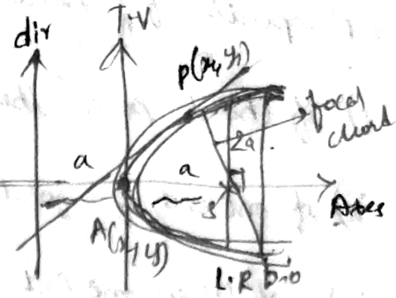
Parabola



→ $e=1$, $h^2=ab$ $\quad \Delta \neq 0$

→ $SP=PM$

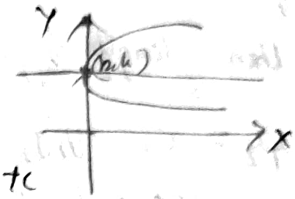
→ STD form ⇒ $y^2=4ax$

→ Length of L.R $=4a$ (Latusrectum), Length of double ordinate $=4\sqrt{ak}$

If makes an angle θ then $\underset{}{\overset{(\phi\cdot 0)}{AB}} = 8a\cot(\theta/2)$

→ $x^2=\pm 4ay$, $y^2=\pm 4ax$

→ Axis of parabola llel to X-axis



$x=ay^2+by+c$ $\qquad$ llly $\quad y=ax^2+bx+c$

$(y-k)^2=\pm 4a(x-h)$ $\qquad (x-h)^2=\pm 4a(y-k)$

$Y^2=\pm 4aX$ $\qquad X^2=4aY$

→ $S\equiv y^2-4ax$

$S_1= yy_1-2a(x+x_1)$

$S_{11}= y_1^2-4ax_1$

$S_{12}= y_1y_2-2a(x_1+x_2)$

for $\quad S\equiv(y-k)^2=4a(x-h)$

$S_1\equiv xx_1+yy_1+\frac{1}{2}(x_1+x_2)+\frac{1}{2}(y_1+y_2)+k$ constant

→ position of point w.r.t parabola

$S_{11}>0$ exterior

$S_{11}=0$ on parabola

$S_{11}<0$ interior

→ parametric co-ordinates of $y^2=\pm 4ax$ is $(\pm at^2, 2at)$

$x^2=\pm 4ay$ is $(2at, \pm at^2)$

$Y^2=4aX$ is $(h+at^2, k+2at)$

→ Eqn of chord is $2x-y(t_1+t_2)+2at_1t_2=0$

$t_2=-2/t_1-t_1$

→ Properties of focal chord :

i) If chord passes through $S(a,0)$ then $\boxed{t_1 t_2 = -1}$

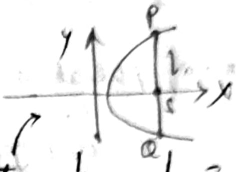
ii) If one end of focal chord is 't_1' other end is '$-1/t_1$'.

iii) If 't' is one end of focal chord then length of focal chord is

$PQ = a(t+1/t)^2$, $y = \frac{2t}{t^2-1}(x-a)$ eqn of focal chord.

Min value of $PQ = 4a$

iv) If focal chord makes an angle 'θ' with +ve direction of x-axis then length of focal chord is $4a\,\mathrm{cosec}^2\theta$.



v) $x_1 x_2 = a^2$, $y_1 y_2 = -4a^2$

vi) If PSQ is focal chord of any conic then $\frac{1}{SP} + \frac{1}{SQ} = \frac{2}{l}$

→ Circle describes focal chord as diameter it touches its directrix

→ Circle describes focal radius as diameter touches tangent at vertex

→ focal distance $SP = |a + at^2|$, $|a+x_1| = SP$, min. distance is from vertex

→ P.O.I of tgt & parabola is $\Delta = 0$.

mostly we use $c = a/m$

Tangent

→ $y = mx + a/m$ → eqn of tgt in slope form

$S_1 = 0$ for tgt at $P(x_1, y_1)$

P.O.C of tgt is $P\left(a/m^2, 2a/m\right)$

for $x^2 = 4ay$
P.O.C $= (2am, am^2)$
$c = -am^2$

angle b/w tgt $\tan\theta = \frac{\sqrt{S_{11}}}{x_1 + a}$ → eqn of director circle

For tgts P lies on directrix

$P(t)$ eqn of tgt is $yt = at^2 + x$ ⇒ $x - yt + at^2 = 0$

Slope of tgt at 't' is $1/t$

P.O.I of two tgt at t_1 & t_2 is $P(at_1t_2, a(t_1+t_2))$

⇒ (G.M of x-coordinate, A.M of y-coordinate)

Midpoint of chord having y-coordinate $= \left(\frac{2am}{l}\right)$.

Area of Δ^{le} inscribed in parabola is $\frac{1}{8a}\left|(y_1-y_2)(y_2-y_3)(y_3-y_1)\right|$

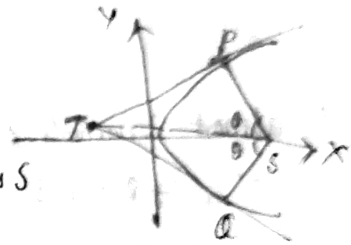
Area of Δ^{le} formed by tgt's is $\frac{1}{16a}\left|(y_1-y_2)(y_2-y_3)(y_3-y_1)\right|$

→ (i) The orthocentre of the Δ^{le} formed by tgt's at t_1, t_2, t_3 to the parabola $y^2 = 4ax$ is $[-a, a(t_1+t_2+t_3+t_1t_2t_3)]$

(ii) The orthocentre of Δ^{le} formed by any three tgt's to the parabola lies on the directrix of the parabola.

(iii) Circumcentre of Δ^{le} formed by any three tgt's to the parabola always passes through focus of parabola.

(iv) Tangent at any point on the parabola bisects the angle b/w the focal distance of the point & the ⊥ar on the directrix from the point



(v) If tangents at P & Q meet at T, then

① TP & TQ will subtends equal angle at focus S

② $ST^2 = SP \cdot PQ$

③ The Δ^{les} STP & STQ are similar

$ST^2 = SP \cdot SQ$
(ST, SP, SQ are in GP)

(vi) If N is foot of ⊥ar from focus S on tgt at point P to the parabola then N lies on tgt at vertex and $SN^2 = SA \cdot SP$ (where A is vertex).

→ Normal: for $y^2 = 4ax$

$$y - y_1 = \frac{-y_1}{2a}(x - x_1)$$

$(x, y) = (am^2, -2am)$ → foot of normal (or) co-normal point

$y - 2at = -xt + at^3$ → parameteric.

$y = mx - 2am - am^3$ → slope form

$c = -2am - am^3$ → condition for normalcy

P.O.I of normal at t_1, t_2 is

$$\left(2a + a(t_1^2 + t_1t_2 + t_2^2),\ -at_1t_2(t_1+t_2)\right)$$

slope of normal $m = -t$

for $x^2 = 4ay$
P.O.N $= \left(\frac{-2a}{m}, \frac{a}{m^2}\right)$
$c = 2a + a/m^2$

Eqn of normal to $(y-k)^2 = 4a(x-h)$ in slope form

$$y - k = m(x-h) - 2am - am^3$$

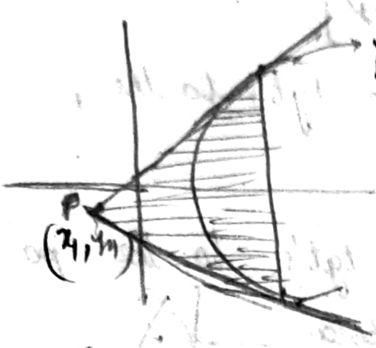
→ Length of latusrectum of parabola

i) $ax^2+by+cx+d=0$ is $\left|\frac{\text{coeff of } y}{\text{coeff of } x^2}\right| = \left|\frac{b}{a}\right|$

ii) $ay^2+by+cx+d=0$

is $\left|\frac{\text{coeff of } x}{\text{coeff of } y^2}\right| = \left|\frac{c}{a}\right|$

→ Area of Δ^{le}

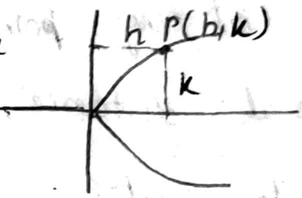


$\frac{1}{2a}(y_1^2-4ax_1)^{3/2} = \text{Area}$

→ (distance of any point on parabola from its axis$)^2$ = L.L.R × distance from tgt at.

$y^2=4ax$

$k^2=4ah$



→ atmost 3 normals can pass through given point

$\because at^3+(2a-x_1)t-y_1=0$

$H=\frac{2a-x_1}{3a}$, $G=-y_1/a$ $\quad \Delta = G^2+4H^3$

i) if $\Delta>0$, only 1 normals

ii) $\Delta=0$, 2 normals

iii) $\Delta<0$, 3 normals.

→ $\perp$ar tgts P.O.I lies on directrix (for $y^2=4ax$, it lies on $x=-a$).

→ condition for a chord passing through 1 point and passes through other is

$t_2=-2/t_1-t_1$