

Assignment 4

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Download all python codes from

<https://github.com/Ganesh-RB/AI1103prob-and-randomvariables/Assignment4/codes>

and latex-tikz codes from

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By defining Moment generating function of X_i

$$M_{X_i}(t) = \frac{1}{2} \times e^{-t} + \frac{1}{2} \times e^t = \frac{e^t + e^{-t}}{2} \quad (2.0.7)$$

Let $Y = X_1 + X_2 + X_3 + X_4$, then by convolution

$$M_Y(t) = M_{X_1}(t) \times M_{X_2}(t) \times M_{X_3}(t) \times M_{X_4}(t) \quad (2.0.8)$$

$$= \left(\frac{e^t + e^{-t}}{2} \right)^4 \quad (2.0.9)$$

$$E(Y^4) = \left[\frac{d^4}{dt^4} M_Y(t) \right]_{t=0} \quad (2.0.10)$$

$$= \left[\frac{256e^{4t} + 64e^{2t} + 64e^{-2t} + 256e^{-4t}}{16} \right]_{t=0} \quad (2.0.11)$$

$$= \frac{640}{16} = 40 \quad (2.0.12)$$

$$\therefore E(Y^4) = 40$$

1 PROBLEM

CSIR UGC NET EXAM (Dec 2012) Q 51

Suppose X_1, X_2, X_3, X_4 are i.i.d random variables taking values 1 and -1 with probability $1/2$ each. Then $E(X_1 + X_2 + X_3 + X_4)^4$ equals

- 1) 4 2) 76 3) 16 4) 12

2 SOLUTION

Definition 1 (Moment generating function). For a discrete random variable X

$$M_X(t) \equiv E(e^{tX}) = \sum_{k=-\infty}^{\infty} e^{tk} \times P_X(k) \quad (2.0.1)$$

Theorem 2.1. n^{th} Moment of X is equivalent to $E(X^n)$

Proof.

$$\left[\frac{d^n}{dt^n} M_X(t) \right]_{t=0} = \left[\sum_{k=-\infty}^{\infty} k^n \times e^{tk} \times P_X(k) \right]_{t=0} \quad (2.0.2)$$

$$= \sum_{k=-\infty}^{\infty} k^n \times P_X(k) \quad (2.0.3)$$

$$= E(X^n) \quad (2.0.4)$$

□

X_i are i.i.d random variable for $i \in \{1, 2, 3, 4\}$ with,

$$\Pr(X_i = +1) = \frac{1}{2} \quad (2.0.5)$$

$$\Pr(X_i = -1) = \frac{1}{2} \quad (2.0.6)$$