Differential Equations

→ Order – highest derivate

degree – power of highest derivate (no power of rational (p/q))

formation of differential eqn

$f(x, y, c_1, c_2, \ldots, c_n) = 0$ be given eqn. diff w.r.t 'x' n times & eliminating $c_1, c_2, \ldots, c_n$ we get required equation

Types of differential eqn.

① Variable & separable

$$\frac{dy}{dx} = \frac{f(x)}{g(y)} \text{ (or) } \frac{dy}{dx} = \frac{g(y)}{f(x)}$$

$$f(x)dx = g(y)dy \text{ (or) } \frac{dx}{f(x)} = \frac{dy}{g(y)}$$

Taking integration on B.S we get required eqn.

② Homogenous differential eqn:

$$\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$$ having same degree ($f(x,y)$, $g(x,y)$)

(i) put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ & convert into variable & separable. (if y/x)

(ii) put $vy = x \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$ & convert into variable & separable (if x/y)

③ Non-homogenous differential eqn.

$$\frac{dy}{dx} = \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}$$

(i) $b_1 = -a_2$ & convert into grouping model.

(ii) $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, put $a_1x + b_1y = t$ & convert variable & separable

(iii) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ put $x = X + h$, $y = Y + k \Rightarrow \frac{dy}{dx} = \frac{dY}{dX}$

$$\frac{dY}{dX} = \frac{a_1(X+h) + b_1(Y+k) + c_1}{a_2(X+h) + b_2(Y+k) + c_2}, \quad \begin{matrix} a_1h + b_1k + c_1 = 0 \\ a_2h + b_2k + c_2 = 0 \end{matrix}$$

④ Linear differential eqn:

(i) $\frac{dy}{dx} + P(x)y = Q(x)$

G.S is $y(I.F) = \int Q(x)(I.F)\,dx + C$ $\quad I.F = e^{\int P(x)dx}$

(ii) $\frac{dx}{dy}+P(y)x=Q(y)$ $\qquad$ $I.F = e^{\int P(y)dy}$

$G.S \Rightarrow x(I.F)=\int Q(y)(I.F)\,dy+C$

(5) **Bernoulli's diff eqn**

(i) $\frac{dy}{dx}+P(x)y=Q(x)y^n$

divide by y^n $\Rightarrow \frac{1}{y^n}\frac{dy}{dx}+P(x)y^{1-n}=Q(x)$

$y^{1-n}=t$ & d w.r.t 'x'

convert into linear

(ii) $\frac{dx}{dy}+P(y)x=Q(y)x^n$

divide by $x^n \Rightarrow \frac{1}{x^n}\frac{dx}{dy}+P(y)x^{1-n}=Q(y)$

put $x^{1-n}=t \Rightarrow$ d. w.r.t 'y'

convert into linear.

Orthogonal trajectory:

(i) $f(x,y,c)=0$

where c is arbitary const.

(ii) d. w.r.t 'x' & eliminate 'c'.

(iii) replace $\frac{dy}{dx}$ by $-\frac{dx}{dy}$

(iv) solve the diff eqn

$\Rightarrow$ (i) $y=c_1e^{\alpha x}+c_2e^{\beta x} \Rightarrow y_2-(\alpha+\beta)y_1+\alpha\beta y=0$

(ii) $y=c_1e^{\alpha x}+c_2e^{\beta x}+c_3e^{\gamma x} \Rightarrow y_3-(\alpha+\beta+\gamma)y_2+(\alpha\beta+\beta\gamma+\gamma\alpha)y_1-\alpha\beta\gamma y=0$

(iii) $y=e^{\alpha x}(c_1x+c_2) \Rightarrow y_2-2\alpha y_1+\alpha^2y=0$

(iv) $y=e^{\alpha x}(c_1x+c_2x^2+c_3) \Rightarrow y_3-3\alpha y_2+3\alpha^2y_1-\alpha^3y=0$

$\rightarrow$ ① $d(x+y)=dx+dy$

② $x\,dy+y\,dx=d(xy)$

③ $x\,dx+y\,dy=\frac{1}{2}d(x^2+y^2)$

④ $\frac{x\,dy-y\,dx}{x^2}=d(y/x)$

⑤ $\frac{y\,dx-x\,dy}{y^2}=d(x/y)$

⑥ $\frac{x\,dy-y\,dx}{x^2+y^2}=d(\tan^{-1}(y/x))$