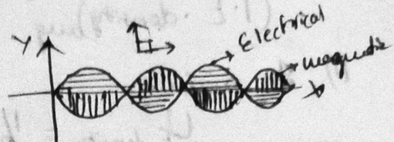
Electromagnetic Waves produced by accelerated charges

ε_0 - permittivity of free space $= 8.854 \times 10^{-12}\, C^2 N^{-1} m^{-2}$

μ_0 = permeability of free space $= 1.257 \times 10^{-6}\, Wb A^{-1} m^{-1}$

→ In free space $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{E}{B} = 3 \times 10^8\, m/s$



In any medium $v = \frac{E}{B} = \frac{1}{\sqrt{\mu \varepsilon}} = c$

R.I, $\mu = c/v = \sqrt{\mu_r \varepsilon_r}$ $\Big|$ $= \frac{c_0}{c_{med}}$ $\quad \mu_r = \frac{\mu}{\mu_0}$, $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$, $\lambda_m = \frac{\lambda}{\mu}$

→ **Wave impedance**

$Z_{free} = \frac{E}{H} = \frac{\mu_0 E}{B} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$, $\quad Z_{med} = \frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_r \mu_0}{\varepsilon_r \varepsilon_0}}$

$\mu = \frac{Z_{med}}{Z_{free\ space}}$

→ **Maxwell's eqn.:**

$I_{displace} = \varepsilon_0 \frac{d\phi_E}{dt} \qquad \phi_E = E A$

Ampere-Maxwell law = $\oint_C \vec{B} \cdot d\vec{l} = \mu_0 (I + I_D) = \mu_0 \left(I + \varepsilon_0 \frac{d\phi_E}{dt}\right)$

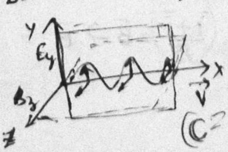
1st eqn: Gauss law in electrostatics $\quad \phi_E = \frac{q}{\varepsilon_0} \quad \oiint \vec{E} \cdot d\vec{s} = q/\varepsilon_0$

2nd eqn: Gauss law in magnetism $\quad \phi_B = 0 \quad \oiint \vec{B} \cdot d\vec{s} = 0$

3rd eqn: Faraday law of EMI $\quad e = \frac{-d\phi_B}{dt} \quad \oint \vec{B} \cdot d\vec{l} = \frac{-d}{dt} \iint \vec{B} \cdot d\vec{s}$

4th eqn: Ampere's law in magnetism $\quad \oint_C \vec{B} \cdot d\vec{l} = \mu_0 (I + I_D) = \mu_0 \varepsilon_0 \frac{d}{dt} \iint \vec{B} \cdot d\vec{s} + \mu_0 I$

$E/B = c$



$\vec{E}$ along y
$\vec{B}$ along z
$(C = \vec{E} \times \vec{B})$ along x.

→ $E_z = E_{z0} \sin(\omega t - ky)$

$B_x = B_{x0} \sin(\omega t - ky)$

Energy of electric field (Medium) $U_E = \frac{1}{2} \varepsilon_0 E^2 dV$

Energy of magnetic field $U_B = \frac{1}{2\mu_0} B^2 dV$

$\Big\}\ T.E = \frac{1}{2} \varepsilon_0 E^2 dV + \frac{B^2}{2\mu_0} dV$

$(\vec{E} = \vec{B} \times C)$

$(\vec{B} = \vec{E} \times C)$

Energy density $U = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2 \qquad E_{rms} = \frac{E_0}{\sqrt{2}}$

$U_{avg} = \frac{1}{4} \varepsilon_0 E_0^2 + \frac{1}{4\mu_0} B_0^2 \qquad \boxed{U_{avg_E} = U_{avg_B}}$

Intensity $I = \frac{U}{A(\Delta t)} = \frac{1}{2} \varepsilon_0 E_0^2 c = P/4\pi r^2 = \frac{\eta P}{4\pi r^2}$ (efficiency) (here $I = I_B + I_E$ $= \frac{1}{2} I_B + \frac{1}{2} I_E$)

Free space

Average electric energy density = Avg magnetic energy density

$\frac{1}{4} \varepsilon_0 E_0^2 = \frac{1}{4} \mu_0 H^2 = \frac{1}{4} B_0^2/\mu_0$

→ i) In free space

$$U_{E_{avg}} = U_{B_{avg}} \Rightarrow \frac{1}{4}\epsilon_0 E_0^2 = \frac{1}{4}\mu_0 H_0^2 = \frac{B_0^2}{4\mu_0}$$

$$(T.E.\ density)_{avg} = \frac{1}{2}\epsilon_0 E_0^2$$

ii) In medium

$$U_{E\,density} = \frac{1}{2}\epsilon E^2 \qquad U_{B\,density} = \frac{B^2}{2\mu}$$

$$(T.E.e)_{avg} = \frac{1}{2}\epsilon E^2 + \frac{B^2}{2\mu}$$

→ Electromagnetic waves carry momentum & exert pressure

i) if radiation absorbed completely

momentum $p = \frac{U}{c}$; $P_r = \frac{I}{V}$, $p = E/c$

ii) if radiation completely reflected

$$p = \frac{2U}{c},\ P_r = \frac{2I}{c}$$

	Range	Production
→ Radiowaves	> 0.1 m	– by rapid acceleration & deceleration of e^-s in aerial
Microwaves	0.1m – 1mm	– by Klystron, magnetron
I.R	700nm – 1mm	– by vibration of atoms & molecules
Light	400nm – 700nm	} e^- transitions
U.v light	1nm – 400nm	} e^- transitions
X-rays	10^{-3}nm – 1nm	– by inner shell e^-s.
γ-rays	$< 10^{-3}$ nm	– by radioactive decay of nucleus

$$\bar{E}\times\bar{B} = \left(\frac{\bar{i}+\bar{j}}{\sqrt{2}}\right)\times\bar{B} \Rightarrow \bar{B} = \frac{-\bar{i}+\bar{j}}{\sqrt{2}},\ \left(\frac{-\bar{i}+\bar{j}}{\sqrt{2}}\right)\times\bar{B} \Rightarrow \bar{B} = \frac{-\bar{i}-\bar{j}}{\sqrt{2}}$$