

ASSIGNMENT 5

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Download all python codes from

<https://github.com/Ganesh-RB/AI1103prob-and-randomvariables/Assignment5/codes>

and latex-tikz codes from

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Now

$$\Pr(X_n > k) = \Pr\left(\frac{X_n - n^2}{n} > \frac{k - n^2}{n}\right) \quad (2.0.4)$$

$$= \Pr\left(\mathcal{N}(0, 1) > \frac{k - n^2}{n}\right) \quad (2.0.5)$$

$$= Q\left(\frac{k - n^2}{n}\right) \quad (2.0.6)$$

1 PROBLEM

For $n \geq 1$, let X_n be a Poisson random variable with mean n^2 . Which of the following's are equal

to $\frac{1}{\sqrt{2\pi}} \int_2^\infty e^{-x^2/2} dx$

- 1) $\lim_{n \rightarrow \infty} \Pr(X_n > (n+1)^2)$
- 2) $\lim_{n \rightarrow \infty} \Pr(X_n \leq (n+1)^2)$
- 3) $\lim_{n \rightarrow \infty} \Pr(X_n < (n-1)^2)$
- 4) $\lim_{n \rightarrow \infty} \Pr(X_n < (n-2)^2)$

2 SOLUTION

Let Y_i be a Poisson random variable with mean 1 for $i \in (1, n^2)$

By additive property of Poisson distribution

$$\sum_i^{n^2} Y_i = X_n \quad (2.0.1)$$

By central limit theorem

$$\lim_{n \rightarrow \infty} \frac{Y_1 + Y_2 + \dots + Y_{n^2} - n^2}{n} = \mathcal{N}(0, 1) \quad (2.0.2)$$

$$\implies \lim_{n \rightarrow \infty} \frac{X_n - n^2}{n} = \mathcal{N}(0, 1) \quad (2.0.3)$$

Here, $\mathcal{N}(0, 1)$ is normal distribution with unit mean and variance

Here

$$Q(X) = 1 - Q(-x) \quad (2.0.7)$$

$$\therefore \frac{1}{\sqrt{2\pi}} \int_{-x}^\infty e^{-x^2/2} dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-x^2/2} dx$$

Also

$$\frac{1}{\sqrt{2\pi}} \int_2^\infty e^{-x^2/2} dx = Q(2) \quad (2.0.8)$$

$$\frac{1}{\sqrt{2\pi}} \int_2^\infty e^{-x^2/2} dx = 1 - Q(-2) \quad (2.0.9)$$

$$1) \lim_{n \rightarrow \infty} \Pr(X_n > (n+1)^2)$$

$$\lim_{n \rightarrow \infty} \Pr(X_n > (n+1)^2) = \lim_{n \rightarrow \infty} Q\left(\frac{(n+1)^2 - n^2}{n}\right) \quad (2.0.10)$$

$$= Q(2) \quad (2.0.11)$$

\therefore Option 1 is correct

$$2) \lim_{n \rightarrow \infty} \Pr(X_n \leq (n+1)^2)$$

$$\lim_{n \rightarrow \infty} \Pr(X_n \leq (n+1)^2) = 1 - \lim_{n \rightarrow \infty} \Pr(X_n > (n+1)^2) \quad (2.0.12)$$

$$= 1 - \lim_{n \rightarrow \infty} Q\left(\frac{(n+1)^2 - n^2}{n}\right) \quad (2.0.13)$$

$$= 1 - Q(2) \quad (2.0.14)$$

\therefore Option 2 is incorrect

$$3) \lim_{n \rightarrow \infty} \Pr(X_n < (n-1)^2)$$

$$\lim_{n \rightarrow \infty} \Pr(X_n < (n-1)^2) = 1 - \lim_{n \rightarrow \infty} \Pr(X_n \geq (n-1)^2) \quad (2.0.15)$$

$$= 1 - \lim_{n \rightarrow \infty} Q\left(\frac{(n-1)^2 - n^2}{n}\right) \quad (2.0.16)$$

$$= 1 - Q(-2) \quad (2.0.17)$$

\therefore Option 3 is also correct

$$4) \lim_{n \rightarrow \infty} \Pr(X_n < (n-2)^2)$$

$$\lim_{n \rightarrow \infty} \Pr(X_n < (n-2)^2) = 1 - \lim_{n \rightarrow \infty} \Pr(X_n \geq (n-2)^2) \quad (2.0.18)$$

$$= 1 - \lim_{n \rightarrow \infty} Q\left(\frac{(n-2)^2 - n^2}{n}\right) \quad (2.0.19)$$

$$= 1 - Q(-4) \quad (2.0.20)$$

\therefore Option 4 is incorrect