

# Assignment 4

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Download all python codes from

<https://github.com/Ganesh-RB/AI1103prob-and-randomvariables/Assignment4/codes>

and latex-tikz codes from

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Let random variables  $X'_i = \frac{X_i+1}{2}$  then,

$$\Pr(X'_i = 1) = \frac{1}{2} \quad (2.0.7)$$

$$\Pr(X'_i = 0) = \frac{1}{2} \quad (2.0.8)$$

So  $X'_i$  are Bernoulli random variables with  $p = 0.5$   
Let assume random variable  $Y$  as

$$Y = \sum_{i=1}^4 X_i \quad (2.0.9)$$

similarly let

$$Z = \sum_{i=1}^4 X'_i \quad (2.0.10)$$

**Corollary 2.3.**  $Z$  is Binomial random variable ,  
from theorem 2.1

$$Z \sim B(4, 0.5)$$

$$Z = \sum_{i=1}^4 \frac{X_i + 1}{2} \quad (2.0.11)$$

$$Z = \frac{Y}{2} + 2 \quad (2.0.12)$$

$$Y^4 = 16(Z - 2)^4 \quad (2.0.13)$$

$$Y^4 = 16(Z^4 - 8Z^3 + 24Z^2 - 32Z + 16) \quad (2.0.14)$$

$$E(Y^4) = 16(E(Z^4) - 8E(Z^3) + 24E(Z^2) - 32E(Z) + 16) \quad (2.0.15)$$

From corollary 2.2 and 2.3

$$E(Y^4) = 40 \quad (2.0.16)$$

$$\therefore E(X_1 + X_2 + X_3 + X_4)^4 = 40$$

## 1 PROBLEM

CSIR UGC NET EXAM (Dec 2012) Q 51

Suppose  $X_1, X_2, X_3, X_4$  are i.i.d random variables taking values 1 and  $-1$  with probability  $1/2$  each. Then  $E(X_1 + X_2 + X_3 + X_4)^4$  equals

- 1) 4                  2) 76                  3) 16                  4) 12

## 2 SOLUTION

**Theorem 2.1.** If  $X_1, \dots, X_n$  are i.i.d. random variables, all Bernoulli trials with success probability  $p$ , then their sum is distributed according to a binomial distribution with parameters  $n$  and  $p$

$$\sum_{k=1}^n X_k \sim B(n, p)$$

**Corollary 2.2.** For a binomial random variable  $X$  with parameters  $n$  and  $p$  and  $q = 1 - p$

$$E(X) = np \quad (2.0.1)$$

$$E(X^2) = np(np + q) \quad (2.0.2)$$

$$E(X^3) = np(n^2p^2 + 3npq - 2pq + q) \quad (2.0.3)$$

$$E(X^4) = np(n^3p^3 + 6n^2p^2q - 11np^2q + 7npq - 6pq^2 + q) \quad (2.0.4)$$

Given that  $X_i$  are i.i.d random variable for  $i \in \{1, 2, 3, 4\}$  with,

$$\Pr(X_i = +1) = \frac{1}{2} \quad (2.0.5)$$

$$\Pr(X_i = -1) = \frac{1}{2} \quad (2.0.6)$$