NCERT-discrete : 10.5.3 - 2

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I. QUESTION

The laplace transform of $x_1(t) = e^{-t}u(t)$ is $X_1(s)$, where $u(t)$ is the unit step function. The laplace transform of $x_2(t) = e^t u(-t)$ is $X_2(s)$. Which one of the following statements is TRUE?

1) The region of convergence of $X_1(s)$ is $Re(s) \geq 0$
2) The region of convergence of $X_2(s)$ is confined to the left half-plane of s.
3) The region of convergence of $X_1(s)$ is confined to the right half-plane of s.
4) the imaginary axis in the s-plane is included in both the region of convergence of $X_1(s)$ and the region of convergence of $X_2(s)$.

Solutions :

Symbols	Description
$X_1(s)$	Laplace transform of $x_1(t)$
$X_2(s)$	Laplace transform of $x_2(t)$

TABLE I

PARAMETERS, DESCRIPTIONS

Laplace transform of $x_1(t)$ is given by :

$$X_1(s) = \int_{-\infty}^{\infty} e^{-t} e^{-st} u(t)\, dt \qquad (1)$$

$$= \left[\frac{-e^{t(s+1)}}{s+1} \right]_0^{\infty} \qquad (2)$$

$$= \frac{1}{s+1}, s > -1 \qquad (3)$$

Laplace transform of $x_2(t)$ is given by :

$$X_2(s) = \int_{-\infty}^{\infty} e^t e^{-st} u(-t)\, dt \qquad (4)$$

$$= \left[\frac{e^{t(1-s)}}{1-s} \right]_{-\infty}^0 \qquad (5)$$

$$= \frac{1}{1-s}, s < 1 \qquad (6)$$

Based on the regions of convergence of $X_1(s)$ and $X_2(s)$, we can conclude that option 4) is correct .