# FLUID MECHANICS

## ① Introductory points

ⓐ density $\rho = m/V$, $\rho_{mix} = \dfrac{m_{total}}{V_{total}}$

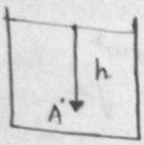
ⓑ Relative density (or) specific gravity $= \rho/\rho_w$ at 4°C

ⓒ Pressure $= \dfrac{F_\perp}{A}$ Pascal or Pa $(N/m^2)$

$P_0 = 1atm = 1.013\times10^5\ Pa \cong 10^5\ Pa$


## ② Pressure variation (vessel at rest)

ⓐ




$P_A = \rho gh + P_0$

↓ Absolute pressure; $\rho gh$ → gauge pressure $(P_g)$

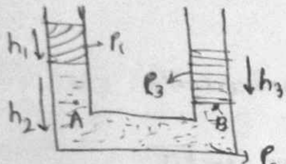
ⓑ


i) $P_A = P_B = \rho gh + P_0$

ii) Pressure at depth h is independent of shape of vessel

iii) At same level, pressure is same (same liquid)

Ex:

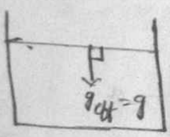



$P_A = P_B$

$P_0 + \rho_1 gh_1 + \rho_2 gh_2 = P_0 + \rho_3 gh_3$

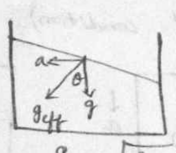
## ③ Free surface → always ⊥ to $g_{eff}$

ⓐ




at rest

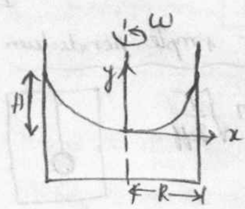
ⓑ




$g_{eff} = \sqrt{a^2+g^2}$

$\tan\theta = a/g$

ⓒ



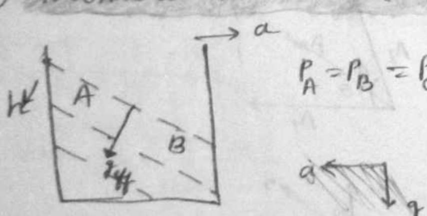

$y = \dfrac{\omega^2 x^2}{2g}$

↳ if $x = R$, $y = H$

$\therefore H = \dfrac{\omega^2 R^2}{2g}$

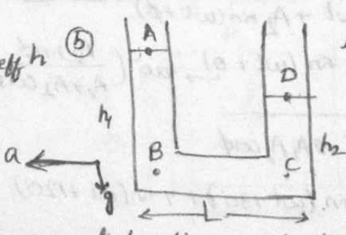
## ④ Pressure variation (accelerated system)

ⓐ




$P_A = P_B = P_0 + \rho g_{eff} h$

ⓑ



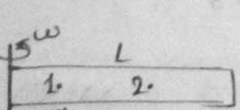

write eqn from A to D:

$P_A + \rho gh_1 - \rho aL - \rho gh_2 = P_D$

$P_A = P_D = P_0$

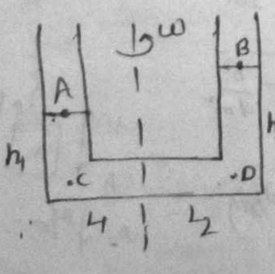
$$\boxed{h_1 - h_2 = \frac{aL}{g}}$$

ⓒ




$P_2 - P_1 = \frac{1}{2}\rho\omega^2(x_2^2 - x_1^2)$ { if $x_1 = 0$ i.e. at axis $P_2 - P_1 = \frac{1}{2}\rho\omega^2 x_2^2$, as you go from axis to location 2, P increases $\boxed{\text{by } \frac{1}{2}\rho\omega^2 x_2^2}$

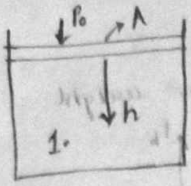
Ex:



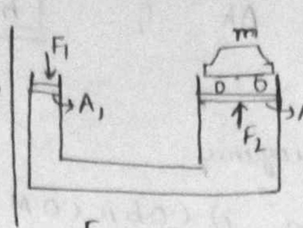

$P_A + \rho gh_1 - \frac{1}{2}\rho\omega^2 L_1^2 + \frac{1}{2}\rho\omega^2 L_2^2 - \rho gh_2 = P_B$ $(P_A = P_B = P_0)$

$h_2 - h_1 = \dfrac{\omega^2}{2g}(L_2^2 - L_1^2)$

⑤ **Pascal's law**: external pressure gets distributed evenly in all directions



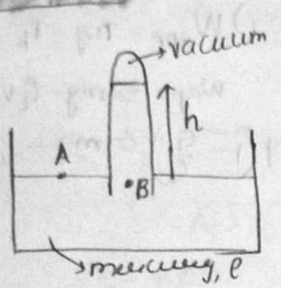

$$P_1 = \rho g h + \frac{mg}{A} + P_0$$




$$\frac{F_1}{r_1^2} = \frac{F_2}{r_2^2}$$

$$F_2 = PA_2$$
$$= \frac{F_1}{A_1} A_2$$
$$F_2 = F_1 (A_2/A_1)$$
$$A_2 >> A_1 \Rightarrow F_2 >> F_1$$
Ex: $mg = F_2 = F_1 (A_2/A_1)$
$$F_1 = \frac{A_1}{A_2} \times mg$$

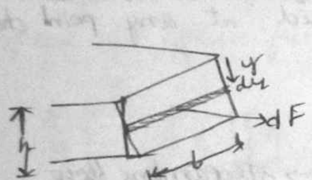
⑥ **Barometer**: setup to measure atmospheric pressure.




$P_A = P_B$
$P_0 = \rho g h$ if $P_0 = 1.013 \times 10^5$ Pa
$h = \frac{P_0}{\rho g} = 760$ mm & 0.76 m
∴ 1 atm is also 760 mm of Hg

NOTE: We don't use $H_2O$ because $h = 10$ m (not practical)

⑦ **Force on side walls (due to liquid)**



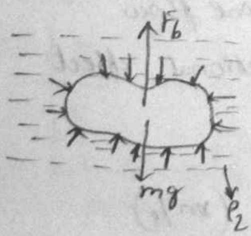

$$dF = P(y) \times dA = \rho g y \times b\,dy$$
$$F = \rho g b \int_0^h y\,dy = \rho g b \frac{h^2}{2} = \frac{\rho g h}{2} \times bh \Rightarrow$$
$$F = P_{av} \times \text{area of wall}$$
$$P_{av} = \rho g \frac{h}{2} \text{ (Pressure at center)}$$

⑧ **Torque on side walls**

$$d\tau = dF \times (h-y) \Rightarrow d\tau = \rho g y \cdot b\,dy\,(h-y)$$
$$\tau = \rho g b \int_0^h y(h-y)\,dy \Rightarrow \frac{\rho g b h^3}{6}$$

⑨ **Archimedes' principle / flotation**

ⓐ




buoyant force, $F_b = \rho_L V g$ { V is volume of displaced liquid

# Body is solid (not hollow from inside)

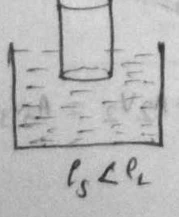
(i) Body sinks: $\rho_s > \rho_L$

(ii) Body floats: $\rho_s \leq \rho_L$

Some portion is out $\rho_s < \rho_L$

$\rho_s = \rho_L$ Completely submerged

ⓑ



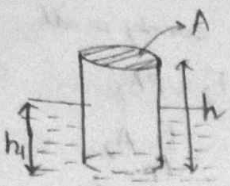

$F_b = mg \Rightarrow \rho_L V_1 g = \rho_s V g$ { $V_1$: displaced liquid vol, V: vol of body

$$\Rightarrow \boxed{\frac{V_1}{V} = \frac{\rho_s}{\rho_L}}$$

Ex: Ice floating in water

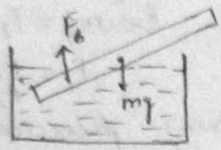
$$\frac{V_1}{V} = \frac{900}{1000} = 0.9$$

∴ 90% of ice is inside water



$$\frac{Ah_1}{Ah} = \frac{\rho_s}{\rho_L} \Rightarrow \boxed{\frac{h_1}{h} = \frac{\rho_s}{\rho_L}}$$ → valid for uniform cross section.

ⓒ centre of buoyancy



i) C.O.b is C.O.M of displaced liquid

ii) $F_b$ passes through C.O.b

ⓓ Apparent weight

i) $W_{app} = mg - F_b$

$W_{app} = mg - \rho_L V g$

Reading $= m\left(1 - \frac{\rho_L}{\rho_S}\right)$ & $mg\left(1 - \frac{\rho_L}{\rho_s}\right) \leftarrow mg - \rho_L \frac{m}{\rho_s} g$

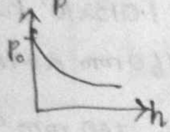
→ Isothermal pressure

$P = P_0 e^{-h/H}$

if $h = H$

$P = P_0 e^{-1}$

$P = 0.37 P_0$



## ⑩ Streamline & turbulent flow:

ⓐ (Laminar & steady flow)

→ velocity of fluid at a point is always same

→ random flow / mixing of flow

→ vel. of fluid at any point changes

ⓑ **Reynolds number ($R_e$)**

$$R_e = \frac{\rho v d}{\eta}$$ → dimensionless quantity

v - velocity

d - cross sec$^n$ dia of pipe

$\eta$ - coeff. of viscosity

$R_e < 1000$ → streamline flow

$R_e > 2000$ → turbulent flow

$1000 < R_e < 2000$ → unsteady flow

ⓒ **Ideal fluid**

i) incompressible ($\rho$ is const)

ii) non-viscous

iii) streamline flow

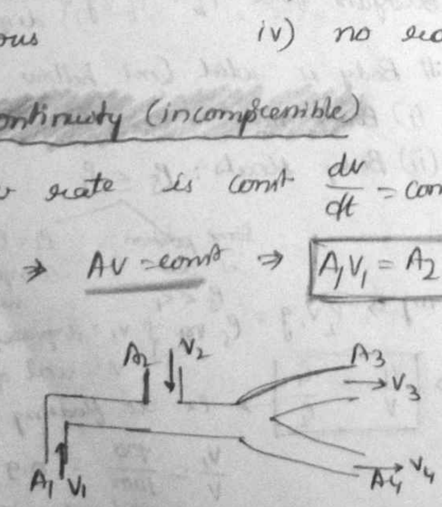
iv) no rotational effect.

ⓓ **Eqn of continuity (incompressible)**

→ volume flow rate is const $\frac{dv}{dt} = \text{const.}$ ($m^3/s$)

$$\frac{A\,dx}{dt} = \text{const} \Rightarrow AV = \text{const} \Rightarrow \boxed{A_1 V_1 = A_2 V_2}$$

Ex 1:

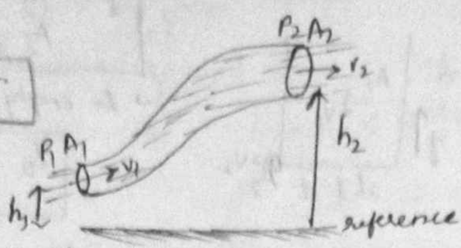


$$A_1 V_1 + A_2 V_2 = A_3 V_3 + A_4 V_4$$

(11) **Bernoulli's eqn:** based on conservation of mechanical energy to flow of ideal fluid.

$$\boxed{P + \tfrac{1}{2}\rho v^2 + \rho g h = \text{const}}$$

$P$ → Pressure energy/vol; $\tfrac{1}{2}\rho v^2$ → KE/vol; $\rho g h$ → PE/vol




$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2$$

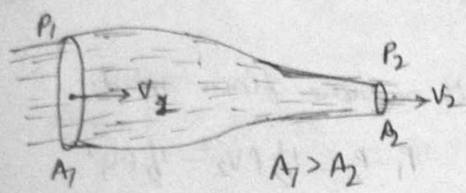
(12) **P-V relation (horizontal flow):** ∵ flow is horizontal

$\Rightarrow P + \tfrac{1}{2}\rho v^2 = \text{const.}$

also $A_1 v_1 = A_2 v_2$, so $v_1 < v_2$

$\therefore \boxed{P_1 > P_2}$

$F = \rho A v^2$

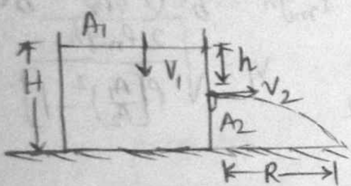



$A_1 > A_2$

Note: loss in pressure energy/vol. = gain in KE/vol for horizontal flow

(13) (i) **Speed of efflux:** Toricelli's law

→ speed of efflux from an open tank is identical to freely falling body




$A_1 >> A_2$

$\Rightarrow V_2 >> V_1$ → speed of efflux

→ Just to left and right of hole
loss in pressure energy/vol = gain in K.E/vol

$$(P_0 + \rho g h) - (P_0) = \tfrac{1}{2}\rho v_2^2 - \tfrac{1}{2}\rho v_1^2 \approx 0$$

$$\boxed{V_2 = \sqrt{2gh}}$$

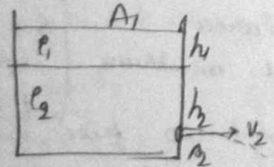
i) Range, $R = V_2 \times \sqrt{\frac{2(H-h)}{g}} = 2\sqrt{h(H-h)}$

ii) $R$ to be max

$\frac{dR}{dh} = 0 \Rightarrow h = H/2$ & $R_{max} = H$

(ii) other cases

(a)



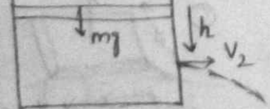

$A_1 >> A_2$

~~loss in pres energy/vol~~ = Gain in KE/vol.

$P_0 + \rho_1 g h_1 + \rho_2 g h_2 - P_0 = \tfrac{1}{2}\rho_2 v_2^2$ {$v_1$ is neglected

$\rho_1 g h_1 + \rho_2 g h_2 = \tfrac{1}{2}\rho_2 v_2^2$

(b)

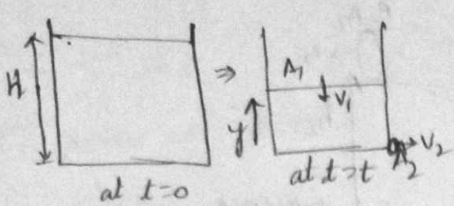



$\Rightarrow$

$\left(P_0 + \rho g h + \frac{mg}{A}\right) - P_0 = \tfrac{1}{2}\rho v_2^2$

$\rho g h + \frac{mg}{A} = \tfrac{1}{2}\rho v_2^2$

## (14) Vessel emptying time:




$A_1 v_1 = A_2 v_2 \Rightarrow A_1\left(-\frac{dy}{dt}\right) = A_2\sqrt{2gy}$

$$\therefore \boxed{t = \frac{A_1}{A_2}\sqrt{\frac{2}{g}}\left(\sqrt{H}-\sqrt{y}\right)}$$

↳ to empty $y=0$ at $t=t_0$

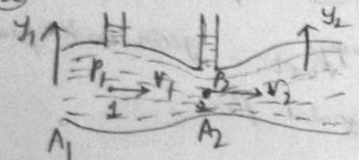
$$\boxed{t_0 = \frac{A_1}{A_2}\sqrt{\frac{2H}{g}}}$$

## (15) Thrust on vessel (liquid coming out):


$F_{th} = \rho A v^2 = \rho A 2gh$

## (16) Venturimeter: a device/setup to measure flow speed.

(a)



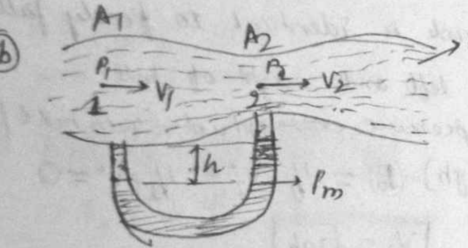

$A_1 > A_2$
$v_1 < v_2$
$P_1 > P_2$
$v_2 = \frac{A_1 v_1}{A_2}$

$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$

$\rho g h = \frac{1}{2}\rho v_1^2\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right)$

$$\boxed{v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}}$$

(b)




$A_1 > A_2$
$v_1 < v_2$
$P_1 > P_2$
$v_2 = \frac{A_1 v_1}{A_2}$

$\rho_m g h = \frac{1}{2}\rho\left(A_1 v_1/A_2\right)^2 - \frac{1}{2}\rho v_1^2$

$$v_1 = \sqrt{\frac{2\rho_m g h}{\rho\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}}$$

## (17) (i) Cohesive vs adhesive force

Cohesive ↓ force of att b/w molecules of same substance

Adhesive ↓ force of att b/w molecules of different substance

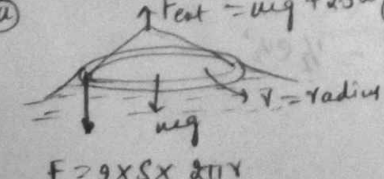
(ii) Surface tension → property of liquid where surface
i) tries have min area.
ii) acts as stretched membrane

$\boxed{S = F/L \text{ (or) } F = SL}$

NOTE: F acts ⊥ar to line and tangential to free surface.

## (18) Lifting of bodies

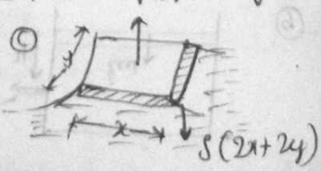
(a) $F_{ext} = mg + 2S\,2\pi r$




$F = 2 \times S \times 2\pi r$

(b) $F_{ext} = mg + S\cdot 2\pi r$


(c) $F_{ext} = mg + S(2x+2y)$



(19) **Surface Energy :** due to intermolecular interaction, free surface of liquid has energy called "surface energy".

$S = E/A$ (or) $E = S\cdot A \rightarrow$ A = surf. area, S – surf. tension, E = surf energy

Ex 1: Energy of drop

$E = S\cdot 4\pi r^2$

(2) Energy of bubble

$E = 2\times S \times 4\pi r^2$

→ standard questions on surface energy

Ex (1) drops coalesce

$OO \cdots O^{r} \rightarrow O$

n drops

volume is same

$\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$ (or) $R = r n^{1/3}$

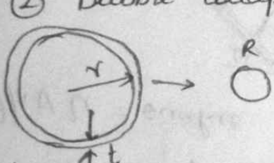
Area ↓ ⇒ energy ↓

$E_i = n\cdot S\cdot 4\pi r^2$

$E_f = S\cdot 4\pi R^2 \Rightarrow 4\pi r^2 n^{2/3}$

$E_i - E_f$ ↑ temp of bigger drop by $\Delta\theta$ ⇒ $\boxed{E_i - E_f = ms\Delta\theta}$

(2) Bubble collapse



$t << r$

$E_i = 2\times S\times 4\pi r^2$ ∵ vol of liq. is same

$E_f = S\cdot 4\pi R^2 \Rightarrow R = (3r^2 t)^{1/3}$

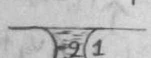
$= S\cdot 4\pi\,(3r^2 t)^{2/3}$

∴ $E_i - E_f = ms\Delta\theta$

(20) Excess pressure

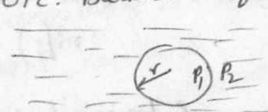
drop bubble

$P_{ext} = P_1 - P_2 = \frac{2T}{r}$  $P_{ext} = P_1 - P_2 = \frac{4T}{r}$



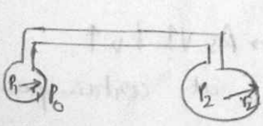
$P_1 - P_2 = \frac{T}{R}$

#NOTE: Bubble in aquarium



$P_1 - P_2 = 2T/R$

Ex:



$P_1 = \frac{4T}{r_1} + P_0$

$P_2 = \frac{4T}{r_2} + P_0$

→ Two bubble in contact

i) Externally

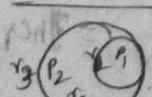
$P_2 - P_0 = \frac{4T}{r_2} \rightarrow (1)$

$P_1 - P_0 = \frac{4T}{r_1} \rightarrow (2)$

$P_1 - P_2 = 4T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

$\boxed{r = \frac{r_1 r_2}{r_2 - r_1}}$

ii) Internally



$P_1 - P_2 = 4T/r_1$

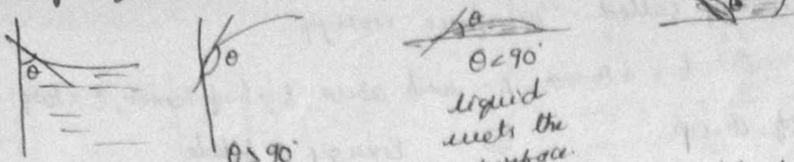
$P_2 - P_0 = 4T/r_2$

$P_1 - P_0 = 4T/r$

$\boxed{r = \frac{r_1 r_2}{r_1 + r_2}}$

r will be smallest

## (21) Angle of contact



$\theta < 90°$ — Ex: $H_2O$

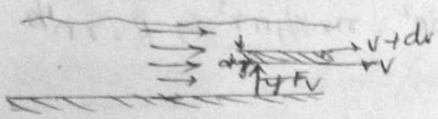
$\theta > 90°$ — Ex: Hg; Cohesive force dominates

$\theta < 90°$ — liquid wets the surface

Note: Generally for distilled $H_2O$ $\theta = 0°$, glass $\theta = 0°$

## (22) Viscous force

fluid oppose the relative motion b/w its different layers



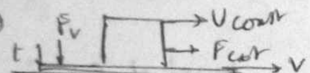
$F_v \propto A \frac{dv}{dy}$

$F_v = -\eta A \frac{dv}{dy}$ — $\eta$: coeff. of viscosity

$[\eta] = ML^{-1}T^{-1}$, SI unit poiseuille, PI

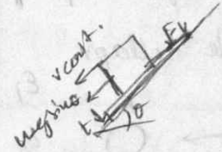
CGS – Poise $1\,PI = 10$ poise

Ex: ①



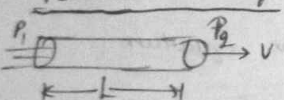
$F_{ext} = F_v = \eta A \frac{dv}{dy} = \eta A \frac{v}{t}$

②



$mg\sin\theta = \eta A v/t$

⇒ Poiseuille's eqn – flow through narrow tube

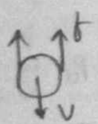


flow rate; $Q = \frac{\pi r^4}{8\eta L}(P_1 - P_2)$ ($m^3/s$)

$\boxed{R = 8\eta l/\pi r^4}$

Parallel: $v = v_1 + v_2$, $1/R_{eq} = 1/R_1 + 1/R_2$

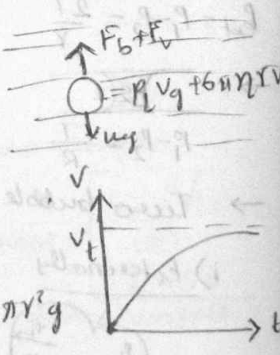
⇒ Stokes' law & terminal velocity



$f = 6\pi\eta r v$

→ As $v\uparrow$ $F_v\uparrow$ & at certain speed

$F_b + F_v = mg \Rightarrow a = 0, v = \text{const.}$ (terminal)

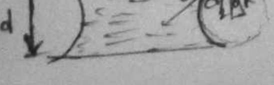


$P_L \cdot \frac{4}{3}\pi r^3 g + 6\pi\eta r v_t = P_s \frac{4}{3}\pi r^3 g$

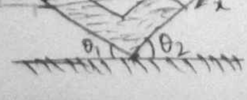
$v_t = \frac{2r^2 g}{9\eta}(P_s - P_L)$

⇒ Capillary rise & fall

$h = \frac{2T}{R\rho g} = \frac{2T\cos\theta}{r\rho g}$ $\left(\text{As } R = \frac{r}{\cos\theta}\right) = \rho h g$



$\frac{d/2}{R} = \cos\theta$



$T = 2\pi\sqrt{\frac{m}{\rho g A(\sin\theta_1 + \sin\theta_2)}}$