3D Geometry

→ distance $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Let distance of the point $P(x,y,z)$ from

ⓐ x-axis $= \sqrt{y^2+z^2}$ ⓑ y-axis $= \sqrt{x^2+z^2}$ ⓒ z-axis $= \sqrt{x^2+y^2}$

xy plane $= |z|$ yz plane $= |x|$ xz plane $= |y|$

→ section formula ⇒ $A(x_1,y_1,z_1)$ $B(x_2,y_2,z_2)$ & $m:n$ ratio

$$\frac{mB \pm nA}{m \pm nA}$$

→ collinear points A, B, C if & iff $AB:BC = (x_1-x_2):(x_2-x_3)$ (or) $\frac{x_1-x_2}{x_2-x_3} = \frac{y_1-y_2}{y_2-y_3} = \frac{z_1-z_2}{z_2-z_3}$

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0 \quad \text{(or)}$$

→ If $A(x_1,y_1,z_1)$ & $B(x_2,y_2,z_2)$ then

YOZ plane divides line segment AB in the ratio $= -x_1 : x_2$ similarly ZOX plane in $-y_1 : y_2$ & XOY plane in AB $= -z_1 : z_2$

→ Internal angular bisector of angle A of △ABC intersect the opp side BC in D and I is incentre of △le then

i) $BD : DC = AB : AC$

ii) $AI : ID = AB + AC : BC$

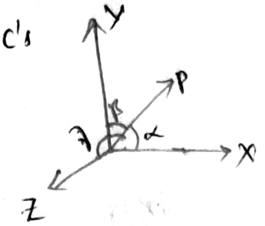
→ G is centroid of △le ABC then $3G = A+B+C$

$(G, OS) = 2:1$ where G is centroid, O is orthocentre, S is circumcentre.

→ G of tetrahedron ABCD divides the line joining any vertex to the centroid of its opp. △le in the ratio 3:1

D.C's & DR's

→ d.c's



$l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma$

i) $l^2+m^2+n^2 = \cos^2\alpha+\cos^2\beta+\cos^2\gamma = 1$

ii) $\sin^2\alpha+\sin^2\beta+\sin^2\gamma = 2$

iii) $\cos 2\alpha+\cos 2\beta+\cos 2\gamma = -1$

→ d.r's - any 3 no's which are proportional to d.c's of line are called d.r's of line. denoted by (a,b,c).

→ (a,b,c) are d.r's & (l,m,n) are d.c's of line. Then

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \pm\frac{1}{\sqrt{a^2+b^2+c^2}}, \quad l = \pm\frac{a}{\sqrt{a^2+b^2+c^2}}$$ similarly m, n also.

→ d.r's of line segment joining $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ is taken as $(x_2-x_1, y_2-y_1, z_2-z_1)$.

d.c's of line segment are $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ are $\pm\left(\frac{x_2-x_1}{AB}, \frac{y_2-y_1}{AB}, \frac{z_2-z_1}{AB}\right)$

every line will have 2 sets of d.c's they are (l,m,n), $(-l,-m,-n)$.

→ Angle b/w 2 lines:

ⓐ $\cos\theta = |l_1l_2+m_1m_2+n_1n_2|$

ⓑ $\sin\theta = \sqrt{\Sigma(l_1m_2-l_2m_1)^2}$

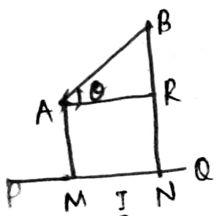
ⓒ if d.r's (a_1,b_1,c_1) and (a_2,b_2,c_2) $\cos\theta = \frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

→ Condition for lines are Lar, llel:

if d.c's or d.r's are given

lines are Lar if $l_1l_2+m_1m_2+n_1n_2=0$, lines are llel if $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

→ Length of projection:



Length of projection of the line segment joining two given points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$

ⓐ X-axis is $p = |x_2-x_1|$

ⓑ Y-axis is $q = |y_2-y_1|$

ⓒ Z-axis is $r = |z_2-z_1|$

ⓓ XY-plane is $d_1 = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ Illy for YZ, ZX planes.

→ $d_1^2 = p^2+q^2,\ d_2^2 = q^2+r^2,\ d_3^2 = p^2+r^2 \Rightarrow d_1^2+d_2^2+d_3^2 = 2(p^2+q^2+r^2)$

$AB^2 = p^2+q^2+r^2,\ AB^2 = \dfrac{d_1^2+d_2^2+d_3^2}{2}$

→ Standard results:

i) D.C's of line equally inclined with coordinate axes are $\left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right)$

ii) Angle b/w any two diagonals of a cube is $\cos^{-1}(1/3)$

iii) Angle b/w a diagonal of cube and diagonal of face is cube is $\cos^{-1}\sqrt{2/3}$

iv) If a line makes angles $\alpha, \beta, \gamma, \delta$ with 4 diagonals of a cube then

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$