NCERT-discrete : 10.5.3 - 2

EE23BTECH11025 - Anantha Krishnan

I. Question

Find the sums given below:

(i) $7 + 10\frac{1}{2} + 14 ... + 84$

(ii) $34 + 32 + 30 ... + 10$

(iii) $-5 + -8 + -11 ... -230$

Symbols	Description	Values
d_i	Common Difference	3.5, -2, -3
$x_i(n)$	Sequence	$(x_i(0) + nd_i)u_{(k)}$
$X_i(z)$	Z-Transform of $x_i(n)$	$zx_i(0)(z-1)^{-1} + d_i z(z-1)^{-2}$
$S_i(n)$	Sum of (n+1)terms	$\frac{(n+1)u_{(u)}}{2}(2x_i(0) + kd_i)$
$h[n]$	Unit step function	$0 \ \forall n < 0, 1 \forall n \geq 0$

Table 1 : Parameters , Descriptions And Values

Solutions:

(i) $7 + 10\frac{1}{2} + 14 ... + 84$

$$S_n = \frac{(n+1)u_{(n)}}{2}(2x_i(0) + nd_i) \tag{1}$$

For number of terms , we use

$$x_i(n) = (x_i(0) + nd_i)u_n \tag{2}$$

Where $x_i(n)$ is the $(n+1)^{th}$ term of the series. Putting the values

$$84 = 7 + \frac{7n}{2} \tag{3}$$

$$n = 22 \tag{4}$$

1) Calculating $S_1(22)$:

$$S_1(22) = \frac{23}{2}(14 + (22)\frac{7}{2})S_1(22) \quad = 1046.5 \tag{5}$$

2) Z-Transform of $x_1(n)$: By the Definition of Z-transform:

$$\sum_{n=-\infty}^{\infty} Z^{-n} x_i(n) = X_i(Z) \tag{6}$$

Putting $x_1(n)$ in (6) , we get

$$\sum_{n=-\infty}^{\infty} (x_1(0) + \frac{7n}{2})u_{(n)}Z^{-n} = X_1(z) \tag{7}$$

$$\sum_{n=-\infty}^{\infty} (7 + \frac{7n}{2})u_{(n)}Z^{-n} = X_1(z) \tag{8}$$

$$7z(z-1)^{-1} + 7z(2(z-1))^{-2} = X_1(z) \tag{9}$$

$$\forall |z| > 1 \tag{10}$$

3) Z-Transform of $S_1(n)$: Using (2) and assuming

$$h(n) = u(n) \tag{11}$$

$$S_1(n) = x_1(n) * h(n) \tag{12}$$

$$S_1(z) = X_1(z) * H_(z) \tag{13}$$

Where $X_1(z)$ comes from (9). For $H(z)$, it is Z-transform of unit-step function

$$H_1(z) = z(z-1)^{-1} \tag{14}$$

For $S_1(z)$:

$$S_1(z) = (7z(z-1)^{-1} + 7z(2(z-1))^{-2})z(z-1)^{-1}$$

ROC:

$$|z| > 1 \tag{15}$$

4) Inversion of $S_1(z)$: By using partial fractions :

$$S_1(z) = (7z^2(z-1)^{-2} + 7z^2(2(z-1))^{-3})$$

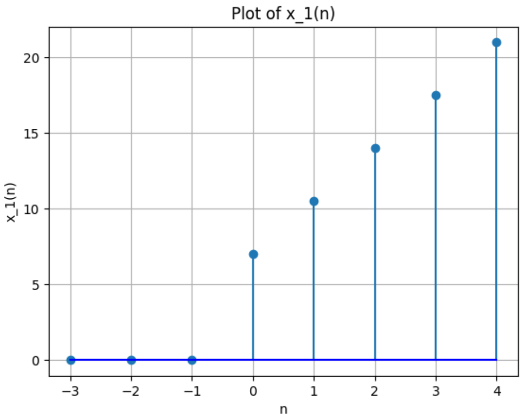
Using known results:
Inverse Z-transform of

$$z^2(z-1)^{-2} \leftrightarrow (n+1)u(n) \tag{16}$$

For $z^2(z-1)^{-3}$

we can differentiate (16) and get the inverse Z-transform as

$$z^2(z-1)^{-3} \leftrightarrow (n(n+1)/2)u(n) \tag{17}$$



Graph:1 $x_1(n)$ vs n

Therefore:

$$S_1(n) = (7(n+1) + 1.75n(n+1))u(n) \tag{18}$$

(ii) 34 + 32 + 30 ... + 10

In this bit $x_2(0)$ = 34 , d_2 = -2.

Using equation (2)

$$10 = 34 - 2n \tag{19}$$
$$n = 12 \tag{20}$$

For $x_2(n)$

$$x_2(n) = x_2(0) + nd_2 \tag{21}$$
$$x_2(n) = x_2(0) - 2n \tag{22}$$

1) Calculating $S_2(12)$: For calculating the sum , we use (1)

$$S_2(12) = \frac{13}{2}(64 + 11(-2)) \tag{23}$$
$$S_2(12) = 286. \tag{24}$$

2) Z-Transform of $x_2(n)$: Using (6)

$$\sum_{n=-\infty}^{\infty} (x_2(0) - 2n)u_{(n)}Z^{-n} = X_2(z) \tag{25}$$

For $X_2(z)$

$$34z(z-1)^{-1} - 2z((z-1))^{-2} = X_2(z) \tag{26}$$
$$|z| > 1 \tag{27}$$

3) Z-Transform of $S_2(n)$: Using (2) and assuming

$$h[n] = u[n] \tag{28}$$
$$S_2(n) = x_2(n) * h(n) \tag{29}$$
$$S_2(z) = X_2(z) * H(z) \tag{30}$$

Where $X_2(z)$ comes from (26) and $H(z)$ from (14). For $S_2(z)$:

$$S_2(z) = (34z(z-1)^{-1} - 2z((z-1))^{-2})z(z-1)^{-1} \tag{31}$$

ROC:

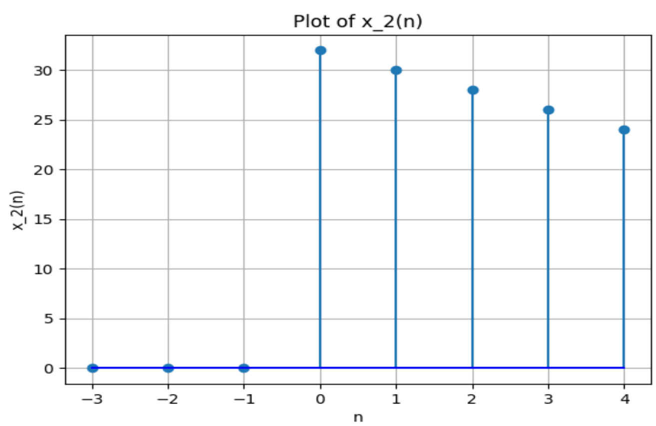
$$|z| > 1 \tag{32}$$

4) Inversion of $S_2(z)$: By using partial fractions

$$S_2(z) = (34z^2(z-1)^{-2} - 2z^2((z-1))^{-3})$$

Using results (16) and (17)

$$S_2(n) = (34(n+1) - n(n+1))u(n) \tag{33}$$



Graph:2 $x_2(n)$ vs n

(iii) -5 + -8 + -11 ... -230

Here $x_3(0)$ = -5, d_3 = -3 From (2)

$$-230 = -5 - 3n \tag{34}$$
$$n = 75 \tag{35}$$

For $x_3(n)$

$$x_3(n) = x_3(0) + nd_3 \tag{36}$$
$$x_{3(n)} = x_3(0) - 3n \tag{37}$$

1) Calculating $S_3(75)$: Using (1) :

$$S_3(75) = \frac{76}{2}(-10 + (76-1)(-3)) \quad (38)$$
$$S_3(75) = -8930 \quad (39)$$

2) Z-Transform of $x_3(n)$: Putting $x_3(n)$ in (6)

$$\sum_{n=-\infty}^{\infty} (x_3(0) - 3n)u_{(n)}Z^{-n} = X_3(z) \quad (40)$$

For $X_3(z)$, we use the same process as in (i) bit

$$-5z(z-1)^{-1} - (1.5)z((z-1))^{-2} = X_3(z) \quad |z| > 1 \quad (41)$$

3) Z-Transform of $S_3(n)$: Using (2) and assuming

$$h(n) = u(n) \quad (42)$$
$$S_3(n) = x_3(n) * h(n)S_3(z) \quad = X_3(z) * H_(z) \quad (43)$$

Where $X_3(z)$ comes from (41) and $H_(z)$ from (14). For $S_3(z)$:

$$S_3(z) = (-5z(z-1)^{-1} - (1.5)z((z-1))^{-2})z(z-1)^{-1} \quad (44)$$

ROC:

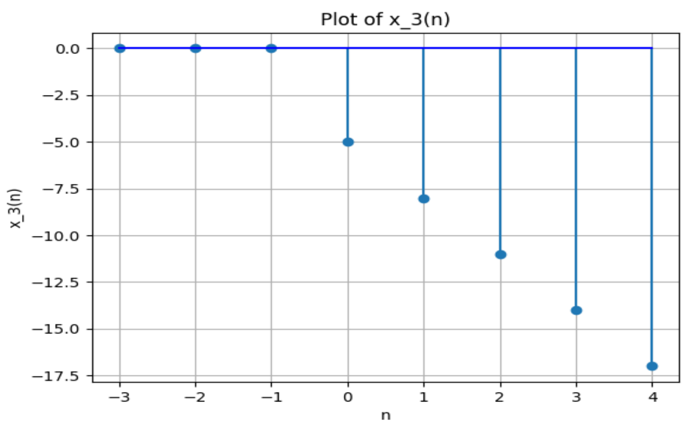
$$|z| > 1 \quad (45)$$

4) Inversion of $S_2(z)$: By using partial fractions

$$S_3(z) = (-5z^2(z-1)^{-2} - 1.5z^2((z-1))^{-3})$$

Using results (16) and (17)

$$S_3(n) = (-5(n+1) - 1.5n(n+1))u(n) \quad (46)$$



Graph:3 $x_3(n)$ vs n