Functions

→ No. of functions from A to B is $\{n(B)\}^{n(A)}$

Here A is domain, B is codomain.

Range of a function f denoted by $f(A)$ & $f(A) \subseteq B$

→ **One-one function (Injection):**

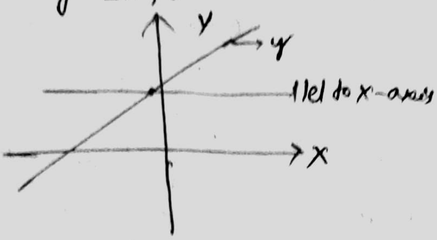
A function $f: A \to B$ is $f(a_1) = f(a_2)$ then $a_1 = a_2$.

no. of one-one functions of $n(A) = r$, $n(B) = n$ $r \leq n$ is ${}^nP_r$.

$r > n$ one-one function are 0.

Horizontal line test – line ||el to X-axis meets graph at 1 point. strictly ↑ & strictly ↓ functions.

$y = 2x + 3$ (one-one)

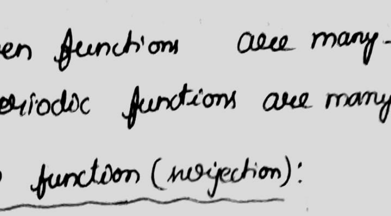


no. of one-one function from A to A if $n(A) = q$ is $q!$

→ **Many-one functions**

Horizontal line test – line ||el to x-axis meets graph at more than 1 points.

$f: R \to R$, $f(x) = (x-1)(x-2)(x-3)$



all even functions are many-one.

all periodic functions are many-one.

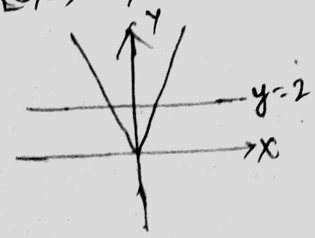
→ **Onto function (surjection):**

Codomain = range $f(A) = B$

if $n(A) \geq n(B)$ then no. of onto functions $= n^r - {}^nC_1(n-1)^r + {}^nC_2(n-2)^r - {}^nC_3(n-3)^r + \cdots + (-1)^{n-1}\,{}^nC_{n-1}$.

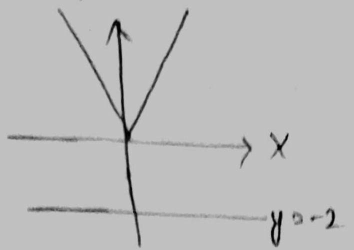
⇒ $n(A) < n(B)$ then no. of onto is zero.

$f: R \to [0, \infty)$, $f(x) = |x|$



Into function:

$f: R \to R$, $f(x) = |x|$ is into function



Range ≠ codomain

Bijection:

i) A, B are finite set then $f: A \to B$ is a bijection then $n(A) = n(B)$

ii) no. of bijections if $n(A) = n(B)$ is $(n(A))!$

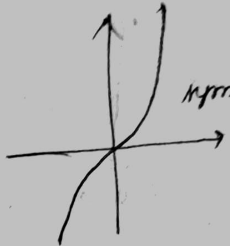
Constant function: no. of const. functions from A to B is $n(B)$.

Even function: $f(-x) = f(x)\ \forall x \in A$

$y = x^2$

symmetric about Y-axis

Odd function: $f(-x) = -f(x)$



symmetric about origin

Modulus function: $|x| = \begin{cases} -x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ x & \text{if } x > 0 \end{cases}$

$|x| \Rightarrow$ domain $= R$, range $= [0, \infty)$

$[x] + [-x] = \begin{cases} 0 & \text{if } x \in I \\ -1 & \text{if } x \notin I \end{cases}$

Periodic function: $f(x)$ is periodic if $f(x+T) = f(x)$

T is period of $f(x)$ here T is least +ve integer

$kx - [kx] = 1/k$

Range: Range of $ax^2 + bx + c = y$ is $\left[\frac{4ac - b^2}{4a}, \infty\right)$ if $a > 0$ or $\left(-\infty, \frac{4ac - b^2}{4a}\right]$ if $a < 0$.

Function	Condition to find out domain
$1/f(x)$	$f(x) \neq 0$
$\sqrt{f(x)}$	$f(x) \geq 0$
$\frac{1}{\sqrt{f(x)}}$	$f(x) > 0$
$\sqrt[3]{f(x)}$	$f(x)$ is real number
$a^{f(x)}$	$f(x)$ is real number
$\log_{g(x)} f(x)$	$f(x) > 0$, $g(x) > 0$, $g(x) \neq 1$

Explicit function: $y=f(x)$ is said to explicit fun. of x if dependent variable y can be expressed in term of independent variable x.

Ex: (i) $e^y - e^{-y} = 2x \Rightarrow e^{2y} - 2xe^y - 1 = 0 \Rightarrow e^y = \frac{2x \pm \sqrt{4x^2+4}}{2}$

$e^y = x + \sqrt{x^2+1}$

$y = \ln(x+\sqrt{x^2+1})$

(ii) $y = x - \cos x$

Implicit function: $y=f(x)$ is said to implicit of x if y can't be written in term of x only.

Ex: (i) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

(ii) $xy = \sin(x+y)$

Properties of modulus:

if (i) $|f(x)| = a$ then $f(x) = \pm a$

(ii) $|f(x)| \geq a$ then $f(x) \leq -a$ (or) $f(x) \geq a$

(iii) $|f(x)| \leq a$ then $-a \leq f(x) \leq a$

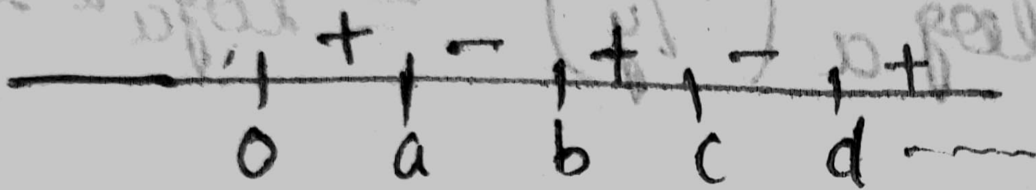
(iv) $a \leq |f(x)| \leq b \Rightarrow x \in [-b,-a] \cup [a,b]$

(v) $|x+y| = |x| + |y|$ if x, y have same sign & either of x, y is zero & $xy \geq 0$

(vi) $|x-y| = |x| - |y| \Rightarrow x \geq 0, y \geq 0$ and $|x| \geq |y|$; or $x \leq 0, y \leq 0$ & $|x| \geq |y|$.

Wavy curve method

$(x-a)(x-b)(x-c) \cdots \; \begin{matrix} \geq 0 \\ < 0 \end{matrix}$



Sub nos b/w 0 & a in above

if (+) keep it as

if negative (-ve)

for +

$(0,a) \cup (b,c) \cup (d,\infty)$

(for −)

$(a,b) \cup (c,d)$

$\sin^{-1}(f(x))$ (or) $\cos^{-1}(f(x))$ $\quad -1 \le f(x) \le 1$

$\sec^{-1}(f(x))$ (or) $\operatorname{cosec}^{-1}(f(x))$ $\quad f(x) \le -1$ (or) $f(x) \ge 1$

$f \pm g,\ fg$ $\quad D(f) \cap D(g)$

$x + 1/x$ for solving use AM ≥ GM

Functional equations

i) $f(x+y) = f(x) + f(y)$ then $f(x) = kx$

ii) $f(x+y) = f(x)f(y)$ then $f(x) = k^x$

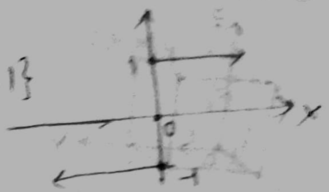
iii) $f(xy) = f(x) + f(y)$ then $f(x) = k \log_a x$

iv) $f(x) + f(1/x) = f(x)f(1/x)$ then $f(x) = 1 \pm x^n$

v) $f(x+y) + f(x-y) = 2f(x)f(y)$ then $f(x) = \frac{k^x + k^{-x}}{2}$

vi) $f(x+y) = f(x) = f(y) \Rightarrow f(x) = k$

→ $\operatorname{Sgn}(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ $\quad$ Range $= \{-1, 0, 1\}$



i) $\operatorname{sgn}(kx) = \operatorname{sgn}(x),\ k \in N$ $\quad$ iii) $|x| \operatorname{sgn}(x) = x$

ii) $x \operatorname{sgn}(x) = |x|$ $\quad$ iv) $x \operatorname{sgn}(x) \cdot \operatorname{sgn}(x) = x$

→ period of $f(x+a) + f(x+b) = \text{const}$ is $2|b-a|$

Ex: $f(x) + f(x+5) = 12 \Rightarrow 2|5-0| = 10$

→ period of $f(x+a) + f(x-a) = f(x)$ is $6|a|$

→ $(x-\alpha)(x-\beta) \ge 0 \Rightarrow x \ge \alpha$ (or) $x \le \beta$ then $x \in (-\infty, \beta] \cup [\alpha, \infty)$

$(x-\alpha)(x-\beta) > 0 \Rightarrow x > \alpha$ & $x < \beta$ then $x \in (-\infty, \beta) \cup (\alpha, \infty)$

$(x-\alpha)(x-\beta) < 0 \Rightarrow \beta < x < \alpha$ then $x \in (\beta, \alpha)$

$(x-\alpha)(x-\beta) \le 0 \Rightarrow \beta \le x \le \alpha$ then $x \in [\beta, \alpha]$

	domain	Range
$\sin x$	R	$[-1,1]$
$\cos x$	R	$[-1,1]$
$\tan x$	$R - \{(2n+1)\pi/2,\ n \in Z\}$	R
$\cot x$	$R - \{n\pi,\ n \in Z\}$	R
$\operatorname{cosec} x$	$R - \{n\pi,\ n \in Z\}$	$R - (-1,1)$
$\sec x$	$R - \{(2n+1)\pi/2,\ n \in Z\}$	$R - (-1,1)$