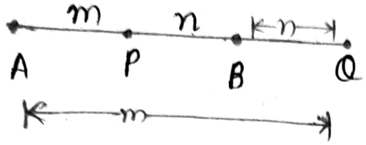
2D-Geometry

→ Section formula $P = \left(\frac{mB \pm nA}{m \pm n}\right)$ if P external $m-n$, $mn<0$.
P internal $m+n$, $mn>0$.

→ $P(x, y)$ passes through $A(x_1, y_1)$ & $B(x_2, y_2)$ then ratio in which P divides $\overline{AB}$ i.e, $AP:PB = x_1 - x : x - x_2$ (or) $y_1 - y : y - y_2$.

⟹ Harmonic Conjugate : P & Q divide AB internally & externally same ratio then P is called harmonic conjugate of Q.



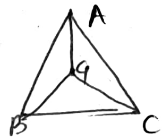
$(m-n):(m+n)$

→ Area of Δ^{le} is if $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ is $\frac{1}{2}\left|\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right|$

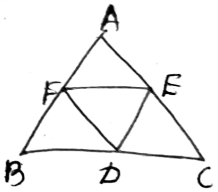
Area of eq. Δ^{le} $= \frac{\sqrt{3}}{4}a^2$

if 'h' altitude then $\frac{h^2}{\sqrt{3}}$.

→ G is centroid then ΔABC; area of $\Delta ABC = 3$ area of ΔABG
$= 3$ area of ΔBCG & ΔCAG



→ area of Δ^{le} formed by midpoints of sides
(1:4) area of Δ^{le} ABC $= 4$(area of ΔDEF)



→ area of quadrilateral $= \frac{1}{2}\begin{vmatrix} x_1 - x_3 & y_1 - y_3 \\ x_2 - x_4 & y_2 - y_4 \end{vmatrix}$

area of pentagon $= \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_5 & y_1 \end{vmatrix}$

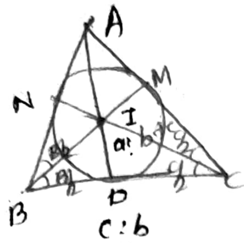
→ Centroid (G): point of concurrency of medians (2:1)

(a) $AB^2 + BC^2 + CA^2 = 3(GA^2 + GB^2 + GC^2)$.

(b) $3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$

→ Incentre (I): internal angular bisectors of Δ^{le} are concurrent & the point of concurrency is called incentre.

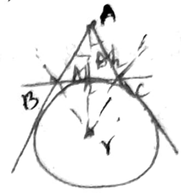
It is equidistant from all the three sides.



i) Δ^{le} ABC if angular bisector of A meets BC at D then BC : DC :: AB : AC.

$$I = \left(\frac{aA+bB+cC}{a+b+c}\right) \text{ where } a=BC,\ b=CA,\ c=AB$$

→ Excentre : internal angular bisector of 1 angle & external angular bisectors of other 2 angles of Δ^{le} are concurrent which is excentre.



excentre opposite ~~vertex~~ to the vertex A is

$$I_1 = \left(\frac{-ax_1+bx_2+cx_3}{-a+b+c},\ \frac{-ay_1+by_2+cy_3}{-a+b+c}\right).$$

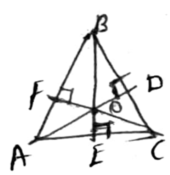
→ Circumcentre(S): point of concurrency of ⊥ar bisectors of Δ^{le}.

circumcentre is equidistant from all three vertices.

(i) S of Δ^{le} formed by $(0,0)$, (x_1,y_1) & (x_2,y_2) is

$$\left(\frac{y_2(x_1^2+y_1^2)-y_1(x_2^2+y_2^2)}{2(x_1y_2-y_1x_2)},\ \frac{x_2(x_1^2+y_1^2)-x_1(x_2^2+y_2^2)}{2(x_2y_1-x_1y_2)}\right)$$

→ Orthocentre(O): point of concurrency of altitudes of Δ^{le}.



BD : DC = tan C : tan B

AO : OD = tan B + tan C : tan A.

→ Δ^{le} formed by feet of altitudes in a Δ^{le} is called orthic or pedal Δ^{le}.

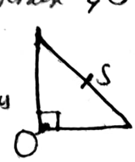
Here, DEF Δ^{le} is orthic Δ^{le} of Δ^{le} ABC

→ orthocentre formed by vertices of Δ^{le} $(ct_1, c/t_1)$, $(ct_2, c/t_2)$ & $(ct_3, c/t_3)$ is $\left(\frac{-c}{t_1t_2t_3},\ -ct_1t_2t_3\right)$.

O for Δ^{le} $(0,0)$, (x_1,y_1) & (x_2,y_2) is $\left(k(y_2-y_1),\ k(x_1-x_2)\right)$

where $k = \dfrac{x_1x_2+y_1y_2}{x_1y_2-y_1x_2}$

⇒ i) for right Δ^{le} S lies on midpoint of hypotenuse & O is at vertex right angled, $\dfrac{\text{hypotenuse dist}}{2}$ = circum radius



ii) for obtuse Δ^{le} both S & O lies outside.

iii) for acute Δ^{le} both S & O lies inside.

→ Length of median through a vertex is

A - $\frac{1}{2}\sqrt{2b^2+2c^2-a^2}$ llly for B & C. $AB = c$, $BC = a$, $CA = b$.

→ Nine point cle

i) The centre of nine point cle, denoted by 'N', N is mid point of orthocentre & circumcentre (ON = NS).

ii) Radius of nine point cle = ½ (circumradius)

iii) ⓐ O·G : G·S = 2 : 1 ($3G = 2S + O$) ⓑ ON : NG : GS = 3 : 1 : 2

→ Relation b/w Orthocentre (O), circumcentre (S), centroid (G) is

$$\boxed{3G = 2S + O}$$

	x	y
X	cos	sin
Y	~~sin~~	cos