Random Variables

11 July 2020 03:56 PM

Starting @ 4:05 PM

Topic 7

→ Random Variables ←

Random variable → Random Variable is a function which will take Sample Space as input & give you some number

'X' : SS → {0, 1, ---}

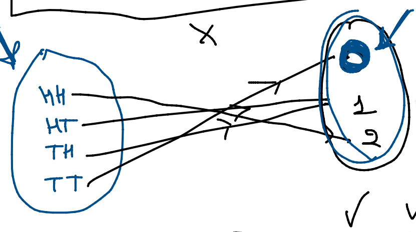
R.E → Tossing two coins

S.S → {HH, TH, HT, TT}

X=2 X=1 X=1 X=0

R.V → 'X' = Count of Heads ← Discrete

$\boxed{X : SS \to \{0,1,2\}}$



$\boxed{S.S}$ = {HH, HT, TH, TT}

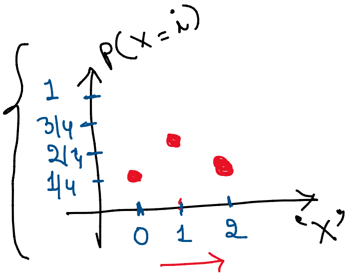
X=2 X=1 X=1 X=0

$P(X=0) = P(\{TT\}) = \frac{1}{4}$

$P(X=1) = P(\{HT, TH\}) = \frac{2}{4}$

$P(X=2) = P(\{HH\}) = \frac{1}{4}$

↓ Distribution of probability of R.V 'X' for



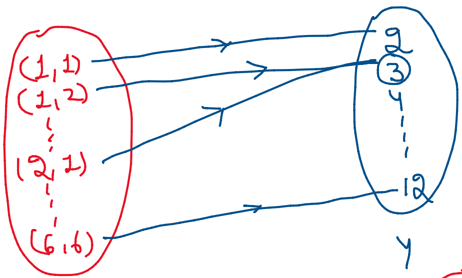
R.E → Rolling two dice

S.S → {(1,1), (1,2) ----(1,6)
(2,1) ------

------ (6,6)}
(6,1)

R.V → 'Y' = Sum the value on both dice

$\boxed{Y : SS \to \{2, 3, --- 12\}}$



$P(Y=2) = P(\{(1,1)\}) = \frac{1}{36}$

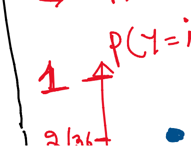
$P(Y=3) = P(\{(1,2),(2,1)\}) = 2/36 = 1/18$

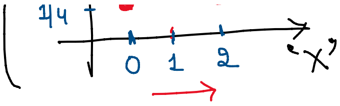
$P(Y=4)$

⋮

$P(Y=12) = P(\{6,6\}) = \frac{1}{36}$

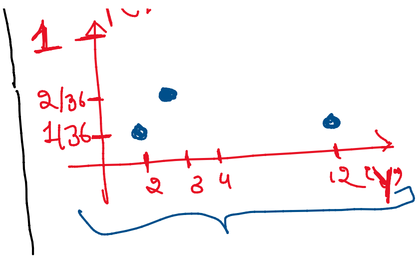
→ Plot for → Prob Dist





[for

$$P(X) = \frac{1}{4} + \frac{2}{4} + \frac{1}{4} = \frac{4}{4} = 1$$



$$\boxed{P(SS) = 1}$$

Step-1 → R.E ⟿ S.S

Step-2 → Define a R.V → X

$X : S.S \to \{x_1, x_2, \dots, x_i\}$

Step-3 → $P(X = x_1)$, $P(X = x_2)$, ⋮, $P(X = x_i)$ } Distribution of probability for Random variable 'X'

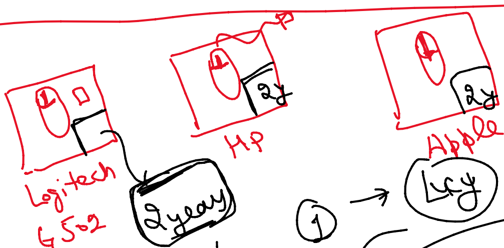
Step-4 → Plot the probability dist. of 'X'

Random Variable
- → Discrete R.V
  - → Bernoulli R.V ✓
  - → Binomial R.V ✓
- → Continuous R.V
  - → Uniform R.V ✓
  - → Normal R.V ✓

$X : SS \to \{\quad\}$
- → Countable set ⇒ Discrete R.V
- → Uncountable set ⇒ Continuous R.V

PMF → Probability Mass function → Discrete R.V

PDF → Probability Density func → Continuous R.V



1 → 1.6y

2years 1day 5hr 30s 15ms 10us 1ns

Logic
4502

2 years

① → Day

2 years 1 day 5 hr 30 s 15 ms 10 us 1 ns

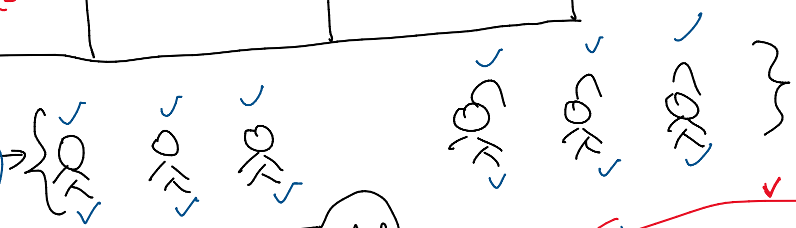
lifetime

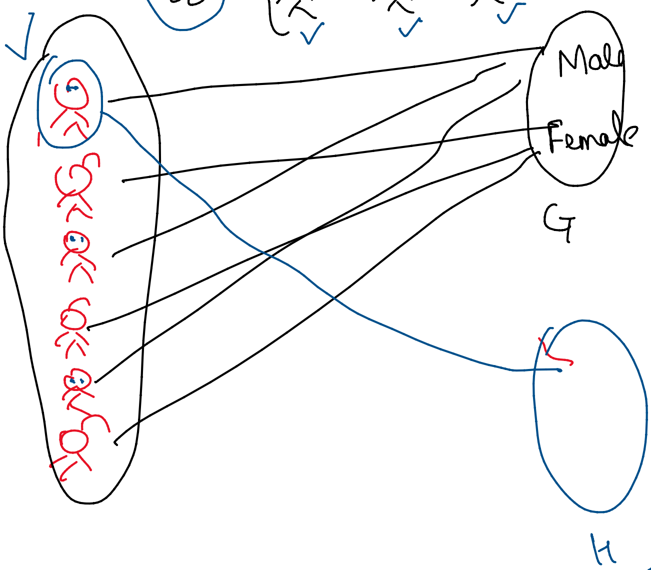
② (2 - 3)

H	W	G
Continuous R.V	Continuous R.V	Discrete R.V

150 - 190 cm

SS →





$G : SS \rightarrow \{M, F\}$

$H : SS \rightarrow [150 - 190]$

S-1 → collecting data → R.E → S.S

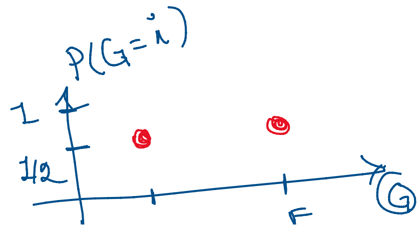
SS → ...

S-2 → $G : SS \rightarrow \{M, F\}$

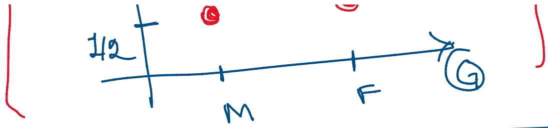
S-3 → $P(G = 'M') = P(\ldots) = \frac{3}{6} = \frac{1}{2}$

$P(G = 'F') = P(\ldots) = \frac{3}{6} = \frac{1}{2}$

S-4



Uniform Distribution



5:00 – 5:05 PM BREAK

→ Discrete R.V

1. Bernoulli R.V ↓

$$\left\{\begin{matrix} 0 & 1 \\ T & F \\ S & F \end{matrix}\right\}$$

$X : SS \to \{\underbrace{0, 1}\}$ → Discrete Set, 2 values

$$\boxed{\begin{aligned} P(X=0) &= p \\ P(X=1) &= 1-p \end{aligned}}$$ → Probability Mass Function

S-1 R.E → Tossing a coin

S.S → {H, T}
H → X=1, T → X=0

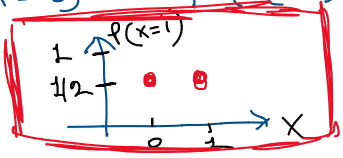
S-2 R.V → X = Getting a Head is Success=1

Bernoulli $X : SS \to \{1, 0\}$ → DISC, → 2 value

S-3 $P(X=1) = P(\{H\}) = \frac{1}{2}$

$P(X=0) = P(\{T\}) = \frac{1}{2}$

S-4



S-1 R.E → Rolling a dice

S.S → {1, 2, 3, 4, 5, 6}
Y=0: 1, 2, 3, 4, 5; Y=1: 6

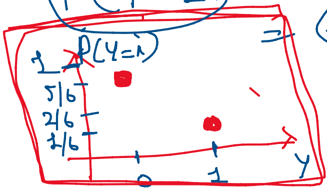
S-2 R.V → Y = Getting a '6' is success

$Y : S.S \to \{0, 1\}$

S-3 $P(Y=0) = P(\{1,2,3,4,5\}) = \frac{5}{6}$

$P(Y=1) = P(\{6\}) = 1/6$

S-4



2. BINOMIAL R.V ↓ Collection of Bernoulli R.V

S-1 R.E → Flip 5 coins

S.S → {HHHHH, HHHHT, ---, HTTTT, TTTTT}

S-1 R.E → Flip 5 coins

→ S.S → { HHHHH , HHHHT , --- HTTTT , TTTTT }

$2^5 = 32$ ✓

S-2

R.V → X = Count no. of Heads

→ X : SS → {0, 1, 2, 3, 4, 5} → Discrete

S-3 R.E → Flipping 5 coins

→ $P(X = 1) = P(\{HTTTT, THTTT, TTHTT, TTTHT, TTTTH\})$

1/32, 1/32, 1/32, 1/32, 1/32

$= \frac{5}{32}$

Getting head

$P(\text{success}) = \frac{1}{2}$

$P(\text{fail}) = \frac{1}{2}$

$P(X=1) = 5/32$

H and T and T and T and T

$p \rightarrow \frac{1}{2} * \frac{1}{2} * \frac{1}{2} * \frac{1}{2} * \frac{1}{2}$

$\left(\frac{1}{2}\right)^1 * \left(\frac{1}{2}\right)^4 \Rightarrow \frac{1}{32} * 5$ ✓

$P(\text{success}) = p$

$P(X=1) \Rightarrow [P(\text{Success})]^1 * [P(\text{fail})]^4 * {}^5C_1$

$$\boxed{P(X=i) = {}^nC_i * p^i * (1-p)^{n-i}}$$

PMF for Binomial R.V

S-1

R.E → Flipping 100 coins

SS → { - , — , - , — — — TTTTTTT } ✓

2^{100}

S-2

R.V → X = Count no. of heads

{0, 1, 2, ... 100}

R.V → X = Count no. of heads

→ X : SS → $\{0, 1, 2, \ldots, 100\}$

S-3

$P(X=0)$

$P(X=1)$

$P(X=100)$

Prob dist

$P(X=60)$

10-20 month

S-4 Plot

$$P(X=60) = {}^{100}C_{60} \left(\frac{1}{2}\right)^{60} * \left(1 - \frac{1}{2}\right)^{100-60}$$

$$^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$\frac{100!}{(60!)\,(40!)}$$

Continuous R.V → Uniform R.V

Normal R.V

Data Analysis

ML → Linear Reg

DL

Numpy

Starting @ 4:10 PM

→ Topic :

- Random Variable } Tuesday
- Numpy ✓
→ - Pandas → } Wed Thur
→ - Matplotlib & Seaborn } Friday
→ - EDA } Sat

Mini-Project

R.V
- → Discrete R.V
  - → Bernoulli R.V
  - → Binomial R.V
- → Cont R.V
  - → Uniform R.V
  - → Normal R.V

3-4 day

MCQ → { Single, Multi }
- → 10 question → 10 marks
- → Maths

Uniform R.V

S-1 R.E
SS

S-2 X : SS → { }

S-3 Prob dist

Binomial

→ { PMF

$$P(X=i) = {}^{n}C_i \, p^i \, (1-p)^{n-i}$$

Bernoulli → PMF →

$P(X=1) = p$
$P(X=0) = 1-p$

Bernoulli → PMF → $P(X=0) = 1-p$

Plot for PMF

S-4

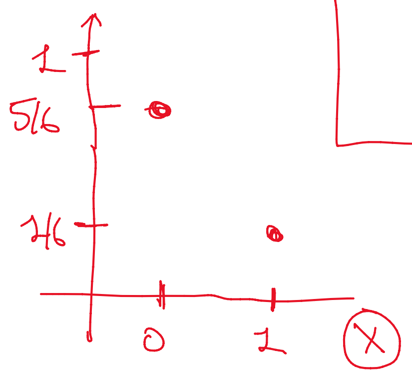
~~Continuous R.V~~ ⇒

UNIFORM R.V ⇒

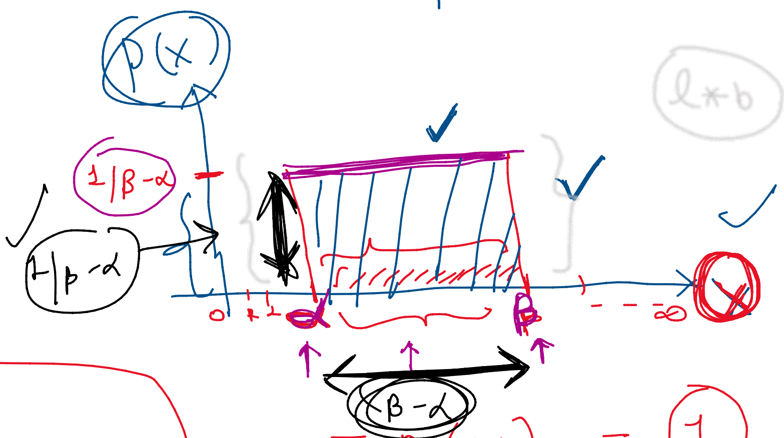
S-3

PDF → $P(X=i) = \begin{cases} \frac{1}{\beta-\alpha}, & \alpha \le i \le \beta \\ 0, & \text{otherwise} \end{cases}$

S-4



$\boxed{\sum P(X = x_i) = 1}$



$\sum P(X) = 1$

$\boxed{\int_{\alpha}^{\beta} \frac{1}{\beta-\alpha} dx} = 1$

L.H.S ⇒

$\frac{1}{\beta-\alpha} \int_{\alpha}^{\beta} dx$

$= \frac{1}{\beta-\alpha} * [x]_{\alpha}^{\beta}$

$= \frac{1}{\cancel{\beta-\alpha}} * \cancel{(\beta-\alpha)}$

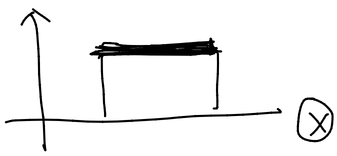
$= 1$

Uniform R.V

PDF

$$P(x=i) = \begin{cases} \frac{1}{\beta-\alpha}, & \alpha \le i \le \beta \\ 0, & \text{otherwise} \end{cases}$$

Plot



Normal R.V

(Height, Weight, marks)

PDF

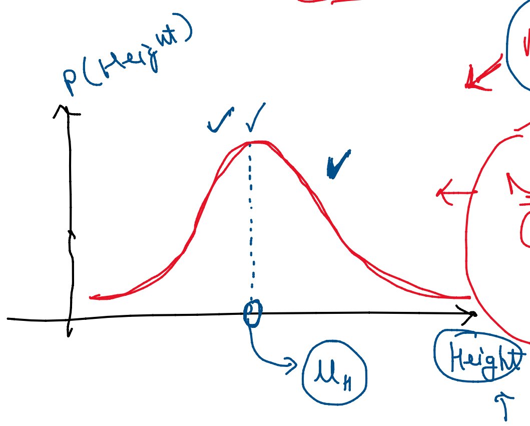
$$P(X=i) = \frac{1}{\sqrt{2\pi}\,\sigma} * e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

mean

variance

std dev

Plot



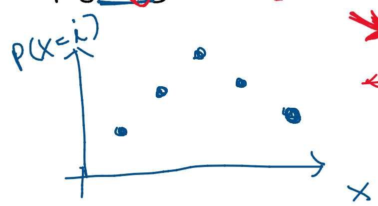
Bell shaped curve

Normal / Gaussian Distribution

Disc **Binomial R.V** PMF

$$P(X=i) = {}^{n}C_{i}\, p^{i} (1-p)^{n-i}$$

follows

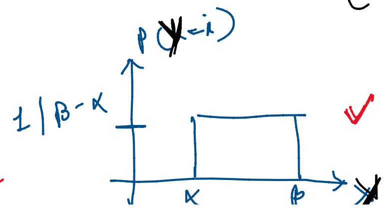


$$\boxed{X \sim B(n, p)}$$

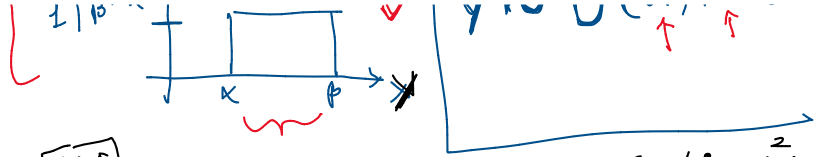
R.V Binomial

Cont **Uniform R.V** PDF

$$P(Y=i) = \begin{cases} \frac{1}{\beta-\alpha}, & (\alpha \le i \le \beta) \\ 0, & \text{otherwise} \end{cases}$$



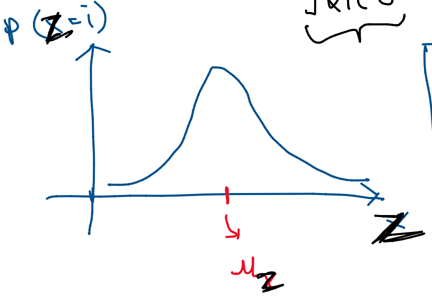
$$Y \sim U(\alpha, \beta)$$



Cont

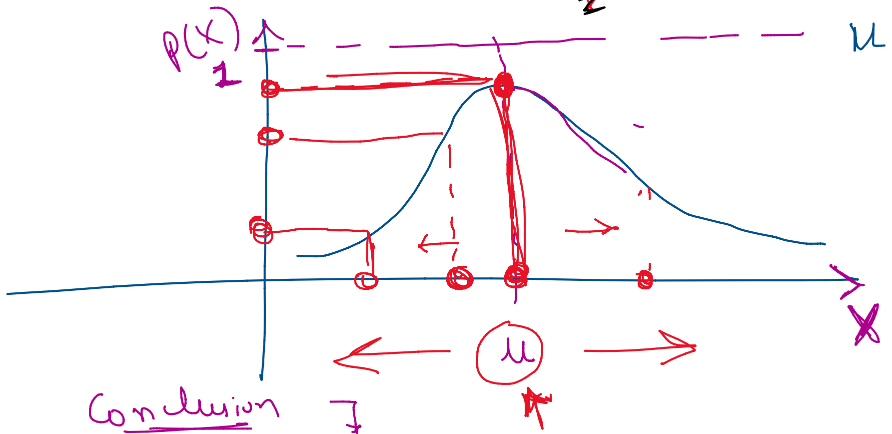
Normal R.V [PDF]

$$P(Z=i) = \frac{1}{\sqrt{2\pi}\,\sigma} * \exp\left\{-\frac{(i-\mu)^2}{2\sigma^2}\right\}$$



$\boxed{Z \sim N(\mu, \sigma^2)}$

μ, σ^2/σ



Conclusion ⟶

→ $\boxed{\text{Symmetric}}$ (Left → 50% } μ)
Right → 50%

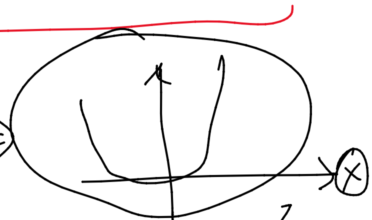
→ As X moves away from μ, P(X) reduce.

Normal R.V (PDF) ⟶ PMF

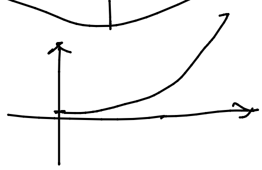
$$\boxed{P(X=i) = \frac{1}{\sqrt{2\pi}\,\sigma} * \exp\left\{-\frac{(i-\mu)^2}{2\sigma^2}\right\}}$$

function

$f(x) = x^2 \Rightarrow$



$f(x) = e^x \Rightarrow$



$f(x) = \log x \Rightarrow$

$$\boxed{f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} * \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}}$$

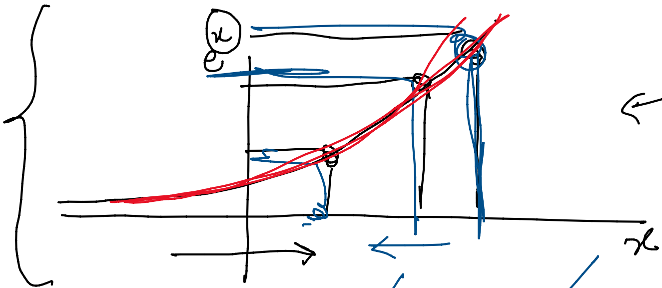
function $\Rightarrow$

$X \sim N(\mu, \sigma^2)$, $\mu \to 0$, $\sigma^2 \to 1$

$$f(x) = \frac{1}{\sqrt{2\pi}} * \exp\left\{-\frac{(x-0)^2}{2}\right\}$$

$$f(x) = \frac{1}{\sqrt{2\pi}} * \exp\left\{-\frac{1}{2}x^2\right\}$$

$$\boxed{f(x) \approx \exp\{-x^2\}} \quad (X \sim N(0, 1))$$



$f(x) = e^x$

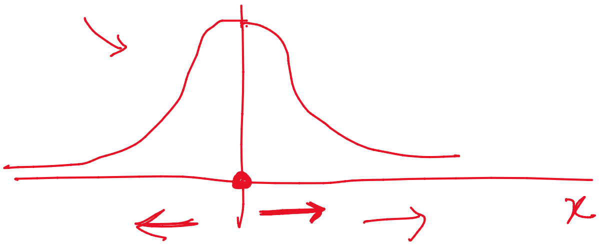
$x\uparrow \Rightarrow e^x\uparrow$

$x\uparrow \Rightarrow e^x\uparrow$

$f(x) = \exp\{-x^2\}$

$x\uparrow \Rightarrow -x^2\downarrow = \exp\{-x^2\}\downarrow$

$x\downarrow \Rightarrow -x^2\downarrow \Rightarrow \exp\{-x^2\}\downarrow$



5:12 — 5:18

BREAK

90% +

70%

80%

90%

Time management

GATE

Time management

IIT

NIT

IIIT

LIFE

2 months

NIT

Career

GF

BF

Iraat

Time managent

→ Family

→ Padhai / Dream

→ Relationships

Time management

ML / DL

→ Learn Something

→ Badminton

→ Team

→ Pubg

→ 8 hrs sleep

→ Development

→ Family

→ Friends

Pubg

School

Clg

Job

Startup

Career

1. System Crashes

2. Chat freeze

1 account → 60k → 50k

1 month

1 year

1 month → 7

2. Chat freeze

3. Bugs (Mute / Unmute)

1 year ✓

1 month

Microsoft Teams

Zoom

1 TB HDD
512 GB SSD
16 GB RAM
2 GB NVIDIA GPU
i7 CPU

~~Gotomeeting~~ → 2/10

~~Webex~~ → CISCO

→ annotation

Google Meet

2 hr

Zoom → 1 year

Zoom → American

Jio Meet ← Beta version

Rondi, OnePlus

Naren
Naitik
Gruksha
Kanav

Jio Meet → Buggy

Sorry

Zoom

Web version → lag

Team → emoji chat

OBS → esports fan
Muted

X OBS → esports fan

youtube → time delay

Monitor

Zoom → youtube → 0 delay

Deadline → easy → 1-2 → Dominos

4 people ✓

2 hr

3 - 10 min

1 - 20-30 min

11:59

100