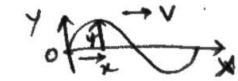
Waves

Wave – Mechanical wave – sound wave
– Non-mechanical wave – electromagnetic waves

Progressive waves – Longitudinal waves – parallel prop
– Transverse waves – perpendicular prop

→ General eqn of wave $\boxed{y = A\sin(\omega t - kx)}$ general form



$y = A\sin(kx - \omega t)$ k = proportionality constant $= \frac{2\pi}{\lambda}$

$\omega = 2\pi f$

$y = -A\sin[kx + \omega t]$ A = -ve, direction of wave = -ve

$y = A\sin(\omega t + kx)$

$y = A\sin[\omega t - kx]$ A = +ve dow = +ve

$y = A\sin[\omega t + kx]$ A = +ve dow = -ve

$y = A\sin(kx + \omega t)$

→ Velocity of wave

$$v = f\lambda = 2\pi f\,\frac{\lambda}{2\pi} = \frac{\omega}{k} \quad (\text{or}) \quad v = \frac{dy}{dx}$$

→ Velocity of wave particle

$$V_p = A\omega\cos(\omega t - kx) \quad (+ve) \text{ of } x \text{ axis}$$

$$(\text{or}) \quad V_p = \omega\sqrt{A^2 - y^2}$$

$$\text{Max } V_p = A\omega$$

$$V_p = -V_{wave} \times \text{slope of wave}$$

→ Differential eqn of wave

$$\frac{d^2y}{dt^2} = V_{wave}^2\,\frac{d^2y}{dx^2}$$

→ Phase difference b/w any two succ. Crests or troughs is 2π

→ $\Delta\phi = k\Delta x = \frac{\omega}{V}\Delta x = \omega\Delta t = \frac{2\pi}{T}\Delta t = \frac{2\pi}{\lambda}\Delta x$

→ Energy of wave

$$KE = \tfrac{1}{2}\rho A^2\omega^2\cos^2(\omega t - kx) \quad \rho = \text{density}$$

$$P.E = (\text{Max. KE})(\text{volume})$$

$$P.E = \tfrac{1}{2}\rho\omega^2A^2 S x$$

S → area, x → length } volume

→ Power of wave (P) = $\frac{1}{2}\rho\omega^2A^2SV$ (V → velocity)

→ Intensity of wave (I) = $\frac{\text{Power}}{\text{Area}} = P/S = \frac{\frac{1}{2}\rho\omega^2A^2SV}{S}$

$$I = \frac{1}{2}\rho\omega^2A^2V$$

$$I = \frac{1}{2}\rho(2\pi f)^2A^2V = 2\pi^2f^2\rho A^2V$$

$$I \propto A^2 \propto f^2A^2$$

(i) Amplitude : Max displacement, SI unit is metre.

(ii) Phase (ϕ) : SI unit is radian

(iii) Wavelength(λ) : SI unit is metre

(iv) Timeperiod (T) : SI unit is second

(v) Frequency (n) : SI unit is hertz (Hz)

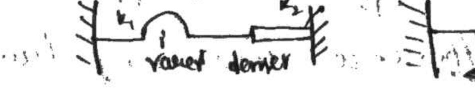
→ In longitudinal wave in region of compression rarefraction density varies.

Reflection of wave :

→ at rigid support

$$y_r = -A\sin(\omega t + kx)$$

→ at free end

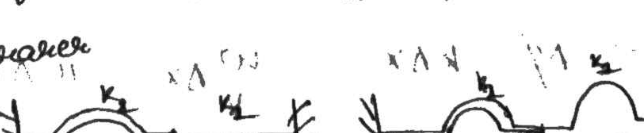


$V_i = V_r$

$$y_r = A\sin(\omega t + kx)$$

→ from rarer to denser

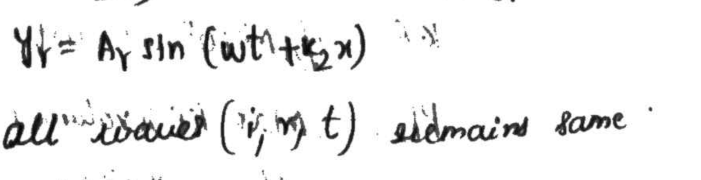
$V_i > V_t$



$$y_t = A_t\sin(\omega t - k_2x) \qquad y_r = A_r\sin(\omega t + k_1x)$$

→ from denser to rarer

$V_i < V_t$



$$y_r = A_r\sin(\omega t + k_2x)$$

frequency of all waves (i, r, t) remains same

$$y_t = y_i + y_r$$

→ $y = A \sin(\omega t \mp kx)$ $\quad y = A \sin 2\pi \left(nt \mp x/\lambda\right)$

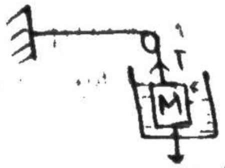
$y = A \sin 2\pi \left(\frac{t}{T} \mp \frac{x}{\lambda}\right)$ $\quad y = A \sin \omega \left(t \mp x/v\right)$

→ $A_t = \left(\frac{2v_2}{v_1+v_2}\right)A_i$ amplitude of transmitted wave

$A_r = \left(\frac{v_2 - v_1}{v_1+v_2}\right)A_i$ amplitude of reflected wave

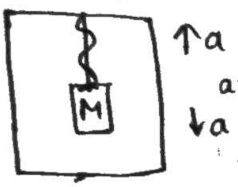
→ Velocity of transverse wave in std. string

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\text{stress}}{\text{density}}} \qquad \mu = \frac{m}{l} = \text{linear density}$$



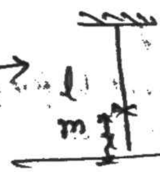
$$V = \sqrt{\frac{Mg\left(1-\frac{d_L}{d_S}\right)}{\mu}}$$

$Mg\left(1-\frac{d_L}{d_s}\right)$

→ 

$a = 0$

$V = \sqrt{\frac{Mg}{\mu}}$, $\quad V = \sqrt{\frac{M(g+a)}{\mu}}$ up +a $\quad V = \sqrt{\frac{M(g-a)}{\mu}}$ up −a, down +a $\quad V = \sqrt{\frac{M(g+a)}{\mu}}$ down −a

→ 

$V = \sqrt{gx}$ $\qquad t = 2\sqrt{l/g}$

→ $V = \sqrt{\frac{\left(\frac{m}{l}x + M\right)g}{\mu}}$  $T_x = \left(\frac{m}{l}x + M\right)g$

$t = 2\sqrt{\frac{l}{mg}}\left[\sqrt{M+m} - \sqrt{M}\right]$ $\qquad \mu = \mu_0 x \quad m = \frac{\mu_0 x^2}{2}$

→ Wave function

$$\frac{d^2 f}{dx^2} = \frac{1}{v^2}\frac{d^2 f}{dt^2}$$

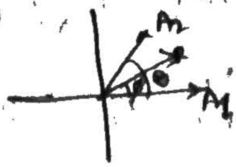
$$y = Ae^{-(ax+bt)^2}$$

$$y = \frac{A}{B + (x-vt)^2} \qquad \frac{A}{B} = \text{amplitude of wave pulse}$$

→ Superposition of wave

$R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$\tan\theta = \dfrac{A_2 \sin\phi}{A_1 + A_2\cos\phi}$



→ Inference

R = resultant wave

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

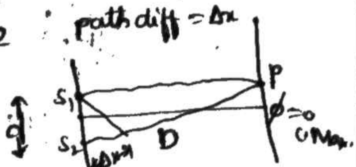
$R = 2A\cos(\phi/2)$ $\qquad I \propto R^2$

$I = 4I_0\cos^2(\phi/2)$

→ Constructive inference $\Delta x = n\lambda$ $\qquad \Delta x = dy/D$

Destructive inference $\Delta x = (2n-1)\lambda/2$ $\qquad$ path diff = Δx

distance of n^{th} minima $y_n = \dfrac{(2n-1)\lambda D}{2d}$



distance of n^{th} maxima $y_n = \dfrac{n\lambda D}{d}$

→ Stationary wave $y = 2A\sin kx\cos\omega t$

Formation of stationary wave in std. string

$f_1 = n_1 = \dfrac{1}{2l}\sqrt{\dfrac{T}{\mu}}$ fundamental frequency 1st harmonic

$f_n = n_p = \dfrac{p}{2l}\sqrt{\dfrac{T}{\mu}}$ p^{th} harmonic (or) $(p-1)$ overtone

Max. λ of length 'l' std. string $\boxed{\lambda = 2L}$

Laws of stationary waves

1st law: $n \propto 1/l$

2nd law: $n \propto \sqrt{T}$

3rd law: $n \propto 1/\sqrt{\mu}$

→ Sound: form of energy cause sense of hear

audible range : 20 Hz – 20,000 Hz

Infrasonic range : 0 Hz – 20 Hz

supersonic range : 20,000 Hz above

audible wavelengths 16.5×10^{-3} m to 16.5 m

velocity of sound = 330 m/s in air

$V_s = \sqrt{Y/d}$ $\quad V_L = \sqrt{K/d}$ $\quad V_g = \sqrt{K/d}$

K = Bulk modulus

Y = Young's modulus

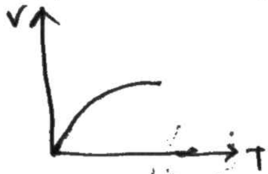
$V_s > V_L > V_g$ as $Y > K > K$

→ Newton formula $V_s = \sqrt{\frac{K}{d}} = \sqrt{P/d}$ at isothermal process $K = P$

Newton's - Laplace formula $V_s = \sqrt{\frac{\gamma RT}{M}}$ adiabatic process $K = \gamma P = \gamma R$

→ Effect of temperature on velocity of sound

$V \propto \sqrt{T}$



For every 1°C rise in temp velocity of sound increases by 0.61 m/s.

Pressure has no effect on velocity of sound

Humidity increases velocity of sound increases.

⇒ **Beats** : $y = 2A \cos\left(\frac{\omega_1 - \omega_2}{2}\right)t \sin\left(\frac{\omega_1 + \omega_2}{2}\right)t$

Tuning fork

(i) If prongs are waxed $\partial\downarrow$ $I\uparrow$

(ii) If holed $\partial\uparrow$

(iii) If heated $\partial\downarrow$

$y = A_b \sin\left(\frac{\omega_1 + \omega_2}{2}\right)t$

frequency (n) $= f = \frac{f_1 + f_2}{2}$

Human ear can hear only 10 beats per sec more than 10 - beats observed is zero.

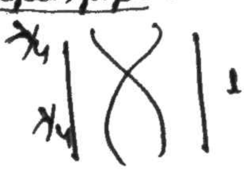
amplitude $(A_b) = 2A \cos\left(\frac{\omega_1 - \omega_2}{2}\right)t$

beat frequency $f = f_1 \sim f_2$

beat period $T = 1/f_1 - f_2$

no. of beats : $\Delta f = f_1 - f_2$

→ Open pipe :



$f = V/2l$

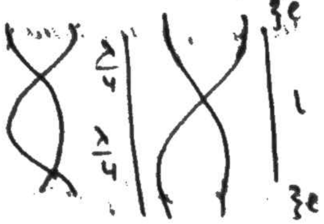
fundamental

$f = \frac{2V}{2l}$

1st overtone

$$n = \frac{1}{2l}\sqrt{\frac{\gamma RT}{M}} \qquad f_1 : f_2 : f_3 : \dots = 1 : 2 : 3 : 4 : \dots$$

End correction



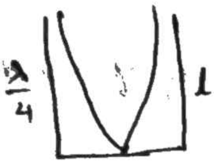
$$\lambda = 2(l + 2e)$$

$$f = \frac{v}{2(l+2e)}$$

If radius of pipe is r, then $e = 0.6r$

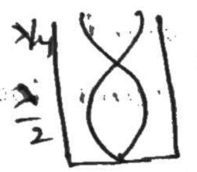
$$f_1 = \frac{v}{2(l + 1.2r)}$$

Closed pipe :



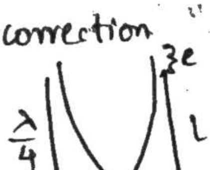
$$f_1 = \frac{v}{4l}$$

$$f_1 : f_2 : f_3 : \dots = 1 : 3 : 5 : \dots : (2p-1)$$



$$f_2 = \frac{3v}{4l}$$

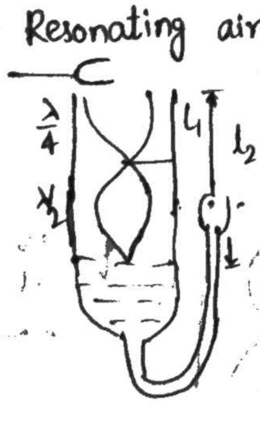
End correction



$$4(l+e) = \lambda \quad f = \frac{v}{4(l+e)} \Rightarrow f = \frac{v}{4(l + 0.6r)}$$

In case of open or closed pipes diff. b/w any 2 successive harmonics is fundamental frequency

→ Resonating air column :



$$v = 2n(l_2 - l_1)$$

$$e = \frac{l_2 - 3l_1}{2}$$

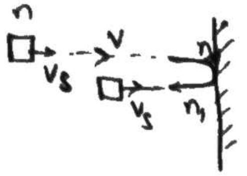
→ Doppler effect

$$n' = \left(\frac{v - v_0}{v - v_s}\right) n$$

v_0 - velocity of observer

v_s - velocity of source

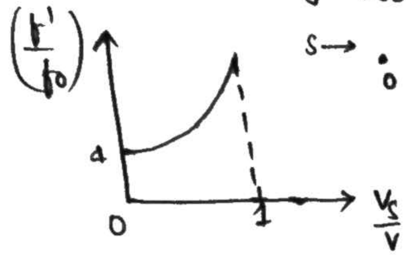
Sign convention - along the direction of sound all velocities are +ve, away from direction of sound all velocities are -ve.

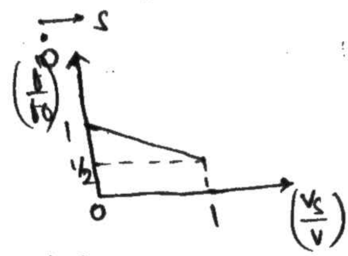


$$n' = n\left(\frac{v + v_s}{v - v_s}\right)$$

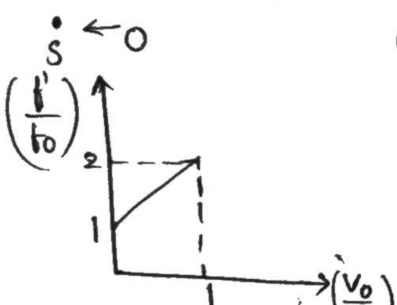
no. of beats $\Delta n = \dfrac{2 n v_s}{v - v_s}$

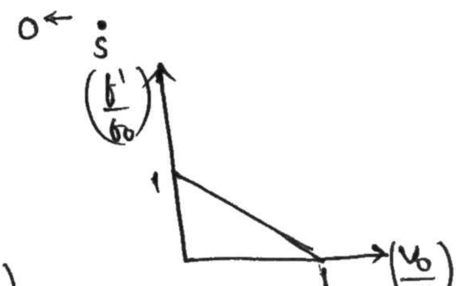
Observer stationary : source moving

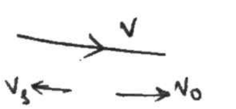




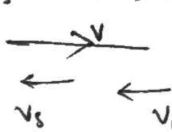
Observer moving: source stationary



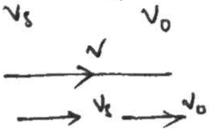




$$n' = n\left(\frac{v - v_0}{v + v_s}\right)$$



$$n' = n\left(\frac{v + v_0}{v + v_s}\right)$$



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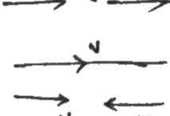

$$n' = n\left(\frac{v + v_0}{v - v_s}\right)$$

Echo

Reflected sound

$V = \frac{2d}{t}$ speed of sound

in sonar system $d_{ocean} = \frac{V_w t}{2}$

V_w = velocity of sound in water

1st echo - t_1

2nd echo - t_2

3rd echo - $t_1 + t_2$

4th echo - $2t_1 + t_2$

5th echo - $t_1 + 2t_2$

6th echo - $2t_1 + 2t_2$

→ Sound level (L) $= 10 \log_{10}\left(\frac{I}{I_0}\right)$