

NCERT-discrete : 10.5.3 - 2

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I. QUESTION

Find the sums given below:

- 1) $7 + 10.5 + 14 \dots + 84$
- 2) $34 + 32 + 30 \dots + 10$
- 3) $-5 + -8 + -11 \dots -230$

Symbols	Description	Values
d_i	Common Difference for i^{th} AP	3.5
		-2
		-3
$x_i(n)$	n^{th} term for i^{th} Sequence	$(x_i(0) + nd_i)u_{(n)}$
$x_i(0)$	First term for i^{th} AP	7
		34
		-5

TABLE I
PARAMETERS , DESCRIPTIONS AND VALUES

Solutions :

- 1) $7 + 10\frac{1}{2} + 14\dots + 84$

$$x_1(n) = (x_1(0) + nd_1)u_{(n)} \quad (1)$$

$$84 = 7 + \frac{7n}{2} \quad (2)$$

$$n = 22 \quad (3)$$

a) z-Transform of $x_1(n)$: Using (??)

$$= \frac{7z}{z-1} + \frac{7z}{2(z-1)^2}, \quad |z| > |1| \quad (4)$$

b) Z-Transform of $y_1(n)$:

$$y_1(n) = x_1(n) * h(n) \quad (5)$$

$$h(n) = u(n) \quad (6)$$

$$H(z) = \frac{z}{z-1} \quad (7)$$

$$Y_1(z) = X_1(z) * H(z) \quad (8)$$

$$= \left(\frac{7z}{z-1} + \frac{7z}{2(z-1)^2} \right) \left(\frac{z}{z-1} \right), \quad |z| > |1| \quad (9)$$

c) Inversion of $Y_1(z)$: Using Contour Integration :

$$y_1(22) = \frac{1}{2\pi j} \oint_C Y(z) z^{21} dz \quad (10)$$

$$= \frac{1}{2\pi j} \oint_C \left(\frac{7z^{23}}{(z-1)^2} + \frac{7z^{23}}{2(z-1)^3} \right) dz \quad (11)$$

For R_2 , $m = 2$

$$R_1 = \frac{1}{(1)!} \lim_{z \rightarrow 1} \frac{d}{dz} \left((z-1)^2 \frac{7z^{23}}{(z-1)^2} \right) \quad (12)$$

$$= 7 \lim_{z \rightarrow 1} \frac{d}{dz} (z^{23}) \quad (13)$$

$$= 161 \quad (14)$$

For R_2 , $m = 3$

$$R_2 = \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{(7z^{13})}{2(z-1)^3} \right) \quad (15)$$

$$= \left(\frac{7}{4} \right) \lim_{z \rightarrow 1} \frac{d^2}{dz^2} (z^{23}) \quad (16)$$

$$= 885.5 \quad (17)$$

$$R_1 + R_2 = 1046.5 \quad (18)$$

$$\Rightarrow y_1(22) = 1046.5 \quad (19)$$

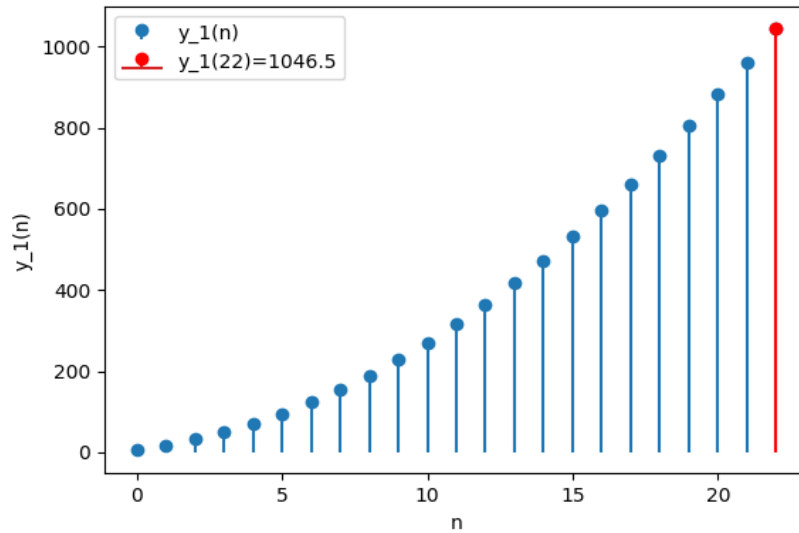


Fig. 1. $y_1(n)$ vs n

2) $34 + 32 + 30 \dots + 10$

$$x_2(n) = (x_2(0) + nd_2) u(n) \quad (20)$$

$$10 = 34 - 2n \quad (21)$$

$$n = 12 \quad (22)$$

a) Z-Transform of $x_2(n)$: Using (??)

$$= \frac{34z}{z-1} - \frac{2z}{(z-1)^2}, \quad |z| > |1| \quad (23)$$

b) Z-Transform of $y_2(n)$:

$$y_2(n) = x_2(n) * h(n) \quad (24)$$

$$h(n) = u(n) \quad (25)$$

$$Y_2(z) = X_2(z) * H(z) \quad (26)$$

$$= \left(\frac{34z}{(z-1)^1} - \frac{2z}{(z-1)^2} \right) \left(\frac{z}{z-1} \right), \quad |z| > |1| \quad (27)$$

c) Inversion of $Y_2(z)$: Using Contour Integration :

$$y_2(12) = \frac{1}{2\pi j} \oint_C Y(z) z^{11} dz \quad (28)$$

$$= \frac{1}{2\pi j} \oint_C \left(\frac{34z^{13}}{(z-1)^2} - \frac{2z^{13}}{(z-1)^3} \right) dz \quad (29)$$

For R_1 , $m = 2$:

$$R_1 = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (30)$$

$$= \frac{1}{(1)!} \lim_{z \rightarrow 1} \frac{d}{dz} \left((z-1)^2 \frac{34z^{13}}{(z-1)^2} \right) \quad (31)$$

$$= 34 \lim_{z \rightarrow 1} \frac{d}{dz} (z^{13}) \quad (32)$$

$$= 442 \quad (33)$$

For R_2 , $m = 3$:

$$R_2 = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (34)$$

$$= \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{(-2z^{13})}{(z-1)^3} \right) \quad (35)$$

$$= - \lim_{z \rightarrow 1} \frac{d^2}{dz^2} (z^{13}) \quad (36)$$

$$= -156 \quad (37)$$

$$R_1 + R_2 = 286 \quad (38)$$

$$\Rightarrow y_2(12) = 286 \quad (39)$$

3) -5 + -8 + -11 ... -230

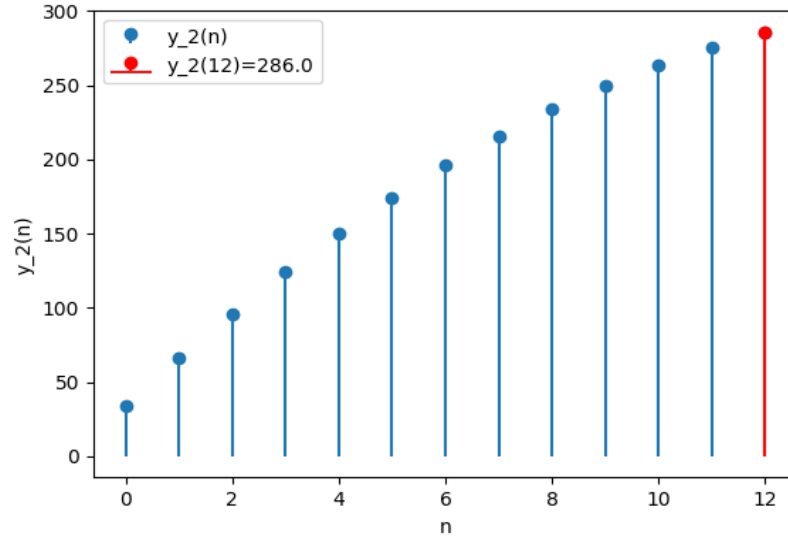


Fig. 2. $y_2(n)$ vs n

$$x_3(n) = (x_3(0) - 3n)u(n) \quad (40)$$

$$-230 = -5 - 3n \quad (41)$$

$$n = 75 \quad (42)$$

a) Z-Transform of $x_3(n)$: Using (??)

$$X_3(z) = \frac{-5z}{(z-1)^1} - \frac{3z}{(z-1)^2}, \quad |z| > |1| \quad (43)$$

b) Z-Transform of $y_3(n)$:

$$y_3(n) = x_3(n) * h(n) \quad (44)$$

$$h(n) = u(n) \quad (45)$$

$$Y_3(z) = X_3(z) * H(z) \quad (46)$$

$$= \left(\frac{-5z}{(z-1)^1} - \frac{3z}{(z-1)^2} \right) \left(\frac{z}{z-1} \right), \quad |z| > |1| \quad (47)$$

c) Inversion of $Y_3(z)$: Using Contour Integration :

$$y_3(75) = \frac{1}{2\pi j} \oint_C Y(z) z^{74} dz \quad (48)$$

$$= \frac{1}{2\pi j} \oint_C \left(\frac{-5z^{76}}{(z-1)^2} - \frac{3z^{76}}{(z-1)^3} \right) dz \quad (49)$$

For R_1 , $m = 2$:

$$R_1 = \frac{1}{(1)!} \lim_{z \rightarrow 1} \frac{d}{dz} \left((z-1)^2 \frac{-5z^{76}}{(z-1)^2} \right) \quad (50)$$

$$= -5 \lim_{z \rightarrow 1} \frac{d}{dz} (z^{76}) \quad (51)$$

$$= -380 \quad (52)$$

For R_2 , $m = 3$:

$$R_2 = \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{3z^{76}}{(z-1)^3} \right) \quad (53)$$

$$= 1.5 \lim_{z \rightarrow 1} \frac{d^2}{dz^2} (z^{76}) \quad (54)$$

$$= -8550 \quad (55)$$

$$R_1 + R_2 = -8930 \quad (56)$$

$$\Rightarrow y_3(75) = -8930 \quad (57)$$

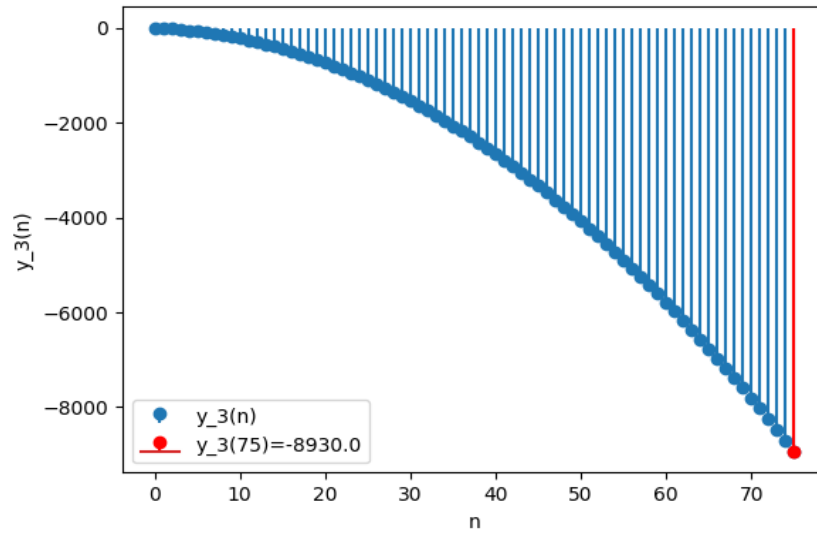


Fig. 3. $y_3(n)$ vs n