

# NCERT-discrete : 10.5.3 - 2

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## I. QUESTION

Find the sums given below:

- (i)  $7 + 10.5 + 14 \dots + 84$
- (ii)  $34 + 32 + 30 \dots + 10$
- (iii)  $-5 + -8 + -11 \dots -230$

Symbols	Description	Values
$d_i$	Common Difference for $i^{th}$ AP	3.5
		-2
		-3
$x_i(n)$	$n^{th}$ term for $i^{th}$ Sequence	$(x_i(0) + nd_i)u_{(n)}$
$s_i(n)$	Sum of (n+1)terms for $i^{th}$ Sequence	$\frac{(n+1)u_{(n)}}{2}(2x_i(0) + kd_i)$
$x_i(0)$	First term for $i^{th}$ AP	7
		34
		-5

Table 1 : Parameters , Descriptions And Values

## Solutions :

- (i)  $7 + 10\frac{1}{2} + 14\dots + 84$

$$x_1(n) = (x_1(0) + nd_1)u_{(n)} \quad (1)$$

$$84 = 7 + \frac{7n}{2} \quad (2)$$

$$n = 22 \quad (3)$$

1. Calculating  $s_1(22)$  :

$$s_1(22) = \frac{23}{2}(14 + (22)\frac{7}{2}) \quad (4)$$

$$= 1046.5 \quad (5)$$

2. z-Transform of  $x_1(n)$  : Using (??)

$$X_1(z) = \sum_{n=-\infty}^{\infty} (7 + \frac{7n}{2})u_{(n)}z^{-n} \quad (6)$$

$$= 7z(z-1)^{-1} + 7z(2(z-1))^{-2}, \quad |z| > |1| \quad (7)$$

3. Z-Transform of  $s_1(n)$  :

$$h(n) = u(n) \quad (8)$$

$$H_1(z) = z(z-1)^{-1} \quad (9)$$

$$y_1(n) = x_1(n) * h(n) \quad (10)$$

$$Y_1(z) = X_1(z) * H_1(z) \quad (11)$$

$$= (7z(z-1)^{-1} + 7z(2(z-1))^{-2})z(z-1)^{-1}, \quad |z| > |1| \quad (12)$$

4. Inversion of  $Y_1(z)$  : Using (??) , (??) for inverse Z-transforms :

$$y_1(n) = (7(n+1) + 1.75n(n+1))u(n) \quad (13)$$

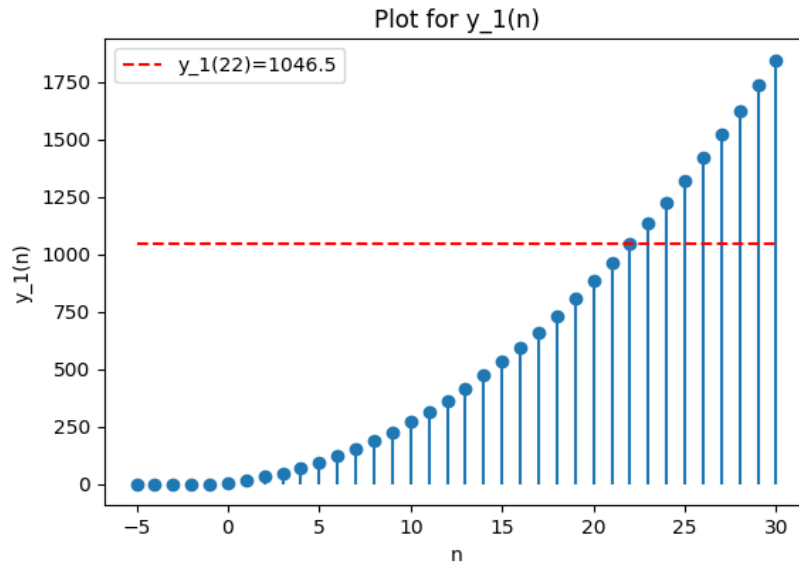


Fig. 1.  $y_1(n)$  vs n

(ii)  $34 + 32 + 30 \dots + 10$

$$x_2(n) = (x_2(0) + nd_2)u_{(n)} \quad (14)$$

$$10 = 34 - 2n \quad (15)$$

$$n = 12 \quad (16)$$

1. Calculating  $s_2(12)$  :

$$s_2(12) = \frac{13}{2}(64 + 11(-2)) \quad (17)$$

$$= 286. \quad (18)$$

2. Z-Transform of  $x_2(n)$  : Using (??)

$$X_2(z) = \sum_{n=-\infty}^{\infty} (x_2(0) - 2n)u_{(n)}z^{-n} \quad (19)$$

$$= 34z(z-1)^{-1} - 2z((z-1))^{-2}, \quad |z| > |1| \quad (20)$$

3. Z-Transform of  $s_2(n)$  :

$$h[n] = u[n] \quad (21)$$

$$y_2(n) = x_2(n) * h(n) \quad (22)$$

$$Y_2(z) = X_2(z) * H(z) \quad (23)$$

$$= 34z(z-1)^{-1} - 2z((z-1))^{-2}z(z-1)^{-1}, \quad |z| > |1| \quad (24)$$

4. Inversion of  $Y_2(z)$  : Using (??) , (??) for inverse Z-transforms :

$$y_2(n) = (34(n+1) - n(n+1))u(n) \quad (25)$$

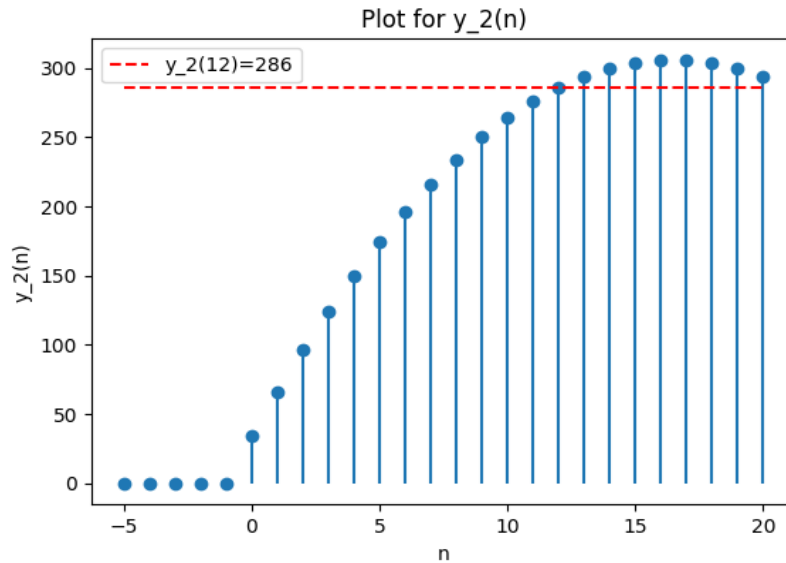


Fig. 2.  $y_2(n)$  vs n

(iii)  $-5 + -8 + -11 \dots -230$

$$x_3(n) = (x_3(0) - 3n)u_{(n)} \quad (26)$$

$$-230 = -5 - 3n \quad (27)$$

$$n = 75 \quad (28)$$

1. Calculating  $s_3(75)$  :

$$s_3(75) = \frac{76}{2}(-10 + (76-1)(-3)) \quad (29)$$

$$= -8930 \quad (30)$$

2. Z-Transform of  $x_3(n)$  : Using (??)

$$X_3(z) = \sum_{n=-\infty}^{\infty} (x_3(0) - 3n)u_{(n)}z^{-n} \quad (31)$$

$$= -5z(z-1)^{-1} - 3z((z-1))^{-2}, \quad |z| > |1| \quad (32)$$

3. Z-Transform of  $s_3(n)$  :

$$h(n) = u(n) \quad (33)$$

$$y_3(n) = x_3(n) * h(n) \quad (34)$$

$$Y_3(z) = X_3(z) * H(z) \quad (35)$$

$$= (-5z(z-1)^{-1} - 3z((z-1))^{-2})z(z-1)^{-1} \quad |z| > |1| \quad (36)$$

4. Inversion of  $Y_3(z)$  : Using (??) , (??) for inverse Z-transforms :

$$y_3(n) = (-5(n+1) - 1.5n(n+1))u(n) \quad (37)$$

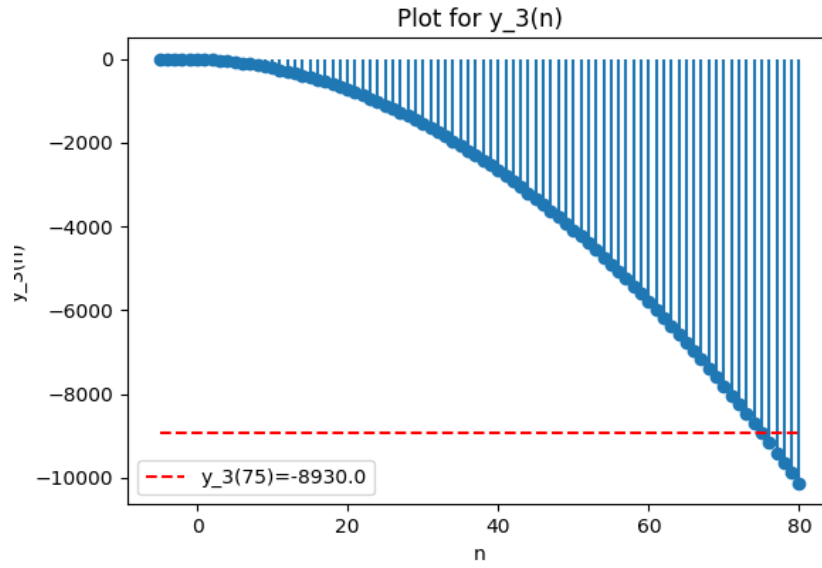


Fig. 3.  $y_3(n)$  vs n