tag{76}$$

$$R = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f\left(z\right)\right) \tag{77}$$

For R_1 , $m = 2$, where m corresponds to number of repeated poles .

$$R_1 = \frac{1}{(1)!} \lim_{z \to 1} \frac{d}{dz} \left((z-1)^2 2z^{20}\left(z-1\right)^{-2}\right) \tag{78}$$

$$= 2 \lim_{z \to 1} \frac{d}{dz}(z^{20}) \tag{79}$$

$$= 40 \tag{80}$$

Solution

$$R_2 = \frac{1}{(2)!} \lim_{z\to 1} \frac{d^2}{dz^2} \left((z-1)^3 4z^{20} (z-1)^{-3} \right) \tag{81}$$

$$= (2) \lim_{z\to 1} \frac{d^2}{dz^2} (z^{20}) \tag{82}$$

$$= 760 \tag{83}$$

$$R_1 + R_2 = 800 \tag{84}$$

$$\implies y(19) = 800 \tag{85}$$

Similarly, the sum for the A.P. $30, 26, 22...$ can be found by the same procedure.

9) In an A.P. of 40 terms, the sum of first 9 terms is 153 and the sum of last 6 terms is 687. Determine the first term and the common difference of the A.P. Also find the sum of all the terms of the A.P.

Given:

$$y(8) = 153 \tag{86}$$
$$y(39) - y(34) = 687 \tag{87}$$

Now, let the first term be $x(0)$ and common difference be d. From (52) and (49)

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2}, \quad |z| > |1| \tag{88}$$

For finding the sum (Assuming $h(n) = u_{(n)}$)

$$y\left(n\right) = x\left(n\right) * h\left(n\right) \tag{89}$$
$$Y\left(z\right) = X\left(z\right) * H\left(z\right) \tag{90}$$
$$= \left(\frac{x(0)}{(1 - z^{-1})} + \frac{dz^{-1}}{(1 - z^{-1})^2}\right)\left(\frac{1}{(1 - z^{-1})}\right), \quad |z| > |1| \tag{91}$$

Solution

$$Y(z) = \frac{(2x(0) + d)\, z^{-1}}{1 - z^{-1}} + \frac{(x(0) + 2d)\, z^{-2}}{(1 - z^{-1})^2} + \frac{dz^{-3}}{(1 - z^{-1})^3} + x(0) \quad (92)$$

Using normal inversion for inverse Z-transform:
Using the results (66),(70) and (72)

$$\begin{aligned} y(n) = (2x_0 + d)u_{(n-1)} + (n-1)u_{(n-1)}(x_0 + 2d) + \\ \frac{d(n-1)(n-2)u_{(n-2)}}{2} + x(0)\delta(n) \end{aligned} \quad (93)$$

Now use (86) and (87) to solve for $x(0)$ and d and put in (103) for the sum of 40 terms.

Solution

Using contour integration for inverse Z-transform

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z)\, z^{n-1}\, dz \tag{94}$$

$$= \frac{1}{2\pi j} \oint_C \left(x(0) z^{n+1} (z-1)^{-2} + dz^{20} (z-1)^{-3} \right) dz \tag{95}$$

$$R = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z) \right) \tag{96}$$

For R_1 , $m = 2$, where m corresponds to number of repeated poles .

$$R_1 = \frac{1}{(1)!} \lim_{z \to 1} \frac{d}{dz} \left((z-1)^2 x(0) z^{n+1} (z-1)^{-2} \right) \tag{97}$$

$$= x(0) \lim_{z \to 1} \frac{d}{dz} (z^{n+1}) \tag{98}$$

$$= (n+1)\, x(0) \tag{99}$$

Solution

For R_2 , $m = 3$

$$R_2 = \frac{1}{(2)!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z-1)^3 dz^{n+1} (z-1)^{-3} \right) \tag{100}$$

$$= \left(\frac{d}{2} \right) \lim_{z \to 1} \frac{d^2}{dz^2} (z^{n+1}) \tag{101}$$

$$= \left(\frac{d}{2} \right) (n) (n+1) \tag{102}$$

$$y(n) = R_1 + R_2 = \left(\frac{n+1}{2} \right) (2x(0) + nd) \tag{103}$$

Now use (86) and (87) to solve for $x(0)$ and d and put in (103) for the sum of 40 terms.

1) If $\overrightarrow{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{b} = \hat{i} + \hat{j} - \hat{k}$, then $\overrightarrow{a}$ and $\overrightarrow{b}$ are:

1. Collinear vectors which are not parallel
2. Parallel vectors
3. Perpendicular vectors
4. Unit vectors

Solution

Let

$$\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \tag{104}$$

Applying concept of rank from (34)

$$\text{rank} \begin{pmatrix} 1 & 1 & -1 \\ 2 & -1 & 1 \end{pmatrix} = 2 \neq 1, \text{Not parallel} \tag{105}$$

Applying condition for perpendicularity:

$$\mathbf{a}^\top \mathbf{b} = \begin{pmatrix} 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 0 \implies \mathbf{a} \perp \mathbf{b} \tag{106}$$

$\implies$ Option (C) is true.

2) If α,β and γ are the angles which a line makes with positive directions of x,y and z axes respectively, then which of the following are **not** true?

1. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
2. $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$
3. $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$
4. $\cos\alpha + \cos\beta + \cos\gamma = 1$

Let **m** represent the direction vector of the line:

$$\mathbf{m} = \begin{pmatrix} \cos\alpha \\ \cos\beta \\ \cos\gamma \end{pmatrix} \tag{107}$$

For option (A)

$$\mathbf{m}\mathbf{m}^\top = 1 \tag{108}$$

The following is the normal equation of a line, where **n** represents the normal vector.

$$\mathbf{n}^\top\mathbf{x} = c \tag{109}$$

From (18)

$$2x + 5y = -4 \tag{110}$$

$$2x = -4 - 5y \tag{111}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} + y \begin{pmatrix} -\frac{5}{2} \\ 1 \end{pmatrix} \tag{112}$$

$$\mathbf{x} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} - \frac{5y}{2} \begin{pmatrix} 1 \\ -\frac{2}{5} \end{pmatrix} \tag{113}$$

$$= \mathbf{A} + K\mathbf{m} \tag{114}$$

The general equation written above is of the parametric form.
Similarly apply the concept of finding **m** for this question and solve.

3) $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$ are three mutually perpendicular unit vectors. If θ is the angle between $\overrightarrow{a}$ and $\left(\overrightarrow{2a} + \overrightarrow{3b} + \overrightarrow{6c}\right)$, find the value of $\cos\theta$.

Solution

Given:

$$\mathbf{a}^\top \mathbf{b} = \mathbf{b}^\top \mathbf{c} = \mathbf{c}^\top \mathbf{a} = 0 \tag{115}$$

$$||\mathbf{a}|| = ||\mathbf{b}|| = ||\mathbf{c}|| = 1 \tag{116}$$

$$\cos\theta = \frac{\mathbf{a}^\top (2\mathbf{a} + 3\mathbf{b} + 6\mathbf{c})}{||\mathbf{a}||||2\mathbf{a} + 3\mathbf{b} + 6\mathbf{c}||} \tag{117}$$

Now,

$$\mathbf{a}^\top (2\mathbf{a} + 3\mathbf{b} + 6\mathbf{c}) = 2\mathbf{a}^\top \mathbf{a} + 3\mathbf{a}^\top \mathbf{b} + 6\mathbf{a}^\top \mathbf{c} = 2 + 0 + 0 = 2 \tag{118}$$

$$||\mathbf{a}||||2\mathbf{a} + 3\mathbf{b} + 6\mathbf{c}|| = ||2\mathbf{a} + 3\mathbf{b} + 6\mathbf{c}|| \tag{119}$$

Solution

From (9) norm definition:

$$(||2\mathbf{a}+3\mathbf{b}+6\mathbf{c}||)^2 = ||4\mathbf{a}^2|| + ||9\mathbf{b}^2|| + ||36\mathbf{c}^2|| = 49 \tag{120}$$

$$\implies ||2\mathbf{a}+3\mathbf{b}+6\mathbf{c}|| = +7 \tag{121}$$

$$\implies \cos\theta = \frac{2}{7} \tag{122}$$

4) Find the position vector of point **C** which divides the line segment joining points **A** and **B** having position vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio 4 : 1 externally. Further, find $|\overrightarrow{AB}| : |\overrightarrow{BC}|$.

Solution

We know that:

$$\mathbf{C} = \frac{4\mathbf{B} - \mathbf{A}}{4 - 1} \tag{123}$$

Simplify the above for **C** (Given in 10^{th} NCERT).

5) Find the equation of the line passing through the point of intersection of the lines $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and $\frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2}$ and perpendicular to these given lines.

Solution

Let the given lines be denoted by $\mathbf{x}_1$ and $\mathbf{x}_2$ respectively. From (114):

$$\mathbf{x}_1 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + k_1 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \mathbf{A} + k_1\mathbf{m}_1 \tag{124}$$

$$\mathbf{x}_2 = \begin{pmatrix} 1 \\ 0 \\ 7 \end{pmatrix} + k_2 \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix} = \mathbf{B} + k_2\mathbf{m}_2 \tag{125}$$

Equation of a line passing through point of intersection of two lines is given by $\mathbf{x}_1 + \lambda\mathbf{x}_2$, let the direction vector of the line be denoted by $\mathbf{m}$

$$(\mathbf{A} + k_1\mathbf{m}_1) + \lambda(\mathbf{B} + k_2\mathbf{m}_2) = 0 \tag{126}$$

Solution

The two equations required to solve for the direction of line are

$$\mathbf{m}_1\mathbf{m}^\top = 0 \tag{127}$$

$$\mathbf{m}_2\mathbf{m}^\top = 0 \tag{128}$$

$$\implies (\mathbf{m}_1\mathbf{m}_2)^\top \mathbf{m} = 0 \tag{129}$$

$$\mathbf{m}\begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & 2 \end{pmatrix} \xleftarrow{R_1 \to 2R_2 + 3R_1} \begin{pmatrix} 3 & 0 & 13 \\ 0 & -3 & 2 \end{pmatrix} \mathbf{m} = 0 \tag{130}$$

$$\begin{pmatrix} 3 & 0 & 13 \\ 0 & -3 & 2 \end{pmatrix} \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} = 0 \tag{131}$$

$$\implies \mathbf{m} = \begin{pmatrix} \frac{-13}{3} \\ \frac{2}{3} \\ 1 \end{pmatrix} \tag{132}$$

Solution

Let the unknown line in its parametric form be denoted as follows from (114).

$$\mathbf{x}_3 = \mathbf{C} + k_3\mathbf{m} \tag{133}$$

For finding the point of intersection of the lines($\mathbf{C}$), equating (124) and (125)

$$\mathbf{A} + k_1\mathbf{m}_1 = \mathbf{B} + k_2\mathbf{m}_2 \tag{134}$$

$$\begin{pmatrix}\mathbf{m}_1 & \mathbf{m}_2\end{pmatrix}\begin{pmatrix}k_1 \\ -k_2\end{pmatrix} = \mathbf{B} - \mathbf{A} \tag{135}$$

From the above k_1 and k_2 can be found by gauss elimination given in (19) and thus $\mathbf{C}$.

6) Two vertices of the parallelogram **ABCD** are given as $\mathbf{A}(-1, 2, 1)$ and $\mathbf{B}(1, -2, 5)$. If the equation if the line passing through **C** and **D** is $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$, then find the distance between the sides **AB** and **CD**. Hence, find the area of parallelogram **ABCD**

Solution

Given:

$$\mathbf{A} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} \tag{136}$$

$$\mathbf{CD} : \mathbf{x} = \begin{pmatrix} 4 \\ -7 \\ 8 \end{pmatrix} + k \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \tag{137}$$

Let the corresponding foot of perpendicular of **A** on line **CD** be **P**, the direction vector of line **CD** be $\mathbf{m_1}$. Therefore the required equations are:

$$(\mathbf{A} - \mathbf{P})^\top \mathbf{m_1} = 0 \tag{138}$$

$$\mathbf{P} = \mathbf{Q} + k\mathbf{m_1} \tag{139}$$

Where **Q** is the given vector lying on **CD**.
From (138) and (139), k can be found out, which when substituted in (139) gives **P**. Distance between **A** and **P** is given by (9).