NCERT-discrete : 10.5.3 - 2

EE23BTECH11025 - Anantha Krishnan

I. Question

Find the sums given below:

(i) $7 + 10\frac{1}{2} + 14 ... + 84$

(ii) 34 + 32 + 30 ... + 10

(iii) -5 + -8 + -11 ... -230

Symbols	Description	Values
d_i	Common Difference for i^{th} AP	3.5
		-2
		-3
$x_i(n)$	n^{th} term for i^{th} Sequence	$(x_i(0) + nd_i)u_{(n)}$
$s_i(n)$	Sum of (n+1)terms for i^{th} Sequence	$\frac{(n+1)u_{(u)}}{2}(2x_i(0) + kd_i)$
$x_i(0)$	First term for i^{th} AP	7
		34
		-5

Table 1 : Parameters , Descriptions And Values

II. Solutions

(i) $7 + 10\frac{1}{2} + 14... + 84$

Using I :

$$x_1(n) = (x_1(0) + nd_1)u_{(n)} \tag{1}$$

$$84 = 7 + \frac{7n}{2} \tag{2}$$

$$n = 22 \tag{3}$$

1) Calculating $s_1(22)$:

$$s_1(22) = \frac{23}{2}(14 + (22)\frac{7}{2}) \tag{4}$$

$$s_1(22) = 1046.5 \tag{5}$$

2) Z-Transform of $x_1(n)$: Using (**??**)

$$X_i(z) = \sum_{n=-\infty}^{\infty} z^{-n} x_i(n) \tag{6}$$

Putting $x_1(n)$ in (6) , we get

$$X_i(z) = \sum_{n=-\infty}^{\infty} (x_1(0) + \frac{7n}{2}) u_{(n)} Z^{-n} \tag{7}$$

$$X_1(z) = \sum_{n=-\infty}^{\infty} (7 + \frac{7n}{2}) u_{(n)} Z^{-n} \tag{8}$$

$$X_1(z) = 7z(z-1)^{-1} + 7z(2(z-1))^{-2} \tag{9}$$

$$|z| > |1| \tag{10}$$

3) Z-Transform of $s_1(n)$: Using table I and assuming

$$h(n) = u(n) \tag{11}$$

$$y_1(n) = x_1(n) * h(n) \tag{12}$$

$$y_1(z) = X_1(z) * H_(z) \tag{13}$$

Where $X_1(z)$ comes from (9). For $H(z)$, it is Z-transform of unit-step function

$$H_1(z) = z(z-1)^{-1} \tag{14}$$

For $y_1(z)$:

$$y_1(z) = (7z(z-1)^{-1} + 7z(2(z-1))^{-2})z(z-1)^{-1}$$

ROC:

$$|z| > |1| \tag{15}$$

4) Inversion of $y_1(z)$: By using partial fractions :

$$y_1(z) = 7(1-z^{-1})^{-1} + 7z^{-1}(1-z^{-1})^{-2} + (1.75)(z^{-2}+z^{-1})(1-z^{-1})^{-3} + (1.75)z^{-1}(1-z^{-1})^{-2}$$

Using (**??**) , (**??**) and (14) for inverse Z-transforms :

$$y_1(n) = (7(n+1) + 1.75n(n+1))u(n) \tag{16}$$

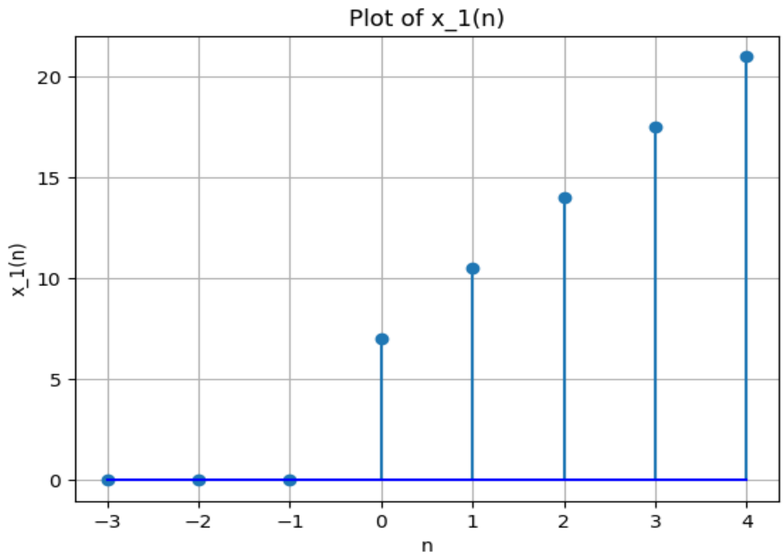

Fig. 1. $x_1(n)$ vs n

(ii) 34 + 32 + 30 ... + 10

Using I :

$$x_2(n) = (x_2(0) + nd_2)u_{(n)} \tag{17}$$
$$10 = 34 - 2n \tag{18}$$
$$n = 12 \tag{19}$$

1) Calculating $s_2(12)$: For calculating the sum , we use the table I

$$s_2(12) = \frac{13}{2}(64 + 11(-2)) \tag{20}$$
$$s_2(12) = 286. \tag{21}$$

2) Z-Transform of $x_2(n)$: Using (**??**)

$$X_2(z) = \sum_{n=-\infty}^{\infty}(x_2(0) - 2n)u_{(n)}Z^{-n} \tag{22}$$
$$X_2(z) = 34z(z-1)^{-1} - 2z((z-1))^{-2} \tag{23}$$
$$|z| > |1| \tag{24}$$

3) Z-Transform of $s_2(n)$: Using table I and assuming

$$h[n] = u[n] \tag{25}$$
$$y_2(n) = x_2(n) * h(n) \tag{26}$$
$$y_2(z) = X_2(z) * H(z) \tag{27}$$

Where $X_2(z)$ comes from (23) and $H(z)$ from (14). For $y_2(z)$:

$$y_2(z) = 34z(z-1)^{-1} - 2z((z-1))^{-2})z(z-1)^{-1} \tag{28}$$

ROC:

$$|z| > |1| \tag{29}$$

4) Inversion of $y_2(z)$: By using partial fractions

$$y_2(z) = 34(1-z^{-1})^{-1} + 34z^{-1}(1-z^{-1})^{-2} - (z^{-2}+z^{-1})(1-z^{-1})^{-3} - z^{-1}(1-z^{-1})^{-2}$$

Using (**??**) , (**??**) and (14) for inverse Z-transforms :

$$y_2(n) = (34(n+1) - n(n+1))u(n) \tag{30}$$

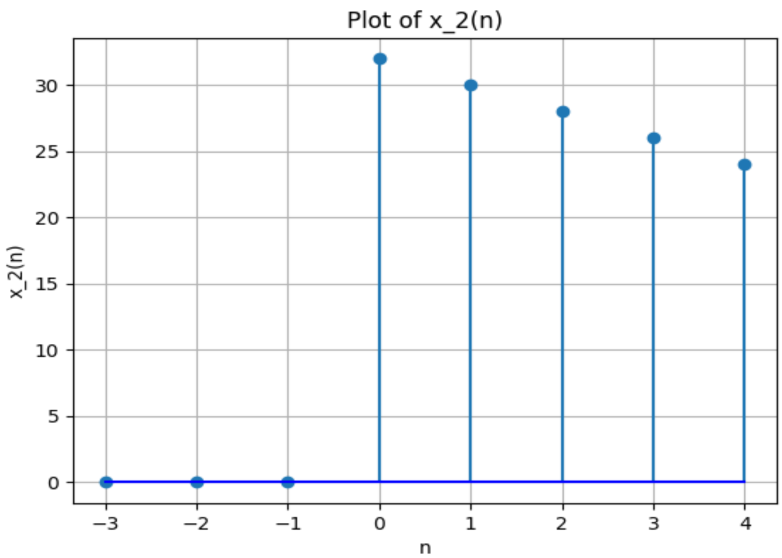

Fig. 2. $x_2(n)$ vs n

(iii) -5 + -8 + -11 ... -230

Using I

$$x_3(n) = (x_3(0) - 3n)u_{(n)} \tag{31}$$
$$-230 = -5 - 3n \tag{32}$$
$$n = 75 \tag{33}$$

Calculating $s_3(75)$: Using I :

$$s_3(75) = \frac{76}{2}(-10 + (76-1)(-3)) \tag{34}$$
$$s_3(75) = -8930 \tag{35}$$

2) Z-Transform of $x_3(n)$: Using (**??**)

$$X_3(z) = \sum_{n=-\infty}^{\infty} (x_3(0) - 3n)u_{(n)}Z^{-n} \tag{36}$$

$$X_3(z) = -5z(z-1)^{-1} - 3z((z-1))^{-2} \tag{37}$$

$$|z| > |1| \tag{38}$$

3) Z-Transform of $s_3(n)$: Using table I and assuming

$$h(n) = u(n) \tag{39}$$

$$y_3(n) = x_3(n) * h(n) \tag{40}$$

$$y_3(z) = X_3(z) * H(z) \tag{41}$$

Where $X_3(z)$ comes from (37) and $H_(z)$ from (14). For $y_3(z)$:

$$y_3(z) = (-5z(z-1)^{-1} - 3z((z-1))^{-2})z(z-1)^{-1} \tag{42}$$

ROC:

$$|z| > |1| \tag{43}$$

4) Inversion of $y_3(z)$:

$$y_3(z) = (-5(1-z^{-1})^{-1} - 5z^{-1}(1-z^{-1})^{-2} - (1.5)(z^{-2}+z^{-1})(1-z^{-1})^{-3} - (1.5)z^{-1}(1-z^{-1})^{-2}$$

Using (**??**) , (**??**) and (14) for inverse Z-transforms :

$$y_3(n) = (-5(n+1) - 1.5n(n+1))u(n) \tag{44}$$

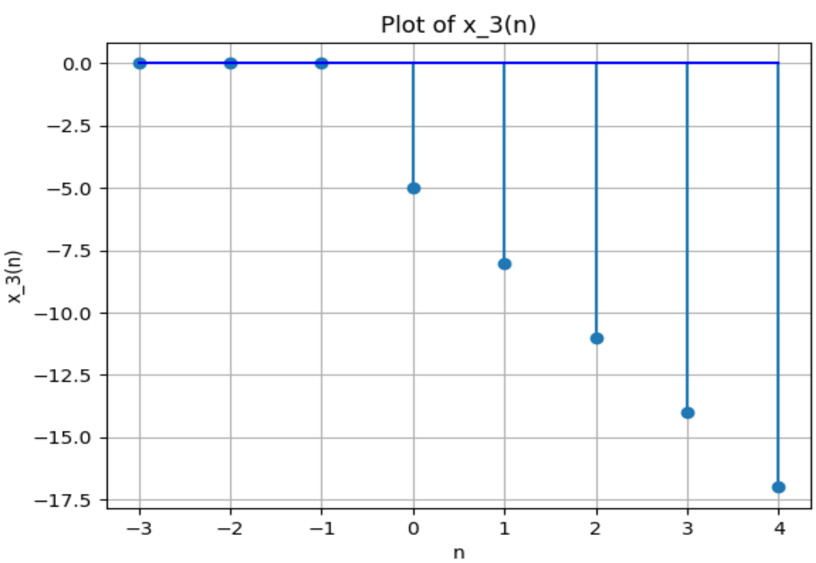

Fig. 3. $x_3(n)$ vs n