NCERT-discrete : 10.5.3 - 2

EE23BTECH11025 - Anantha Krishnan

I. Question

The laplace transform of $x_1(t) = e^{-t}u(t)$ is $X_1(s)$, where $u(t)$ is the unit step function. The laplace transform of $x_2(t) = e^{t}u(-t)$ is $X_2(s)$. Which one of the following statements is TRUE?

1) The region of convergence of $X_1(s)$ is $Re(s) \geq 0$
2) The region of convergence of $X_2(s)$ is confined to the left half-plane of s.
3) The region of convergence of $X_1(s)$ is confined to the right half-plane of s.
4) the imaginary axis in the s-plane is included in both the region of convergence of $X_1(s)$ and the region of convergence of $X_2(s)$.

Solutions :

Symbols	Description
$X_1(s)$	Laplace transform of $x_1(t)$
$X_2(s)$	Laplace transform of $x_2(t)$
$u(t)$	Unit step function

TABLE I
Parameters, Descriptions

Laplace transform of $x_1(t)$ is given by :

$$X_1(s) = \int_{-\infty}^{\infty} e^{-t}e^{-st}u(t)\,dt \tag{1}$$

Let $s = \sigma + j\omega$:

$$X_1(s) = \int_{0}^{\infty} e^{-t(\sigma+1)}e^{-tj\beta}\,dt \tag{2}$$

$$= \left[\frac{-e^{-t(\sigma+1)}e^{-tj\beta}}{(\sigma+1)+j\beta}\right]_0^{\infty} \tag{3}$$

$$\tag{4}$$

For $X_1(s)$ to be convergent,$-e^{-t(\sigma+1)}$ and $e^{-tj\beta}$ must converge $\forall t\epsilon\,(0,\infty)$, so:

$$\sigma + 1 > 0 \implies Re\,(s) > -1 \tag{5}$$

$$\left|e^{-tj\beta}\right| < |1|, \forall\beta\epsilon\mathbb{R} \implies Im\,(s)\,\epsilon\mathbb{R} \tag{6}$$

Putting the limits :

$$X_1(s) = \frac{1}{s+1}, \tag{7}$$

$$\text{ROC of } X_1(s)\ :Re\,(s) > -1 \tag{8}$$

Laplace transform of $x_2(t)$ is given by :

$$X_2(s) = \int_{-\infty}^{\infty} e^t e^{-st} u(-t)\, dt \tag{9}$$

$$= \int_{-\infty}^{0} e^{t(1-\sigma)} e^{-tj\beta}\, dt \tag{10}$$

$$= \left[\frac{e^{t(1-\sigma)} e^{-tj\beta}}{(1-\sigma) - j\beta} \right]_{-\infty}^{0} \tag{11}$$

For $X_2(s)$ to be convergent,$e^{-t(1-\sigma)}$ and $e^{-tj\beta}$ must converge $\forall t \epsilon\, (-\infty, 0)$, so:

$$1 - \sigma > 0 \implies Re\,(s) < 1 \tag{12}$$

$$\left| e^{-tj\beta} \right| < |1|\,, \forall \beta \epsilon \mathbb{R} \implies Im\,(s)\, \epsilon \mathbb{R} \tag{13}$$

Putting the limits

$$X_2(s) = \frac{1}{1-s}, \tag{14}$$

$$\text{ROC of } X_2(s) : Re\,(s) < 1 \tag{15}$$

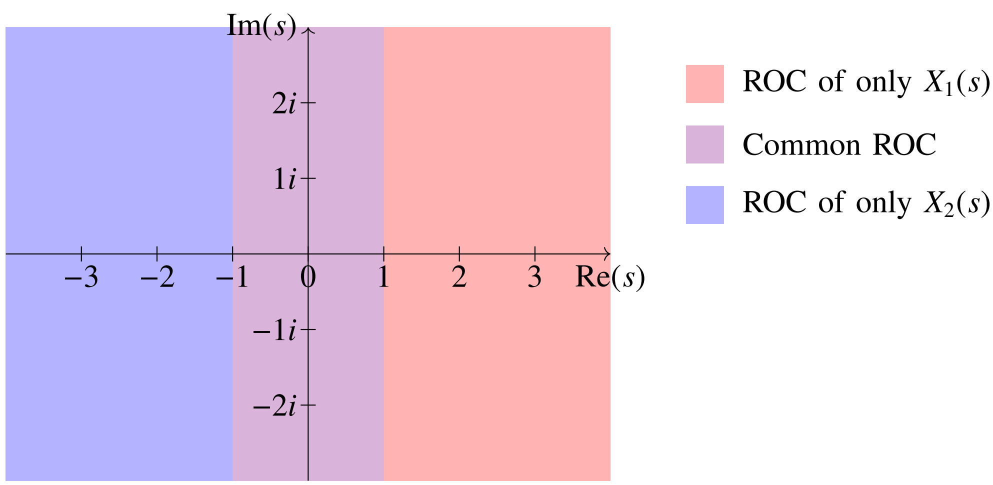

Fig. 1. Representation of ROCs of $X_1(s)$ and $X_2(s)$

Based on the overlap of regions of convergence of $X_1(s)$ and $X_2(s)$ from **??** , we can conclude that option 4) is correct .