Assignment 4

Ganesh Bombatkar - CS20BTECH11016

Download all python codes from

```
https://github.com/Ganesh-RB/AI1103prob-and-randomvariables/Assignment4/codes
```

and latex-tikz codes from

```
https://github.com/Ganesh-RB/AI1103prob-and-randomvariables/Assignment4
```

1 Problem

CSIR UGC NET EXAM (Dec 2012) Q 51

Suppose X_1, X_2, X_3, X_4 are i.i.d random variables taking values 1 and -1 with probability $1/2$ each. Then $E\left(X_1 + X_2 + X_3 + X_4\right)^4$ equals

1) 4
2) 76
3) 16
4) 12

2 Solution

Definition 1 (Moment generating function)**.** *For a discrete random variable X*

$$M_X(t) \equiv E\left(e^{tX}\right) = \sum_{k=-\infty}^{\infty} e^{tk} \times P_X(k) \tag{2.0.1}$$

Theorem 2.1. *n^{th} Moment of X is equivalent to $E\left(X^n\right)$*

Proof.

$$\left[\frac{d^n}{dt^n} M_X(t)\right]_{t=0} = \left[\sum_{k=-\infty}^{\infty} k^n \times e^{tk} \times P_X(k)\right]_{t=0} \tag{2.0.2}$$

$$= \sum_{k=-\infty}^{\infty} k^n \times P_X(k) \tag{2.0.3}$$

$$= E\left(X^n\right) \tag{2.0.4}$$

$\square$

X_i are i.i.d random variable for $i \in \{1, 2, 3, 4\}$ with,

$$\Pr\left(X_i = +1\right) = \frac{1}{2} \tag{2.0.5}$$

$$\Pr\left(X_i = -1\right) = \frac{1}{2} \tag{2.0.6}$$

By defining Moment generating function of X_i

$$M_{X_i}(t) = \frac{1}{2} \times e^{-t} + \frac{1}{2} \times e^{t} = \frac{e^t + e^{-t}}{2} \tag{2.0.7}$$

Let $Y = X_1 + X_2 + X_3 + X_4$, then by convolution

$$M_Y(t) = M_{X_1}(t) \times M_{X_2}(t) \times M_{X_3}(t) \times M_{X_4}(t) \tag{2.0.8}$$

$$M_Y(t) = \left(\frac{e^t + e^{-t}}{2}\right)^4 \tag{2.0.9}$$

Now

$$E\left(Y^4\right) = \left[\frac{d^4}{dt^4} M_Y(t)\right]_{t=0} \tag{2.0.10}$$

$$= \left[\frac{d^4}{dt^4}\left(\frac{e^t + e^{-t}}{2}\right)^4\right]_{t=0} \tag{2.0.11}$$

$$= \left[\frac{256e^{4t} + 64e^{2t} + 64e^{-2t} + 256e^{-4t}}{16}\right]_{t=0} \tag{2.0.12}$$

$$E\left(Y^4\right) = \frac{640}{16} = 40 \tag{2.0.13}$$

$$\therefore \mathbf{E\left(X_1 + X_2 + X_3 + X_4\right)^4 = 40}$$