Plane surfaces

→ Laws of reflection

1. $\angle i = \angle r$
2. Incident ray, reflected ray, normal all lie in the same plane $\bar{R}\cdot(\bar{I}\times\bar{N}) = \bar{N}(\bar{I}\times\bar{R}) = \bar{I}(\bar{N}\times\bar{R}) = 0$
3. $\hat{e}_1$ is unit vector along incident ray, $\hat{e}_2$ unit vector along reflected ray, $\hat{n}$ along normal

$$\hat{e}_2 = \hat{e}_1 - 2(\hat{e}_1 \cdot \hat{n})\hat{n}$$

$$\delta = 180^\circ - 2i = 180^\circ - 2r$$

Satisfies all types of reflecting surfaces

→ Plane mirror :

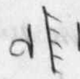
Due to plane mirror i) erect & upright image is formed.

ii) Object distance = Image distance

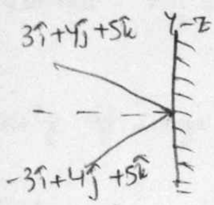
iii) Size of object = size of image

Line joining b/w object & image must be ⊥ar to each other

→ Lateral inversion: due to plane mirror lateral inversion was occured.



→ If a light is incident on plane mirror & it directions represented by vector, after reflection the direction of vector along the normal is inverted.



→ Magnification of plane mirror = 1

If a light is incident normally on a plane surface it retraces its path.

Focal length of plane mirror is infinity.

Focal power of plane mirror is zero.

$$\vec{V}_{I_x} = 2\vec{V}_{M_x} - \vec{V}_{O_x}$$

$$\vec{a}_{I_x} = 2\vec{a}_{M_x} - \vec{a}_{O_x}$$

→ During the above case velocity of image is ∥el to mirror does not change.

→ If a plane mirror is rotated by an angle θ without changing the incident ray then reflected ray rotates by 2θ.

ω becomes 2ω.

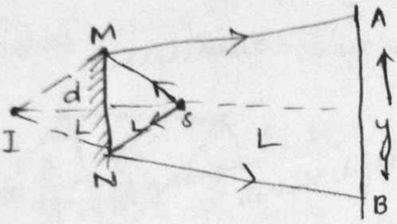
→ Whatever the size of the mirror may be it forms the complete image of the object lying infront of it.

Large mirror gives more bright image

To observe full object height or person height, the size of mirror must be half of height of the person is required

$$\frac{h_p}{2} = \text{height of mirror}$$

→ A point source of light 'S' placed at a distance 'L'. Infront of the centre of a mirror of width 'd'. A man walks infront of the mirror at a distance 2l from it as shown in fig. The distance over which he can see the image of light is 3d



→ No of images formed due to inclined mirrors is $\frac{360}{\theta}$.

i) if even, no of images $= \frac{360}{\theta} - 1$

ii) if fraction, no of images $= [\cdot]$

iii) if odd, no of images $= \frac{360}{\theta} = n$ if object is unsymmetric

$n = \frac{360}{\theta} - 1$ if object is symmetric

Spherical mirror

1. If ||el beam of light is incident, they converge at focus in a concave mirror.

2. If ||el beam of light is incident, they appear to diverge from focus in a convex mirror.

If object is at infinity we get ||el beam of light

$$f = R/2, \quad R = 2f$$

$$\frac{2}{R} = \frac{1}{u} + \frac{1}{v} \Rightarrow \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Sign Convection

1. Generally all distances can be calculated from pole.
2. Distances along the direction of propagation of light will be taken as +ve.
3. Distances opposite to the propagation of light will be taken as -ve.

→ Magnification (m)

Lateral $m = \frac{-h_i}{h_o} = \frac{-v}{u} = \frac{f-v}{f} = \frac{-f}{u-f}$

Areal $m_a = m^2 = \frac{\text{area of image}}{\text{area of object}}$

Longitudinal $m_L = -(v/u)^2 = -\left(\frac{f}{u-f}\right)^2 = -\left(\frac{v-f}{f}\right)^2$

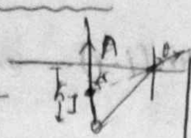
→ Velocity of image $V_{IM} = -\left(\frac{v}{u}\right)^2 \vec{V}_{OM}$

$$\vec{V}_I - \vec{V}_M = -\left(\frac{v}{u}\right)^2 \left(\vec{V}_O - \vec{V}_M\right)$$

RAYOPTICS

A·d – apparent depth
R·d – real depth

Refraction



Object is in denser medium –

① $A{\cdot}d = \dfrac{R{\cdot}d}{{}_R\mu_D}$, ${}_R\mu_D = \dfrac{\mu_{denser}}{\mu_{rarer}}$

② Image velocity = $\dfrac{\text{object velocity}}{\mu}$

③ $A{\cdot}S = R{\cdot}d\left[1 - 1/\mu\right]$

$\angle\delta = \angle r - \angle i$

Object is in Rarer medium –

① $Ad = \mu R{\cdot}d$

② Image velocity = $\mu \times$ object velocity

③ $A{\cdot}S = R{\cdot}d[\mu - 1]$

$\angle\delta = \angle i - \angle r$

Shift produced by glass slab $\Delta x = t\left[1 - 1/\mu\right]$

Normal shift $\Delta x = t\left[1 - 1/\mu_{rel}\right] = t\left[1 - \dfrac{\mu_{surr}}{\mu_{slab}}\right]$

$\angle i + \angle r + \angle\delta = 180^\circ$

Upwards velocity +ve
downwards velocity −ve

Refractive index (R·I)

x = thickness

① μ (or) $n = \dfrac{c}{v} = \dfrac{cx}{time} = \dfrac{f\lambda_{free}}{f\lambda_{medium}} = \dfrac{\lambda_{free}}{\lambda_{medium}} = \dfrac{x_1}{x_2}\left(\dfrac{t_2}{t_1}\right) = \sqrt{\mu_r \varepsilon_r}$

② $t = \dfrac{\mu x}{c}$, $\Delta t = t_1 - t_2 = \dfrac{\mu_1 x_1}{c} - \dfrac{\mu_2 x_2}{c}$

③ no. of waves = $\dfrac{\mu_{med}\, x_{med}}{\lambda_{free}}$ [∴ x = thickness]

④ $n_1 \sim n_2 = \dfrac{\mu_1 x_1}{\lambda_{free}} - \dfrac{\mu_2 x_2}{\lambda_{free}}$

⑤ Optical path = μd , optical path difference = $(\mu - 1)d$

⑥ ${}_1\mu_2 = \dfrac{1}{{}_2\mu_1}$

→ $\mu = \tan i$ from rarer to denser

→ $\mu = \cancel{\tan}\ i$ from denser to rarer

⑦ Conditions for no Refraction

ⓐ light hits the interface normally

ⓑ $\mu_1 = \mu_2$

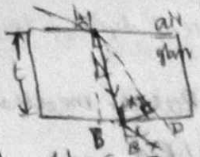
⑧ Snell's law: ⓐ $\mu \sin\theta$ = constant ⓑ $\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$

ⓒ $\mu_1 \sin i = \mu_2 \sin r$ ⓓ $\dfrac{\sin i}{v_1} = \dfrac{\sin r}{v_2}$ ⓔ $\dfrac{\sin i}{\lambda_1} = \dfrac{\sin r}{\lambda_2}$

ⓕ $\mu_1\left[\hat{i} \times \hat{n}\right] = \mu_2\left[\hat{r} \times \hat{n}\right]$

→ During refraction of light frequency, phase angle and colour doesn't change.

(9) Lateral shift by glass slab



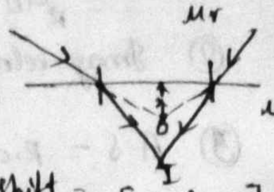
$x = \frac{t}{\cos r} \sin(i-r)$ (or) for small angles $x = t(i)\left[1 - \frac{1}{\mu_g}\right]$

(10) $\sin C = \frac{1}{{}_R\mu_D} = \frac{\mu_R}{\mu_D} = \frac{1}{\mu}$

$\delta = \pi/2 - C = \pi/2 - \sin^{-1}(1/\mu)$

$\delta_{max} = \pi - 2C = \pi - 2\sin^{-1}(1/\mu)$

(15) (a)

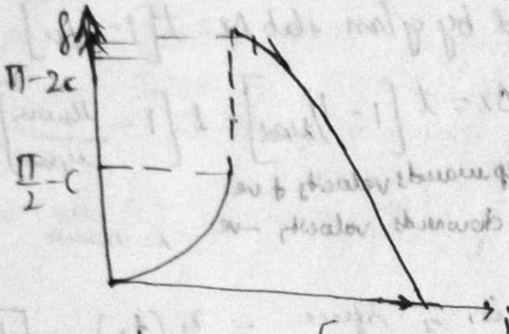


shift $= x\left[1 - 1/\mu_{rel}\right]$

(16)

shift $= x\left[1 - 1/\mu_{rel}\right]$

(11)



upwards velocity +ve
downwards velocity −ve

$i_{max} = \sin^{-1}\left[\sqrt{\mu_1^2 - \mu_2^2}\right]$ μ_1 = denser medium, μ_2 = rarer medium

(12) Radius of circular path viewed by fish $x = h\tan C = h/\sqrt{\mu_2^2 - 1}$

(13) Area illuminated by fish $\cdot A = \pi r^2 = \frac{\pi h^2}{\mu^2 - 1}$

$A = \frac{\pi h^2}{{}_r\mu_D^2 - 1}$

(14) During refraction frequency remains constant

Refraction at curved surfaces:

$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ Gaussian relation

Lateral magnification $(m) = \frac{h_i}{h_o} = \frac{\mu_1}{\mu_2}(v/u)$

Longitudinal magnification $(m_L) = \frac{\mu_1}{\mu_2}\left[\frac{v^2}{u^2}\right]$

$V_{Image} = \frac{\mu_1}{\mu_2}\left(\frac{v^2}{u^2}\right) V_{object}$

Image velocity = $m \times$ object velocity

Principle focus

$f_2 = \frac{\mu_2 R}{\mu_2 - \mu_1}$, $f_1 = \frac{-\mu_1 R}{\mu_2 - \mu_1}$, $\frac{f_1}{f_2} = \frac{-\mu_1}{\mu_2}$

$\frac{f_1}{\mu_1} + \frac{f_2}{\mu_2} = 0$

focal power $P_1 = \frac{\mu_1}{f_1}$, $P_2 = \frac{\mu_2}{f_2}$

→ For real image, image distance (v) is +ve. For virtual image, image distance (v) is -ve.

→ Depending on μ_1, μ_2, u, R, the image may be real or virtual.

→ Power of lens $= \tan\delta = 1/f$

diapotre (D) units $\quad P = \frac{100}{f(\text{in cm})}$

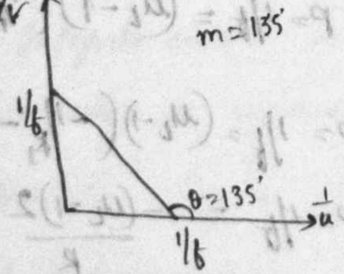
$P_{med} = \frac{\mu}{f}$

→ For plane surface, focal length $= \infty$, focal power $= 0$

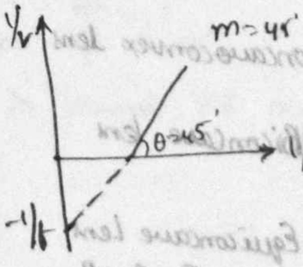
Real image $+v$, virtual image $-v$, Real object $= -u$, virtual object $= +u$

→ $1/u$ and $1/v$ graph

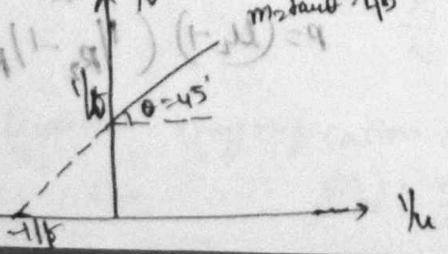
Convex lens - real image



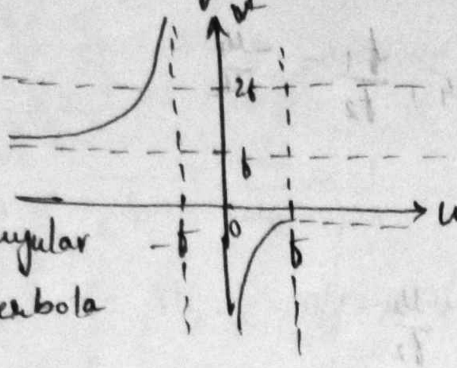
Convex lens - Virtual image



Concave lens - Virtual image

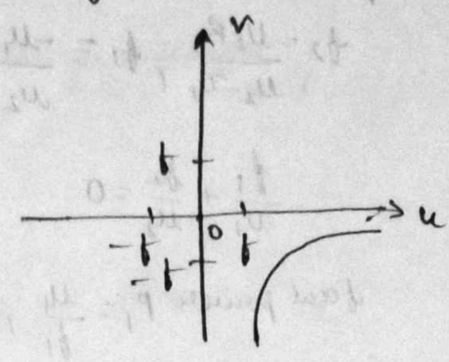


→ U-V convex lens graph



Rectangular hyperbola

U-V graph for concave lens



→ Lens makers formula

$$P = 1/f = \left[\frac{\mu_L - \mu_m}{\mu_m}\right]\left(1/R_1 - 1/R_2\right)$$ μ_L = Lens RI

$$P = 1/f = \left[\frac{\mu_L}{m} - 1\right]\left(1/R_1 - 1/R_2\right)$$

Lens	Lens makers formula
① Biconvex lens $(R_1 \neq R_2)$	$P = 1/f = (\mu_L - 1)(1/R_1 + 1/R_2)$
② Equiconvex lens $(R_1 = R_2)$	$P = 1/f = \frac{(\mu_L - 1)2}{R}$
③ Planoconvex lens $(R_1 = \infty, R_2 = R)$	$P = 1/f = \frac{(\mu_L - 1)}{R}$
④ Concavoconvex lens	$P = 1/f = (\mu_L - 1)(1/R_1 - 1/R_2)$
⑤ Biconcave lens	$P = 1/f = (\mu_L - 1)(-1/R_1 - 1/R_2)$
⑥ Equiconcave lens $R_1 = R_2 = R$	$P = 1/f = -\frac{(\mu_L - 1)2}{R}$
⑦ Planoconcave lens $R_1 = \infty, R_2 = R$	$P = 1/f = -\frac{(\mu_L - 1)}{R}$
⑧ Convexo concave lens	$P = (\mu_L - 1)(1/R_2 - 1/R_1)$

→ Lens immersed in liquid

$$\frac{P_{air}}{P_{med}} = \frac{f_{med}}{f_{air}} = \frac{\mu_{med}\left[\mu_L - 1\right]}{\mu_L - \mu_{med}}$$

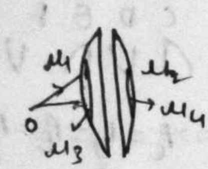
(a) $\mu_L > \mu_m$ nature of lens does n't change but f and Power changes

(b) $\mu_L = \mu_m$ Lens acts as plane glass plate

(c) $\mu_L < \mu_m$ natuere of lens, f and Poweer of lens changes.

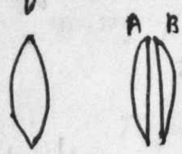
→ No. of images formed by lens

is no. of differeent horizontal R.I



No. of images formed = 1

→ Cutting of lens



P f $P_1 = P/_2$ $P_2 = P/_2$

$f_1 = 2f$ $f_2 = 2f$

→ Vaeriable refeactive index of surfaces

Step I – Find $\angle i$ & $\angle r$ by Snell's law

Step II – Find slope of line $m = \tan\theta = \frac{dy}{dx}$

Step III – Integrate on B.S with respect of limits

Light teavels in cuerved paths in vaeriable R.I media.

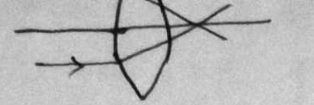
→ Lens equation $1/f = 1/v - 1/u$

→ Lateeral/ teansveese/ lineaer magnification $m = \frac{h_i}{h_o} = v/u$

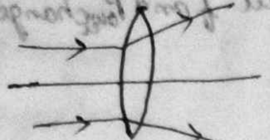
$$m = \frac{h_i}{h_o} = \frac{v}{u} = \frac{f}{f+u} = \frac{f-v}{f}$$

Longitudinal magnification of lens

$$(m_L) = \frac{dv}{du} = \frac{v^2}{u^2} = m^2$$

(a) 

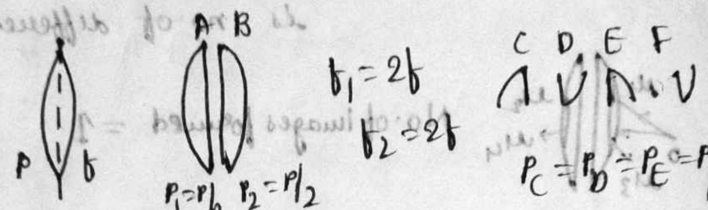
Nature of lens [illegible]
focal power changes.

(b) 

$\mu_L < \mu_m$
Nature of lens changes, focal lengths & focal power changes.

(c) 

$\mu_L = \mu_m$
Lens acts as plane glass plate.

→ Cutting of lens : 

$f_1 = 2f$
$f_2 = 2f$

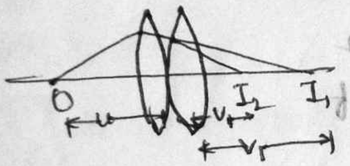
$P_1 = P/2 \quad P_2 = P/2$

$f_1 = 2f \quad f_2 = 2f$

$P_C = P_D = P_E = P_F = P/2$

$f_C = f_D = f_E = f_F = 2f$

→ 2 thin lenses are kept in contact



$1/f_{net} = 1/f_1 + 1/f_2$

$P_{net} = P_1 + P_2$

$1/f_{net} = 1/f_1 + 1/f_2 + \cdots + 1/f_n$

$P_{net} = P_1 + P_2 + \cdots + P_{net}$

→ For convex lens both focal power & focal lengths are +ve

→ For concave lens both focal power & focal lengths are -ve

→ If two lenses are separated a distance 'd' in a R.I of medium then.

$$1/f_{net} = 1/f_1 + 1/f_2 - \frac{d}{\mu f_1 f_2}$$

$$P_{net} = P_1 + P_2 - \left(\frac{d}{\mu}\right)(P_1 P_2)$$

(a) If D<4f

(b) If D=4f

(c) D>4f

Magn

→ Silvering

i) Plane

ii) Con

→ Lens-displacement method: $u = \frac{D \pm \sqrt{D(D-4f)}}{2}$

(a) If $D < 4f$ the term 'u' will be imaginary. This is not possible.

(b) If $D = 4f$ $\boxed{u = D/2 = v}$

(c) $D > 4f$ there are two possibilites of object distance

$$u_1 = \frac{D - \sqrt{D(D-4f)}}{2} \quad \& \quad u_2 = \frac{D + \sqrt{D(D-4f)}}{2}$$

displacement of lens $x = \sqrt{D(D-4f)}$

focal length of lens $f = \frac{D^2 - x^2}{4D}$

$$u_1 = \frac{D-x}{2}, \; u_2 = \frac{D+x}{2} \qquad v_1 = \frac{D+x}{2}, \; v_2 = \frac{D-x}{2}$$

Magnification $\frac{m_1}{m_2} = \left[\frac{D+x}{D-x}\right]\left[\frac{D+x}{D-x}\right] = \left[\frac{D+x}{D-x}\right]^2$

$$m_1 m_2 = \frac{I_1 I_2}{O^2} = 1 \quad \Rightarrow O = \sqrt{I_1 I_2}$$

$$m_1 - m_2 = \frac{4Dx}{D^2 - x^2} = \frac{x}{f}$$

→ Silvering of ~~mirror~~ lens:

$$P_{eff} = 2P_{Lens} + P_{mirror} \qquad f_{eff} = \frac{-1}{P_{eff}}$$

i) Plane surface of plano convex lens is silvered

$$P_{eff} = \frac{2[\mu_L - 1]}{R} \qquad f_{eff} = \frac{-R}{2[\mu_L - 1]}$$

ii) Convex surface of planoconvex lens is silvered

$$P_{eff} = \frac{2(\mu_L)}{R} \qquad f_{eff} = \frac{-R}{2(\mu_L)}$$

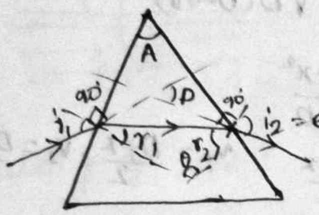
3) One side of equiconvex lens is silvered

$$P_{eff} = \frac{4\mu_2 - 2}{R} \qquad f_{eff} = \frac{-R}{4\mu_2 - 2}$$

→ Prism: Angle b/w two refracting surfaces.

For equilateral prism angle of prism is 60°.

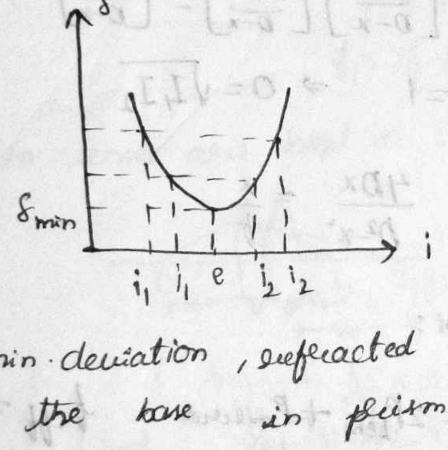
Refraction of light through prism.



$A = r_1 + r_2$

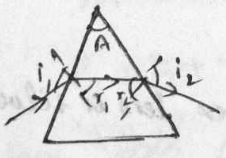
$A + D = i_1 + i_2$

$\mu_p = \frac{\sin i_1}{\sin r_1} \qquad \mu_p = \frac{\sin i_2}{\sin r_2}$



At min deviation $i_1 = i_2 \Rightarrow i = e$

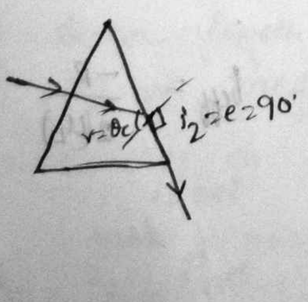
→ At min. deviation, refracted ray passes symmetrically (‖el) to the base in prism (regular).



$r_1 = r_2$

$$i = \frac{A + D_{min}}{2} \qquad \mu_{prism}^{med} = \frac{\sin\left(\frac{A + D_{min}}{2}\right)}{\sin(A/2)}$$

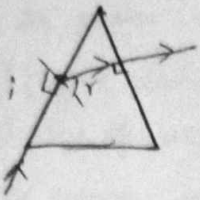
→ Normal incidence - grazing emergence



$A + D = 90°$

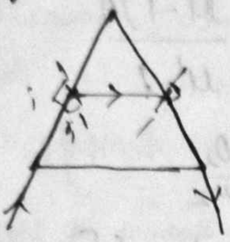
$D = 90° - A = 90° - \sin^{-1}(1/\mu)$

→ Grazing incidence and normal emergence



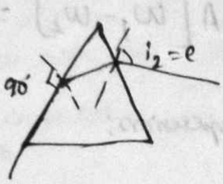
$$D = 90^\circ - \sin^{-1}(1/\mu) = 90^\circ - \sin^{-1}\left(\frac{\mu_{med}}{\mu_{prism}}\right)$$

→ Grazing incidence - Grazing emergence



$$D = 180^\circ - 2\sin^{-1}(1/\mu)$$

→ Max. deviation



$$D_{max} = 90^\circ + \sin^{-1}\left[\mu \sin(A - \theta_c)\right] - A$$

$$D_{max} = 90^\circ + \sin^{-1}\left[\sin A\left(\sqrt{\mu_p^2 - 1}\right) - \cos A\right] - A$$

If $i_1 = \angle i$ & $\theta = \sin^{-1}[\mu \sin(A - \theta_c)]$ then

$i_1 = \theta$ then emergent ray grazes the 2nd surface

$i_1 > \theta$ then emergent ray comes out from 2nd surface

$i_1 < \theta$ then ~~emergent~~ light ray undergoes T.I.R inside the prism

For T.I.R $\mu \geq \operatorname{cosec}(A/2)$

→ δ produced by small angle prism:

$i_1 = \mu r_1$, $i_2 = \mu r_2$

$$\delta_{min} = \left[\frac{\mu_{pri}}{\mu_{med}} - 1\right] A = [\mu_p - 1] A$$

→ Dispersion of light

$$\theta = \delta_V - \delta_R = (\mu_V - \mu_R) A$$

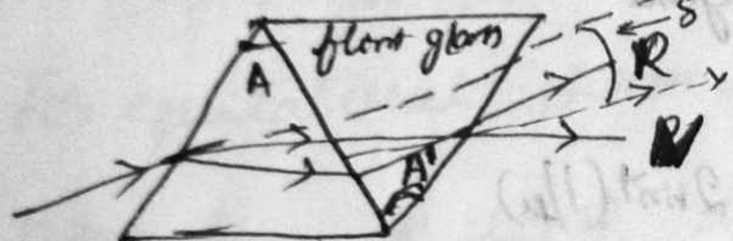
$$\text{Dispersive power } (\omega) = \frac{\text{Angular dispersion b/w 2 colours}}{\text{mean deviation of 2 colours}}$$

$$\omega = \theta / \delta \qquad \omega = \frac{\delta_V - \delta_R}{\delta_Y} = \frac{(\mu_V - \mu_R)}{(\mu_Y - 1) A} = \frac{(\mu_V - \mu_R)}{\mu_Y - 1}$$

→ Mean deviation is max. for yellow coloured light ray

$$\omega = \frac{(\mu_2 - \mu_1)}{\left(\frac{\mu_2 + \mu_1}{2} - 1\right)}$$

→ Dispersion without deviation:



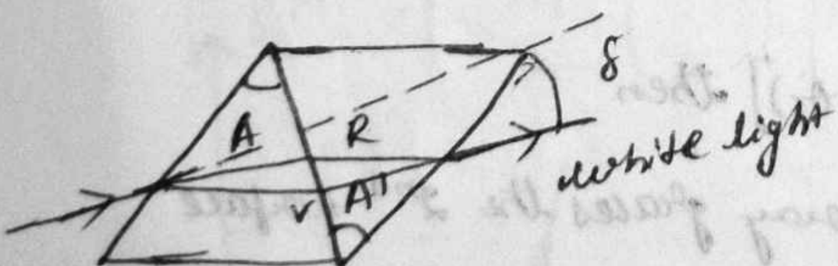
[Direct vision prism]

$$A' = -\frac{(\mu - 1)A}{\mu' - 1}$$

$$\frac{\theta_1}{\omega_1} = \frac{-\theta_2}{\omega_2}$$

$$\theta_{net} = (\mu - 1) A [\omega_1 - \omega_2] = \delta\, [\omega_1 - \omega_2]$$

→ Deviation without dispersion:



$$A' = -\left[\frac{\mu_V - \mu_R}{\mu'_V - \mu'_R}\right] A$$

$$\omega_1 \delta_1 = -\omega_2 \delta_2$$

$$\delta_{net} = \delta\left[1 - \frac{\omega_1}{\omega_2}\right]$$