1. Big omega notation: prove that $g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$

A) $g(n) \geq c \cdot n^3$

$g(n) = n^3 + 2n^2 + 4n$

For finding constants c and n_0

$n^3 + 2n^2 + 4n \geq c \cdot n^3$

Divide both sides with n^3

$$1 + \frac{2n^2}{n^3} + \frac{4n}{n^3} \geq c$$

$$1 + \frac{2}{n} + \frac{4}{n^2} \geq c$$

Here $2/n$ and $4/n^2$ appraches 0

$1 + 2/n + 4/n^2 \approx 1$

Example $c = 1/2$

$1 + 2/n + 4/n^2 \geq 1/2$

$1 + 2/n + 4/n^2 \geq 1$ $\quad (1 \geq 1/2, n \geq 1)$

$1 + 2/n + 4/n^2 \geq 1/2$ $\quad (n \geq 1, n_0 = 1)$

Thus, $g(n) = n^3 + 2n^2 + 4n$ is indeeded $\Omega(n^3)$

2. Big theta notation: Determine a shether $h(n) = 4n^2 + 3n$ is $\Theta(n^2)$ or not

A. $c_1 n^2 \leq h(n) \leq c_2 n^2$

In upper bound $h(n)$ is $O(n^2)$

In lower bound $h(n)$ is $\Omega(n^2)$

upper bound $(O(n^2))$:

$h(n) = 4n^2 + 3n$

$h(n) \leq c_2 n^2$

$4n^2 + 3n \leq c_2 n^2$

$4n^2 + 3n \leq 5n^2$

Let's $C_2 = 5$

Divide both sides by n^2

$4 + 3/n \leq 5$

$h(n) = 4n^2 + 3n$ is $O(n^2)$ $(C_2 = 5, n_0 = 1)$

lower bound: $h(n) = 4n^2 + 3n$

$h(n) \geq C_1 n^2$

$4n^2 + 3n \geq C_1 n^2$

Let's $C_1 = 4 \Rightarrow 4n^2 + 3n \geq 4n^2$

Divide both sides by n^2

$4 + 3/n \geq 4$

$h(n) = 4n^2 + 3n$ $(C_1 = 4, n_0 = 1)$

$h(n) = 4n^2 + 3n$ is $\Theta(n^2)$

3. let $F(n) = n^3 - 2n^2 + n$ and $g(n) = n^2$ show whether $F(n) = \Omega(g(n))$ is true or False and justify your answer

A. $F(n) \geq C \cdot g(n)$

Substituting $F(n)$ and $g(n)$ into this inequality we get

$n^3 - 2n^2 + n \geq C(-n^2)$

Find C and n_0 holds $n \geq n_0$

$n^3 - 2n^2 + n \geq -Cn^2$

$n^3 - 2n^2 + n + Cn^2 \geq 0$

$n^3 + (C-2)n^2 + n \geq 0$

$n^3 + (C-2)n^2 + n \geq 0$ $n^3 \geq 0$

$n^3 + (1-2)n^2 + n = n^3 - n^2 + n \geq 0$ $(C = 2)$

$F(n) = n^3 - 2n^2 + n$ is $\Omega(g(n)) = \Omega(-n^2)$

Therefore the statement $F(n) = \Omega(g(n))$ is True.

4. Determine whether $h(n) = n\log n + n$ is in $\Theta(n\log n)$. Prove a rigorous proof for your conclusion.

A

$$c_1 n\log n \le h(n) \le c_2 n\log n$$

upper bound:

$$h(n) \le c_2 n\log n$$

$$h(n) = n\log n + n$$

$$n\log n + n \le c_2 n\log n$$

Divide both sides by $n\log n$

$$1 + n/n\log n \le 2$$

$$1 + 1/\log n \le c_2 \quad \text{(Simplify)}$$

$$1 + 1/\log n \le 2 \quad (c_2 = 2).$$

Then $h(n)$ is $O(n\log n)$ $(c_2 = 2, n_0 = 2)$

Lower bound:

$$h(n) \ge c_1 n\log n$$

$$h(n) = n\log n + n$$

$$n\log n + n \ge c_1 n\log n$$

Divide both sides by $n\log n$

$$1 + n/n\log n \ge c_1$$

$$1 + 1/\log n \ge c_1 \quad \text{(simplify)}$$

$$1 + 1/\log n \ge 1 \quad c_1 = 1$$

$$1/\log n \ge 0 \quad \text{For all } n > 1$$

$h(n)$ is $\Omega(n\log n)$ $(c_1 = 1, n_0 = 1)$

$h(n) = n\log n + n$ is $\Theta(n\log n)$

15. Solve the following Recurrence Relations and find the order of growth for solution $T(n) = 4T(n/2) + n^2$, $T(1) = 1$.

A.

$T(n) = 4T(n/2) + n^2$, $T(1) = 1$

$T(n) = aT(n/2) + F(n)$

$a = 4$, $b = 2$, $F(n) = n^2$

Applying master theorem

$T(n) = aT(n/b) + F(n)$

$F(n) = O(n^{\log_b a - 1})$. $\left(\begin{matrix} \epsilon > 0 \\ T(n) = \Theta(n^{\log_b a}) \end{matrix} \right)$

$F(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} \log n)$

$F(n) = \Omega(n^{\log_b a + 1})$, then $T(n) = F(n)$

Calculating $\log_b a$:

$\log_b a = \log_2 4 = 2$

$F(n) = n^2 = \Theta(n^2)$ (Comparing $F(n)$ with $n^{\log_b a}$)

$F(n) = \Theta(n^2) = \Theta(n^{\log_b a})$. (case 2)

$T(n) = 4T(n/2) + n^2$

$T(n) = \Theta(n^{\log_b a} \log n) = \Theta(n^2 \log n)$

order of growth

$T(n) = 4T(n/2) + n^2$ with $T(1) = 1$ is $\Theta(n^2 \log n)$