NCERT-discrete : 10.5.3 - 2

EE23BTECH11025 - Anantha Krishnan

I. Question

Find the sums given below:

(i) $7 + 10\frac{1}{2} + 14 ... + 84$

(ii) $34 + 32 + 30 ... + 10$

(iii) $-5 + -8 + -11 ... -230$

Solutions:

(i) $7 + 10\frac{1}{2} + 14 ... + 84$

Let $S_i(k)$ denote the sum of first k terms in the i^{th} series , $x_i(0)$ denotes its first term , d_i denotes the common difference and $u_{(k)}$ denote thte unit step function.

$$S_k = \frac{ku_{(k)}}{2}(2x_i(0) + (k-1)d_i) \tag{1}$$

For number of terms , we use

$$x_i(n) = (x_i(0) + nd_i)u_n \tag{2}$$

Where $x_i(n)$ is the $(n+1)^{th}$ term of the series. Putting the values

$$84 = 7 + \frac{7n}{2} \tag{3}$$

$$n = 22 \tag{4}$$

1) **Calculating $S_1(k)$ for $x_1(n)$** : We now use this result for calculating $S_1(23)$

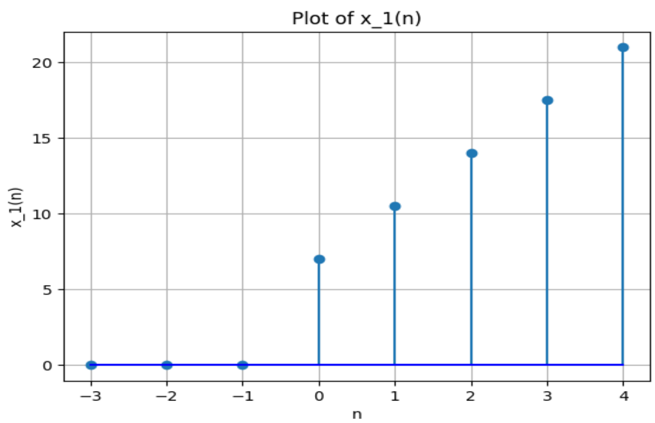
$$S_1(23) = \frac{23}{2}(14 + (22)\frac{7}{2}) \tag{5}$$

Solving this yields $S_1(23) = 1046.5$ We are now required to calculate $X_1(z)$ in terms of $u_(n)$ and $x_1(n)$.Where $u_{(n)}$ is the unit step function.

$$x_1(n) = u_{(n)}(x_1(0) + \frac{7n}{2}) \tag{6}$$

2) **Z-Transform of $x_1(n)$** : By the Definition of Z-transform:

$$\sum_{n=-\infty}^{\infty} Z^{-n} x_i(n) = X_i(Z) \tag{7}$$



Graph:1 $x_1(n)$ vs n

Putting $x_1(n)$ in (7) , we get

$$\sum_{n=-\infty}^{\infty} (x_1(0) + \frac{7n}{2})u_{(n)}Z^{-n} = X_1(z) \tag{8}$$

$$\sum_{n=-\infty}^{\infty} (7 + \frac{7n}{2})u_{(n)}Z^{-n} = X_1(z) \tag{9}$$

$$\begin{aligned} 7z(1-z)^{-1} + \\ 7z((1-z))^{-1} + \\ 7z(2(1-z))^{-2} = X_1(z) \end{aligned} \tag{10}$$

3) **Region of Convergence**

$$|z| < 1 \tag{11}$$

(ii) $34 + 32 + 30 ... + 10$

In this bit $x_2(0) = 34$, d_2 = -2.

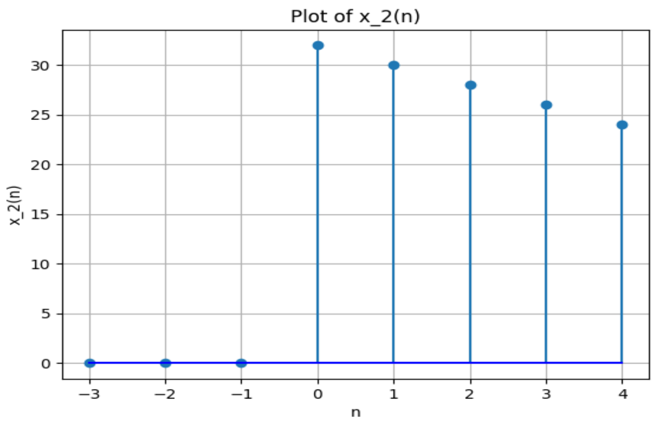
Using equation (2)

$$10 = 34 - 2n \tag{12}$$

$$n = 12 \tag{13}$$

For $x_2(n)$

$$x_2(n) = x_2(0) + nd_2 \tag{14}$$
$$x_2(n) = x_2(0) - 2n \tag{15}$$



Graph:2 $x_2(n)$ vs n

1) **Z-Transform of** $x_2(n)$: Using (7)

$$\sum_{n=-\infty}^{\infty} (x_2(0) - 2n)u_{(n)}Z^{-n} = X_2(z) \tag{16}$$

For $X_2(z)$

$$34z(1-z)^{-1} - 34z((1-z))^{-1} - 2z((1-z))^{-2} = X_2(z) \tag{17}$$

2) **Calculating** $S_2(k)$ **of** $x_2(n)$: For calculating the sum , we use (1)

$$S_2(13) = \frac{13}{2}(64 + 11(-2)) \tag{18}$$
$$S_2(13) = 286. \tag{19}$$

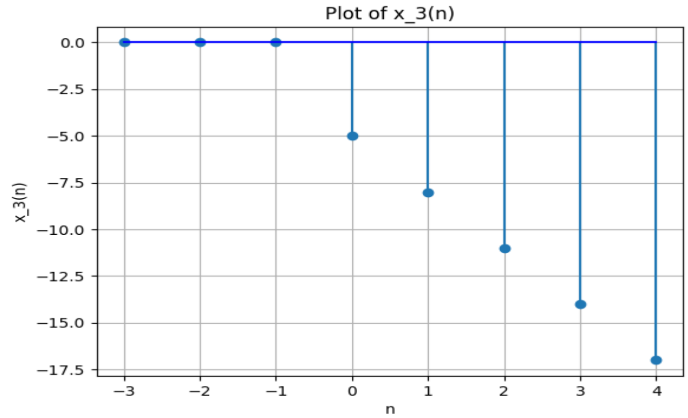
3) **Region of Convergence**

$$|z| < 1 \tag{20}$$

(iii) -5 + -8 + -11 ... -230

Here $x_3(0)$ = -5, d_3 = -3 From (2)

$$-230 = -5 - 3n \tag{21}$$
$$n = 75 \tag{22}$$



Graph:3 $x_3(n)$ vs n

For $x_3(n)$

$$x_3(n) = x_3(0) + nd_3 \tag{23}$$
$$x_{3(n)} = x_3(0) - 3n \tag{24}$$

1) **Z-Transform of** $x_3(n)$: Putting $x_3(n)$ in (7)

$$\sum_{n=-\infty}^{\infty} (x_3(0) - 3n)u_{(n)}Z^{-n} = X_3(z) \tag{25}$$

For $X_3(z)$, we use the same process as in (i) bit

$$-5z(1-z)^{-1} - 5z((1-z))^{-1} - 3z((1-z))^{-2} = X_3(z) \tag{26}$$

2) **Calculating** $S_3(k)$ **of** $x_3(n)$: Using (1) :

$$S_3(76) = \frac{76}{2}(-10 + (76-1)(-3)) \tag{27}$$
$$S_3(76) = -8930 \tag{28}$$

3) **Region of Convergence**

$$|z| < 1 \tag{29}$$

Symbols	Description	Values
d_i	Common Difference	3.5, -2, -3
$x_i(n)$	Sequence	$(x_i(0) + nd_i)u_{(k)}$
$X_i(z)$	Z-Transform of $x_i(n)$	$2x_i(0)(1-z)^{-1} + (d_i)z(1-z)^{-2}$
$S_i(k)$	Sum of k-terms in i^{th} series	$\frac{ku_{(k)}}{2}(2x_i(0) + (k-1)d_i)$

Table 1 : Symbols ,Description and Values