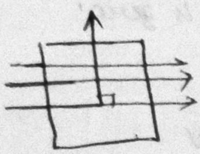
Electromagnetic Induction

Magnetic flux (ϕ) $= \vec{B} \cdot \vec{A}$

SI: Weber (W), C.G.S: Maxwell (M)

1 weber $= 10^8$ Maxwell

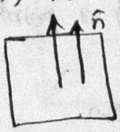
Case(i): Plane of the coil is ||el to M.F



$\phi = BA\cos 90^\circ$

$\boxed{\phi = 0}$

Case(ii): Plane of coil is $\perp$ar to M.F



$\phi = BA\cos 0^\circ$

$\phi = BA$

Change in magnetic flux ($\Delta\phi$)

$$\Delta\phi = BAN\left[\cos\theta_2 - \cos\theta_1\right] \quad \theta_2 > \theta_1$$

Faraday's laws:

1st law: Magnetic flux linked with the coil changes it produces induced E.M.F

2nd law: Induced E.M.F $\propto$ rate of change of magnetic flux linked with coil.

$$e = \frac{-N\,d\phi}{dt} \quad (N \text{ turns}), \qquad \Delta q = \frac{-\Delta\phi}{R}$$

$-$ve sign indicates oppose the change in magnetic flux linked with the coil.

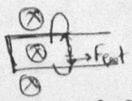
Lenz's law: Direction of induced E.M.F to produce a current it opposes change in magnetic flux linked with coil.

Follows law of conservation of energy.

Magnetic field in non-uniform field:

$$d\phi = \int \vec{B} \cdot \vec{A}$$

Case(i): 'B' value changes $\quad e = A\left[\frac{B_2 - B_1}{T_2 - T_1}\right]$



Case(ii): 'A' value changes $\quad e = -Blv$

$$i = \frac{Blv}{R} \qquad F_{ext} = \frac{B^2 l^2 v}{R}$$

$$F_{ext} = F_M = Bil$$

$dq = \frac{d\phi}{R}$ $\therefore$ charge is independent on time

power $P = F \cdot V = \frac{B^2 l^2 v^2}{R}$

Heat generated $\int dH = \int d\phi \, i$

case (iii): θ value changes $e = e_0 \sin\omega t = BAN\omega \sin\omega t$

Working of AC generator.

ME → E.E

Motional E.M.F: Forefinger → magnetic field direction

Central finger → induced current (-ve to +ve)

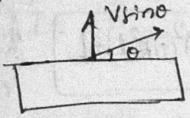
Thumb → direction of motion of conductor.

Induced electric field

$\boxed{\begin{matrix} \parallel E \\ \nabla B \end{matrix}}$ Potential diff b/w ends of conductor is applicable if & only if

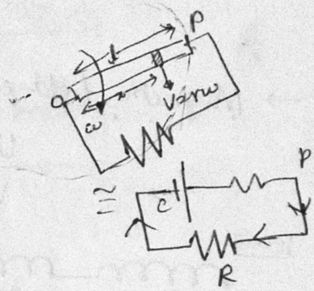
we use only ⊥ lar.

$e = +[\vec{V} \times \vec{B}] \cdot \vec{l}$. V, B, l are mutually ⊥ lar



$e = Blv$

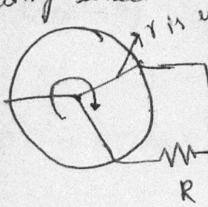
$V_P - V_Q = Blv\sin\theta$ | $V_P - V_O = B\int_0^L x \, dx \, \omega$



Rotational conductor: $e = \dfrac{Bl^2\omega}{2}$

with resistor $i = \dfrac{\frac{1}{2} Bl^2\omega}{R+r}$

→ Rotating wheel:



$E_{eff} = \dfrac{e/r + e/r + e/r}{1/r + 1/r + 1/r} = e$ $i = \dfrac{\frac{1}{2} Bl^2\omega}{R + r/3}$

e is independent on no. of spokes

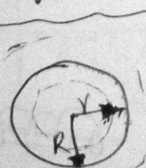
→ **Induced electric field:**

Non-conservative nature, Workdone $W \neq 0$, $\frac{dB}{dt}$ responsible for induced electric field, $\frac{dB}{dt}$ is not produce electric charge.

Always formed in closed loops. (time varying)

$\oint \vec{E} \cdot d\vec{l} = \dfrac{-d\phi_B}{dt}$ of conservative $\oint \vec{E} \cdot d\vec{l} = 0$

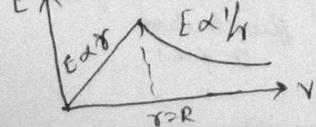
$\vec{E}$ cylindrical regions,



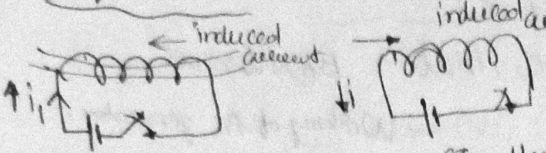
(i) $r < R$ $E = \dfrac{r}{2} \dfrac{dB}{dt}$

(ii) $r = R$ $E = \dfrac{R}{2} \dfrac{dB}{dt}$

(iii) $r > R$ $E = \dfrac{R^2}{2r} \dfrac{dB}{dt}$



→ Self induction:



$N\phi \propto i$

$N\phi = Li$

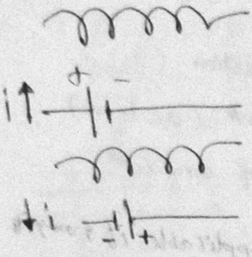
$\boxed{L = \frac{N\phi}{i} \qquad \phi = Li}$

L = coeff of self induction

SI = Henry

$\boxed{e = -L\frac{di}{dt}}$

$L = \mu_0 n^2 Al$



→ Coeff. of self induction of circular coil:

$L = \frac{\mu_0 N^2 \pi R}{2}$

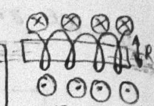
→ Energy stored in a inductor: $U = W = \frac{Li^2}{2}$

$U = \frac{1}{2}\phi i$

$U = \frac{\phi^2}{2L}$

→ for solenoid:

$\boxed{\mu_0 n^2 (\pi R^2 l) = L}$



$B = \mu_0 ni$

$n = N/l$

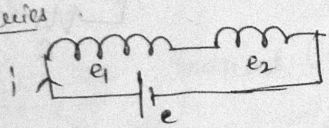
→ Magnetic field energy density

$$\frac{U}{V} = \frac{B^2}{2\mu_0} = \frac{1}{2}\varepsilon_0 E^2$$

→ for toroid

$\boxed{L = \frac{\mu_0 N^2 R}{2}}$

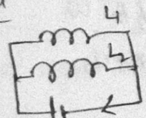
series



$L_S = L_1 + L_2$

$V = L\frac{di}{dt}$

$\parallel$el



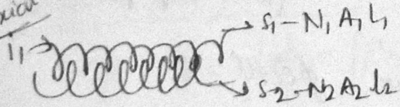
$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2}$$

→ Mutual inductance: $M_{21} = \frac{\phi_2}{i_1}$ S.I = Henry

$e_s = \frac{-M\,di}{dt}$

depends upon no of turns, area of the coil, distance b/w the coils, orientation of coils

Co-axial



$B_1 = \frac{\mu_0 N_1 i_1}{l_1}$

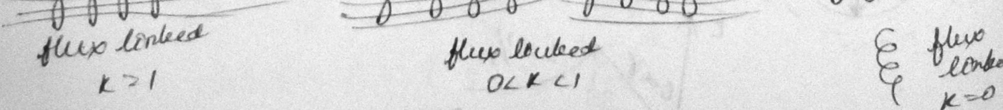
$N_2 \phi_2 = M_{21} i_1$

$B_2 = 0, \quad M_{21} = \frac{\mu_0 N_1 N_2 A_2}{l_1}$

$M = \frac{\mu_0 N_1 N_2 \pi r^2}{2R}$

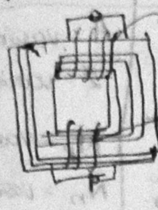
$M \propto l^2/R$

Coupling factor (k) $= \frac{M}{\sqrt{L_1 L_2}}$



Transformers

principle : Mutual induction



Ideal

$\frac{V_S}{V_P} = \frac{i_P}{i_S}$

as power = const

$V_1 i_1 = V_2 i_2$

$P_{leakage} = 0$

$K = \frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{i_P}{i_S}$

K = transformer ratio

$K > 1$, $N_S > N_P$ step up

$K < 1$, $N_S < N_P$ step down [Some power lost due to eddy currents, hysteresis etc.]

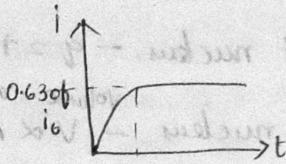
It works only on A Current

L-R ckt

Growth: $T = L/R$

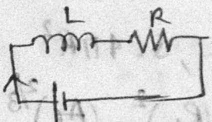
$i = i_0[1 - e^{-t/T}]$

$i = 63\%$ of i_0



Voltage across R

$V_R = V_0[1 - e^{-t/T}]$

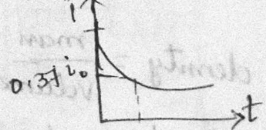


Voltage across inductor (L)

$V_L = V_0 e^{-t/T}$

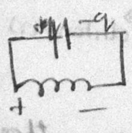
Energy of inductor

$U = U_0[1 - e^{-t/T}]^2$



Decay $i = i_0 e^{-t/T}$ $i = 37\%$ of i_0

L-C oscillations



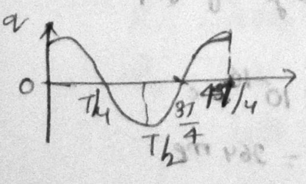
$\frac{d^2q}{dt^2} + \frac{q}{CL} = 0$

$\omega = \frac{1}{\sqrt{LC}}$ $n = \frac{1}{T} = \frac{1}{2\pi\sqrt{LC}}$

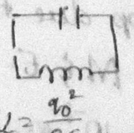
Energy $\frac{q_0^2}{2C}$

$i = q_0(-\omega)\sin\omega t$

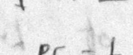
Comparision of L-C ckts with spring block system:



t=0

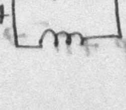


$U_E = \frac{q_0^2}{2C}$

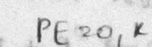


$PE = \frac{1}{2}KA^2$

t=T/4



$U_E = 0$



$U_B = \frac{1}{2}Li^2$

PE = 0, $KE = \frac{1}{2}mv^2$