# 3. KINEMATICS

Always distance $\geq$ displacement

$$\text{Speed} = \frac{\text{Distance}}{\text{time}}$$

$$\text{Avg. speed} = \frac{\text{total distance}}{\text{total time}}$$

$$= \frac{\Delta x}{\Delta t}$$

$$V_{instant} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}}$$

$$\vec{V} = \frac{\vec{S}}{t}$$

$$\text{Avg. velocity} = \frac{\Delta S}{\Delta t} \Rightarrow \frac{S_1 + S_2}{t_1 + t_2} \Rightarrow \frac{V_1 t_1 + V_2 t_2}{t_1 + t_2}$$

when $S_1$ in $t_1$ & $S_2$ in $t_2$

$$\boxed{\text{when } t_1 = t_2 = t \quad V_{avg} = \frac{V_1 + V_2}{2}}$$

$$\boxed{V_{avg} = \frac{2V_1 V_2}{V_1 + V_2} \quad S_1 = S_2 = S}$$

$$\text{Time taken } (t) = \frac{t_1 t_2}{t_2 + t_1}$$

$$\text{acceleration } (a) = \frac{V - u}{t}$$

$$\text{Instantaneous acceleration} = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t}$$

$$a = \frac{dv}{dt}$$

$$a = v \cdot \frac{dv}{ds}$$

## Equations of motion:

i) $V = u + at$

ii) $S = ut + \frac{1}{2}at^2$

iii) $S_n = u + a(n - \frac{1}{2})$

iv) $V^2-u^2=2as$ [$\because S$ - displacement]

v) when avg. velocity is given

$$S=\left(\frac{V+u}{2}\right)t$$

vi) $S=\frac{1}{2}at^2$ when $u=0$

Shortcuts

$u=\frac{3S_1-S_2}{2t}$ when $t$ is same

$a=\frac{S_2-S_1}{t^2}$

$a=\frac{S_m-S_n}{m-n}$ when different $S$ and $t$ are given

The distance travelled by bullet before to rest $ux=\left(\frac{u^2}{v^2-u^2}\right)x$

velocity of a body at midpoint $\boxed{V=\sqrt{\frac{V_1^2+V_2^2}{2}}}$

Max velocity $\boxed{(V_{max})=\left(\frac{\alpha\beta}{\alpha+\beta}\right)t}$ where $\alpha,\beta$ are accelerat

$\boxed{S=\frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2}$

Starting from rest covered $\frac{1}{2}$ of distance in last sec is

$n=\frac{1+\sqrt{1-f}}{f}$ where $f$ is fraction

**Freely falling body:**

1. $V=gt$ 2. $S=h=\frac{1}{2}gt^2$

S in 1st 1sec, 1st 2sec, 1st 3sec, 1st 4sec $=h_1:h_2:h_3:h_4:\ldots=1:4:9:16:\ldots$

S travelled in 1st 1sec $=\frac{1}{2}\times 9.8\times 1^2=4.9m$

in 1st 2sec $=19.6$, in 1st 3sec $=44.1m$, 1st 4sec $=78.4m$,

in 1st 5sec $=122.5m$

3. $V=\sqrt{2gh}$ when $S=h$ is given

Ratio b/w S travelled by freely falling body in 1st sec, 2nd sec, 3rd sec, 4th sec, 5th sec, 6th sec, ...... = 1:3:5:7:9:11:...

S travelled by freely falling body in nth sec $S_n = \frac{g}{2}(2n-1)$

in 1st sec = $S_1 = \frac{9.8}{2}(2(1)-1)$
$= 4.9m$

in 2nd sec = 14.7m, in 3rd sec = 24.5m, in 4th sec = 34.3m

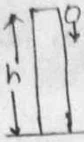
$\downarrow\downarrow\downarrow$ -ve $\uparrow\uparrow$ +ve

## Vertically projected body

$t_a = u/g$ , $t_d = \frac{u}{g}$

$T = t_a + t_d = \frac{2u}{g}$

$a = -g$



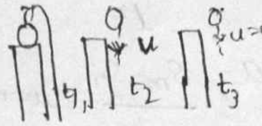
$\boxed{t = h/u}$

## top of tower -

$h = -ut + \frac{1}{2}gt^2$

$t = \frac{u \pm \sqrt{u^2+2gh}}{g}$

$a = g$



$h = \frac{1}{2}gt_1t_2$

$t_3 = \sqrt{t_1 t_2}$

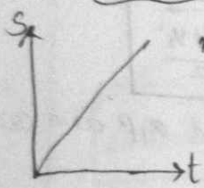
$x = (u\cos\theta)t$

$v = \sqrt{u^2+(gt)^2-2ugt\sin\theta}$

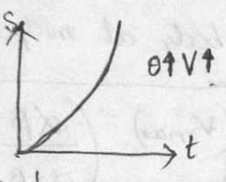
## S-t graph

$m = \frac{ds}{dt} = v$

$\theta = 0, v = 0$

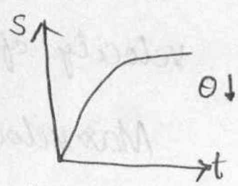


particle moves with uniform velocity



$\theta\uparrow V\uparrow$

moves with uniform acceleration

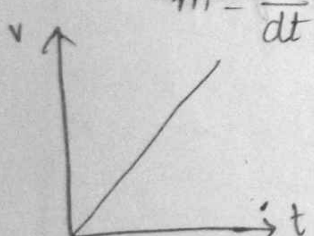


$\theta\downarrow V\downarrow$

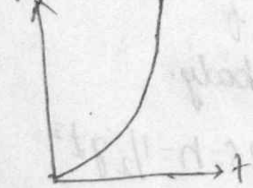
moves with constant retardation

## V-t graph

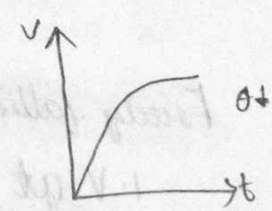
$m = \frac{dv}{dt} = a$ Area gives displacement.



particle moves with uniform acceleration



moves with ↑ acceleration



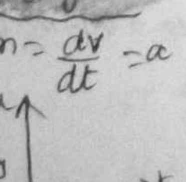
$\theta\downarrow V\downarrow$

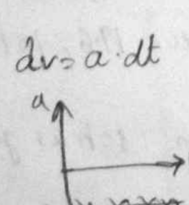
moves with retardation

## a-t graph

$m = \frac{dv}{dt} = a \rightarrow dv = a \cdot dt$

uniform velocity

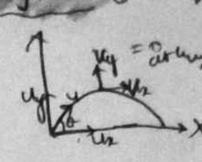




with retardation

## Projectile

① oblique projectile



$\vec{u} = u\cos\theta$

at t sec

$\vec{V} = u\cos$

$H_{max} = \frac{u^2\sin^2\theta}{2g}$

② Complementary angles:

$H_1 + H_2 = \frac{u^2}{2g}$

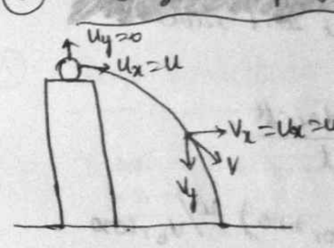
③ Eqn of trajectory

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

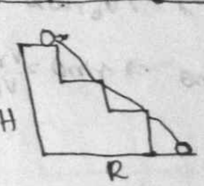
$y = x\tan\theta\left[1 - \frac{x}{R}\right.$

④ $KE = \frac{1}{2}mu^2\cos^2\theta$

⑤ Horizontal proj



ⓐ



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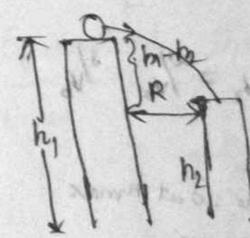
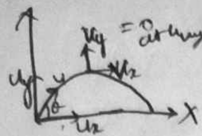


⑥ Inclined plan

$H = \frac{u_y^2}{2a_y}$

## Projectile

① Oblique projectile – ground to ground




$\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$ $\qquad \tan\alpha = \dfrac{u\sin\theta - gt}{u\cos\theta}$

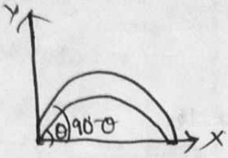
at t sec

$\vec{V} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\hat{j}$

$H_{max} = \dfrac{u^2\sin^2\theta}{2g}$ $\quad R = \dfrac{u^2\sin 2\theta}{g}$ $\quad t_a = t_d = \dfrac{u\sin\theta}{g}$ $\quad T = \dfrac{2u\sin\theta}{g}$

② Complementary angles:

$H_1 + H_2 = \dfrac{u^2}{2g}$




Ranges and time of flights are not same for any projectile

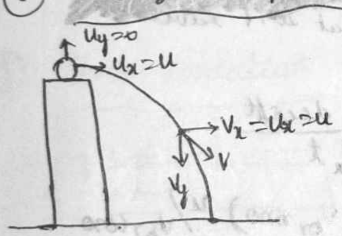
③ Eqn of trajectory

$y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$ $\qquad \langle v \rangle = \dfrac{u}{2}\sqrt{3\cos^2\theta + 1}$

$y = x\tan\theta\left[1 - \dfrac{x}{R}\right]$ $R \to$ Range

④ $KE = \frac{1}{2}mu^2\cos^2\theta$, $PE = \frac{1}{2}mu^2\sin^2\theta \Rightarrow TE = PE + KE = \frac{1}{2}mu^2$

⑤ Horizontal projectile:




$\vec{V} = u_x\hat{i} + u_y\hat{j}$

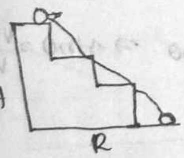
$\vec{V} = u\hat{i} + gt\hat{j}$

$\tan\alpha = \dfrac{V_y}{V_x} = \dfrac{gt}{u}$

$t_d = \sqrt{\dfrac{2h}{g}}$

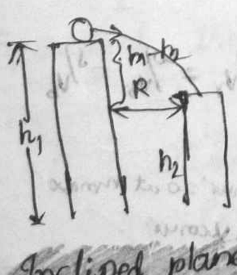
$\boxed{R = u\sqrt{\dfrac{2h}{g}}}$

ⓐ



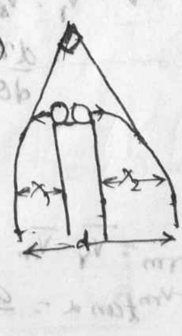

$R = u\sqrt{\dfrac{2h}{g}}$ $\qquad \boxed{n = \dfrac{2u^2h}{gw^2}}$

ⓑ




$\boxed{R = u\sqrt{\dfrac{2(h_1 - h_2)}{g}}}$

ⓒ




$t = \dfrac{\sqrt{u_1u_2}}{g}$

$d = \dfrac{(x_1 + x_2)\sqrt{u_1u_2}}{g}$

⑥ Inclined plane:

$H = \dfrac{u_y^2}{2a_y} = \dfrac{u^2\sin^2\alpha}{2g\cos\theta}$ $\qquad T = \dfrac{2u\sin\alpha}{g\cos\theta}$

$R = (u\cos\alpha)T - \frac{1}{2}g(\sin\theta)T^2$

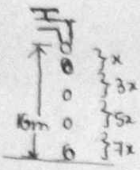
(b) downwards

$$H = \frac{u^2 \sin^2 \alpha}{2g\cos\theta}$$

$$T = \frac{2u\sin\alpha}{g\cos\theta}$$

$$R = (u\cos\alpha)t + \tfrac{1}{2}(g\sin\theta)t^2$$

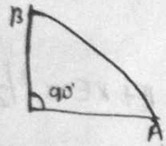
→ Water drop model




$x + 3x + 5x + 7x = 16$

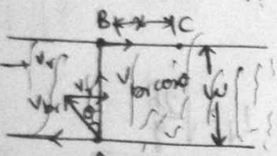
$x = 1m$

distance from ground of 3rd drop $= 5x + 7x = 12x = 12m$




distance $= R\theta$

displacement $= 2R\sin\theta/2$




$v_r$ - velocity of river

$v_b$ - velocity of boat

$v_{br}$ - velocity of boat w.r.t river

min time $= d/v_b$

$(\vec{v}_{br}) + \vec{v}_r = \vec{v}_b$

$v_{bx} = v_r - v_{br}\sin\theta$

$v_{by} = 0 + v_{br}\cos\theta$

No drift

$x = (v_b)_x t$

$x = (v_r - v_{br}\sin\theta)\, w/v_{br}\cos\theta$

$v_r = v_{br}\sin\theta \Rightarrow \sin\theta = \frac{v_r}{v_{br}}$

i) Min drift

$v_r = v_{br}\sin\theta$

$\frac{dx}{d\theta} = 0, \quad \sin\theta = \frac{v_{br}}{v_r}$

$v_b = \sqrt{v_{br}^2 + v_r^2}$

$x/v_r = w/v_{br} = d/v_b$

Rain umbrella

$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$


$\tan\alpha = \frac{a\,v_m}{b}$