Assignment 4

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Download all python codes from

https://github.com/Ganesh−RB/AI1103prob−and−randomvariables/Assignment4/codes

and latex-tikz codes from

https://github.com/Ganesh−RB/AI1103prob−and−randomvariables/Assignment4

1 Problem

CSIR UGC NET EXAM (Dec 2012) Q 51
Suppose X_1, X_2, X_3, X_4 are i.i.d random variables taking values 1 and -1 with probability $1/2$ each. Then $E\left(X_1 + X_2 + X_3 + X_4\right)^4$ equals

1) 4 2) 76 3) 16 4) 12

2 Solution

Theorem 2.1. *If $X_1, \ldots, X_n$ are i.i.d. random variables, all Bernoulli trials with success probability p, then their sum is distributed according to a binomial distribution with parameters n and p*

$$\sum_{k=1}^{n} X_k \sim B(n, p)$$

Corollary 2.2. *For a binomial random variable X with parameters n and p and $q = 1 - p$*

$$E\left(X\right) = np \tag{2.0.1}$$
$$E\left(X^2\right) = np\left(np + q\right) \tag{2.0.2}$$
$$E\left(X^3\right) = np\left(n^2p^2 + 3npq - 2pq + q\right) \tag{2.0.3}$$
$$E\left(X^4\right) = np\Big(n^3p^3 + 6n^2p^2q - 11np^2q + 7npq - 6pq^2 + q\Big) \tag{2.0.4}$$

Given that X_i are i.i.d random variable for $i \in \{1, 2, 3, 4\}$ with,

$$\Pr\left(X_i = +1\right) = \frac{1}{2} \tag{2.0.5}$$
$$\Pr\left(X_i = -1\right) = \frac{1}{2} \tag{2.0.6}$$

Lemma 2.1. *Let random variables $M_i = \frac{X_i+1}{2}$ then, M_i are Bernoulli random variables with $p = 0.5$*

Proof.

$$\Pr\left(M_i = 1\right) = \Pr\left(X_i = 1\right) = \frac{1}{2} \tag{2.0.7}$$
$$\Pr\left(M_i = 0\right) = \Pr\left(X_i = -1\right) = \frac{1}{2} \tag{2.0.8}$$

$\therefore$ M_i are Bernoulli random variables with $p = 0.5$ □

Let assume random variable Y as

$$Y = \sum_{i=1}^{4} X_i \tag{2.0.9}$$

similarly let

$$Z = \sum_{i=1}^{4} M_i \tag{2.0.10}$$

Corollary 2.3. *Z is Binomial random variable , from theorem 2.1*

$$Z \sim B(4, 0.5)$$

$$Z = \sum_{i=1}^{4} \frac{X_i + 1}{2} \tag{2.0.11}$$
$$Z = \frac{Y}{2} + 2 \tag{2.0.12}$$

$$Y^4 = 16(Z - 2)^4 \tag{2.0.13}$$

$$Y^4 = 16\left(Z^4 - 8z^3 + 24Z^2 - 32Z + 16\right) \tag{2.0.14}$$

$$E\left(Y^4\right) = 16\Big(E\left(Z^4\right) - 8E\left(z^3\right) + 24E\left(Z^2\right) - 32E\left(Z\right) + 16\Big) \tag{2.0.15}$$

From corollary 2.2 and 2.3

$$E\left(Y^4\right) = 40 \tag{2.0.16}$$

$$\therefore \mathbf{E}\,(\mathbf{X_1} + \mathbf{X_2} + \mathbf{X_3} + \mathbf{X_4})^4 = \mathbf{40}$$