

# Machine Learning

## Assignment 11.1

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### Learning Vector Quantization (batch)

points	$C_1^{dist}$	$C_2^{dist}$	Cluster
A	<b>2</b>	5.65	$C_1$
B	<b>2.82</b>	4.47	$C_1$
C	4.47	<b>4</b>	$C_2$
D	<b>1</b>	6.4	$C_1$
E	<b>4.12</b>	5	$C_1$

Table 1: Winning Cluster

centroids	x	y
$C_1$	1	0
$C_2$	5	6

Table 2: Initial centers

#### Update rule

$$r^{(new)} = r^{(old)} + \eta \sum_{winner(p)=r^{(old)}} (p - r^{(old)})$$

#### a) Using $\eta = 0.2$ for 2 iterations

##### Iteration-1

$$\begin{aligned} \vec{C}_1^{(new)} &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0.2 \left[ \left( \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) + \left( \begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) + \left( \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) + \left( \begin{bmatrix} 5 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) \right] \\ \therefore \vec{C}_1^{(new)} &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0.2 \left[ \begin{bmatrix} 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 4 \\ 1 \end{bmatrix} \right] \\ \therefore \vec{C}_1^{(new)} &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1.2 \\ 1.2 \end{bmatrix} = \begin{bmatrix} 2.2 \\ 1.2 \end{bmatrix} \end{aligned}$$

Similarly,

$$\vec{C}_2^{(new)} = \begin{bmatrix} 5 \\ 6 \end{bmatrix} + 0.2 \left( \begin{bmatrix} 5 \\ 2 \end{bmatrix} - \begin{bmatrix} 5 \\ 6 \end{bmatrix} \right) = \begin{bmatrix} 5.0 \\ 5.2 \end{bmatrix}$$

**Iteration-2**

points	$C_1^{dist}$	$C_2^{dist}$	Cluster
A	<b>1.44</b>	5.12	$C_1$
B	<b>1.13</b>	3.77	$C_1$
C	<b>2.91</b>	3.2	$C_1$
D	<b>1.21</b>	5.8	$C_1$
E	<b>2.8</b>	4.2	$C_1$

Table 3: Winning Cluster

centroids	x	y
$C_1$	2.2	1.2
$C_2$	5.0	5.2

Table 4: Iteration-1

$$\begin{aligned}
C_1^{r(new)} &= \begin{bmatrix} 2.2 \\ 1.2 \end{bmatrix} + 0.2 \left[ \left( \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 2.2 \\ 1.2 \end{bmatrix} \right) + \left( \begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 2.2 \\ 1.2 \end{bmatrix} \right) + \left( \begin{bmatrix} 5 \\ 2 \end{bmatrix} - \begin{bmatrix} 2.2 \\ 1.2 \end{bmatrix} \right) + \left( \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 2.2 \\ 1.2 \end{bmatrix} \right) + \left( \begin{bmatrix} 5 \\ 1 \end{bmatrix} - \begin{bmatrix} 2.2 \\ 1.2 \end{bmatrix} \right) \right] \\
\therefore C_1^{r(new)} &= \begin{bmatrix} 2.2 \\ 1.2 \end{bmatrix} + 0.2 \left[ \begin{bmatrix} -1.2 \\ 0.8 \end{bmatrix} + \begin{bmatrix} 0.8 \\ 0.8 \end{bmatrix} + \begin{bmatrix} 2.8 \\ 0.8 \end{bmatrix} + \begin{bmatrix} -1.2 \\ -0.2 \end{bmatrix} + \begin{bmatrix} 2.8 \\ -0.2 \end{bmatrix} \right] \\
\therefore C_1^{r(new)} &= \begin{bmatrix} 2.2 \\ 1.2 \end{bmatrix} + 0.2 \begin{bmatrix} 4.0 \\ 2.0 \end{bmatrix} = \begin{bmatrix} 2.2 \\ 1.2 \end{bmatrix} + \begin{bmatrix} 0.8 \\ 0.4 \end{bmatrix} = \begin{bmatrix} \mathbf{3.0} \\ \mathbf{1.6} \end{bmatrix}
\end{aligned}$$

centroids	x	y
$C_1$	3.0	1.6
$C_2$	5.0	5.2

Table 5: Iteration-2

**b) Using  $\eta = 0.5$  for 4 iterations**

centroids	x	y
$C_1$	4.0	3.0
$C_2$	5.0	4.0

Table 6: Iteration-1

centroids	x	y
$C_1$	1.0	1.5
$C_2$	4.0	0.5

Table 8: Iteration-3

centroids	x	y
$C_1$	1.5	-0.5
$C_2$	5.0	4.0

Table 7: Iteration-2

centroids	x	y
$C_1$	1.0	1.5
$C_2$	4.5	2.25

Table 9: Iteration-4

## Observations

- With  $\eta = 0.2$ , the cluster updation process get stuck in local minima.

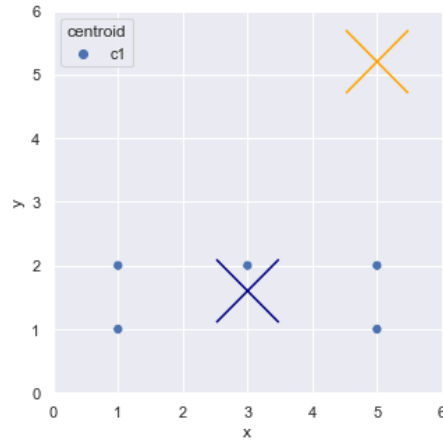


Figure 1: Forms chain-like structures

- With  $\eta = 0.5$ , produces fairly good clustering but requires more iterations to converge.

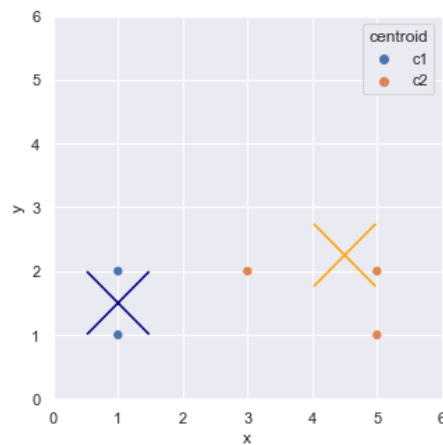


Figure 2: Forms compact clusters

- Use learning rate schedule.
- Use LVQ+neighborhood, so updating will affect nearby points as well.