Assignment 4

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Download all python codes from

https://github.com/Ganesh−RB/AI1103prob−and−randomvariables/Assignment4/codes

and latex-tikz codes from

https://github.com/Ganesh−RB/AI1103prob−and−randomvariables/Assignment4

1 Problem

CSIR UGC NET EXAM (Dec 2012) Q 51
Suppose $X1, X2, X3, X4$ are i.i.d random variables taking values 1 and -1 with probability $1/2$ each. Then $E\left(X_1 + X_2 + X_3 + X_4\right)^4$ equals

1) 4
2) 76
3) 16
4) 12

2 Solution

Moment generating function : For a discrete random variable X

$$M_X(t) \equiv E\left(e^{tX}\right) = \sum_{k=-\infty}^{\infty} e^{tk} \times \Pr_X(k) \tag{2.0.1}$$

n^{th} Moment of X

$$\left[\frac{d^n}{dt^n} Y(t)\right]_{t=0} = \left[\sum_{k=-\infty}^{\infty} k^n \times e^{tk} \times \Pr_X(k)\right]_{t=0} \tag{2.0.2}$$

$$= \sum_{k=-\infty}^{\infty} k^n \times \Pr_X(k) \tag{2.0.3}$$

$$= E\left(X^n\right) \tag{2.0.4}$$

By defining Moment generating function of X_i for $i \in \{1, 2, 3, 4\}$

$$M_{X_i}(t) = \frac{1}{2} \times e^{-t} + \frac{1}{2} \times e^{t} = \frac{e^t + e^{-t}}{2} \tag{2.0.5}$$

Let $Y = X_1 + X_2 + X_3 + X_4$, then by convolution

$$M_Y(t) = M_{X_1}(t) \times M_{X_2}(t) \times M_{X_3}(t) \times M_{X_4}(t) \tag{2.0.6}$$

$$= \left(\frac{e^t + e^{-t}}{2}\right)^4 \tag{2.0.7}$$

$$E\left(Y^4\right) = \left[\frac{d^4}{dt^4} Y(t)\right]_{t=0} \tag{2.0.8}$$

$$= \left[\frac{256e^{4t} + 64e^{2t} + 64e^{-2t} + 256e^{-4t}}{16}\right]_{t=0} \tag{2.0.9}$$

$$= \frac{640}{16} = 40 \tag{2.0.10}$$

$$\therefore \mathbf{E}\left(\mathbf{Y^4}\right) = \mathbf{40}$$