Assignment 4

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Download all python codes from

```
https://github.com/Ganesh−RB/AI1103prob−and−
    randomvariables/Assignment4/codes
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and latex-tikz codes from

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https://github.com/Ganesh−RB/AI1103prob−and−
    randomvariables/Assignment4
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1 Problem

CSIR UGC NET EXAM (Dec 2012) Q 51
Suppose X_1, X_2, X_3, X_4 are i.i.d random variables taking values 1 and -1 with probability $1/2$ each. Then $E\left(X_1 + X_2 + X_3 + X_4\right)^4$ equals

1) 4
2) 76
3) 16
4) 12

2 Solution

Theorem 2.1. *If $X_1, \ldots, X_n$ are i.i.d. random variables, all Bernoulli trials with success probability p, then their sum is distributed according to a binomial distribution with parameters n and p*

$$\sum_{k=1}^{n} X_k \sim B(n, p)$$

Corollary 2.2. *For a binomial random variable X with parameters n and p and $q = 1 - p$*

$$E(X) = np \tag{2.0.1}$$

$$E\left(X^2\right) = np\left(np + q\right) \tag{2.0.2}$$

$$E\left(X^3\right) = np\left(n^2p^2 + 3npq - 2pq + q\right) \tag{2.0.3}$$

$$E\left(X^4\right) = np\Big(n^3p^3 + 6n^2p^2q - 11np^2q + 7npq - 6pq^2 + q\Big) \tag{2.0.4}$$

Given that X_i are i.i.d random variable for $i \in \{1, 2, 3, 4\}$ with,

$$\Pr\left(X_i = +1\right) = \frac{1}{2} \tag{2.0.5}$$

$$\Pr\left(X_i = -1\right) = \frac{1}{2} \tag{2.0.6}$$

Let random variables $X_i' = \frac{X_i + 1}{2}$ then,

$$\Pr\left(X_i' = 1\right) = \frac{1}{2} \tag{2.0.7}$$

$$\Pr\left(X_i' = 0\right) = \frac{1}{2} \tag{2.0.8}$$

So X_i' are Bernoulli random variables with $p = 0.5$
Let assume random variable Y as

$$Y = \sum_{i=1}^{4} X_i \tag{2.0.9}$$

similarly let

$$Z = \sum_{i=1}^{4} X_i' \tag{2.0.10}$$

$$Z = \sum_{i=1}^{4} \frac{X_i + 1}{2} \tag{2.0.11}$$

$$Z = \frac{Y}{2} + 2 \tag{2.0.12}$$

Here Z will be binomial random variable with parameter $n = 4$, $p = 0.5$ and $q = 0.5$
Thus follows

$$E(Z) = 2 \tag{2.0.13}$$

$$E\left(Z^2\right) = 5 \tag{2.0.14}$$

$$E\left(Z^3\right) = 14 \tag{2.0.15}$$

$$E\left(Z^4\right) = 42.5 \tag{2.0.16}$$

Now

$$Y^4 = 16(Z - 2)^4 \tag{2.0.17}$$

$$Y^4 = 16\left(Z^4 - 8z^3 + 24Z^2 - 32Z + 16\right) \tag{2.0.18}$$

$$E\left(Y^4\right) = 16\Big(E\left(Z^4\right) - 8E\left(z^3\right) + 24E\left(Z^2\right) - 32E\left(Z\right) + 16\Big) \tag{2.0.19}$$

$$E\left(Y^4\right) = 16\left(42.5 - 8 \times 14 + 24 \times 5 - 32 \times 2 + 16\right) \tag{2.0.20}$$

$$E\left(Y^4\right) = 16 \times 2.5 \tag{2.0.21}$$

$$E\left(Y^4\right) = 40 \tag{2.0.22}$$

$$\therefore \mathbf{E\left(X_1 + X_2 + X_3 + X_4\right)^4 = 40}$$