

Distributed Control Systems-EE601

Course Project

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Problem statement:

Apply the cooperative control law in on a group of two-input-two-output double integrator systems (represents the feedback-linearized version of a two-wheeled mobile robot having non-holonomic constraints) to achieve group formation tracking specified by the formation reference vector **r** and formation configuration vector **h**.

The cooperative control law is given by,

$$\begin{cases} u_i = (c_i + \rho_i)K\xi_i + \gamma_i - \mu f(\xi_i) \\ \dot{c}_i = \xi_i^T \Gamma \xi_i \end{cases} \quad \forall i \in F$$

Theory:

The objective of our project is to perform the coordination control of a multi-agent MIMO system. The above-mentioned control law is applied to achieve group formation tracking specified by the formation reference vector **r** and formation configuration vector **h**.

In the cooperative control law,

u_i = exogenous input given to the i^{th} leader which is independent of all agents in the system

ζ_i = group formation tracking error

c_i = coupling weight assigned to the i^{th} agent in the system

$\rho_i = \zeta_i^T P \zeta_i$ =continuously differentiable function of ζ_i

$K = R^{-1}B^T P$ =optimal parameter of the controller

$\Gamma = PBR^{-1}B^T P$ =parameter of the controller

where **P** is the solution of the algebraic Riccati equation (ARE) and **R**=1

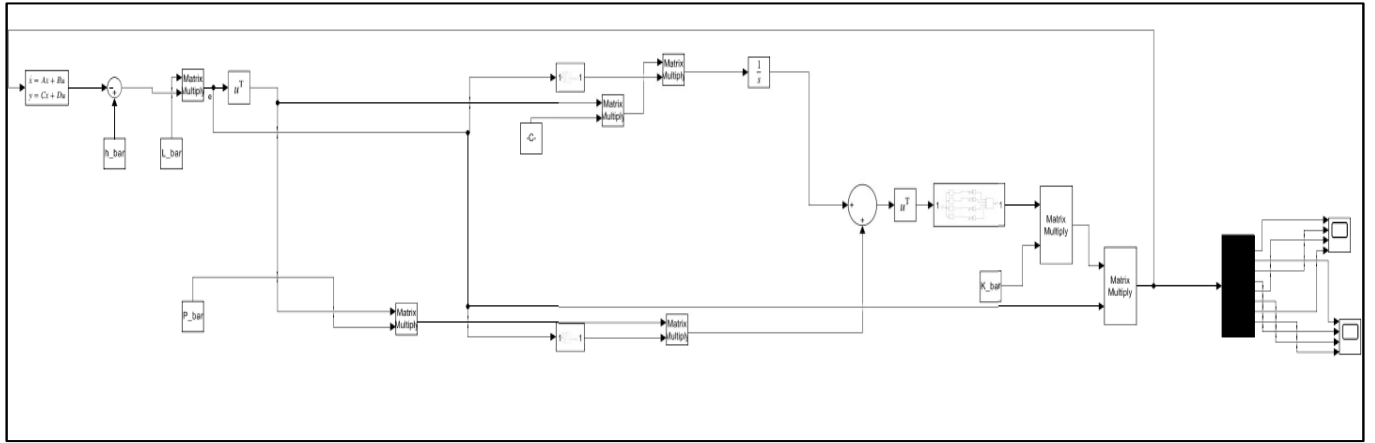
MATLAB Code:

Steps followed for writing the code:

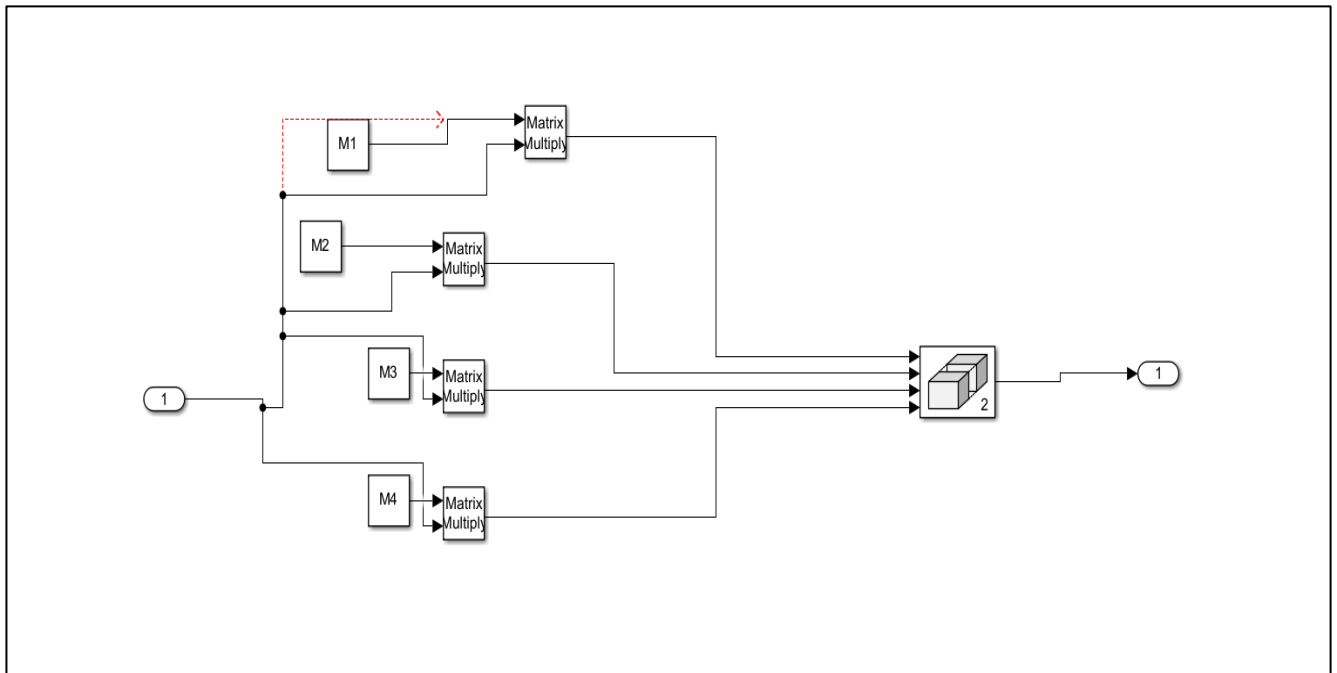
- N denotes the number of agents in the double integrator system. In our case, $N=4$.
- $n=2$ denotes the number of states of each agent that are being tracked in our double integrator system.
- By representing the system in state space representation, we obtained the matrices A, B, C and D which we later converted to A_bar , B_bar , C_bar and D_bar by applying the Kronecker product with an identity matrix of order N.
- We assumed a fully connected graph for representing the formation and computed the Laplacian matrix of the system.
- Here, h represents the formation vector which has a displacement input and a velocity input.
- We implemented the first part of the control where we computed K, c_i , ρ_i by using the formulae from the reference paper provided.
- Here K is computed by using solving ARE for A, B, $Q=Identity$ matrix, $R=1$. We used the icare formula in MATLAB for solving the ARE.
- P matrix which is the solution of ARE is then used to compute Γ matrix. Here, $\Gamma=PBR^{-1}B^TP$.
- c_i which denotes coupling gain is computed by using derivative $(c_i)'=\zeta^T\Gamma\zeta$ which represents time-varying coupling gain.
- This is sent through the integrator to obtain c_i .
- ρ_i is computed by using $\rho_i=\zeta^TP\zeta$.
- These are also adjusted by using the Kronecker product.
- After this, a Simulink model was built for this system and the results were obtained.

Simulation in Simulink:

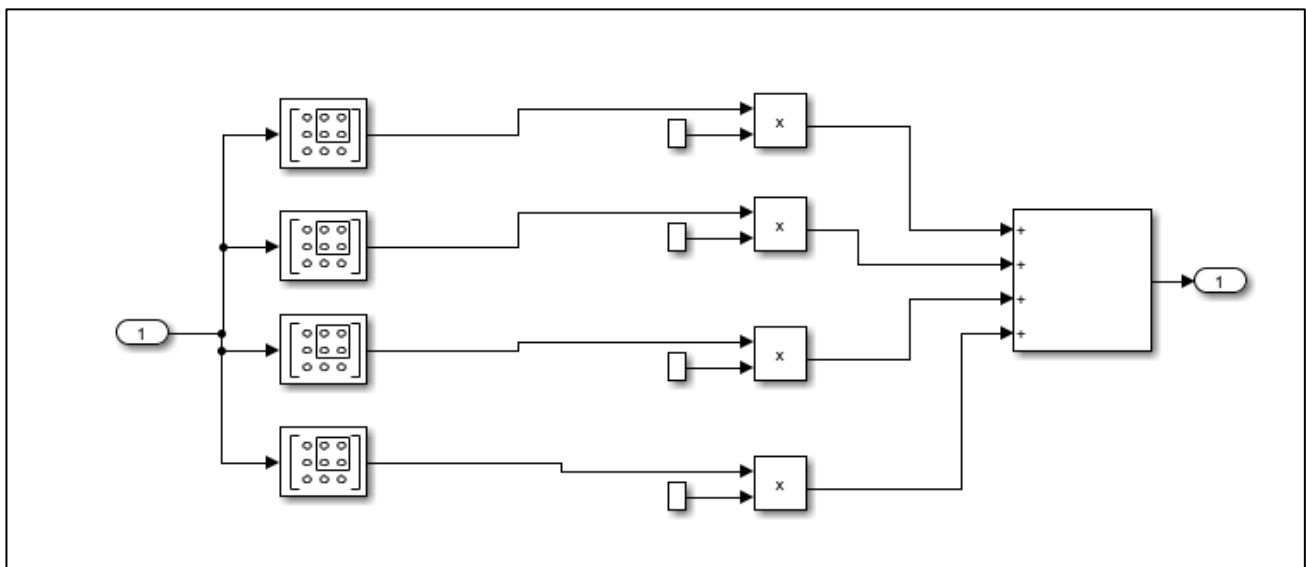
The Simulink model is as follows:



Simulink Model



Block for obtaining individual c_i 's



Matrix Multiplication block

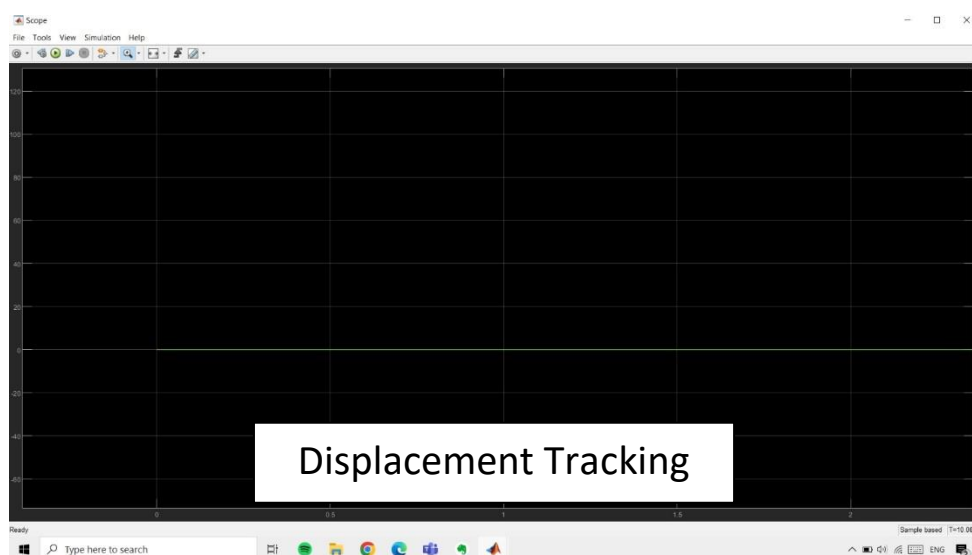
Each block required for the feedback system is added to the simulation.

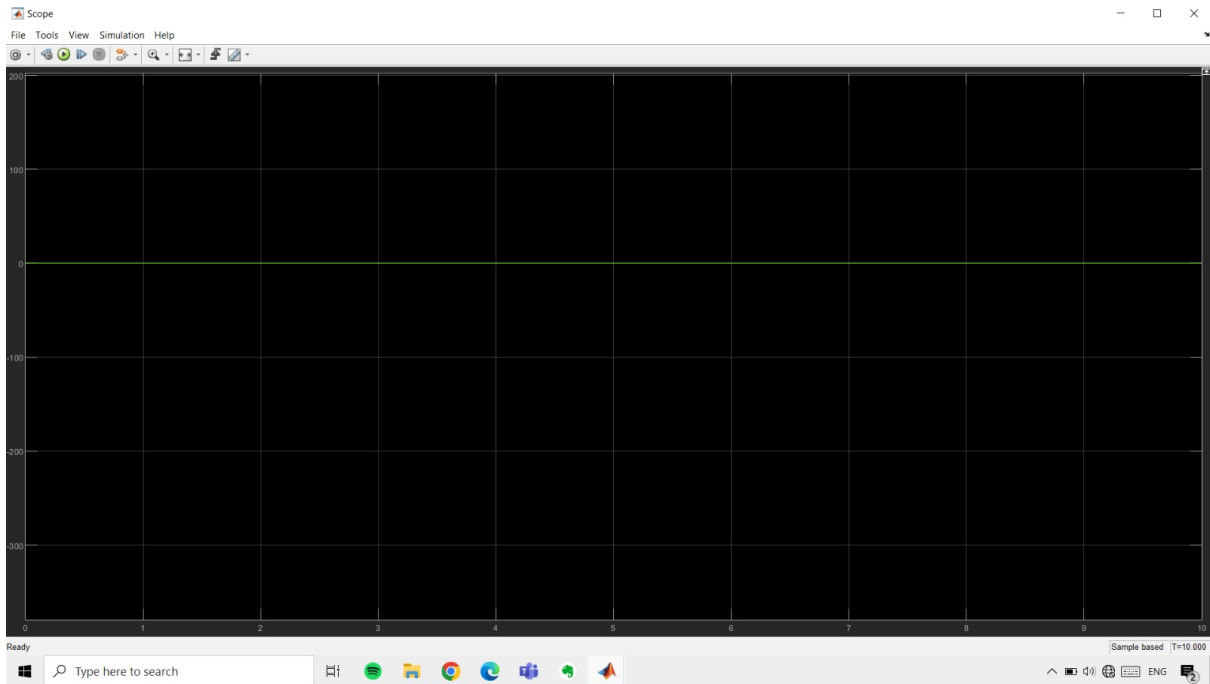
Steps followed in simulation formation:

- The state space block is added with matrices A_{bar} , B_{bar} , C_{bar} and D_{bar} .
- Similarly, we placed h_{bar} and L_{bar} blocks wherever necessary.
- While implementing the equation of c_i since c_i is for each (i) all the c_i 's were getting added after the product.
- So, we have used the subsystem to make sure that we get each c_i separately.
- A similar process was followed to get the matrix of ρ_i 's.
- There are four agents, so we got 4 c_i 's. But to multiply $c_i + \rho_i$ with K_{bar} which is 8×8 we need the c_i matrix in the form of $[c_1 \ c_1 \ c_2 \ c_2 \ c_3 \ c_3 \ c_4 \ c_4]$ and ρ_i similarly.
- For this, we used a subsystem whose formation is shown in the above figures in this report.
- This made sure that the ultimate Kronecker product which we get is correct.
- Then we added the scope to separately track the displacement and velocity of each agent.

Simulation result for different inputs:

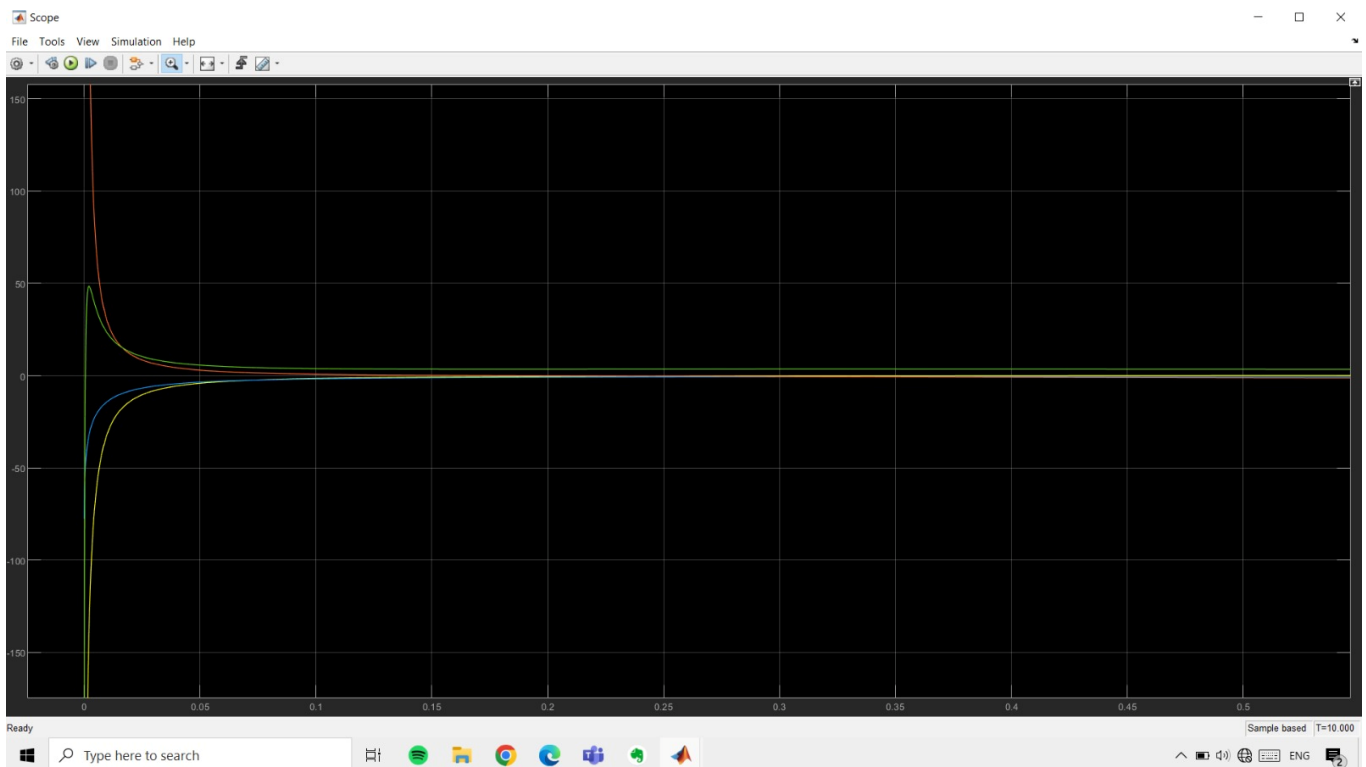
1. When we take h_{bar} as $[1;2;1;2;1;2;1;2;]$, we will get product of h_{bar} and L_{bar} as zero. So, we can track that displacement and velocity to be zero at every point.



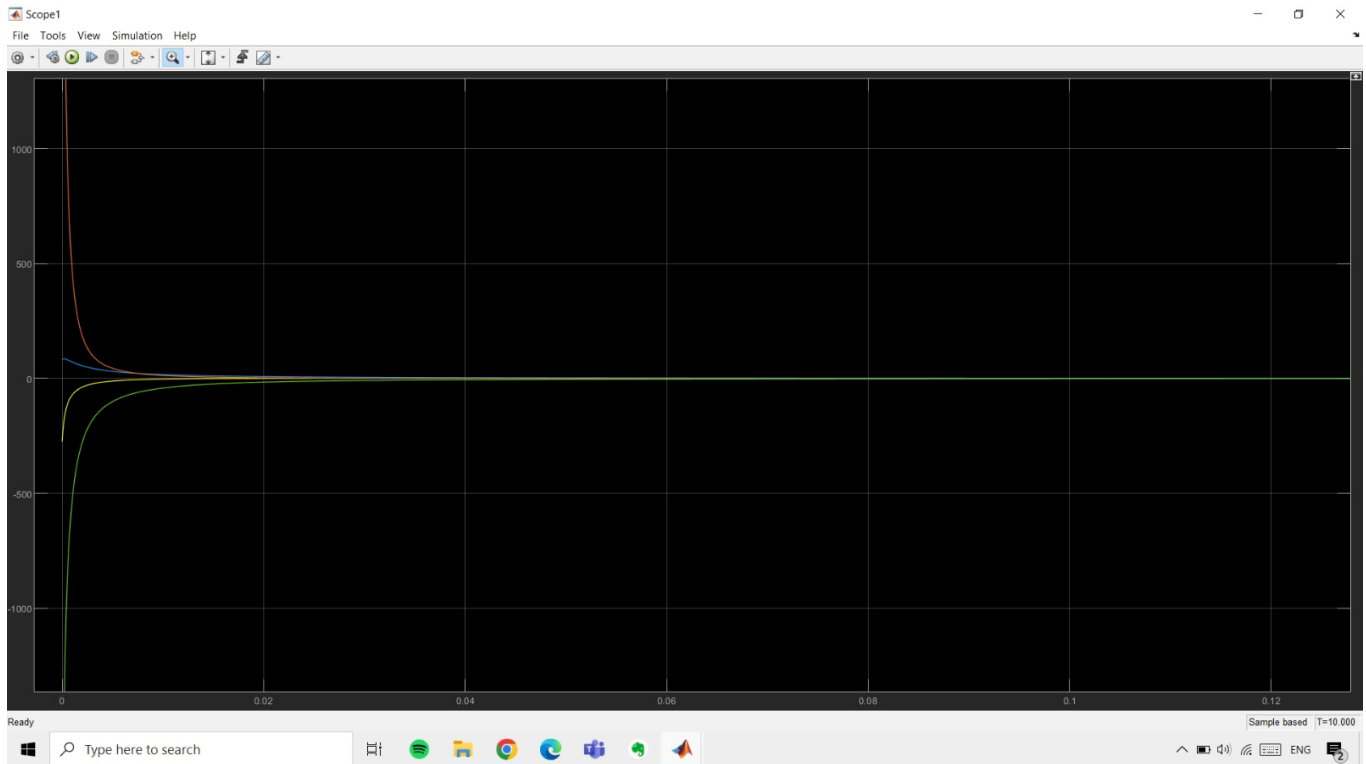


Velocity Tracking

- When $h_bar = [2;4;3;5;7;6;5;1;]$ then we would get a non-zero product of L_bar and h_bar . So, agents will reach a consensus from non-zero initial conditions.



Displacement Tracking



Velocity Tracking

Conclusion:

- Using the above code and Simulink model, the results were obtained.
- It is evident that the consensus condition is achieved depending on the formation configuration vector.

References:

1. Junyan Hu, Parijat Bhowmick, and Alexander Lanzon, *"Distributed Adaptive Time-Varying Group Formation Tracking for Multiagent Systems With Multiple Leaders on Directed Graphs"*, *IEEE Transactions on Control of Network Systems*, Vol. 7, No. 1, March 2020.

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