29/08/22

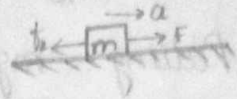
6. Friction

Kinetic friction: Horizontal surface $\quad \mu$ = coefficient of friction

$N = mg$

$f_k = \mu_k N$

$F = \mu_k mg + ma$



retarding force:

$f_k = \mu_k mg$

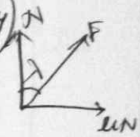
$ma = \mu_k mg$

$a = -\mu_k g \Rightarrow$ retardation

Stopping distance: $S = \dfrac{u^2}{2\mu_k g}$

stopping time: $t = \dfrac{u}{\mu_k g}$

rolling friction $\propto$ (1/radius of rolling body)



angle of friction $(\phi) \Rightarrow \mu_s = \tan\phi$

contact force $F_C = N\sec\phi \qquad R = \sqrt{f^2 + N^2}$

Relation b/w kinetic, rotational, static friction

$f_r < f_k < f_s \qquad \rightarrow f$ is independent on area of contact.

$\mu_r < \mu_k < \mu_s$

A block of mass 'm' is intially at rest. We applied a force F along horizontal direction

Case 1: If $F < f_L$ then body is at rest $f_s = F$

Case 2: If $F = F_L$ then body is ready to slide

$F_L = F$

Case 3: If $F > f_L$ then body is in motion

frictional force

$f_k = \mu_k N$

$F - f_k = ma$

$\boxed{F = \mu_k mg + ma}$

Chain is ready to slide then

mass of hanging part $= \dfrac{mg}{n}$

mass of chain lying on table = $mg(1-1/n)$

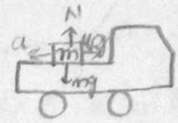
l is length of hanging part L length of chain

$$\frac{l}{L} = \frac{\mu_s}{\mu_s+1} \times 100\%$$

Pulling: $\dfrac{mg\sin\phi}{\cos(\theta-\phi)}$

Pushing: $\dfrac{mg\sin\phi}{\cos(\theta+\phi)}$

Consider a block of mass 'm' is placed on a truck at a distance l from its rear end.



Case 1: $ma < f_L$, $f_s = ma$, $a_T = \mu g$

Case 2: $ma > f_L$ block is in motion

$f_k = \mu_k N = \mu_k mg$

acceleration $a' = a - \mu_k g$

block leaves the truck in a time t

$$t = \sqrt{\frac{2l}{a-\mu_k g}}$$

$$a = (\mu_s - \mu_k)g$$

Contact with front part of truck so that the block should not fall acceleration is

$$a = g/\mu$$

Vertical surface

A block of mass m is pressed against wall without falling

$f_s = mg$, $N = F$

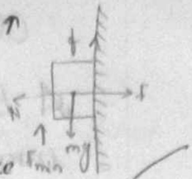
$\mu_s N = mg$

$F = \dfrac{mg}{\mu_s}$ $\qquad a = \dfrac{mg - f_k}{m}$

A block pressed by 2 hands

$mg = 2f_s \Rightarrow F = \dfrac{mg}{2\mu_s}$

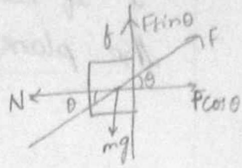
Min. force applied parallel to the wall body move ↑



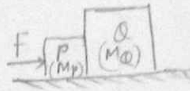
$$F_{min} = (mg + f) = (mg + \mu N) = (mg + \mu F)$$

Min. work done to move up the body to S distance

$$W = F_{min} S = (mg + f) S$$



$$F = \frac{mg}{(\sin\theta + \mu\cos\theta)}$$

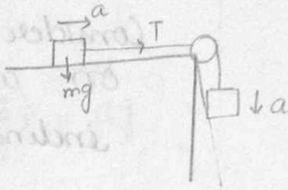


F on Q $\qquad$ F on P

$$F_Q = \frac{M_Q F}{M_P + M_Q} \qquad F_P = \frac{M_P F}{M_P + M_Q}$$

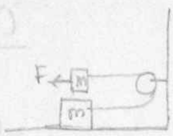
$$a = \frac{F}{M_P + M_Q}$$

<u>Connected bodies</u>



$$a = \left(\frac{m_2 - \mu_k m_1}{m_1 + m_2}\right) g$$

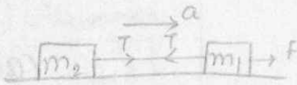
Tension $T = \dfrac{m_2 m_1 g}{m_1 + m_2}(1 + \mu_k)$



On upper block $F = T + f \Rightarrow \mu mg + T$

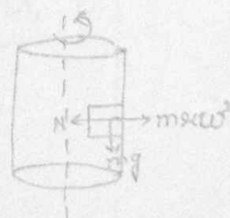
On lower block $T = \mu mg$

$$F_R = 2\mu mg$$



$$a = \frac{F - (f_1 + f_2)}{m_1 + m_2}$$

<u>Deathwell</u>



$$\mu_s m r \omega^2 = mg$$

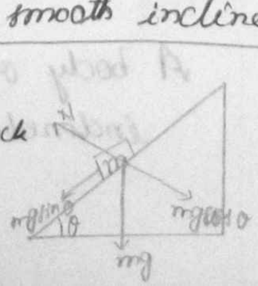
$$\omega = \sqrt{\frac{g}{\mu_s R}}$$

max. time of revolution $T = 2\pi\sqrt{\dfrac{\mu_s R}{g}}$

<u>Motion of a body on smooth inclined plane</u>

$$N = mg\cos\theta$$

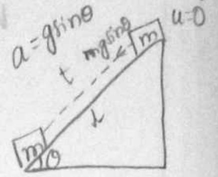
resultant force acting on the block to slide down the block



$$\boxed{F = mg\sin\theta}$$

acceleration $\boxed{a = g\sin\theta}$

A body is released from rest from top of the smooth inclined plane of inclination θ, inclined length l. If body takes t sec velocity becomes v at bottom of the plane



$$\boxed{V = \sqrt{2gl\sin\theta}}$$

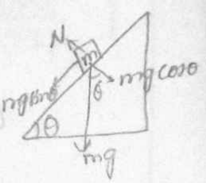
$$t = \sqrt{\frac{2l}{g\sin\theta}}$$

Motion of a body on rough inclined plane

Angle of repose (ϕ): $\tan\phi = \mu_s$

$$\mu_k = \tan\phi$$

Consider a body of mass 'm' is present on a rough inclined plane of inclination θ



$$N = mg\cos\theta$$

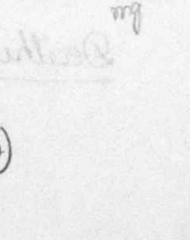
Case 1: $mg\sin\theta < f_L$

$$f_s = mg\sin\theta$$

Case 2: $mg\sin\theta = f_L$ body is ready to slide

$$f_L = \mu mg\cos\theta$$

$$\mu = \tan\theta$$



Case 3: $mg\sin\theta > f_L$

$$f_k = \mu_k N$$

$$f_k = \mu_k mg\cos\theta$$

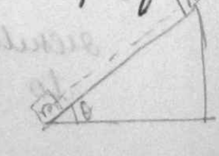
$$F = mg(\sin\theta - \mu_k\cos\theta)$$

$$a = g(\sin\theta - \mu_k\cos\theta)$$

$$W = mgl(\sin\theta - \mu_k\cos\theta)$$

A body of mass m is released from top of inclined plane.

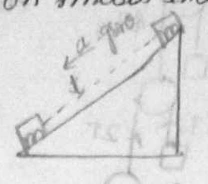
$$V = \sqrt{2gl(\sin\theta - \mu_k\cos\theta)}$$



time $t = \sqrt{\dfrac{2l}{g(\sin\theta - \mu_k \cos\theta)}}$

Time taken for body to slide down on smooth inclined plane

$$t = \sqrt{\frac{2l}{g\sin\theta}}$$



$$(t_d)_{rough} = n\,(t_d)_{smooth}$$

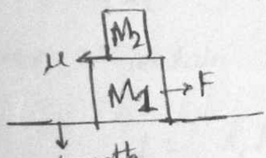
$$\sqrt{\frac{2l}{g(\sin\theta - \mu_k\cos\theta)}} = n\sqrt{\frac{2l}{g\sin\theta}}$$

$$\mu_k = \tan\theta\left(\frac{n^2-1}{n}\right)$$

Block on Block

$$(a_2)_{max} = \frac{f}{M_2} = \frac{\mu \cancel{M_2} g}{\cancel{M_2}} = \mu g$$

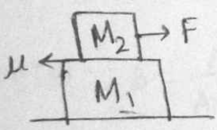
(i)



$$\boxed{F = (M_1 + M_2)\mu g}$$

$$(a_1)_{max} = \frac{f}{M_1} = \frac{\mu M_2 g}{M_1}$$

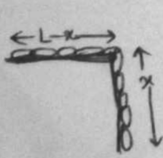
(ii)



$$F = (M_1 + M_2)\,(a_1)_{common}$$

$$\boxed{F = (M_1 + M_2)\,\frac{\mu M_2 g}{M_1}}$$

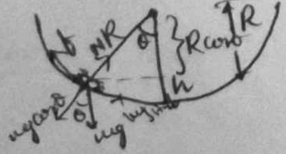
Sliding of chain



$$\mu = \frac{\text{hanging part}}{\text{remaining part}}$$

$$\mu = \frac{x}{L-x}$$

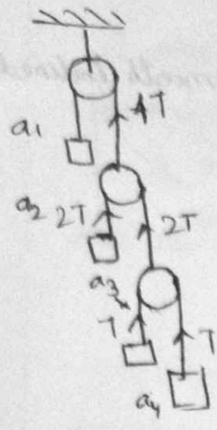
$$x = \frac{\mu L}{\mu + 1}$$



$$\mu = \tan\theta$$

$$h = R\left[1 - \frac{1}{\sqrt{\mu^2+1}}\right]$$

Connected pulleys

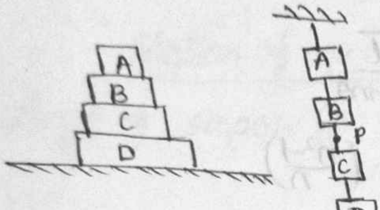


$$4a_1+2a_2+a_3+a_4=0$$

$g_{eff}=g\pm a$

$g_{eff}=g$ if $a=0$

→

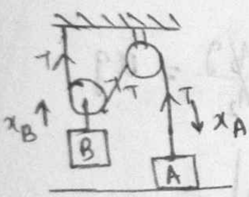


$N_{rxn} = (\sum \text{mass above } P)\times g_{eff}$

$T = \sum(\text{mass above } P)\times g_{eff}$

if string is massless then $T=(m_C+m_D)g$

→



$x_B = 2x_A$

$V_B = 2x_B$

$a_B = 2a_B$

$Bg - 2T = Ba$

$T - Ag = Aa$