Errors

1. $\Delta y = \delta y = f(x+\delta x) - f(x)$

2. $dy = f'(x)\,\delta x$

3. $f(x+\delta x) = f(x) + f'(x)\,\delta x$

4. $\frac{\delta y}{y} \times 100\% = \%$ error

Cauchy Mean value theorem:

$$\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$$

(or)

$$\frac{f(a+h)-f(a)}{g(a+h)-g(a)} = \frac{f'(a+\theta h)}{g'(a+\theta h)}$$

Mean value theorems

⇒ Rolle's theorem

$f : [a,b] \to R$ such that i) f is continuous on $[a,b]$ ii) f is derivable on (a,b)

(iii) $f(a) = f(b) \to \exists$ atleast one value of c in $(a,b) \to f'(c) = 0$

(OR)

$f : [a, a+h] \to R$ suchthat i) f is continuous on $[a, a+h]$ & it is derivable on $(a, a+h)$

(iii) $f(a) = f(a+h) \to \exists$ atleast one no. θ $(0<\theta<1) \to f'(a+\theta h) = 0$

⇒ Lagrange's Mean value theorem:

$f(x) : [a,b] \to R$ i) $f(x)$ is continuous $[a,b]$, ii) derivable on (a,b)

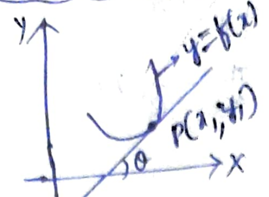
iii) $f(a) \neq f(b)$ $\exists$ $x = c$ in (a,b), $f'(c) = \frac{f(b)-f(a)}{b-a}$

(OR)

$f(a+h) = f(a) + h f'(a+\theta h)$

Tangents & Normals

→ **Slope of tgt & normals**

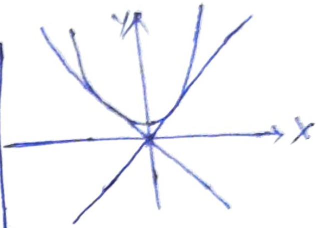


$\boxed{m = \tan\theta = \left(\frac{dy}{dx}\right)_P}$ slope of normal $= -1/m$

→ If $m = \left(\frac{dy}{dx}\right)_P = 0$ then the tgt is ||el to X-axis.

→ If $m = \left(\frac{dy}{dx}\right)_P \to \infty$ then the tgt is ||el to Y-axis & ⊥ar to X-axis

→ If the tgt to the curve $y=f(x)$ at $P(x_1,y_1)$ is makes equal angles with co-ordinate axes then $m = \pm 1$.



→ Eqn of tgt to curve $y = f(x)$ at $P(x_1, y_1)$ is $\boxed{y - y_1 = m(x - x_1)}$ where $m = \frac{dy}{dx}$

Eqn of normal $\boxed{y - y_1 = -1/m\,(x - x_1)}$

→ Condition that the line $y = mx + c$ may be a tgt to

i) parabola $y^2 = 4ax$ is $c = a/m$

ii) an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $c^2 = a^2m^2 + b^2$

iii) hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $c^2 = a^2m^2 - b^2$

→ $y = f(x)$, $y = g(x)$, $P(x_1, y_1)$ $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$ $m_1 = (f'(x))_P$ $m_2 = (g'(x))_P$

→ 2 curves cut each other orthogonally $\boxed{m_1 \cdot m_2 = -1}$

→ If 2 curves touch each other $\boxed{m_1 = m_2}$

→ Area bounded by the tgt, normal at $P(x_1, y_1)$ on the curve $y = f(x)$ & the line

i) x-axis is $\frac{y_1^2}{2}\left|m + 1/m\right|$

ii) Y-axis is $\frac{x_1^2}{2}\left|m + 1/m\right|$

iii) $x = k$ is $\frac{(x_1 - k)^2}{2}\left|m + 1/m\right|$

iv) $y = k$ is $\frac{(y_1 - k)^2}{2}\left|m + 1/m\right|$

→ If the curves $xy = c^2$, $y^2 = 4ax$ cut each other orthogonally then $\boxed{c^4 = 32a^4}$

→ If the 2 curves $a_1x^2 + b_1y^2 = 1$ & $a_2x^2 + b_2y^2 = 1$ cut each other orthogonally

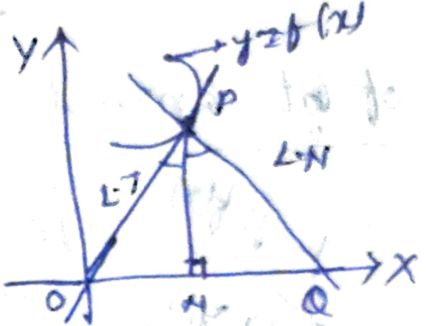
$\boxed{1/a_1 - 1/b_1 = 1/a_2 - 1/b_2}$

→ Length of tangent (PM) = $\left|\frac{y_1}{m}\sqrt{1+m^2}\right|$

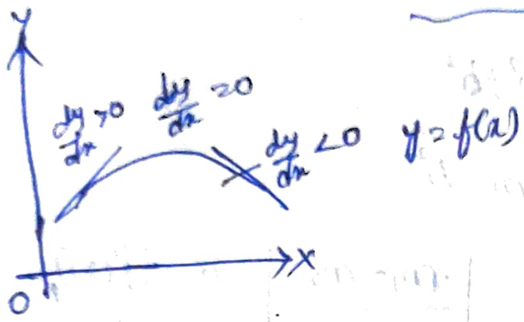
Length of normal (PQ) = $\left|y_1\sqrt{1+m^2}\right|$

Length of sub.tgt (MN) = $|y_1/m|$

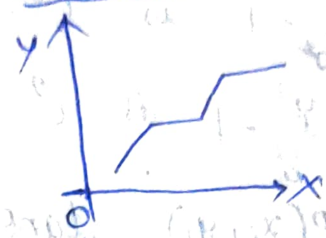
Length of sub normal (NQ) = $|y_1 m|$



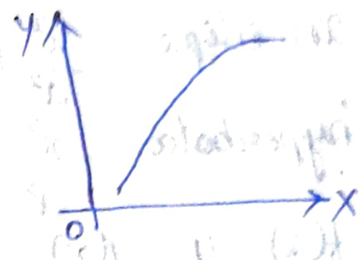
Maxima & Minima



Increasing:



Strictly increasing



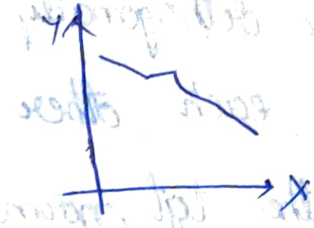
$I = f'(x) \geq 0$

$S.I = f'(x) > 0$

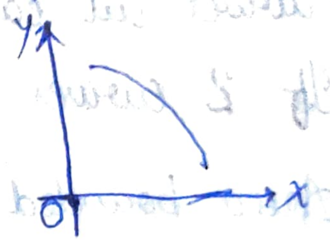
$D = f'(x) \leq 0$

$S.D = f'(x) < 0$

Decreasing



Strictly decreasing:



→ f is ↑ function (a,b): if $x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)\ \forall\ x_1, x_2 \in (a,b)$

→ f is strictly ↑ function (a,b) if $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)\ \forall\ x_1, x_2 \in (a,b)$

→ if $x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2)\ \forall\ x_1, x_2 \in (a,b)$ for decreasing function.

for strictly ↓ function (a,b) if $x_1 < x_2 \in (a,b)$ $f(x_1) > f(x_2)$

for local maxima (or) minima check $f''(x)$ for the values of x exists or not ($f''(a) \neq 0$).

at $x = a$, $f'(a) = 0$ then 'a' is critical point of $f(x)$