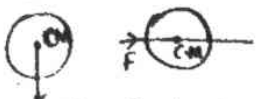
# Centre of Mass

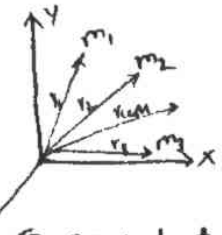
## ① Understanding of COM




only translation motion

Mg: COM depends on external forces only but not internal forces

## ② COM of multiple system:



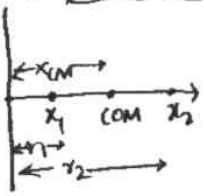

$$r_{COM} = \frac{m_1 r_1 + m_2 r_2 + m_3 r_3}{m_1 + m_2 + m_3}$$

$$x_{COM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

→ Sum of moments of mass of the system of particles about centre of mass is zero

$$\sum_{i=1}^{n} m_i r_i = 0$$

## ③ COM of two particle system:




$$x_{COM} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

if COM is taken as origin then

$$m_1 r_1 = m_2 r_2$$

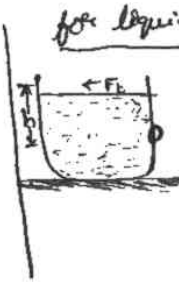
## ④ COM of continuous mass distribution:

$$r_{COM} = \frac{\int r\,dm}{\int dm} \qquad x_{COM} = \frac{\int x\,dm}{\int dm} \qquad y_{COM} = \frac{\int y\,dm}{\int dm}$$

## ⑤ COM of uniform mass

| cone | Triangular plate/cone | Hollow hemisphere | semi circular ring | solid hemisphere | disc |
|---|---|---|---|---|---|
| C | T | H | R | S | D |
| $\frac{3l}{4}$ | $\frac{H}{3}$ | $\frac{R}{2}$ | $\frac{2R}{\pi}$ | $\frac{3R}{8}$ | $\frac{4R}{3\pi}$ |

for liquid

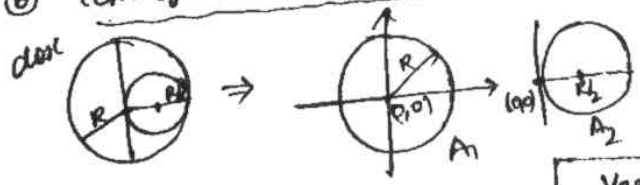



$V_r = V$

$\left(\frac{-dm}{dt}\right) = \rho A v$

$\left(\frac{-dm}{dt}\right) = \rho\left(\frac{dv}{dt}\right)$

## ⑥ COM of ~~combined structure~~ cavity:




$$\frac{A_1 x_1 - A_2 x_2}{A_1 - A_2} = x_{COM}$$

$$V_{COM} = \frac{m_1 v_1 + m_2 v_2 + m_3 v_3}{m_1 + m_2 + m_3}$$

$$a_{COM} = \frac{m a_1 + m_2 a_2 + m_3 a_3}{m_1 + m_2 + m_3}$$

### Variable mass

$v_r$ = relative velocity

$$F_t = v_r\left(\pm \frac{dm}{dt}\right)$$

**Rocket**

$$v = u - gt + v_r \ln\left(\frac{m_0}{m}\right)$$

$v_r$ = exhaust gases velocity

$m_0$ = initial mass of rocket

$m$ = mass of rocket at time $t$.