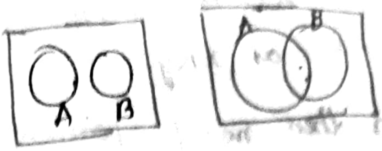
Sets & Relations

Power set (P(A)): ~~No of~~ $n(A)=m$, set formed by subsets of A

$P(A)$ contains 2^m subsets.

disjoint $A\cap B=\phi$



$A\cap B\neq\phi$

→ $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

→ $(A\cup B)'$ or $\overline{A\cup B} = \bar{A}\cap\bar{B}$ [De Morgan laws]

$\overline{A\cap B} = \bar{A}\cup\bar{B}$

→ $(A-B)\cap B=\phi$, $(A-B)\cup B = A\cup B$

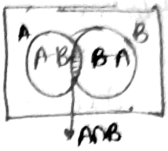
Symmetric difference

→ $A\Delta B = (A-B)\cup(B-A) \Rightarrow (A\cup B)-(A\cap B)$

$\bar{A} = U-A$, $\overline{A\cup B} = U-(A\cup B)$

→ If $A\subset B$ then $(A\cap B) = A$

→



i) $(A-B)\cap B=\phi$

$(A-B)\cup B = A\cup B$

ii) $A-(A-B) = A\cap B$

→ $n(A)=p$, $n(B)=q$

$\max\{p,q\} \le n(A\cup B) \le p+q$, $0\le n(A\cap B)\le \min\{p,q\}$

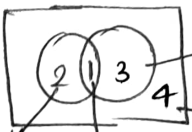
⇒ A, B, C $A\cap B'\cap C' = n(A)+n(B)+n(C)-2n(A\cap B)-2n(B\cap C)-2n(A\cap C)+3n(A\cap B\cap C)$

$A'\cap B\cap C'$ or $A'\cap B'\cap C$

$A-B = A-(A\cap B)$

$B-A = B-(A\cap B)$

→



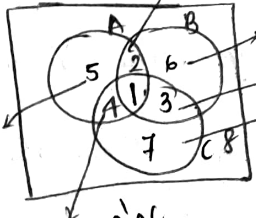
→ $B-A = B-(A\cap B)$

→ $A'\cap B'$

$A-B = A-(A\cap B)$ $A\cap B$

$n(A\cup B\cup C) = n(A)+n(B)+n(C)-n(A\cap B)-n(B\cap C)-n(C\cap A)+n(A\cap B\cap C)$

$A\cap B\cap C'$



$A'\cap B\cap C'$

$A\cap B\cap C$

$A'\cap B'\cap C$

$A\cap B'\cap C'$ (only A)

$A\cap B'\cap C$

Relations:

⇒ A & B be non empty sets. (a, b) is carteisan product of A & B if $a\in A$, $b\in B$ product of A & B. denoted by $A\times B$.

i) General $A\times B \neq B\times A$

ii) $n(A)=p$, $n(B)=q$ then $n(A\times B) = pq = n(B\times A)$

iii) $n(A\cap B)=m$ then $n\{(A\times B)\cap(B\times A)\} = m^2$

→ A & B be non empty sets then every sub set of $A \times B$ is a relation from A to B

$$R \subseteq A \times B.$$

→ R is a relation from A to B & $(x, y) \in R$, then x is related to y under the relation R as xRy;

→ If $n(A) = m$, $n(B) = n$ then no of possible relations from A to B is 2^{mn}

→ Inverse (R^{-1}) → $(a,b) \Rightarrow (b,a)$

i) $aRb \Rightarrow bR^{-1}a$ (ii) $(R^{-1})^{-1} = R$

→ Identity Relation – $I_A = \{(a,a), (b,b), (c,c), (d,d)\}$

→ Reflexive   A relation R on a set A is related to itself.

$$\forall a \in A \Rightarrow (a,a) \in R$$

No. of reflexive relations from A to A is $2^{n(n-1)}$.

→ Symmetric : $(a,b) \in R \Leftrightarrow (b,a) \in R$

No. of symmetric relations from A to A is $2^{\frac{n(n+1)}{2}}$

→ Anti symmetric relation : aRb & bRa then $a = b$

→ Transitive : aRb & $bRc \Rightarrow aRc$ $\forall a, b, c \in A$.

$aRb \Rightarrow a < b$ & $b < c \Rightarrow a < c$

→ Equivalence : i) Reflexive ii) Symmetric iii) Transitive

Partial Order relation : i) Reflexive ii) Anti symmetric iii) Transitive.

NOTE: In real numbers $<, \leq$, in sets $\subset, \subseteq$ are not equivalence relations.

→ no. of matrices formed by $a, b, c \in (0, 1, \ldots, p-1)$ either symmetric or skew symmetric or both and their $\det|A|$ is divisible by p is

$$\begin{bmatrix} a & b \\ c & a \end{bmatrix} = \underline{2p-1}$$