1 Solve the following recurrence relation.

a) $x(n) = x(n-1) + 5$ for $n > 1$ with $x(1) = 0$

A. Step1: write down the first two terms to identify the pattern

$x(1) = 0$

$x(2) = x(1) + 5 = 5$

$x(3) = x(2) + 5 = 10$

$x(4) = x(3) + 5 = 15$

Step2: Identify the pattern (or) the general term

→ the first term $x(1) = 0$

the common difference $d = 5$

The general formula for the n^{th} term of an AP is

$$x(n) = x(1) + (n-1) \cdot d$$

Substituting the given values

$$x(n) = 0 + (n-1) \cdot 5 = 5(n-1)$$

The solution is $x(n) = 5(n-1)$

b) $x(n) = 3x(n-1)$ for $n > 1$ with $x(1) = 4$

A Step1: write down the first two terms to identify the pattern

$x(1) = 4$

$x(2) = 3x(1) = 3 \cdot 4 = 12$

$x(3) = 3x(2) = 24$

$x(4) = 3x(3) = 36$

Step2: Identify the general term

→ the first term $x(1) = 4$

→ The common ratio $r = 3$

The general formula for the n^{th} term of a GP is

$$x(n) = x(1) \cdot r^{n-1}$$

Substituting the given values

$$x(n) = 4 \cdot 3^{n-1}$$

The solution is $x(n) = 4 \cdot 3^{n-1}$

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c) $x(n) = x(n/2) + n$ for $n > 1$ with $x(1) = 1$ (Solve for $n = 2^k$)

For $n = 2^k$, we can write recurrence in terms of k.

1. substitute $n = 2^k$ in the recurrence

$$x(2^k) = x(2^{k-1}) + 2^k$$

2. write down the first few terms to identify the pattern

$$x(1) = 1$$
$$x(2) = x(2^1) = x(1) + 2 = 3$$
$$x(4) = x(2^2) = x(2) + 4 = 3 + 4 = 7$$
$$x(8) = x(2^3) = x(4) + 8 = 7 + 8 = 15$$

3. Identify the general term by finding the pattern we observe that. $x(2^k) = x(2^{k-1}) + 2^k$

we sum the series.

$$x(2^k) = 2^k + 2^{k-1} + 2^{k-2} + \cdots$$

Since $x(1) = 1$

$$x(2^k) = 2^k + 2^{k-1} + 2^{k-2} + \cdots$$

The geometric series with the term $a = 2$ and the last term 2^k except for the additional +1 term. The sum of a geometric series s with ratio $r = 2$ is given by

$$S = a\frac{r^n - 1}{r - 1}$$

Here $a = 2$, $r = 2$ and $n = k$

$$S = \frac{2^{2k} - 1}{2 - 1} = 2(2^k - 1) = 2^{k+1}$$

adding the +1 term

$$x(2^k) = 2^{k+1} - 2 + 1 = 2^{k+1} - 1$$

solution is

$$x(2^k) = 2^{k+1} - 1$$

d) $x(n) = x(n/3) + 1$ for $n > 1$ with $x(1) = 1$ (solve for $n = 3^k$)

For $n = 3k$, we can write the recurrence in terms of k

1) Substitute $n = 3^k$ in the recurrence $x(3^k) = x(3^{k-1}) + 1$

2. write down the first few terms to identify the pattern

$x(1) = 1$

$x(3) = x(3^1) = x(1)+1 = 1+1 = 2$

$x(9) = x(3^2) = x(3)+1 = 2+1 = 3$

$x(27) = x(3^3) = x(9)+1 = 3+1 = 4$

3. identify the general term:

we observe that

$x(3^k) = x(3^{k-1})+1$

Sum up the series

$x(3^k) = 1+1+1+\cdots$

$x(3^k) = k+1$

The solution is $x(3^k) = k+1$

2. Evaluate the following recurrence complexity

i) $T(n) = T(n/2)+1$ where $n = 2^k$ for all $k \geq 0$

The recurrence relation can be solved using iteration method

1) substitute $n = 2^k$ in the recurrence

2) iterate the recurrence

For $k=0$: $T(2^0) = T(1) = T(1)$

$k=1$: $T(2^1) = T(1)+1$

$k=2$: $T(2^2) = T(8) = T(n)+1 = T(1)+2+1 = T(1)+2$

$k=3$: $T(2^3) = T(8) = T(n)+1 = (T(1)+2)+1 = T(1)+3$

3) generalize the pattern

$T(2^k) = T(1)+k$

Since $n = 2^k$, $k = \log_2 n$

$\therefore T(n) = T(2^k) = T(1)+\log_2 n$

4) Assume $T(1)$ is a constant c

$T(n) = c+\log_2 n$

The solution is $T(n) = O(\log n)$

ii) $T(n) = T(n/3) + T(2n/3) + cn$. where c is constant and n is input size

The recurrence can be solved using the master's theorem

For divide and conquer reccurrence of the form

$T(n) = aT(n/b)+F(n)$

Where $a=2$, $b=3$ and $F(n) = cn$

Lets determine the value of $\log_b^a$

$$\log_b a = \log_3 2$$

using the Properties OF logarithm

$$\log_3 2 = \frac{\log 2}{\log 3}$$

now we Compare $F(n) = Cn$ with $n^{\log_3 2}$

$$F(n) = O(n)$$

$$n = n^1$$

Since $\log_3^2$ we are in the third case OF the master's theorem

$F(n) = O(n^c)$ with $c > \log_b a$

The solution is : $T(n) = O(F(n)) = O(cn) = O(n)$

3. Consider the Following reccurence algorithm ?

```
min [A(0... n-2)]
if n=1 return A[0]
else temp = min ( A[0... n-2])
  if temp <= A(n-1) return temp
  else  return A[n-1]
```

a) what does this algorithm compute?

The given algorithm min[A[0,...n-2]] Computes the minimum value in the array 'A' From index '0' For $n-1$. it does this by recursively Finding the minimum value in the sub array A[0..n-2] and then Comparing it with the last element A[n-1] to determine the overall maximum value.

b) setup a reccurence relation For the algorithm basic operation count and solve it

The solution is

$$T(n) = n$$

This means the algorithm perform n basic operations For an input array OF size n.

4.) Analyze the order of growth

i) $F(n) = 2n^2 + 5$ and $g(n) = 7n$ use the $\Omega\, g(n)$ notation

To analyze the order of growth and use the Ω notation, we need to compare the given function $F(n)$ and $g(n)$

given Functions:

$$F(n) = 2n^2 + 5$$

$$g(n) = 7n$$

order of growth using $\Omega(g(n))$ notation

The notation $\Omega(g(n))$ describes a lower bound on the growth rate that for sufficiently large n, $F(n)$, grows at least as Far as $g(n)$

$$F(n) = c \cdot g(n)$$

less analyze $F(n) = 2n^2 + 5$ with respect to $g(n) = 7n$

1) identify Dominant terms:

→ The dominant term in $F(n)$ is $2n^2$ since it grows faster then the constant term as n increases

→ the dominant term in $(g(n))$ is $7n$

2) establish the inequality.

→ we want to Find constants c and n_0 such that:

$$2n^2 + 5 \geq c \cdot 7n \text{ For all } n \geq n_0$$

3) simplify the inequality.

→ ignore the lower order term 5 For larger

$$2n^2 \geq 7cn$$

→ Divide both sides by n

$$2n \geq 7c$$

→ solve For n:

$$n \geq 7c/2$$

4) choose constants

let $c = 1$

$$n \geq \frac{7 \cdot 1}{2} = 3.5$$

∴ For $n \geq n_0$ the inequality holds.

$2n^2+5 \geq 7n$ for all $n \geq n$

we have shown that there exist constant $c=1$ and $n_0=n$

such that for all $n \geq n_0$

$$2n^2+5 \geq 7n$$

Thus, we can conclude that:

$$f(n) = 2n^2+5 = \Omega(7n)$$

in Ω notation, the dominant term $2n^2$ in $f(n)$ clearly grows

faster than $7n$ Hence $f(n) = \Omega(n^2)$

However, for the specific comparison asked $f(n) = \Omega(7n)$ is

also correct

showing that $f(n)$ grows at least as fast as $7n$.