Continuity & Differentiability

→ $f(x)$ is continuous at $x=a$ if $\lim_{x\to a} f(x) = f(a)$

→ $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a) \Rightarrow f(x)$ is continuous at $x=a$.

→ $\lim_{x\to a^+} f(x) = f(a) \Rightarrow f(x)$ is right continuous at $x=a$

→ $\lim_{x\to a^-} f(x) = f(a) \Rightarrow f(x)$ is left continuous at $x=a$

→ $f(x)$ is continuous in (a,b) if $f(x)$ is continuous every point in (a,b)

→ $f(x)$ is continuous in $[a,b]$ if (i) $f(x)$ is continuous at every point in (a,b)

(ii) $f(x)$ is right continuous at $x=a$

(iii) $f(x)$ is left continuous at $x=b$

Discontinuity:

→ $f(x)$ is discontinuous if $\lim_{x\to a} f(x) \neq f(a)$

→ $f(x)$ will be discontinue in any 1 of the following cases

(i) $\lim_{x\to a^+} f(x) \neq \lim_{x\to a^-} f(x)$

(ii) $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) \neq f(a)$

(iii) $f(a)$ is not defined

(iv) atleast 1 of the limit doesn't exist.

Differentiability

→ A function $f(x)$ is said to be differentiable at $x=a$ if $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ exist & finite. It is denoted by $f'(a)$.

$$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}, \quad f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

→ $f(x)$ is right hand derivate at $x=a \Rightarrow f'(a^+) = \lim_{x\to a^+} \frac{f(x)-f(a)}{x-a}$

$f(x)$ is left hand derivate at $x=a \Rightarrow f'(a^-) = \lim_{x\to a^-} \frac{f(x)-f(a)}{x-a}$

→ $f(x)$ is differentiable at $x=a$ if $f'(a^+) = f'(a^-)$

$f(x)$ is not differentiable $\Leftrightarrow f'(a^+) \neq f'(a^-)$

→ $f(x)$ is differentiable in $(a,b) \Leftrightarrow$ if $f(x)$ is differentiable in every point (a,b).

→ $f(x)$ is differentiable in $[a,b] \Rightarrow$ (i) if $f(x)$ is differentiable in every point (a,b)

(ii) $f(x)$ is right differentiable at $x=a$

(iii) $f(x)$ is left differentiable at $x=b$

→ if $f(x)$ is differentiable then $f(x)$ is continuous converse need not to be true.

→ if $f(x)$ is not continuous then $f(x)$ is not differentiable.

→ A function $f(x)$ is said to be differentiable if it is differentiable in its domain.

→ Exponential, logarithmic, trigonometric, inverse trigonometric function are differentiable in their domain.

→ Polynomial, constant functions are differentiable $\forall\ x \in R$

→ $\frac{d}{dx}\left[\int_{\phi(x)}^{\psi(x)} f(t)\,dt\right] = f(\psi(x))\psi'(x) - f(\phi(x))\phi'(x)$

→ $x \sin 1/x$, $x \cos 1/x$ are continuous at $x=0$ but not differentiable at $x=0$