

Current Electricity

→ Rate of flow of charges through any cross sectional area, is current.

It is a scalar quantity. SI = Ampere (A).

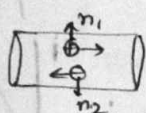
→ $I_{avg} = \frac{\Delta Q}{\Delta t}$, $I_{instant} = \lim_{\Delta t \rightarrow 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$

→ Translation charge carriers $i = q/t = ne/t$

Rotational

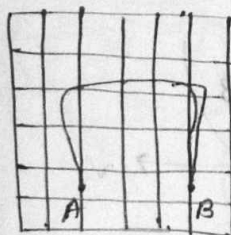
$$i = \frac{q \omega}{2\pi} = \frac{Vq}{2\pi r} \quad (V = r\omega)$$

→ +ve charge direction is conventional current direction.

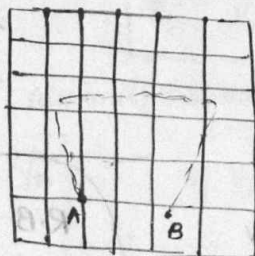


$$I = (n_1 + n_2) e/t$$

→ Drift velocity: avg velocity of a free e^- in conductor get drifted under the influence of external E field.



absence of E field



presence of E field

$$\frac{\vec{u}_1 + \vec{u}_2 + \dots + \vec{u}_n}{n} = 0$$

$$a = \frac{-eE}{m}$$

e^- move in curved path from lower to higher potential

Relaxation time (τ): Avg. time b/w 2 successive collisions.

$$\vec{v} = \frac{u_1 + a\tau_1 + u_2 + a\tau_2 + \dots + u_n + a\tau_n}{n} = \frac{u_1 + u_2 + \dots + u_n}{n} + a \left(\frac{\tau_1 + \tau_2 + \dots + \tau_n}{n} \right)$$

$$\vec{v} = a\tau = \frac{-eE}{m} \tau$$

At S.T.P order of v_d is 10^{-4} m/sec.

Mobility (μ): v_d per unit E field.

$$\mu = \frac{e\tau}{m} = \frac{v_d}{E}$$

Current density (\vec{J}): Current passing per unit cross section of normal cross section.

It is a vector quantity, SI = A/m²

$$i = \vec{J} \cdot \vec{A} = JA \cos \theta$$

direction of \vec{J} is same as current flow.

$$\text{momentum (p) of } e^- = \frac{m_e v_d}{e}$$



→ Relation b/w current & drift velocity

$$i = neAv_d, J = nev_d \quad e = \text{charge of } e^- = 1.6 \times 10^{-19} \text{ C}$$

→ Ohm's law

$$V = iR$$

$$R = \frac{m l}{ne^2 A \tau}$$

vector form

$$\vec{J} = \sigma \vec{E}$$

$$\sigma = \frac{ne^2 \tau}{m} = \frac{1}{\rho}$$

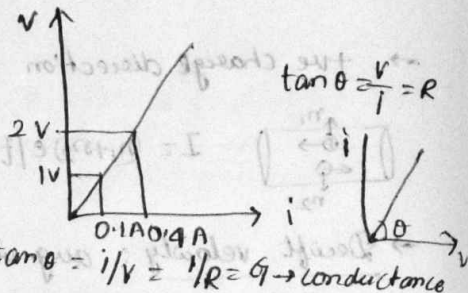
σ = conductivity — Siemen/m

ρ = resistivity

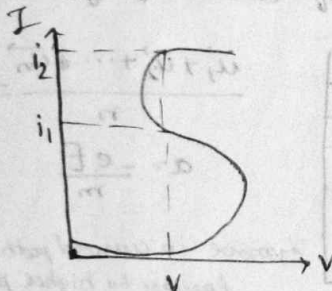
→ Resistance: Opposition of flow of current.

SI = ohm Ω

→ Ohmic conductors

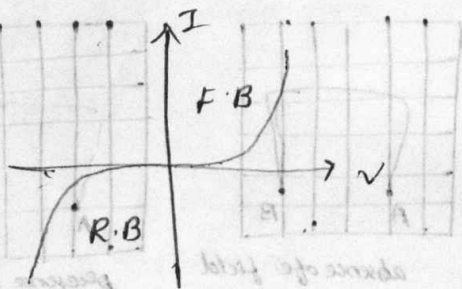


→ Non-ohmic conductors:



Thermistor graph

Resistance varies with temp.



semiconductor graph

→ Resistivity or specific resistance

$$R = \frac{\rho l}{A}$$

SI — $\Omega \cdot m$ (ohm-metre)

$$\text{Conductivity } (\sigma) = \frac{1}{\rho}$$

$$R = \frac{\rho l}{A} = \frac{\rho l^2}{A} \cdot \frac{1}{l} \rightarrow \text{density}$$

if $l \uparrow$ ρ remains same.

→ Resistivity is a specific property, resistance is a bulk property.

→ It depends on temp, nature of material & impurities

→ Nichrome wire has high resistivity & m.p. (Heaters)

→ Tungsten filament has low resistivity & high m.p. (Bulb)

$$\frac{R_1}{R_2} = \left[\frac{l_1}{l_2} \right]^2 = \left[\frac{A_2}{A_1} \right]^2$$

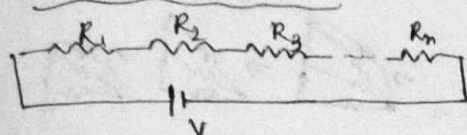
→ Temp. dependence of resistance

$$\alpha = \frac{R_t - R_0}{R_0 \Delta t} = \frac{l_2 - l_1}{l_1 \Delta t}$$

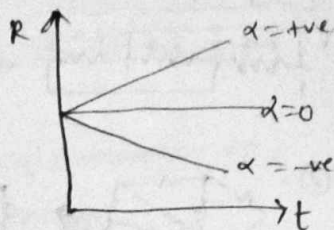
$$R_t = R_0 [1 + \alpha \Delta t] \quad \gamma = \frac{1}{\alpha}$$

$\alpha = +ve$ for metals $= -ve$ for non metal (semiconductor) $= 0$ for magnetic material

→ Resistors in series



($i = \text{constant}$)



Potential of resistance = p.d. ends of the resistor

If 'n' identical resistors are connected in series combination

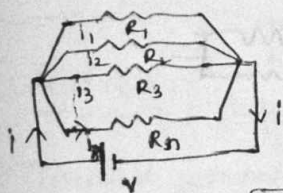
$$R_s = nR$$

$$R_s = R_1 + R_2 + R_3 \dots$$

$$V_1 : V_2 : V_3 = R_1 : R_2 : R_3$$

$$V_1 = \left(\frac{R_1}{R_1 + R_2 + R_3} \right) V \quad \text{||ly for } V_2 \text{ \& } V_3$$

Resistors in parallel



[$V = \text{constant}$]

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \dots$$

→ eq. resistance of 'n' sided polygon at adj. corner = $\frac{(n-1)R}{n^2}$

'n' identical resistors are connected in parallel combination

$$R_p = R/n$$

$$V_1 : V_2 : V_3 = 1 : 1 : 1$$

$$I_1 : I_2 : I_3 = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3}$$

$$I_1 = \left(\frac{1/R_1}{1/R_p} \right) I \quad \text{||ly for } I_2, I_3$$

→ If the wire is cut into 'n' equal parts. If they are connected in parallel combination.

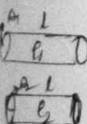
$$R_{\text{eff}} = \frac{R}{n^2}$$

→ Possible arrangement of resistor = 2^{n-1} of different

→ for same kind = 2^n

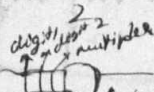
$$\sigma \propto 1/\rho \quad \left(\frac{1}{\rho_2} \parallel \frac{1}{\rho_1} \right)$$

$$R_{\text{eff}} (\text{series}) = \frac{\rho_1 + \rho_2}{2}$$



$$R_{\text{eff}} (\text{||el}) = \frac{2l_1 l_2}{l_1 + l_2}$$

$$\sigma_{\text{eff}} = \frac{\sigma_1 + \sigma_2}{2}$$



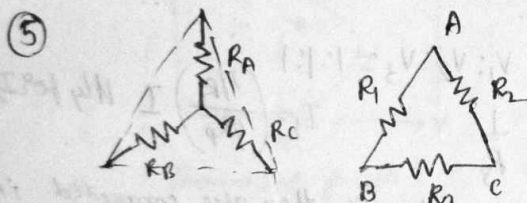
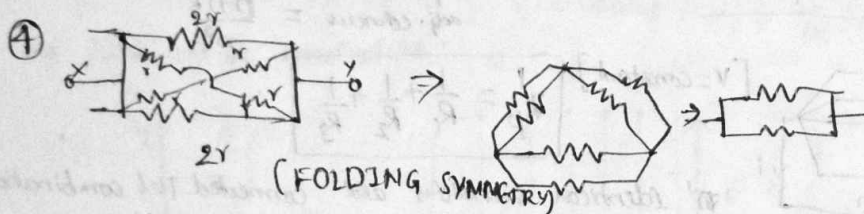
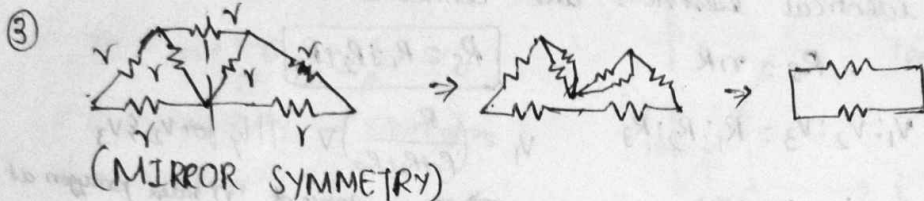
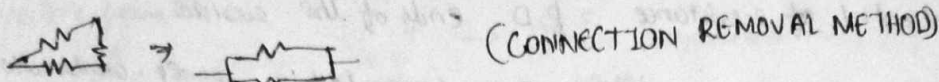
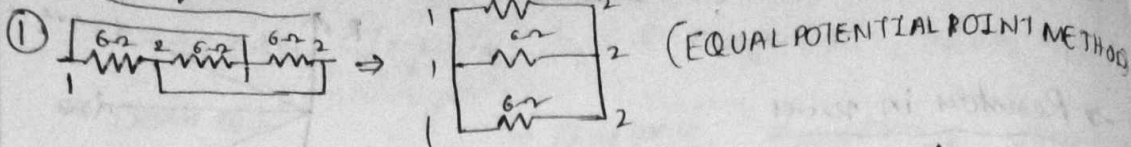
Colour coding of resistors:

Colour	digit	Multiplier
Black	0	10^0
Brown	1	10^1
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5

Blue	6	10^6
Violet	7	10^7
Grey	8	10^8
White	9	10^9
Gold		10^{-1} 5%
Silver		10^{-2} 10%
No colour		20%

BIB ROY Great Britain
tolerance very good wife

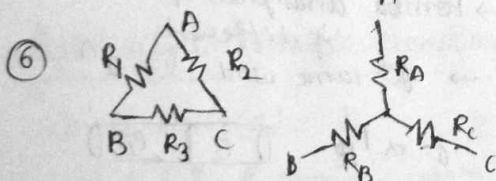
Group of resistors



$$R_1 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

$$R_2 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}$$

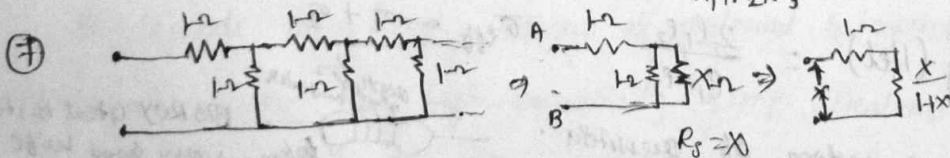
$$R_3 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$$



$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

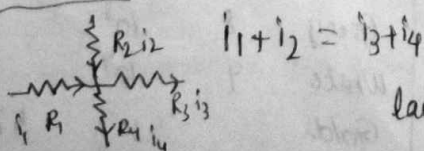
$$R_B = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

$$R_C = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$



Kirchoff's laws:

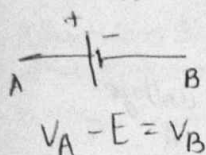
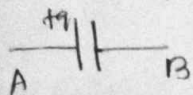
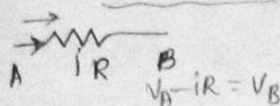
1st law (KCL)



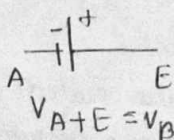
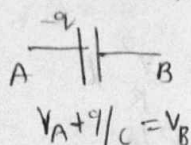
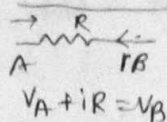
law of conservation of charges

2nd Law (Ohm) KVL

Potential fall



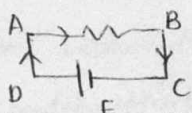
Potential rise



Law of conservation of energy

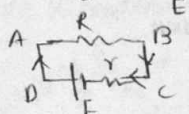
→ E.M.F of a cell: $E = V$ SI = volt (V)

Ideal cell:



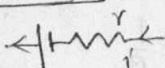
$$i = \frac{E}{R}$$

Real cell:



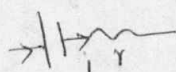
$$i = \frac{E}{R+r}$$

Terminal potential difference:



$$V = E - ir$$

Charging cell



$$V = E + ir$$

Relation b/w external & internal resistance + $r = \left(\frac{E-V}{V} \right) R$

Grouping of cells

Series:

$$V_A - V_C = E_{eq} - i r_{eq}$$

$$i = \frac{nE}{R + nr}$$

$$i_1 = \frac{E - V}{r_1} \text{ in cell}$$

$$E_{eq} = E_1 + E_2, r_{eq} = r_1 + r_2$$

Parallel:

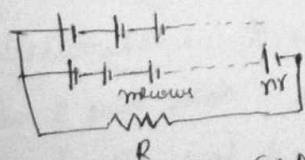
$$E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

$$\frac{E_1/r_1 + E_2/r_2 + E_3/r_3}{1/r_1 + 1/r_2 + 1/r_3}$$

$$i = \frac{E_1 r_2 + E_2 r_1}{R(r_1 + r_2) + r_1 r_2}$$

$$i = \frac{nE}{nR + r}$$

Mixed grouping of cells:



$$i = \frac{mnE}{mR + nr}$$

$$\text{Max current: } i = \frac{nE}{2R} \text{ (or)} \frac{mE}{2r}$$

total no. of cells (N) = mn

Wrongly connected cells

$$i = \frac{(n-2m)E}{R+nR} \quad (n-2m)E = E_{\text{total}}$$

Joule's heating effect:

$$Q = W = V i t = i^2 R t = \frac{V^2}{R} t$$

$$\text{Power (P)} = V i = i^2 R = V^2 / R$$

Bulb:

$$R = \frac{V_Y^2}{P_Y}$$

V_Y & P_Y are rated
 V_C & P_C are consumed

V_a - applied voltage

$$P_C = \left[\frac{V_a}{V_Y} \right]^2 P_Y$$

$$P_C \propto R \propto 1/P_Y$$

→ In series Less rated power has more brightness

$$\frac{1}{P_{\text{total}}} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3}$$

→ In parallel more rated power has more brightness

$$P_C \propto 1/R \propto P_Y$$

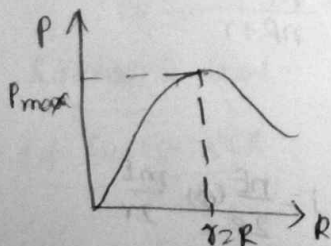
$$P_C = P_1 + P_2 + P_3$$

$$P_S = P/n, \quad P_P = nP, \quad \frac{P_S}{P_P} = \frac{1}{n^2}$$

→ Max. power transfer theorem

$$P_{\text{max}} = \frac{E^2}{4R}$$

Power dissipated in an external resistance is maxima. If $r = R$
 $R=0 \rightarrow P=0$ (Short ckt)



$$r > R \rightarrow P \uparrow$$

$$r = R \rightarrow P_{\text{max}}$$

$$r < R \rightarrow P \downarrow$$

$$R = \infty \rightarrow P = 0 \text{ (Open ckt)}$$

→ Heater :

$$MSDT = \frac{V_a^2}{R} t = i^2 R t$$

$R \propto t$

$t = \text{time}$

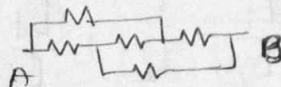
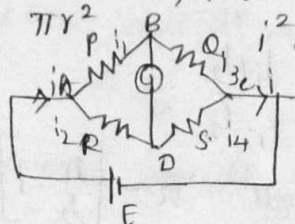
Series : $R_s = R_1 + R_2 \Rightarrow t_s = t_1 + t_2$

Par : $R_p = \frac{R_1 R_2}{R_1 + R_2} \Rightarrow t_p = \frac{t_1 t_2}{t_1 + t_2}$

→ Fuse wire :

$$H = \frac{i^2 \rho L}{\pi r^2} \Rightarrow i^2 \rho = 2\pi^2 r^3 H_0$$

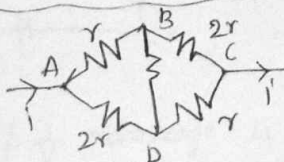
→ Wheatstone Bridge :



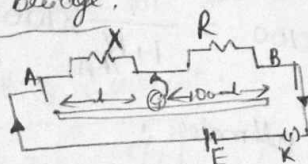
$$\frac{P}{Q} = \frac{R}{S}$$

* Current flows due to potential difference. In B & D pdiff = 0
so we can remove B & D.

Unbalanced Wheatstone Bridge :



Meter bridge :

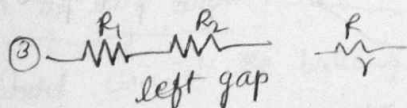


$$\frac{\text{left gap}}{\text{right gap}} = \frac{R}{100 - l}$$

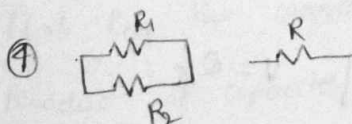
Magnitude of area of connection is constant.

End correction

$$\frac{X}{R} = \frac{l + \alpha}{100 - l + \beta}$$



$$\frac{R_1 + R_2}{R} = \frac{l}{100 - l}$$



$$\frac{R_1 R_2}{(R_1 + R_2) R} = \frac{l}{100 - l}$$

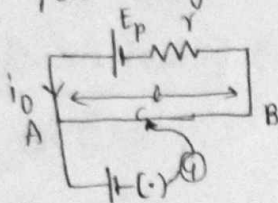
Use
→ To find unknown resistance of wire.

→ Potentiometer : It is a device measure potential difference b/w 2 given points without drawing a current.

Principle : Potential drop across any connection of the wire is directly proportional to balancing length.

$$\frac{V_{AC}}{L} \approx K \text{ potential gradient}$$

→ Potential gradient is potential drop per unit length.



$$E_s = \left[\frac{E_p}{R_0 + R} \right] \left[\frac{R_0}{L} \right] l$$

R_0 = resistance of wire

R = internal resistance of cell

L = length of wire

l = balancing length

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}, \quad \frac{E_1}{E_2} = \frac{l_1 + l_2}{l_1 - l_2}$$

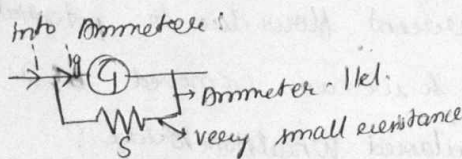
if null deflection in galvanometer at l_2 ,

$$E_{unknown} = K l_1$$

→ Internal resistance of cell $R = \left[\frac{l_1}{l_2} - 1 \right] R$

→ Conversion of Galvanometer into Ammeter:

$$\text{error } \Delta i = i - i'$$

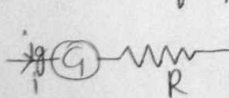


$$S = \frac{G}{n-1} = \frac{i_g G}{i - i_g}$$

$$R_{eff} = \frac{GS}{G+S}$$

$$\% \text{ error in ammeter} \Rightarrow \frac{\Delta i'}{i} \times 100 = \frac{1}{1 + R/R_A} \times 100 \quad [R_A = \text{resistance of ammeter}]$$

→ Conversion of Galvanometer into voltmeter:

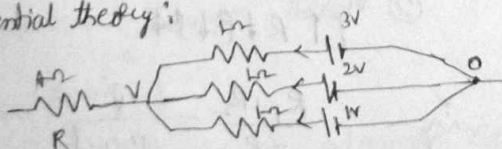


$$V_{new} = i_g [G + R]$$

$$R = (n-1)G$$

$$\frac{V_g}{V} = \frac{V_R}{V} = \frac{G}{R}, \quad R_{eff} = R + G$$

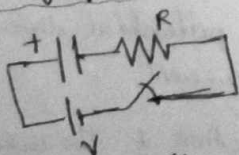
→ Point potential theory:



$$\frac{V-3}{1} + \frac{V-2}{1} + \frac{V-1}{1} = 0$$

E-R circuits

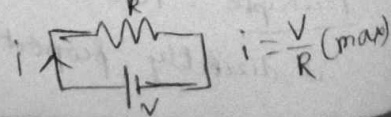
Charging of capacitor



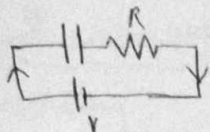
$$t = 0 \text{ sec}, q = 0$$

$$V_c = \frac{q}{C} = 0$$

Capacitor acts as short circuit. It acts as a conducting wire.



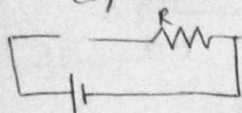
ii)



$$t = \infty \text{ sec}, q = q_0, i = 0$$

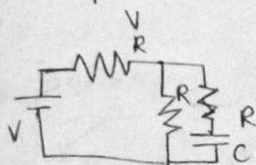
fully charged capacitor is called steady state

Capacitor is open circuit.



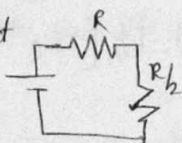
$$V = V_C + V_R \Rightarrow V_R = 0$$

$$V = V_C$$

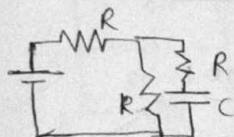


$t = 0 \text{ sec}$, short ckt

$$i = \frac{2V}{3R}$$

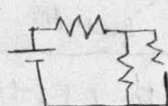


	i	i_1	i_2
$t = 0$ short ckt	$\frac{2V}{3R}$	$\frac{V}{3R}$	$\frac{V}{3R}$
$t = \infty \text{ sec}$ open ckt	$\frac{V}{2R}$	$\frac{V}{2R}$	0

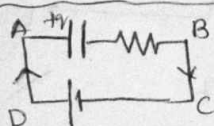


$t = \infty \text{ sec}$, open ckt

$$i = \frac{V}{2R}$$



Growth of charge:



$$q_0 = CV \rightarrow \text{final charge}$$

$$q = q_0 [1 - e^{-t/CR}]$$

τ = time const

$$q = 63\% q_0$$

63% of max charge is stored in the capacitor at one time const.

τ capacitor has fast charging.

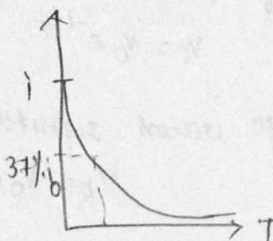
Growth of current:

$$q = q_0 [1 - e^{-t/\tau}]$$

$$i = \frac{dq}{dt}$$

$$i = \frac{d}{dt} [q_0 - q_0 e^{-t/\tau}]$$

$$i = i_0 e^{-t/\tau}$$



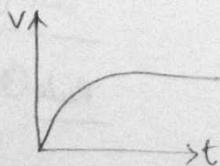
Find the time const (τ):

1. Short circuit the battery
2. Find R_{eff} b/w capacitor plates
3. Product of capacitor & resistance

$$\tau = CR$$

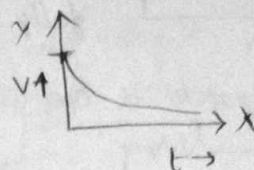
Potential across capacitor varying with time:

$$V_C = V_0 [1 - e^{-t/\tau}]$$



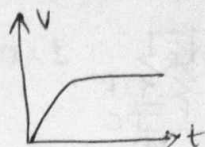
Potential across a resistor varying with time:

$$V_R = V_0 e^{-t/\tau}$$



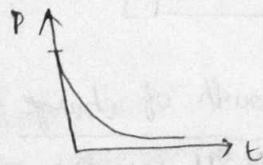
Energy stored in capacitor varying with time:

$$U = U_0 [1 - e^{-t/\tau}]^2$$



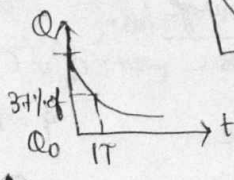
Power across a resistor varying with time:

$$P = P_0 [e^{-t/\tau}]^2$$



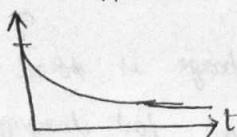
Decay of charge: $Q = \frac{Q_0}{e}$

$$Q = Q_0 e^{-t/\tau}$$



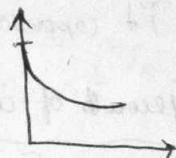
Decay of current:

$$i = i_0 e^{-t/\tau}$$



Voltage across capacitor varying with time (t):

$$V_C = V_0 e^{-t/\tau}$$



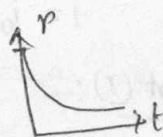
Voltage across resistor varying with time (t):

$$V_R = V_0 e^{-t/\tau}$$

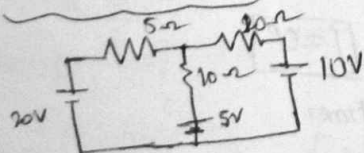


Power across resistor

$$P = P_0 (e^{-t/\tau})^2$$



Thevenin's analysis:

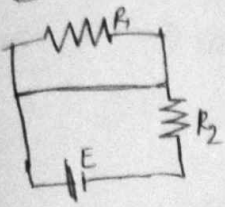


Rule ①: Open circuit at a resistor ~~to be removed~~
do you want to find current.
Measure current in next loop.

Rule ②: Find potential b/w ends of R .

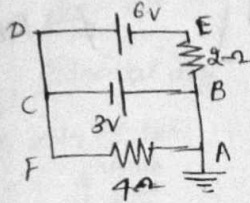
Rule ③: Find equivalent resistance by removing all resistors.

→ Short circuiting: 2 point in electric circuit connected by conducting wire are called short circuited. (both potential same) - no current flow.



→ Here R_1 is short circuit i.e., $R_1 = 0$. No current flow through R_1 & current through R_2 is E/R_2 .

→ Earthing: If a point is earthed then its potential is zero.



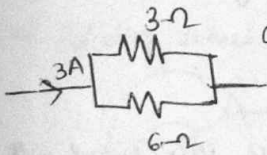
$$V_A = V_B = 0, \quad V_F = V_C = V_D = -3V$$

$$V_E = -9V, \quad V_B - V_E = 9V$$

i) Current through 2Ω is

$$\frac{V_B - V_E}{2} (\text{or}) \frac{9}{2} \text{ A} \quad (\text{from B to E})$$

ii) Current through 4Ω is $\frac{V_A - V_F}{4} (\text{or}) \frac{3}{4} \text{ A} \quad (\text{from A to F})$



current through 3Ω

$$A) \quad i_3 = \left(\frac{6}{3+6} \right) \times 3 = 2 \text{ A}$$