Statistics

(I) Arithematic Mean (A.M):

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} = \frac{\sum x_i}{n} \quad (\text{or}) \quad \bar{x} = A + \frac{\sum (x_i - A)}{n}$$

where A = assumed mean

→ for discrete frequency

$$\bar{x} = A + \frac{\sum f_i d_i}{\sum f_i} \quad \text{where } d_i = x_i - A$$

$$A.M = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

→ for combined arithmetic mean

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2 + \cdots + n_k \bar{x}_k}{n_1 + n_2 + \cdots + n_k} = \frac{\sum n_i \bar{x}_i}{\sum n_i}$$

(II) G.M

$$G.M = \sqrt[n]{x_1 x_2 \cdots x_n}$$

G.M cannot be calculated if the size of any of item is zero (or) -ve

(III) H.M

$$H.M = \frac{n}{1/x_1 + 1/x_2 + \cdots + 1/x_n}$$

$$H.M = \frac{\sum_{i=1}^{n} f_i}{\sum_{i=1}^{n} \frac{f_i}{x_i}}$$

Median

for individual series $M = \left(\frac{n+1}{2}\right)^{th}$, n is odd, $M = \frac{1}{2}\left(\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}\right)$ if n is even

$$M = l + \left(\frac{N/2 - F}{f}\right) C$$

l = lower limit of median class
C = width of the median
f = frequency of median class
F = cumulative frequency of preceding median class

3 Median - 2 Mean = Mode,

Mean - mode = 3(Mean - median)

Mode (Z) - $Z = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) C$

l: lower limit of modal class
C = width of modal class
f_0 = frequency of class just preceding the modal class
f_1 = frequency of modal class
f_2 = frequency of class just succeeding the modal class

→ Range = Maximum - Minimum

coeff. of range = $\frac{\text{Range}}{\text{Maximum} + \text{Minimum}}$

→ Quartile Deviation: (semi inter quartile range)

i) Quartile deviation = $\frac{Q_3 - Q_1}{2}$

ii) coeff. of quartile deviation = $\frac{Q_3 - Q_1}{Q_3 + Q_1}$

→ Mean deviation

i) $\bar{x}$ is mean of n observations $x_1, x_2, \ldots, x_n$ then mean deviation = $\frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$

ii) Mean deviation = $\frac{1}{n}\sum|x_i - m|$, where m = median.

$= \frac{1}{n}\sum|x_i - Z|$, where Z = mode

$= \frac{\sum f_i |x_i - M|}{\sum f_i}$

coeff. of Mean deviation = $\frac{\text{Mean deviation}}{M}$

→ Variance (σ^2): $x_1, x_2, \ldots, x_n$ are n items & $\bar{x}$ is A.M

i) $\sigma^2 = \frac{\sum(x_i - \bar{x})^2}{n} = \frac{\sum x_i^2}{n} - (\bar{x})^2 = \frac{\sum(x_i)^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = \frac{\sum f_i x_i^2}{\sum f_i} - (\bar{x})^2$

mid value

ii) S.D $= \sigma = \sqrt{\sigma^2}$

Moderately asymmetrical deviation mean deviation = $\frac{4}{5}$ (S.D).

→ coeff of S.D = $\sigma/\bar{x}$

coeff of variation = $\frac{\sigma}{\bar{x}} \times 100$

→ if n_1, n_2 are sizes, $\bar{x}_1, \bar{x}_2$ means & σ_1, σ_2 are S.D of 2 series then

$$\sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$$

$d_1 = \bar{x} - \bar{x}_1$; $d_2 = \bar{x} - \bar{x}_2$, $\bar{x}$ = combined mean

→ $V(x)$ is variance of X, then

i) $V(x+a) = V(x)$

ii) $V(ax) = a^2 V(x)$

iii) $V(ax+b) = a^2 V(x)$.

[S.D = standard deviation]

→ if variance varies from a to b then variance(x) $\leq \left(\frac{b-a}{2}\right)^2$

→ for series $a, a+d, a+2d, \ldots, a+(n-1)d$

ⓐ $\bar{x} = a + \frac{(n-1)d}{2}$

ⓑ $\sigma^2 = \frac{n^2-1}{12} d^2$

ⓒ $S.D = \sqrt{\frac{n^2-1}{12}}\, |d|$

ⓓ S.D of n consecutive natural nos. $= \sqrt{\frac{n^2-1}{12}}$ $(\because d = 1)$