Roll No.:.........................

# National Institute of Technology, Delhi

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**Name of the Examination: B. Tech. End Semester Examination February 2023**
**(Delayed Autumn Semester)**

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| **Branch** | **: CSE, ECE, EE, ME** | **Semester** | **: 1st** |
| **Title of the Course** | **: Advanced Calculus / Engineering Mathematics I** | **Course Code** | **: MALB 101 / MALB 103** |

**Time: Three Hours** **Maximum Marks: 50**

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## Section A (10 Marks)

**Section A contains Question number 1 to 10 of 01 Mark each.**
**Multiple options may be correct in MCQ's.**

Q.1. Which of the following series are divergent

(A). $\sum_{n=1}^{\infty} \frac{1}{2n(2n+1)}$ (B). $\sum_{n=1}^{\infty} \frac{1+3n^2}{1+n^2}$ (C). $\sum_{n=1}^{\infty} \frac{n}{\sqrt{4n^2+1}}$ (D). $\sum_{n=1}^{\infty} \frac{3n+1}{3n^2-2}$

Q.2. Radius of convergence of series $\sum_{n=1}^{\infty} (ln n)x^n$ is

(A). 2 (B). 0 (C). 1 (D). Infinite

Q.3. The norm of the partition P = {0, 0.2, 0.5, 0.7, 0.9, 1.4, 1.7, 2} is

(A). 1.4 (B). 0.2 (C). 0.5 (D). 0.9

Q.4. The value of the integral $\int_0^1 \int_0^{2-x} \int_0^{2-x-y} dz dy dx$ is

(A). 7/4 (B). 7/6 (C). 5/6 (D). 1/4

Q.5. Which of the following fields are conservative

(A). $\vec{F} = yz\vec{i} + xz\vec{j} + xy\vec{k}$ (B). $\vec{F} = y\sin z\vec{i} + x\sin z\vec{j} + xy\cos z\vec{k}$

(C). $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ (D). $\vec{F} = (y+z)\vec{i} + (x+z)\vec{j} + (x+y)\vec{k}$

Q.6. The inflection points for the function $y = \frac{4}{3}x - \tan x$ are __________.

Q.7. Given $f(x,y) = x^2 + kxy + y^2$. For what values of k the second derivative test locating extreme values is inconclusive?

Q.8. The double integral $\int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3} dy dx$ by changing the order of integration is written as _________.

Q.9. The directional derivative of the function $f(x, y, z) = xy + yz + zx$ at the point $P(1, -1, 2)$ in the direction of $\vec{u} = 3\vec{i} + 6\vec{j} - 2\vec{k}$ is ____________

Q.10. The curl of the vector $\vec{F}(x, y, z) = 2x[y^2 + z^3]\vec{i} + 2x^2y\vec{j} + 3x^2z^2\vec{k}$ is ______.

## Section B (20 Marks)

**Section B contains 5 questions (Question number 11 to 15) of 04 Marks each.**

Q.11. Find the absolute maxima and minima of the function $f(x, y) = x^2 + y^2 + xy - 6x$ on the rectangular plate $0 \leq x \leq 5, -3 \leq y \leq 3$.

Q.12. Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1 + 2 + 3 + \ldots + n}{1^2 + 2^2 + 3^2 + \ldots + n^2} x^n$.

Q.13. Find the volume of the solid generated by revolving the region bounded by curves $y = e^{-x}$, $y = 0, x = 0, x = 1$ about the x-axis.

Q.14. Find the volume of the region bounded above by the paraboloid $z = x^2 + y^2$ and below by the triangle enclosed by the lines $y = x,\ x = 0, x + y = 2$ in the xy-plane.

Q.15. Evaluate the surface integral of the vector field $\vec{F} = z^2\vec{i} + x\vec{j} - 3z\vec{k}$ in the outward normal direction away from z-axis through the cone $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq 1$.

## Section C (20 Marks)

**Section C contains 4 questions (Question number 16 to 19) of 05 Marks each.**

Q.16. Define ratio and root test for convergence of series. Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{2^n n! n!}{(2n)!}$.

Q.17. Convert the integral $\int_{-1}^{1} \int_{0}^{\sqrt{1-y^2}} \int_{0}^{x} (x^2 + y^2) dz dx dy$ into an equivalent integral in cylindrical coordinates and hence evaluate the integral.

Q.18. Using Green's theorem in a plane calculate the counterclockwise circulation and outward flux for the force field $\vec{F} = x^3y^2\vec{i} + \frac{1}{2}x^4y\vec{j}$ on the region bounded by curves $y = x,\ y = x^2 - x$.

Q.19. Calculate the flux of the curl of the field $\vec{F} = 2z\vec{i} + 3x\vec{j} + 5y\vec{k}$ across the surface $S : \vec{r}(r, \theta) = r\cos\theta\vec{i} + r\sin\theta\vec{j} + (4 - r^2)\vec{k},\ 0 \leq r \leq 2, 0 \leq \theta \leq 2\pi$ in the direction of the outward unit normal.