Machine Learning

Assignment 8.3

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Play Tennis example

day	outlook	temperature	humidity	wind	play tennis
1	rain	mild	high	strong	no
2	overcast	hot	normal	weak	yes
3	overcast	mild	high	strong	yes
4	sunny	mild	normal	strong	yes
5	rain	mild	normal	weak	yes
6	sunny	cool	normal	weak	yes
7	sunny	mild	high	weak	no
8	overcast	cool	normal	strong	yes
9	rain	cool	normal	strong	no
10	rain	cool	normal	weak	yes

Table 1: Training Data

P(yes)=$\frac{7}{10}$, P(no)=$\frac{3}{10}$

a) Classification

By using Naive Bayes

$i_1 = (sunny, cool, high, strong)$

$$P(i_1) = \frac{3}{10}.\frac{4}{10}.\frac{3}{10}.\frac{5}{10} = 0.018$$

$$P(i_1|yes) = P(sunny|yes).P(cool|yes).P(high|yes).P(strong|yes)$$

$$\therefore P(i_1|yes) = \frac{2}{7}.\frac{3}{7}.\frac{1}{7}.\frac{3}{7} = 7.49 \times 10^{-3}$$

probability	value
$P(sunny\|yes)$	2/7
$P(cool\|yes)$	3/7
$P(high\|yes)$	1/7
$P(strong\|yes)$	3/7

Table 2: Probabilities for *yes* class

probability	value
$P(sunny\|no)$	1/3
$P(cool\|no)$	1/3
$P(high\|no)$	2/3
$P(strong\|no)$	2/3

Table 3: Probabilities for *no* class

Now by Bayes theorem,

$$P(yes|i_i) = \frac{P(i_1|yes).P(yes)}{P(i_1)} = \frac{7.49 \times 10^{-3}(\frac{7}{10})}{0.018} = \mathbf{0.29}$$

Similarly,

$$P(i_1|no) = P(sunny|no).P(cool|no).P(high|no).P(strong|no)$$

$$\therefore P(i_1|no) = \frac{1}{3}.\frac{1}{3}.\frac{2}{3}.\frac{2}{3} = 0.049$$

Now by Bayes theorem,

$$P(no|i_i) = \frac{P(i_1|no).P(no)}{P(i_1)} = \frac{0.049(\frac{3}{10})}{0.018} = \mathbf{0.82}$$

For exact posterior probabilities we normalize the above quantities,

$$v_{NB}^{yes} = \frac{0.29}{0.29 + 0.82} = 0.26$$

and

$$v_{NB}^{no} = \frac{0.82}{0.29 + 0.82} = 0.73$$

Since, $v_{NB}^{no} > v_{NB}^{yes}$, i_1 is therefore classified as **no**.

By using ML estimate

Let, $h_1 = yes$ and $h_2 = no$

$$\boldsymbol{P(i_1|yes).P(yes)} = \frac{2}{7}.\frac{3}{7}.\frac{1}{7}.\frac{3}{7}.\frac{7}{10} = 5.24 \times 10^{-3}$$

Similarly,

$$\boldsymbol{P(i_1|no).P(no)} = \frac{1}{3}.\frac{1}{3}.\frac{2}{3}.\frac{2}{3}.\frac{3}{10} = 0.0148$$

Since,

$$h_{ML} = \underset{h_1,h_2 \in H}{\operatorname{argmax}} \quad P(i_1|yes).P(yes), P(i_1|no).P(no) = P(i_1|no).P(no)$$

i_1 is therefore classified as **no**.

By using Naive Bayes

$i_2 = (overcast, mild, normal, weak)$

probability	value
$P(overcast\|yes)$	3/7
$P(mild\|yes)$	3/7
$P(normal\|yes)$	6/7
$P(weak\|yes)$	4/7

Table 4: Probabilities for *yes* class

probability	value
$P(overcast\|no)$	0/3
$P(mild\|no)$	2/3
$P(normal\|no)$	1/3
$P(weak\|no)$	1/3

Table 5: Probabilities for *no* class

$$P(i_2) = \frac{3}{10}.\frac{5}{10}.\frac{7}{10}.\frac{5}{10} = 0.0525$$

$$P(i_2|yes) = P(overcast|yes).P(mild|yes).P(normal|yes).P(weak|yes).P(yes)$$

$$\therefore P(i_2|yes) = \frac{3}{7}.\frac{3}{7}.\frac{6}{7}.\frac{4}{7} = 0.09$$

Now by Bayes theorem,

$$P(yes|i_2) = \frac{P(i_2|yes).P(yes)}{P(i_2)} = \frac{0.09(\frac{7}{10})}{0.0525} = \mathbf{1.2}$$

Similarly,

$$P(no|i_2) = P(overcast|no).P(mild|no).P(normal|no).P(weak|no)$$

$$\therefore P(no|i_2) = \frac{0}{3}.\frac{2}{3}.\frac{1}{3}.\frac{1}{3} = 0.0$$

Now by Bayes theorem,

$$P(no|i_2) = \frac{P(i_2|no).P(no)}{P(i_2)} = \frac{0.0(\frac{3}{10})}{0.0525} = \mathbf{0.0}$$

For exact posterior probabilities we normalize the above quantities,

$$v_{NB}^{yes} = \frac{1.2}{1.2 + 0.0} = 1$$

and

$$v_{NB}^{no} = \frac{0.0}{1.2 + 0.0} = 0$$

Since, $v_{NB}^{yes} > v_{NB}^{no}$, i_2 is therefore classified as **yes**.

By using ML estimate

Let, $h_1 = yes$ and $h_2 = no$

$$\boldsymbol{P(i_2|yes).P(yes)} = \frac{3}{7}.\frac{3}{7}.\frac{6}{7}.\frac{4}{7}.\frac{7}{10} = 0.063$$

Similarly,

$$\boldsymbol{P(i_2|no).P(no)} = \frac{0}{3}.\frac{2}{3}.\frac{1}{3}.\frac{1}{3}.\frac{3}{10} = 0.0$$

and since,

$$h_{ML} = \underset{h_1,h_2 \in H}{\text{argmax}} \quad P(i_2|yes).P(yes), P(i_2|no).P(no) = P(i_2|yes).P(yes)$$

i_2 is therefore classified as **yes**.

b) with Laplace correction

Given by:

$$\hat{P}(a_i|v_j) \leftarrow \frac{n_c + m.p}{n + m}$$

$p = \frac{1}{k}, m = 1$

n_c	**count**
$(sunny\|yes)$	2
$(cool\|yes)$	3
$(high\|yes)$	1
$(strong\|yes)$	3

Table 6: n_c for *yes* class

n_c	**count**
$(sunny\|no)$	1
$(cool\|no)$	1
$(high\|no)$	2
$(strong\|no)$	2

Table 7: n_c for *no* class

For i_1

$p_{outlook} = \frac{1}{3}, p_{temperature} = \frac{1}{3}, p_{humidity} = \frac{1}{2}, p_{wind} = \frac{1}{2}, n_{yes} = 7, n_{no} = 3$
Therefore,

$$P(sunny|yes) = \frac{2 + (1.\frac{1}{3})}{7 + 1} = \frac{7}{24}$$

$$P(cool|yes) = \frac{3 + (1.\frac{1}{3})}{7 + 1} = \frac{5}{12}$$

$$P(high|yes) = \frac{1 + (1.\frac{1}{2})}{7 + 1} = \frac{3}{16}$$

$$P(strong|yes) = \frac{3 + (1.\frac{1}{2})}{7 + 1} = \frac{7}{16}$$

So,

$$P(yes|i_1) = P(sunny|yes).P(cool|yes).P(high|yes).P(strong|yes).P(yes)$$

$$P(yes|i_1) = \frac{7}{24}.\frac{5}{12}.\frac{3}{16}.\frac{7}{16}.\frac{7}{10} = \mathbf{6.9 \times 10^{-3}}$$

Similarly,

$$P(sunny|no) = \frac{1 + (1.\frac{1}{3})}{3+1} = \frac{1}{3}$$

$$P(cool|no) = \frac{1 + (1.\frac{1}{3})}{3+1} = \frac{1}{3}$$

$$P(high|no) = \frac{2 + (1.\frac{1}{2})}{3+1} = \frac{5}{8}$$

$$P(strong|no) = \frac{2 + (1.\frac{1}{2})}{3+1} = \frac{5}{8}$$

So,

$$P(no|i_1) = P(sunny|no).P(cool|no).P(high|no).P(strong|no).P(no)$$

$$P(no|i_1) = \frac{1}{3}.\frac{1}{3}.\frac{5}{8}.\frac{5}{8}.\frac{3}{10} = \mathbf{0.01}$$

Since, $P(no|i_1) > P(yes|i_1)$ i_1 is therefore classified as **no**.

For i_2

n_c	**count**
$(overcast|yes)$	3
$(mild|yes)$	3
$(normal|yes)$	6
$(weak|yes)$	4

Table 8: n_c for *yes* class

n_c	**count**
$(overcast|no)$	0
$(mild|no)$	2
$(normal|no)$	1
$(weak|no)$	1

Table 9: n_c for *no* class

$p_{overcast} = \frac{1}{3}, p_{mild} = \frac{1}{3}, p_{normal} = \frac{1}{2}, p_{weak} = \frac{1}{2}, n_{yes} = 7, n_{no} = 3$
Therefore,

$$P(overcast|yes) = \frac{3 + (1.\frac{1}{3})}{7+1} = \frac{5}{12}$$

$$P(mild|yes) = \frac{3 + (1.\frac{1}{3})}{7+1} = \frac{5}{12}$$

$$P(normal|yes) = \frac{6 + (1.\frac{1}{2})}{7+1} = \frac{13}{16}$$

$$P(weak|yes) = \frac{4 + (1.\frac{1}{2})}{7 + 1} = \frac{9}{16}$$

So,

$$P(yes|i_2) = P(overcast|yes).P(mild|yes).P(normal|yes).P(weak|yes).P(yes)$$

$$P(yes|i_2) = \frac{5}{12}.\frac{5}{12}.\frac{13}{16}.\frac{9}{16}.\frac{7}{10} = \mathbf{0.055}$$

Similarly,

$$P(overcast|no) = \frac{0 + (1.\frac{1}{3})}{3 + 1} = \frac{1}{12}$$

$$P(mild|no) = \frac{2 + (1.\frac{1}{3})}{3 + 1} = \frac{7}{12}$$

$$P(normal|no) = \frac{1 + (1.\frac{1}{2})}{3 + 1} = \frac{1}{3}$$

$$P(weak|no) = \frac{1 + (1.\frac{1}{2})}{3 + 1} = \frac{1}{3}$$

So,

$$P(no|i_1) = P(overcast|no).P(mild|no).P(normal|no).P(weak|no).P(no)$$

$$P(no|i_1) = \frac{1}{12}.\frac{7}{12}.\frac{1}{3}.\frac{1}{3}.\frac{3}{10} = \mathbf{1.6 \times 10^{-3}}$$

Since, $P(yes|i_2) > P(no|i_2)$ i_2 is therefore classified as **yes**.