

# ASSIGNMENT 5

Ganesh Bombatkar - CS20BTECH11016

Download all python codes from

<https://github.com/Ganesh-RB/AI1103prob-and-randomvariables/Assignment5/codes>

and latex-tikz codes from

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Now

$$\Pr(X_n > k) = \Pr\left(\frac{X_n - n^2}{n} > \frac{k - n^2}{n}\right) \quad (2.0.4)$$

$$= \Pr\left(\mathcal{N}(0, 1) > \frac{k - n^2}{n}\right) \quad (2.0.5)$$

$$= Q\left(\frac{k - n^2}{n}\right) \quad (2.0.6)$$

## 1 PROBLEM

For  $n \geq 1$ , let  $X_n$  be a Poisson random variable with mean  $n^2$ . Which of the following's are equal

to  $\frac{1}{\sqrt{2\pi}} \int_2^\infty e^{-x^2/2} dx$

- 1)  $\lim_{n \rightarrow \infty} \Pr(X_n > (n+1)^2)$
- 2)  $\lim_{n \rightarrow \infty} \Pr(X_n \leq (n+1)^2)$
- 3)  $\lim_{n \rightarrow \infty} \Pr(X_n < (n-1)^2)$
- 4)  $\lim_{n \rightarrow \infty} \Pr(X_n < (n-2)^2)$

Here

$$Q(X) = 1 - Q(-x) \quad (2.0.7)$$

$$\therefore \frac{1}{\sqrt{2\pi}} \int_{-x}^\infty e^{-x^2/2} dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-x^2/2} dx$$

$$\text{and } \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty e^{-x^2/2} dx = 1$$

Also

$$\frac{1}{\sqrt{2\pi}} \int_2^\infty e^{-x^2/2} dx = Q(2) \quad (2.0.8)$$

## 2 SOLUTION

Let  $Y_i$  be a Poisson random variable with mean 1 for  $i \in (1, n^2)$

By additive property of Poisson distribution

$$\sum_i^{n^2} Y_i = X_n \quad (2.0.1)$$

By central limit theorem

$$\lim_{n \rightarrow \infty} \frac{Y_1 + Y_2 + \dots + Y_{n^2} - n^2}{n} = \mathcal{N}(0, 1) \quad (2.0.2)$$

$$\implies \lim_{n \rightarrow \infty} \frac{X_n - n^2}{n} = \mathcal{N}(0, 1) \quad (2.0.3)$$

Here,  $\mathcal{N}(0, 1)$  is normal distribution with unit mean and variance

$$1) \lim_{n \rightarrow \infty} \Pr(X_n > (n+1)^2)$$

$$\lim_{n \rightarrow \infty} \Pr(X_n > (n+1)^2) = \lim_{n \rightarrow \infty} Q\left(\frac{(n+1)^2 - n^2}{n}\right) \quad (2.0.9)$$

$$= Q(2) \quad (2.0.10)$$

$\therefore$  **Option 1 is correct**

$$2) \lim_{n \rightarrow \infty} \Pr(X_n \leq (n+1)^2)$$

$$\begin{aligned} & \lim_{n \rightarrow \infty} \Pr(X_n \leq (n+1)^2) \\ &= 1 - \lim_{n \rightarrow \infty} \Pr(X_n > (n+1)^2) \end{aligned} \quad (2.0.11)$$

$$= 1 - \lim_{n \rightarrow \infty} Q\left(\frac{(n+1)^2 - n^2}{n}\right) \quad (2.0.12)$$

$$= 1 - Q(2) \quad (2.0.13)$$

$$= Q(-2) > Q(2) \quad (2.0.14)$$

**$\therefore$  Option 2 is incorrect**

$$3) \lim_{n \rightarrow \infty} \Pr(X_n < (n-1)^2)$$

$$\begin{aligned} & \lim_{n \rightarrow \infty} \Pr(X_n < (n-1)^2) \\ &= 1 - \lim_{n \rightarrow \infty} \Pr(X_n \geq (n-1)^2) \end{aligned} \quad (2.0.15)$$

$$= 1 - \lim_{n \rightarrow \infty} Q\left(\frac{(n-1)^2 - n^2}{n}\right) \quad (2.0.16)$$

$$= 1 - Q(-2) \quad (2.0.17)$$

$$= Q(2) \quad (2.0.18)$$

**$\therefore$  Option 3 is also correct**

$$4) \lim_{n \rightarrow \infty} \Pr(X_n < (n-2)^2)$$

$$\begin{aligned} & \lim_{n \rightarrow \infty} \Pr(X_n < (n-2)^2) \\ &= 1 - \lim_{n \rightarrow \infty} \Pr(X_n \geq (n-2)^2) \end{aligned} \quad (2.0.19)$$

$$= 1 - \lim_{n \rightarrow \infty} Q\left(\frac{(n-2)^2 - n^2}{n}\right) \quad (2.0.20)$$

$$= 1 - Q(-4) \quad (2.0.21)$$

$$= Q(4) < Q(2) \quad (2.0.22)$$

**$\therefore$  Option 4 is incorrect**