

NCERT-discrete : 10.5.3 - 2

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I. QUESTION

The laplace transform of $x_1(t) = e^{-t}u(t)$ is $X_1(s)$, where $u(t)$ is the unit step function. The laplace transform of $x_2(t) = e^t u(-t)$ is $X_2(s)$. Which one of the following statements is TRUE?

- 1) The region of convergence of $X_1(s)$ is $Re(s) \geq 0$
- 2) The region of convergence of $X_2(s)$ is confined to the left half-plane of s .
- 3) The region of convergence of $X_1(s)$ is confined to the right half-plane of s .
- 4) the imaginary axis in the s -plane is included in both the region of convergence of $X_1(s)$ and the region of convergence of $X_2(s)$.

Solutions :

Symbols	Description
$X_1(s)$	Laplace transform of $x_1(t)$
$X_2(s)$	Laplace transform of $x_2(t)$
$u(t)$	Unit step function

TABLE I

PARAMETERS, DESCRIPTIONS

Laplace transform of $x_1(t)$ is given by :

$$X_1(s) = \int_{-\infty}^{\infty} e^{-t} e^{-st} u(t) dt \quad (1)$$

Let $s = \sigma + j\omega$:

$$X_1(s) = \int_0^{\infty} e^{-t(\sigma+1)} e^{-tj\beta} dt \quad (2)$$

$$= \left[\frac{-e^{-t(\sigma+1)} e^{-tj\beta}}{(\sigma+1) + j\beta} \right]_0^{\infty} \quad (3)$$

$$(4)$$

For $X_1(s)$ to be convergent, $|-e^{-t(\sigma+1)} e^{-tj\beta}|$ must converge $\forall t \in (0, \infty)$, so:

$$|e^{-tj\beta}| = |1|, \forall \beta \in \mathbb{R} \implies Im(s) \in \mathbb{R} \quad (5)$$

$$\sigma + 1 > 0 \implies Re(s) > -1 \quad (6)$$

Putting the limits :

$$X_1(s) = \frac{1}{s+1}, \quad (7)$$

$$\text{ROC of } X_1(s) : Re(s) > -1 \quad (8)$$

Laplace transform of $x_2(t)$ is given by :

$$X_2(s) = \int_{-\infty}^{\infty} e^t e^{-st} u(-t) dt \quad (9)$$

$$(10)$$

Let $s = \sigma + j\omega$:

$$= \int_{-\infty}^0 e^{t(1-\sigma)} e^{-tj\beta} dt \quad (11)$$

$$= \left[\frac{e^{t(1-\sigma)} e^{-tj\beta}}{(1-\sigma) - j\beta} \right]_{-\infty}^0 \quad (12)$$

For $X_2(s)$ to be convergent, $|e^{t(1-\sigma)} e^{-tj\beta}|$ must converge $\forall t \in (-\infty, 0)$, so:

$$|e^{-tj\beta}| = |1|, \forall \beta \in \mathbb{R} \implies \text{Im}(s) \in \mathbb{R} \quad (13)$$

$$1 - \sigma > 0 \implies \text{Re}(s) < 1 \quad (14)$$

Putting the limits

$$X_2(s) = \frac{1}{1-s}, \quad (15)$$

$$\text{ROC of } X_2(s) : \text{Re}(s) < 1 \quad (16)$$

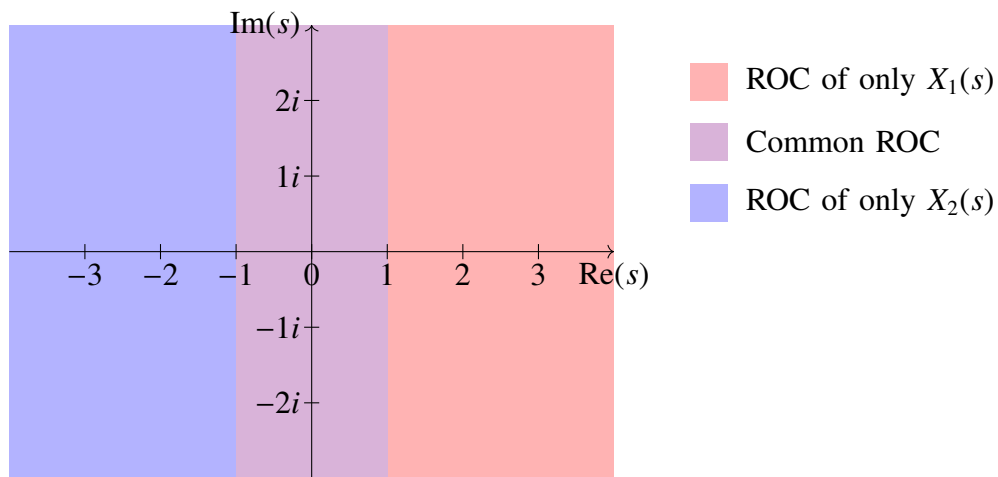


Fig. 1. Representation of ROCs of $X_1(s)$ and $X_2(s)$

Based on the overlap of regions of convergence of $X_1(s)$ and $X_2(s)$ from 1 , we can conclude that option 4) is correct .