Infinite Integration

→ Reverse process of differentiation is called integration.

1. $\int 0\,dx = C$
2. $\int x^n dx = \frac{x^{n+1}}{n+1} + C$
3. $\int 1\,dx = x + C$
4. $\int \frac{1}{x} dx = \log|x| + C$
5. $\int \frac{1}{x^2} dx = -\frac{1}{x} + C$
6. $\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$
7. $\int \frac{|x|}{x} dx = |x| + C$
8. $\int e^x dx = e^x + C$
9. $\int a^x dx = \frac{a^x}{\log a} + C$
10. $\int \sin x\,dx = -\cos x + C$
11. $\int \cos x\,dx = \sin x + C$
12. $\int \sec^2 x\,dx = \tan x + C$
13. $\int \operatorname{cosec}^2 x\,dx = -\cot x + C$
14. $\int \sec x \tan x\,dx = \sec x + C$
15. $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C$
16. $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}x + C = -\cos^{-1}x + C$
17. $\int \frac{1}{1+x^2} dx = \tan^{-1}x + C = -\cot^{-1}x + C$
18. $\int \frac{1}{|x|\sqrt{x^2-1}} dx = \sec^{-1}x + C = -\operatorname{cosec}^{-1}x + C$
19. $\int \sinh x\,dx = \cosh x + C$
20. $\int \cosh x\,dx = \sinh x + C$
21. $\int \operatorname{sech}^2 x\,dx = \tanh x + C$
22. $\int \operatorname{sech} x \tanh x\,dx = -\operatorname{sech} x + C$
23. $\int \operatorname{cosech} x \coth x\,dx = -\operatorname{cosech} x + C$
24. $\int \operatorname{cosech}^2 x\,dx = -\coth x + C$
25. $\int \frac{1}{\sqrt{1+x^2}} dx = \sinh^{-1}x + C = \log|x+\sqrt{x^2+1}| + C$
26. $\int \frac{1}{\sqrt{x^2-1}} dx = \cosh^{-1}x + C$ if $x > 1$; $-\cosh^{-1}x + C$ if $x < -1$; $= \log|x+\sqrt{x^2-1}| + C$ ($x < -1$ or $x > 1$)
27. $\int \frac{1}{1-x^2} dx = \tanh^{-1}x + C$, $|x| < 1$
28. $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx + C$
29. $\int f(ax+b)dx = \frac{1}{a} F(ax+b) + C$

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30. $\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + C$
31. $\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C$
32. $\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + C$
33. $\int \tan x\,dx = -\log|\cos x| + C = \log|\sec x| + C$
34. $\int \cot x\,dx = -\log|\operatorname{cosec} x| + C = \log|\sin x| + C$
35. $\int \sec x\,dx = \log|\sec x + \tan x| + C = \log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| + C = \log\left|\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right| + C$
36. $\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x| + C = \log|\tan \frac{x}{2}| + C$
37. $\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
38. $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$
39. $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

40. $\int \frac{1}{\sqrt{a^2+x^2}} dx = \sinh^{-1}(x/a) + c$

41. $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}(x/a) + c$

42. $\int \frac{1}{\sqrt{x^2-a^2}} dx = \cosh^{-1}(x/a) + c$

43. $\int \sqrt{a^2+x^2}\, dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\sinh^{-1}(x/a) + c$

44. $\int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}(x/a) + c$

45. $\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\cosh^{-1}(x/a) + c$

→ If in the denominator contains both sin (or) both cos multiply & divided by sin (a-b) otherwise with cos (a-b).

→ Type-1: $\int \frac{1}{ax^2+bx+c} dx$ will be of the form $\int \frac{1}{x^2+a^2} dx$ (or) $\int \frac{1}{x^2-a^2} dx$ (or) $\int \frac{1}{a^2-x^2}$

Shortcut: (a) $b^2-4ac < 0,\ a > 0 \Rightarrow \int \frac{1}{ax^2+bx+c} dx = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) + k$

(b) $b^2-4ac > 0,\ a > 0 \Rightarrow \int \frac{1}{ax^2+bx+c} dx = \frac{1}{\sqrt{b^2-4ac}} \log\left|\frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right| + k$

(c) $b^2-4ac > 0,\ a < 0 \Rightarrow \int \frac{1}{ax^2+bx+c} dx = \frac{1}{\sqrt{b^2-4ac}} \log\left|\frac{\sqrt{b^2-4ac}-(2ax+b)}{\sqrt{b^2-4ac}+(2ax+b)}\right| + k$

→ Integration of Rational Trigonometric functions :

Type 1: $\int \frac{1}{a+b\cos^2 x}dx$ (or) $\int \frac{1}{a+b\sin^2 x}dx$ (or) $\int \frac{1}{a\cos^2 x+b\sin^2 x}dx$ (or)

$\int \frac{1}{a\cos^2 x+b\sin x\cos x+c\sin^2 x}dx$, multiply both Nr & Dr with $\sec^2$

& $\tan x = t$

Type 2: $\int \frac{1}{a+b\cos x}dx$ (or) $\int \frac{1}{a+b\sin x}dx$ (or) $\int \frac{1}{a\cos x+b\sin x+c}dx$

put $\tan x/2 = t$

$dx = \frac{2dt}{1+t^2}$ $\sin x = \frac{2t}{1+t^2}$ $\cos x = \frac{1-t^2}{1+t^2}$

Type 3: $\int \frac{a\cos x+b\sin x}{c\cos x+d\sin x}dx$

only short cut = $\frac{ac+bd}{c^2+d^2}x + \frac{ad-bc}{c^2+d^2}\log|Dr| + C$

for $I = \int \frac{ae^x+be^{-x}}{ce^x+de^{-x}}dx$

Short cut is $\frac{1}{2}\left(\frac{a}{c}+\frac{b}{d}\right)x + \frac{1}{2}\left(\frac{a}{c}-\frac{b}{d}\right)\log|Dr| + C$

By parts :

$\int u\,dv = uv - \int v\frac{du}{dx}dx$ ILATE (e^x)

I – Inverse, L – Log, A – Alg, T – Trig, E – Exp

$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \ldots$

$\rightarrow \int e^x [f(x) + f'(x)]\, dx = e^x f(x) + c$

$\int e^{ax} \cos(bx+c)\, dx = \frac{e^{ax}}{a^2+b^2} [a\cos(bx+c) + b\sin(bx+c)] + k$

$\int e^{ax} \sin(bx+c)\, dx = \frac{e^{ax}}{a^2+b^2} [a\sin(bx+c) - b\cos(bx+c)] + k$

$\int [f(x) + xf'(x)]\, dx = xf(x) + c$

$\int e^{g(x)} (g'(x) f(x) + f'(x))\, dx = e^{g(x)} f(x) + c$

$\rightarrow$ By reduction :

$\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} I_{n-1} \quad (n \in N)$

$\int \sin^n x\, dx = -\frac{\sin^{n-1}x \cos x}{n} + \frac{n-1}{n} I_{n-2}$

$\int \cos^n x\, dx = \frac{\cos^{n-1}x \sin x}{n} + \frac{n-1}{n} I_{n-2}$

$\int \tan^n x\, dx = \frac{\tan^{n-1}x}{n-1} - I_{n-2}$

$\int \cot^n x\, dx = -\frac{\cot^{n-1}x}{n-1} - I_{n-2}$

$\int \text{cosec}^n x\, dx = \frac{-\text{cosec}^{n-2}x \cot x}{n-1} + \frac{n-2}{n-1} I_{n-2}$

$\int \sec^n x\, dx = \frac{\sec^{n-2}x \tan x}{n-1} + \frac{n-2}{n-1} I_{n-2}$

$\int \frac{\sin nx}{\sin x} dx = \frac{2}{n-1} \sin(n-1)x + I_{n-2}$

Shortcuts -

$\rightarrow \int \frac{dx}{ax^2+bx+c} = \frac{2}{\sqrt{\Delta}} \tan^{-1}\left(\frac{\frac{d}{dx}(ax^2+bx+c)}{\sqrt{4ac-b^2}}\right) + c$

$\rightarrow \int \frac{dx}{a+b\cos^2 x} = \frac{1}{a}\left(\sqrt{\frac{a}{a+b}} \tan^{-1}\left(\sqrt{\frac{a}{a+b}} \tan x\right)\right) + c$

for $\sin^2 x$ replace it by $1-\cos^2 x$

$\rightarrow \int \frac{ae^{kx}+b}{ce^{kx}+d} dx = \frac{bx}{d} + \frac{1}{k} \frac{(ad-bc)}{cd} \log_e |ce^{kx}+d| + c$

$\rightarrow \int e^{-x} f(x)\, dx = e^{-x} [f(x) + f'(x) + f''(x) + \ldots]$

$\rightarrow \int \frac{f'(x)}{[f(x)+a][f(x)+b]} dx = \frac{1}{b-a} \log \left|\frac{f(x)+a}{f(x)+b}\right| + c$