Probability

→ Simple event: event having only a outcome.

Ex: When a coin is tossed $P(H) = 1/2$.

→ Equally like events: if there is no reason to except one of them in preference.

$\boxed{P(A) = P(B)}$

→ Mutually exclusive event: if happening of one of event prevents happening of others. $\boxed{A \cap B = \phi}$, $\boxed{P(A) + P(B) = 1}$

→ Exhaustive events: List of all possible outcomes are exhaustive if one of them happen.

$A \cup B = S$

→ Classical definition of probability: If there are 'n' mutually exclusive, exhaustive, equally like, elementary events of an experiment & 'm' of them are favourable to an event 'A' then $P(A) = m/n$.

i) $0 \leq P(A) \leq 1$

ii) $P(A) = 0$ impossible event, $P(A) = 1$ sure event.

→ disjoint event – $A \cap B = \phi$, complementary event $(\bar{A}) \Rightarrow 1 - P(A) = P(\bar{A})$

→ Odds in favour – $P(A) : P(\bar{A})$, $P(A) = \frac{m}{m+n}$, $P(\bar{A}) = \frac{n}{m+n}$

Odds against of an event – $P(\bar{A}) : P(A)$

NOTATIONS:

① $A \cup B$ – either A or B occurs (or) atleast 1 of A, B occurs.

② $A \cap B$ – both A & B occur.

③ $\overline{A \cup B} = \bar{A} \cap \bar{B}$ – neither A nor B occur.

④ $\overline{A \cap B} = \bar{A} \cup \bar{B}$ – either A does not happen or B does not happen.

⑤ $A \cap \bar{B}$ – A occur but B does not occur

⑥ $(A \cap \bar{B}) \cup (\bar{A} \cap B) = (A - B) \cup (B - A) = (A \cup B) - (A \cap B)$ mean exactly 1 of A & B occur(s)

symmetrical difference b/w A & B

→ Addition theorem: i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

ii) $P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$

iii) $P(\text{exactly 1 of } A, B \text{ occur}) = P(A \cap \bar{B}) + P(\bar{A} \cap B) = P(A \cup B) - P(A \cap B)$

iv) $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B)$ $\qquad P(A \cap \bar{B}) = P(A) - P(A \cap B)$

v) $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$ $\qquad P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$P(A\cup B\cup C) = P(A)+P(B)+P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$.

i) P(Exactly 1 of A, B, C occur) $= P(A)+P(B)+P(C) - 2P(A\cap B) - 2P(B\cap C) - 2P(A\cap C) + 3P(A\cap B\cap C)$.

ii) P(A, B, C exactly 2 occur) $= P(A\cap B)+P(B\cap C)+P(C\cap A) - 3P(A\cap B\cap C)$.

iii) P(Atleast 2 of A, B, C occur) $= P(A\cap B)+P(B\cap C)+P(C\cap A) - 2P(A\cap B\cap C)$.

Conditional probability: A, B are events in sample space 'S' then the event of happening of B after happening of A is called C.P. It is denoted by $P(B/A)$.

$$P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{n(A\cap B)}{n(A)}, \quad P(A/B) = \frac{n(A\cap B)}{n(B)}$$

Multiplication theorem: $P(A) \neq 0$, $P(B) \neq 0$.

i) $P(A\cap B) = P(A)\cdot P(B/A)$, $\quad P(A\cap B) = P(B)\cdot P(A/B)$

ii) $P(A\cap B\cap C\cap D) = P(A)\cdot P(B/A)\cdot P(C/A\cap B)\cdot P(D/A\cap B\cap C)$

→ **Idependent events**: Occurance of A cannot influence occurence of B.

$$P(A\cap B) = P(A)\cdot P(B).$$

$$P(A\cap B\cap C) = P(A)\cdot P(B)\cdot P(C)$$

→ If A, B are independent events then

(i) $\bar{A}$ & B are independent, (ii) A & $\bar{B}$ are independent, (iii) $\bar{A}$ & $\bar{B}$ are independent.

Baye's theorem: If $A_1, A_2, \sim, A_n$ are 'n' mutually exclusive & exhaustive events in sample space (S) & 'E' be event connected then

$$P(A_k/E) = \frac{P(A_k)\,P(E/A_k)}{\sum_{i=1}^{n} P(A_i)\,P(E/A_i)}$$

S.C tree diagram

Ex: $P(\text{truth}) = 2/3$, $P(6 \text{ in die}) = 1/6$

$P(\text{false}) = 1/3$, $P(\bar{6}) = 5/6$ then $P(6 \text{ i.e true}) =$

T $2/3$ — $1/6$

F $1/3$ — $5/6$

$$\Rightarrow R.P = \frac{2/3 \cdot 1/6}{2/3\cdot 1/6 + 1/3\cdot 5/6} = 2/7$$

i) When tossing 'n' coins – $n(S) = 2^n$

ii) 'n' fair coins are tossed $P(\text{exactly } r (\leq n) \text{ heads (or tails)}) = \frac{{}^nC_r}{2^n}$

iii) $P(\text{atleast 1 head (or 1 tail)}) = 1 - 1/2^n$

iv) a coin is tossed $(m+n)$ times $(m \geq n)$ then probability of getting atleast 'm' consecutive heads $= \frac{n+2}{2^{m+1}}$

v) Coin is tossed $(m+n)$ times $P(\text{exactly 'm' consecutive heads}) = \frac{n+3}{2^{m+2}}$

→ Dices : $n(S) = 6^n$

i) Range of sum of no.s on $= \{2, 3, \ldots, 12\}$ 2 dice are rolled

r	$n(A)$	$P(A)$
$2 \leq r \leq 7$	$r-1$	$\frac{r-1}{6^2}$
$7 \leq r \leq 12$	$13-r$	$\frac{13-r}{6^2}$

ii) range of sum of no.s on $= \{3, 4, \ldots, 18\}$ 3 dice are rolled

r	$n(A)$	$P(A)$
$3 \leq r \leq 8$	$\frac{(r-1)(r-2)}{2}$	$\frac{(r-1)(r-2)}{2(6)^3}$
$13 \leq r \leq 18$	$\frac{(19-r)(20-r)}{2}$	$\frac{(19-r)(20-r)}{2(6^3)}$

→ if 'n' dices are thrown $P(\text{sum on 'n' dices is } r) = $ coeff of x^r in

$$\frac{(x + x^2 + x^3 + \cdots + x^6)^n}{6^n}$$

→ Leap year – $366 = 52 \times 7 + 2$

Non-leap year – $365 = 52 \times 7 + 1$

→ Two persons game – If A starts game if p are $P(\text{success})$ & q are $P(\text{failure})$ then

$P(\text{A's win}) = \frac{p}{1-q^2}$

$P(\text{B's win}) = \frac{pq}{1-q^2}$

$P(\text{A's win}) = p/1-q^3$

$P(\text{B's win}) = pq/1-q^3$

$P(\text{C's win}) = pq^2/1-q^3$

** works only if $P(A)+P(B)=1$

otherwise

$P(A) = P + qqP + qqqqP + \ldots$

$P(B) = Pq + qqqP + qqqqqP + \ldots$

→ $P(x^4 - y^4 \div 5) = \frac{17n-5}{5(5n-1)}$ for $\{1, 2, 3, \ldots, 5n\}$