Machine Learning

Assignment 10.4

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Fuzzy-c-Means

points	A	B	C	D	E
x	1	3	5	1	5
y	2	2	2	1	1

Table 1: Data

centroids	**x**	**y**
C_1	2.75	3
C_2	3.25	0

Table 2: Initialization

For $m = 2$, $\quad u_{\alpha i} = \frac{1}{\sum_{\beta=1}^{c}(\frac{d_{i\alpha}^2}{d_{i\beta}^2})} = \frac{d_{i\alpha}^{\frac{2}{1-m}}}{\sum_{\beta=1}^{c} d_{i\beta}^{\frac{2}{1-m}}}$ and $d_{i\alpha}^2 = \sum_{j=1}^{k}(x_{ij} - v_{\alpha j})^2$ by L2 norm.

Iteration-1

points	A	B	C	D	E
$d_{i\alpha}^{-2}$	$\frac{16}{65}$	$\frac{16}{17}$	$\frac{16}{97}$	$\frac{16}{113}$	$\frac{16}{145}$
$d_{i\beta}^{-2}$	$\frac{16}{145}$	$\frac{16}{65}$	$\frac{16}{113}$	$\frac{16}{97}$	$\frac{16}{65}$

Table 3: Distance values

points	A	B	C	D	E
$u_{i\alpha}^1$	$\frac{29}{42}$	$\frac{65}{82}$	$\frac{113}{210}$	$\frac{97}{210}$	$\frac{13}{42}$
$u_{i\alpha}^2$	$\frac{13}{42}$	$\frac{17}{82}$	$\frac{97}{210}$	$\frac{113}{210}$	$\frac{29}{42}$
Cluster	C_1	C_1	C_1	C_2	C_2

Table 4: Membership values

Calculating,

$$J_2^1(U,V) = \sum_{i=1}^{N}\sum_{\alpha=1}^{c} u_{i\alpha}^2 d_{i\alpha}^2$$

$$\therefore J_2^1(U,V) = [(\frac{65}{16})*(\frac{13}{42})^2+(\frac{17}{16})*(\frac{17}{82})^2+(\frac{97}{16})*(\frac{97}{210})^2+(\frac{113}{16})*(\frac{113}{210})^2+(\frac{145}{16})*(\frac{29}{42})^2]$$

$$+$$

$$[(\frac{145}{16})*(\frac{29}{42})^2+(\frac{65}{16})*(\frac{65}{82})^2+(\frac{113}{16})*(\frac{113}{210})^2+(\frac{97}{16})*(\frac{97}{210})^2+(\frac{65}{16})*(\frac{13}{42})^2] = 8.09+10.60 = \mathbf{18.69}$$

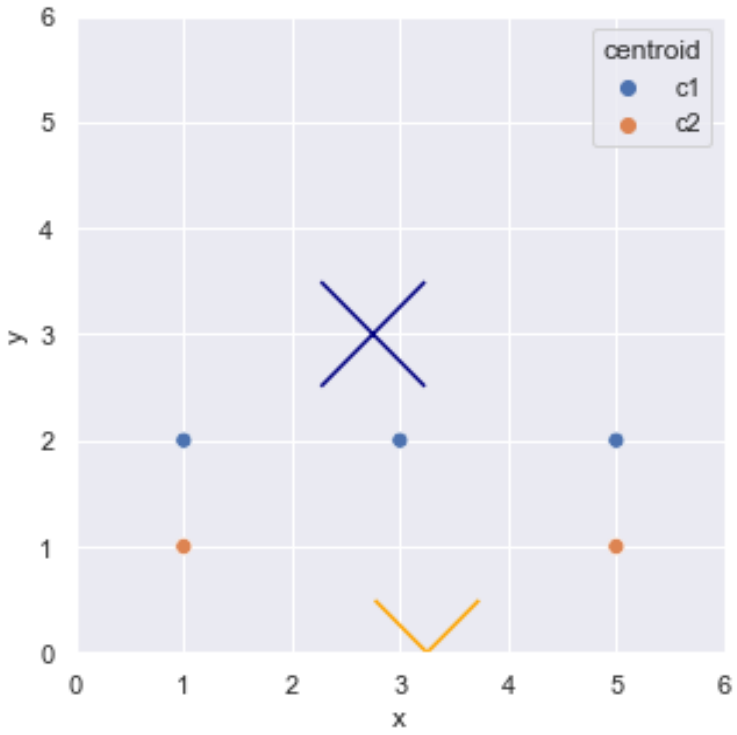

Figure 1: Visualization-1

Now for,

$$v_\alpha = \frac{\sum_{i=1}^{N} u_{\alpha i}^2 . x_i}{\sum_{i=1}^{N} u_{\alpha i}^2}$$

$$C_2^x = \left(\frac{1*(\frac{13}{42})^2 + 3*(\frac{17}{82})^2 + 5*(\frac{97}{210})^2 + 1*(\frac{113}{210})^2 + 5*(\frac{29}{42})^2}{(\frac{13}{42})^2 + (\frac{17}{82})^2 + (\frac{97}{210})^2 + (\frac{113}{210})^2 + (\frac{29}{42})^2}\right)$$

$$C_2^y = \left(\frac{2*(\frac{13}{42})^2 + 2*(\frac{17}{82})^2 + 2*(\frac{97}{210})^2 + 1*(\frac{113}{210})^2 + 1*(\frac{29}{42})^2}{(\frac{13}{42})^2 + (\frac{17}{82})^2 + (\frac{97}{210})^2 + (\frac{113}{210})^2 + (\frac{29}{42})^2}\right)$$

$$\therefore C_2 = \left(\frac{3.96}{1.11}, \frac{1.47}{1.11}\right) = \mathbf{(3.56,\ 1.32)}$$

Similarly ,

$$C_1^x = \left(\frac{1*(\frac{29}{42})^2 + 3*(\frac{65}{82})^2 + 5*(\frac{113}{210})^2 + 1*(\frac{97}{210})^2 + 5*(\frac{13}{42})^2}{(\frac{29}{42})^2 + (\frac{65}{82})^2 + (\frac{113}{210})^2 + (\frac{97}{210})^2 + (\frac{13}{42})^2}\right)$$

$$C_1^y = \left(\frac{2*(\frac{29}{42})^2 + 2*(\frac{65}{82})^2 + 2*(\frac{113}{210})^2 + 1*(\frac{97}{210})^2 + 1*(\frac{13}{42})^2}{(\frac{29}{42})^2 + (\frac{65}{82})^2 + (\frac{113}{210})^2 + (\frac{97}{210})^2 + (\frac{13}{42})^2}\right)$$

$$\therefore C_1 = \left(\frac{4.5}{1.7}, \frac{3.09}{1.7}\right) = \mathbf{(2.64,\ 1.82)}$$

Iteration-2

[1] Calculating,

points	A	B	C	D	E
$d^2_{i\alpha}$	7.01	0.77	2.53	6.65	2.17
$d^2_{i\beta}$	2.72	0.16	5.6	3.36	6.24

Table 5: Distance values

points	A	B	C	D	E
$u^1_{i\alpha}$	$\frac{877}{1217}$	$\frac{388}{469}$	$\frac{317}{1017}$	$\frac{208}{313}$	$\frac{68}{263}$
$u^2_{i\alpha}$	$\frac{340}{1217}$	$\frac{81}{469}$	$\frac{700}{1017}$	$\frac{105}{313}$	$\frac{195}{263}$
Cluster	C_1	C_1	C_2	C_1	C_2

Table 6: Membership values

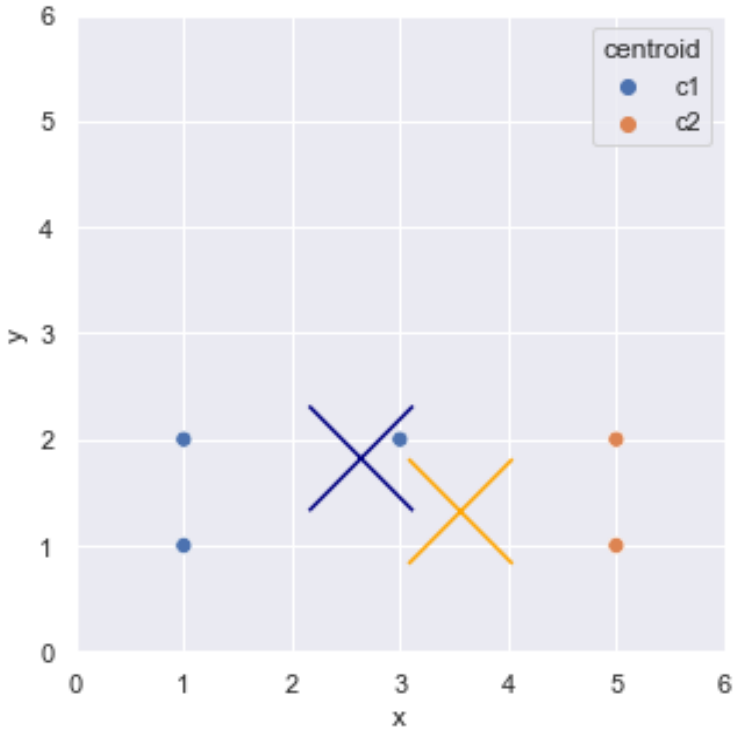

Figure 2: Visualization-2

$$J_2^2(U,V) = 7.49 + 6.67 = \mathbf{14.17}$$

$$C_1 = \mathbf{(2.12,\ 1.72)}, \quad C_2 = \mathbf{(4.33,\ 1.46)}$$

[1] Converges at 10 iterations with a SSE of 2.9912