Sequence & Series

(I) A.P :

→ $a_{n+1} - a_n = d$

i) General term $T_n = a + (n-1)d$

ii) Sum of n terms in A.P $\Rightarrow S_n = \frac{n}{2}(2a + (n-1)d) = \frac{n}{2}(a+l)$

iii) $2b = a + c$

→ Insertion of $A_1, A_2, A_3, \ldots, A_n$ btw a & b then

sum of n AM's $= n\left(\frac{a+b}{2}\right)$

→ no. of common terms

n^{th} term = L.C.M of common diff's $(n-1)$

(II) G.P :

→ $T_n = ar^{n-1}$

i) $\frac{a_2}{a_1} = \frac{a_3}{a_2} = r \rightarrow$ common ratio

ii) n^{th} term from end of G.P with last term "l" & common ratio is $l\left(1/r\right)^{n-1}$

iii) Sum of 'n' terms in G.P $\Rightarrow S_n = \frac{a(r^n - 1)}{r-1}$ if $(r > 1)$

$S_n = \frac{a(1-r^n)}{1-r}$ if $(r < 1)$

for ∞ series $|r| < 1$ is $S_\infty = \frac{a}{1-r}$

iv) $b^2 = ac$

→ Insertion of $G_1, G_2, G_3, \ldots G_n$ terms btw a & b then

$G_1 \cdot G_2 \cdot G_3 \cdots G_n = (\sqrt{ab})^n = G^n$

→ Properties of AM, G.M & H.M :

Let $A = AM$, $G = GM$, $H = HM$ then btw 2 nos a & b !

$A = \frac{a+b}{2}$, $H = \frac{2ab}{a+b}$! $G = \sqrt{ab}$

i) $A \geq G \geq H$

ii) $G^2 = A \cdot H$

iii) $x^2 - 2Ax + G^2 = 0$

→ i) $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

ii) $1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$

iii) $1 + 3 + 5 + 7 \cdots + 2n+1 = n^2$