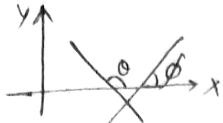
Straight lines

**→ Inclination of line:



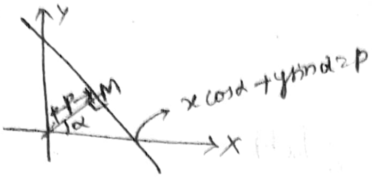
$\boxed{0 \le \theta < 180}$ Range ⟹ slope of line is $\boxed{m = \tan\theta}$

$\tan\theta$ b/w 2 lines of slopes m_1, m_2

$\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$ for acute

obtuse $\theta = \pi - \theta$.

→ Normal form of line



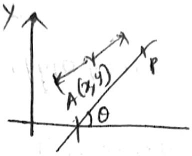
→ symmetric form

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta}$$

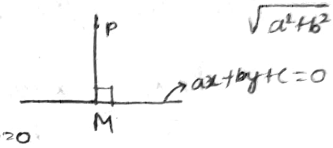
→ Parametric form:

$x = x_1 + r\cos\theta, \quad y = y_1 + r\sin\theta$

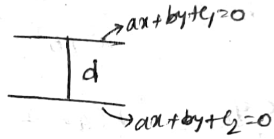
$P(x_1 + r\cos\theta,\ y_1 + r\sin\theta)$



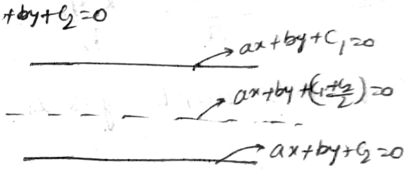
→ ⊥ar distance from $P(x_1, y_1)$ to $ax + by + c = 0$ is $\dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$



→ distance b/w ∥lel lines $d = \dfrac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$



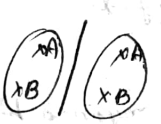
→ Midway eqn is $ax + by + \left(\dfrac{c_1 + c_2}{2}\right) = 0$



→ Ratio:

① ratio $= -L_{11} : L_{22}$, $L_{11} = ax_1 + by_1 + c$; $L_{22} = ax_2 + by_2 + c$

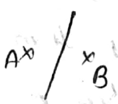
(ii)



$L_{11} \cdot L_{22} = 0 \leftrightarrow L_{11} > 0,\ L_{22} > 0$

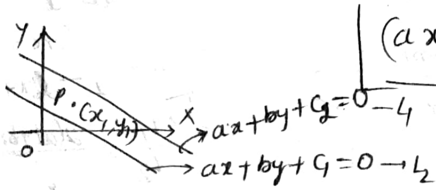
$L_{11} < 0,\ L_{22} < 0$

(iii)



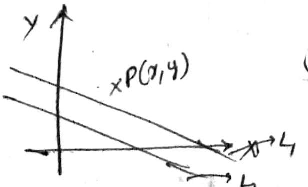
$L_{11} \cdot L_{22} < 0 \leftrightarrow L_{11} < 0,\ L_{22} > 0$ & $L_{11} > 0,\ L_{22} < 0$

(iv)



$\boxed{(ax_1 + by_1 + c_1)(ax_2 + by_2 + c_2) < 0}$

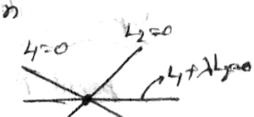
(v)



$(ax_1 + by_1 + c_1)(ax_2 + by_2 + c_2) > 0$

→ Intercept form — $x/a + y/b = 1$, $m:n \Rightarrow \dfrac{nx}{x_1} + \dfrac{my}{y_1} = m + n$

→ Eqn of line passing through P.O.I of $L_1 = 0$ & $L_2 = 0$ is



represents a family of st lines. $\boxed{L_1 + \lambda L_2 = 0}$ (λ is parameter)

→ $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$, $a_3x+b_3y+c_3=0$ are concurrent iff

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

→ angle b/w lines – $\cos\theta = \left|\dfrac{a_1a_2+b_1b_2}{\sqrt{a_1^2+b_1^2}\sqrt{a_2^2+b_2^2}}\right|$ & $\tan\theta = \left|\dfrac{a_1b_2-a_2b_1}{a_1a_2+b_1b_2}\right|$

if lines are ⊥ar then $a_1a_2+b_1b_2=0$

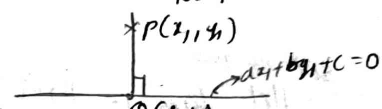
→ Area:

i) by $x/a + y/b = 1$ & co-ordinate axis is $\frac{1}{2}|ab|$

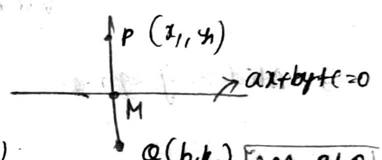
ii) by $ax+by+c=0$ & co-ordinate axes is $\dfrac{c^2}{2|ab|}$

iii) of rhombus formed by $a|x|+b|y|+c=0$ is $\dfrac{2c^2}{|ab|}$

→ foot: $\dfrac{h-x_1}{a} = \dfrac{k-y_1}{b} = \dfrac{-(ax_1+by_1+c)}{a^2+b^2}$

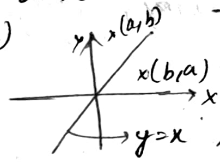


→ Image: $\dfrac{h-x_1}{a} = \dfrac{k-y_1}{b} = \dfrac{-2(ax_1+by_1+c)}{a^2+b^2}$

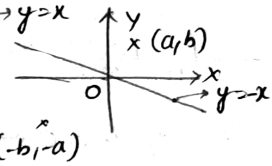


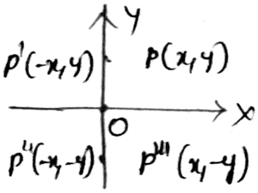
$\boxed{2M=P+Q}$

i) of (a,b) w.r.t $y=x$ is (b,a)



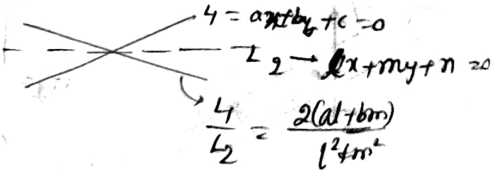
ii) of (a,b) w.r.t $y=-x$ is $(-b,-a)$





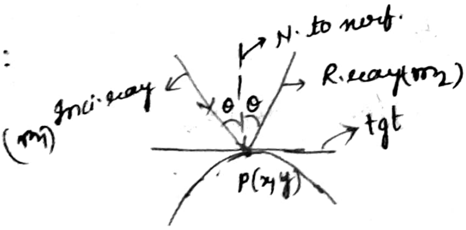
→ Image of $ax+by+c=0$ w.r.t $lx+my+n=0$ bisects an angle b/w 2 lines which 1 of them $ax+by+c=0$ then eqn of other line is

$$\frac{L_1}{L_2} = \frac{2(al+bm)}{l^2+m^2}$$



$\dfrac{L_1}{L_2} = \dfrac{2(al+bm)}{l^2+m^2}$

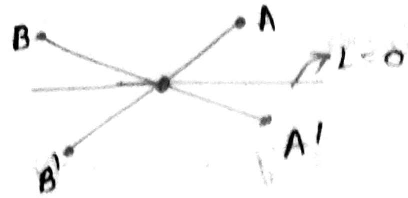
→ Reflection:



$\boxed{m_1 = m_2}$

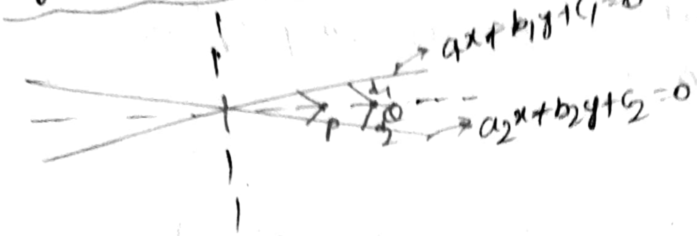
→ Optimization: A & B are 2 points on same st. line $L = ax+by+c=0$
point P such that $PA+PB=\min$, Intersection of $L=0$ & line joining A to image B or line joining B to image of A w.r.t $L=0$

$L = ax+by+c=0$

1) 

ii) point P such that $|PA-PB|$ - max, is point of I. $L=0$ & line joining A & B

Angular bisectors of line :



$d_1=d_2 \Rightarrow \dfrac{|a_1x_1+b_1y_1+c_1|}{\sqrt{a_1^2+b_1^2}} = \dfrac{|a_2x_1+b_2y_1+c_2|}{\sqrt{a_2^2+b_2^2}}$

$$\frac{a_1x_1+b_1y_1+c_1}{\sqrt{a_1^2+b_1^2}} = \pm\frac{a_2x_1+b_2y_1+c_2}{\sqrt{a_2^2+b_2^2}}$$

locus of $P(x_1,y_1)$ is $\dfrac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \dfrac{\pm\, a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$

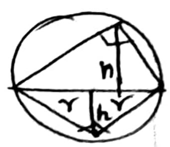
→ $\boxed{c_1>0,\quad c_2>0}$

condition	acute	obtuse
$a_1a_2+b_1b_2>0$	−	+
$a_1a_2+b_1b_2<0$	+	−

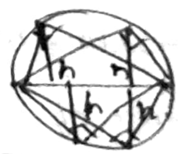
→ If $c_1c_2>0$ then $\dfrac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \dfrac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$ is bisector of angle.

→ No. of right Δ^{le} in Cle.

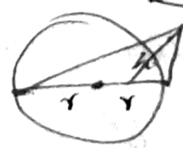
i) if $h=r$ then 2 right Δ^{le}



ii) if $h<r$ then no. of right Δ^{le}s = 4



iii) if $h>r$ then no. of right Δ^{le} = 0



$a = 2h/\sqrt{3}$ $\Delta = h^2/\sqrt{3}$