NCERT-discrete : 10.5.3 - 2

EE23BTECH11025 - Anantha Krishnan

I. Question

Find the sums given below:
(i) 7 + 10.5 + 14 ... + 84
(ii) 34 + 32 + 30 ... + 10
(iii) -5 + -8 + -11 ... -230

Symbols	Description	Values
d_i	Common Difference for i^{th} AP	3.5
		-2
		-3
$x_i(n)$	n^{th} term for i^{th} Sequence	$(x_i(0)\ +nd_i)u_{(n)}$
$s_i(n)$	Sum of (n+1)terms for i^{th} Sequence	$\frac{(n+1)u_{(u)}}{2}(2x_i(0)+kd_i)$
$x_i(0)$	First term for i^{th} AP	7
		34
		-5

TABLE I
Parameters , Descriptions And Values

Solutions :

(i) $7 + 10\frac{1}{2} + 14... + 84$

$$x_1(n) = (x_1(0) + nd_1)\, u_{(n)} \tag{1}$$
$$84 = 7 + \frac{7n}{2} \tag{2}$$
$$n = 22 \tag{3}$$

1. Calculating $s_1(22)$:

$$s_1(22) = \frac{23}{2}\left(14 + (22)\frac{7}{2}\right) \tag{4}$$
$$= 1046.5 \tag{5}$$

2. z-Transform of $x_1(n)$: Using (**??**)

$$X_1(z) = \sum_{n=-\infty}^{\infty}\left(7 + \frac{7n}{2}\right)u_{(n)}z^{-n} \tag{6}$$
$$= 7z(z-1)^{-1} + 7z(2(z-1))^{-2}, \quad |z| > |1| \tag{7}$$

3. Z-Transform of $s_1(n)$:

$$h(n) = u(n) \tag{8}$$
$$H_1(z) = z(z-1)^{-1} \tag{9}$$
$$y_1(n) = x_1(n) * h(n) \tag{10}$$
$$Y_1(z) = X_1(z) * H_{(z)} \tag{11}$$
$$= \left(7z(z-1)^{-1} + 7z(2(z-1))^{-2}\right) z(z-1)^{-1}, \quad |z| > |1| \tag{12}$$

4. Inversion of $Y_1(z)$: Using Contour Integration :

$$y_1(22) = \frac{1}{2\pi j} \oint_C Y(z)\, z^{21}\, dz \tag{13}$$
$$= \frac{1}{2\pi j} \oint_C \left(\frac{7z^{23}}{(z-1)^2} + \frac{7z^{23}}{2(z-1)^3} \right) dz \tag{14}$$

We can observe that the pole is repeated 2 times and thus $m = 2$,

$$R_1 = \frac{1}{(m-1)!} \lim_{z\to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z)\right) \tag{15}$$
$$= \frac{1}{(1)!} \lim_{z\to 1} \frac{d}{dz} \left((z-1)^2 \frac{7z^{23}}{(z-1)^2} \right) \tag{16}$$
$$= 7 \lim_{z\to 1} \frac{d}{dz}(z^{23}) \tag{17}$$
$$= 161 \tag{18}$$

We can observe that the pole is repeated 3 times and thus $m = 3$,

$$R_2 = \frac{1}{(m-1)!} \lim_{z\to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z)\right) \tag{19}$$
$$= \frac{1}{(2)!} \lim_{z\to 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{\left(7z^{13}\right)}{2(z-1)^3} \right) \tag{20}$$
$$= \left(\frac{7}{4}\right) \lim_{z\to 1} \frac{d^2}{dz^2}(z^{23}) \tag{21}$$
$$= 885.5 \tag{22}$$
$$R_1 + R_2 = 1046.5 \tag{23}$$
$$\implies y_1(22) = 1046.5 \tag{24}$$

(ii) 34 + 32 + 30 ... + 10

$$x_2(n) = (x_2(0) + nd_2)\, u_{(n)} \tag{25}$$
$$10 = 34 - 2n \tag{26}$$
$$n = 12 \tag{27}$$

1. Calculating $s_2(12)$:

$$s_2(12) = \frac{13}{2}(64 + 12(-2)) \tag{28}$$
$$= 286. \tag{29}$$

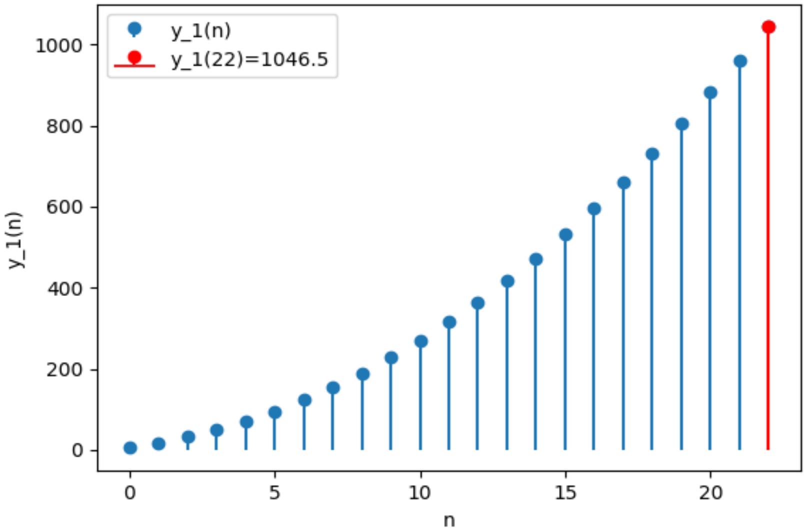

Fig. 1. $y_1(n)$ vs n

2. Z-Transform of $x_2(n)$: Using (**??**)

$$X_2(z) = \sum_{n=-\infty}^{\infty} (x_2(0) - 2n)\, u_{(n)} z^{-n} \tag{30}$$

$$= 34z(z-1)^{-1} - 2z((z-1))^{-2}, \quad |z| > |1| \tag{31}$$

3. Z-Transform of $s_2(n)$:

$$h[n] = u[n] \tag{32}$$

$$y_2(n) = x_2(n) * h(n) \tag{33}$$

$$Y_2(z) = X_2(z) * H(z) \tag{34}$$

$$= \left(34z(z-1)^{-1} - 2z((z-1))^{-2}\right) z(z-1)^{-1}, \quad |z| > |1| \tag{35}$$

4. Inversion of $Y_2(z)$: Using Contour Integration :

$$y_2(12) = \frac{1}{2\pi j} \oint_C Y(z)\, z^{11}\, dz \tag{36}$$

$$= \frac{1}{2\pi j} \oint_C \left(\frac{34z^{13}}{(z-1)^2} - \frac{2z^{13}}{(z-1)^3} \right) dz \tag{37}$$

We can observe that the pole is repeated 2 times and thus $m = 2$,

$$R_1 = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z)\right) \tag{38}$$

$$= \frac{1}{(1)!} \lim_{z \to 1} \frac{d}{dz} \left((z-1)^2 \frac{34z^{13}}{(z-1)^2} \right) \tag{39}$$

$$= 34 \lim_{z \to 1} \frac{d}{dz}(z^{13}) \tag{40}$$

$$= 442 \tag{41}$$

We can observe that the pole is repeated 3 times and thus $m = 3$,

$$R_2 = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z)\right) \tag{42}$$

$$= \frac{1}{(2)!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{\left(-2z^{13}\right)}{(z-1)^3}\right) \tag{43}$$

$$= -\lim_{z \to 1} \frac{d^2}{dz^2}(z^{13}) \tag{44}$$

$$= -156 \tag{45}$$

$$R_1 + R_2 = 286 \tag{46}$$

$$\implies y_2(12) = 286 \tag{47}$$

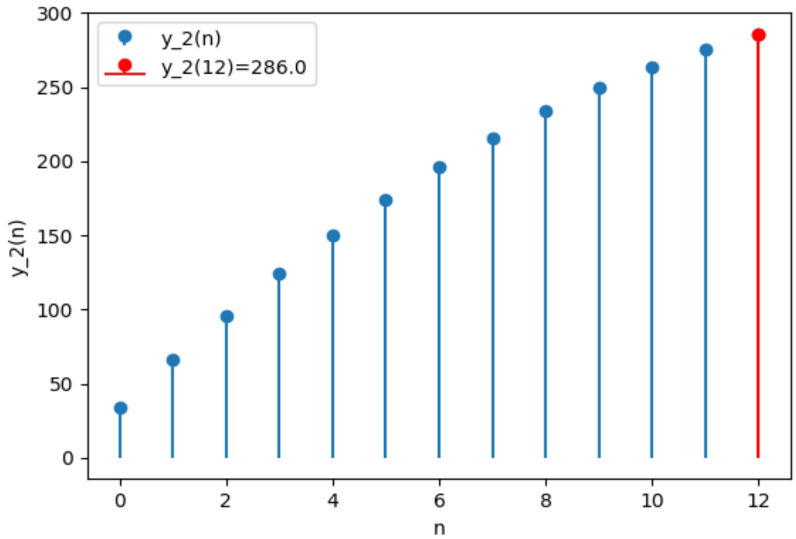

Fig. 2. $y_2(n)$ vs n

(iii) -5 + -8 + -11 ... -230

$$x_3(n) = (x_3(0) - 3n)\, u_{(n)} \tag{48}$$

$$-230 = -5 - 3n \tag{49}$$

$$n = 75 \tag{50}$$

1. Calculating $s_3(75)$:

$$s_3(75) = \frac{76}{2}\left(-10 + (76-1)(-3)\right) \tag{51}$$

$$= -8930 \tag{52}$$

2. Z-Transform of $x_3(n)$: Using (**??**)

$$X_3(z) = \sum_{n=-\infty}^{\infty} (x_3(0) - 3n)\, u_{(n)} z^{-n} \tag{53}$$

$$= -5z(z-1)^{-1} - 3z\left((z-1)\right)^{-2}, \quad |z| > |1| \tag{54}$$

3. Z-Transform of $s_3(n)$:

$$h(n) = u(n) \tag{55}$$
$$y_3(n) = x_3(n) * h(n) \tag{56}$$
$$Y_3(z) = X_3(z) * H(z) \tag{57}$$
$$= \left(-5z(z-1)^{-1} - 3z((z-1))^{-2}\right) z(z-1)^{-1} \quad |z| > |1| \tag{58}$$

4. Inversion of $Y_3(z)$: Using Contour Integration :

$$y_3(75) = \frac{1}{2\pi j} \oint_C Y(z)\, z^{74}\, dz \tag{59}$$
$$= \frac{1}{2\pi j} \oint_C \left(\frac{-5z^{76}}{(z-1)^2} - \frac{3z^{76}}{(z-1)^3} \right) dz \tag{60}$$

We can observe that the pole is repeated 2 times and thus $m = 2$,

$$R_1 = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z)\right) \tag{61}$$
$$= \frac{1}{(1)!} \lim_{z \to 1} \frac{d}{dz} \left((z-1)^2 \frac{-5z^{76}}{(z-1)^2} \right) \tag{62}$$
$$= -5 \lim_{z \to 1} \frac{d}{dz} (z^{76}) \tag{63}$$
$$= -380 \tag{64}$$

We can observe that the pole is repeated 3 times and thus $m = 3$,

$$R_2 = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z)\right) \tag{65}$$
$$= \frac{1}{(2)!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{3z^{76}}{(z-1)^3} \right) \tag{66}$$
$$= 1.5 \lim_{z \to 1} \frac{d^2}{dz^2} (z^{76}) \tag{67}$$
$$= -8550 \tag{68}$$
$$R_1 + R_2 = -8930 \tag{69}$$
$$\implies y_3(75) = -8930 \tag{70}$$

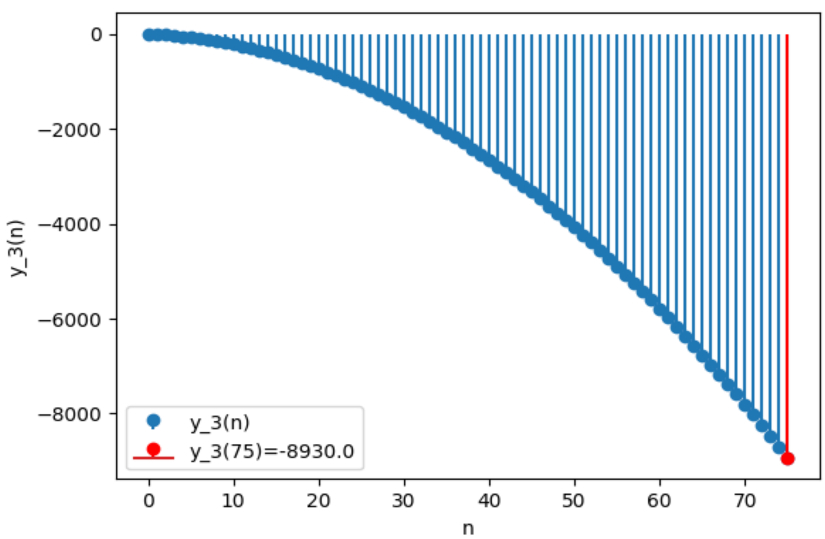

Fig. 3. $y_3(n)$ vs n