# NCERT-discrete : 10.5.3 - 2

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## I. QUESTION

Find the sums given below:

(i) 7 + 10.5 + 14 ... + 84

(ii) 34 + 32 + 30 ... + 10

(iii) -5 + -8 + -11 ... -230

| Symbols | Description | Values |
|---|---|---|
| $d_i$ | Common Difference for $i^{th}$ AP | 3.5 |
| | | -2 |
| | | -3 |
| $x_i(n)$ | $n^{th}$ term for $i^{th}$ Sequence | $(x_i(0) + nd_i)u_{(n)}$ |
| $s_i(n)$ | Sum of (n+1)terms for $i^{th}$ Sequence | $\frac{(n+1)u_{(u)}}{2}(2x_i(0) + kd_i)$ |
| $x_i(0)$ | First term for $i^{th}$ AP | 7 |
| | | 34 |
| | | -5 |

Table 1 : Parameters , Descriptions And Values

**Solutions :**

(i) $7 + 10\frac{1}{2} + 14... + 84$

$$x_1(n) = (x_1(0) + nd_1)u_{(n)} \tag{1}$$

$$84 = 7 + \frac{7n}{2} \tag{2}$$

$$n = 22 \tag{3}$$

1. Calculating $s_1(22)$ :

$$s_1(22) = \frac{23}{2}(14 + (22)\frac{7}{2}) \tag{4}$$

$$= 1046.5 \tag{5}$$

2. z-Transform of $x_1(n)$ : Using (**??**)

$$X_1(z) = \sum_{n=-\infty}^{\infty}(7 + \frac{7n}{2})u_{(n)}z^{-n} \tag{6}$$

$$= 7z(z-1)^{-1} + 7z(2(z-1))^{-2} \tag{7}$$

$$|z| > |1| \tag{8}$$

3. Z-Transform of $s_1(n)$ :

$$h(n) = u(n) \tag{9}$$
$$H_1(z) = z(z-1)^{-1} \tag{10}$$
$$y_1(n) = x_1(n) * h(n) \tag{11}$$
$$y_1(z) = X_1(z) * H_(z) \tag{12}$$
$$= (7z(z-1)^{-1} + 7z(2(z-1))^{-2})z(z-1)^{-1}, \quad |z| > |1| \tag{13}$$

4. Inversion of $y_1(z)$ :

$$y_1(z) = 7(1-z^{-1})^{-1} + 7z^{-1}(1-z^{-1})^{-2} + (1.75)(z^{-2}+z^{-1})(1-z^{-1})^{-3} + (1.75)z^{-1}(1-z^{-1})^{-2} \tag{14}$$

Using (**??**) , (**??**) and (13) for inverse Z-transforms :

$$y_1(n) = (7(n+1) + 1.75n(n+1))u(n) \tag{15}$$

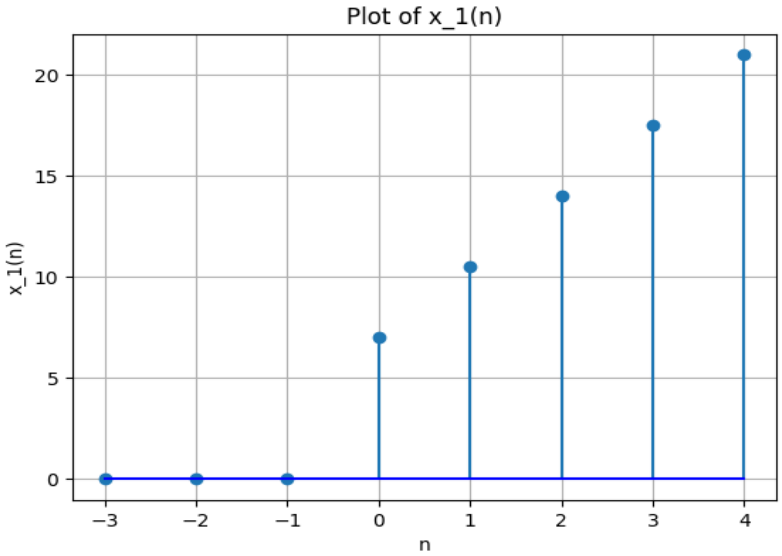


Fig. 1. $x_1(n)$ vs n

(ii) 34 + 32 + 30 ... + 10

$$x_2(n) = (x_2(0) + nd_2)u_{(n)} \tag{16}$$
$$10 = 34 - 2n \tag{17}$$
$$n = 12 \tag{18}$$

1. Calculating $s_2(12)$ :

$$s_2(12) = \frac{13}{2}(64 + 11(-2)) \tag{19}$$
$$= 286. \tag{20}$$

2. Z-Transform of $x_2(n)$ : Using (**??**)

$$X_2(z) = \sum_{n=-\infty}^{\infty} (x_2(0) - 2n)u_{(n)}z^{-n} \tag{21}$$

$$= 34z(z-1)^{-1} - 2z((z-1))^{-2}, \quad |z| > |1| \tag{22}$$

3. Z-Transform of $s_2(n)$ :

$$h[n] = u[n] \tag{23}$$

$$y_2(n) = x_2(n) * h(n) \tag{24}$$

$$y_2(z) = X_2(z) * H(z) \tag{25}$$

$$= 34z(z-1)^{-1} - 2z((z-1))^{-2})z(z-1)^{-1}, \quad |z| > |1| \tag{26}$$

4. Inversion of $y_2(z)$ :

$$y_2(z) = 34(1-z^{-1})^{-1} + 34z^{-1}(1-z^{-1})^{-2} - (z^{-2}+z^{-1})(1-z^{-1})^{-3} - z^{-1}(1-z^{-1})^{-2} \tag{27}$$

Using (**??**) , (**??**) and (13) for inverse Z-transforms :

$$y_2(n) = (34(n+1) - n(n+1))u(n) \tag{28}$$

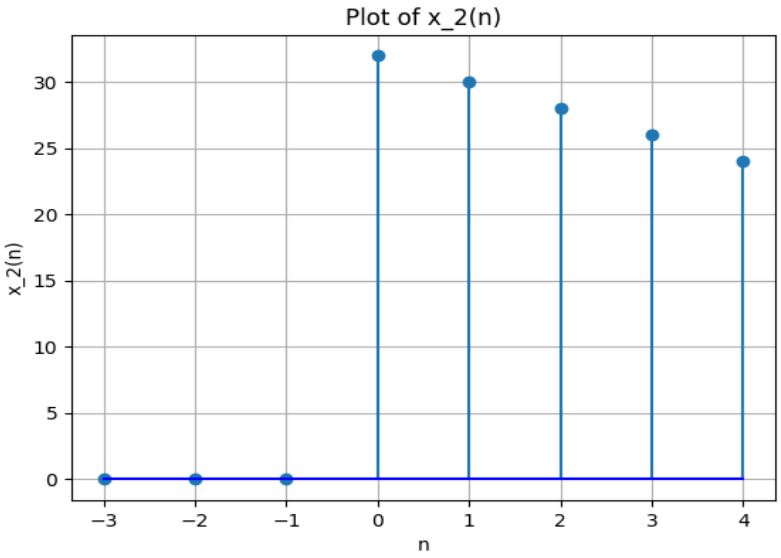


Fig. 2. $x_2(n)$ vs n

(iii) -5 + -8 + -11 ... -230

$$x_3(n) = (x_3(0) - 3n)u_{(n)} \tag{29}$$

$$-230 = -5 - 3n \tag{30}$$

$$n = 75 \tag{31}$$

1. Calculating $s_3(75)$ :

$$s_3(75) = \frac{76}{2}(-10 + (76-1)(-3)) \tag{32}$$
$$= -8930 \tag{33}$$

2. Z-Transform of $x_3(n)$ : Using (**??**)

$$X_3(z) = \sum_{n=-\infty}^{\infty} (x_3(0) - 3n)u_{(n)}z^{-n} \tag{34}$$
$$= -5z(z-1)^{-1} - 3z((z-1))^{-2}, \quad |z| > |1| \tag{35}$$

3. Z-Transform of $s_3(n)$ :

$$h(n) = u(n) \tag{36}$$
$$y_3(n) = x_3(n) * h(n) \tag{37}$$
$$y_3(z) = X_3(z) * H(z) \tag{38}$$
$$= (-5z(z-1)^{-1} - 3z((z-1))^{-2})z(z-1)^{-1} \quad |z| > |1| \tag{39}$$

4. Inversion of $y_3(z)$ :

$$y_3(z) = (-5(1-z^{-1})^{-1} - 5z^{-1}(1-z^{-1})^{-2} - (1.5)(z^{-2}+z^{-1})(1-z^{-1})^{-3} - (1.5)z^{-1}(1-z^{-1})^{-2} \tag{40}$$

Using (**??**) , (**??**) and (13) for inverse Z-transforms :

$$y_3(n) = (-5(n+1) - 1.5n(n+1))u(n) \tag{41}$$

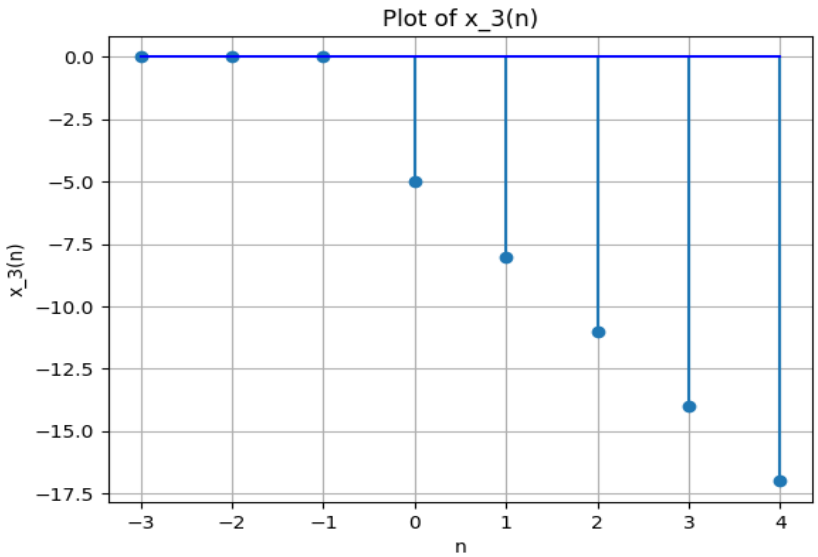


Fig. 3. $x_3(n)$ vs n