Quadratic Equations

→ $x^2 - (\alpha+\beta)x + \alpha\beta = 0$

→ $ax^2 + bx + c = 0$ if α, β are roots for this then

i) $\alpha+\beta = \frac{-b}{a}$, $\alpha\beta = c/a$, $1/\alpha + 1/\beta = -b/c$

ii) $|\alpha-\beta| = \frac{\sqrt{b^2-4ac}}{|a|}$

iii) $(\alpha+k)(\beta+k) = \frac{ak^2-bk+c}{a} = \frac{f(-k)}{a}$

→ If $a+b+c=0$ then roots of eqn are 1 & c/a, if $a-b+c=0$ then roots are -1 & $-c/a$

→ $\Delta = b^2 - 4ac$

$\Delta > 0 \leftrightarrow$ real & unequal

$\Delta = 0 \leftrightarrow$ real & equal, rational roots (perfect sq)

$\Delta < 0 \leftrightarrow$ imaginary roots

→ If $f(x)=0$ is Q. eqn then roots are

i) reciprocals of roots of $f(x)=0$ is $f(1/x)=0$

ii) increased by k than that of $f(x)=0$ is $f(x-k)=0$

iii) α, β are roots of Q. eqn $f(x)=0$ eqn whose roots are $a\alpha+b, a\beta+b$ is $f\left(\frac{x-b}{a}\right)=0$

→ condition for the roots of $ax^2+bx+c=0$ to be in the ratio $m:n$ is $(m+n)^2 ac = mnb^2$

condition for one root of $ax^2+bx+c=0$ may be the square of the other is $b^3+a^2c+ac^2 = 3abc$

→ one root of Q. eqn is n^{th} root of other then $(ac^n)^{1/n+1} + (a^nc)^{1/n+1} + b = 0$

→ if difference of roots is same as difference of other Q. eqn then
$(ax^2+bx+c=0)$ $(px^2+qx+r=0)$

$$\frac{\Delta_1}{\Delta_2} = \frac{a^2}{p^2}$$

→ common roots of $a_1x^2+b_1x+c_1=0$, $a_2x^2+b_2x+c_2=0$ where $a_1b_2-a_2b_1 \neq 0$ have a common root if $(c_1a_2-c_2a_1)^2 = (a_1b_2-b_1a_2)(b_1c_2-b_2c_1)$.

∴ Root is $\frac{c_1a_2-c_2a_1}{a_1b_2-b_1a_2}$ (or) $\frac{b_1c_2-b_2c_1}{c_1a_2-c_2a_1}$

→ $a_1x^2+b_1x+c_1=0$ and $a_2x^2+b_2x+c_2=0$ have same roots then $a_1:a_2=b_1:b_2=c_1:c_2$

→ If the eqn. $ax^2+bx+c=0$ have complex roots then $\forall x\in R$, ax^2+bx+c & a will have same sign.

→ $ax^2+bx+c=0$ has real roots α & β $(\alpha>\beta)$ then

i) $\beta<x<\alpha \Rightarrow ax^2+bx+c$ & a will have opp. sign.

ii) $x>\alpha$ (or) $x<\beta \Rightarrow ax^2+bx+c$ & a will have same sign.

→ vertex of parabola of eqn. $y=ax^2+bx+c$ is $\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$.

Length of latusrectum is $1/|a|$

→ If $a>0$, the min. value of ax^2+bx+c occurs at $x=\frac{-b}{2a}$ it is $\frac{4ac-b^2}{4a}$ $\forall x\in$

If $a<0$, the max. value of ax^2+bx+c occurs at $x=\frac{-b}{2a}$, it is $\frac{4ac-b^2}{4a}$, $\forall x\in$

→ If $f(x)=\frac{ax^2+bx+c}{ax^2-bx+c}$ (or) $\frac{ax^2-bx+c}{ax^2+bx+c}$ $(b^2-4ac<0)$ then min & max. values of $f(x)$ at $f\left(\pm\sqrt{\frac{c}{a}}\right)$.

→ $ax^2+2hxy+by^2+2gx+2fy+c$ to be expressed as a product of two linear factors is $abc+2fgh-af^2-bg^2-ch^2=0$ (or) $\boxed{\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c\end{vmatrix}=0,\ h^2\geq ab,\ g^2\geq ac,\ f^2\geq bc}$