NCERT-discrete : 10.5.3 - 2

EE23BTECH11025 - Anantha Krishnan

I. QUESTION

Find the sums given below:

(i) 7 + 10.5 + 14 ... + 84

(ii) 34 + 32 + 30 ... + 10

(iii) -5 + -8 + -11 ... -230

Symbols	Description	Values
d_i	Common Difference for i^{th} AP	3.5
		-2
		-3
$x_i(n)$	n^{th} term for i^{th} Sequence	$(x_i(0) + nd_i)u_{(n)}$
$s_i(n)$	Sum of (n+1)terms for i^{th} Sequence	$\frac{(n+1)u_{(u)}}{2}(2x_i(0) + kd_i)$
$x_i(0)$	First term for i^{th} AP	7
		34
		-5

Table 1 : Parameters , Descriptions And Values

Solutions :

(i) $7 + 10\frac{1}{2} + 14... + 84$

$$x_1(n) = (x_1(0) + nd_1)u_{(n)} \tag{1}$$

$$84 = 7 + \frac{7n}{2} \tag{2}$$

$$n = 22 \tag{3}$$

1. Calculating $s_1(22)$:

$$s_1(22) = \frac{23}{2}(14 + (22)\frac{7}{2}) \tag{4}$$

$$= 1046.5 \tag{5}$$

2. z-Transform of $x_1(n)$: Using (**??**)

$$X_1(z) = \sum_{n=-\infty}^{\infty}(7 + \frac{7n}{2})u_{(n)}z^{-n} \tag{6}$$

$$= 7z(z-1)^{-1} + 7z(2(z-1))^{-2}, \quad |z| > |1| \tag{7}$$

3. Z-Transform of $s_1(n)$:

$$h(n) = u(n) \tag{8}$$
$$H_1(z) = z(z-1)^{-1} \tag{9}$$
$$y_1(n) = x_1(n) * h(n) \tag{10}$$
$$Y_1(z) = X_1(z) * H_(z) \tag{11}$$
$$= (7z(z-1)^{-1} + 7z(2(z-1))^{-2})z(z-1)^{-1}, \quad |z| > |1| \tag{12}$$

4. Inversion of $Y_1(z)$: Using (**??**) , (**??**) for inverse Z-transforms :

$$y_1(n) = (7(n+1) + 1.75n(n+1))u(n) \tag{13}$$

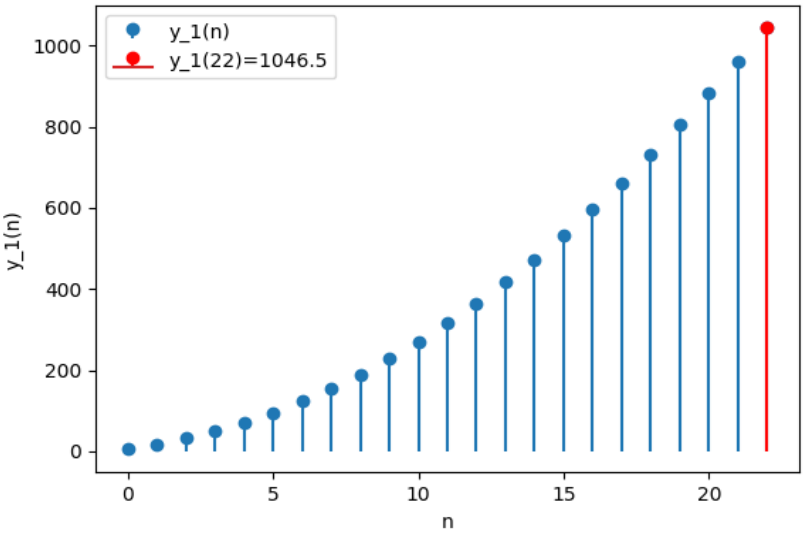

Fig. 1. $y_1(n)$ vs n

(ii) 34 + 32 + 30 ... + 10

$$x_2(n) = (x_2(0) + nd_2)u_{(n)} \tag{14}$$
$$10 = 34 - 2n \tag{15}$$
$$n = 12 \tag{16}$$

1. Calculating $s_2(12)$:

$$s_2(12) = \frac{13}{2}(64 + 11(-2)) \tag{17}$$
$$= 286. \tag{18}$$

2. Z-Transform of $x_2(n)$: Using (**??**)

$$X_2(z) = \sum_{n=-\infty}^{\infty} (x_2(0) - 2n)u_{(n)}z^{-n} \tag{19}$$

$$= 34z(z-1)^{-1} - 2z((z-1))^{-2}, \quad |z| > |1| \tag{20}$$

3. Z-Transform of $s_2(n)$:

$$h[n] = u[n] \tag{21}$$

$$y_2(n) = x_2(n) * h(n) \tag{22}$$

$$Y_2(z) = X_2(z) * H(z) \tag{23}$$

$$= 34z(z-1)^{-1} - 2z((z-1))^{-2})z(z-1)^{-1}, \quad |z| > |1| \tag{24}$$

4. Inversion of $Y_2(z)$: Using (**??**) , (**??**) for inverse Z-transforms :

$$y_2(n) = (34(n+1) - n(n+1))u(n) \tag{25}$$

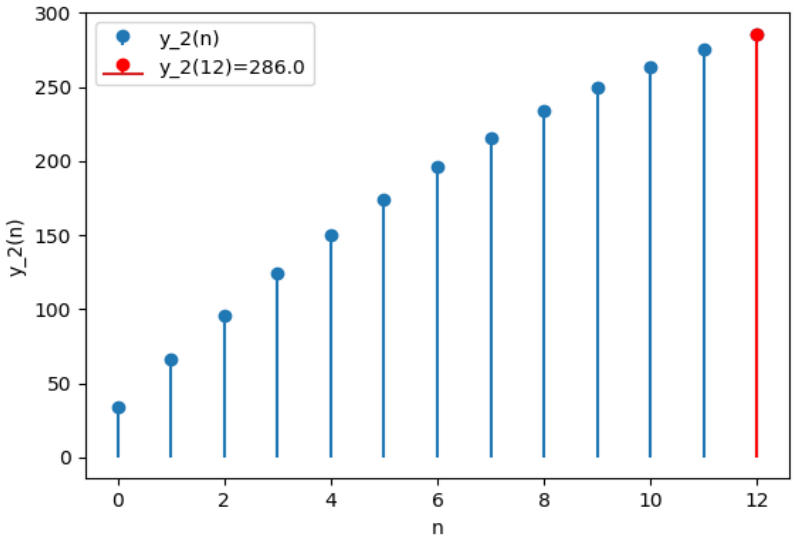

Fig. 2. $y_2(n)$ vs n

(iii) -5 + -8 + -11 ... -230

$$x_3(n) = (x_3(0) - 3n)u_{(n)} \tag{26}$$

$$-230 = -5 - 3n \tag{27}$$

$$n = 75 \tag{28}$$

1. Calculating $s_3(75)$:

$$s_3(75) = \frac{76}{2}(-10 + (76-1)(-3)) \tag{29}$$

$$= -8930 \tag{30}$$

2. Z-Transform of $x_3(n)$: Using (**??**)

$$X_3(z) = \sum_{n=-\infty}^{\infty} (x_3(0) - 3n)u_{(n)}z^{-n} \tag{31}$$

$$= -5z(z-1)^{-1} - 3z((z-1))^{-2}, \quad |z| > |1| \tag{32}$$

3. Z-Transform of $s_3(n)$:

$$h(n) = u(n) \tag{33}$$

$$y_3(n) = x_3(n) * h(n) \tag{34}$$

$$Y_3(z) = X_3(z) * H(z) \tag{35}$$

$$= (-5z(z-1)^{-1} - 3z((z-1))^{-2})z(z-1)^{-1} \quad |z| > |1| \tag{36}$$

4. Inversion of $Y_3(z)$: Using (**??**) , (**??**) for inverse Z-transforms :

$$y_3(n) = (-5(n+1) - 1.5n(n+1))u(n) \tag{37}$$

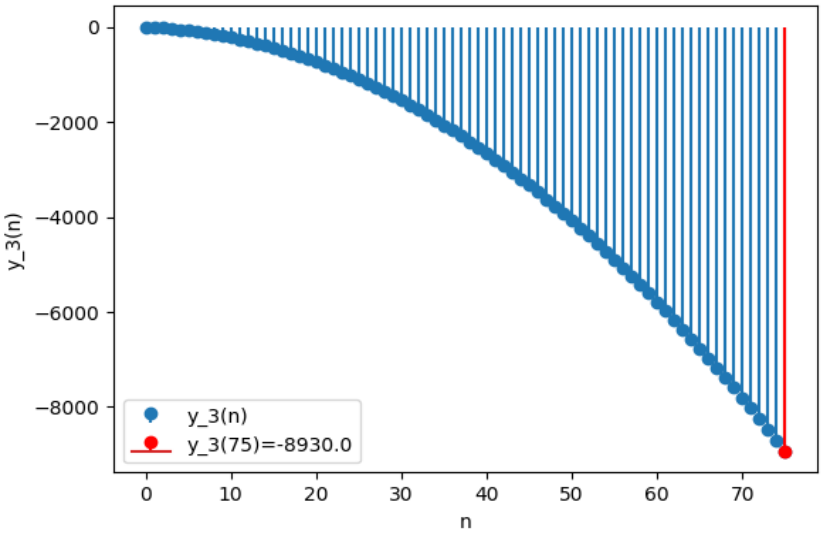

Fig. 3. $y_3(n)$ vs n