Theory of Equations

→ If $\alpha_1, \alpha_2, \ldots, \alpha_n$ are roots of eqn $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_n = 0$ then

(i) sum of roots — $\sum \alpha_1 = S_1 = -\frac{a_1}{a_0}$

(ii) sum of product of roots taken 2 at a time: $\sum \alpha_1\alpha_2 = S_2 = \frac{a_2}{a_0}$

(iii) $\alpha_1 \cdot \alpha_2 \cdot \alpha_3 \cdots \alpha_n = (-1)^n \frac{a_n}{a_0} = S_n$

→ If α, β, γ are roots of $ax^3+bx^2+cx+d=0$ then

(i) $S_1 = \alpha+\beta+\gamma = -b/a$

(ii) $S_2 = \alpha\beta+\beta\gamma+\gamma\alpha = c/a$

(iii) $S_3 = \alpha\beta\gamma = -d/a$

→ Eqn. having roots α, β, γ is $(x-\alpha)(x-\beta)(x-\gamma) = 0$ is

$$x^3 - S_1x^2 + S_2x - S_3 = 0$$

→ (i) Eqn. of lowest degree with rational coeff having a root $\sqrt{a}+\sqrt{b}$ is $x^4 - 2(a+b)x^2 + (a-b)^2 = 0$.

(ii) Eqn. of lowest degree with rational coeff. having a root is $\sqrt{a}+i\sqrt{b}$ is $x^4 - 2(a-b)x^2 + (a+b)^2 = 0$.

→ (i) Eqn. $ax^3+bx^2+cx+d=0$ have roots in A.P then $2b^3+27a^2d = 9abc$.

(ii) if $ax^3+bx^2+cx+d=0$ have roots in G.P then $ac^3 = b^3d$.

(iii) if $ax^3+bx^2+cx+d=0$ have roots in H.P then $2c^3+27ad^2 = 9bcd$.

→ Transformed equations:

$f(x)=0$ is an eqn. of degree n then to eliminate r^{th} term in $f(x)=0$ can be transformed to $f(y+h)=0$ where h = const. such that $f^{(n-r+1)}(h)=0$

→ Newton's method

$x^4 + P_1x^3 + P_2x^2 + P_3x + P_4 = 0$ ($\alpha, \beta, \gamma, \delta$ roots of this eqn.)

$S_1 + P_1 = 0$ — $S_1 = \alpha+\beta+\gamma+\delta$

$S_2 + S_1P_1 + 2P_2 = 0$ — $S_2 = \alpha^2+\beta^2+\gamma^2+\delta^2$

$S_3 + S_2P_1 + S_1P_2 + 3P_3 = 0$ — $S_3 = \alpha^3+\beta^3+\gamma^3+\delta^3$

$S_4 + S_3P_1 + S_2P_2 + S_1P_3 + 4P_4 = 0$ — $S_4 = \alpha^4+\beta^4+\gamma^4+\delta^4$