WORK, POWER, ENERGY

① Work

Const force | Variable force

$W = Fs\cos\theta$

$dW = \int_{x_1}^{x} F(x)\,dx$

$1J = 10^7$ erg

$W = -ve$ at $\theta = 180°$

→ Work depends on frame of reference.

② Work-energy theorem

Net work done = change in K.E

$W_C + W_{N\cdot C} + W_{ext\,F} = \Delta K.E$ $\quad \boxed{W_C = -\Delta U}$

$W_{N\cdot C} + W_{ext\,F} = \underbrace{\Delta KE + \Delta U}_{\text{change in mechanical energy}}$

→ Conservative forces are path independent of work.

Ex: coulombic, gravitational, electrostatic, magnetic, spring forces

→ Non-conservative forces are path dependent. ← $W \neq 0$

Ex: normal, viscous, frictional forces, etc.

→ Applicable in both inertial & non-inertial frames

③ Law of conservation of energy:

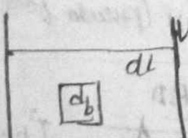
$W_{N\cdot C} = W_{ext} = 0$

$\Delta KE + \Delta U = 0$

$(T\cdot E)_i = (T\cdot E)_f$

④ Application:

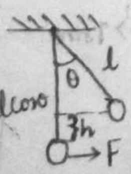
ⓐ



$W_g + W_b + W_{F\cdot l} = 0$

$W_{Fl} = Mg\left[1 - {}^{1}/_{R.D}\right]$ $\quad \ell = \frac{d_b}{d_l}$

ⓑ



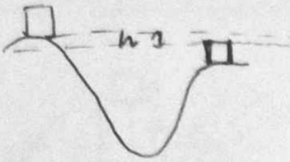
$W_g + W_T + W_F = 0$

$\boxed{W_g = -Mgl[1-\cos\theta]}$

$W_f = Fl\sin\theta$

$\boxed{W_f = mgl(1-\cos\theta) - Fl\sin\theta}$

⑤ How to apply ΔU

ⓐ 

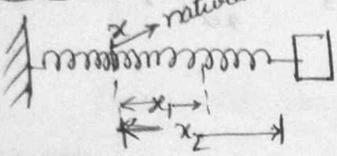
$A \to B \Rightarrow \Delta U = U_f - U_i = -Mgh$

$B \to A \Rightarrow \Delta U = Mgh$

$W_C = -\Delta U$

$W_{Ext} = +\Delta U$

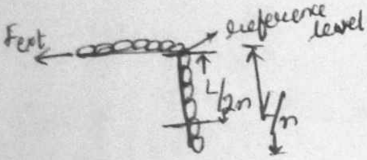
ⓑ Spring force:



$\Delta U = \frac{1}{2}K(x_1^2 - x_2^2)$

$W_{spring} = -\Delta U$
$= \frac{1}{2}Kx_1^2 - \frac{1}{2}Kx_2^2$

ⓒ external force required to pull chain:



$\Delta U = U_2 - U_1$

$W_{ext} = +\Delta U$

$W = \int_y^0 mgy\, dy$

U has two parts

part 1 – hanging part

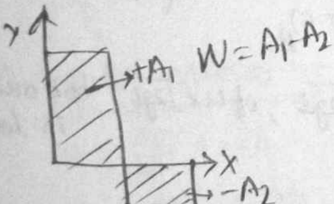
$U_1 = -\frac{Mgl}{2N^2}$

part 2 → zero

$U_2 = 0 \Rightarrow U_2 - U_1 = \frac{Mgl}{2N^2}$

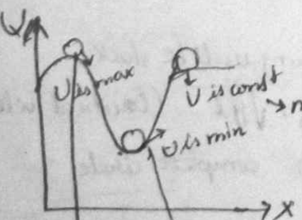
mass of hanging part $m = \frac{M}{L}$

⑦ Graph



$W = A_1 - A_2$

⑧ Conservative force $F = -\frac{dU}{dx}$



↓
NO SHM
↓
$\frac{d^2U}{dx^2} = 0$

U is max
unstable eq.
⇓
NO SHM

$\frac{dU}{dx} = 0$

$\frac{d^2U}{dx^2} < 0 = -ve$

U is min
stable eq.
⇓
SHM

$\frac{dU}{dx} = 0$

$\frac{d^2U}{dx^2} > 0$ / +ve

⑨ Power

ⓐ

$P_{avg} = \frac{\Delta W}{\Delta t}$ $\qquad$ $P_{inst} = \frac{dW}{dt}$

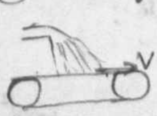
$P_{avg} = \frac{P_{inst}}{2}$

ⓑ Power of machine gun

$P = \frac{\frac{1}{2}nmv^2}{t}$

ⓒ $\eta = \frac{o/p\ power}{i/p\ power} \times 100$

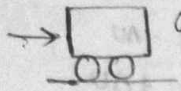
ⓓ Conveyor belt



dm/dt $\qquad$ $P = Fv$

$\boxed{P = v^2 \frac{dm}{dt}}$

(a) const power [not const force]



const power $u=0$

In time t

$P = f \cdot v$

$P = m \frac{dv}{dt} v$

$v = \sqrt{\frac{2Pt}{M}} \qquad x = \frac{2}{3}\sqrt{\frac{2P}{M}}\, t^{3/2} \qquad x \propto t^{3/2}$

(b) variable power

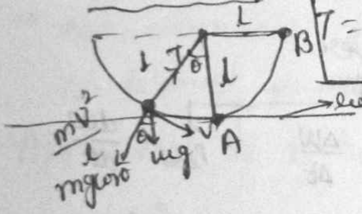
if $P = 2t$

$v = \sqrt{\frac{2}{M}}\, t \quad v \propto t$

$x = \sqrt{\frac{2}{M}}\, \frac{t^2}{2} \quad x \propto t^2$

Vertical circular motion

① Lower half:



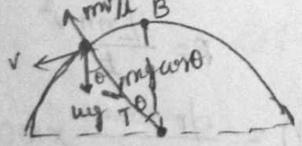
$\boxed{T = \frac{mv^2}{l} + mg\cos\theta}$

$v = \sqrt{u^2 - 2gl(1-\cos\theta)}$

$(T \cdot E)_A = (T \cdot E)_B$

$u = \sqrt{2gl}$, if $u < \sqrt{2gl}$ bob oscillates in lower half

② Upper half:



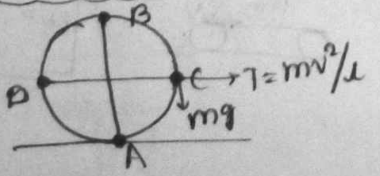
$T = \frac{mv^2}{l} - mg\cos\theta$

if $T=0$, string will be slack

$v_B = \sqrt{gl}$ (critical velocity)

$T \geq 0$ complete circle

③ Complete circle:



At A

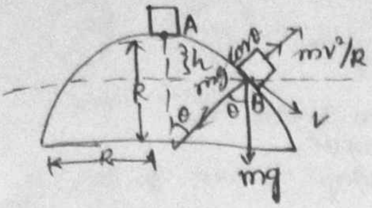
$u = \sqrt{5gl}$

$T_A = 6Mg$

At C

$v_C = \sqrt{3gl}$

④ Hemispherical bowl:



$(T \cdot E)_A = (T \cdot E)_B$

$h = R(1 - \cos\theta)$

if $N = 0$

here $\frac{Mv^2}{R} = Mg\cos\theta$

$v = \sqrt{Rg\cos\theta}$

Horizontal circular motion

① Kinematics

ⓐ angular displacement (θ), ang. velocity (ω), ang. accl. (α)

ⓑ $\omega_{avg} = \frac{\Delta\theta}{\Delta t}$ $\omega_{inst} = \frac{d\theta}{dt}$ $1\,rpm = \frac{2\pi}{60}\,rad/s$

ⓒ $\alpha = \frac{d\omega}{dt}$ SI = rad/s²

ⓓ $V = r\omega$

$\frac{dv}{dt} = r\frac{d\omega}{dt} \Rightarrow a_t = r\alpha$ → tangential accel.

ⓔ a_r (or) $N = \frac{V^2}{R} = R\omega^2$

a_r changes direction.

a_t changes the speed of the particle.

② Angular velocity of seconds hand:

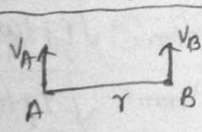
$\omega = \frac{2\pi}{60}\,rad/s$ → seconds

$\omega = \frac{2\pi}{60 \times 60}\,rad/s$ → minutes

$\omega = \frac{2\pi}{12 \times 60 \times 60}\,rad/s$ → hours

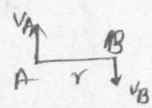
③ Relative angular velocity

ⓐ



$\omega_{AB} = \frac{V_A - V_B}{r}$

ⓑ



$\omega_{AB} = \frac{V_A + V_B}{r}$

④ Uniform circular motion & non-uniform circular motion:

(only direction changes)

$a_t = 0,\ \alpha = 0$

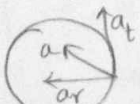
ω = const

V = const.

$\omega \neq$ const.

$V \neq$ const. [speed changes]

$a_t \neq 0,\ \alpha \neq 0$



$a = \sqrt{a_r^2 + a_t^2}$

$a_r = r\omega^2$

$a_t = r\alpha$

$a_t = dv/dt$

(5) Non uniform C.M (α = const)

$$v = u + at \qquad \omega_f = \omega_i + \alpha t$$
$$s = ut + \tfrac{1}{2}at^2 \qquad \theta = \omega_i t + \tfrac{1}{2}\alpha t^2$$
$$v^2 - u^2 = 2as \qquad \omega_f^2 - \omega_i^2 = 2\alpha\theta$$

→ if ω & α are same sense A.C.W (or) C.W α = +ve

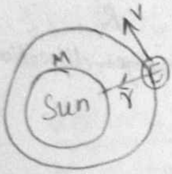
→ if ω & α are opp. sense α = −ve.

($\alpha \neq$ const)

if $\alpha = yt$ rad/s²

$$\frac{d\omega}{dt} = yt \rightarrow \int d\omega = y\int_0^t t\,dt$$

(6) Centripetal force:



$$F_G = mv^2/r$$

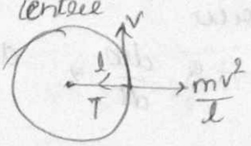
$$\frac{GMm}{r^2} = \frac{mv^2}{\cancel{r}}$$

force acting towards centre

(7) Centrifugal force

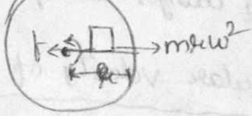
Force acting away from centre

(a)



$$\boxed{T = mv^2/l}$$

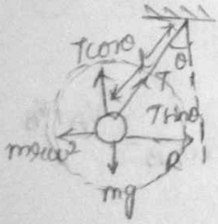
(b)



$$f \geq mR\omega^2$$

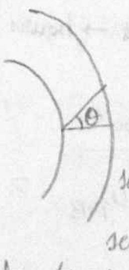
$$\omega = \sqrt{\frac{\mu g}{R}}$$

(8) Conical pendulum:



$$\omega = \sqrt{\frac{g}{l\cos\theta}}$$

(9) Banking of roads:



$\tan\theta = \frac{v^2}{rg}$ → desired velocity

↓ no friction

safe $v_{max} > \sqrt{rg\tan\theta}$

safe $v_{min} < \sqrt{rg\tan\theta}$

No slipping

$$\sqrt{\frac{Rg(\tan\theta - \mu)}{1 + \mu\tan\theta}} \leq v \leq \sqrt{\frac{Rg(\tan\theta + \mu)}{1 - \mu\tan\theta}}$$