NCERT-discrete : 10.5.3 - 2

EE23BTECH11025 - Anantha Krishnan

I. Question .

Find the sums given below:

(i) $7 + 10\frac{1}{2} + 14 ... + 84$
(ii) $34 + 32 + 30 ... + 10$
(iii) $-5 + -8 + -11 ... -230$

Symbols	Description	Values
d_i	Common Difference for i^{th} AP	3.5
		-2
		-3
$x_i(n)$	n^{th} term for i^{th} Sequence	$(x_i(0)\ +nd_i)u_{(n)}$
$X_i(z)$	Z-Transform of $x_i(n)$	$zx_i(0)(z-1)^{-1} + d_i z(z-1)^{-2}$
$S_i(n)$	Sum of (n+1)terms for i^{th} Sequence	$\frac{(n+1)u_{(u)}}{2}(2x_i(0) + kd_i)$
$h(n)$	Unit step function ($u_{(n)}$)	$0\ \forall n < 0, 1 \forall n \geq 0$
$x_i(0)$	First term for i^{th} AP	7
		34
		-5

Table 1 : Parameters , Descriptions And Values

Solutions:

(i) $7 + 10\frac{1}{2} + 14 ... + 84$.

For number of terms :

$$x_i(n) = (x_i(0) + nd_i)u_{(n)} \tag{1}$$
$$84 = 7 + \frac{7n}{2} \tag{2}$$
$$n = 22 \tag{3}$$

1) Calculating $S_1(22)$:

$$S_1(22) = \frac{23}{2}(14 + (22)\frac{7}{2})S_1(22) \quad = 1046.5 \tag{4}$$

2) Z-Transform of $x_1(n)$: By the Definition of Z-transform:

$$\sum_{n=-\infty}^{\infty} z^{-n} x_i(n) = X_i(z) \tag{5}$$

Putting $x_1(n)$ in (5) , we get

$$\sum_{n=-\infty}^{\infty} (x_1(0) + \frac{7n}{2})u_{(n)} Z^{-n} = X_1(z) \tag{6}$$

$$\sum_{n=-\infty}^{\infty} (7 + \frac{7n}{2})u_{(n)} Z^{-n} = X_1(z) \tag{7}$$

$$7z(z-1)^{-1} + 7z(2(z-1))^{-2} = X_1(z) \tag{8}$$
$$\forall |z| > 1 \tag{9}$$

3) Z-Transform of $S_1(n)$: Using (1) and assuming

$$h(n) = u(n) \tag{10}$$
$$S_1(n) = x_1(n) * h(n) \tag{11}$$
$$S_1(z) = X_1(z) * H_(z) \tag{12}$$

Where $X_1(z)$ comes from (8). For $H(z)$, it is Z-transform of unit-step function

$$H_1(z) = z(z-1)^{-1} \tag{13}$$

For $S_1(z)$:

$$S_1(z) = (7z(z-1)^{-1} + 7z(2(z-1))^{-2})z(z-1)^{-1}$$

ROC:

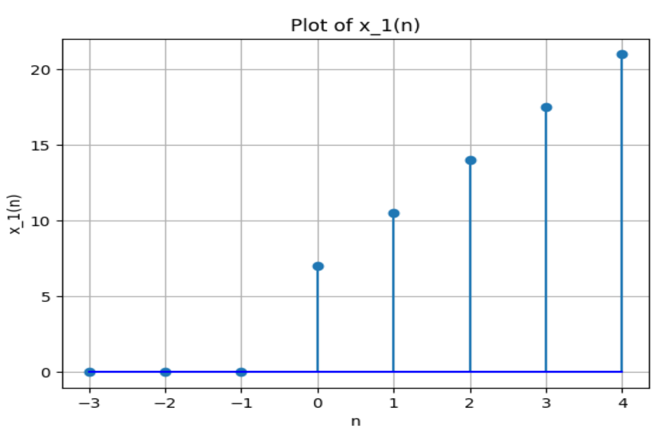
$$|z| > 1 \tag{14}$$

4) Inversion of $S_1(z)$: By using partial fractions :

$$S_1(z) = (7(1-z^{-1})^{-1} + 7z^{-1}(1-z^{-1})^{-2}$$
$$+(1.75)(z^{-2} + z^{-1})(1-z^{-1})^{-3} + (1.75)z^{-1}(1-z^{-1})^{-2} \tag{15}$$

Using (**??**) , (**??**) and (13) for inverse Z-transforms :

$$S_1(n) = (7(n+1) + 1.75n(n+1))u(n) \tag{16}$$



Graph:1 $x_1(n)$ vs n

(ii) 34 + 32 + 30 ... + 10

In this bit $x_2(0)$ = 34 , d_2 = -2.
Using equation (1)

$$10 = 34 - 2n \tag{17}$$
$$n = 12 \tag{18}$$

For $x_2(n)$

$$x_2(n) = x_2(0) + nd_2 \tag{19}$$
$$x_2(n) = x_2(0) - 2n \tag{20}$$

1) Calculating $S_2(12)$: For calculating the sum , we use the table I

$$S_2(12) = \frac{13}{2}(64 + 11(-2)) \tag{21}$$
$$S_2(12) = 286. \tag{22}$$

2) Z-Transform of $x_2(n)$: Using (5)

$$\sum_{n=-\infty}^{\infty} (x_2(0) - 2n)u_{(n)}Z^{-n} = X_2(z) \tag{23}$$

For $X_2(z)$

$$34z(z-1)^{-1} - 2z((z-1))^{-2} = X_2(z) \tag{24}$$
$$|z| > 1 \tag{25}$$

3) Z-Transform of $S_2(n)$: Using (1) and assuming

$$h[n] = u[n] \tag{26}$$
$$S_2(n) = x_2(n) * h(n) \tag{27}$$
$$S_2(z) = X_2(z) * H(z) \tag{28}$$

Where $X_2(z)$ comes from (24) and $H(z)$ from (13). For $S_2(z)$:

$$S_2(z) = 34z(z-1)^{-1} - 2z((z-1))^{-2})z(z-1)^{-1} \tag{29}$$

ROC:

$$|z| > 1 \tag{30}$$

4) Inversion of $S_2(z)$: By using partial fractions

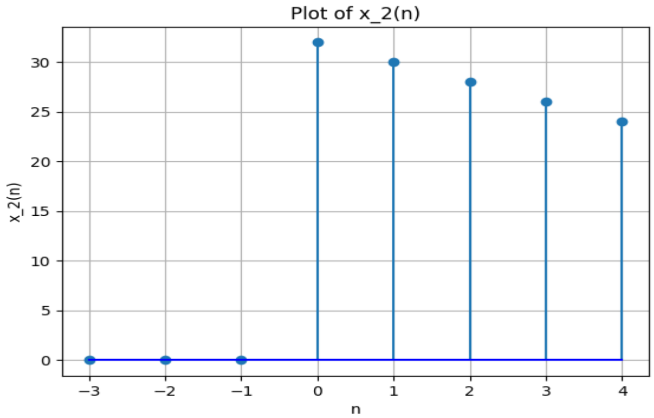
$$S_2(z) = 34(1-z^{-1})^{-1} + 34z^{-1}(1-z^{-1})^{-2}$$
$$-(z^{-2} + z^{-1})(1-z^{-1})^{-3} - z^{-1}(1-z^{-1})^{-2} \tag{31}$$

Using (**??**) , (**??**) and (13) for inverse Z-transforms :

$$S_2(n) = (34(n+1) - n(n+1))u(n) \tag{32}$$

(iii) -5 + -8 + -11 ... -230

Here $x_3(0)$ = -5, d_3 = -3 From (1)



Graph:2 $x_2(n)$ vs n

$$-230 = -5 - 3n \tag{33}$$
$$n = 75 \tag{34}$$

For $x_3(n)$

$$x_3(n) = x_3(0) + nd_3 \tag{35}$$
$$x_{3(n)} = x_3(0) - 3n \tag{36}$$

1) Calculating $S_3(75)$: Using I :

$$S_3(75) = \frac{76}{2}(-10 + (76-1)(-3)) \tag{37}$$
$$S_3(75) = -8930 \tag{38}$$

2) Z-Transform of $x_3(n)$: Putting $x_3(n)$ in (5)

$$\sum_{n=-\infty}^{\infty} (x_3(0) - 3n)u_{(n)}Z^{-n} = X_3(z) \tag{39}$$

For $X_3(z)$, we use the same process as in (i) bit

$$-5z(z-1)^{-1} - 3z((z-1))^{-2} = X_3(z) \tag{40}$$
$$|z| > 1 \tag{41}$$

3) Z-Transform of $S_3(n)$: Using (1) and assuming

$$h(n) = u(n) \tag{42}$$
$$S_3(n) = x_3(n) * h(n) \tag{43}$$
$$S_3(z) = X_3(z) * H(z) \tag{44}$$

Where $X_3(z)$ comes from (40) and $H_(z)$ from (13). For $S_3(z)$:

$$S_3(z) = (-5z(z-1)^{-1} - 3z((z-1))^{-2})z(z-1)^{-1} \tag{45}$$

ROC:

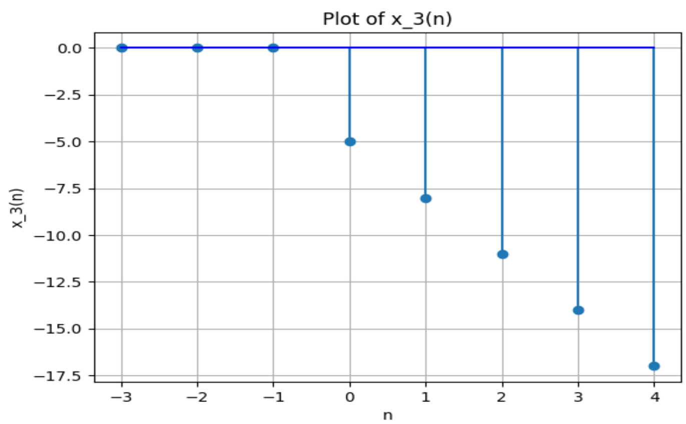
$$|z| > 1 \tag{46}$$

4) Inversion of $S_3(z)$:

$$S_3(z) = (-5(1-z^{-1})^{-1} - 5z^{-1}(1-z^{-1})^{-2} \\ -(1.5)(z^{-2} + z^{-1})(1-z^{-1})^{-3} - (1.5)z^{-1}(1-z^{-1})^{-2} \tag{47}$$

Using (**??**) , (**??**) and (13) for inverse Z-transforms :

$$S_3(n) = (-5(n+1) - 1.5n(n+1))u(n) \tag{48}$$



Graph:3 $x_3(n)$ vs n