Electrostatics

→ Charges at rest (electrostatics)

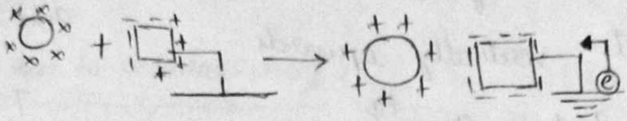
SI - coulomb (C), C.G.S = 3×10^{9} e.s.u

→ charge is conserved, invariant.

$q = \pm ne$

→ conduction; Ⓐ—Ⓑ q_{A} q_{B} $\underset{\text{on both}}{q_{net}} = \frac{q_A + q_B}{2}$ conduction preceeds repulsion.

induction :



$$q' = -q\,(1 - 1/\kappa)$$

→ Coulomb's inverse square law:

for point charges & stationary charges only.

(+q₁) ←r→ (+q₂)

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

(applicable for all bodies)

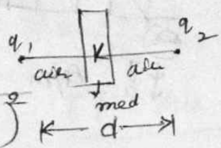
→ dielectric const:

$\varepsilon_r (\text{or}) \kappa = \frac{\varepsilon}{\varepsilon_0}$ $\kappa=1$ for air, $\kappa=\infty$ for metals

$$\boxed{\varepsilon_{med} = \kappa\varepsilon_0}$$

→ $F_{med} = \frac{F_{air}}{\kappa}$

→ $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{\left((d-t)_{air} + \sqrt{\kappa}\,t\right)^2}$



$\vec{F_{12}} = -\vec{F_{21}}$

→ vector form $\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^3}\vec{r_{12}}$

→ Electric field intensity $(\vec{E})$:

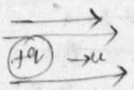
① point charge in vacuum $E_{vacu} = \frac{1}{4\pi\varepsilon_0}\,q/r^2 = \frac{F_{vacu}}{q_0}$ $F_{med} < F_{vacu}$

in medium $E_{med} = \frac{1}{4\pi\varepsilon}\,q/r^2 = \frac{1}{4\pi\kappa\varepsilon_0}\,q/r^2 = \frac{F_{vacuum}}{\kappa}$

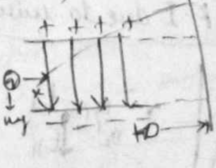
→ Force acting on stationary charge

$$\vec{F} = q\vec{E}$$

→ When charged particle is moving either along st line (or) opposite to field — st. line motion.



→ charged particle is moving ⊥ar then it moves in parabolic path.



$$\vec{F} = q\vec{E}$$

$m\vec{a} = q\vec{E} \Rightarrow a = \frac{q\vec{E}}{m}$

→ Work done by particle in field $W = \Delta KE$

$$W = \frac{q^2 E^2 t^2}{2m}$$

→ Time period of pendulum

i) if field acts vertically downwards

$$a_{net} = g + \frac{Eq}{m}$$

$$T' = 2\pi\sqrt{\frac{l}{g + Eq/m}} \qquad T' < T$$

ii) field acts vertically upwards

$$a_{net} = g - \frac{Eq}{m}$$

$$T' = 2\pi\sqrt{\frac{l}{g - Eq/m}} \qquad T' > T$$

iii) If field acts horizontally

$$a_{net} = \sqrt{g^2 + \frac{E^2q^2}{m^2}} > g$$

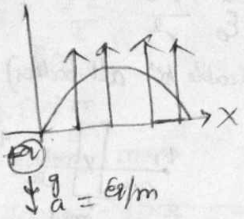
$$T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \frac{E^2q^2}{m^2}}}}$$

→ Projectile motion

$$R = \frac{u^2 \sin 2\theta}{g + Eq/m}$$

$$H = \frac{u^2 \sin^2\theta}{2(g + Eq/m)}$$

$$T = \frac{2u\sin\theta}{g + Eq/m}$$



→ Neutral point (or) null point - no charge at that point

① Like charges

$\overset{+}{q_1}$ $\overset{+}{q_2}$

from $+q_1$

$$x = \frac{r}{\sqrt{\frac{q_2}{q_1}} + 1}$$

from q_2

$$= r - x$$
$$= r - \frac{r}{\sqrt{\frac{q_2}{q_1}} + 1}$$

② Unlike charges

$\overset{+}{q_1}$ $\overset{-}{q_2}$

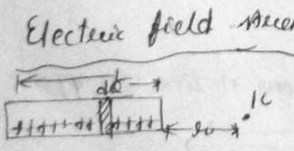
from q_1

$$x = \frac{r}{\sqrt{\frac{q_2}{q_1}} - 1}$$

from q_2

$$r + x = r + \frac{r}{\sqrt{\frac{q_2}{q_1}} - 1}$$

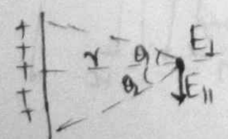
→ Electric field strength (E.F.I) due to uniform rod:



$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{l}\left(\frac{1}{r} - \frac{1}{l+r}\right)$$

→ E.F.I due to wire:

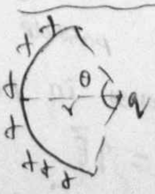
Q, l



$$E_\perp = \frac{\lambda}{4\pi\varepsilon_0 r}(\sin\theta_1 + \sin\theta_2)$$

$$E_\parallel = \frac{\lambda}{4\pi\varepsilon_0 r}(\cos\theta_2 - \cos\theta_1)$$

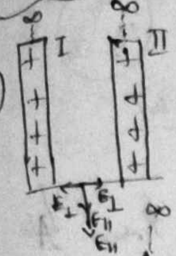
→ E.F.I due to arc:



$$E = \frac{\lambda}{2\pi\varepsilon_0 r}\sin\theta/2$$

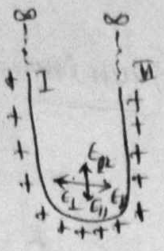
Combination of rods & arcs :

i)



$$E_{net} = \frac{\lambda}{2\pi\varepsilon_0 r}$$

ii)



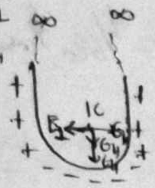
$E_{net} = 0$

$(E_{\parallel})_I + (E_{\parallel})_{II} = \lambda/2\pi\varepsilon_0 r$

$E_{rc} = \frac{\lambda}{2\pi\varepsilon_0 r}$

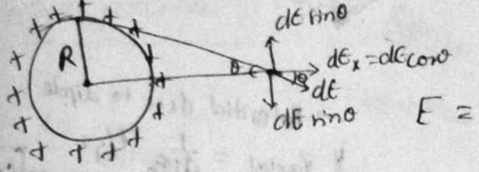
$E_{net} = (E_{\parallel})_I + (E_{\parallel})_{II} - E_{rc}$

iii)

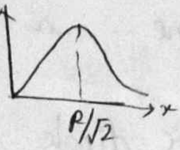


$$E_{net} = \frac{\lambda}{\pi\varepsilon_0 r}$$

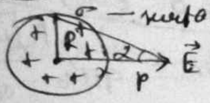
Electric field due to circular ring :



$$E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(x^2+R^2)^{3/2}}$$



E·F·I due to circular disc



σ — surface charge density

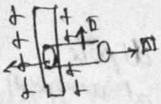
$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)$$

$$E = \frac{\sigma}{2\varepsilon_0}(1 - \cos\alpha)$$

E·F·I due to long sheet :

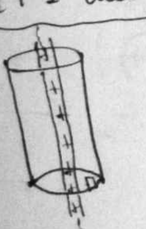
α = 90° $E = \frac{\sigma}{2\varepsilon_0}$

E·F·I due to conducting sheet :



$$E = \frac{\sigma}{\varepsilon_0}$$

E·F·I due to infinite line charge :



$$E = \frac{\lambda}{2\pi R\varepsilon_0}$$

E·F·I due to conducting sphere :

(hollow)

i) r < R

$E_{inside} = 0$

$E_{out} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

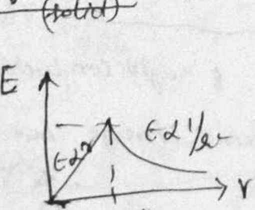
$E_{surf} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$

E·F·I due ~~non~~-conducting sphere : ~~(solid)~~

$E_{out} = \frac{1}{4\pi\varepsilon_0} q/r^2$

$E_{surf} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$

$E_{inside} = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}$



(When 2 bodies connected p.diff V ~~is~~ equal)

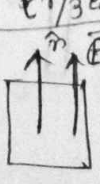
*** Electric field at a distance 'r' (r<R) from centre of sphere is $\rho r/3\varepsilon_0$

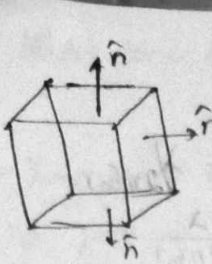
Electric flux :

$\phi = \vec{E}\cdot\vec{A}$

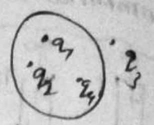
area vector is ⊥ar to the surface (open)

$\phi = E \cdot A\cos\theta$





Gauss law: $\oint \vec{E}\cdot d\vec{s} = \frac{q_{enclosed}}{\varepsilon_0}$



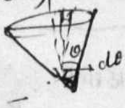
$\oint \vec{E}\cdot d\vec{s} = \frac{(q_1+q_2+q_4)}{\varepsilon_0}$

i) $\phi_{\substack{cube \\ (total)}} = \frac{q}{\varepsilon_0}$, $\phi_{\substack{each \\ face}} = q/6\varepsilon_0$, $\phi_{corner} = q/4\varepsilon_0$

→ Symmetric charge distribution only.

Solid angle (3 D) $\Omega = 4\pi$ (steradian)

in cone – $\Omega = 2\pi(1-\cos\theta)$



⇒ Electric dipole

$\vec{P} = q \times 2a$

→ always -ve to +ve.

→ Potential due to dipole

$V_{axial} = \frac{1}{4\pi\varepsilon_0} P/r^2$

$V_{point} = 1/4\pi\varepsilon_0 \frac{P\cos\theta}{r^2}$ $\boxed{(\vec{P}\cdot\vec{r})}$

$V_{eq} = 0$ $(\theta = 90°)$

→ E·F·I due to any point due to dipole $E_{point} = \frac{1}{4\pi\varepsilon_0} \frac{P\sqrt{1+3\cos^2\theta}}{r^3}$

E_{axial} $(\theta = 0°)$, $E_{equatorial}$ $(\theta = 90°)$

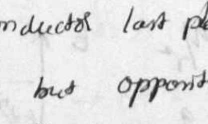
due to dipole at anypoint) $E = 1/4\pi\varepsilon_0 \frac{2P\cos\theta}{r^3}$ (axi) axial $E = k\cdot\frac{2pr}{(r^2-a^2)^2}$

$E = 1/4\pi\varepsilon_0 \frac{P\sin\theta}{r^3}$ (eq) Eq. $E = k\cdot\frac{P}{(r^2+a^2)^{3/2}}$

→ τ(torque) = $\vec{P}\times\vec{E} = \overset{dipole}{PE}\sin\theta$

Work done(W) = PE($\cos\theta_1 - \cos\theta_2$)

$U = -\vec{P}\cdot\vec{E}$

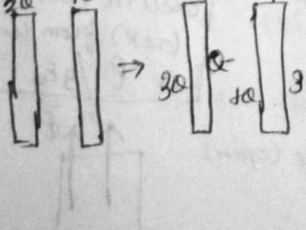


→ Charge distribution due to induction

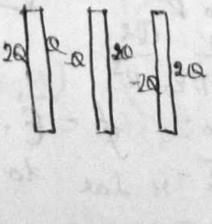
i) first conductor left plate & right conductor last plate same charge.

ii) in b/w conductor same charge but opposite sign

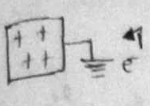
Exs ①



②



Grounding (or) Earthing



$V = 0$

$q \neq 0$

$V_{in} = 0$

$Q_{out} = Q$

$q' = \frac{Q}{b} a$

$\because V_{in} = 0$

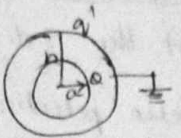
$0 = \frac{1}{4\pi\varepsilon_0}\frac{q'}{a} + \frac{1}{4\pi\varepsilon_0} Q/b$

$q'/a = -Q/b$

$V_{out} - V_{in} = \Delta V$

$\Delta V = \frac{Q}{4\pi\varepsilon_0}\left(1 - a/b\right)$

$V_{out} = \frac{1}{4\pi\varepsilon_0} q'/b + \frac{1}{4\pi\varepsilon_0} Q/b$



$V_{out} = 0$

$Q_{in} = Q$

$q' = -Q$

$\Delta V = \frac{Q}{4\pi\varepsilon_0}\left(1/a - 1/b\right)$

$V_{out} = \frac{1}{4\pi\varepsilon_0}\frac{q'}{b} + \frac{1}{4\pi\varepsilon_0} Q/b$

$0 = \frac{1}{4\pi\varepsilon_0} q'/b + \frac{1}{4\pi\varepsilon_0} Q/b$

$q' = -Q$

$V_{in} = \frac{1}{4\pi\varepsilon_0} Q/a$

$\Leftarrow \frac{1}{4\pi\varepsilon_0} Q/b$

Electrostatic potential energy (U) $= \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}$

for $+q_1 \xrightarrow{r} +q_2$ $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = Vq_0$ for $+q_1 \xrightarrow{r} -q_2$ $U = \frac{1}{4\pi\varepsilon_0}\frac{-q_1 q_2}{r}$

Electric potential (V):

$W_{all\,forces} = \Delta KE$

$W_c + W_{ext} + W_{N.C} = \Delta KE$

$V = \frac{W}{q_0}$

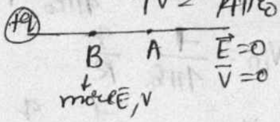
$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

i) for $(+q) \xrightarrow{r} p$ $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ ii) for $(-q) \xrightarrow{r} p$ $V = \frac{-1}{4\pi\varepsilon_0} q/r$

Relation b/w electric field ($\vec{E}$) & potential diff (V):

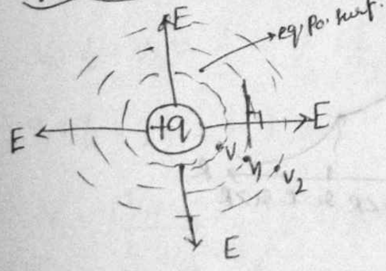
$V_B - V_A = -\int_A^B \vec{E}\, dl$

$\Delta V = \frac{1}{4\pi\varepsilon_0} q/r \downarrow$



$\bar{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$

Equipotential surface:



→ these never be intersect.

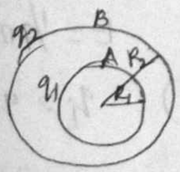
→ E is normal to the surface.

→ P. diff b/w any two points on surf. is 0.

$\boxed{W = 0}$ by electric force.

$W = \Delta V . q = 0$

Concentric spherical shells:



$\Delta V = \frac{q}{4\pi\varepsilon_0}\left(1/R_1 - 1/R_2\right)$

→ Potential due to system of point charges

$V = V_1 + V_2 + \dots + V_n = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$

→ Potential due to electric dipole:

i) $V_{point} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{(\vec{p}.\vec{r})}{r^3}$ (iii) $V_{eq} \equiv 0$ $(\theta = 90°)$

ii) $V_{axial} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$ $(\theta = 0°)$

Null (or) zero potential:

→ $V_{net} = 0$, exist b/w 2 dissimilar charges only.

① Inside zero potential

$+q$ $\dot{y}$ $-q$ $\dot{x}$

$x = \frac{r}{\frac{q_2}{q_1}+1}$ from q_1, $r - x = r - \frac{r}{\frac{q_2}{q_1}+1}$ from q_2

(ii) Outside zero potential

$y = \frac{r}{\frac{q_2}{q_1}-1}$ from q_1, $r + y = r + \frac{r}{\frac{q_2}{q_1}-1}$ from q_2.

Potential of different cases

① line of charge $V = \frac{-\lambda}{2\pi\varepsilon_0} \log_e r$

② non conducting sheet $V = \frac{-\sigma r}{2\varepsilon_0}$

③ conducting sheet $V = \frac{-\sigma r}{\varepsilon_0}$

④ Spherical shell i) $V_{out} = \frac{-1}{4\pi\varepsilon_0} \frac{q}{r}$

ii) $V_{in} = \frac{-1}{4\pi\varepsilon_0} \frac{q}{R}$

iii) $V_{surf} = \frac{-1}{4\pi\varepsilon_0} \frac{q}{R}$

if n bubbles collase to form big drop

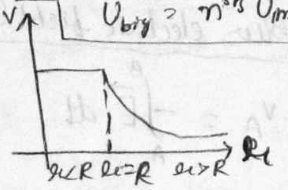
$Q_{big} = n Q_{small}$

$C_{big} = n^{1/3} C_{small}$

$V_{big} = n^{2/3} V_{small}$

$E_{big} = n^{2/3} E_{small}$

$U_{big} = n^{5/3} U_{small}$

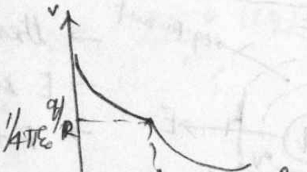


⑤ Solid sphere

(i) $V_{out} = \frac{-1}{4\pi\varepsilon_0} q/r$

(ii) $V_{in} = \frac{-q}{4\pi\varepsilon_0 r}\left(3/2 - \frac{r^2}{2R^2}\right)$

(iii) $V_{surf} = \frac{-1}{4\pi\varepsilon_0} \frac{q}{R}$

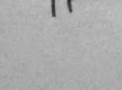


⇒ Capacitors – storge of charge.

→ Effect of dielectrics – $E = \frac{E_0}{K} < E_0$ $E = 1/4\pi\varepsilon_0 \, q/r^2$

$E_i = E_0 - E = E_0(1 - 1/K)$ $q_i = q(1 - 1/K)$

Capacitance of isolated conductor ⇒ $C = \frac{Q}{V}$ SI = Coulomb/volt (or) Faraday



Capacitance of isolated sphere ⇒ $C = 4\pi\varepsilon_0 R$

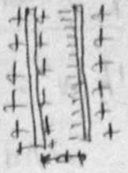
Capacitance depends upon ⓘ size, ⓘⓘ shape (area) ⓘⓘⓘ insulating media

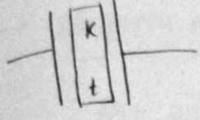
$C = 4\pi\varepsilon_0 \frac{r_1 r_2}{r_2 - r_1}$

→ Parallel plate capacitors

i) air medium $C_0 = \frac{\varepsilon_0}{d} A$ (A = area)

ii) any medium $C = \frac{K\varepsilon_0 A}{d} = \frac{\varepsilon A}{d} = KC_0$

$C > C_0$

→ When dielectric const 'k' & thickness 't' placed partially b/w plates



$$C = \frac{\varepsilon_0 A}{d - t(1 - 1/k)}$$

→ When 'n' no. of dielectric slabs with diff thickness introduced partially.

$$\frac{Q}{V} = C = \frac{\varepsilon_0 A}{(d - (t_1 + t_2 + \cdots + t_n))\left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + \cdots + \frac{t_n}{k_n}\right)}$$

→ When 'n' no. of different dielectric with different thickness arranged completely.

$$C = \frac{\varepsilon_0 A}{\left(\frac{t_1}{k_1} + \frac{t_2}{k_2} + \cdots + \frac{t_n}{k_n}\right)}$$

→ Series

If 3 capacitors

$1/C_s = 1/C_1 + 1/C_2 + 1/C_3$

$V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$

$q_1 : q_2 : q_3 = 1 : 1 : 1$

$V_1 = \left(\frac{1/C_1}{1/C_2 + 1/C_1 + 1/C_3}\right)V$, V_2 & V_3 lly.

for n identical capacitors

$C_s = C/n < n$

$\rightarrow \frac{C_s}{C_p} = 1/n^2$

→ Parallel

$C_p = C_1 + C_2 + C_3$

$V_1 : V_2 : V_3 = 1 : 1 : 1$

$q_1 : q_2 : q_3 = C_1 : C_2 : C_3$

$q_1 = \left(\frac{C_1}{C_1 + C_2 + C_3}\right)q$, q_2 & q_3 lly

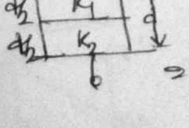
$C_p = nC > C$

$V_A - V_B = \int \bar{E} \cdot d\bar{l}$ for non-uniform e⁻ field

→ Combination of dielectrics in capacitors:

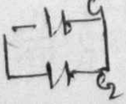
$t_1 = t_2 = d/2$, $A_1 = A_2 = A$

i)



$=$

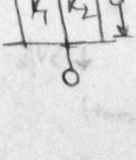
$$\boxed{K = \frac{2K_1K_2}{K_1 + K_2}}$$



$\Rightarrow \frac{1}{C} = 1/C.$

$A_1 = A_2 = A/2$, $t_1 = t_2 = d$

ii)



$K = \frac{K_1 + K_2}{2}$

→ **Energy stored in a capacitor**

$Q = CV$

$\sigma = \frac{Q}{A}$, $C = \frac{\varepsilon_0 A}{d}$

$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$

$U = \frac{\sigma^2 A d}{2\varepsilon_0}$ Energy density $= \frac{U}{V} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{1\varepsilon_0 V^2}{2 d^2} = \frac{1}{2}\varepsilon_0 E^2$

→ When 2 charged capacitors in connected such that +ve plate to +ve plate and vice versa.

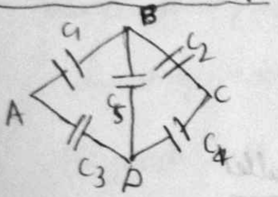
Common potential $V = \frac{Q}{C} = \frac{Q_1+Q_2}{C_1+C_2} = \frac{C_1V_1+C_2V_2}{C_1+C_2}$

for reversed $V = \frac{C_1V_1 - C_2V_2}{C_1+C_2}$

→ Loss of energy $\Delta U = U_i - U_f$

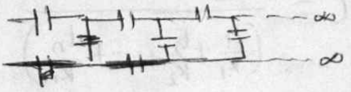
$= \frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)(V_1-V_2)^2$

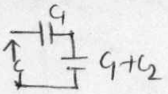
Wheatstone bridge



$\frac{C_1}{C_2} = \frac{C_3}{C_4}$

→ **Infinite ladder network**

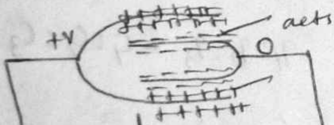




$1/C = 1/C_1 + 1/(C_1+C_2)$

$C^2 + CC_2 - C_1C_2 = 0$

→ **Arrangement of plates:**



$C_{eff} = 2C$

→ Placing dielectric slab without disconnecting battery (at const. V.)

i) $C = KC_0$ ↑

ii) $V = V_0$ const.

iii) $Q = CV = KC_0V_0 = KQ_0$ ↑

iv) $E = V/d = V_0/d = E_0$ const

v) $U = ↑ = \frac{1}{2}CV^2 = KU$

→ Placing dielectric slab with disconnecting battery (at const q)

i) $C = KC_0$ ↑

ii) $V = Q/C$ ↓ → V/k

iii) $Q = Q_0$ (const)

iv) $E = V/d = \frac{V_0}{Kd} = \frac{E_0}{k}$ ↓

v) $U = \frac{1}{2}CV^2$ ↓

$U = U/k$