DETERMINATION OF NATURAL FREQUENCIES OF A COMPOSITE PLATE BY PERFORMING FINITE ELEMENT MODELING ANALYSIS AND STUDYING EFFECTS ON NATURAL FREQUENCIES BY VARYING DIFFERENT PARAMETERS OF A COMPOSITE PLATE

A Thesis submitted to University College of Engineering Kakinada, JNTUK- Kakinada in partial fulfilment of the requirements for the award of the degree of

BACHELOR OF TECHNOLOGY

IN

MECHANICAL ENGINEERING

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CERTIFICATE

This is to certify that the thesis entitled **"DETERMINATION OF NATURAL FREQUENCIES OF A COMPOSITE PLATE BY PERFORMING FINITE ELEMENT MODELING ANALYSIS AND STUDYING EFFECTS ON NATURAL FREQUENCIES BY VARYING DIFFERENT PARAMETERS OF A COMPOSITE PLATE"** Submitted by **M.GANESH KUMAR (RollNo.14021A0313), G.VIVEK (RollNo.14021A0319), D.NIKHIL (RollNo.14021A0313) and CH.SURAKSHITH (Roll No. 15025A0354)** in partial fulfilment of the requirement for the award of the degree of **Bachelor of Technology** in **Mechanical Engineering** to the University College of Engineering Kakinada (Autonomous), JNTU Kakinada is a record of bonafide work carried out by them under my guidance and supervision.

The results embodied in this thesis have not been submitted to any other university or institute for the award of any degree or diploma.

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DECLARATION

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ABSTRACT

Conventional materials such as steel, aluminium etc. are used in industries because of their high strength and stiffness. But composite materials have taken their places because they are giving excellent strength and stiffness with low weight. Currently, many industries such as automobile, aerospace, trains, buildings are using sandwich materials to reduce noise level. These sandwich materials consist of sheets of conventional materials which are bonded by polymers, plastics to reduce vibration and noise.

In this study, vibration analysis of laminated composite plate is carried out . Carbon fibre reinforced polymer and glass fibre reinforced polymer plates are used to study low frequency vibration and their effect of surrounding air medium. Combined modes shapes are formed because of resonance of natural frequencies of the structure. These combined mode shapes generally occur in low frequency region and possesses both high-order displacement. The finite element simulation model is developed to obtain the results.

An orthotropic plate with symmetric fiber orientation was considered for this study. The material properties were fixed. The natural frequencies were computed for different boundary conditions, stacking sequence, lay up's, fiber volume ratio, and fiber orientation. The effect of these variables on the nature of vibration is analysed and discussed. ANSYS 18.0 is used for the computation of natural frequencies.

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NOTATIONS USED

E_1 = YOUNG'S MODULUI IN LONGITUDNAL DIRECTION

E_2 = YOUNG'S MODULUI IN TRANSVERSE DIRECTION

G_{12} = SHEAR MODULUI IN 1-2 PLANE

PR_{12} = POISSION'S RATIO IN 1-2 PLANE

GPa = GIGA PASCALS (UNITS OF STRENGTH)

Hz = HERTZ (UNITS OF FREQUENCY)

E_f = YOUNG'S MODULUI OF FIBER

E_m = YOUNG'S MODULUI OF MATRIX

G_f = SHEAR MODULUI OF FIBER

G_m = SHEAR MODULUI OF MATRIX

PR_f = POISSION'S RATIO OF FIBER

PR_m = POISSION'S RATIO OF MATRIX

ABREVATIONS USED

C - CARBON FIBER REINFORCED POLYMERS

E - E-GLASS FIBER REINFORCED POLYMERS

CLT - CLASSICAL LAMINATE THEORY

FSDT - FIRST ORDER SHEAR DEFORMATION THEORY

ANSYS - ANALYSIS SYSTAMATICALLY

CHAPTER 1
INTRODUCTION

1.1 WHY WE USE COMPOSITES?

The biggest advantage of modern composite materials is that they are light as well as strong. By choosing an appropriate combination of matrix and reinforcement material, a new material can be made that exactly meets the requirements of a particular application. Composites also provide design flexibility because many of them can be moulded into complex shapes. The downside is often the cost. Although the resulting product is more efficient, the raw materials are often expensive.

1.2 WHAT IS A COMPOSITE?

A composite material is made by combining two or more materials – often ones that have very different properties. The two materials work together to give the composite unique properties. However, within the composite you can easily tell the different materials apart as they do not dissolve or blend into each other.

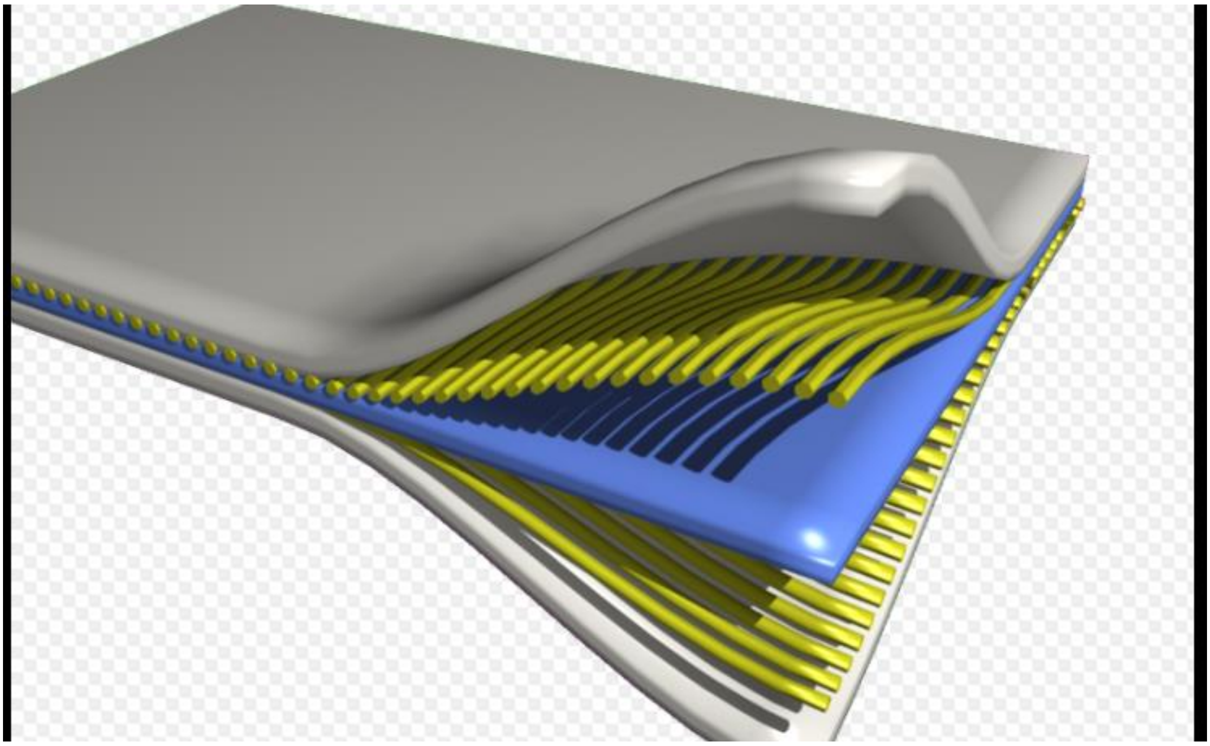


FIG 1 A COMPOSITE MATERIAL

1.3 CLASSIFICATION OF COMPOSITES

There are three main types of composite matrix materials:

Ceramic matrix - Ceramic matrix composites (CMCs) are a subgroup of composite materials. They consist of ceramic fibers embedded in a ceramic matrix, thus forming a ceramic fiber reinforced ceramic (CFRC) material. The matrix and fibers can consist of any ceramic material. CMC materials were designed to overcome the major disadvantages such as low fracture toughness, brittleness, and limited thermal shock resistance, faced by the traditional technical ceramics.

Metal matrix - Metal matrix composites (MMCs) are composite materials that contain at least two constituent parts – a metal and another material or a different metal. The metal matrix is reinforced with the other material to improve strength and wear. Where three or more constituent parts are present, it is called a hybrid composite. In structural applications, the matrix is usually composed of a lighter metal such as magnesium, titanium, or aluminum. In high temperature applications, cobalt and cobalt-nickel alloy matrices are common. Typical MMC's manufacturing is basically divided into three types: solid, liquid, and vapor. Continuous carbon, silicon carbide, or ceramic fibers are some of the materials that can be embedded in a metallic matrix material. MMCs are fire resistant, operate in a wide range of temperatures, do not absorb moisture, and possess better electrical and thermal conductivity. They have also found applications to be resistant to radiation damage, and to not suffer from outgassing. Most metals and alloys make good matrices for composite applications.

Polymer matrix - Polymer matrix composites (PMCs) can be divided into three sub-types, namely, thermoset, thermoplastic, and rubber. Polymer is a large molecule composed of repeating structural units connected by covalent chemical bonds. PMC's consist of a polymer matrix combined with a fibrous reinforcing dispersed phase. They are cheaper with easier fabrication methods. PMC's are less dense than metals or ceramics, can resist atmospheric and other forms of corrosion, and exhibit superior resistance to the conduction of electrical current.

1.4 CHARACTERISTICS OF COMPOSITES

HIGH STRENGTH TO WEIGHT RATIO

Fibre composites are extremely strong for their weight. By refining the laminate many characteristics can be enhanced. A common laminate of say 3mm Chopped strand mat, is quite flexible compared to say a 3 mm ply. However it will bend a long way more than the ply before yielding. Stiffness should not be confused with Strength. A carbon fibre laminate on the other hand, will have a stiffness of many times that of mild steel of the same thickness, increased ultimate strength, yet only be less than 1/4 of it's weight.

LIGHTWEIGHT

A standard Fibreglass laminate has a specific gravity in the region of 1.5, compared to Alloy of 2.7 or steel of 7.8. When you then start looking at Carbon laminates, strengths can be many times that of steel, but only a fraction of the weight.

A DVD case lid was produced using carbon fibre to reduce the case's overall weight so that it could be carried as cabin baggage whilst traveling, and for improved security. It was used by support crew for the All Blacks during their 1999 Rugby World Cup campaign.

FIRE RESISTANCE

The ability for composites to withstand fire has been steadily improving over the years. There is two types of systems to be considered:

Fire Retardant - Are self extinguishing laminates, usually made with chlorinated resins and additives such as Antimony trioxide. These release CO2 when burning so when the flame source is removed, the self extinguish.

Fire Resistant - More difficult and made with the likes of Phenolic Resins. These are difficult to use, are cured with formaldehyde, and require a high degree of post curing to achieve true fire resistance.

Other materials are also becoming more readily available to be used as in layers, which expand and blanket the surface, preventing spread of flame. There is a paint on coating usually applied to the back of the product laminate, plus a thin fibre film to go under the Gel coat giving the outer surface a blanketing coat as well.

CHEMICAL & WEATHERING RESISTANCE

Composite products have good weathering properties and resist the attack of a wide range of chemicals. This depends almost entirely on the resin used in manufacture, but by careful selection resistance to all but the most extreme conditions can be achieved. Because of this, composites are used in the manufacture of chemical storage tanks, pipes, chimneys and ducts, boat hulls and vehicle bodies. Composite panels were chosen because of their ability to withstand salty sea side conditions without corrosion.

COLOUR

Almost any shade of any colour can be incorporated into the product during manufacture by pigmenting the gel coat used. Costs are therefore reduced by no further finishing or painting. Soluble dyes can be used if a translucent product is desired.

We do not however, recommend dark colours. These produce excessive heat on the surface which can lead to the surface deteriorating and showing print through, where the Resin matrix cures more and shrinks, bringing the fibres to the surface. In extreme cases de lamination can occur.

TRANSLUCENCY

Polyester resins are widely used to manufacture translucent mouldings and sheets. Light transmission of up to 85% can be achieved.

1.5 ADVANTAGES OF COMPOSITES

Composite materials are relatively unknown and are often regarded as high tech materials for modern applications. Almost every material has previously gone through this phase; even wood used for building ships was once regarded as revolutionary and later the same applied to steel. This process was not straightforward. A certain motivation was required for adopting new materials (e.g. strength, stiffness, shortage of existing materials) and new construction methods became necessary. Shipyards, for example, disappeared or had to be completely reorganised in order to process the new material. Design methods and computational procedures changed, often through a process of bitter experience; for example, unexpected brittle fractures in American Liberty ships eventually helped improve steel ship design. It can be disputed that composites are revolutionary materials. As Figure 2 illustrates, modern composites have been used in structures for more than 60 years. However, a 'transition to composite' seems to be gradually taking place in more and more industries

Engineers are considered to be familiar with available materials and – based on project requirements – to be capable of selecting the right materials for the job. Know-how can be obtained through modern software, such as CES Edupack [3], which is able to compare the properties of countless materials. In addition, it is necessary to know both the advantages and disadvantages of a material.

FIG 2 *THE 1961 CHEVROLET CORVETTE WITH FIBER GLASS BODY PARTS*

FIG 3 TRANSITION FROM METAL TO COMPOSITE IN TRANSPORTATION SECTOR

Light Weight - Composites are light in weight, compared to most woods and metals. Their lightness is important in automobiles and aircraft, for example, where less weight means better fuel efficiency (more miles to the gallon). People who design airplanes are greatly concerned with weight, since reducing a craft's weight reduces the amount of fuel it needs and increases the speeds it can reach. Some modern airplanes are built with more composites than metal including the new Boeing 787, Dream liner.

High Strength - Composites can be designed to be far stronger than alluminum or steel. Metals are equally strong in all directions. But composites can be engineered and designed to be strong in a specific direction.

Strength Related to Weight - Strength-to-weight ratio is a material's strength in relation to how much it weighs. Some materials are very strong and heavy, such as steel. Other materials can be strong and light, such as bamboo poles. Composite materials can be designed to be both strong and light. This property is why composites are used to build airplanes—which need a very high strength material at the lowest possible weight. A composite can be made to resist bending in one direction, for example. When something is built with metal, and greater strength is needed in one direction, the material usually must be made thicker, which adds weight. Composites can be strong without being heavy. Composites have the highest strength-to-weight ratios in structures today.

Corrosion Resistance - Composites resist damage from the weather and from harsh chemicals that can eat away at other materials. Composites are good choices where chemicals are handled or stored. Outdoors, they stand up to severe weather and wide changes in temperature.

High-Impact Strength - Composites can be made to absorb impacts—the sudden force of a bullet, for instance, or the blast from an explosion. Because of this property, composites are used in bulletproof vests and panels, and to shield airplanes, buildings, and military vehicles from explosions.

Design Flexibility - Composites can be moulded into complicated shapes more easily than most other materials. This gives designers the freedom to create almost any shape or form. Most recreational boats today, for example, are built from fiber glass composites because these materials can easily be moulded into complex shapes, which improve boat design while lowering costs. The surface of composites can also be moulded to mimic any surface finish or texture, from smooth to pebbly.

Part Consolidation - A single piece made of composite materials can replace an entire assembly of metal parts. Reducing the number of parts in a machine or a structure saves time and cuts down on the maintenance needed over the life of the item.

Dimensional Stability - Composites retain their shape and size when they are hot or cool, wet or dry. Wood, on the other hand, swells and shrinks as the humidity changes. Composites can be a better choice in situations demanding tight fits that do not vary. They are used in aircraft wings, for example, so that the wing shape and size do not change as the plane gains or losses altitude.

Nonconductive - Composites are nonconductive, meaning they do not conduct electricity. This property makes them suitable for such items as electrical utility poles and the circuit boards in electronics. If electrical conductivity is needed, it is possible to make some composites conductive.

Nonmagnetic - Composites contain no metals; therefore, they are not magnetic. They can be used around sensitive electronic equipment. The lack of magnetic interference allows large magnets used in MRI (magnetic resonance imaging) equipment to perform better. Composites are used in both the equipment housing and table. In addition, the construction of the room uses composites rebar to reinforced the concrete walls and floors in the hospital.

Radar Transparent - Radar signals pass right through composites, a property that makes composites ideal materials for use anywhere radar equipment is operating, whether on the ground or in the air. Composites play a key role in stealth aircraft, such as the U.S. Air Force's B-2 stealth bomber, which is nearly invisible to radar.

<u>Low Thermal Conductivity</u> - Composites are good insulators—they do not easily conduct heat or cold. They are used in buildings for doors, panels, and windows where extra protection is needed from severe weather.

<u>Durable</u> - Structures made of composites have a long life and need little maintenance. We do not know how long composites last, because we have not come to the end of the life of many original composites. Many composites have been in service for half a century.

1.6 DISADVANTAGES OF COMPOSITES

1. Composites are more brittle than wrought metals and thus are more easily damaged. Cast metals also tend to be brittle.

2. Repair introduces new problems, for the following reasons: Materials require refrigerated transport and storage and have limited shelf lives. . Hot curing is necessary in many cases, requiring special equipment. Curing either hot or cold takes time. The job is not finished when the last rivet has been installed.

3. If rivets have been used and must be removed, this presents problems of removal without causing further damage.

4. Repair at the original cure temperature requires tooling and pressure.

5. Composites must be thoroughly cleaned of all contamination before repair.

6. Composites must be dried before repair because all resin matrices and some fibers absorbmoisture

TABLE 1 PROS AND CONS OF COMPOSITE MATERIALS

Advantages	Disadvantages
Weight saving	High material costs
High degree of freedom in form, material and process	Sophisticated computational methods sometimes required
Easy to colour	Colour and gloss preservation not always predictable
Translucent	Relatively limited knowledge on structural behaviour of details and connection methods
High degree of integration of functions possible	Finishing not yet well developed
Strength, stiffness, thermal and electrical resistance can be designed	Stiffness and failure behaviour can be undesirable; sensitive to temperature, fire and lightning strike
Low total maintenance costs	High costs of raw materials
Water- and chemically resistant	Sensitive to UV light
Use of durable materials possible	Recycling not yet well developed
Automated manufacturing possible	Sometimes capital intensive production methods (e.g. automated methods)

The above-mentioned advantages and disadvantages relate to a ‘current’ material that is not further specified. Careful distinctions should be made per design, because characteristics are not applicable or incompatible in some cases. An example is the lower weight; for objects that do not need to be moved often and are not appreciably loaded by their own weight, it would not make sense to opt for lightweight design. Furthermore, costs and sustainability of a design should always be considered throughout the life cycle. The costs of components or life-cycle phases can soar (e.g. investments in a mould) or materials may not be sustainable. For example, the production of carbon fibres requires high amounts of energy. This may sometimes be compensated for by the total costs of use due to lower maintenance costs or energy consumption during transport or in energy generation processes such as wind turbines. The relative number of advantages and disadvantages means little in respect to the general applicability of a composite. A particular aspect of a design may either lead to guaranteed success or be show-stopper

1.7 FIBERS

The role of fibres in a composite

The fibres generally determine the strength and stiffness of the composite material . A polymer to which directional fibres have been added is much stronger in the fibre direction than the polymer without fibres. Perpendicular to the fibre direction, the increase in stiffness is less pronounced. The strength in that direction is smaller, since the fibres act as stress concentrators. In practice, fibres are often incorporated in different directions. Glass and carbon – the most commonly used fibres Although many types of fibres exist that are suitable as reinforcement in a composite material, glass fibres and carbon fibres (carbon/graphite) are most commonly used. Production methods of these fibres differ. To produce glass fibres, silicon oxide (SiO2, from sand) is heated together with various additives above its melting point. The molten material is then fed to small channels with small holes in the bottom (of approx. 2 mm diameter) through which the molten material passes. The viscous melt is wound on a coil. This is done at high speed (tens of metres per second). As a result, the molten fibres are stretched and become much thinner – approximately 20 micrometers in diameter. Immediately after leaving the extrusion sleeves (which are made of a platinum-rhodium alloy that is capable of withstanding high temperatures), the fibres are sprinkled with water so that they solidify at high speed. The water contains an additive to facilitate further processing of the fibres. See Figure 4.

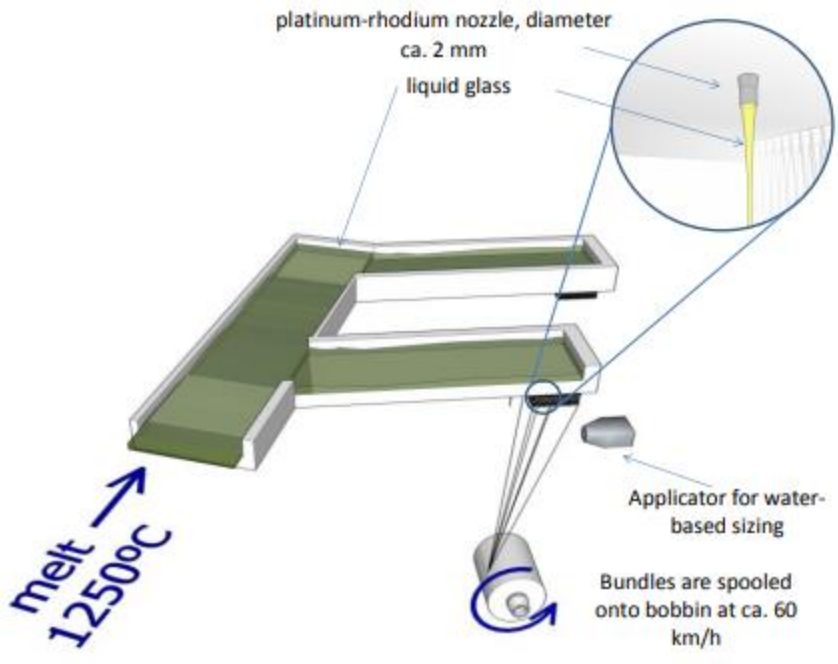


FIG 4 OVERVIEW OF GLASS FIBER PRODUCTION

There are various types of glass, each with different material properties. Each type is designated by a letter. The most common type is E-glass. Other types include S-glass (increased strength and stiffness), C-glass (chemically resistant), D-glass (low dielectric constant and thus highly suitable for application in radars, for example). The type of glass is determined by its chemical composition. The process described above is relatively cheap but involves a number of complicating factors. The constituents must be mixed in the correct ratios. Also, changing the chemical composition requires time and material, since the furnace must first be purged. The material of the glass furnace has a limited lifetime of a few years.

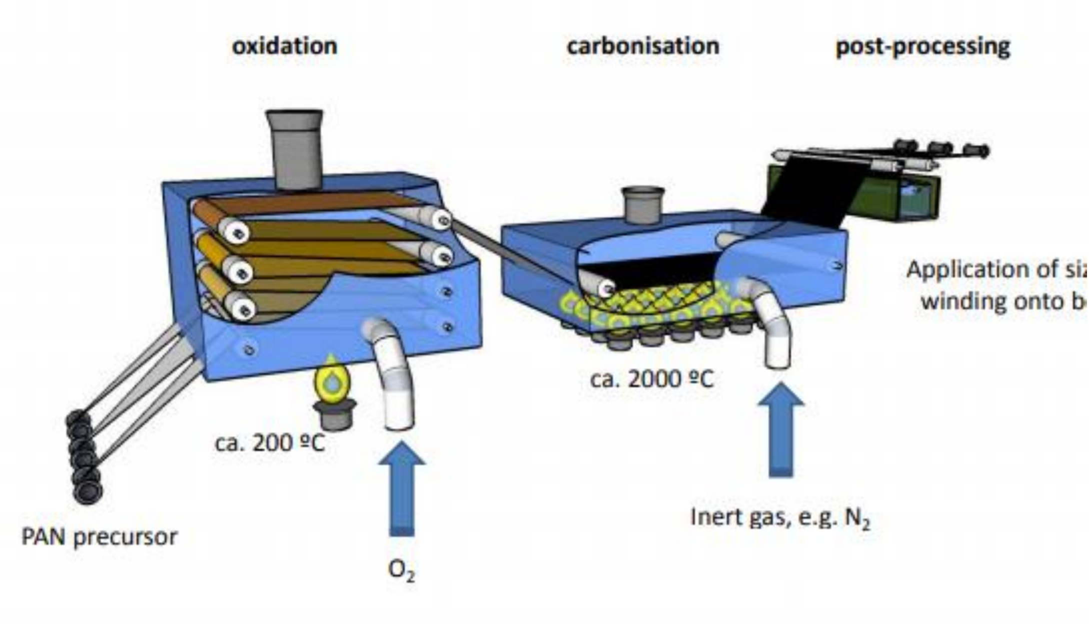


FIG 5 OVERVIEW OF CARBON FIBER PRODUCTION

Carbon fibres are produced in a completely different way (Figure 5). Various raw materials can be used, provided their chemical compound has high carbon atom content. In general, PAN (poly acrylnitrile), pitch or rayon/viscose (now used in e.g. bio composites) are used. PAN is a manufactured product that has well defined properties. Pitch, on the other hand, is a natural product. PAN is preferred for consistent quality, while pitch is cheaper. Threads drawn from PAN or pitch pass through three stages: oxidation at approximately 200ºC (the fibres obtain their characteristic black colour in this stage); carbonisation at 800- 1600ºC (various components, such as nitrogen atoms, are removed in an inert atmosphere); and ultimately graphitisation (where the fibre obtains its definitive composition). The fibres are stretched during this process, so that the orientation of the carbon chains in the material runs parallel to the fibre direction as much as possible, and an anisotropic fibre is formed. Carbon fibres are often transversally isotropic and have a much higher stiffness in the axial direction than in the transverse direction

Other fibres

In addition to glass and carbon, many other fibre reinforcements are used. With respect to the manufacturing method, basalt fibres closely resemble glass fibres. Basalt (volcanic rock) is heated in a furnace, similar to glass, after which threads are drawn. The basalt is ready for processing 'as is' – which means components do not need to be mixed beforehand. The composition of basalt depends on the site where it is mined, however. This means that ultimately there is only a limited supply. Furthermore, basalt is more difficult to melt than glass and more abrasive. Therefore, extrusion sleeves must be replaced more frequently. This leads to basalt being more expensive than E-glass, although it is still cheaper than the more expensive kinds of glass and carbon. Other commonly applied fibres are aromatic polyamides, known by the brand names Kevlar and Twaron. The polymer chains in these fibres are strongly directed during the manufacturing process, resulting in the formation of a stiff fibre. The specific gravity of these fibres is very low, resulting in good specific properties. An important advantage of these fibres is their great tenacity, making them very suitable for application in bullet-proof vests. During processing and in use, natural fibres (from plants such as flax, hemp, bamboo and wood) have the disadvantage of being sensitive to moisture absorption and rotting. Another disadvantage of plant fibres is that they are fairly short. Their strength and stiffness, certainly in relation to their weight, can be of the same order as that of synthetic fibres.

1.8 PLIES AND LAMINATES

Key terms in working with composites are ply and laminate. A layer of impregnated fibre reinforcement is called a lamina or ply; a stack of plies is called a laminate.

TABLE 2 PROPERTIES OF SOME FIBERS

Property	E-glass	Carbon*	Aramid	Bamboo
Stiffness [GPa]	70-80	160-440	60-180	10-15
Breaking strength [MPa]	2400	2000-5300	3100-3600	100-200
Failure strain [%]	2.6	1-1.5	1.7	-
Density [kg/m^3]	2500-2600	1800-2000	1540	400-800
Fracture length** [km]	96	187	238	25

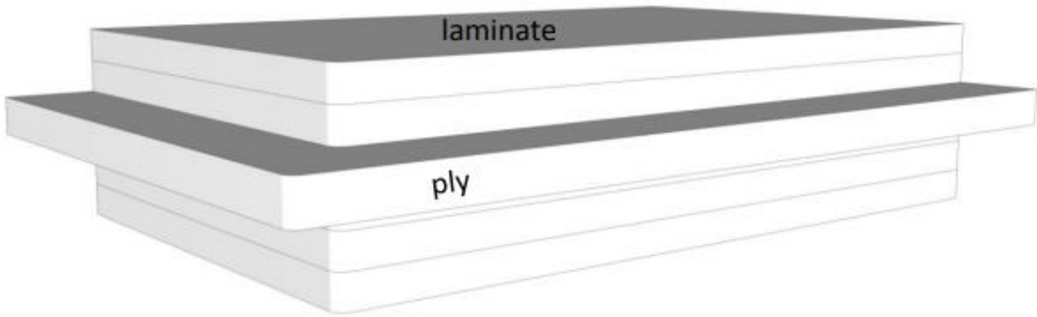


FIG 6 PLY AND LAMINATE

1.9 POLYMERS

The world around us is full of polymer products. At first glance, the differences between most polymers are not obvious. But just as different wood species were used for different purposes in the past (e.g. oaks for robust furniture, tropical hardwood for window frames and sandalwood for carved images), different polymer materials are used for different purposes. For example, plastic coffee cups are made of polystyrene (as is the insulation in a refrigerator), the vacuum cleaner in a corner of your student accommodation is made of impact-resistant ABS and your water bottle is made out of inert polyethylene. In composites, fibres are embedded in a polymer, which is called the 'matrix' (originating from the Latin word for 'womb'). It may not seem logical to partly compromise the high strength and stiffness characteristics of the fibre materials by mixing them with polymer materials. Indeed, most production techniques are at least partially based on including as little polymers to the composite as possible. The most important and most obvious reason for including the polymer is that the polymer acts as an 'adhesive' and binds the fibres together. By encapsulating a fibre with polymer, the fibre can absorb higher compressive loads; it is supported by the resin. A somewhat less obvious, but very important, function of the polymer is that the fibres can work together better, because the polymer transfers loads from one fibre to the other through shear stresses. External loads are thus better distributed over the fibres in a composite than in a dry fibre bundle. When one filament breaks, the load is distributed over the other filaments. In addition, the polymer largely determines the sensitivity of the composite to external influences such as moisture, chemicals, and ultraviolet light. It often determines colour and surface quality, opacity, and fire safety. In summary, the role of the polymer in a composite should not be underestimated. The term 'fibre-reinforced plastics' does not justify the role of the polymer, because the polymer itself plays a crucial role in the success of a composite material.

Various kinds of polymers are used in composites. The most relevant are discussed in the following sections. But first we shall classify polymers into two categories, since most of the processing methods depend on the category the polymers belong to. These categories are the thermoplastics and the thermosets.

1.10 THERMOPLASTICS

Thermoplastics are polymers that melt upon heating, becoming formable and regain their solid shape upon cooling. Most commonly used unreinforced polymers are thermoplastics. In molecular terms, thermoplastics consist of long entangled chains. Upon heating, some freedom of movement is gained through the molecular movements. Exceptions aside, thermoplastics are not generally suitable for impregnation of a fibre reinforcement due to their viscosity (high viscosity in liquid state, related to the molecular state). This prevents the thermoplastic from wetting the fibres adequately (impregnating), thus a good composite cannot be formed. To produce composite materials using thermoplastics, high pressures and temperatures are necessary. A commonly used method is to alternate dry plies with thermoplastic films and produce a composite by means of a heated mould (e.g. compression moulding). Alternatively, thermoplastic yarns or fibre bundles are co-spun with the fibre reinforcement. Then, a lower external pressure is needed to arrive at a good impregnation. A recent process development is the infusion of thermoplastics. The infusion process uses monomers that polymerise during curing. Since monomers have short chains they do not get entangled, leading to a low viscosity in liquid state and making them suitable for infusion.

1.11 THERMOSETS

Thermosetting resins (in short: thermosets) do not melt on heating, but ultimately disintegrate. From a molecular point of view, most thermosets consist of relatively short chains ensuring the non-cured polymers to have very low viscosity . Curing is carried out by initiating a chemical reaction, in which the short chains form bonds and create a three-dimensional 'cross-linked' network. The temperature is often regulated during curing. This can also apply for the pressure (depending on the fabrication method). The distinction between thermoplastics and thermosets is not always clear. Polyesters, which are classified as thermosets below, can also be thermoplastic. Phenolic resins behave as thermoplastics up to a particular temperature.

1.12 OTHER POLYMERS

1. The most commonly used thermosetting plastics are polyesters, vinylesters and epoxies. A comparison of these three immediately reveals:

2. Polyesters and vinylesters are cheaper to produce than epoxies.

3. Epoxies shrink less during curing than polyesters and vinylesters

4. Polyesters are more sensitive to damage due to osmosis

5. All polymers require at least two components to be mixed: in the case of polyester and vinylester this is the monomer and a catalyst (and accelerator) to cure; epoxies require a hardener to be mixed with the main component.

6. An exothermic reaction takes place in all systems

1.13 APPLICATIONS OF COMPOSITES

Space craft:

Antenna structures, Solar reflectors, Satellite structures, Radar, Rocket engines, etc.

Aircraft:

Jet engines, Turbine blades, Turbine shafts, Compressor blades, Airfoil surfaces, Wing box structures, Fan blades, Flywheels, Engine bay doors, Rotor shafts in helicopters, Helicopter transmission structures, etc

Miscellaneous:

Bearing materials, Pressure vessels, Abrasive materials, Electrical machinery, Truss members, Cutting tools, Electrical brushes, etc.

Automobile:

Engines ,bodies , piston, cylinder, connecting rod, crankshafts, bearing materials, etc.

FIG 7 COMPOSITE ARTIFICIAL LIMBS

FIG 8 FRP TOILETS FOR RAILWAY COACHES

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

Composite plates are typically used in aircraft and automobiles. In most of the structures, these plates are subjected to dynamic loads. Therefore, composite plates are expected to have adequate stiffness to resist failure due to vibration, buckling etc. The frequency of vibration of the plates should be within a certain limit so that it does not affect the function of nearby parts in the structure and does not produce any discomfort. Therefore, it is important to predict the natural frequencies of composite plates to study the behaviour of the structure and to avoid resonance of large structures under dynamic loading. The modal analysis can be used as a non-destructive technique for the assessment of stiffness of structures against vibration. The free vibration characteristics of laminated composite plates have been analyzed by many investigators and a number of theoretical and experimental methods have been proposed to predict their natural mode of vibration. The vibrational behaviour of plates with cut-outs has also attracted the attention of researchers over the past few decades. The related literature was critically reviewed so as to provide background information on the problems to be considered in the project work and to emphasize the relevance of the present study.

Cawley and Adams (1978) studied the natural modes for square composite plates for free-free boundary conditions. Crawley (1979) practically determined vibration frequencies of composite plates for various aspect ratios and compared with FEM. Palardy and Palazotto (1990) studied frequency response of laminated cross ply using Levy approach based on shear deformation. Lee and Lim (1993) used numerical method based on the Rayleigh method for predicting the natural frequencies of a rectangular composite plate with a centrally located crack. Sinha and Maiti (1996) used the FSDT theory to develop methodology for vibration response of thick composite plates. Chakraborty et al. (2000) studied GFRP plates for vibration properties and used NISA to validate the results. Hwang and Chang (2000) utilized impulse technique for frequency testing of composite plates. Guan-Yuan Wu and Yan-Shin Shih (2005) studied the dynamic uncertainty of rectangular composite plate having a crack at one side.

Rajamani et al. [5] and [6] considered homogeneous orthotropic composite plates, the laminations of which were assumed to be symmetrical about the mid-plane He investigated the effects of central circular holes and square cut-outs on the natural frequencies of the plate under two end conditions-simply supported (Part 1)and clamped-clamped (Part 2). The effects of cutout were considered equivalent to an external loading.

Reddy J.N. [7] carried investigations on the large amplitude vibration of anisotropic rectangular laminated composite plates. Also, he varied side-to-thickness ratio, aspect ratio and plate side to cut-out side ratio and observed the variation in nature of vibration.

Lee et al. [8] studied the simply supported orthotropic rectangular composite plates with central rectangular cut-outs and double square cut-outs. He used Rayleigh principle to predict the natural frequencies, fundamental modes and selected higher modes of the composite plates.

Bicos et al. [9] applied finite element method to formulate equations to describe the free damped vibrations of plates and shells. He also developed computer code to calculate the natural frequencies, mode shapes and damping factors of rectangular plates, cylinders and cylindrical panels with different boundary conditions – free, clamped and simply supported. He considered both plates with cutouts and without cutouts.

Ramakrishna et al. [11] considered a laminated composite plate with a central circular hole. He developed a computer program for predicting the natural frequencies of vibration of the plate by using a hybrid-stress finite element. Also, he studied the effects of fiber orientation, width-tothickness ratio, aspect ratio and hole-size on the first four natural frequencies.

Jwalamalini et al. [12] considered a simply supported square plate with openings and analyzed its stability under in-plane loading. He used BUCSAP, a Finite Element Program. He took central and square openings for the main study. The tension and compression were assumed as initial pre-stress in the transverse direction before the longitudinal stress was applied.

Chai Gin Boay [13] published a paper presenting finite element results on free vibration of laminated composite plates containing a central circular hole considering the aspect ratio and hole-size as variables. The material properties and stacking sequence were kept constant. He considered materials that are typically used in aircraft structural application.

Sivakumar et al. [14] investigated the free vibration of laminated composite plates with cut-outs undergoing large oscillations. They used Ritz finite element model and obtained results for plates with cut-outs of various geometries- circle, square, rectangular and ellipse in the large amplitude range.

Liew et al. [15] developed a semi analytical procedure to predict natural frequencies of plates with discontinuities in cross-section. He assumed a square element as a basic building element. He used Ritz procedure to extract the frequencies and mode shapes.

Myung Jo Jhung et al. [18] analyzed the free vibration of circular plate with eccentric hole. He assumed the plate was submerged in fluid. He developed an analytical method, based on finite Fourier-Bessel series expansion and Rayleigh-Ritz method. He also varied the hole-size and studied its effects on the vibration characteristics of the plate.

2.2 OBJECTIVE OF PRESENT STUDY

Hence after a critical review of literature, it was decided to study the free vibration of laminated composite plates and variation of natural frequencies of laminated composite plate for various changes in parameters like

1. Fiber volume ratio (0.4 ,0.5 ,0.6)

2. Fiber orientation (0 ,30 ,45 ,60)

3. Boundary conditions (Cantilever , Simply Supported)

4. Stacking Sequence

5. Lay up (Unidirectional ,Cross ply Symmetric ,Cross ply Anti symmetric ,Angle ply)

2.3 SCOPE FOR FUTURE WORK

The current study of vibration analysis of laminated composite plates can be expanded to the following arenas:

- Study of Hygrothermal effects on the natural vibration frequency of composite plates.
- Study of buckling characteristics of composite plate and study of Hygrothermal effects on the buckling characteristics of the plate.
- Study of vibration characteristics of composite plate in many more boundary conditions and other different crack parameters.
- Study of vibration characteristics of composite plates with holes and cracks.

CHAPTER 3

THEORY BEHIND COMPOSITES

3.1 ISOTROPIC, ANISOTROPIC, AND ORTHOTROPIC MATERIALS:

Materials can be classified as either isotropic or anisotropic. Isotropic materials have the same material properties in all directions, and normal loads create only normal strains. By comparison, anisotropic materials have different material properties in all directions at a point in the body. There are no material planes of symmetry, and normal loads create both normal strains and shear strains. A material is isotropic if the properties are independent of direction within the material.

For example, consider the element of an isotropic material shown. If the material is loaded along its 0°, 45°, and 90° directions, the modulus of elasticity (E) is the same in each direction ($E_0 = E_{45} = E_{90}$). However, if the material is anisotropic (for example, the composite ply shown, it has properties that vary with direction within the material. In this example, the moduli are different in each direction ($E_0 \neq E_{45} \neq E_{90}$). While the modulus of elasticity is used in the example, the same dependence on direction can occur for other material properties, such as ultimate strength, Poisson's ratio, and thermal expansion coefficient.

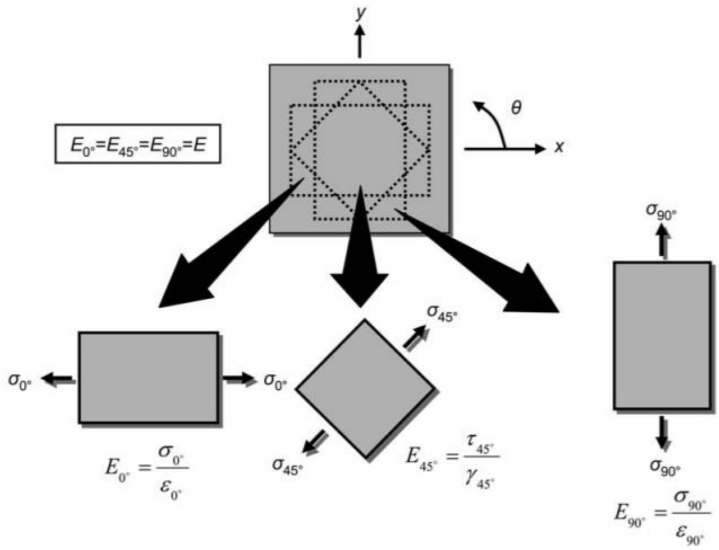


FIG 9 ELEMENT OF ISOTROPIC MATERIAL UNDER STRESS

Bulk materials, such as metals and polymers, are normally treated as isotropic materials, while composites are treated as anisotropic. However, even bulk materials such as metals can become anisotropic––for example, if they are highly cold worked to produce grain alignment in a certain direction. Consider the unidirectional fiber-reinforced composite ply (also known as a lamina) shown. The coordinate system used to describe the ply is labeled the 1-2-3 axes. In this case, the 1-axis is defined to be parallel to the fibers (0°), the 2-axis is defined to lie within the plane of the plate and is perpendicular to the fibers (90°), and the 3-axis is defined to be normal to the plane of the plate. The 1-2-3 coordinate system is referred to as the principal material coordinate system. If the plate is loaded parallel to the fibers (one- or zero-degree direction), the modulus of elasticity E_{11} approaches that of the fibers. If the plate is loaded perpendicular to the fibers in the two- or 90-degree direction, the modulus E_{22} is much lower, approaching that of the relatively less stiff matrix. Since $E_{11} >> E_{22}$ and the modulus varies with direction within the material, the material is anisotropic.

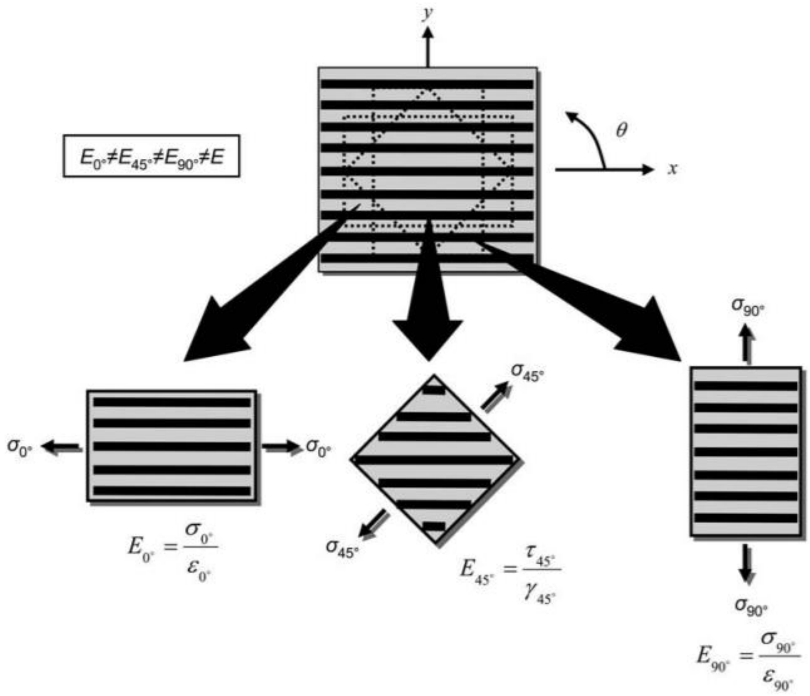


FIG 10 ELEMENT OF COMPOSITE PLY MATERIAL UNDER STRESS

Composites are a subclass of anisotropic materials that are classified as orthotropic. Orthotropic materials have properties that are different in three mutually perpendicular directions. They have three mutually perpendicular axes of symmetry, and a load applied parallel to these axes produces only normal strains. However, loads that are not applied parallel to these axes produce both normal and shear strains. Therefore, orthotropic mechanical properties are a function of orientation.

3.2 LAMINA, LAMINATE; CHARACTERISTICS AND CONFIGURATIONS:

3.2.1 LAMINA:

A Lamina or ply is a plane (or curved) layer of unidirectional fibers or woven fabric in a matrix. In the case of unidirectional fibers, it is also referred to as unidirectional lamina(UD). The lamina is an orthotropic material with principal material axes in the direction of the fibers (longitudinal), normal to the fibers in the plane of the lamina (in-plane transverse), and normal to the plane of the lamina. These principal axes are designated as 1,2, and 3, respectively. In the case of a woven fabric composite, the wrap and fill directions are the in-plane principal directions.

3.2.2 LAMINATE:

A laminate is made up of two or more unidirectional laminate or plies stacked together at various orientations. The laminate (or plies, or layers) can be of various thicknesses and consist of different materials. Since the principal material axes differ from ply to ply, it is more convenient to analyse laminates using a common fixed system of coordinates (x, y, z), as shown. The orientations of a given ply is given by the angle between the reference x-axis and the major principal material axis (fiber orientation) of the ply, measured in a counterclockwise direction on the x-y plane.

Composite laminates containing plies of two or more different types of materials are called hybrid composites and more specifically, interplay hybrid composites. For example, a composite laminate may be made up of unidirectional glass/epoxy and aramid/epoxy layers stacked together in a specified sequence. In some cases it may be advantageous to intermingle different types of fibers, such as glass and carbon or aramid and carbon, within the same unidirectional ply. Such composites are called intraply hybrid composites. Of course, one could combine intraply hybrid layers with other layers to from an intraply-interply hybrid composite.

Composite laminates are designated in a manner indicating the number, type, orientation, and stacking sequence of the plies. The configuration of the laminate indicating its ply composition is called lay-up. The configuration indicating, in addition to the composition, the exact location or sequence of the various plies is called the stacking sequence. Following are some examples of laminate designations:

3.2.3BASIC CONCEPTS AND CHARACTERISTICS:

Unidirectional 6-ply	$[0/0/0/0/0/0]=[0_6]$
Cross ply symmetric	$[0/90/90/0]=[0/90]_s$
	$[0/90/0]=[0/90]_s$
Angle-ply symmetric	$[+45/-45/-45/+45] = [\pm45]_s$
	$[30/-30/30/-30/30/-30/30/-30] = [\pm30]_{2s}$
Angle-ply symmetric	$[30/-30/30/-30/30/-30/30/-30] = [\pm30]_4$
Multi directional	$[0/45/-45/-45/45/0] = [0/\pm45]_s$
	$[0/0/45/-45/0/0/0/0/-45/45/0/0] = [0_2/\pm45/0_2]_s$
	$[0/15/-15/15/-15/0] = [0/\pm15/\pm15/0]_T= [0/(\pm15)_2/0]_T$
Hybrid	$[0^K/0^k/45^C/-45^C/90^G/-45^C/45^C/0^K/0^K]_T= [0_2^k/\pm45^C/90^G]_s$

Where subscripts and symbols signify the following:

Number subscript = Multiple of plies or group of plies

s = Symmetric sequence

T = Total (number of plies)

In the case of the hybrid laminate, superscripts K, C, and G denote Kevlar (aramid), carbon (graphite), and glass fibers, respectively.

3.2.4 BASIC LAMINA PROPERTIES:

The approach followed here is based on macromechanics. The unidirectional lamina or ply is considered the basic building block of any laminate or composite structure. The basic material properties necessary for analysis and design are the average ply properties. With reference to figure 11, the unidirectional ply is characterized by the following properties:

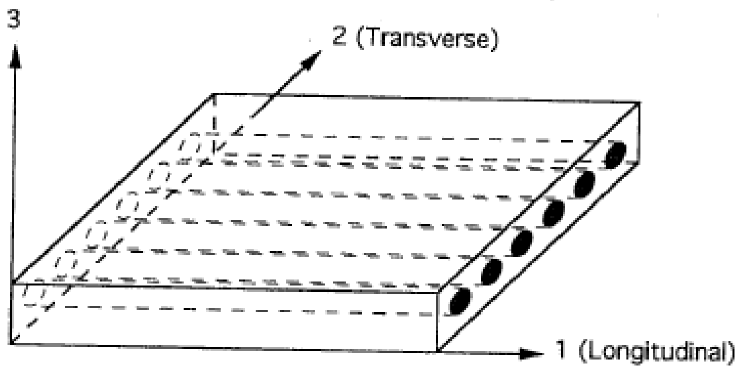


FIG 11 . UNIDIRECTIONAL LAMINA AND PRINCIPLE COORDINATE AXIS

E_1, E_2, E_3 = Young's moduli along the principal ply directions

G_{12}, G_{23}, G_{13} = Shear moduli in 1-2, 2-3, and 1-3 planes, respectively

(these are equal to G_{21}, G_{32}, and G_{31}, respectively)

Pr_{12}, Pr_{23}, Pr_{13} = Poisson's ratios

(the first subscript denotes the loading direction,

and the second subscript denotes the strain direction, these Poisson's ratio are different from Pr_{12}, Pr_{23}, and Pr_{13}, i.e., subscripts are not interchangeable)

$\alpha_1, \alpha_2, \alpha_3$ = Coefficients of thermal expansion

$\beta_1, \beta_2, \beta_3$ = Coefficient of moisture expansion

k_1, k_2, k_3 = Coefficient of thermal conductivity

In addition to the above, the composite lamina is characterized by the following properties:

Fiber volume ratio $V_f = \frac{\text{volume of fiber}}{\text{volume of composites}}$

Fiber weight ratio $W_f = \frac{\text{weight of fiber}}{\text{weight of composite}}$

Matrix volume ratio $V_m = \frac{\text{volume of matrix}}{\text{volume of composite}}$

Matrix weight ratio $W_m = 1 - W_f = \frac{\text{weight of matrix}}{\text{weight of composite}}$

Void volume ratio $V_v = 1 - V_f - V_m = \frac{\text{volume of voids}}{\text{volume of composite}}$

3.3 FUNDAMENTAL PROPERTY RELATIONSHIPS:

When a unidirectional continuous-fiber lamina or laminate is loaded in a direction parallel to its fibers (0° or 11-direction), the longitudinal modulus E_{11} can be estimated from its constituent properties by using what is known as the rule of mixtures:

$$E_1 = E_f V_f + E_m V_m$$

Where E_f is the fiber modulus, V_f is the fiber volume percentage, E_m is the matrix modulus, and V_m is the matrix volume percentage.

The longitudinal tensile strength σ_{11} also can be estimated by the rule of mixtures:

$$\sigma_1 = \sigma_f V_f + \sigma_m V_m$$

where σ_f and σ_m are the ultimate fiber and matrix strengths, respectively. Because the properties of the fiber dominate for all practical volume percentages, the values of the matrix can often be ignored; therefore:

$$E_1 = E_f V_f$$

When the lamina shown in Fig. 1.11 is loaded in the transverse (90° or 22-direction), the fibers and the matrix function in series, with both carrying the same load. The transverse modulus of elasticity E_2 is given as:

$$1/E_2 = V_f /E_f + V_m/E_m$$

Other rule of mixture expressions for lamina properties include those for the Poisson's ratio PR_{12} and for the shear modulus G_{12}:

$$Pr_{12} = Pr_f V_f + Pr_m V_m$$

$$1/G_{12} = V_f /G_f + V_m/G_m$$

These expressions are somewhat less useful than the previous ones, because the values for Poisson's ratio (Pr_f) and the shear modulus (G_f) of the fibers are usually not readily available.

Physical properties, such as density (r), can also be expressed using rule of mixture relations:

$$f_{12} = f_f V_f + f_m V_m$$

While these micromechanics equations are useful for a first estimation of lamina properties when no data are available, they generally do not yield sufficiently accurate values for design purposes. For design purposes, basic lamina and laminate properties should be determined using actual mechanical property testing.

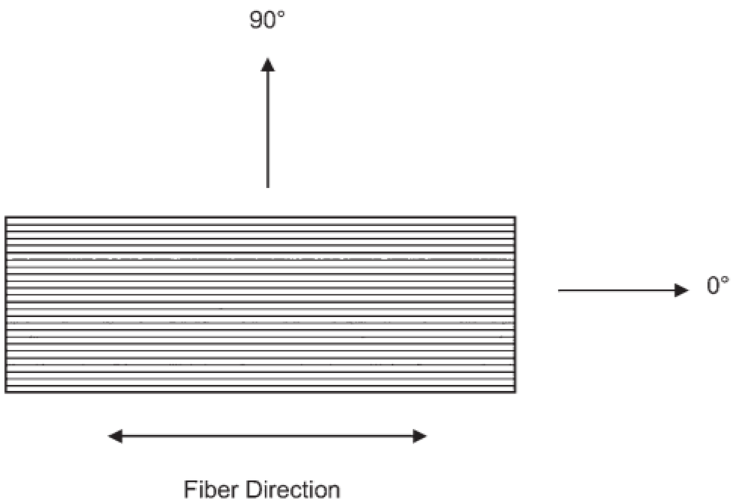


FIG 12 . UNIDIRECTIONAL CONTINOUS FIBER LAMINA OR LAMINATE

3.4 CLASSICAL LAMINATE THEORY :

A laminate is built up from different plies, each with its own properties. For laminates with plies that are all equal and lie in the same direction, the laminate properties – the relationship between external load/stress and strain – can be easily derived. But when plies are stacked in different directions, the stiffness in a particular laminate direction will differ per ply. This is because an external load then results in different internal ply stresses. A 'slack' ply will stretch just as much as a 'rigid' one, since the plies are bonded to each other. Due to the difference in elasticity modulus, however, the slack ply will be under less tension (see Figure 13).

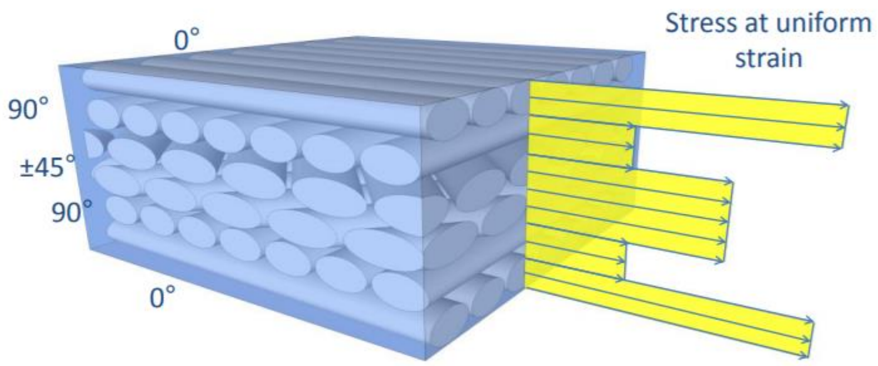


FIG 13 STRESS IN A PLY

This has an important consequence: if you want to determine the strength of a laminate, you need to calculate the stresses per ply on the basis of the strain values and test them with a failure criterion. However, the strain of the laminate depends again on the combined stiffness values of the plies!

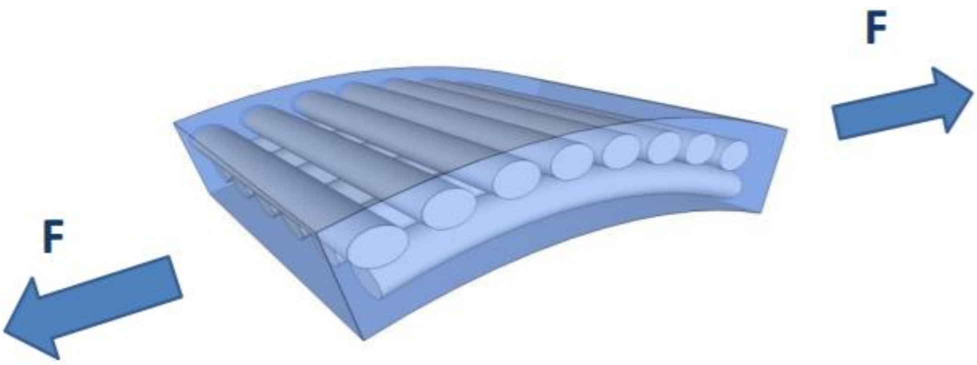


FIG 14 COUPLING EFFECTS

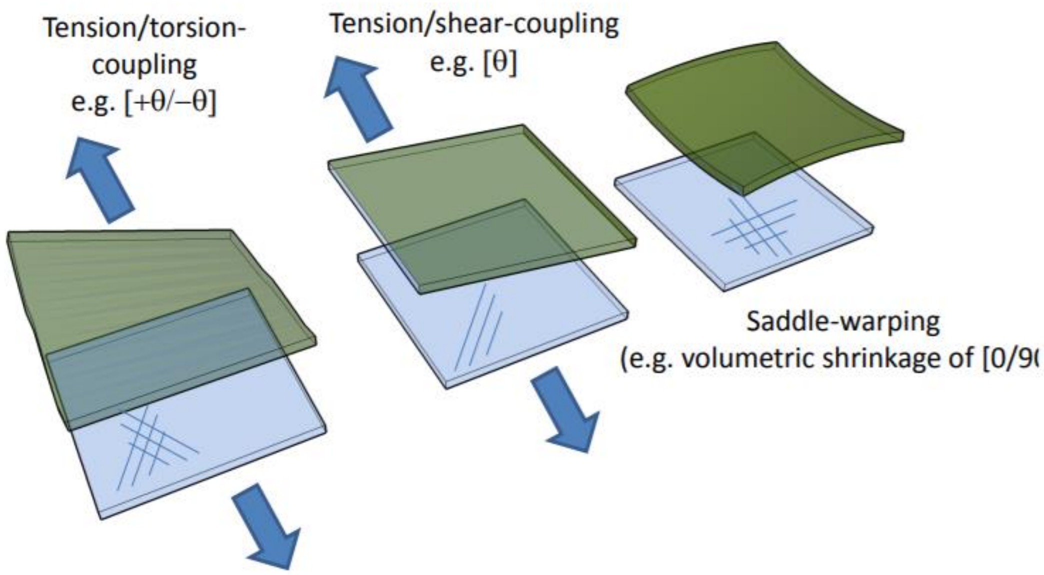


FIG 15 COUPLING EFFECTS THAT CAN OCCUR AS A CONSEQUENCE OF LAMINATE STRUCTURE

In addition, coupling effects can occur, e.g. when the ply stiffness values do not lie symmetrically relative to the centre of the laminate (see Figures 14&15), or when the laminate is not balanced. In Figure, the top layer is provided with fibres transversely to the bottom layer. When a load F is applied, the top layer (which is less rigid in the load direction) will elongate more than the bottom layer. This will cause the laminate to curve. Thus there is a 'coupling' between the load direction and the deformation in other directions.

Such coupling effects can be desirable. Experiments have been carried out on aircraft wings, for example, with a built-in relationship between lift and the angle of incidence. In these experiments, use was made of the possibilities of a composite to improve the stability of an inherently unstable wing configuration (see Figure 15).

When designing a laminate, it is important to know its strength and stiffness, and to control possible coupling effects. To this end, 'Classical Lamination Theory' (CLT) has been developed. This is very suitable for calculating the stress and elongation of each ply under an external load (force or moment). The application requires fairly extensive calculations however, involving heavy use of linear algebra. For manual calculations, this theory is less suitable. CLT is used particularly in software applications.

3.4.1 ASSUMPTIONS:

Classical lamination theory is based on the following assumptions. These directly indicate limitations and possibilities:

- ‘Smeared properties’: the structure of the fibres and resin is not modelled. For each ply, 'smeared' properties are used. This means you cannot use the theory to determine what takes place at a microscopic level within a ply. It is also assumed that the fibre content is constant;

- All plies stick to each other: the theory is not valid for delamination;

- The theory is linear elastic: any non-linear behaviour of the participating plies is not taken into account;

- The laminate has a constant thickness: the theory is not valid in the vicinity of ply-drops or other thickness jumps;

- The laminate is undisturbed: the theory is not valid in the vicinity of holes, inclusions, inserts, corners and edges;

- The plies are thin compared to the laminate.

3.4.2 OVERVIEW OFCLT:

The flowchart in Figure 16 shows broadly how CLT works.

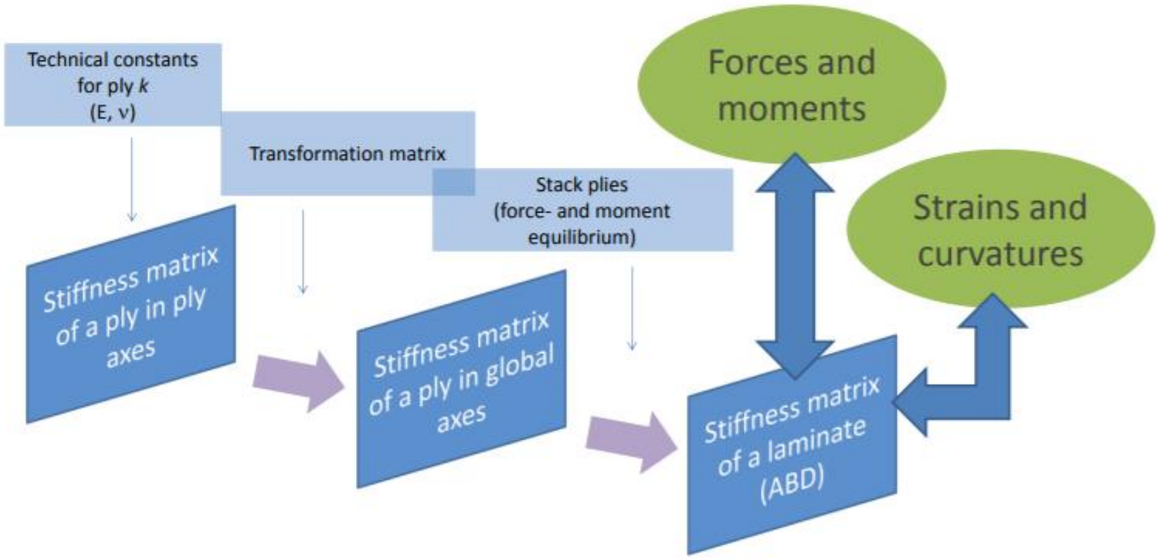


FIG 16 OVERVIEW OF CLASSICAL LAMINATE THEORY

First of all, the 'technical constants' per ply are determined. These are the values of the elasticity modulus and the Poisson ratio in each direction of the ply. Then the stiffness matrix and/or the compliance matrix of each ply is determined in the main directions of the ply, e.g. in and perpendicular to the fibre direction. This matrix establishes the connection between stress and strain and contains the technical constants.

Because a ply can be processed in an arbitrary direction in a laminate, the main direction of the ply is not always parallel to the main direction of the laminate. This is often the load direction and the direction perpendicular to it. In a UD ply, the fibres will not always lie in the load direction. Thus the ply is applied in a 'rotated' direction in the laminate plane. The properties of the ply must be determined in the main direction of the laminate. This takes place using the transformation matrix. Since a rotation in the plane of the laminate is always involved in these considerations, you could also refer to this as a rotation matrix.

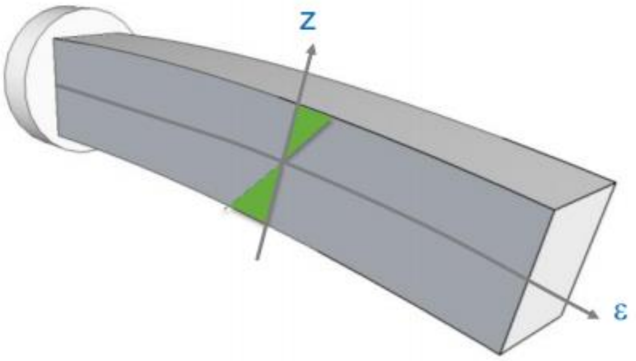


FIG 17 CURVATURE UNDER THE INFLUENCE OF BENDING;RESULTING STRAIN PATTERN

When the plies are stacked, the stresses can be calculated in each ply as a function of the external load and a combination of the (transformed) ply properties – at least if the external load is a normal load in parallel to the plane of the laminate. To this end, the stiffness matrix of the laminate is determined from the stiffness matrices of the plies. In order to calculate the response at a bending moment as well, curvature of the plate must be introduced. This curvature is in fact again translated back to the strain values of the plies: plies that lie further from the neutral – elastic – line, stretch further than the plies that are near it (see Figure 17). The curvature here is curvature under the influence of load, not curvature that was present beforehand.

The matrix that shows the relationship between external perpendicular and moment loads and the strains and curvatures in the laminate is called an ABD matrix. The 'A' part describes the strains ε in the plane of the laminate as a consequence of in-plane loads N (and vice versa); The 'B' part describes the strains in the plane as a consequence of bending moments M and the curvatures κ as a result of the in-plane loads. The D matrix describes the relationship between curvatures and bending moments. The terms that do not lie on the main diagonal describe the relationships between strains/curvatures and loads that do not lie mutually in the same direction. These are the coupling effects.

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \kappa \end{Bmatrix}$$

The CLT 'components' are discussed below in greater detail

3.4.3 CONSTITUTIVE EQUATION OF A PLY:

The relationship between stress and strain is called a 'constitutive' relationship. For a one-dimensional bar, this is simple and equal to Hooke's law. For an element in a flat panel of an anisotropic material that can be loaded in several directions, the constitutive relationship is somewhat more complicated. The constitutive equation of an orthotropic ply is as follows:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & 0 \\ -\frac{\nu_{21}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

Here the compliance matrix has been written out. You can write this in a similar form, specifying stress as a function of the strain:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} \frac{1}{1-\nu_{12}\nu_{21}}E_1 & \frac{\nu_{12}}{1-\nu_{12}\nu_{21}}E_2 & 0 \\ \frac{\nu_{12}}{1-\nu_{12}\nu_{21}}E_2 & \frac{1}{1-\nu_{12}\nu_{21}}E_2 & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

To do this, you must invert the compliance matrix to the stiffness matrix.

3.4.4 POSITIONING OF PLY IN LAMINATE – THE TRANSFORMATION MATRIX:

There are two variants of the transformation matrix: one for stress and the other for strains. The formulation you should use depends on whether the constitutive equations of a ply have been written in terms of a stiffness matrix or a compliance matrix.

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} cos^2\theta & sin^2\theta & 2sin\theta cos\theta \\ sin^2\theta & cos^2\theta & -2sin\theta cos\theta \\ -sin\theta cos\theta & sin\theta cos\theta & cos^2\theta - sin^2\theta \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

for stresses or, simplified:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = [T]_\sigma \begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}$$

You can use this equation to calculate the stresses in the main direction of the ply as a function of the rotation in the plane of the laminate of an angle θ. For strains:

$$\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \frac{1}{2}\gamma_{12}\end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \frac{1}{2}\gamma_{xy}\end{Bmatrix} \rightarrow$$

Using the preceding formulas, you can now provide the relationship between stresses and strains for a single ply – at an arbitrary angle relative to the laminate stress directions (in the global laminate axis system) – by starting with the constitutive equation for a ply:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix} = [E] \begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix}$$

and ‘rotating’ this by means of the transformation matrices to obtain the constitutive equation of that same ply in a global axis system:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = [T]_\sigma^{-1}[E][T]_\varepsilon \begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = [\bar{E}] \begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}$$

3.4.5 STACKING PLIES – INTRODUCTION TO CURVATURE:

The relationship between the strains and stresses in a ply is thus known in an arbitrary axis system (x, y). When a number of plies is stacked to form a laminate, the external forces in the plane will have to be in equilibrium with the total ply forces. The elongations in a ply are jointly determined by the curvature of the laminate. For equilibrium between internal and external forces, the following then applies:

$$[N] = A\{\varepsilon\} + B\{\kappa\}$$

Here N represents the external load (line load), A represents the ply stress (line stress) as a result of the elongations in the plane and B represents the contribution of the curvature of the laminate (κ). A similar relationship as above can be written for the equilibrium between external moments and internal strains:

$$[M] = B\{\varepsilon\} + D\{\kappa\}$$

3.4.6 THE ABD MATRIX:

Combination provides the simplified ABD matrix:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \kappa \end{Bmatrix}$$

of in extended form:

$$\begin{Bmatrix} n_x \\ n_x \\ n_{xy} \\ m_x \\ m_y \\ m_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} \\ A_{21} & A_{22} & A_{23} & B_{21} & B_{22} & B_{23} \\ A_{31} & A_{32} & A_{33} & B_{31} & B_{32} & B_{33} \\ B_{11} & B_{12} & B_{13} & D_{11} & D_{12} & D_{13} \\ B_{21} & B_{22} & B_{23} & D_{21} & D_{22} & D_{23} \\ B_{31} & B_{32} & B_{33} & D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_x \\ \gamma_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

3.5COMPOSITES USED IN THIS STUDY:

3.5.1CARBON / EPOXY RESIN:

Carbon fibre reinforced composites have exceptional mechanical properties. These strong, stiff and lightweight materials are an ideal choice for applications where lightweight & superior performance are important, such as components for aircraft, automotive, rail and high quality consumer products.

Composite materials are produced by combining a reinforcing fibre with a resin matrix system such as epoxy. This combination of fibre and resin provides characteristics superior to either of the materials alone and are increasingly being used as replacements for relatively heavy metallic materials. In a composite material, the fibre carries the majority of the load and is the major contributor to the composite material properties. The resin helps to transfer load between fibres, prevents them from buckling and binds the materials together. The range offered is based upon composite sheets produced by stacking carbon fibre fabrics one upon another and then infusing the stack with resin under vacuum. This process produces sheets with one smooth glossy resin rich side and the other rougher side showing the fabric weave detail.

Carbon fibres are produced from polymer fibres such as polyacrylonitrile and from pitch. The initial fibre material is drawn under tension whilst it is heated to around 1000°C causing 2 dimensional carbon-carbon crystals (graphite) to be formed when hydrogen is driven out. The carbon-carbon chain has extremely strong molecular bonds and this is what gives the fibres their high strength.

3.5.2 E-GLASS / EPOXY RESIN:

Glass/epoxy UDF laminates are used in various industries like aerospace, automotive and marine due to their appealing factors such as strength-to weight ratio, fatigue properties and corrosion resistance. The laminates that are used in the above mentioned industries undergo a variety of loadings and damage. When the fibre-reinforced composites undergo damage, two kinds of approaches are followed depending upon the severity of damage:

1. Replacing the damaged structure.

2. Repairing the damaged structure.

Of these two, the former is a costlier process, and the latter is an economical one and needs more theoretical knowledge on materials, understanding of magnitude of loads at the local damaged zone and suitable repair practices. As this study would be a preliminary contribution towards optimum usage of π/4 quasiisotropic E-glass/epoxy laminate repair practices, it is important to understand the material properties. Therefore, the tensile properties of UDF E-Glass/ epoxy laminate have been studied experimentally. In order to understand the tensile properties of E-glass/epoxy laminate, many similar studies have been carried out and reported. Moreover, ASTM (American Society for Testing and Materials) described the standard test method for tensile properties of polymer matrix composite materials in the specification of ASTM D 3039/ D3039M-001 .

CHAPTER-4

MODELLING USING ANSYS 18.0

4.1 INTRODUCTION

ANSYS (acronym for Analysis System) is a general purpose Finite Element Analysis (FEA) program that solves a vast area of solid and structural mechanics problems in geometrically complicated regions. In the present work, ANSYS 18.0 is used to model the plate, to compute natural frequencies and to plot deformed shapes.

In the following sub-sections, details of the modelling are presented. First, some terms related to this topic are explained. Then, the procedure of modelling is presented.

4.2 TERMINOLOGIES

Shell 281: Shell 281 is used as an element type for thin to moderately thick shell structures. It has eight nodes with six degrees of freedom at each node: translations in the x, y, and z axes, and rotations about the x, y, and z-axes. It follows first order shear deformation theory. The geometry, nodes and co-ordinate system of a shell element is shown below .

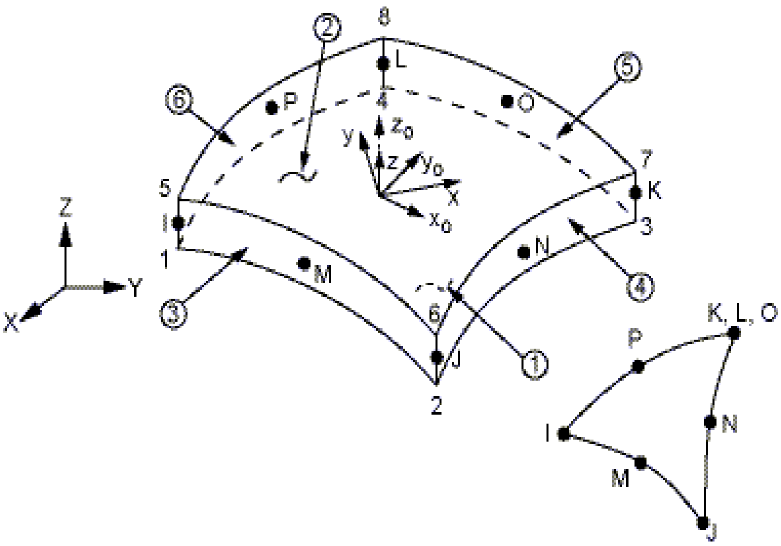


FIG 18 SHELL 281 ELEMENT

SHELL281 Input Summary

Nodes

I, J, K, L, M, N, O, P

Degrees of Freedom

UX, UY, UZ, ROTX, ROTY, ROTZ if KEYOPT(1) = 0

UX, UY, UZ if KEYOPT(1) = 1

SHELL281 may be used for layered applications for modeling composite shells or sandwich construction. The accuracy in modeling composite shells is governed by the first-order shear-deformation theory.

4.3 FIRST ORDER SHEAR DEFORMATION THEORY

Reissner and Mindlin [29, 32] is a well-known theory for the analysis of composite structures. This theory is also known as first order shear deformation theory (FSDT) and takes the displacement field as linear variations of mid plane displacements. Here the relation between the resultant shear forces and the shear strains is affected by the shear correction factors. This theory has some advantages as its simplicity and low computational cost.

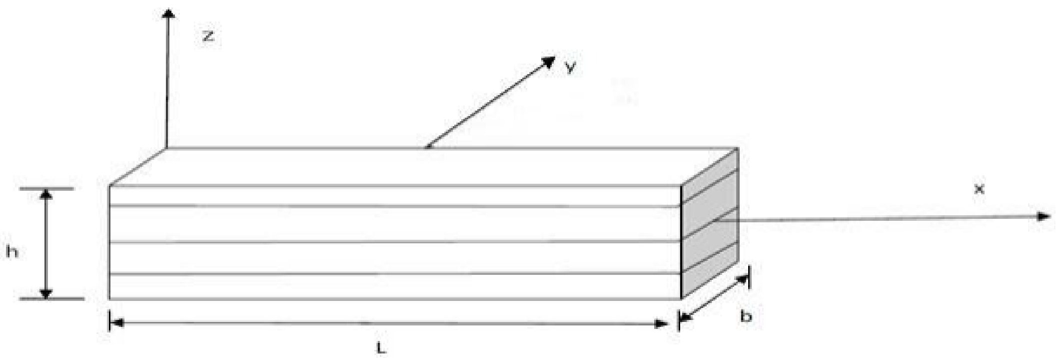


FIG 19 COMPOSITE BEAM IN FIRST ORDER SHEAR DEFORMATION THEORY

Here, L, b and h are length, breadth and thickness of the laminated composite beam.

To approximate a 3D elasticity problem into a 2D beam problem, the displacement functions u, v and w of the laminate at a point x, y and z are expanded in a Taylor series in terms of thickness co-ordinate as [1]. Where the displacement functions u, v and w can be written as

$u = u_0(x,y) + z\, \Theta_x(x,y)$

$v = v_0(x,y) + z\, \Theta_y(x,y)$ and

$w = w_0(x,y)$ respectively

Here u, v and w are in-plane and transverse displacement components at any point along the x, y and z-axis in the laminate. The displacements along x, y and z axis are u0, v0 and w0 respectively. Also middle plane slopes are θ_x and θ_y.

4.3.1 ELASTICITY MATRIX (D)

Strain – displacement relations for the lamina are

$\varepsilon_x = \varepsilon_{x0} + zk_x$, $\varepsilon_y = \varepsilon_{y0} + zk_y$,

$\gamma_{xy} = \varepsilon_{xy0} + zk_{xy}$, $\gamma_{yz} = \varepsilon_{yz0}$, $\gamma_{xz} = \varepsilon_{xz0}$

Stress-strain relations for a lamina with respect to the fiber-matrix co-ordinate axis (1, 2, 3) are

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \\ \tau_{23} \\ \tau_{13} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \\ \gamma_{23} \\ \gamma_{13} \end{Bmatrix}$$

Here (σ_1, σ_2, τ_{12}, τ_{23}, τ_{31}) are the stresses and (ε_1, ε_2, γ_{12}, γ_{23}, γ_{31}) are the strain components corresponding to the lamina co-ordinates (1, 2, 3). Cij is the compliance matrix with respect to lamina axis (1, 2, 3) and is defined in appendix A. In laminate co-ordinates (x, y, z) the stress-strain relations for the lamina are given as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{14} & 0 & 0 \\ Q_{12} & Q_{22} & Q_{24} & 0 & 0 \\ Q_{14} & Q_{24} & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & KQ_{55} & KQ_{56} \\ 0 & 0 & 0 & KQ_{56} & KQ_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}.$$

Here (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{zx}) are the stresses and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{zx}) are the strain components corresponding to the laminate co-ordinates (x, y, z). Q_{ij}'s are the transformed elasticity constants.

Here K is shear correction factor [34]. It appears as a coefficient in the expression for the transverse shear stress resultant to consider the shear deformation effect with good approximation. Due to low shear modulus in multilayered plate and shell finite elements, there is an appreciable constant shear deformation. As the transverse shear stresses are zero at top and bottom faces and maximum at neutral axis. So the constant shear distribution across the thickness causes a decrease in accuracy. So, one has to multiply shear correction factor with transverse shear stress components. Numerical value of K depends upon Poisson's ratio, shape of the cross section and ply angle for the composite beam. Shear correction factor considers the effect of extension-shear coupling. Shear correction factor make neutral axis of the beam coincide with its geometric axis. From here the elasticity matrix [D] is derived as

Here [T] is the thickness co-ordinate matrix. So, D-matrix for FSDT can be written as

$$[D]=\begin{bmatrix} A_{ij} & B_{ij} & 0 \\ B_{ij} & D_{ij} & 0 \\ 0 & 0 & AA_{ij} \end{bmatrix}.$$

Here

$$\left(A_{ij}, B_{ij}, D_{ij}\right)=\int_{-h/2}^{h/2} Q_{ij}\left(1, Z, Z^2\right) dZ \rightarrow i, j=1,2,4 \text{ and}$$

$$\left(AA_{ij}\right)=\int_{-h/2}^{h/2} Q_{ij}(1) dZ \rightarrow i, j=5,6.$$

Here [Aij] is extensional stiffness matrix, [Bij] is stretching-bending coupling matrix and [Dij] is flexural stiffness matrix.

4.3.2 STRAIN-DISPLACEMENT MATRIX

The strain-displacement relation is

{ε} = [B]{δ}.

Here [L] is operator matrix, {ε} is the in-plane strain matrix and {δ} is the displacement at any point on the mid-plane of the element. The strain-displacement matrix [B] is obtained by multiplying operator matrix by shape functions as

$[B]=[L]*N_r = \rightarrow 1,2,3,4...9$.

So, the B-matrix for FSDT can be given by

$$[B]=\begin{bmatrix} \frac{\delta N_r}{\delta x} & 0 & 0 & 0 & 0 \\ 0 & \frac{\delta N_r}{\delta y} & 0 & 0 & 0 \\ \frac{\delta N_r}{\delta y} & \frac{\delta N_r}{\delta x} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\delta N_r}{\delta x} & 0 \\ 0 & 0 & 0 & 0 & \frac{\delta N_r}{\delta y} \\ 0 & 0 & 0 & \frac{\delta N_r}{\delta y} & \frac{\delta N_r}{\delta x} \\ 0 & 0 & \frac{\delta N_r}{\delta y} & 0 & N_r \\ 0 & 0 & \frac{\delta N_r}{\delta x} & N_r & 0 \end{bmatrix}.$$

4.4 GOVERNING EQUATIONS

The mathematical modelling in ANSYS is generally based on the concept of the FSDT and it is demonstrated as follows:

$$U'(X',Y',Z') = U0'(X',Y') + Z'\Theta X'(X',Y')$$

$$V'(X',Y',Z') = V0'(X',Y') + Z'\Theta X'(X',Y')$$

$$W'(X',Y',Z') = W0'(X',Y') + Z'\Theta X'(X',Y')$$

The displacements are presented and derived in terms of Shape Functions (Ni).

$\delta = \sum Ni\ \delta i$, i varies from 1 to j

where $\delta i = [U'0i\ V'0i\ W'0i\ \phi X'i\ \phi Y'i\ \phi Z'i]^T$

The shape functions for eight-noded shell element are as stated:

N1 = 1 4 $(1 - \varepsilon)(1 - n)(-\varepsilon - n - 1)$

N2 = 1 4 $(1 + \varepsilon)(1 - n)(\varepsilon - n - 1)$

N3 = 1 4 $(1 + \varepsilon)(1 + n)(\varepsilon + n - 1)$

N4 = 1 4 $(1 - \varepsilon)(1 + n)(-\varepsilon + n - 1)$

N5 = 1 2 $(1 - \varepsilon\ 2\ )(1 - n)$

N6 = 1 2 $(1 + \varepsilon)(1 - n\ 2\ )$

N7 = 1 2 $(1 - \varepsilon\ 2\ )(1 + n)$

N8 = 1 2 $(1 - \varepsilon)(1 - n\ 2\ )$

The above equations are based on natural coordinates and U', V', W' represent the displacement of any point along X', Y', Z' coordinate axes.

Strains along different axes are determined by derivation of displacements along respective directions.

The strain vector expressed in terms of nodal displacement vector is as follows:

$\{ \varepsilon \} = [B] \{ \delta \}$, where

[B] is the strain displacement matrix comprising of interpolation functions and their derivatives $\{ \delta \}$ is the nodal displacement vector .

4.5 MODAL ANALYSIS

Modal analysis is the study of the dynamic properties of systems in the frequency domain. A typical example would be testing structures under vibrational excitation.

Modal analysis is the field of measuring or calculating and analyzing the dynamic response of structures and/or fluids or other systems during excitation. Examples would include measuring the vibration of a car's body when it is attached to an electromagnetic shaker, analysis of unforced vibration response of vehicle suspension,[2] or the noise pattern in a room when excited by a loudspeaker. Modern day experimental modal analysis systems are composed of 1) sensors such as transducers (typically accelerometers, load cells), or non contact via a Laser vibrometer, or stereophotogrammetric cameras 2) data acquisition system and an analog-to-digital converter front end (to digitize analog instrumentation signals) and 3) host PC (personal computer) to view the data and analyze it.

In structural engineering, modal analysis uses the overall mass and stiffness of a structure to find the various periods at which it will naturally resonate. These periods of vibration are very important to note in earthquake engineering, as it is imperative that a building's natural frequency does not match the frequency of expected earthquakes in the region in which the building is to be constructed. If a structure's natural frequency matches an earthquake's frequency, the structure may continue to resonate and experience structural damage. Modal analysis is also important in structures such as bridges where the engineer should attempt to keep the natural frequencies away from the frequencies of people walking on the bridge. This may not be possible and for this reasons when groups of people are to walk along a bridge, for example a group of soldiers, the recommendation is that they break their step to avoid possibly significant excitation frequencies. Other natural excitation frequencies may exist and may excite a bridge's natural modes. Engineers tend to learn from such examples (at least in the short term) and more modern suspension bridges take account of the potential influence of wind through the shape of the deck, which might be designed in aerodynamic terms to pull the deck down against the support of the structure rather than allow it to lift. Other aerodynamic loading issues are dealt with by minimizing the area of the structure projected to the oncoming wind and to reduce wind generated oscillations of, for example, the hangers in suspension bridges.

Once a set of modes has been calculated for a system, the response at any frequency (within certain bounds) in response to many inputs at many points with different time histories can be calculated by superimposing the result from each mode. This assumes the system is linear.

Modal analysis in ANSYS is a linear analysis. Several mode extraction methods, e.g. Block Lanczos, Supernode, PCG Lanczos, reduced, unsymmetric, damped, and QR damped are available. Block Lanczos is used in the present analysis. It is used for large symmetric eigenvalue problems. This method uses the sparse matrix solver. Similarly all the methods have their own limitations. For more information on the methods of mode extraction, the reader may follow ANSYS Technical Guide.

4.6 PROCEDURE

The following flow chart shows an overview of the steps to be followed in ANSYS.

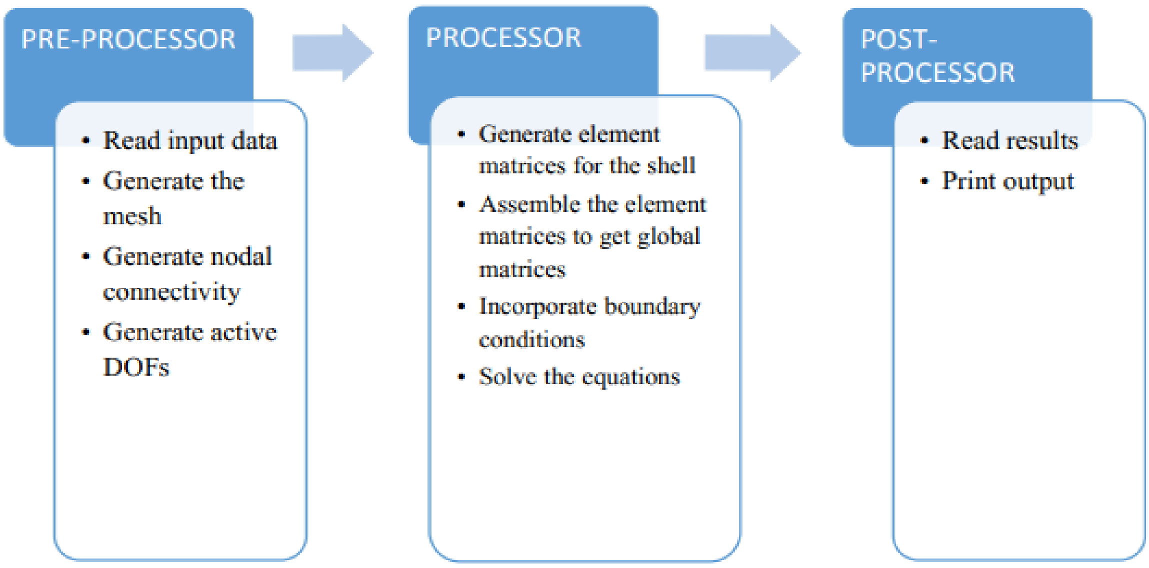
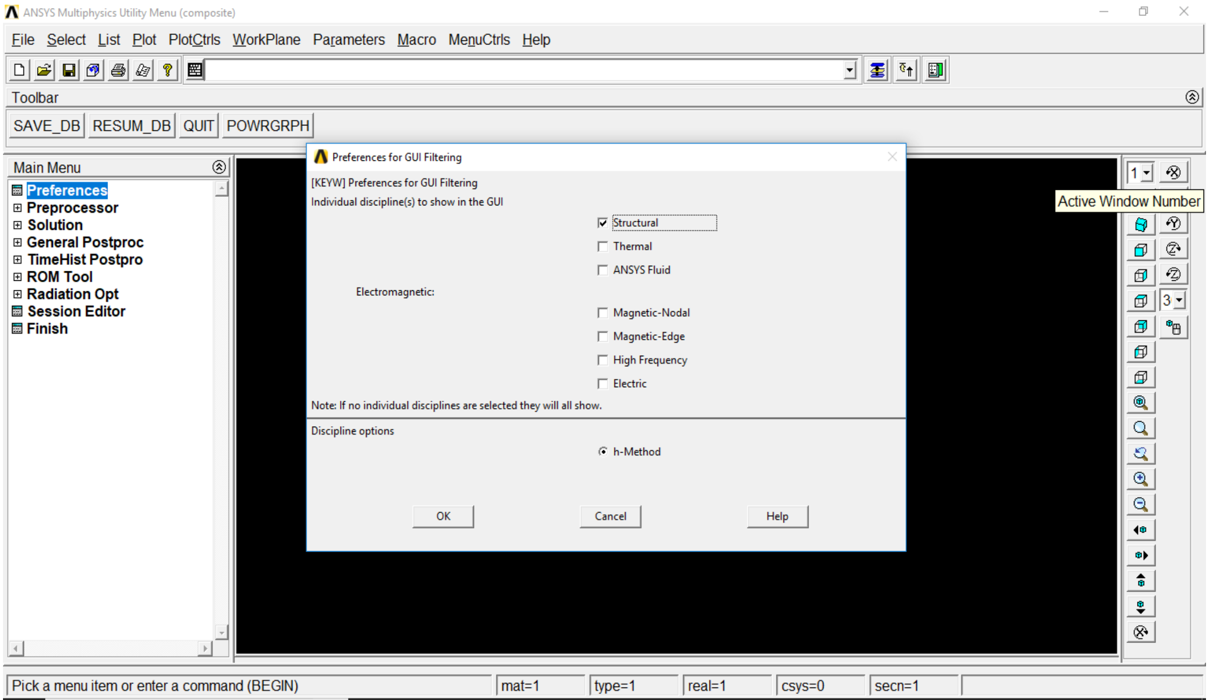


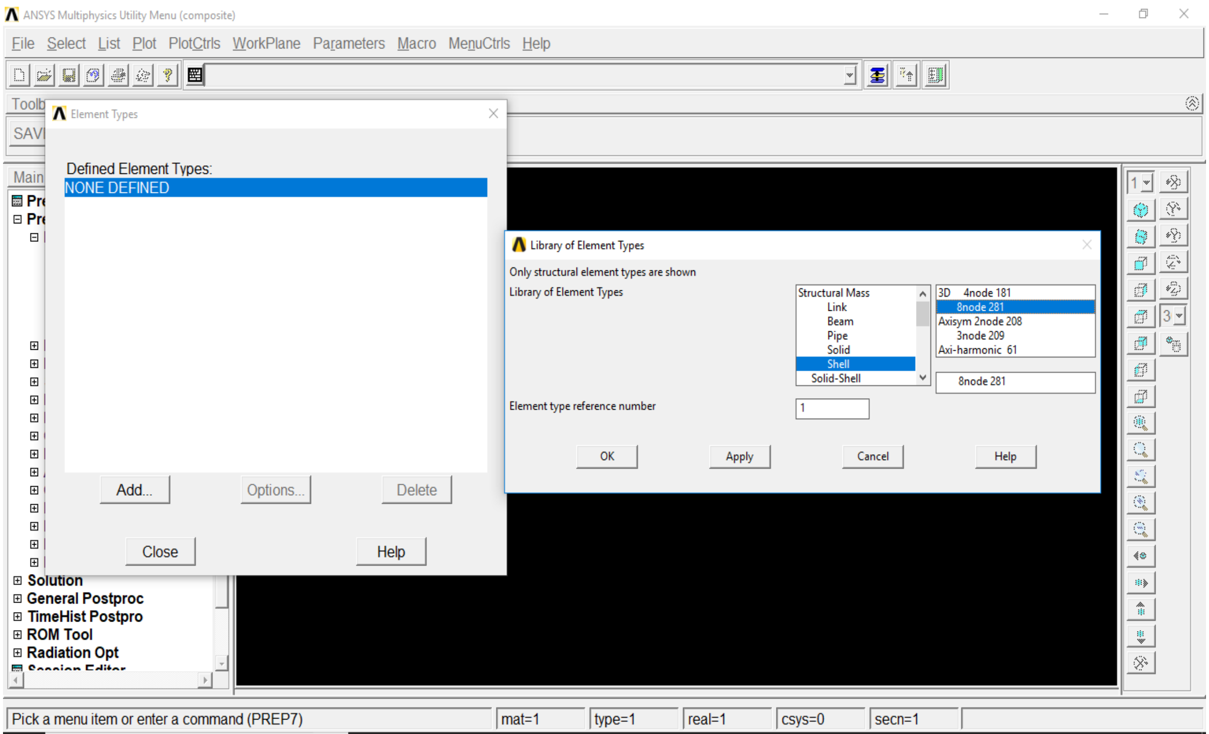
FIG 20 FLOW CHART FOR MODELING COMPOSITE PLATE IN ANSYS

4.6.1 PREPROCESSOR

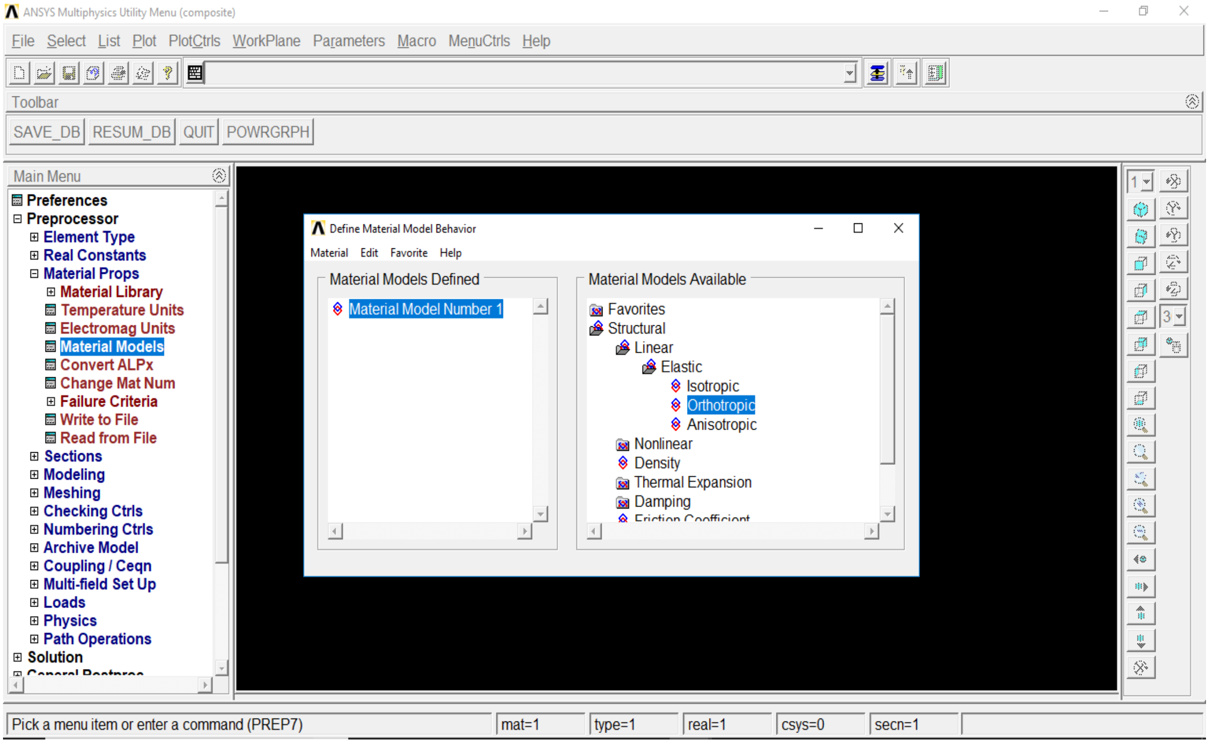
It involves the following steps. The following tabs can be found in 1.Utility menu>Preferences> Structral>ok.



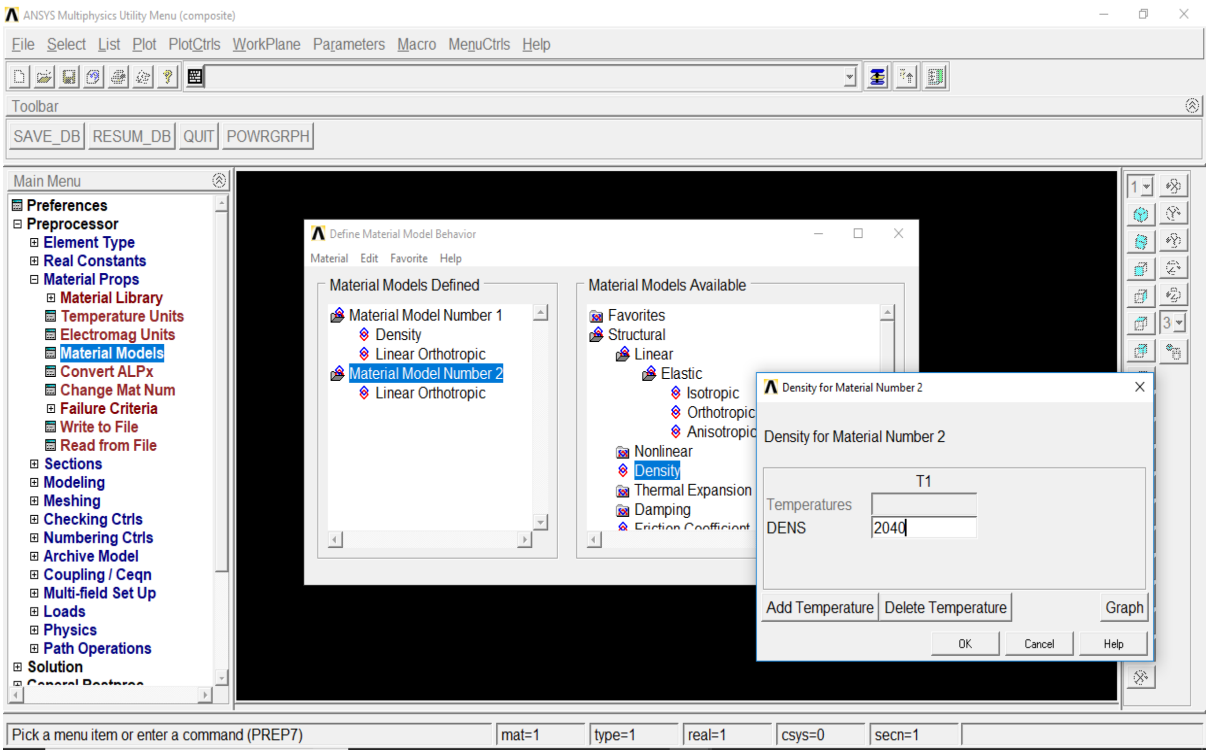
2.Utility Menu > Preprocessor>Element Type > Add > Structural Mass > Shell > 8 node 281



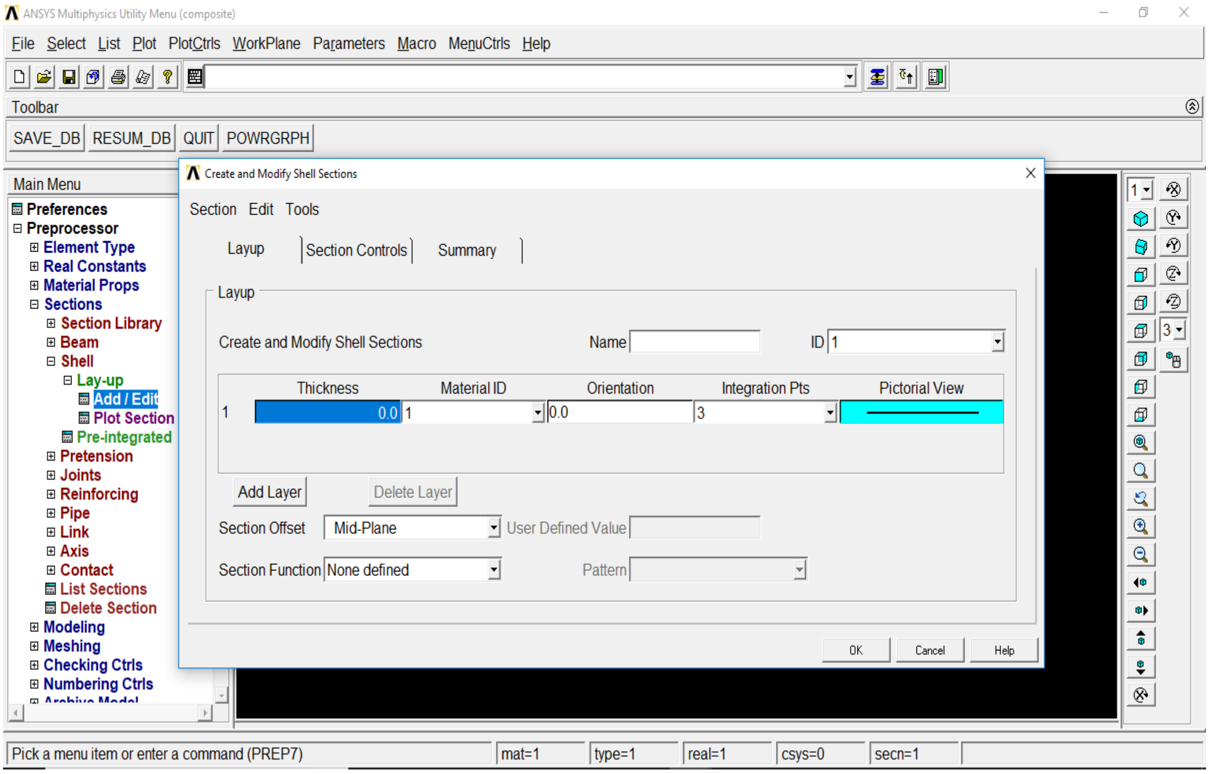
3. Material models > Material properties > Structural > Linear > Elastic > Orthotropic – Enter the linear orthotropic properties



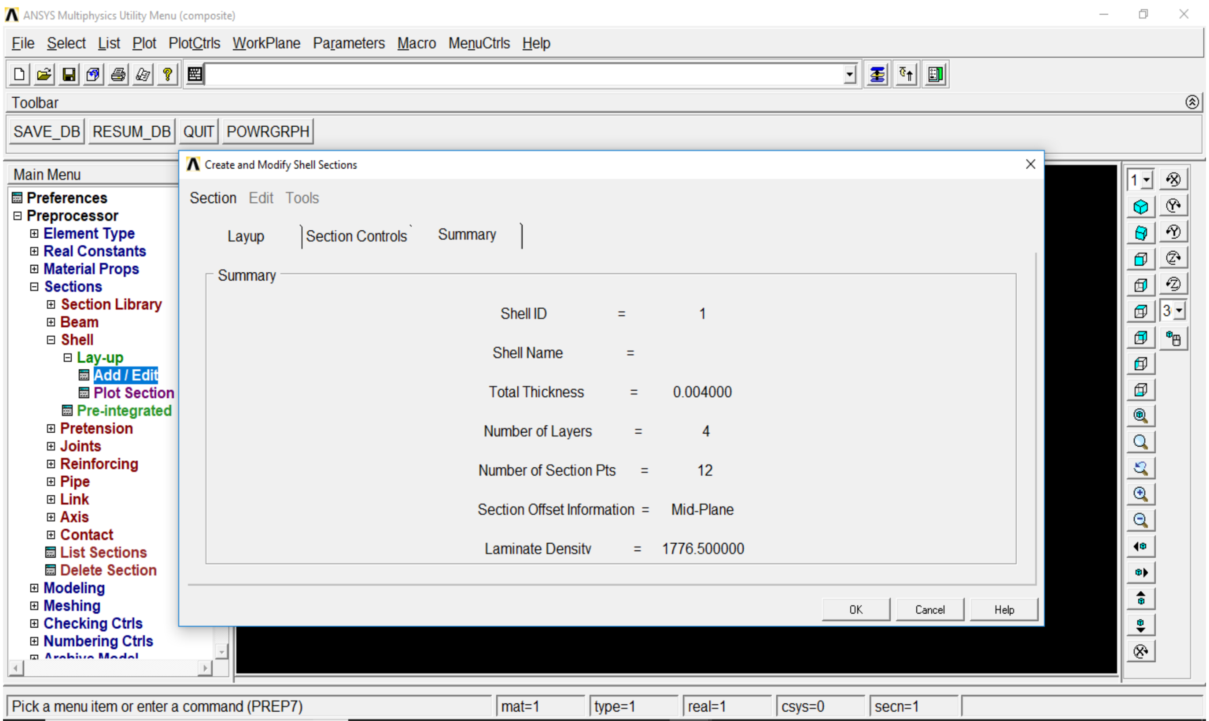
4. Material models > Density > Enter the value of density



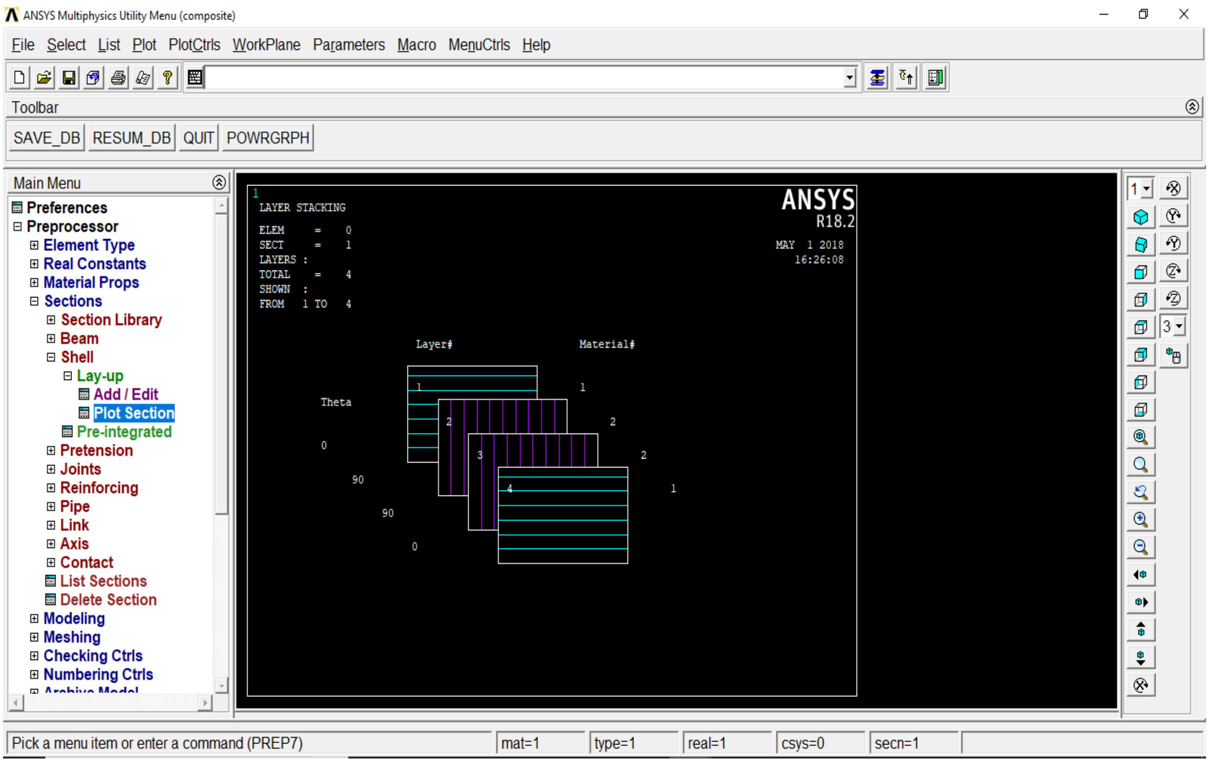
5. Sections > Shell > Lay-up > Add/Edit > Enter thickness and fiber orientation of each layer



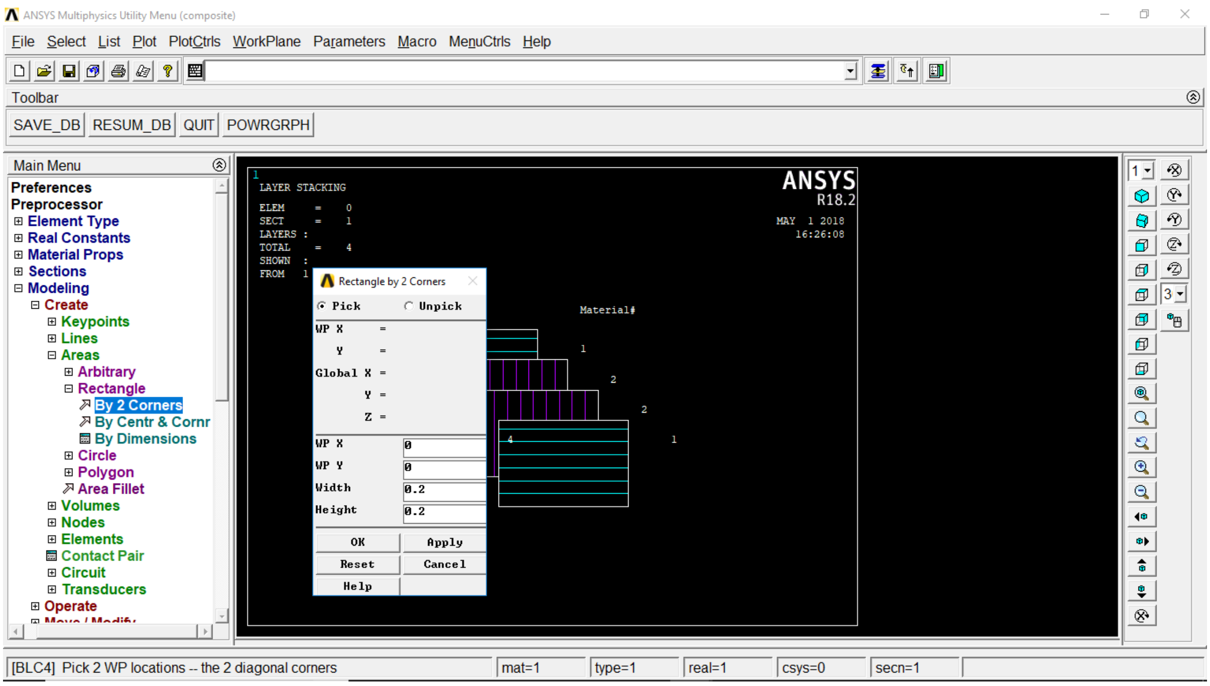
Enter the thickness of each layer,no of layers,orientation of fibers,and verify by clicking on summary



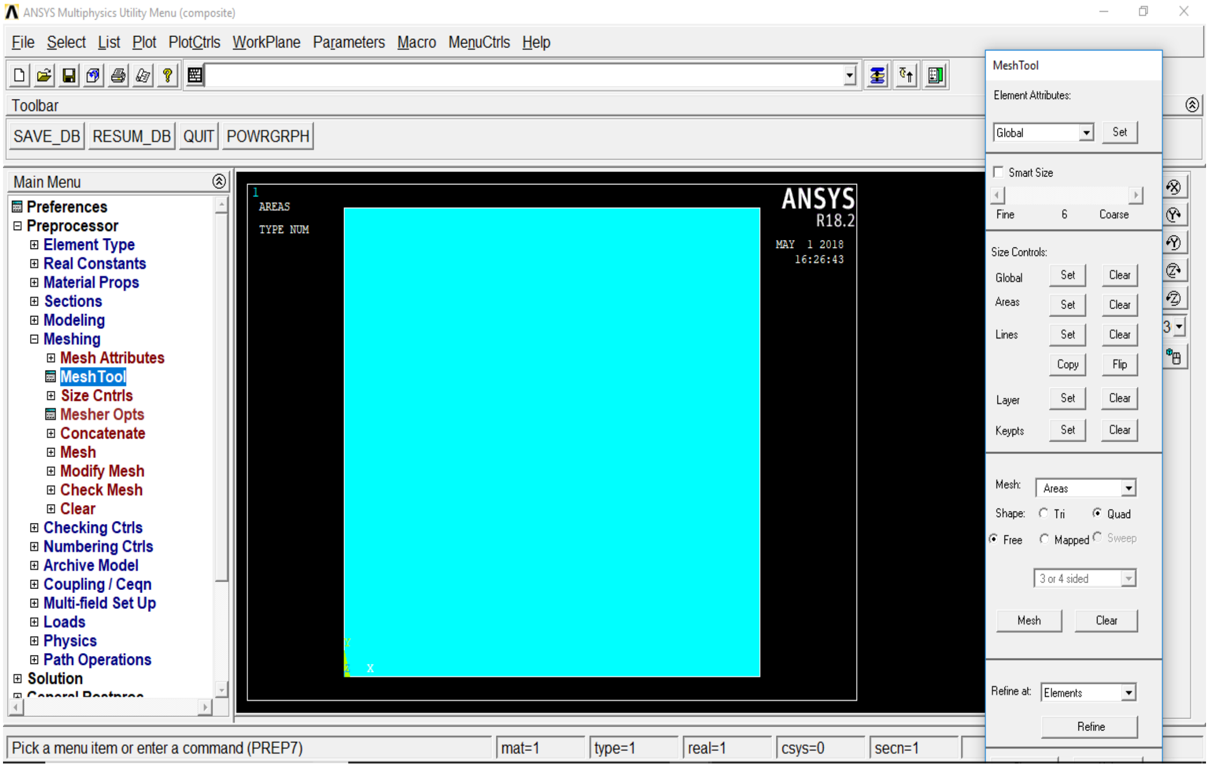
6. Sections > Shell > Lay-up > plot sections > Enter layer range LAY1,LAY2



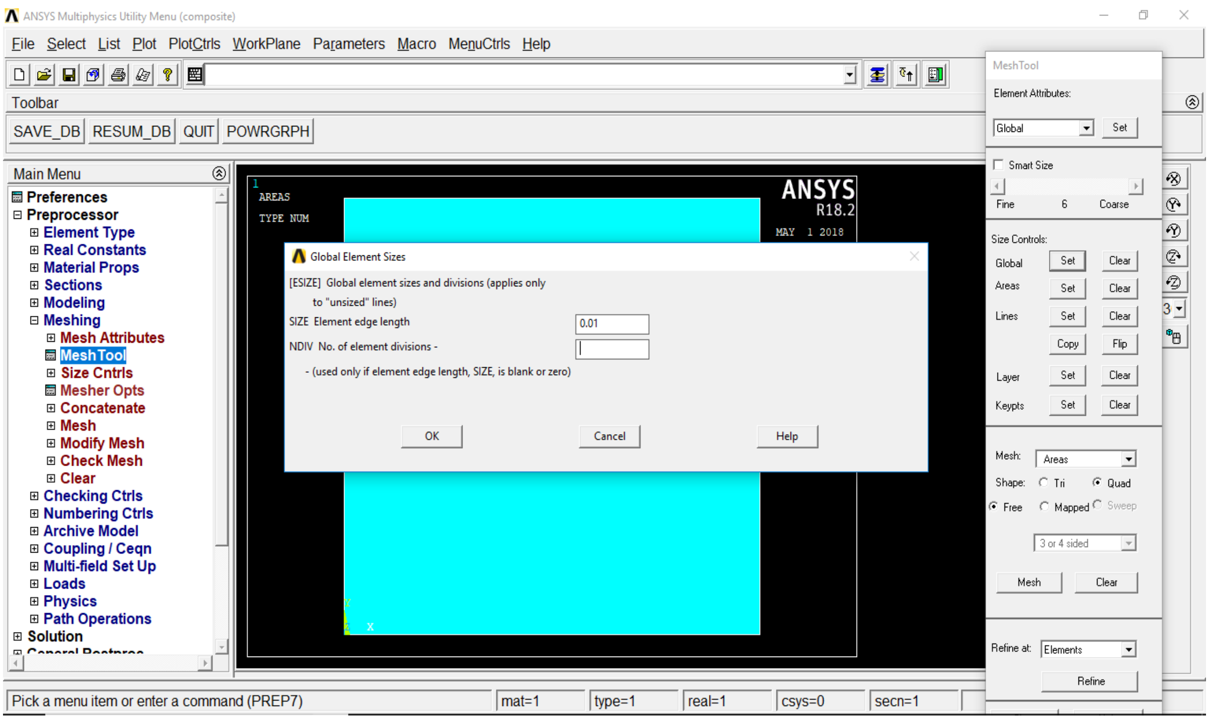
7. Modelling > Create > Areas > Rectangle > By dimension - Enter the co-ordinates of the corner points (There are 3 possible ways in which rectangle can be created)



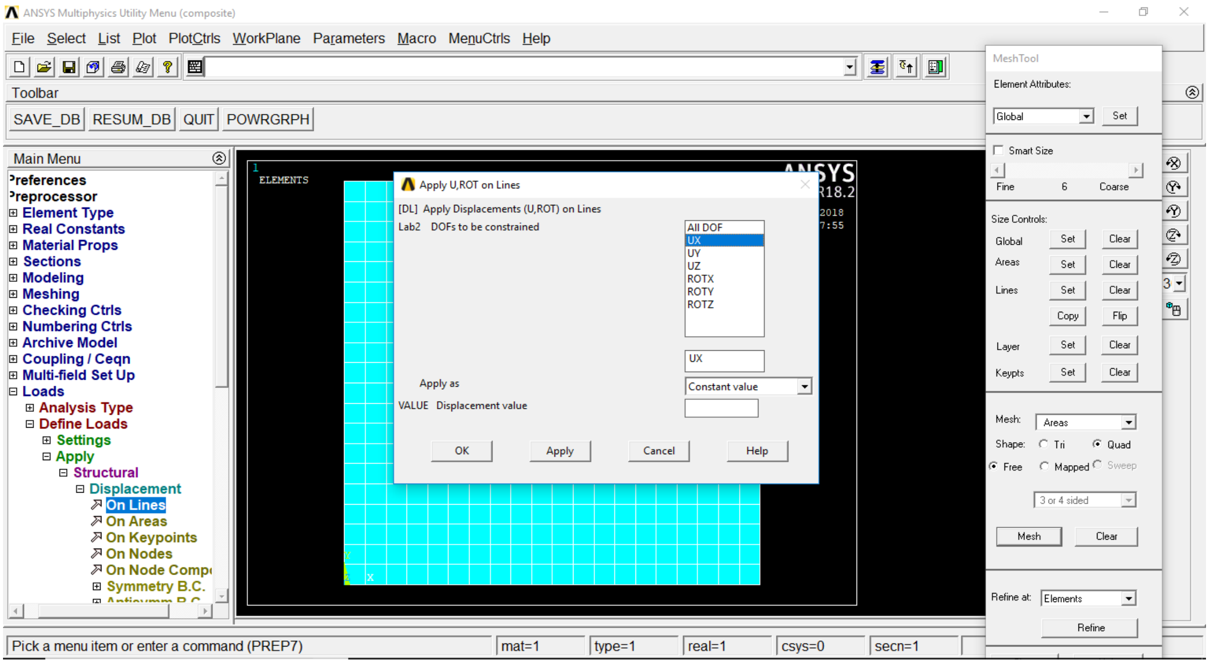
The plate is shown below



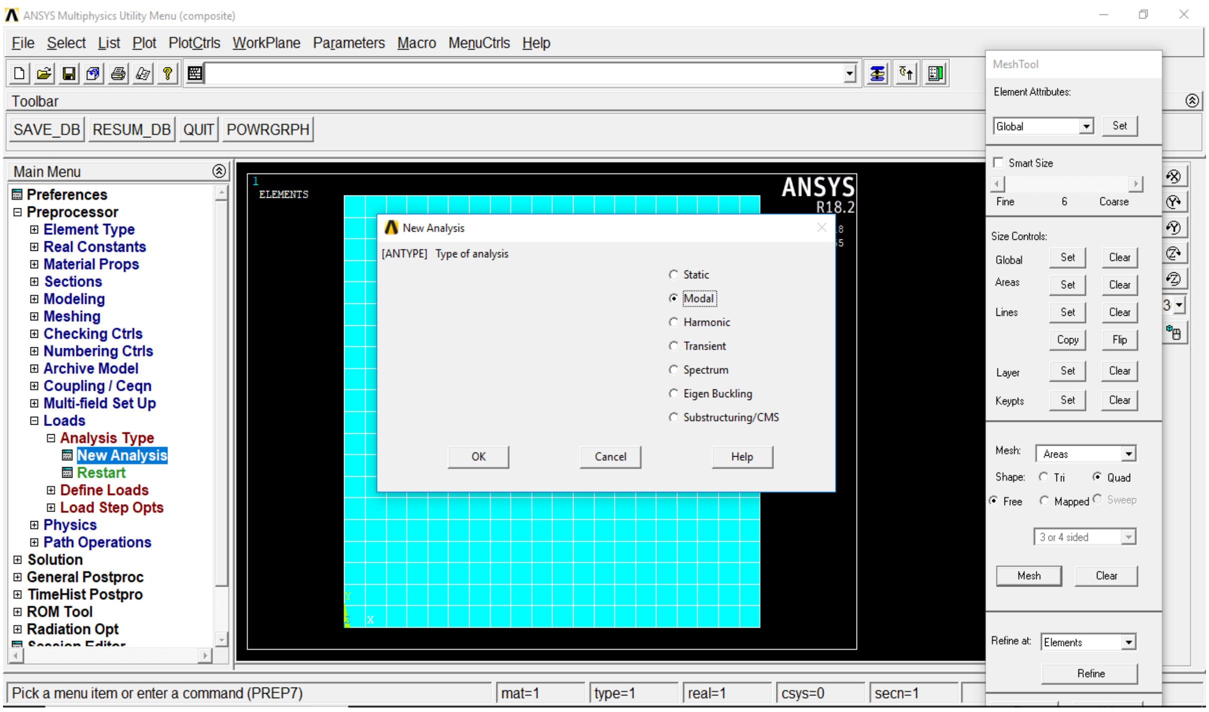
8. Meshing > Mesh Tools > Size controls : Global > Set > Enter number of divisions



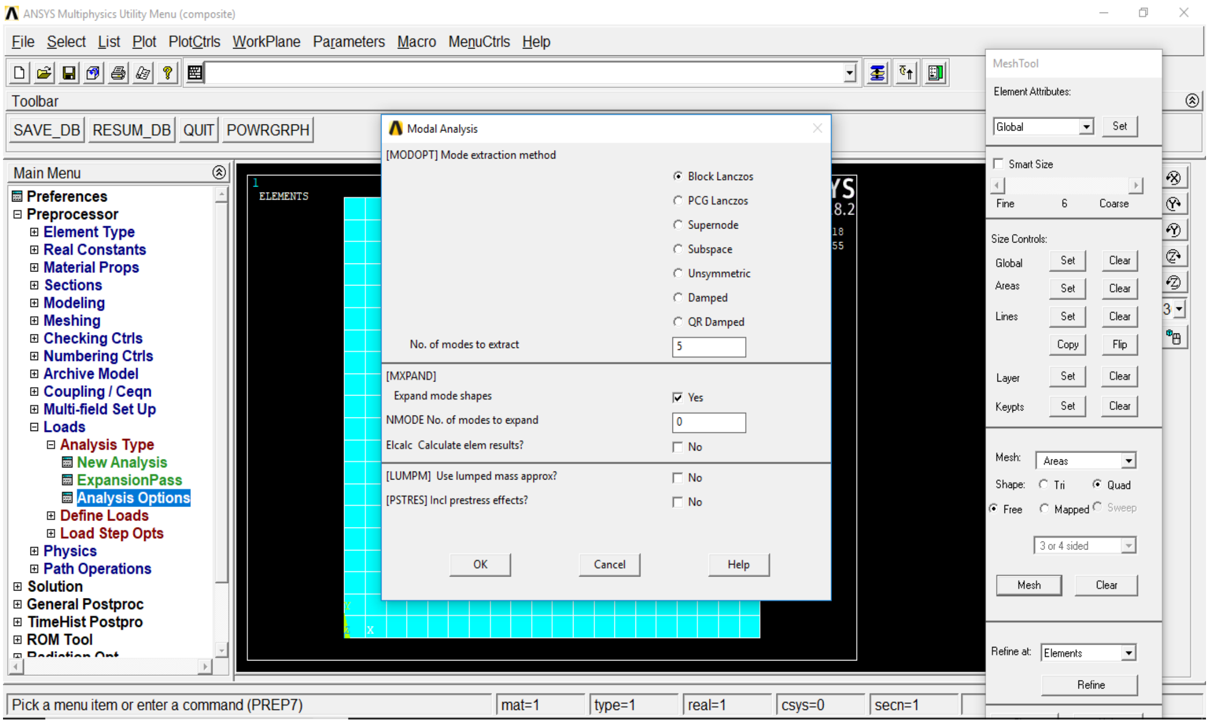
9. Loads > Define loads > Apply > Structural > Displacement > On lines > Pick lines > Apply displacement on lines



10. Loads > Analysis type > New analysis > Type of analysis : Modal



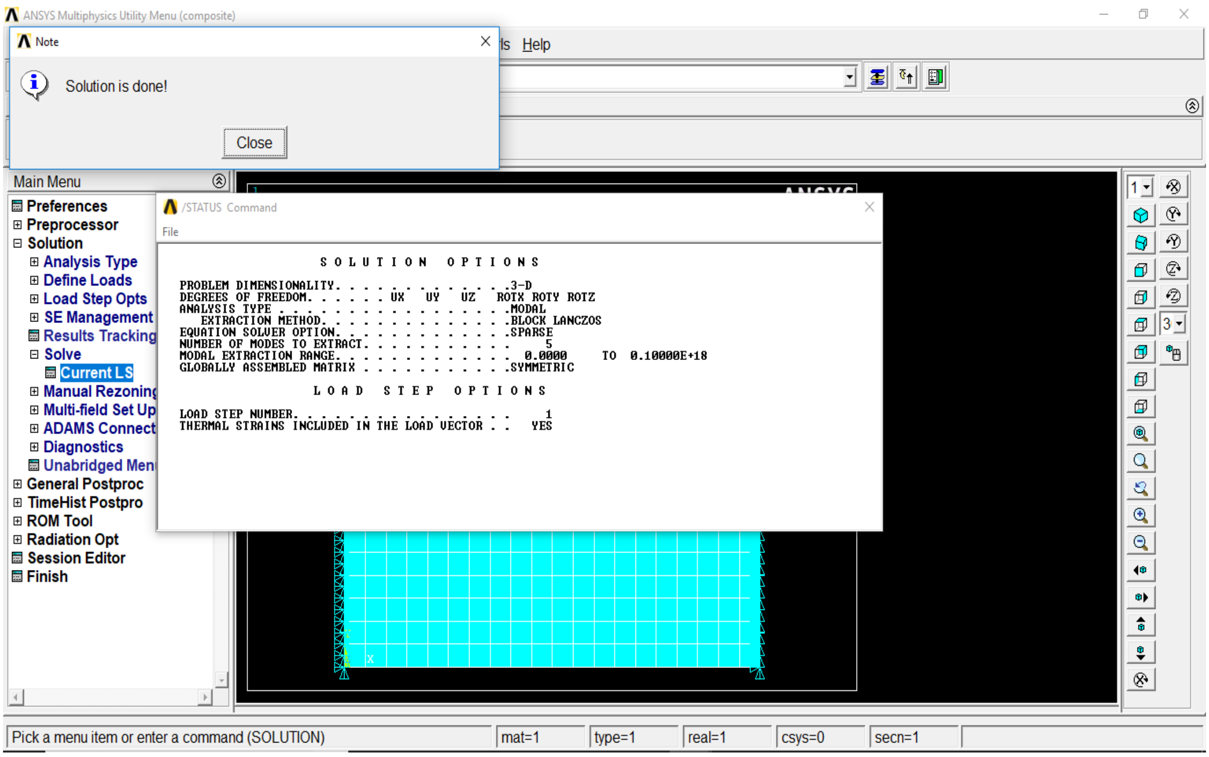
11. Loads > Analysis type > Analysis options > Mode-extraction method – Block Lanczos (select any one of the available options) > Number of modes to extract-5 (enter a number > 0)



4.6.2 PROCESSOR

It is the solution stage. The program solves the concerning equations using the input data provided in preprocessor stage.

1. Utility Menu > Solution > Solve > Current LS



4.6.3 POSTPROCESSOR

Here, the results can be viewed, mode shapes can be plotted. A number of forms of graphical representation of the results are available, including deformed shape, deformed + undeformed shape, contour plot, vector plot etc.

1. Utility Menu > General Postproc > Result Summary

2. Utility Menu > General Postproc > Plot Results > Deformed shape – Select the desired shape

CHAPTER-5

RESULTS AND DISCUSSION

An attempt has been made to study the free vibration characteristics of square laminated composite plates with cut-outs. For simplification of the analysis, the plate is assumed to be orthotropic and symmetric with respect to mid-plane. The effect of fibre volume ratio,stacking sequence,lay up,boundary conditions and fiber orientation on the natural frequencies of vibration is studied. As mentioned earlier, the studies have been done with the finite element package ANSYS 18.0.

5.1 EFFECTIVE PROPERTIES OF LAMINATE AT DIFFERENT FIBER VOLUME RATIO

In this section, Material properties are calculated using Halphin-Tsai equation for different fiber volume ratios.Two different laminas are used for this purpose.They are

1.Carbon Epoxy Lamina

2. E-glass Epoxy Lamina

These laminas are assumed to be transversely isotropic in nature.Hence,

$E_2=E_3$,$PR_{23}=PR_{13}$,$G_{23}=G_{13}$

5.1.1 CARBON EPOXY LAMINATE PROPERTIES

Matrix=Epoxy   Fiber=carbon

Youngs modulus of matrix E_m = 3.45Gpa

Shear modulus of matrix G_m=1.28Gpa

Poissions ratio of matrix PR_m=0.35

Youngs modulus of fiber E_f =220Gpa

Shear modulus of fiber G_f=9Gpa

Poissions ratio of fiber PR_f=0.2

TABLE 3 EFFECTIVE PROPERTIES OF CARBON EPOXY LAMINATE

Fiber Volume Ratio(V_f)	E_1(GPa)	E_2(GPa)	G_{12}(GPa)	PR_{12}
0.4	90.07	9.84	2.67	0.29
0.5	111.725	12.896	3.204	0.275
0.6	133.38	17.31	3.84	0.26

5.1.2 E-GLASS EPOXY PROPERTIES

Matrix=Epoxy Fiber=E-glass

Youngs modulus of matrix E_m = 3.45Gpa

Shear modulus of matrix G_m=1.28Gpa

Poissions ratio of matrix PR_m=0.35

Youngs modulus of fiber E_f =75Gpa

Shear modulus of fiber G_f=30Gpa

Poissions ratio of fiber PR_f=0.2

TABLE 4 EFFECTIVE PROPERTIES OF E-GLASS EPOXY LAMINATE

Fiber Volume Ratio(V_f)	E_1(GPa)	E_2(GPa)	G_{12}(GPa)	PR_{12}
0.4	32.07	9.00	3.37	0.29
0.5	39.22	13.27	4.3	0.275
0.6	46.38	14.85	5.59	0.26

5.2 MODE SHAPES OF A COMPOSITE

The following are the different mode shapes of a cross ply symmetric (0/90/90/0) at following parameters

Fiber volume ratio = 0.4

Fiber-orientation = 0°/90°/90°/0^o

Lay up = Cross ply

Boundary condition = Simply Supported

Stacking sequence = Carbon/E-Glass/E-Glass/Carbon

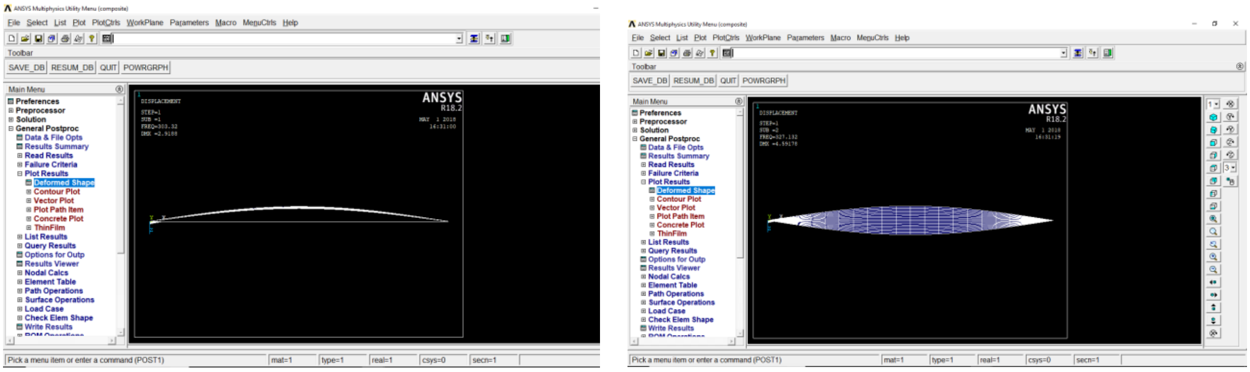


FIG 21 MODE SHAPE FOR 1ST AND 2ND NATURAL FREQUENCIES

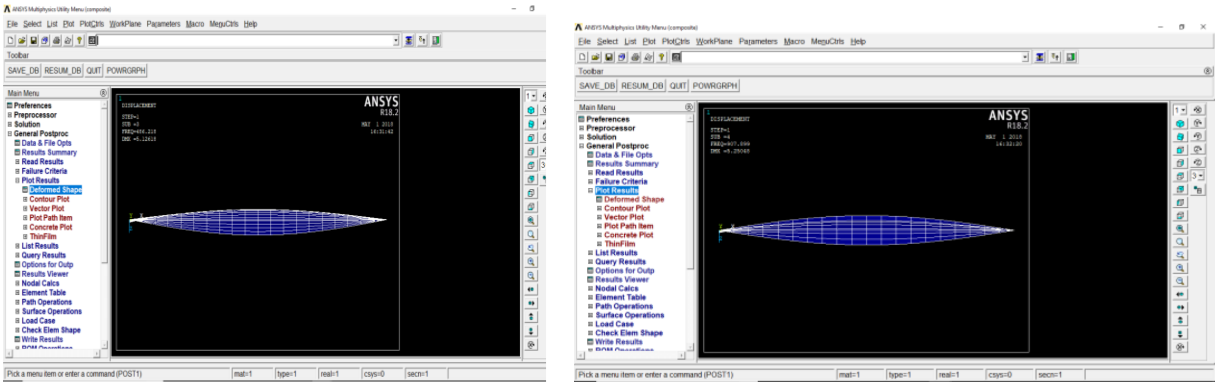


FIG 22 MODE SHAPE FOR 3RD AND 4TH NATURAL FREQUENCIES

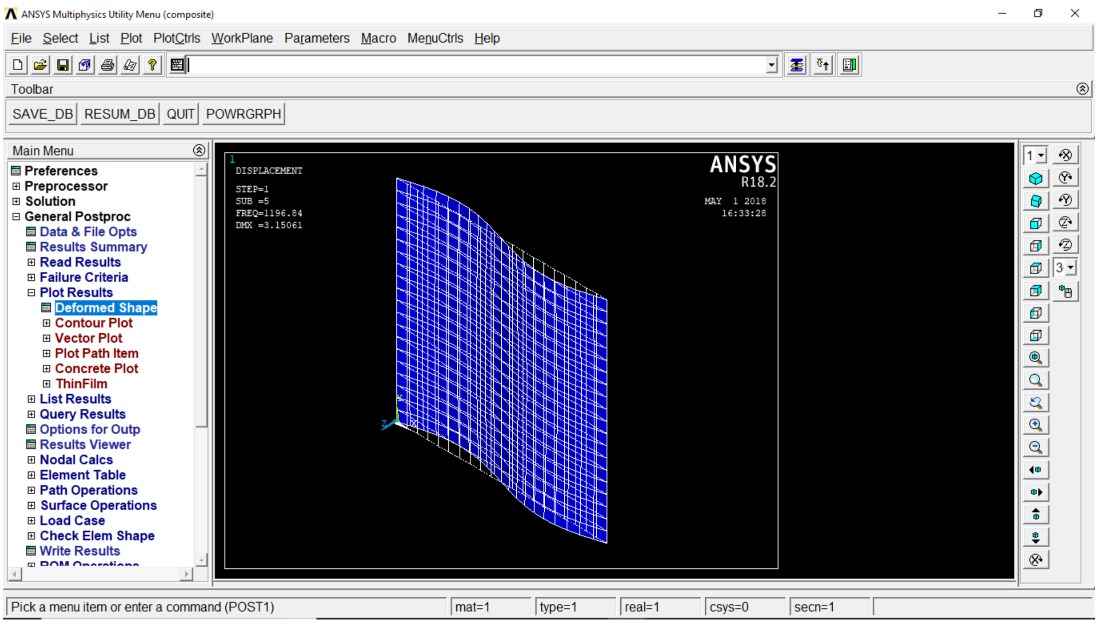


FIG 23 MODE SHAPE FOR 5TH NATURAL FREQUENCY

5.3 NODAL SOLUTION FOR DIFFERENT MODE SHAPES

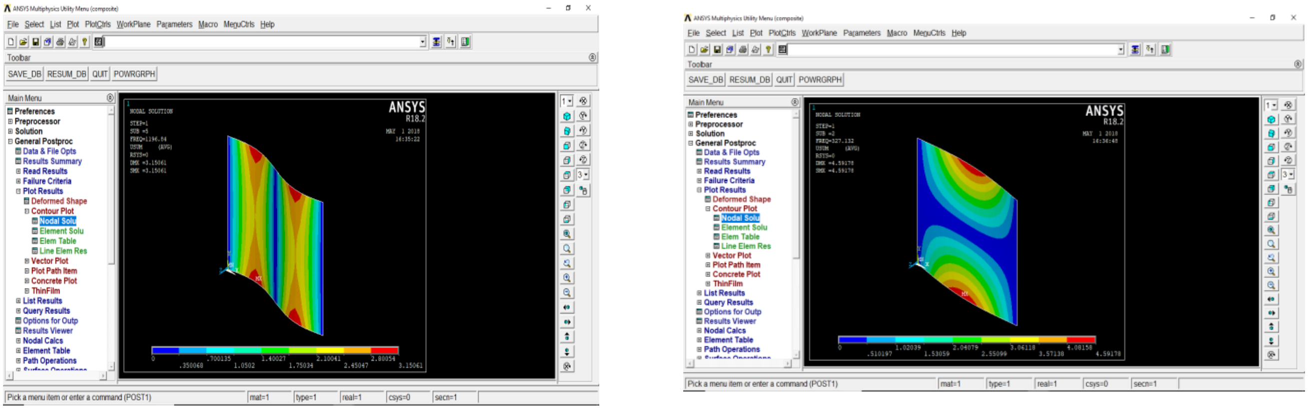


FIG24 NODAL DISPLACEMENT SOLUTION FOR 1ST AND 2ND NATURAL FREQUENCIES

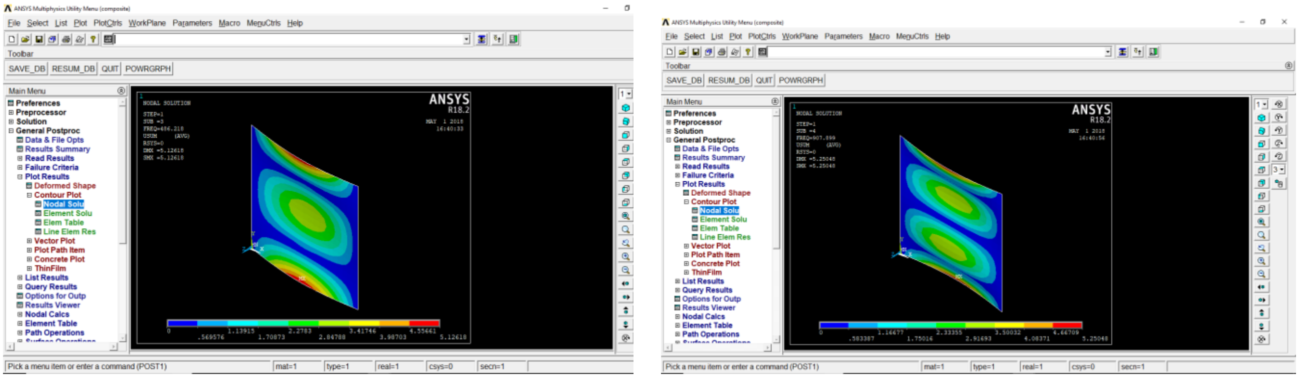


FIG25 NODAL DISPLACEMENT SOLUTION FOR 3RD AND 4TH NATURAL FREQUENCIES

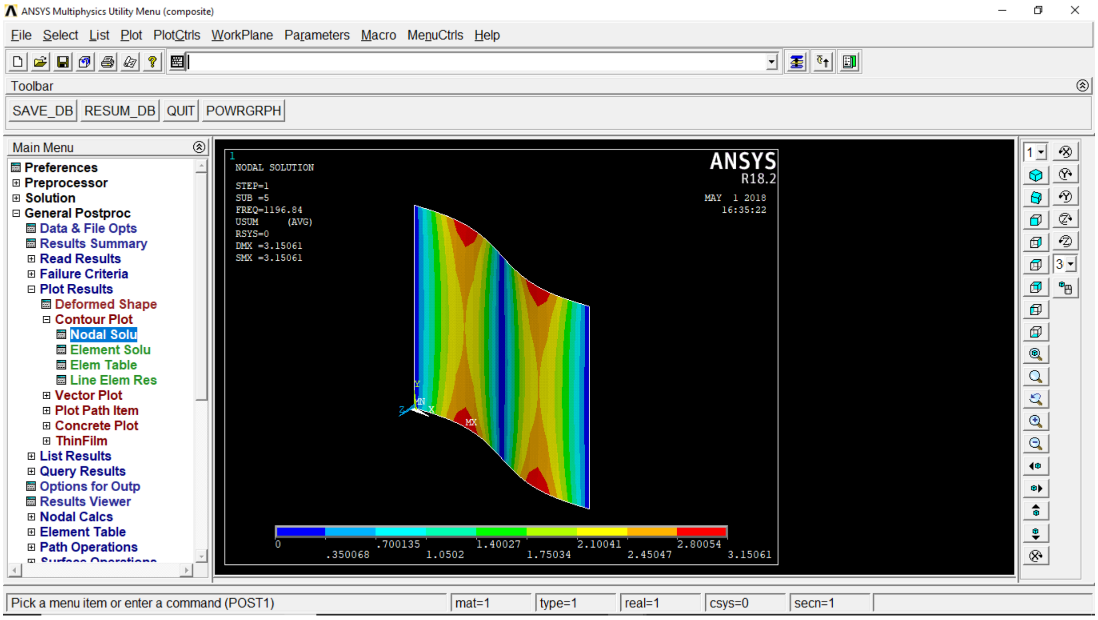


FIG26 NODAL DISPLACEMENT SOLUTION FOR 5^{TH} NATURAL FREQUENCY

5.4 EFFECT OF DIFFERENT PARAMETERS ON NATURAL FREQUENCIES OF A COMPOSITE PLATE

A laminated composite plat is considered and the natural frequencies are computed. Also, the effect of Boundary conditions,Stacking sequence,Lay up,Fiber volume ratio and Fiber orientation on the free vibration characteristics is analyzed. The results are presented through following sub-sections.

1. Effect of Boundary Conditions
2. Effect of Stacking Sequence
3. Effect of Lay up
4. Effect of Fiber volume ratio
5. Effect of Fiber-orientation

5.4.1 Effect of Boundary Conditions

A laminated unidirectional square plate (0°/0°/0°/0°) with two different boundary conditions as given below is studied.

1 Cantilever Plate (CFFF)
2 Simply Supported Plate (CFFC)

Other parameters remain constant and takes values as

Fiber volume ratio = 0.4

Fiber-orientation = 0°/0°/0°/0°

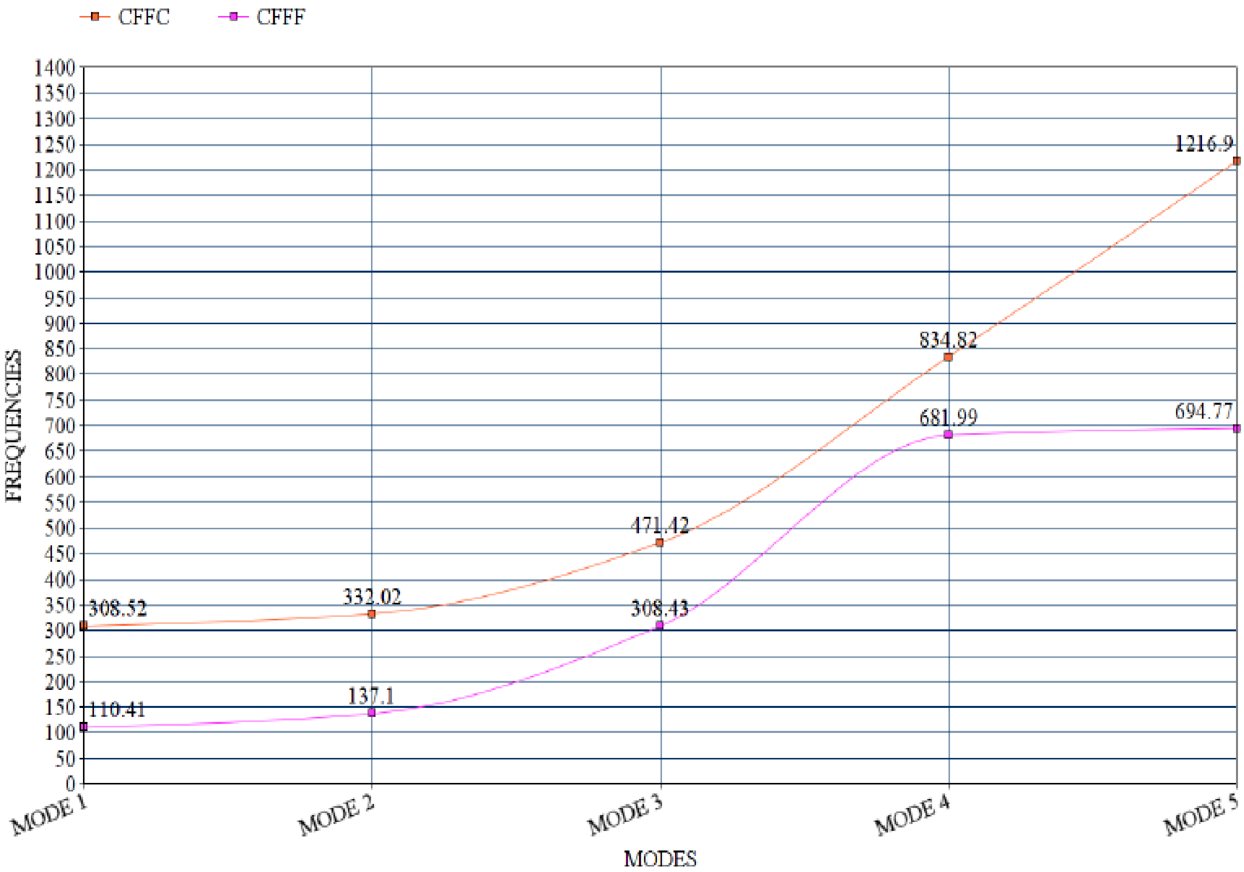
Lay up = Unidirectional

Stacking Sequence = Carbon/E-glass/E-glass/Carbon

The first five natural frequencies are presented in Table 5.

TABLE 5 NATURAL FREQUENCIES FOR DIFFERENT BOUNDARY CONDITIONS IN Hz

MODE	FREQUENCY(CFFF)	FREQUENCY(CFFC)
1	110.41	308.52
2	137.10	332.02
3	308.43	471.42
4	681.99	834.82
5	694.77	1216.9



MODES VS FREQUENCIES FOR DIFFERENT BOUNDARY CONDITIONS

5.4.2 Effect of Stacking Sequence

A laminated unidirectional square plate (0°/0°/0°/0°) with six different stacking sequences as given below is studied.

1 CARBON/CARBON/CARBON/CARBON
2 E-GLASS/E-GLASS/E-GLASS/E-GLASS
3 CARBON/E-GLASS/E-GLASS/CARBON
4 E-GLASS/CARBON/CARBON/E-GLASS
5 CARBON/E-GLASS/CARBON/E-GLASS
6 CARBON/CARBON/E-GLASS/E-GLASS

Other parameters remain constant and takes values as

Fiber volume ratio = 0.4

Fiber-orientation = 0°/0°/0°/0°

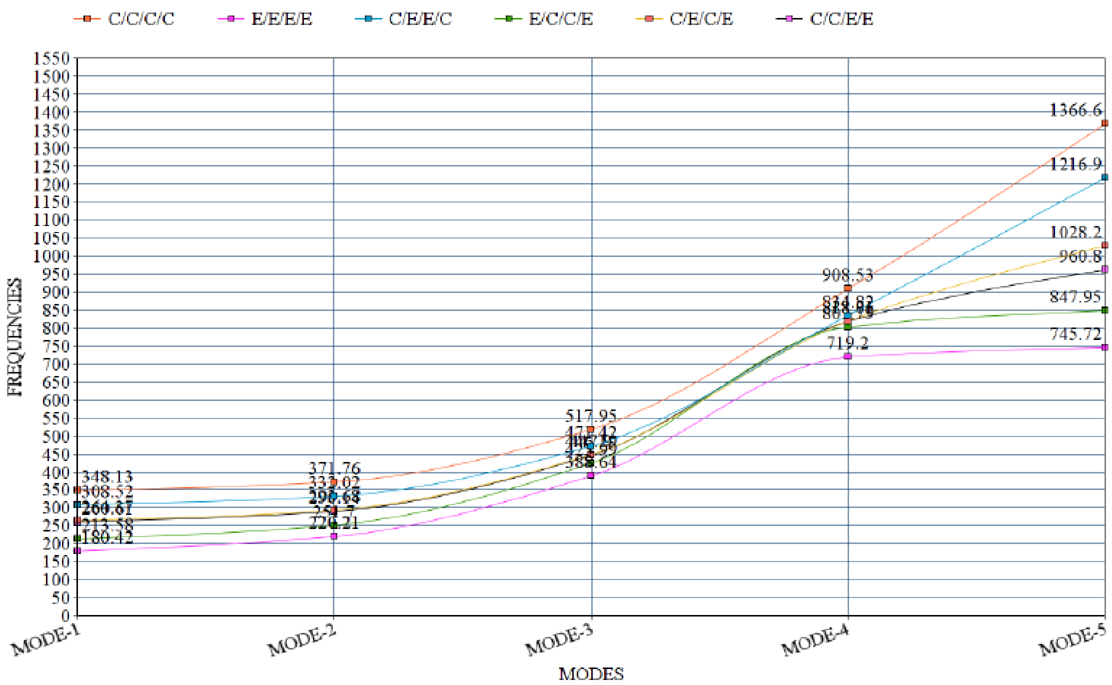
Lay up = Unidirectional

Boundary condition = Simply Supported

The first five natural frequencies are presented in Table 6.

TABLE 6 NATURAL FREQUENCIES FOR DIFFERENT STACKING SEQUENCES IN Hz

MODE	C/C/C/C	E/E/E/E	C/E/E/C	E/C/C/E	C/E/C/E	C/C/E/E
1	348.13	180.42	308.52	213.58	264.37	260.61
2	371.76	220.21	332.02	251.70	293.68	290.14
3	517.95	388.64	471.42	423.99	447.90	446.19
4	908.53	719.20	834.82	802.79	818.76	818.01
5	1366.6	745.72	1216.9	847.95	1028.2	960.80



MODES VS FREQUENCIES FOR DIFFERENT STACKING SEQUENCES

5.4.3 Effect of Lay up

A laminated unidirectional square plate (0°/0°/0°/0°) with four different lay up's as given below is studied.

1 UNIDIRECTIONAL
2 CROSS PLY SYMMETRIC
3 CROSS PLY ANTI SYMMETRIC
4 ANGLE PLY

Other parameters remain constant and takes values as

Fiber volume ratio = 0.4

Fiber-orientation = 0°/0°/0°/0°

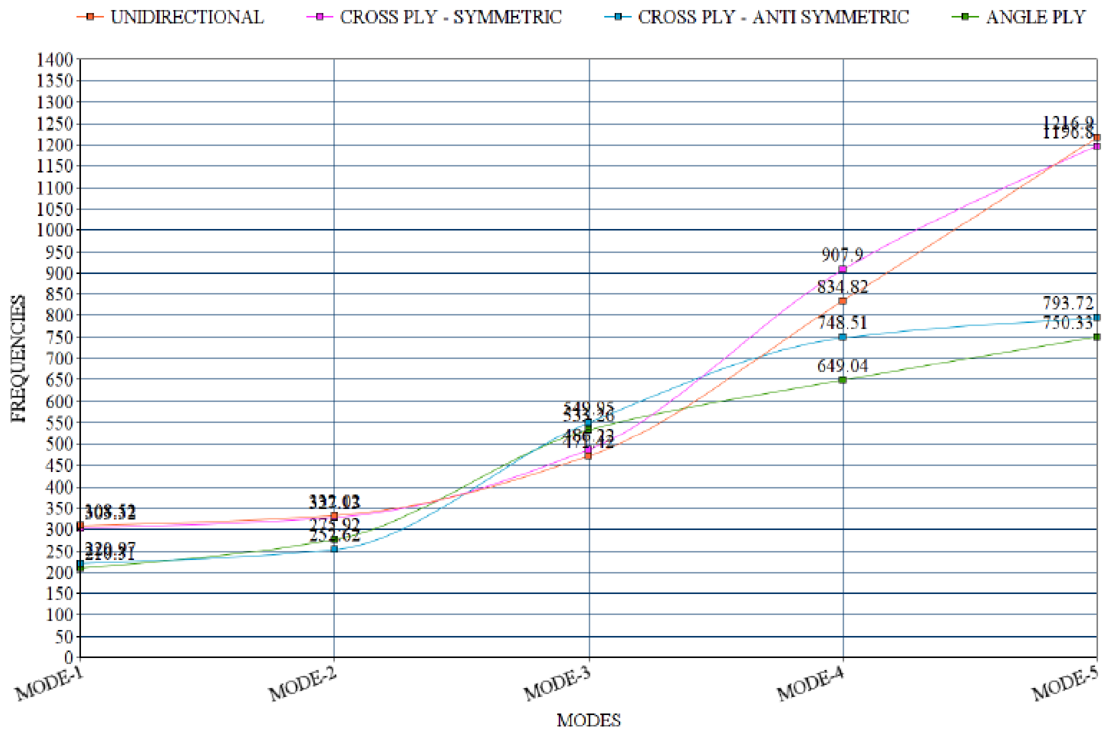
Stacking Sequence = Carbon/E-glass/E-glass/Carbon

Boundary condition = Simply Supported

The first five natural frequencies are presented in Table 7.

TABLE 7 NATURAL FREQUENCIES FOR DIFFERENT LAY UP'S IN Hz

MODE	UNIDIRECTIONAL	CROSSPLY SYM	CROSSPLY ANTI SYM	ANGLE PLY
1	308.52	303.32	220.97	210.31
2	332.02	327.13	252.62	275.92
3	471.42	486.22	549.95	533.16
4	834.82	907.90	748.51	649.04
5	1216.9	1196.8	793.72	750.33



MODES VS FREQUENCIES FOR DIFFERENT LAY UP'S

5.4.4 Effect of Fiber volume ratio

A laminated unidirectional square plate (0°/0°/0°/0°) with three different fiber volume ratios as given below is studied.

1 Vf = 0.4
2 Vf = 0.5

3 Vf = 0.6

Other parameters remain constant and takes values as

Fiber-orientation = 0°/0°/0°/0°

Stacking Sequence = Carbon/E-glass/E-glass/Carbon

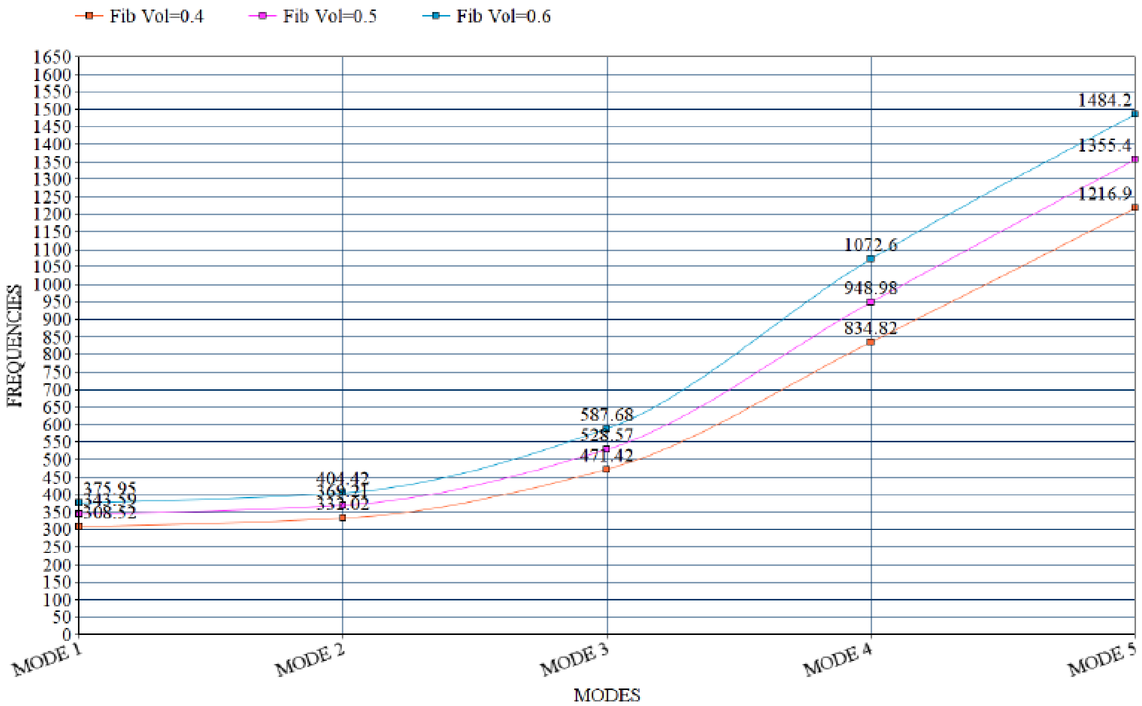
Boundary condition = Simply Supported

Lay up = Unidirectional

The first five natural frequencies are presented in Table 8.

TABLE 8 NATURAL FREQUENCIES FOR DIFFERENT FIBER VOLUME RATIO IN Hz

MODE	Vf=0.4	Vf=0.5	Vf=0.6
1	308.52	343.59	375.95
2	332.02	369.21	404.42
3	471.42	528.57	587.68
4	834.82	948.98	1072.6
5	1216.9	1355.4	1484.2



MODES VS FREQUENCIES FOR DIFFERENT FIBER VOLUME RATIOS

5.4.5 Effect of Fiber Orientation

A laminated unidirectional square plate (0°/0°/0°/0°) with four different fiber orientations as given below is studied.

1 0°
2 30°
3 45°
4 60°

Other parameters remain constant and takes values as

Stacking Sequence = Carbon/E-glass/E-glass/Carbon

Boundary condition = Simply Supported

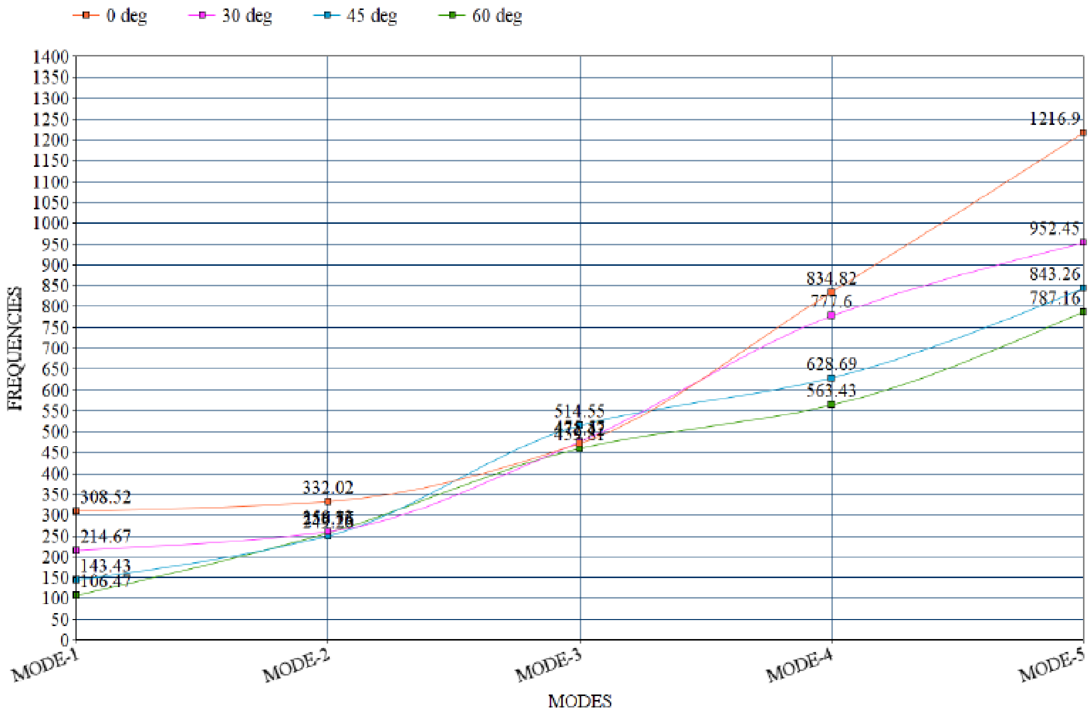
Lay up = Unidirectional

Fiber volume ratio = 0.4

The first five natural frequencies are presented in Table 9.

TABLE 9 NATURAL FREQUENCIES FOR DIFFERENT FIBER ORIENTATIONS IN Hz

MODE	0°	30°	45°	60°
1	308.52	214.67	143.43	106.47
2	332.02	259.13	249.26	256.76
3	471.42	475.57	514.55	459.81
4	834.82	777.60	628.69	563.43
5	1216.9	952.45	843.26	787.16



MODES VS FREQUENCIES FOR DIFFERENT FIBER ORIENTATIONS

5.5 CONCLUSIONS:

Thus, the free vibration of a laminated composite plate is analyzed for different boundary conditions, different fiber volume ratios,lay up's,stacking sequencies and fiber-orientation.

- The frequency of a plate is reduced owing to the reduction in its fiber volume ratio.
- The plate is found to be sensitive to boundary conditions applied. The plate showed highest frequencies when its edges were simply supported. Lowest frequencies were obtained for the cantilever plate.
- With increasing number of layers of the laminated composite plate, the frequencies were observed to increase, irrespective of the boundary conditions applied.
- The fiber-orientation was varied from (0°/0°/0°/0°) to (60°/60°/60°/60°). The fundamental frequencies were found to decrease with increase in angle of orientation of fibers.
- Graphs were plotted for understanding the behaviour of composite plates for various stacking sequences and Lay up's.

CHAPTER 6
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