# Differentiation

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$$ First principle

→ Product rule $(uv)$

$$\frac{d}{dx}(uv) = \frac{du}{dx}(v) + \frac{dv}{dx}(u)$$

Quotient rule $(u/v)$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\,du - u\,(dv)}{v^2}$$

Chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dx}$$

→

| S.No | $f(x)$ | $f'(x)$ |
|---|---|---|
| 1 | constant | 0 |
| 2 | $x^n$ | $nx^{n-1}$ |
| 3 | $e^x$ | $e^x$ |
| 4 | $a^x$ | $a^x \log a$ |
| 5 | $\lvert x\rvert$ | $\frac{\lvert x\rvert}{x}$ if $x \neq 0$ |
| 6 | $\log_e \lvert x\rvert$ | $1/x$ |
| 7 | $\log_a \lvert x\rvert$ | $\frac{1}{x} \frac{1}{\log a}$ |
| 8 | $\sqrt{x}$ | $\frac{1}{2\sqrt{x}}$ |
| 9 | $1/x$ | $-1/x^2$ |
| 10 | $1/x^n$ | $\frac{-n}{x^{n+1}}$ |
| 11 | $[x]$ | $\begin{cases} 0, & \forall x \notin Z \\ d.N.E, & \forall x \in Z \end{cases}$ |
| 12 | $\sin x$ | $\cos x$ |
| 13 | $\cos x$ | $-\sin x$ |
| 14 | $\tan x$ | $\sec^2 x$ |
| 15 | $\cot x$ | $-\csc^2 x$ |
| 16 | $\sec x$ | $\sec x \tan x$ |
| 17 | $\csc x$ | $-\csc x \cot x$ |
| 18 | $\sin^{-1} x$ | $\frac{1}{\sqrt{1-x^2}}$ |
| 19 | $\cos^{-1} x$ | $-1/\sqrt{1-x^2}$ |
| 20 | $\tan^{-1} x$ | $\frac{1}{1+x^2}$ |
| 21 | $\cot^{-1} x$ | $\frac{-1}{1+x^2}$ |
| 22 | $\sec^{-1} x$ | $\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$ |
| 23 | $\csc^{-1} x$ | $\frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$ |
| 24 | $\sinh x$ | $\cosh x$ |
| 25 | $\cosh x$ | $\sinh x$ |
| 26 | $\tanh x$ | $\text{sech}^2 x$ |
| 27 | $\coth x$ | $-\text{cosech}^2 x$ |
| 28 | $\text{sech}\, x$ | $-\text{sech}\, x \tanh x$ |
| 29 | $\text{cosech}\, x$ | $-\text{cosech}\, x \coth x$ |
| 30 | $\sinh^{-1} x$ | $1/\sqrt{1+x^2}$ |
| 31 | $\cosh^{-1} x$ | $1/\sqrt{x^2-1}$ |
| 32 | $\tanh^{-1} x$ | $1/1-x^2$ |
| 33 | $\coth^{-1} x$ | $1/1-x^2$ |
| 34 | $\text{sech}^{-1} x$ | $\frac{-1}{x\sqrt{1-x^2}}$ |
| 35 | $\text{cosech}^{-1} x$ | $\frac{-1}{\lvert x\rvert\sqrt{1+x^2}}$ |

→ Parametric differentiation:

If $x = f(t)$, $y = g(t)$ are para. eqn. of curve then

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{g'(t)}{f'(t)}$$

→ $$\frac{dy}{dx} = -\frac{\left(\frac{\partial f}{\partial x}\right)}{\left(\frac{\partial f}{\partial y}\right)} \qquad \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) \ldots$$

→ Standard Derivates:

i) $y = \sqrt[n]{f(x) + \sqrt[n]{f(x) + \sqrt[n]{f(x) + \cdots \infty}}}$ then $\frac{dy}{dx} = \frac{f'(x)}{ny^{n-1} - 1}$

ii) $y = \sqrt{f(x) + \sqrt{f(x) + \sqrt{f(x) + \cdots \infty}}}$ then $\frac{dy}{dx} = \frac{f'(x)}{(2y-1)}$

iii) $y = f(x)^{g(x)}$ then $\frac{dy}{dx} = y\left(g'(x) \log f(x) + g(x) \frac{f'(x)}{f(x)}\right)$

iv) $f(x+y) = f(x) \cdot f(y)$ then $f(x) \neq 0$, $f'(x) = f'(0) \cdot f(x)$

v) $$\frac{d}{dx}\left(\frac{af(x)+b}{cf(x)+d}\right) = \frac{(ad-bc)f'(x)}{[cf(x)+d]^2}$$

→ $$\frac{d}{dx}\left[\tan^{-1}\left(\frac{a\cos f(x) + b\sin f(x)}{b\cos f(x) - a\sin f(x)}\right)\right] = f'(x)$$

$$\frac{d}{dx}\left[\tan^{-1}\left(\frac{a\cos f(x) - b\sin f(x)}{b\cos f(x) + a\sin f(x)}\right)\right] = -f'(x)$$

Derivate of complex function ← $y = (g \circ f)(x) = g[f(x)] = \frac{dy}{dx} = g'(f(x)) f'(x)$

→ If $f(y/x) = c$ then $\frac{dy}{dx} = y/x$ $(c = \text{const})$

If $f(x^m y^n) = c$ then $\frac{dy}{dx} = \frac{-my}{nx}$

→ $x^m y^n = (x+y)^{m+n}$ then $\boxed{dy/dx = y/x}$

→ Derivate of determinant: D – differentiate

$$\frac{d}{dx}\begin{vmatrix} - \\ - \\ - \end{vmatrix} = \begin{vmatrix} D \\ - \\ - \end{vmatrix} + \begin{vmatrix} - \\ D \\ - \end{vmatrix} + \begin{vmatrix} - \\ - \\ D \end{vmatrix}$$

→ $y = (f(x))^{(f(x))^{f(x)) \cdots \infty}}$, then $\frac{dy}{dx} = \frac{y^2 f'(x)}{f(x)\{1 - y \log_e(f(x))\}}$

→ $y = f(x) + \cfrac{1}{f(x) + \cfrac{1}{f(x) + \cdots \infty}}$, then $\frac{dy}{dx} = \frac{y f'(x)}{2y - f(x)}$

→ if $f(x)$ is inverectiable, then $g(x) = f^{-1}(x)$ then $g'(x)$ is given

$$\boxed{g'(f(x))\, f'(x) = 1}$$