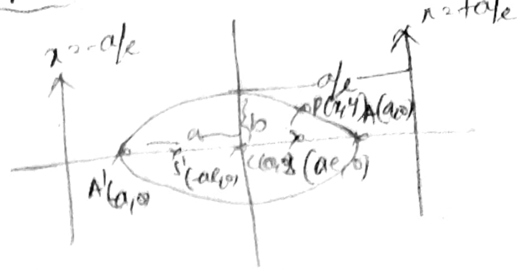
Ellipse

$S \equiv \frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad (a>b)$

$\rightarrow b^2 < ab,$

$\rightarrow$ STD form $\quad (a>b) \quad \Delta \neq 0:$



$C=(0,0)$

foci $S=(\pm ae,0)$

vertices $A=(\pm a,0)$

$e=\sqrt{\frac{a^2-b^2}{a^2}}$

Major axes, $y=0$, distance of major axes is $2a$

Minor axes, $x=0$ Length of minor axes is $2b$

$L.L.R = 2b^2/a$

Eqn of directrix $\Rightarrow x=\pm a/e$

Eqn of latusrecta $\rightarrow x=\pm ae$, ends of $L.R = (\pm ae, \pm b^2/a)$

Distance b/w foci $SS' = 2ae$

Distance b/w directrix $ZZ' = 2a/e$

$l = b^2/a, \; 2l = 2b^2/a$

$b^2 = a^2(1-e^2)$

$\rightarrow$ for $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \quad (a>b)$ (Major axes is ||el X-axis)

$C=(h,k)$

foci $S=(\pm ae+h, k)$

Eqn of directrix, $x=\pm a/e+h$

$e=\sqrt{\frac{a^2-b^2}{a^2}}$

distance b/w foci $SS' = 2ae$

distance b/w directrix $ZZ' = 2a/e$

Eqn of major axes, $y=k$

Eqn of minor axes, $x=h$

$L.L.R = 2b^2/a$

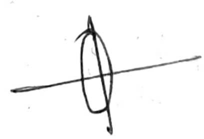
Length of major axes $=2a$

Length of minor axes $=2b$

ends of $L.R = (\pm ae+h, \pm b^2/a+k)$

Eqn of $L.R \Rightarrow x=\pm ae+h$

$\rightarrow$ for $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad (a<b)$



$l = a^2/b$

$L.L.R = 2l = 2a^2/b$

$C=(0,0)$

$e=\sqrt{\frac{b^2-a^2}{b^2}}$

$S=(0,\pm be)$

Eqn of directrix; $y=\pm b/e$

vertices $=(0,\pm b)$

$L.L.R = \frac{2a^2}{b}$

distance of foci $=2be$

distance b/w directrix $=2b/e$

Eqn of major axes is $x=0$

Eqn of minor axes is $y=0$

Eqn of $L.R \Rightarrow y=\pm be$

Ends of L.R are

$(\pm a^2/b, \pm be)$

L. of major axes $=2b$

L. of minor axes $=2a$

$a^2=b^2(1-e^2)$

→ for $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ $(a<b)$

$C=(h,k)$

$e=\sqrt{\frac{b^2-a^2}{b^2}}$

$S=(h, k\pm be)$

$L.L.R = 2a^2/b$

vertices $=(h, k\pm b)$

Eqn of directrices $\Rightarrow y=k\pm b/e$

Eqn of L.L.R $\Rightarrow y=k\pm be$

Eqn of Major axes $\Rightarrow x=h$

Minor axes $\Rightarrow y=k$

L. of major axes $=2b$

L of minor axes $=2a$

→ Area of rectangle formed by ends of $LL', L''L'''$ is $4b^2e$

Area of rectangle formed by ends of vertices AA', BB' is $4ab$

→ Maximum area of rectangle ~~sides~~ $(a\sqrt{2}, b\sqrt{2})$

→ Auxiliary circle:

Circle with major axis as diameter.

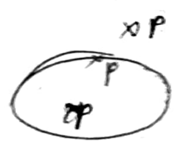
→ Locus of feet ⊥ar drawn ($\perp$) from foci upon any tgt is $x^2+y^2=a^2$ if $(a>b)$ $=b^2$ if $(a<b)$

Eqn is $x^2+y^2=a^2$ if $(a>b)$, $x^2+y^2=b^2$ if $(a<b)$

$P(\theta)=(a\cos\theta, b\sin\theta)$

$S\equiv \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $S_1\equiv \frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1$, $S_{11}\equiv \frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1$

$S_{12}\equiv \frac{x_1x_2}{a^2}+\frac{y_1y_2}{b^2}-1$

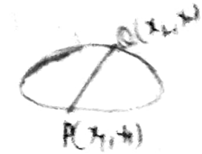


→ Position of point:

$S_{11}>0$ P is outside

$S_{11}=0$ P is on ellipse

$S_{11}<0$ P is inside.



→ Eqn of chord joining $P(x_1,y_1)$ & $Q(x_2,y_2)$ is $S_1+S_2=S_{12}$

Eqn of chord joing $P(\alpha), Q(\beta)$ is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ⇒

$$\frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right)+\frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right)=\cos\left(\frac{\alpha-\beta}{2}\right)$$

→ Condition for focal chord $S(ae,0)$ is $\dfrac{\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)} = \dfrac{1}{e}$

$$\tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2} = \frac{e-1}{e+1}$$

for $S'(-ae,0)$ is $\tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2} = \dfrac{e+1}{e-1}$

→ If chord is at a distance of d units from centre then

$$\tan\frac{\alpha}{2}\,\tan\frac{\beta}{2} = \frac{d-a}{d+a}$$

Tangent :

Point form ⇒ $S_1 = 0$

slope form → $\Delta = 0$ point of contact is $(x_1,y_1) = \left(\dfrac{-a^2m}{c}, \dfrac{+b^2}{c}\right)$

$$c^2 = a^2m^2+b^2$$

$$y = mx \pm \sqrt{a^2m^2+b^2}$$

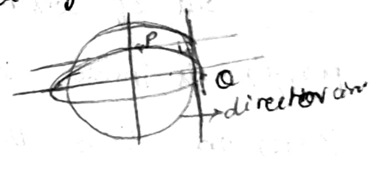
$P(\alpha)$ ⇒ $\dfrac{x}{a}\cos\alpha + \dfrac{y}{b}\sin\alpha = 1$

from external points we can draw two tgts to ellipse

$$m_1+m_2 = \frac{2x_1y_1}{x_1^2-a^2}, \quad m_1m_2 = \frac{y_1^2-b^2}{x_1^2-a^2}$$

Angle b/w tgts is $\tan\theta = \left|\dfrac{2ab\sqrt{S_{11}}}{x_1^2+y_1^2-a^2-b^2}\right|$

→ Director circle : Locus of P.O.I of ⊥ar tgts is called director circle.



Eqn is $x^2+y^2 = a^2+b^2$

$(x-h)^2+(y-k)^2 = a^2+b^2$

→ If $c^2 < a^2m^2+b^2$ it is a chord

→ If $c^2 > a^2m^2+b^2$ it neither touches nor intersects.

→ point of intersection of tgt drawn from $P(\alpha)$ & $Q(\beta)$ is

$$\left(\frac{a\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}, \frac{b\sin\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}\right)$$

→ locus of foot of ⊥ar from centre upon on any tgt is

$$(x^2+y^2)^2 = a^2x^2+b^2y^2$$

→ Product of ⊥ar distances drawn from foci upon any tgt is equal to b^2 if $(a>b)$

a^2 if $(a<b)$

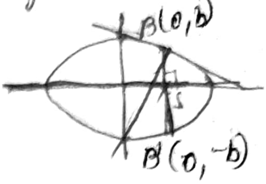
Normal:

→ $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2-b^2$ (point form)

→ $\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2-b^2$ (P(t) form)

→ $y = mx \pm \frac{m(a^2-b^2)}{\sqrt{a^2+b^2m^2}}$ (slope form)

→ Normal at one end of latus rectum passes through other end of minor axis then $e^4+e^2-1=0$



$e^2 = \frac{\sqrt{5}-1}{2}$ $e = \sqrt{2\sin 18^\circ}$

→ If $lx+my+n=0$ touches $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then

Condition for the line be normal to ellipse is

$$\frac{a^2}{l^2}+\frac{b^2}{m^2} = \frac{(a^2-b^2)^2}{n^2}$$

→ Eqn of chord of ellipse $S=0$ having (x_1,y_1) as its midpoint is

$S_1 = S_{11}$

Midpoint of chord $lx+my+n=0$ of ellipse is $\left(\frac{-a^2ln}{a^2l^2+b^2m^2}, \frac{-b^2mn}{a^2l^2+b^2m^2}\right)$

→ Eqn of chord of contact $P(x_1,y_1)$ w.r.t $S=0$ is $S_1=0$

$P(x_1,y_1)$ is inside then chord of contact does not exist.

P lies on $S=0$ then tgt is chord of contact.

Co-normal points (or) feet of normals:

$$P(x_1,y_1) = \left(\frac{\pm a^2}{\sqrt{a^2+m^2b^2}}, \frac{\pm b^2m}{\sqrt{a^2+b^2m^2}}\right)$$

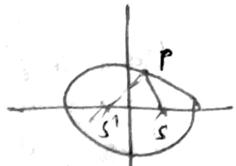
→ 4 normals can be drawn to ellipse from any point.

→ Sum of eccentric angles of co-normal points drawn from any point to an ellipse is equal to $(2n+1)\pi$

→ Tgt & Normal are external & internal angular bisectors of $\angle$le SPS'.

$$\frac{SN}{NS'} = \frac{1+e\cos\theta}{1-e\cos\theta}$$

→ **Reflection property:** If incident ray passes through S then reflected ray passes through S'.



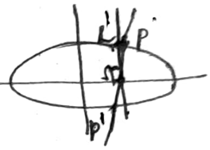
→ Eqn of an ellipse formed (or) referred to 2 ⊥ar lines is

$$\frac{(PN)^2}{a^2}+\frac{(PM)^2}{b^2}=1 \Rightarrow \frac{\left(\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right)^2}{a^2}+\frac{\left(\frac{bx_1-ay_1+k}{\sqrt{a^2+b^2}}\right)^2}{b^2}=1$$

→ **Diameter:** locus of midpoints of system of ‖el chords is called diameter.

$$y_1=\frac{-b^2x_1}{a^2m}$$

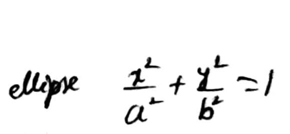
→ If PSP' is focal chord, SL is semilatusrectum then $\frac{1}{SP}+\frac{1}{S'P}=\frac{2}{SL}$



→ If α, β are the eccentric angle of extremities of a focal chord of ellipse S=0 (a>b) then

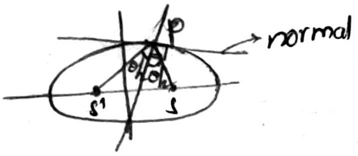
ⓐ $\cos\left(\frac{\alpha-\beta}{2}\right)=e\cos\left(\frac{\alpha+\beta}{2}\right)$ ⓑ $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{e-1}{e+1}$

$\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{d-a}{d+a}$ ⇐



CR=d

→



S, S' are foci, normal at P is internal angular bisector of ∠S'PS.

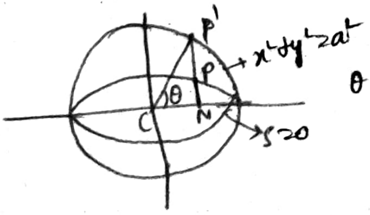
→ If $P(x_1,y_1)=(a\cos\theta, b\sin\theta)$ is a point on ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then

$SP=|a-ex_1|=|a-ea\cos\theta|$, $S'P=|a+ex_1|=|a+ae\cos\theta|$

$SP+S'P=2a$

$SP\cdot S'P'=b^2$

→ ✱ Eccentric angle:



θ = eccentric angle at centre.

✱✱

P by section formula

→ Length of latus rectum of an ellipse for $ax^2+by^2+cx+dy+e=0$

$$= \frac{2(\text{coeff of } y^2)}{\sqrt{\text{coeff of } x^2}} \quad (a>b), \quad = \frac{2(\text{coeff of } x^2)}{\sqrt{\text{coeff of } y^2}} \quad (b>a).$$

→ eccentricity of ellipse $a>b$ is $\sqrt{1-\frac{\text{coeff of } y^2}{\text{coeff of } x^2}}$

$b>a$ is $\sqrt{1-\frac{\text{coeff of } x^2}{\text{coeff of } y^2}}$