NCERT-discrete : 10.5.3 - 2

EE23BTECH11025 - Anantha Krishnan

I. QUESTION

Find the sums given below:

(i) $7 + 10\frac{1}{2} + 14 \ldots + 84$

(ii) $34 + 32 + 30 \ldots + 10$

(iii) $-5 + -8 + -11 \ldots -230$

Solutions:

(i) $7 + 10\frac{1}{2} + 14 \ldots + 84$

Let "S_k" denote the sum of first k terms in a series ,"a" denotes its first term and "d" denotes the common difference.

$$S_k = \frac{k}{2}(2a + (k-1)d) \tag{1}$$

For number of terms , we use

$$x(n) = x(0) + nd \tag{2}$$

Where x(n) is the $(n+1)^{th}$ term of the series. Putting the values

$$84 = 7 + \frac{7n}{2} \tag{3}$$

$$n = 22 \tag{4}$$

1) **Calculating S_n for $x_{1(n)}$** : We now use this result for calculating S_{23}

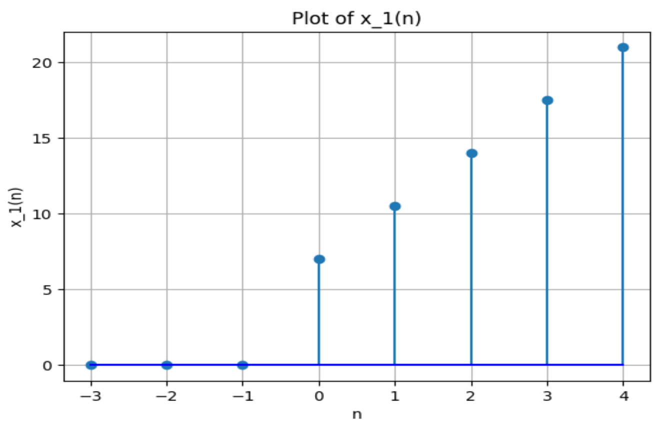
$$S_{23} = \frac{23}{2}(14 + (22)\frac{7}{2}) \tag{5}$$

Solving this yields $S_{23} = 1046.5$ We are now required to calculate $X_{1(z)}$ in terms of u_n and $x_{1(n)}$.Where $u_{(n)}$ is the unit step function.

$$x_{1(n)} = u_{(n)}(x_{1(0)} + \frac{7n}{2}) \tag{6}$$

2) **Z-Transform of $x_{1(n)}$** : By the Definition of Z-transform:

$$\sum_{n=-\infty}^{\infty} Z^{-n}x(n) = X(Z) \tag{7}$$



Graph:1 $x_{1(n)}$ vs n

Putting $x_{1(n)}$ in (7) , we get

$$\sum_{n=-\infty}^{\infty} (x_{1(0)} + \frac{7n}{2})u_{(n)}Z^{-n} = X_{1(z)} \tag{8}$$

$$\sum_{n=-\infty}^{\infty} (7 + \frac{7n}{2})u_{(n)}Z^{-n} = X_{1(z)} \tag{9}$$

$$\begin{aligned} 7z(1-z)^{-1} + \\ 7z((1-z))^{-1} + \\ 7z(2(1-z))^{-2} = X_{1(z)} \end{aligned} \tag{10}$$

Region of Convergence

$$|z| < 1 \tag{11}$$

(ii) $34 + 32 + 30 \ldots + 10$

In this bit a=34 , d=-2 Using equation (2)

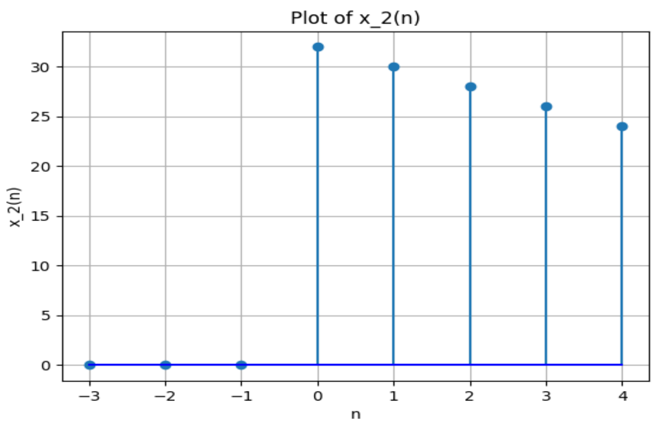
$$10 = 34 - 2n \tag{12}$$

$$n = 12 \tag{13}$$

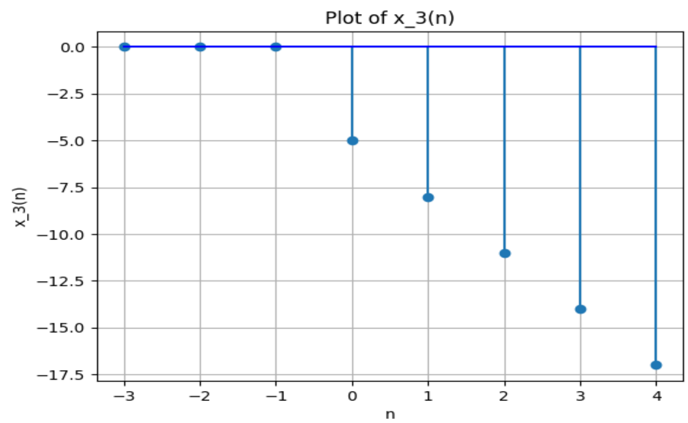
For $x_{2(n)}$

$$x_{2(n)} = x_{2(0)} + nd \tag{14}$$

$$x_{2(n)} = x_{2(0)} - 2n \tag{15}$$



Graph:2 $x_{2(n)}$ vs n



Graph:3 $x_{3(n)}$ vs n

1) **Z-Transform of** $x_{2(n)}$: Using (7)

$$\sum_{n=-\infty}^{\infty}(x_{2(0)} - 2n)u_{(n)}Z^{-n} = X_{2(z)} \tag{16}$$

Region of convergence does not change as the nature of the function remains the same as (i)bit For $X_{2(z)}$

$$\begin{aligned} 34z(1-z)^{-1} - \\ 34z((1-z))^{-1} - \\ 2z((1-z))^{-2} = X_{1(z)} \end{aligned} \tag{17}$$

2) **Calculating** S_n **of** $x_{2(n)}$: For calculating the sum , we use (1)

$$S_{13} = \frac{13}{2}(64 + 11(-2)) \tag{18}$$

$$S_{13} = 286. \tag{19}$$

(iii) -5 + -8 + -11 ... -230

Here a=-5, d=-3 From (2)

$$-230 = -5 - 3n \tag{20}$$

$$n = 75 \tag{21}$$

For $x_{3(n)}$

$$x_{3(n)} = x_3(0) + nd \tag{22}$$

$$x_{3(n)} = x_3(0) - 3n \tag{23}$$

1) **Z-Transform of** $x_{3(n)}$: Putting $x_{3(n)}$ in (7)

$$\sum_{n=-\infty}^{\infty}(x_{3(0)} - 3n)u_{(n)}Z^{-n} = X_{3(z)} \tag{24}$$

Region of convergence does not change as the nature of the function remains the same as (i)bit For $X_{3(z)}$, we use the same process as in (i)bit

$$\begin{aligned} -5z(1-z)^{-1} - \\ 5z((1-z))^{-1} - \\ 3z((1-z))^{-2} = X_{1(z)} \end{aligned} \tag{25}$$

2) **Calculating** S_n **of** $x_{3(n)}$: Using (1) :

$$S_{76} = \frac{76}{2}(-10 + (76-1)(-3)) \tag{26}$$

$$S_{76} = -8930 \tag{27}$$

Parameters	Description	Values
$x_{1(n)}$	Sequence labelled (i)	$x_{1(0)}$ +3.5n
$x_{2(n)}$	Sequence labelled (ii)	$x_{2(0)}$ -2n
$x_{3(n)}$	Sequence labelled (iii)	$x_{3(0)}$ -3n
$x_{1(0)}$	1^{st} term of sequence $x_{1(n)}$	7
$x_{2(0)}$	1^{st} term of sequence $x_{2(n)}$	34
$x_{3(0)}$	1^{st} term of sequence $x_{3(n)}$	-5

TABLE 1 : PARAMETERS , DESCRIPTIONS AND VALUES