Assignment 4

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Download all python codes from

https://github.com/Ganesh−RB/AI1103prob−and−randomvariables/Assignment4/codes

and latex-tikz codes from

https://github.com/Ganesh−RB/AI1103prob−and−randomvariables/Assignment4

1 Problem

CSIR UGC NET EXAM (Dec 2012) Q 51
Suppose $X1, X2, X3, X4$ are i.i.d random variables taking values 1 and -1 with probability $1/2$ each. Then $E\left(X_1 + X_2 + X_3 + X_4\right)^4$ equals

1) 4 2) 76 3) 16 4) 12

2 Solution

By defining Z transform of X_i as $X_i\left[z\right]$ for $i \in \{1, 2, 3, 4\}$

$$X_i\left[z\right] = \frac{1}{2}\cdot z + \frac{1}{2}\cdot z^{-1} = \frac{z + z^{-1}}{2} \tag{2.0.1}$$

Let $Y = X_1 + X_2 + X_3 + X_4$, then

$$Y\left[z\right] = X_1\left[z\right]\cdot X_2\left[z\right]\cdot X_3\left[z\right]\cdot X_4\left[z\right] \tag{2.0.2}$$

$$= \left(\frac{z + z^{-1}}{2}\right)^4 \tag{2.0.3}$$

Taking derivatives of $Y\left[z\right]$

$$\left[\frac{d}{dz}Y\left[Z\right]\right]_{z=1} = E\left[Y\right] \tag{2.0.4}$$

$$= \frac{z^3}{4} + \frac{z}{2} - \frac{1}{2z^3} - \frac{1}{4z^5} \tag{2.0.5}$$

$$= 0 \tag{2.0.6}$$

$$\left[\frac{d^2}{dz^2}Y\left[Z\right]\right]_{z=1} = E\left[Y^2\right] - E\left[Y\right] \tag{2.0.7}$$

$$= \frac{3z^2}{4} + \frac{1}{2} + \frac{3}{2z^4} + \frac{5}{4z^6} \tag{2.0.8}$$

$$= 4 \tag{2.0.9}$$

$$\left[\frac{d^3}{dz^3}Y\left[Z\right]\right]_{z=1} = E\left[Y^3\right] - 3E\left[Y^2\right] + 2E\left[Y\right] \tag{2.0.10}$$

$$= \frac{3z}{2} - \frac{6}{z^5} - \frac{15}{2z^7} \tag{2.0.11}$$

$$= -12 \tag{2.0.12}$$

$$\left[\frac{d^4}{dz^4}Y\left[Z\right]\right]_{z=1} = E\left[Y^4\right] - 6E\left[Y^3\right] + 11E\left[Y^2\right] - 6E\left[Y\right] \tag{2.0.13}$$

$$= \frac{3}{2} + \frac{30}{z^6} + \frac{105}{2z^8} \tag{2.0.14}$$

$$= 84 \tag{2.0.15}$$

By manipulating equation (2.0.4) ,(2.0.7) ,(2.0.10) and (2.0.13)

$$E\left[Y^4\right] = \left[\frac{d^4}{dz^4}Y\left[Z\right]\right]_{z=1} + 6\cdot\left[\frac{d^3}{dz^3}Y\left[Z\right]\right]_{z=1} + 7\cdot\left[\frac{d^2}{dz^2}Y\left[Z\right]\right]_{z=1} + 25\cdot\left[\frac{d}{dz}Y\left[Z\right]\right]_{z=1} \tag{2.0.16}$$

$$\therefore \mathbf{E\left[Y^4\right] = 84 + 6\cdot(-12) + 7\cdot 4 + 25\cdot 0 = 40}$$