NCERT-discrete : 10.5.3 - 2

EE23BTECH11025 - Anantha Krishnan

I. Question

Find the sums given below:

(i) $7 + 10\frac{1}{2} + 14 ... + 84$

(ii) 34 + 32 + 30 ... + 10

(iii) -5 + -8 + -11 ... -230

Solutions:

(i) $7 + 10\frac{1}{2} + 14 ... + 84$

Let "S_k" denote the sum of first k terms in a series ,"a" denotes its first term and "d" denotes the common difference.

$$S_k = \frac{k}{2}(2a + (k-1)d) \tag{1}$$

For number of terms , we use

$$x(n) = x(0) + nd \tag{2}$$

Where x(n) is the $(n+1)^{th}$ term of the series. Putting the values

$$84 = 7 + \frac{7n}{2} \tag{3}$$

$$n = 22 \tag{4}$$

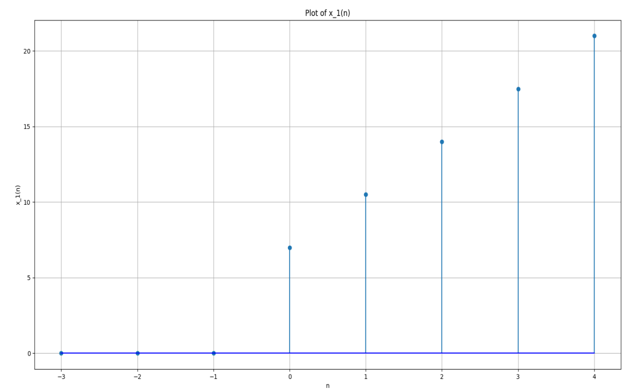
Calculating S_n for $x_{1(n)}$
We now use this result for calculating S_{23}

$$S_{23} = \frac{23}{2}(14 + (22)\frac{7}{2}) \tag{5}$$

Solving this yields $S_{23} = 1046.5$

We are now required to calculate $X_{1(z)}$ in terms of u_n and $x_{1(n)}$.Where $u_{(n)}$ is the unit step function.

$$x_{1(n)} = u_{(n)}(x_{1(0)} + \frac{7n}{2}) \tag{6}$$



Graph:1 $x_{1(n)}$ vs n

Z-Transform of $x_{1(n)}$
By the Definition of Z-transform:

$$\sum_{n=-\infty}^{\infty} Z^{-n}x(n) = X(Z) \tag{7}$$

Putting $x_{1(n)}$ in (7) , we get

$$\sum_{n=-\infty}^{\infty} (x_{1(0)} + \frac{7n}{2})u_{(n)}Z^{-n} = X_{1(z)} \tag{8}$$

$$\sum_{n=-\infty}^{\infty} (7 + \frac{7n}{2})u_{(n)}Z^{-n} = X_{1(z)} \tag{9}$$

For the **region of convergence**
$X_{1(z)}$ = convergent $\forall$ z $\epsilon(-\infty, -1)U(1, \infty)$

This can be proved from ratio test .
We now calculate the sum (Here $(k-1)^{th}$ term is the last term and $x_{1(0)}$ is the first term).

$$7(1 - z^k)(z^k(1 - z))^{-1} + (7(z^k - 1)z)(2z^k(z - 1)^2)^{-1} - (7kz)(2z^{k+1}(z - 1))^{-1} = X_{1(z)} \tag{10}$$

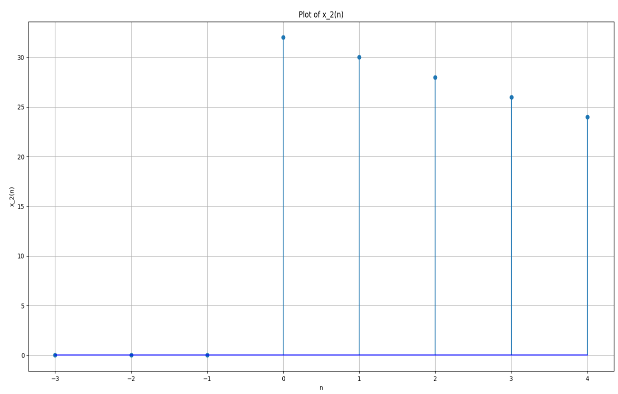
(ii) 34 + 32 + 30 ... + 10

In this bit
a=34 , d=-2 Using equation (2)

$$10 = 34 - 2n \tag{11}$$
$$n = 12 \tag{12}$$

For $x_{2(n)}$

$$x_{2(n)} = x_{2(0)} + nd \tag{13}$$
$$x_{2(n)} = x_{2(0)} - 2n \tag{14}$$



Graph:2 $x_{2(n)}$ vs n

Z-Transform of $x_{2(n)}$
Using (7)

$$\sum_{n=-\infty}^{\infty} (x_{2(0)} - 2n)u_{(n)}Z^{-n} = X_{2(z)} \tag{15}$$

Region of convergence does not change as the nature of the function remains the same as (i)bit
For $X_{2(z)}$, we use the same process as in (i)bit.

$$\begin{aligned} &32(1 - z^k)(z^k(1 - z))^{-1} - \\ &(2(z^k - 1)z)(z^k(z - 1)^2)^{-1} + \\ &(2kz)(z^{k+1}(z - 1))^{-1} = X_{2(z)} \end{aligned} \tag{16}$$

Calculating S_n of $x_{2(n)}$
For calculating the sum , we use (1)

$$S_{13} = \frac{13}{2}(64 + 11(-2)) \tag{17}$$
$$S_{13} = 286. \tag{18}$$

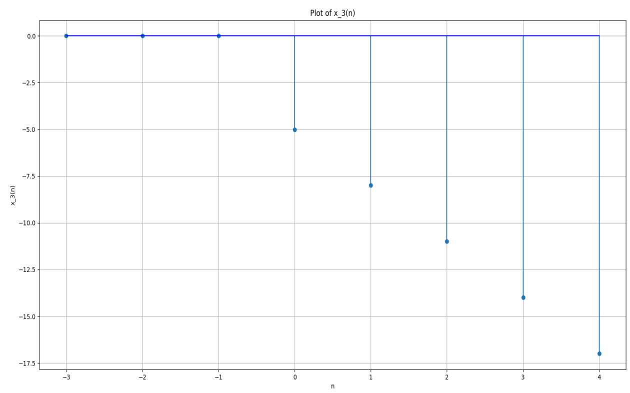
(iii) -5 + -8 + -11 ... -230

Here a=-5, d=-3
From (2)

$$-230 = -5 - 3n \tag{19}$$
$$n = 75 \tag{20}$$

For $x_{3(n)}$

$$x_{3(n)} = x_3(0) + nd \tag{21}$$
$$x_{3(n)} = x_3(0) - 3n \tag{22}$$



Graph:3 $x_{3(n)}$ vs n

Z-Transform of $x_{3(n)}$
Putting $x_{3(n)}$ in (7)

$$\sum_{n=-\infty}^{\infty} (x_{3(0)} - 3n)u_{(n)}Z^{-n} = X_{3(z)} \tag{23}$$

Region of convergence does not change as the nature of the function remains the same as (i)bit
For $X_{3(z)}$, we use the same process as in (i)bit

$$\begin{aligned} &-5(1 - z^k)(z^k(1 - z))^{-1} - \\ &(3(z^k - 1)z)(z^k(z - 1)^2)^{-1} + \\ &(3kz)(z^{k+1}(z - 1))^{-1} = X_{3(z)} \end{aligned} \tag{24}$$

Calculating S_n of $x_{3(n)}$
Using (1) :

$$S_{76} = \frac{76}{2}(-10 + (76 - 1)(-3)) \tag{25}$$
$$S_{76} = -8930 \tag{26}$$

Parameters	Description	Values
$x_{1(n)}$	Sequence labelled (i)	$x_{1(0)}$ +3.5n
$x_{2(n)}$	Sequence labelled (ii)	$x_{2(0)}$ -2n
$x_{3(n)}$	Sequence labelled (iii)	$x_{3(0)}$ -3n
$x_{1(0)}$	1^{st} term of sequence $x_{1(n)}$	7
$x_{2(0)}$	1^{st} term of sequence $x_{2(n)}$	34
$x_{3(0)}$	1^{st} term of sequence $x_{3(n)}$	-5
$X_{1(z)}$	Z-Transform of $x_{1(n)}$	$7(1-z^k)(z^k(1-z))^{-1} + (7(z^k-1)z)(2z^k(z-1)^2)^{-1} - (7kz)(2z^{k+1}(z-1))^{-1}$
$X_{2(z)}$	Z-Transform of $x_{2(n)}$	$32(1-z^k)(z^k(1-z))^{-1} - (2(z^k-1)z)(z^k(z-1)^2)^{-1} + (2kz)(z^{k+1}(z-1))^{-1}$
$X_{3(z)}$	Z-Transform of $x_{3(n)}$	$-5(1-z^k)(z^k(1-z))^{-1} - (3(z^k-1)z)(z^k(z-1)^2)^{-1} + (3kz)(z^{k+1}(z-1))^{-1}$

TABLE 1 : PARAMETERS , DESCRIPTIONS AND VALUES