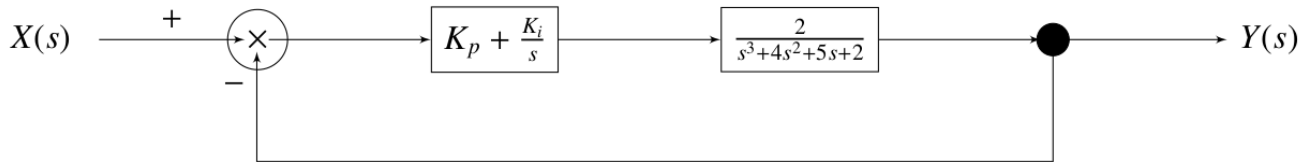


GATE:2021 - EC 48

EE23BTECH11025 - Anantha Krishnan

I. QUESTION

A unity feedback system that uses proportional-integral (PI) control is shown in the figure. The stability



of the overall system is controlled by tuning the PI control parameters K_p and K_i . The maximum value of K_i that can be chosen so as to keep the overall system stable or, in the worst case, marginally stable (rounded off to three decimal places) is? (GATE EC 2021)

Solutions :

Symbols	Description	Values
$P(s)$	Plant transfer function	$\frac{2}{s^3 + 4s^2 + 5s + 2}$
$C(s)$	PI controller transfer function	$K_p + \frac{K_i}{s}$
$G(s)$	Closed loop transfer function	$\frac{P(s)C(s)}{1 + P(s)C(s)}$

TABLE I

PARAMETERS, DESCRIPTIONS, AND VALUES

From table I, the characteristic equation is given as:

$$1 + C(s)P(s) = 0 \quad (1)$$

$$1 + \left(K_p + \frac{K_i}{s}\right) \left(\frac{2}{s^3 + 4s^2 + 5s + 2}\right) = 0 \quad (2)$$

$$s^4 + 4s^3 + 5s^2 + (2 + 2K_p)s + 2K_i = 0 \quad (3)$$

For the system to be stable, there must be no sign changes in the first column of the routh array for the above equation. From (3)

$$\begin{array}{c|ccc} s^4 & 1 & 5 & 2K_i \\ s^3 & 4 & (2 + 2K_p) & 0 \\ s^2 & \frac{18 - 2K_p}{4} & 2K_i & 0 \\ s^1 & \frac{\left(\frac{18 - 2K_p}{4}\right)(2 + 2K_p) - 8K_i}{\frac{18 - 2K_p}{4}} & 0 & 0 \\ s^0 & 2K_i & 0 & 0 \end{array} \quad (4)$$

(5)

$$\frac{18 - 2K_p}{4} > 0 \quad (6)$$

$$\implies K_p < 9 \quad (7)$$

$$\frac{\left(\frac{18-2K_p}{4}\right)(2+2K_p) - 8K_i}{\frac{18-2K_p}{4}} > 0 \quad (8)$$

$$K_i > 0 \quad (9)$$

For marginal stability, assuming 3 cases while maximising K_i and checking if the above inequalities hold.

1) $K_p = 9$

$$\left(\lim_{K_p \rightarrow 9^-} \frac{\left(\frac{18-2K_p}{4}\right)(2+2K_p) - 8K_i}{\frac{18-2K_p}{4}} > 0 \right) \cap (K_i > 0) \quad (10)$$

$$\left(\lim_{K_p \rightarrow 9^-} -8K_i > 0 \right) \cap (K_i > 0) \quad (11)$$

$$\implies K_p = 9, \forall K_i \in (\phi) \quad (12)$$

2) $K_i = 0$

$$\left(\left(\frac{18 - 2K_p}{4} \right) (2 + 2K_p) > 0 \right) \cap (K_p < 9) \quad (13)$$

$$\implies K_i = 0, \forall K_p \in (-1, 9) \quad (14)$$

$$3) \frac{\left(\frac{18-2K_p}{4}\right)(2+2K_p) - 8K_i}{\frac{18-2K_p}{4}} = 0$$

$$\left(\frac{18 - 2K_p}{4} \right) (2 + 2K_p) = 8K_i \quad (15)$$

$$-K_p^2 + 8K_p + 9 = 8K_i \quad (16)$$

Vertex ($K_p = 4$) satisfies (7):

$$K_i = 3.125 \forall (K_p = 4, K_i > 0) \quad (17)$$

Based on the three cases for marginal stability, the maximum value of K_i is 3.125, for $K_p = 4$.

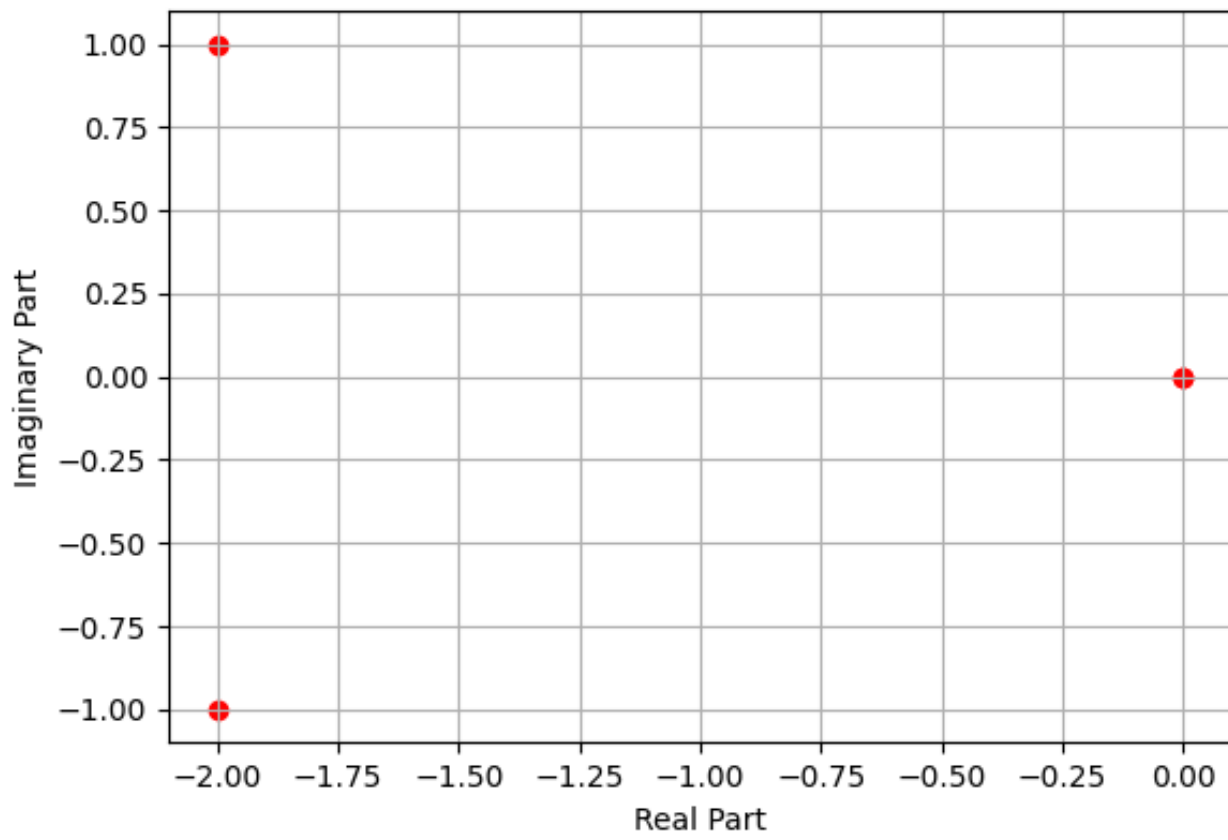


Fig. 1. Location of roots for $k_i = 0, k_p = -1$

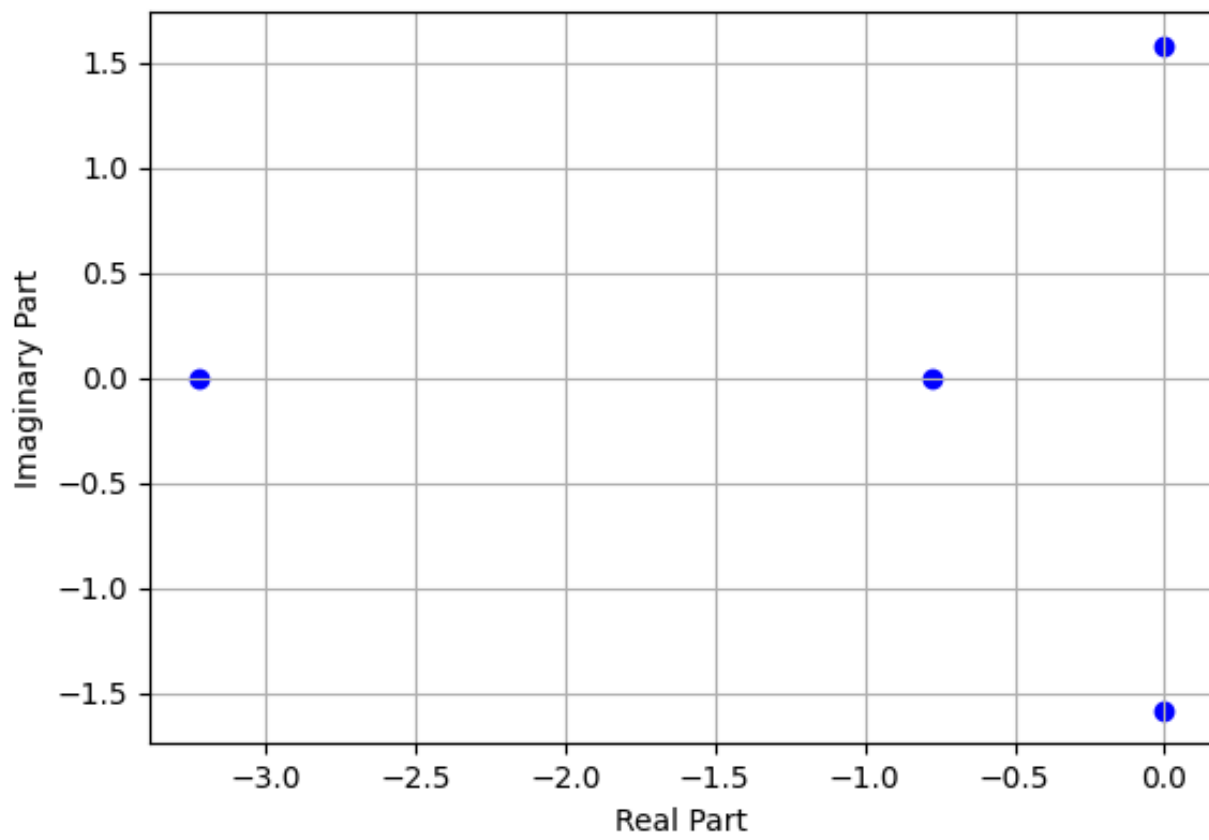


Fig. 2. Location of roots for $k_i = 0, k_p = 9$

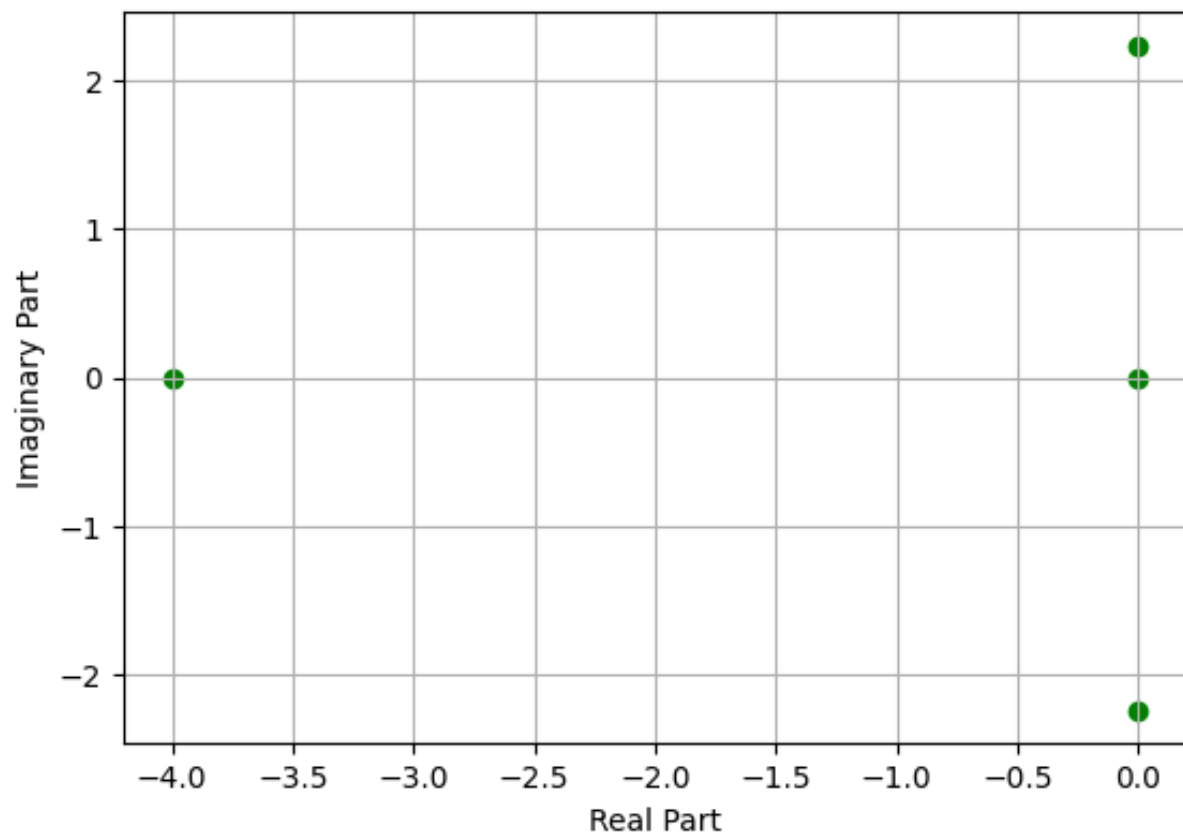


Fig. 3. Location of roots for $k_i = 3.125, k_p = 4$