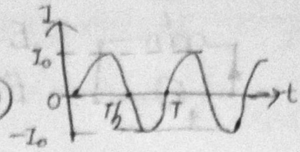
Alternating Current

→ $E = E_0 \sin\omega t$, $E_0 \cos\omega t$, $E_0 \sin(\omega t \pm \phi)$, $E_0 \cos(\omega t \pm \phi)$

$I = I_0 \sin\omega t$, $I_0 \cos\omega t$, $I_0 \sin(\omega t \pm \phi)$, $I_0 \cos(\omega t \pm \phi)$



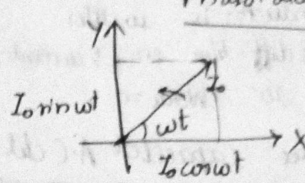
$P = E_{inst} I_{inst}$

→ **average voltage** – $V_{avg} = E_{avg} = \dfrac{\int_{t_1}^{t_2} E_{inst}\, dt}{\int_{t_1}^{t_2} dt} = \dfrac{\int_0^t E_0 \sin\omega t\, dt}{\int_0^t dt}$

i) for complete cycle $t_1 = 0,\ t_2 = T = 2\pi/\omega$

$V_{avg} = 0$

Phasor diagram



ii) for half cycle $t_1 = 0,\ t_2 = T/2 = \pi/\omega$

$$V_{avg} = \frac{\int_0^{T/2} E_{inst}\, dt}{\int_0^{T/2} dt} = \frac{2V_0}{\pi} = 0.63 V_0$$

→ **average current** i) for complete cycle $\langle I \rangle = I_{avg} = \dfrac{\int_0^T I_0 \sin\omega t\, dt}{\int_0^T dt} = 0$

ii) for half cycle $\langle I \rangle = \dfrac{2I_0}{\pi} = 0.63 I_0$

→ **RMS voltage** $V_{rms}^2 = \dfrac{\int_0^T V_{inst}^2\, dt}{\int_0^T dt} = \dfrac{V_0^2}{2} \Rightarrow V_{rms} = \dfrac{V_0}{\sqrt{2}} = 0.707 V_0$

$I_{rms}^2 = \dfrac{\int_0^T I^2\, dt}{\int_0^T dt}$ $\quad I_{rms} = \dfrac{I_0}{\sqrt{2}} = 0.707 I_0$ * (ammeters reads only I_{rms}) *

→ **average power** $\langle P \rangle = E_{rms} I_{rms} \cos\phi$ = true power

$$= \frac{E_0 I_0}{2} \cos\phi$$

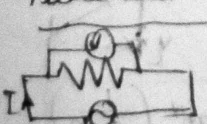
$$P_{app} = E_{rms} I_{rms} = \frac{E_0 I_0}{2}$$

Power factor $= \cos\phi = \dfrac{P_{true}}{P_{app}} = \dfrac{V_R}{V} = \dfrac{V_C}{V} = \dfrac{V_L}{V} = R/Z$

form factor $= \dfrac{I_{rms}}{I_{avg}} = \dfrac{V_{rms}}{V_{avg}} = \dfrac{\pi}{2\sqrt{2}} = 1.11$

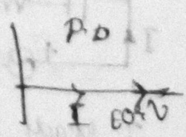
peak factor $= \dfrac{V_0}{V_{rms}} = \dfrac{I_0}{I_{rms}} = \dfrac{V_0}{\frac{V_0}{\sqrt{2}}} = \sqrt{2}$

Pure resistor AC ckt



$V_R = E = E_0 \sin\omega t$

$IR = E_0 \sin\omega t \Rightarrow I = I_0 \sin\omega t$

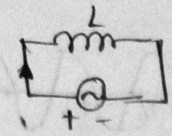


$Z = R$

$\Delta\phi = \phi_v - \phi_I = 0$, power factor, $\cos\phi = 1$

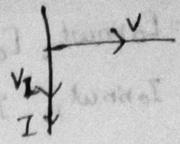
$$\langle P \rangle = \frac{E_0 I_0}{2}$$

Pure inductor A.C ckt



$E = E_0 \sin\omega t$

$E - L\dfrac{di}{dt} = I(Z)$, $I = I_0 \sin(\omega t - \pi/2)$



$\Delta\phi = \phi_V - \phi_I = -\pi/2$ ($\pi/2$ lagging)

Power factor, $\cos\phi = 0$, $\langle P \rangle = 0$ # induced emf = applied emf

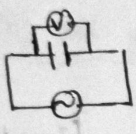
→ current through pure inductor is wattless.
phase diff. b/w emf & current is 90°. Work = 0

$I_0 = \dfrac{E_0}{L\omega}$

$X_L = Z = L\omega$ X_L – resistance offered by inductor

$\sin\phi = \dfrac{\omega L}{Z}$ ~~at~~ high

Pure capacitor A.C ckt

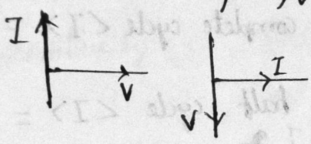


$E = E_0 \sin\omega t$

$\phi_V < \phi_S \Rightarrow I = I_0 \sin(\omega t + \pi/2)$

$\phi = \phi_V - \phi_I = -\pi/2$ (leading)

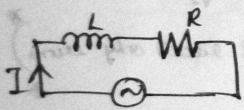
X_C – resistance offered by capacitor



$\langle P \rangle = 0$

$Z = X_C = 1/C\omega$

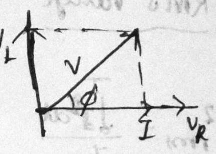
L-R ckt



$E = E_0 \sin\omega t$

$V_L = IR$

$E - V_L = V_R \Rightarrow E - L\dfrac{di}{dt} = IR$



$I = I_0 \sin(\omega t - \phi)$

$\longrightarrow V = \sqrt{V_R^2 + V_L^2}$, $Z = \sqrt{R^2 + L^2\omega^2}$

→ I lags E by ϕ

$\tan\phi = \dfrac{V_L}{V_R} \Rightarrow \phi = \tan^{-1}\left(\dfrac{V_L}{V_R}\right) \Rightarrow \tan^{-1}\left(\dfrac{I\cancel{Z}X_L}{ZR}\right) = \tan^{-1}\left(\dfrac{L\omega}{R}\right)$

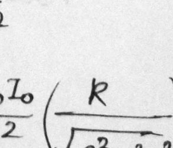
power factor, $\cos\phi = \dfrac{V_R}{V} = \dfrac{V_R}{\sqrt{V_R^2 + V_L^2}}$

$\langle P \rangle = E_{rms} I_{rms} \cos\phi = \dfrac{E_0 I_0}{2}\left(\dfrac{R}{\sqrt{R^2 + L^2\omega^2}}\right)$

C-R ckt

$E = E_0 \sin\omega t$

$E - V_L = IR \Rightarrow E - q/C = IR$



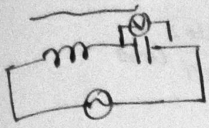
$Z = \sqrt{R^2 + (1/C\omega)^2}$

$\tan\phi = \dfrac{V_C}{V_R} = \dfrac{1}{C\omega R} \Rightarrow \phi = \tan^{-1}\left(\dfrac{1}{C\omega R}\right)$

→ I leads E by ϕ

power factor $\cos\phi = \dfrac{V_R}{\sqrt{V_R^2 + V_C^2}}$

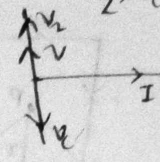
L-C ckt



$E = E_0 \sin\omega t$

$E = V_L + V_C \Rightarrow I = I_0 \sin(\omega t - \pi/2)$ if $(V_L > V_C)$

$I = I_0 \sin(\omega t + \pi/2)$ if $(V_L < V_C)$



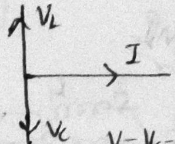
$Z = (X_L - X_C)$

$X_L > X_C$ I lags

$X_L < X_C$ I leads

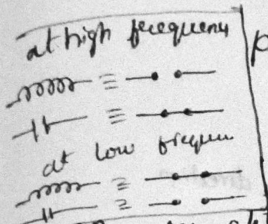
$\phi = +\pi/2$ (if $V_L < V_C$ (or) $X_L < X_C$)

$= -\pi/2$ (if $V_L > V_C$ (or) $X_L > X_C$)



$V = V_C - V_L$

$V = I(X_C - X_L)$

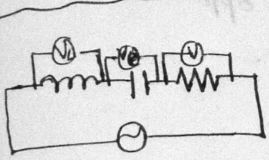


power factor $= \cos\phi = 0$

$\langle P \rangle = E_{rms} I_{rms} \cos\phi = 0$

$$\boxed{Z = (X_C - X_L) \\ Z = \frac{1}{C\omega} - L\omega}$$

LCR series ckt:

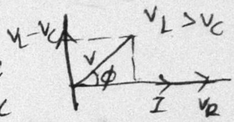


$E = E_0 \sin\omega t$

$E - V_L - V_C = IR$

$I = I_0 \sin(\omega t + \phi)$ if $V_L < V_C$, $X_L < X_C$

$I = I_0 \sin(\omega t - \phi)$ if $V_L > V_C$ (or) $X_L > X_C$



$V = \sqrt{V_R^2 + (V_L - V_C)^2}$, $Z = \sqrt{R^2 + (L\omega)^2 + (1/C\omega)^2 - 2L/C}$

power factor $= \cos\phi = \frac{R}{(L\omega - 1/C\omega)^2}$

$\phi = \tan^{-1}\left(\frac{L\omega - 1/C\omega}{R}\right)$

$P_{avg} = E_{rms} I_{rms} \cos\phi = \frac{E_0 I_0}{2}\left(\frac{R}{X_L - X_C}\right) \neq 0$

LCR resonance:

$f_{applied} = f_{ckt}$ then ckt is at resonance

$I \to I_{max}$

$Z \to Z_{min}$

at resonance

$X_C = X_L$

$Z = \sqrt{R^2 + (L\omega - 1/C\omega)^2}$

$Z_{LCR} = R$

Power $= V_{rms} I_{rms}$

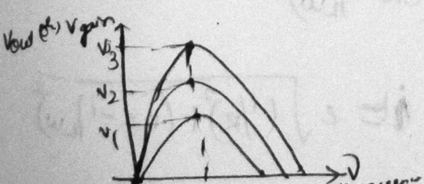
$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

$\omega^2 = 1/LC$, $\omega = 1/\sqrt{LC} = \omega_r$

it is independent on resistor X_L

$V_{gain} = \frac{V_{out}}{V_{in}}$, $I_{gain} = \frac{I_{out}}{I_{in}}$

$V_3 > V_2 > V_1 \Leftrightarrow R_3 < R_2 < R_1$



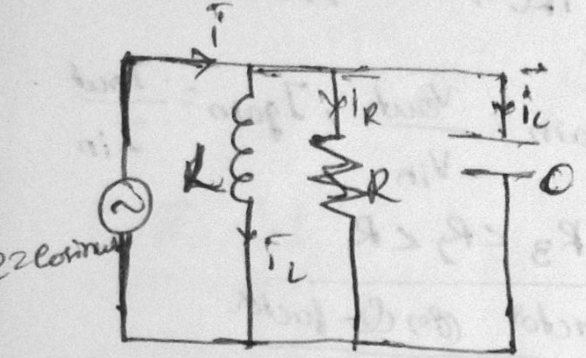
Quality factor (or) Q-factor

$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{F_r}{\text{Band width } (F_H - F_L)}$

$\Delta\omega = \omega_2 - \omega_1 = R/L$

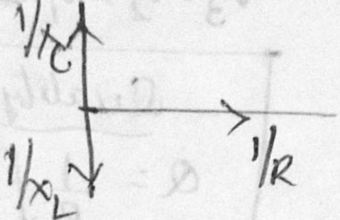
$Q = \frac{\omega_R}{\Delta\omega}$

||el RLC ckt



$$\vec{i} = \vec{i}_R + \vec{i}_L + \vec{i}_C = \sqrt{i_R^2 + (i_L - i_C)^2}$$

$$1/Z = \sqrt{(1/R)^2 + (C\omega - 1/L\omega)^2}$$



$$i = e\sqrt{(1/R)^2 + (\omega C - 1/L\omega)^2}$$

$\omega_R = 1/\sqrt{LC}$, $i_1 e_1 = i_2 e_2$ for ideal transformer

$$\eta = \frac{v_s i_s}{i_p v_p} \times 100\% = \frac{v_p i_p - P_{loss}}{v_p i_p} \times 100\%$$