Assignment 4

Ganesh Bombatkar - CS20BTECH11016

Download all python codes from

```
https://github.com/Ganesh−RB/AI1103prob−and−
    randomvariables/Assignment4/codes
```

and latex-tikz codes from

```
https://github.com/Ganesh−RB/AI1103prob−and−
    randomvariables/Assignment4
```

1 Problem

CSIR UGC NET EXAM (Dec 2012) Q 51
Suppose $X1, X2, X3, X4$ are i.i.d random variables taking values 1 and -1 with probability $1/2$ each. Then $E(X_1 + X_2 + X_3 + X_4)^4$ equals

1) 4
2) 76
3) 16
4) 12

2 Solution

Z Transform : For a discrete random variable X , Z transform of $X \equiv E\left(z^X\right)$

$$X(z) \equiv E\left(z^X\right) = \sum_{i=-\infty}^{\infty} z^{x_i} \times \Pr(x_i) \tag{2.0.1}$$

By defining Z transform of X_i for $i \in \{1, 2, 3, 4\}$

$$X_i(z) = \frac{1}{2} \times z + \frac{1}{2} \times z^{-1} = \frac{z + z^{-1}}{2} \tag{2.0.2}$$

Let $Y = X_1 + X_2 + X_3 + X_4$, then

$$Y(z) = X_1(z) \times X_2(z) \times X_3(z) \times X_4(z) \tag{2.0.3}$$
$$= \left(\frac{z + z^{-1}}{2}\right)^4 \tag{2.0.4}$$

derivatives of z Transform

$$\left[\frac{d}{dz} Y(Z)\right]_{z=1} = \left[\sum_{i=-\infty}^{\infty} y_i z^{y_i - 1} \times \Pr(y_i)\right]_{z=1} \tag{2.0.5}$$
$$= \sum_{i=-\infty}^{\infty} y_i \times \Pr(y_i) \tag{2.0.6}$$
$$= E(Y) \tag{2.0.7}$$
$$= \frac{z^3}{4} + \frac{z}{2} - \frac{1}{2z^3} - \frac{1}{4z^5} \tag{2.0.8}$$
$$= 0 \tag{2.0.9}$$

$$\left[\frac{d^2}{dz^2} Y(Z)\right]_{z=1} = \sum_{i=-\infty}^{\infty} y_i (y_i - 1) \times \Pr(y_i) \tag{2.0.10}$$
$$= \sum_{i=-\infty}^{\infty} y_i^2 \times \Pr(y_i) - \sum_{i=-\infty}^{\infty} y_i \times \Pr(y_i) \tag{2.0.11}$$
$$= E\left(Y^2\right) - E(Y) \tag{2.0.12}$$
$$= \frac{3z^2}{4} + \frac{1}{2} + \frac{3}{2z^4} + \frac{5}{4z^6} \tag{2.0.13}$$
$$= 4 \tag{2.0.14}$$

$$\left[\frac{d^3}{dz^3} Y(Z)\right]_{z=1} = \sum_{i=-\infty}^{\infty} \left(y_i^3 - 3y_i^2 + 2y_i\right) \times \Pr(y_i) \tag{2.0.15}$$
$$= E\left(Y^3\right) - 3E\left(Y^2\right) + 2E(Y) \tag{2.0.16}$$
$$= \frac{3z}{2} - \frac{6}{z^5} - \frac{15}{2z^7} \tag{2.0.17}$$
$$= -12 \tag{2.0.18}$$

$$\left[\frac{d^4}{dz^4}Y(Z)\right]_{z=1}$$
$$= \sum_{i=-\infty}^{\infty}\left(y_i^4 - 6y_i^3 + 11y_i^2 - 6y_i\right) \times \Pr(y_i) \tag{2.0.19}$$
$$= E\left(Y^4\right) - 6E\left(Y^3\right) + 11E\left(Y^2\right) - 6E(Y) \tag{2.0.20}$$
$$= \frac{3}{2} + \frac{30}{z^6} + \frac{105}{2z^8} \tag{2.0.21}$$
$$= 84 \tag{2.0.22}$$

By manipulating equation (2.0.7) ,(2.0.12) ,(2.0.16) and (2.0.20)

$$E\left(Y^4\right) = \left[\frac{d^4}{dz^4}Y(Z)\right]_{z=1} + 6 \times \left[\frac{d^3}{dz^3}Y(Z)\right]_{z=1}$$
$$+ 7 \times \left[\frac{d^2}{dz^2}Y(Z)\right]_{z=1} + 25 \times \left[\frac{d}{dz}Y(Z)\right]_{z=1} \tag{2.0.23}$$

$$\therefore \mathbf{E}\left(\mathbf{Y^4}\right) = \mathbf{84} + \mathbf{6} \times (\mathbf{-12}) + \mathbf{7} \times \mathbf{4} + \mathbf{25} \times \mathbf{0} = \mathbf{40}$$