Machine Learning

Assignment 9.3

Submitted By: Ranji Raj

December 27, 2020

Given:

points	coordinates	class
A	(1;1;1)	1
B	(9;1;2)	1
C	(4;2;1)	1
D	(6;5;4)	2
E	(3;4;3)	2
F	(1;4;4)	2

Table 1: Data

Classification for query point $P(20; 4; 4)$

a) Regular kNN classification (Euclidean distance)

$$d(i,j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2} \quad , for \quad i \neq j$$

distance	value
$d(P, A)$	$\sqrt{379}$
$d(P, B)$	$\sqrt{134}$
$d(P, C)$	$\sqrt{269}$
$d(P, D)$	$\sqrt{197}$
$d(P, E)$	$\sqrt{290}$
$d(P, F)$	$\sqrt{361}$

Table 2: distance calculations

Weight is defined as,

$$w_i = \frac{1}{d(i,j)^2}$$

weight	value
w_A	2.36×10^{-3}
w_B	7.46×10^{-3}
w_C	3.71×10^{-3}
w_D	5.07×10^{-3}
w_E	3.44×10^{-3}
w_F	2.77×10^{-3}

Table 3: weight calculations

$$f(x_1) = \sum_{A,B,C \in 1} = 0.01353$$

$$f(x_2) = \sum_{D,E,F \in 2} = 0.01128$$

Since, $f(x_1) > f(x_2)$ therefore P belongs to class 1 and its nearest neighbor is B.

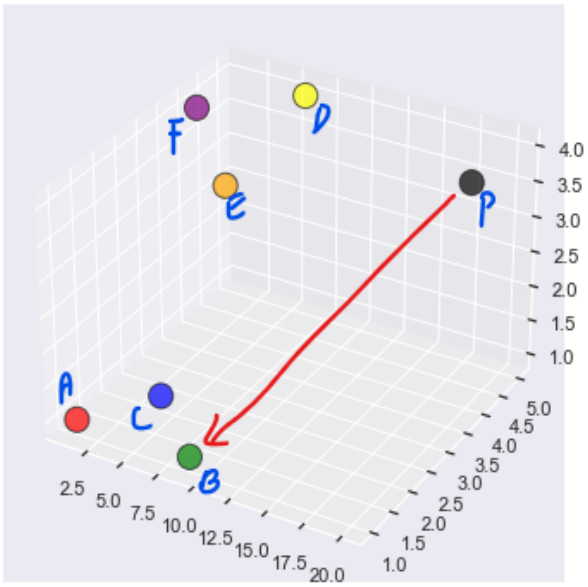

Figure 1: Nearest Neighbour

b) Classification by attribute weighting

Given: $\Delta = 0.5$, $w_x = w_y = w_z = 1$ To each attribute weight is assigned by,

$$d(x, y) = \sqrt{\sum_{i=1}^{n} w_i (x_i - y_i)^2}$$

correct classification	**incorrect classification**
largest distance → decrease	largest distance → increase
tie → decrease	tie → increase
smallest distance → increase	smallest distance → decrease

Table 4: Rule for weight update

distance	**calculation**	**value**
$d(A, B)$	$\sqrt{1(9-1)^2 + 1(1-1)^2 + 1(2-1)^2}$	$\sqrt{65}$
$d(A, C)$	$\sqrt{1(4-1)^2 + 1(2-1)^2 + 1(1-1)^2}$	$\sqrt{10}$
$d(A, D)$	$\sqrt{1(6-1)^2 + 1(5-1)^2 + 1(4-1)^2}$	$5\sqrt{2}$
$d(A, E)$	$\sqrt{1(3-1)^2 + 1(4-1)^2 + 1(3-1)^2}$	$\sqrt{17}$
$d(A, F)$	$\sqrt{1(1-1)^2 + 1(4-1)^2 + 1(4-1)^2}$	$3\sqrt{2}$

Table 5: attribute-weight calculations for A(1;1;1)

From table 5 it can be inferred that point A is **closest** to C. And C belongs to class 1. Thus, **correct** classification. For non-negative weights we take the absolute difference between the dimensions, New weights $w_x = 0.5$, $w_y = 1$, $w_z = 1.5$

comparison	**rule**	**learned weight**
$\lvert a_x - c_x \rvert = 3$	$w_i = w_i - \Delta$	$w_x^{new} = 0.5$
$\lvert a_y - c_y \rvert = 1$	$w_i = w_i$	$w_y^{new} = 1$
$\lvert a_z - c_z \rvert = 0$	$w_i = w_i + \Delta$	$w_z^{new} = 1.5$

Table 6: weight tuning for A

From table 7 it can be inferred that point B is **closest** to C. And C belongs to class 1. Thus, **correct** classification. New weights $w_x = 0$, $w_y = 1.5$, $w_z = 2$
Similarly, for C(4;2;1)

From table 9 it can be inferred that point C is **closest** to A. And A belongs to class 1. Thus, **correct** classification. New weights $w_x = 0$, $w_y = 1.5$, $w_z = 2.5$

distance	**calculation**	**value**
$d(B, A)$	$\sqrt{0.5(9-1)^2 + 1(1-1)^2 + 1.5(2-1)^2}$	5.78
$d(B, C)$	$\sqrt{0.5(4-9)^2 + 1(2-1)^2 + 1.5(1-2)^2}$	3.87
$d(B, D)$	$\sqrt{0.5(6-9)^2 + 1(5-1)^2 + 1.5(4-2)^2}$	5.14
$d(B, E)$	$\sqrt{0.5(3-9)^2 + 1(4-1)^2 + 1.5(3-2)^2}$	5.33
$d(B, F)$	$\sqrt{0.5(1-9)^2 + 1(4-1)^2 + 1.5(4-2)^2}$	6.85

Table 7: attribute-weight calculations for B(9;1;2)

comparison	**rule**	**learned weight**
$\lvert b_x - c_x \rvert = 5$	$w_i = w_i - \Delta$	$w_x^{new} = 0$
$\lvert b_y - c_y \rvert = 1$	$w_i = w_i$	$w_y^{new} = 1.5$
$\lvert b_z - c_z \rvert = 1$	$w_i = w_i$	$w_z^{new} = 2$

Table 8: weight tuning for B

Similarly, for D(6;5;4)

From table 11 it can be inferred that point D is **closest** to F. And F belongs to class 2. Thus, **correct** classification. New weights $w_x = 0$, $w_y = 1.5$, $w_z = 3$
Similarly, for E(3;4;3)

From table 13 it can be inferred that point E is **closest** to F. And F belongs to class 2. Thus, **correct** classification. New weights $w_x = 0$, $w_y = 2$, $w_z = 3$
Similarly, for F(1;4;4)

From table 15 it can be inferred that point F is **closest** to D. And D belongs to class 2. Thus, **correct** classification. New weights $w_x = 0$, $w_y = 2$, $w_z = 3.5$
Similarly, for P(20;4;4)

From table 17 it can be inferred that point P is **closest** to F. And D belongs to class 2. Thus, P is classified as class 2.

distance	calculation	value
$d(C,A)$	$\sqrt{0(4-1)^2+1.5(2-1)^2+2(1-1)^2}$	1.22
$d(C,B)$	$\sqrt{0(4-9)^2+1.5(2-1)^2+2(1-2)^2}$	1.87
$d(C,D)$	$\sqrt{0(4-6)^2+1.5(2-5)^2+2(1-4)^2}$	5.61
$d(C,E)$	$\sqrt{0(4-3)^2+1.5(2-4)^2+2(1-3)^2}$	3.74
$d(C,F)$	$\sqrt{0(4-1)^2+1.5(2-4)^2+2(1-4)^2}$	4.89

Table 9: attribute-weight calculations for C(4;2;1)

comparison	rule	learned weight
$\lvert a_x - c_x\rvert = 3$	$w_i = w_i$	$w_x^{new} = 0$
$\lvert a_y - c_y\rvert = 1$	$w_i = w_i$	$w_y^{new} = 1.5$
$\lvert a_z - c_z\rvert = 0$	$w_i = w_i + \Delta$	$w_z^{new} = 2.5$

Table 10: weight tuning for C

distance	calculation	value
$d(D,A)$	$\sqrt{0(6-1)^2+1.5(5-1)^2+2.5(4-1)^2}$	6.81
$d(D,B)$	$\sqrt{0(6-9)^2+1.5(5-1)^2+2.5(4-2)^2}$	5.83
$d(D,C)$	$\sqrt{0(6-4)^2+1.5(5-2)^2+2.5(4-1)^2}$	6.00
$d(D,E)$	$\sqrt{0(6-3)^2+1.5(5-4)^2+2.5(4-3)^2}$	2.00
$d(D,F)$	$\sqrt{0(6-1)^2+1.5(5-4)^2+2.5(4-4)^2}$	1.22

Table 11: attribute-weight calculations for D(6;5;4)

comparison	rule	learned weight
$\lvert d_x - f_x\rvert = 5$	$w_i = w_i$	$w_x^{new} = 0$
$\lvert d_y - f_y\rvert = 1$	$w_i = w_i$	$w_y^{new} = 1.5$
$\lvert d_z - f_z\rvert = 0$	$w_i = w_i + \Delta$	$w_z^{new} = 3$

Table 12: weight tuning for D

distance	calculation	value
$d(E,A)$	$\sqrt{0(3-1)^2+1.5(4-1)^2+3(3-1)^2}$	5.04
$d(E,B)$	$\sqrt{0(3-9)^2+1.5(4-1)^2+3(3-2)^2}$	4.06
$d(E,C)$	$\sqrt{0(3-4)^2+1.5(4-2)^2+3(3-1)^2}$	4.24
$d(E,D)$	$\sqrt{0(3-6)^2+1.5(4-5)^2+3(3-4)^2}$	2.12
$d(E,F)$	$\sqrt{0(3-1)^2+1.5(4-4)^2+3(3-4)^2}$	1.73

Table 13: attribute-weight calculations for E(3;4;3)

comparison	rule	learned weight
$\lvert e_x - f_x \rvert = 2$	$w_i = w_i$	$w_x^{new} = 0$
$\lvert e_y - f_y \rvert = 0$	$w_i = w_i + \Delta$	$w_y^{new} = 2$
$\lvert e_z - f_z \rvert = 1$	$w_i = w_i$	$w_z^{new} = 3$

Table 14: weight tuning for E

distance	calculation	value
$d(F, A)$	$\sqrt{0(1-1)^2 + 2(4-1)^2 + 3(4-1)^2}$	6.70
$d(F, B)$	$\sqrt{0(1-9)^2 + 2(4-1)^2 + 3(4-2)^2}$	5.47
$d(F, C)$	$\sqrt{0(1-4)^2 + 2(4-2)^2 + 3(4-1)^2}$	5.91
$d(F, D)$	$\sqrt{0(1-6)^2 + 2(4-5)^2 + 3(4-4)^2}$	1.414
$d(F, E)$	$\sqrt{0(1-3)^2 + 2(4-4)^2 + 3(4-3)^2}$	1.73

Table 15: attribute-weight calculations for F(1;4;4)

comparison	rule	learned weight
$\lvert d_x - f_x \rvert = 5$	$w_i = w_i$	$w_x^{new} = 0$
$\lvert d_y - f_y \rvert = 1$	$w_i = w_i$	$w_y^{new} = 2$
$\lvert d_z - f_z \rvert = 0$	$w_i = w_i + \Delta$	$w_z^{new} = 3.5$

Table 16: weight tuning for F

distance	calculation	value
$d(P, A)$	$\sqrt{0(20-1)^2 + 2(4-1)^2 + 3.5(4-1)^2}$	7.03
$d(P, B)$	$\sqrt{0(20-9)^2 + 2(4-1)^2 + 3.5(4-2)^2}$	5.65
$d(P, C)$	$\sqrt{0(20-4)^2 + 2(4-2)^2 + 3.5(4-1)^2}$	6.28
$d(P, D)$	$\sqrt{0(20-6)^2 + 2(4-5)^2 + 3.5(4-4)^2}$	1.414
$d(P, E)$	$\sqrt{0(20-3)^2 + 2(4-4)^2 + 3.5(4-3)^2}$	1.87
$d(P, F)$	$\sqrt{0(20-3)^2 + 4(4-4)^2 + 3.5(4-4)^2}$	0.0

Table 17: attribute-weight calculations for P(20;4;4)