NCERT-discrete : 10.5.3 - 2

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I. QUESTION

Find the sums given below:

1) $7 + \frac{21}{2} + 14... + 84$
2) $34 + 32 + 30... + 10$
3) $-5 + -8 + -11... - 230$

Solutions :

Symbols	Description	Values
d_i	Common Difference for i^{th} AP	3.5
		-2
		-3
$x_i(n)$	n^{th} term for i^{th} Sequence	$(7 + \frac{7n}{2})u_{(n)}$
		$(34 - 2n)u_{(n)}$
		$(-5 + -3n)u_{(n)}$
$x_i(0)$	First term for i^{th} AP	7
		34
		-5

TABLE I
PARAMETERS , DESCRIPTIONS AND VALUES

1) $7 + \frac{21}{2} + 14... + 84$

$$x_1(n) = (x_1(0) + nd_1)\, u_{(n)} \tag{1}$$

$$\implies 84 = 7 + \frac{7n}{2} \tag{2}$$

$$\implies n = 22 \tag{3}$$

a) z-Transform of $x_1(n)$: Using (**??**)

$$X_1(z) = \frac{7z}{z-1} + \frac{7z}{2(z-1)^2}, \quad |z| > |1| \tag{4}$$

b) Z-Transform of $y_1(n)$:

$$y_1(n) = x_1(n) * h(n) \tag{5}$$

$$h(n) = u(n) \tag{6}$$

$$H(z) = \frac{z}{z-1} \tag{7}$$

$$Y_1(z) = X_1(z) * H(z) \tag{8}$$

$$= \left(\frac{7z}{z-1} + \frac{7z}{2(z-1)^2}\right)\left(\frac{z}{z-1}\right), \quad |z| > |1| \tag{9}$$

c) Inversion of $Y_1(z)$: Using Contour Integration :

$$y_1(22) = \frac{1}{2\pi j}\oint_C Y(z)\, z^{21}\, dz \tag{10}$$

$$\implies = \frac{1}{2\pi j}\oint_C \left(\frac{7z^{23}}{(z-1)^2} + \frac{7z^{23}}{2(z-1)^3}\right) dz \tag{11}$$

$$R = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{12}$$

For R_1 , $m = 2$, where m corresponds to number of repeated poles .

$$R_1 = \frac{1}{(1)!}\lim_{z\to 1}\frac{d}{dz}\left((z-1)^2\frac{7z^{23}}{(z-1)^2}\right) \tag{13}$$

$$= 7\lim_{z\to 1}\frac{d}{dz}(z^{23}) \tag{14}$$

$$= 161 \tag{15}$$

For R_2 , $m = 3$

$$R_2 = \frac{1}{(2)!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3\frac{\left(7z^{13}\right)}{2(z-1)^3}\right) \tag{16}$$

$$= \left(\frac{7}{4}\right)\lim_{z\to 1}\frac{d^2}{dz^2}(z^{23}) \tag{17}$$

$$= \frac{1771}{2} \tag{18}$$

$$R_1 + R_2 = \frac{2093}{2} \tag{19}$$

$$\implies y_1(22) = \frac{2093}{2} \tag{20}$$

2) 34 + 32 + 30 ... + 10

$$x_2(n) = (x_2(0) + nd_2)\, u_{(n)} \tag{21}$$

$$\implies 10 = 34 - 2n \tag{22}$$

$$\implies n = 12 \tag{23}$$

a) Z-Transform of $x_2(n)$: Using (**??**)

$$X_2(z) = \frac{34z}{z-1} - \frac{2z}{(z-1)^2}, \quad |z| > |1| \tag{24}$$

b) Z-Transform of $y_2(n)$:

$$y_2(n) = x_2(n) * h(n) \tag{25}$$

$$h(n) = u(n) \tag{26}$$

$$Y_2(z) = X_2(z) * H(z) \tag{27}$$

$$= \left(\frac{34z}{(z-1)^1} - \frac{2z}{(z-1)^2}\right)\left(\frac{z}{z-1}\right), \quad |z| > |1| \tag{28}$$

c) Inversion of $Y_2(z)$: Using Contour Integration :

$$y_2(12) = \frac{1}{2\pi j}\oint_C Y(z)\, z^{11}\, dz \tag{29}$$

$$\implies = \frac{1}{2\pi j}\oint_C \left(\frac{34z^{13}}{(z-1)^2} - \frac{2z^{13}}{(z-1)^3}\right) dz \tag{30}$$

Using (12) For R_1 , $m = 2$:

$$R_1 = \frac{1}{(1)!}\lim_{z\to 1}\frac{d}{dz}\left((z-1)^2\frac{34z^{13}}{(z-1)^2}\right) \tag{31}$$

$$= 34\lim_{z\to 1}\frac{d}{dz}(z^{13}) \tag{32}$$

$$= 442 \tag{33}$$

For R_2 , $m = 3$:

$$R_2 = \frac{1}{(2)!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3\frac{\left(-2z^{13}\right)}{(z-1)^3}\right) \tag{34}$$

$$= -\lim_{z\to 1}\frac{d^2}{dz^2}(z^{13}) \tag{35}$$

$$= -156 \tag{36}$$

$$R_1 + R_2 = 286 \tag{37}$$

$$\implies y_2(12) = 286 \tag{38}$$

3) -5 + -8 + -11 ... -230

$$x_3(n) = (x_3(0) - 3n)\, u_{(n)} \tag{39}$$

$$\implies -230 = -5 - 3n \tag{40}$$

$$\implies n = 75 \tag{41}$$

a) Z-Transform of $x_3(n)$: Using (**??**)

$$X_3(z) = \frac{-5z}{(z-1)^1} - \frac{3z}{(z-1)^2}, \quad |z| > |1| \tag{42}$$

b) Z-Transform of $y_3(n)$:

$$y_3(n) = x_3(n) * h(n) \tag{43}$$

$$h(n) = u(n) \tag{44}$$

$$Y_3(z) = X_3(z) * H(z) \tag{45}$$

$$= \left(\frac{-5z}{(z-1)^1} - \frac{3z}{(z-1)^2}\right)\left(\frac{z}{z-1}\right), \quad |z| > |1| \tag{46}$$

c) Inversion of $Y_3(z)$: Using Contour Integration :

$$y_1(75) = \frac{1}{2\pi j}\oint_C Y(z)\, z^{74}\, dz \tag{47}$$

$$\implies = \frac{1}{2\pi j}\oint_C \left(\frac{-5z^{76}}{(z-1)^2} - \frac{3z^{76}}{(z-1)^3}\right) dz \tag{48}$$

Using (12) For R_1 , $m = 2$:

$$R_1 = \frac{1}{(1)!} \lim_{z \to 1} \frac{d}{dz} \left((z-1)^2 \frac{-5z^{76}}{(z-1)^2} \right) \tag{49}$$

$$= -5 \lim_{z \to 1} \frac{d}{dz} (z^{76}) \tag{50}$$

$$= -380 \tag{51}$$

For R_2 , $m = 3$:

$$R_2 = \frac{1}{(2)!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{3z^{76}}{(z-1)^3} \right) \tag{52}$$

$$= 1.5 \lim_{z \to 1} \frac{d^2}{dz^2} (z^{76}) \tag{53}$$

$$= -8550 \tag{54}$$

$$R_1 + R_2 = -8930 \tag{55}$$

$$\implies y_3(75) = -8930 \tag{56}$$

Fig. 1. $y_1(n)$ vs n

Fig. 2. $y_2(n)$ vs n

Fig. 3. $y_3(n)$ vs n