NCERT-discrete : 10.5.3 - 2

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I. QUESTION

Find the sums given below:
1) 7 + 10.5 + 14 ... + 84
2) 34 + 32 + 30 ... + 10
3) -5 + -8 + -11 ... -230

Symbols	Description	Values
d_i	Common Difference for i^{th} AP	3.5
		-2
		-3
$x_i(n)$	n^{th} term for i^{th} Sequence	$(x_i(0) + nd_i)u_{(n)}$
$x_i(0)$	First term for i^{th} AP	7
		34
		-5

TABLE I
PARAMETERS , DESCRIPTIONS AND VALUES

Solutions :
1) $7 + 10\frac{1}{2} + 14... + 84$

$$x_1(n) = (x_1(0) + nd_1)\, u_{(n)} \tag{1}$$
$$84 = 7 + \frac{7n}{2} \tag{2}$$
$$n = 22 \tag{3}$$

a) z-Transform of $x_1(n)$: Using (**??**)

$$= \frac{7z}{z-1} + \frac{7z}{2(z-1)^2}, \quad |z| > |1| \tag{4}$$

b) Z-Transform of $y_1(n)$:

$$y_1(n) = x_1(n) * h(n) \tag{5}$$
$$h(n) = u(n) \tag{6}$$
$$H(z) = \frac{z}{z-1} \tag{7}$$
$$Y_1(z) = X_1(z) * H(z) \tag{8}$$
$$= \left(\frac{7z}{z-1} + \frac{7z}{2(z-1)^2}\right)\left(\frac{z}{z-1}\right), \quad |z| > |1| \tag{9}$$

c) Inversion of $Y_1(z)$: Using Contour Integration :

$$y_1(22) = \frac{1}{2\pi j}\oint_C \left(\frac{7z^{23}}{(z-1)^2} + \frac{7z^{23}}{2(z-1)^3}\right) dz \tag{10}$$

For R_2 , $m = 2$

$$R_1 = \frac{1}{(1)!}\lim_{z\to 1}\frac{d}{dz}\left((z-1)^2\frac{7z^{23}}{(z-1)^2}\right) \tag{11}$$

$$= 7\lim_{z\to 1}\frac{d}{dz}(z^{23}) \tag{12}$$

$$= 161 \tag{13}$$

For R_2 , $m = 3$

$$R_2 = \frac{1}{(2)!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3\frac{\left(7z^{13}\right)}{2(z-1)^3}\right) \tag{14}$$

$$= \left(\frac{7}{4}\right)\lim_{z\to 1}\frac{d^2}{dz^2}(z^{23}) \tag{15}$$

$$= 885.5 \tag{16}$$

$$R_1 + R_2 = 1046.5 \tag{17}$$

$$\implies y_1(22) = 1046.5 \tag{18}$$

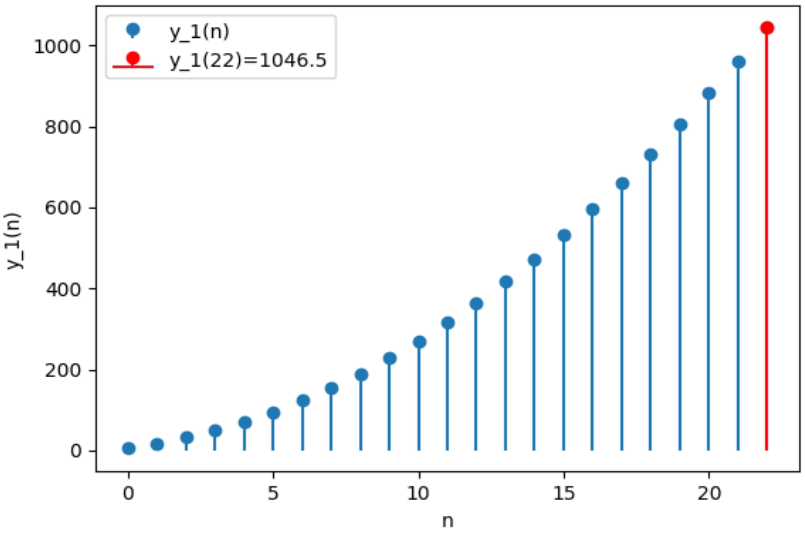

Fig. 1. $y_1(n)$ vs n

2) 34 + 32 + 30 ... + 10

$$x_2(n) = (x_2(0) + nd_2)\,u_{(n)} \tag{19}$$

$$10 = 34 - 2n \tag{20}$$

$$n = 12 \tag{21}$$

a) Z-Transform of $x_2(n)$: Using (**??**)

$$= \frac{34z}{z-1} - \frac{2z}{(z-1)^2}, \quad |z| > |1| \tag{22}$$

b) Z-Transform of $y_2(n)$:

$$y_2(n) = x_2(n) * h(n) \tag{23}$$
$$h(n) = u(n) \tag{24}$$
$$Y_2(z) = X_2(z) * H(z) \tag{25}$$
$$= \left(\frac{34z}{(z-1)^1} - \frac{2z}{(z-1)^2}\right)\left(\frac{z}{z-1}\right), \quad |z| > |1| \tag{26}$$

c) Inversion of $Y_2(z)$: Using Contour Integration :

$$y_2(12) = \frac{1}{2\pi j}\oint_C \left(\frac{34z^{13}}{(z-1)^2} - \frac{2z^{13}}{(z-1)^3}\right) dz \tag{27}$$

For R_1 , $m = 2$:

$$R_1 = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{28}$$
$$= \frac{1}{(1)!}\lim_{z\to 1}\frac{d}{dz}\left((z-1)^2\frac{34z^{13}}{(z-1)^2}\right) \tag{29}$$
$$= 34\lim_{z\to 1}\frac{d}{dz}(z^{13}) \tag{30}$$
$$= 442 \tag{31}$$

For R_2 , $m = 3$:

$$R_2 = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{32}$$
$$= \frac{1}{(2)!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3\frac{\left(-2z^{13}\right)}{(z-1)^3}\right) \tag{33}$$
$$= -\lim_{z\to 1}\frac{d^2}{dz^2}(z^{13}) \tag{34}$$
$$= -156 \tag{35}$$
$$R_1 + R_2 = 286 \tag{36}$$
$$\implies y_2(12) = 286 \tag{37}$$

3) -5 + -8 + -11 ... -230

$$x_3(n) = (x_3(0) - 3n)\,u_{(n)} \tag{38}$$
$$-230 = -5 - 3n \tag{39}$$
$$n = 75 \tag{40}$$

a) Z-Transform of $x_3(n)$: Using (**??**)

$$X_3(z) = \frac{-5z}{(z-1)^1} - \frac{3z}{(z-1)^2}, \quad |z| > |1| \tag{41}$$

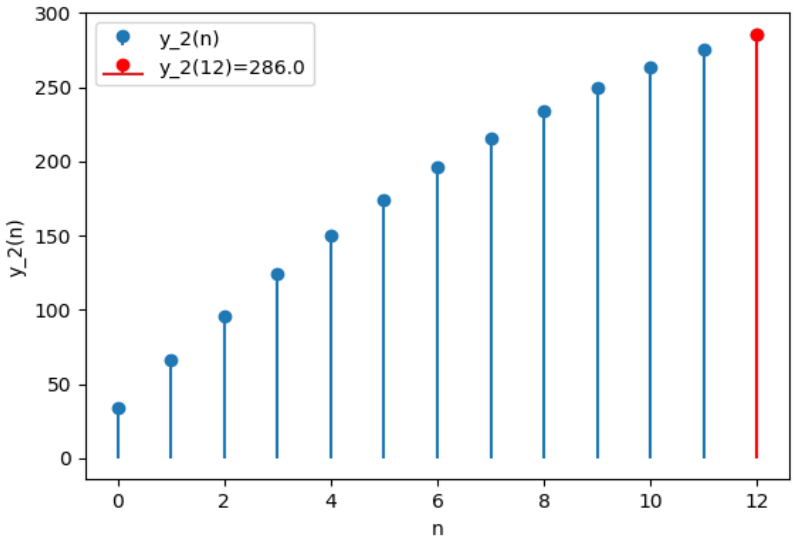

Fig. 2. $y_2(n)$ vs n

b) Z-Transform of $y_3(n)$:

$$y_3(n) = x_3(n) * h(n) \tag{42}$$
$$h(n) = u(n) \tag{43}$$
$$Y_3(z) = X_3(z) * H(z) \tag{44}$$
$$= \left(\frac{-5z}{(z-1)^1} - \frac{3z}{(z-1)^2}\right)\left(\frac{z}{z-1}\right), \quad |z| > |1| \tag{45}$$

c) Inversion of $Y_3(z)$: Using Contour Integration :

$$y_3(75) = \frac{1}{2\pi j}\oint_C \left(\frac{-5z^{76}}{(z-1)^2} - \frac{3z^{76}}{(z-1)^3}\right) dz \tag{46}$$

For R_1 , $m = 2$:

$$R_1 = \frac{1}{(1)!}\lim_{z\to 1}\frac{d}{dz}\left((z-1)^2\frac{-5z^{76}}{(z-1)^2}\right) \tag{47}$$
$$= -5\lim_{z\to 1}\frac{d}{dz}(z^{76}) \tag{48}$$
$$= -380 \tag{49}$$

For R_2 , $m = 3$:

$$R_2 = \frac{1}{(2)!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3\frac{3z^{76}}{(z-1)^3}\right) \tag{50}$$
$$= 1.5\lim_{z\to 1}\frac{d^2}{dz^2}(z^{76}) \tag{51}$$
$$= -8550 \tag{52}$$
$$R_1 + R_2 = -8930 \tag{53}$$
$$\implies y_3(75) = -8930 \tag{54}$$

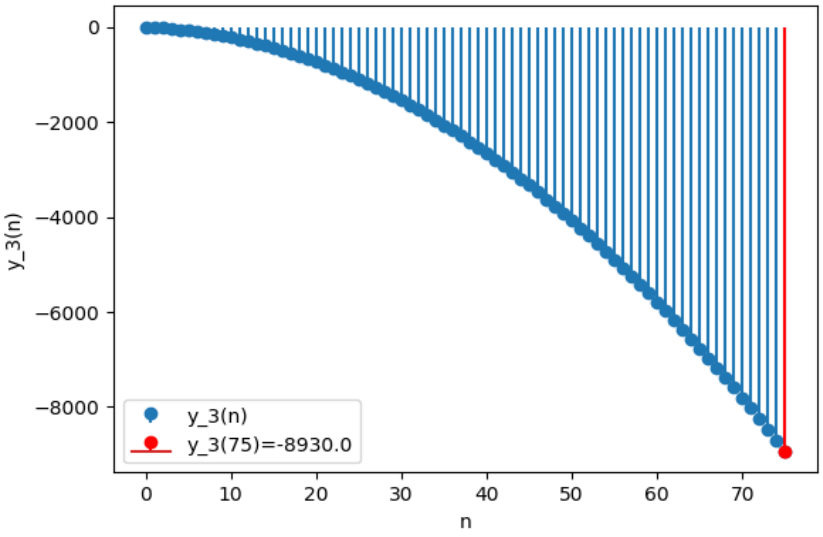

Fig. 3. $y_3(n)$ vs n