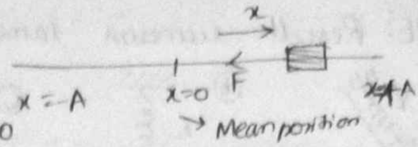
SIMPLE HARMONIC MOTION

① Kinematics / energy of SHM

$F \propto -x \Rightarrow F = -kx$



$a = -\left(\frac{k}{m}\right)x \Rightarrow \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$

$A = \sqrt{\frac{v_1^2x_2^2 - v_2^2x_1^2}{v_1^2 - v_2^2}}$

$\boxed{x = A\sin(\omega t + \phi)}$

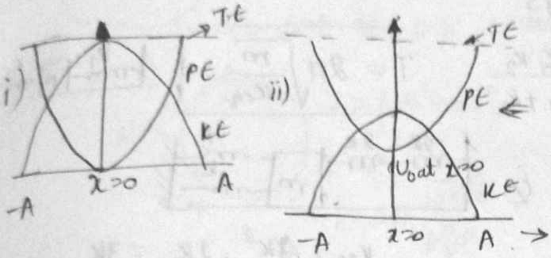
$\omega^2 = k/m$ (or) $T = 2\pi\sqrt{m/k}$

ⓐ $v(x) = \omega\sqrt{A^2 - x^2}$

$v(t) = A\omega\cos\omega t$

ⓑ $a(x) = -\omega^2 x$

$a(t) = -\omega^2 A\sin\omega t$

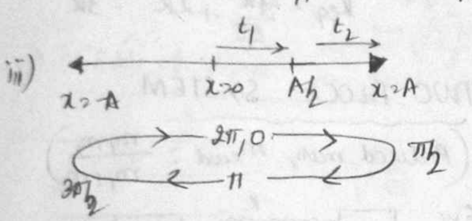


ⓒ $K.E(x) = \frac{1}{2}m\omega^2(A^2 - x^2)$

$PE(x) = \frac{1}{2}m\omega^2x^2$

$T.E(x) = KE + PE = \frac{1}{2}m\omega^2A^2 = \text{const}$

→ at meanposition system has PE



Ⓘ $t_{x=0 \to x=A} = T/4$

Ⓘ $x = A\sin\omega t$

$A/2 = A\sin\omega t_1 \Rightarrow \frac{1}{2} = \sin\omega t \Rightarrow \boxed{t_1 = T/12}$

Ⓘ $t_2 = T/4 - t_1 = T/6$

② STEPS to find time period

Linear SHM

① Given linear displacement of x from mean position

② Find linear acceleration

$F = ma$

③ You will get $a = -\underbrace{(\text{some const.})}_{\omega^2} \times x$

$T = 2\pi/\omega$

Angular SHM

① Given angular displacement of θ (small) from mean position

② Find angular acceleration

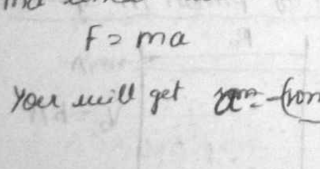
$\tau = I\alpha$

$\frac{d^2\theta}{dt^2} = -\omega^2\theta$

③ You will get $\alpha = -\underbrace{(\text{some const.})}_{\omega^2} \times \theta$

$T = 2\pi/\omega$

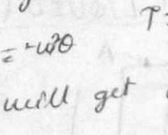
③ SPRING BLOCK AND SIMPLE PENDULUM



$F = -kx$

$a = -\underbrace{(k/m)}_{\omega^2}x$

$\therefore T = 2\pi\sqrt{m/k}$



θ is very small

$\sin\theta \cong \theta$

$\tau = -mg\sin\theta \times l$

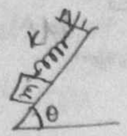
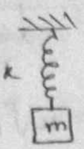
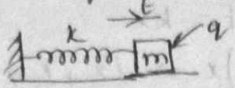
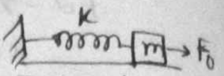
$ml^2\alpha = -mg\sin\theta \times l$

$\alpha = -\underbrace{\left(\frac{g}{l}\right)}_{\omega^2}(\theta)$

$\therefore T = 2\pi\sqrt{l/g}$

④ SPRING BLOCK (under presence of constant force)

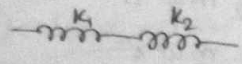
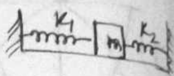
NOTE: Result remains same (only mean position changes)

ⓐ  ⓑ  ⓒ  ⓓ 

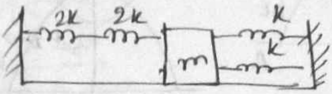
F_0 is external const force.

$T = 2\pi\sqrt{\frac{m}{K \sin\theta}}$ i) In all cases $T = 2\pi\sqrt{m/K}$, mean position changes.

⑤ COMBINATION OF SPRINGS

ⓐ  $\equiv$ $K_{eq} = \frac{K_1 K_2}{K_1 + K_2}$ $\boxed{T = 2\pi\sqrt{\frac{m}{K_{eq}}}}$ 

ⓑ  $\equiv K_{eq} = K_1 + K_2$

Ex. 

$K_{eq} = \frac{4K^2}{4K} + 2K = 3K$

⑥ SPRING CUT ($K \cdot l = $ const)

$\underset{K, l}{\text{spring}} = \underset{K_1, l_1}{\text{spring}} + \underset{K_2, l_2}{\text{spring}}$

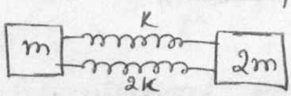
$Kl = K_1 l_1 = K_2 l_2$

$K_1 = \frac{Kl}{l_1}, \; K_2 = \frac{Kl}{l_2}$

Ex: K, l cut into three pieces K', K', K' of lengths $l/3$, $l/3$, $l/3$ $\therefore K' = 3K$

⑦ TWO BLOCK SYSTEM

(Reduced mass, $m_{red} = \frac{m_1 m_2}{m_1 + m_2}$)

Ex: 

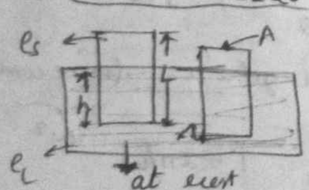
$T = 2\pi\sqrt{\frac{m_{red}}{K_{eq}}}$

$m_{red} = \frac{m \times 2m}{m + 2m} = \frac{2m}{3}$

$K_{eq} = K + 2K = 3K$

$T = 2\pi\sqrt{\frac{2m}{9K}}$

⑧ BLOCK IN LIQUID ($\rho_s < \rho_L$)

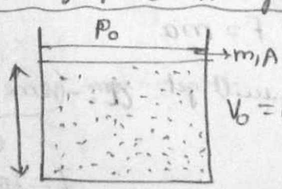


$T = 2\pi\sqrt{h/g}$ (or) $T = 2\pi\sqrt{\frac{m}{A\rho_L g}}$

A = area cross-section

at equi $\frac{\rho_s}{\rho_L} = \frac{h}{L}$

⑨ SHM of piston in cylinder



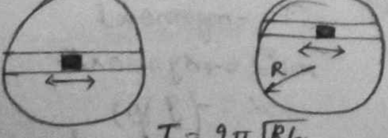
$V_0 = hA$

Piston given small displacement

$T = 2\pi\sqrt{\frac{mV_0}{P_0 A^2}}$ $T = 2\pi\sqrt{\frac{mV_0}{\gamma P_0 A^2}}$

$\gamma = C_p/C_v$

⑩ SHM in tunnel in a planet



$T = 2\pi\sqrt{R/g}$

where $g = \frac{GM}{R^2}$

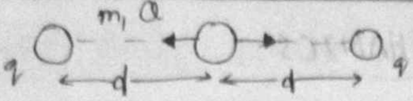
⑪ SHM of charge

ⓐ 

for small disturbance

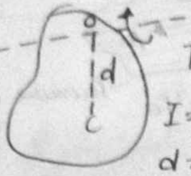
$T = 2\pi\sqrt{\frac{md^3}{2KQq}}$ $K = \frac{1}{4\pi\epsilon_0}$

(6)



$$T = 2\pi\sqrt{\frac{md^3}{4kQq}} \qquad k = \frac{1}{4\pi\varepsilon_0}$$

(12) Physical pendulum

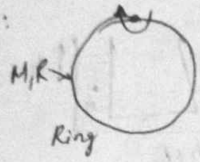


$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

I = M.O.I about O

d = dist. b/w O & C (C.O.M)

Ex:



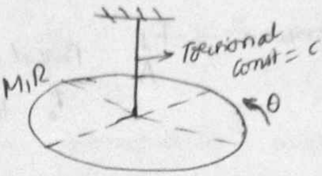
$$I = MR^2 + MR^2 = 2MR^2$$

$d = R$

$$T = 2\pi\sqrt{\frac{2MR^2}{MgR}}$$

$$T = 2\pi\sqrt{\frac{2R}{g}}$$

(13) Torsional pendulum

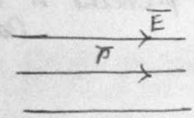


$\tau = -C\theta$

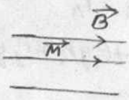
$\alpha = -\left(\frac{C}{I}\right)\theta$, $\frac{C}{I} \to \omega^2$

$$\therefore T = 2\pi\sqrt{\frac{I}{C}}$$

(14) SHM of dipole in field (small angular displacement)



$$T = 2\pi\sqrt{\frac{I}{PE}}$$

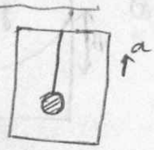


$$T = 2\pi\sqrt{\frac{I}{MB}}$$

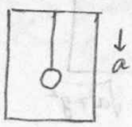
I = Moment of inertia

(15) T of simple pendulum (different condition)

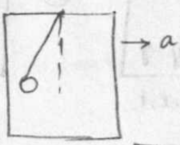
$$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$



$g_{eff} = g + a$



$g_{eff} = g - a$



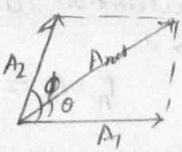
$g_{eff} = \sqrt{g^2 + a^2}$

(16) Superposition of SHM

$Y = A_1 \sin\omega t + A_2 \sin(\omega t + \phi)$

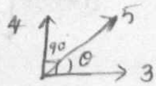
$= A_{net} \sin(\omega t + \theta)$, $\theta = \tan^{-1}\left(\frac{A_2 \sin\phi}{A_1 + A_2\cos\phi}\right)$

$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

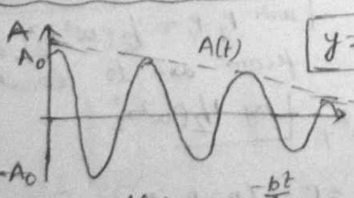


Ex: $Y = 3\sin(\omega t + 30°) + 4\sin(\omega t + 120°)$

$Y = 5\sin(\omega t + 30° + \theta)$ where $\theta = \tan^{-1}(4/3)$



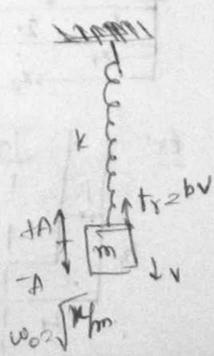
(17) Damped oscillation



$$\boxed{y = A(t)\cos(\omega' t + \phi)}$$

$$\to A(t) = A_0 e^{-\frac{bt}{2m}}$$

$$\to \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$



$$A(t) = A_0 e^{-\frac{bt}{2m}}$$

b = damping const (depends on medium)

$(k = \omega^2 m)$

$$\text{Energy }(E) = \frac{1}{2}k\left(A_0 e^{-bt/2m}\right)^2$$