# Pair of straight lines

→ $ax^2+2hxy+by^2=0$ are $ax+(h+\sqrt{h^2-ab})y=0$, $ax+(h-\sqrt{h^2-ab})y=0$

→ nature of pair of lines:

i) $h^2>ab$ — two real & distinct. (for $ax^2+2hxy+by^2=0$)
ii) $h^2=ab$ — two coincident lines
iii) $h^2<ab$ — imaginary lines

→ slope b/w 2 lines — $m_1+m_2=-2h/a$, $m_1m_2=a/b$, $|m_1-m_2|=\frac{2\sqrt{h^2-ab}}{|b|}$

→ angle b/w lines —

$$\cos\theta=\frac{|a+b|}{\sqrt{(a-b)^2+4h^2}},\quad \sin\theta=\frac{2\sqrt{h^2-ab}}{\sqrt{(a-b)^2+4h^2}},\quad \tan\theta=\frac{2\sqrt{h^2-ab}}{|a+b|}$$

if $a+b=0$ then ⊥ar lines.

→ product of ⊥ars :

~~from~~ $(\alpha,\beta)$ to $ax^2+2hxy+by^2=0$ is $\frac{|a\alpha^2+2h\alpha\beta+b\beta^2|}{\sqrt{(a-b)^2+4h^2}}$

→ Area of $\Delta^{le}$ formed by $lx+my+n=0$ & $ax^2+2hxy+by^2=0$ is

$$\frac{n^2\sqrt{h^2-ab}}{|am^2-2hlm+bl^2|}$$

→ eqn of pair of lines passes through origin and making an angle '$\alpha$' with the line $lx+my+n=0$ is $(lx+my)^2-\tan^2\alpha\,(mx-ly)^2=0$

then area of $\Delta^{le}$ is $\frac{n^2}{\tan\alpha\,(l^2+m^2)}$

→ $ax^2+2hxy+by^2=0$ represents 2 sides of $\Delta^{le}$ & $(x_1,y_1)$ be its centroid. the midpoint of third side of $\Delta^{le}$ is $\frac{3}{2}G$ i.e, $\left(\frac{3x_1}{2},\frac{3y_1}{2}\right)$

→ if $(kl,km)$ is orthocentre of $\Delta^{le}$ formed by lines $ax^2+by^2+2hxy=0$ and $lx+my+n=0$ then $k=\frac{-n(a+b)}{am^2-2hlm+bl^2}$

→ $S\equiv ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of lines then

$\Delta=abc+2fgh-af^2-bg^2-ch^2=0$ i.e $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=0$, $h^2\geq ab$, $g^2\geq ac$, $f^2\geq bc$.

→ area of $\Delta^{le}$ formed by $S$ & $lx+my+n=0$ is

$$\frac{(lx+my+n)^2\sqrt{h^2-ab}}{|am^2-2hlm+bl^2|}$$

→ Intercepts for pair of lines :

ⓐ x-axis is $\dfrac{2\sqrt{g^2-ac}}{|a|}$ ⓑ y-axis is $\dfrac{2\sqrt{f^2-bc}}{|b|}$

if lines intersect on ⓐ x-axis then $g^2=ac$ & $2fgh=af^2+ch^2$

ⓑ y-axis then $f^2=bc$ & $2fgh=bg^2+ch^2$

→ area of ||elogram is $\dfrac{|c|}{2\sqrt{h^2-ab}}$

→ eqn. to the pair of lines passing through the origin & each is at a distance of d from $(\alpha,\beta)$ is $(\beta x-\alpha y)^2=d^2(x^2+y^2)$

→ If $ax^2+2hxy+by^2=0$, $ax^2+2hxy+by^2+2gx+2fy+c=0$ are p.o lines

diagonality condition

| | | |
|---|---|---|
| condition for square | $a+b=0$ , | $\dfrac{g^2-f^2}{gf}=\dfrac{a-b}{h}$ |
| rhombus | $a+b\neq0$ , | $\dfrac{g^2-f^2}{gf}=\dfrac{a-b}{h}$ |
| ||elogram | $a+b\neq0$, | $g^2-f^2/gf\neq a-b/h$ |
| rectangle | $a+b=0$, | $\dfrac{g^2-f^2}{gf}\neq\dfrac{a-b}{h}$ |

Area $=\dfrac{|c|}{2\sqrt{h^2-ab}}$

→ eqn. of angular bisector

$$\frac{x^2-y^2}{a-b}=\frac{xy}{h}$$

→ if $ax^2+2hxy+by^2=0$ are 2 sides of ||elogram and $lx+my=1$ be its one of diagonal then other diagonal is $y(bl-hm)=x(am-hl)$

→ eqn. of pair of lines making an angle θ with origin & line is $lx+my+n=0$

$$(lx+my)^2-\tan^2\theta\,(mx-ly)^2=0$$