Circles

→ **Cle** :

$r=0$, point cle, $r=1$ → unit circle, $r>0$ → real circle, $r<0$, imaginary

→ $C(h,k)$, r → $(x-h)^2+(y-k)^2=r^2$

→ $x^2+y^2=r^2$ — stand. form of cle

$x^2+y^2+2gx+2fy+c=0$ → General form

$c=(-g,-f)$ $\quad r=\sqrt{g^2+f^2-c}$

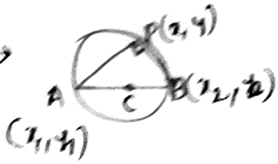
→ $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a cle if

i) $a=b$, i.e, coeff of x^2 = coeff of y^2

ii) $h=0$ ie, coeff of $xy=0$

iii) $r\ge 0$

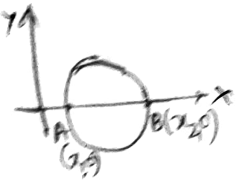
→ 

→ 

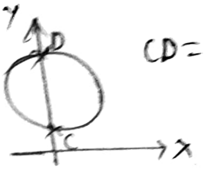
$m_{AP}\cdot m_{PB}=-1$

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

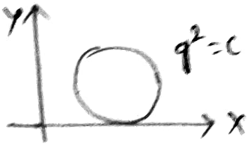
→ **Length of intercepts of cle** :

(i) 

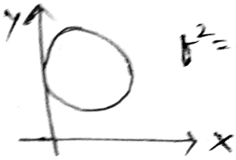
$AB=2\sqrt{g^2-c}$

(ii) 

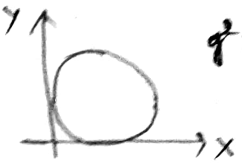
$CD=2\sqrt{f^2-c}$

(iii) 

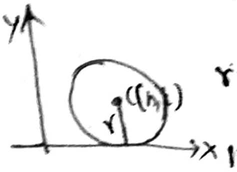
$g^2=c$

(iv) 

$f^2=c$

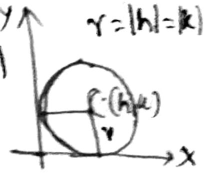
(v) 

$g^2=f^2=c$

(vi) 

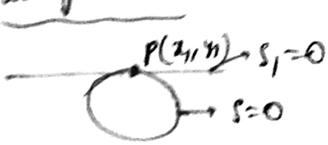
$r=|k|$

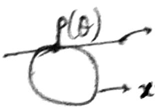
||ly for Y-axis



$r=|h|=|k|$

→ **tangent line** :





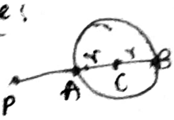
→ $x\cos\theta+y\sin\theta=r$

→ $x^2+y^2=r^2$

for $x^2+y^2+2gx+2fy+c=0$

$r=(x+g)\cos\theta+(y+f)\sin\theta$

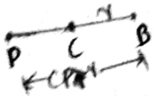
→ **Max & Min distances from a point to cle** :



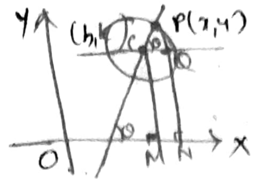
(i) Min distance $(PA)=|CP-CA|=|CP-r|$

(ii) Max distance $(PB)=CP+CB=CP+r$

(iii) 

(iv) 

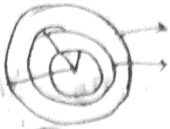
→ **Parametric eqn. of Ole :**



$x = h + r\cos\theta$, $y = k + r\sin\theta$

$P(\theta) = (r\cos\theta, r\sin\theta)$ for $x^2+y^2=r^2$

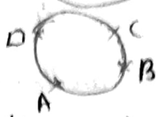
→ Concentric circles –

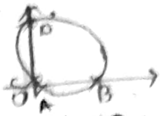


$x^2+y^2+2gx+2fy+c=0$

$x^2+y^2+2gx+2fy+k=0$

→ Concyclic points –

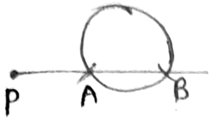




$a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$ cuts co-ordinate axes at A, B, C, & D then $\boxed{a_1a_2 = b_1b_2}$

$\because$ $OA \cdot OB = OC \cdot OD = S_{11}$

→



$\Rightarrow$ $PA \cdot PB = S_{11}$

→ eqn. of Ole through these lines is $(a_1x+b_1y+c_1)(a_2x+b_2y+c_2) - (a_1b_2+a_2b_1)xy = 0$

→ Notations –

$S \equiv x^2+y^2+2gx+2fy+c$

$S_1 \equiv xx_1+yy_1+g(x+x_1)+f(y+y_1)+c$

$S_{11} = x_1^2+y_1^2+2gx_1+2fy_1+c$

$\leftarrow S_{12} = x_1x_2+y_1y_2+g(x_1+x_2)+f(y_1+y_2)+C$

* for conjugate points $S_{12}=0$

→ Length of chord

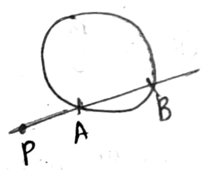
$L = 2\sqrt{r^2-d^2}$

$L = 2\sqrt{|S_{11}|}$

→ Power of the point

$P \cdot P = CP^2 - r^2$

$P \cdot S = S_{11}$



$PA \cdot PB = S_{11}$

(i) P → inside ⟷ P.P is –ve

(ii) P → outside ⟷ P.P is +ve

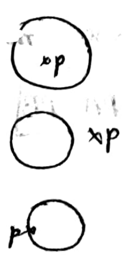
(iii) P → on the Ole ⟷ P.P is zero

→ Position of point wrt Ole –

(i) P → inside ⟷ $S_{11} < 0$

(ii) P → outside ⟷ $S_{11} > 0$

(iii) P → on the Ole ⟷ $S_{11} = 0$



→ Position of line wrt Ole –

(i)

⟷ $r = d$

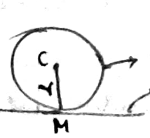
(ii)

⟷ $r > d$

(iii)

⟷ $r < d$

→ conditions of tangency –



$x^2+y^2=r^2$

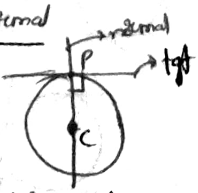
$mx-y+c=0$

$(r=d)$ $\boxed{c^2 = r^2(1+m^2)}$

$y = mx \pm r\sqrt{1+m^2}$

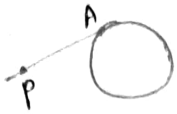
→ M is ⊥ar distance i.e, foot of ⊥ar. from C to tgt.

→ Normal



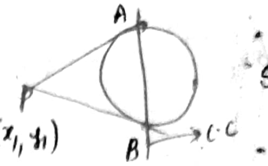
Normal always passes through centre of Ole.

→ **Length of tgt** -



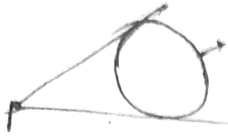
$PA = L.T = \sqrt{S_{11}}$

→ **Chord of contact**



$S_1 = 0$

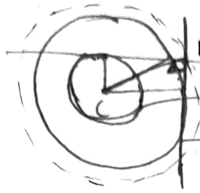
→



$m_1 + m_2 = \dfrac{2x_1y_1}{x_1^2 - r^2}$, $m_1 m_2 = \dfrac{y_1^2 - r^2}{x_1^2 - r^2}$

→ **Locus of P.O.I of ⊥ar tgts to cle to another cle** -

(i)

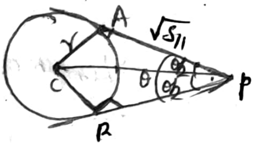


$\to x^2+y^2 = b^2 \Rightarrow x^2+y^2 = a^2+b^2$

$\to x^2+y^2 = a^2$

(ii) $(x-x_1)^2 + (y-y_1)^2 = a^2$, $(x-x_1)^2 + (y-y_1)^2 = b^2 \Rightarrow \boxed{(x-x_1)^2 + (y-y_1)^2 = a^2+b^2}$

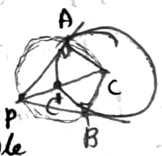
→ **Area of quadrilateral formed by tgts.**



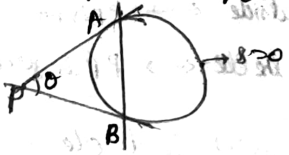
$\boxed{\text{Area} = r\sqrt{S_{11}}}$ $\tan(\theta/2) = r/\sqrt{S_{11}}$

→ PA, PB are tgts to cle $S=0$ then circumcentre of Δ^{le} PAB is $C' = \dfrac{P+C}{2}$

circumradius $(R) = \dfrac{PC}{2}$.

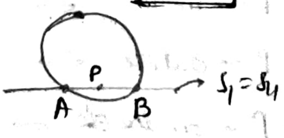


→ Area of Δ^{le} formed by pair of tgts & chord of contact of cle $S=0$ is $\boxed{\Delta = \dfrac{r(S_{11})^{3/2}}{S_{11}+r^2}}$.



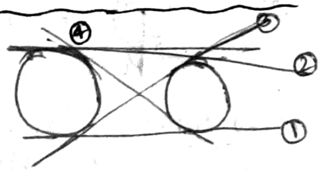
→ Eqn of chord cle $S=0$ whose midpoint $P(x_1,y_1)$ is $\boxed{S_1 = S_{11}}$ & its length $2\sqrt{|S_{11}|}$.

$\boxed{PA = PB}$



→ **Relative positions of 2 cles**

(1)



$C_1C_2 > r_1 + r_2$

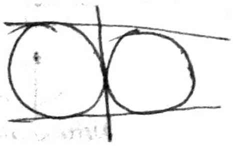
$n = 4$

* for common tgt eqn

for external only.

$S_1 - S_2 = 0$

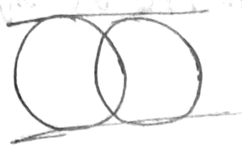
(2)



$C_1C_2 = r_1 + r_2$

$n = 3$.

* for P.O. contact use section formulae -

③ 

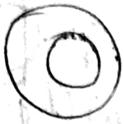
$r_1 - r_2 < C_1C_2 < r_1 + r_2$

$n = 2$

④ 

$C_1C_2 = |r_1 - r_2|$ ← P.O.C ⇒ $r_1 : r_2$ externally

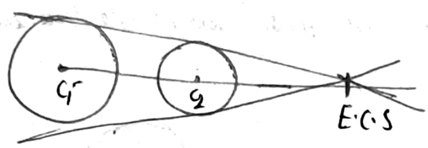
$n = 1$

⑤ 

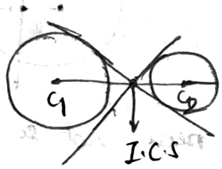
$r_1 - r_2 > C_1C_2$

$n = 0$

→ Centres of similitude –

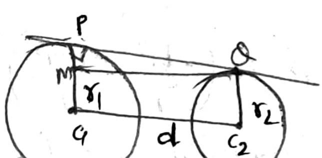
① 

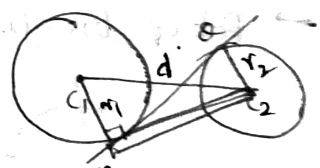
$\overline{C_1C_2}$ is divided into $r_1 : r_2$ externally by E.C.S

(ii) 

I.C.S divides $\overline{C_1C_2}$ in the ratio $r_1 : r_2$ internally

→ Length of direct common tgt: $L = \sqrt{d^2 - (r_1 - r_2)^2}$

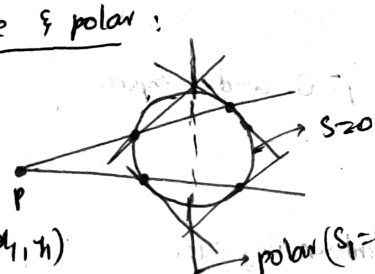




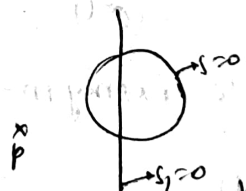
$C_1C_2 = d$

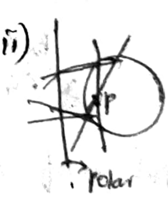
→ Length of transverse common tgt: $L = \sqrt{d^2 - (r_1 + r_2)^2}$

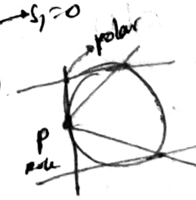
→ Pole & polar:



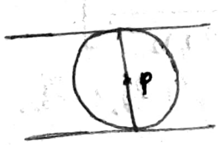
(x_1, y_1)
(pole)

i) 

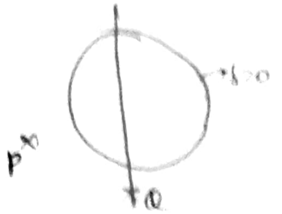
ii) 

iii) 

if pole(P) = centre(C) then polar does not exist



→ **Conjugate points** – if polar of P passes through Q then P & Q are conjugate pts



P, Q conjugate points if $S_{12}=0$

→ **Conjugate lines**: $l_1x+m_1y+n_1=0$ & $l_2x+m_2y+n_2=0$ are conjugate lines w.r.t

(i) $x^2+y^2=r^2$ is $r^2(l_1l_2+m_1m_2)=n_1n_2$

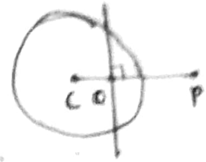
(ii) $x^2+y^2+2gx+2fy+c=0$ is $r^2(l_1l_2+m_1m_2)=(l_1g+m_1f-n_1)(l_2g+m_2f-n_2)$

→ **Inverse points** – P, Q are said to inverse points w.r.t cle S=0 with centre C

i) if P, Q lies on same side

ii) if C, P, Q are collinear points → Q lies on polar of P

iii) $CP \cdot CQ = r^2$



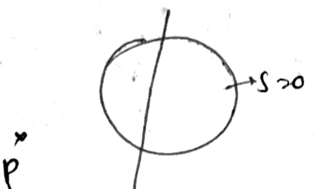
⇒ inverse points are conjugate points converse need not be true.

Q is foot of ⊥ar drawn from P to polar) Q is foot of ⊥ar drawn from C to polar.

→ Pole of line $lx+my+n=0$ w.r.t cle $x^2+y^2=r^2$ is $\left(\frac{-lr^2}{n}, \frac{-mr^2}{n}\right)$

w.r.t cle $x^2+y^2+2gx+2fy+c=0$ is $\left(-g+\frac{-lr^2}{N}, -f+\frac{-mr^2}{N}\right)$

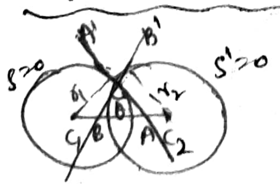
where $N = l(-g)+m(-f)+n$



(or) $\frac{x_1+g}{l} = \frac{y_1+f}{m} = \frac{r^2}{lx_1+my_1+n}$

Best equate $lx+my+n=0$ to $S_1=0$ and compare $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

System of Cles



angle b/w 2 intersecting cles

(i) C_1, C_2 are centres & radii r_1, r_2 is

$$\cos^{-1}\left[\frac{C_1C_2^2-r_1^2-r_2^2}{2r_1r_2}\right]$$

(ii) for $x^2+y^2+2g_1x+2f_1y+c_1=0$ & $x^2+y^2+2g_2x+2f_2y+c_2=0$ is

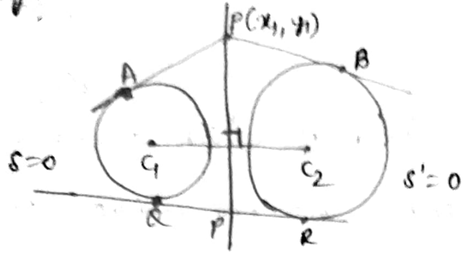
$$\cos^{-1}\left[\frac{c_1+c_2-2(g_1g_2+f_1f_2)}{2\sqrt{g_1^2+f_1^2-c_1}\sqrt{g_2^2+f_2^2-c_2}}\right]$$

→ Orthogonal intersection of circles ($\theta = 90$)

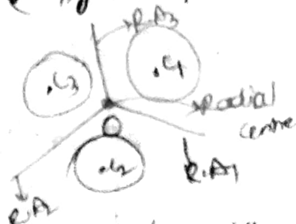
$r_1^2 + r_2^2 = d^2$

$2g_1g_2 + 2f_1f_2 = c_1 + c_2$

→ Radial axis – The axis of 2 cles which moves such that the length of tangents drawn from it to the two cles are equal.

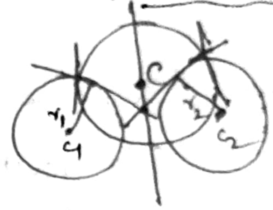


$2(g-g_1)x + 2(f-f_1)y + c - c_1 = 0$

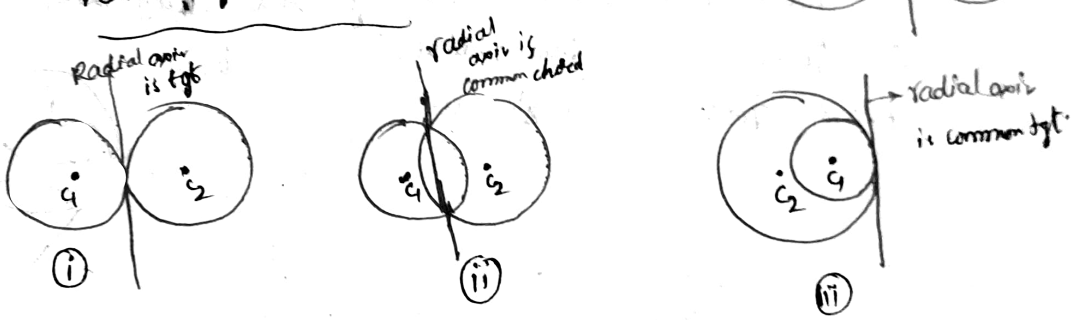


→ R.A always bisects the common tgts of 2 cles

→ 2 cles cuts 3rd cle orthogonally then radial axis of 2 cles will pass through centre of 3rd cle.



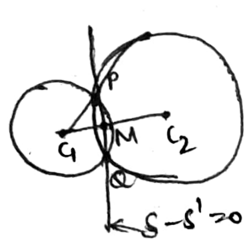
Position of radial axis



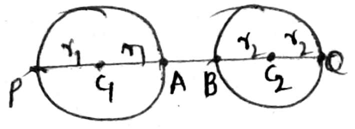
→ no. of radial axes of 'n' circles, no three of their centres are collinear is $^nC_2$.

→ Power of radial centre w.r.t 3 cles are equal. (S_{11}).

→ Length of common chord:



$PQ = 2(PM) = 2\sqrt{(C_1P)^2 - (C_1M)^2}$



Min. D = AB

$= C_1C_2 - (r_1 + r_2)$

Max D = PQ

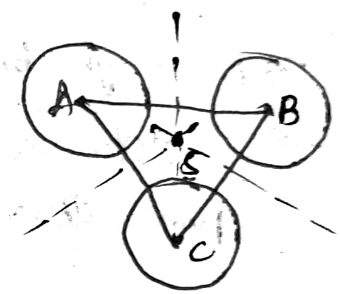
$= C_1C_2 + (r_1 + r_2)$

→ If 1^{st} ○le bisects the 2^{nd} ○le circumcentre the centre of 2^{nd} ○le lies on radical axis.

→ If A, B, C are centres of 3 ○les touching mutually externally then radical centre of ○les is incentre of △le ABC.

→ Radical centre of 3 ○les described on 3 sides of △le as diameter is orthocentre of △le ABC.

→ A radical centre of 3 ○les of equal radii which doesn't touch externally pairwise whose centres are non collinear is circumcentre of △le ABC.



→ A,B,C are centres of 3 ○les C_1, C_2, C_3 such that C_1C_2 touch each other externally and they both touch C_3 from inside, then radical centre of ○les is excentre opp. to C for △le ABC.