# Assignment 4

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Download all python codes from

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https://github.com/Ganesh−RB/AI1103prob−and−randomvariables/Assignment4/codes
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and latex-tikz codes from

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https://github.com/Ganesh−RB/AI1103prob−and−randomvariables/Assignment4
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## 1 Problem

CSIR UGC NET EXAM (Dec 2012) Q 51

Suppose $X_1, X_2, X_3, X_4$ are i.i.d random variables taking values 1 and $-1$ with probability $1/2$ each. Then $E(X_1 + X_2 + X_3 + X_4)^4$ equals

1) 4
2) 76
3) 16
4) 12

## 2 Solution

**Theorem 2.1.** *If $X_1, \ldots, X_n$ are i.i.d. random variables, all Bernoulli trials with success probability $p$, then their sum is distributed according to a binomial distribution with parameters $n$ and $p$*

$$\sum_{k=1}^{n} X_k \sim B(n, p)$$

**Corollary 2.2.** *For a binomial random variable $X$ with parameters $n$ and $p$ and $q = 1 - p$*

$$E(X) = np \tag{2.0.1}$$

$$E\left(X^2\right) = np(np + q) \tag{2.0.2}$$

$$E\left(X^3\right) = np\left(n^2p^2 + 3npq - 2pq + q\right) \tag{2.0.3}$$

$$E\left(X^4\right) = np\Big(n^3p^3 + 6n^2p^2q - 11np^2q + 7npq - 6pq^2 + q\Big) \tag{2.0.4}$$

Given that $X_i$ are i.i.d random variable for $i \in \{1, 2, 3, 4\}$ with,

$$\Pr(X_i = +1) = \frac{1}{2} \tag{2.0.5}$$

$$\Pr(X_i = -1) = \frac{1}{2} \tag{2.0.6}$$

Let random variables $X'_i = \frac{X_i+1}{2}$ then,

$$\Pr(X'_i = 1) = \frac{1}{2} \tag{2.0.7}$$

$$\Pr(X'_i = 0) = \frac{1}{2} \tag{2.0.8}$$

So $X'_i$ are Bernoulli random variables with $p = 0.5$
Let assume random variable $Y$ as

$$Y = \sum_{i=1}^{4} X_i \tag{2.0.9}$$

similarly let

$$Z = \sum_{i=1}^{4} X'_i \tag{2.0.10}$$

By theorem 2.1 $Z$ is Binomial random variable

$$Z = \sum_{i=1}^{4} \frac{X_i + 1}{2} \tag{2.0.11}$$

$$Z = \frac{Y}{2} + 2 \tag{2.0.12}$$

$$Y^4 = 16(Z - 2)^4 \tag{2.0.13}$$

$$Y^4 = 16\left(Z^4 - 8z^3 + 24Z^2 - 32Z + 16\right) \tag{2.0.14}$$

$$E\left(Y^4\right) = 16\Big(E\left(Z^4\right) - 8E\left(z^3\right) + 24E\left(Z^2\right) - 32E(Z) + 16\Big) \tag{2.0.15}$$

Here $Z \sim B(4, 0.5)$ thus by 2.2

$$E\left(Y^4\right) = 40 \tag{2.0.16}$$

$$\therefore \mathbf{E}\left(\mathbf{X_1} + \mathbf{X_2} + \mathbf{X_3} + \mathbf{X_4}\right)^{\mathbf{4}} = \mathbf{40}$$