Probability

→ Probability (Study of uncertainity)

Random Experiment	Sample Space	Event
- It is a process for which outcome cannot be predicted with certainity.	- It is a set of all possible outcome of a 'RANDOM EXPERIMENT'.	- It is a subset of a 'SAMPLE SPACE'.
Example ⇁ R.E → Tossing a coin	Example ⇁ S.S = {H,T}	Example ⇁ Event - Getting a Head {H}

→ Probability →

$$P(\text{Event}) = \frac{\text{No. of favourable Outcomes}}{\text{Total no. of Outcomes}} = \frac{|\text{Event}|}{|\text{Sample Space}|}$$

→ Example-1 :

R.E → Tossing 2 coins

S.S → { HH, HT, TH, TT }

Event ⇁

A : Getting atleast 1 Head

A → { HH, HT, TH }

$P(A) = 3/4$

→ Example-2 :

R.E → Rolling a dice

S.S → {1, 2, 3, 4, 5, 6}

Event ⇁

B : Getting a Prime no.

B → {2, 3, 5}

$P(B) = 3/6$

→ In case of two or more events ⇁

Event A & B

- Marginal Probability i.e $P(A)$ & $P(B)$
- Joint Probability i.e $P(A \cap B)$
- Conditional Probability i.e $P(A|B)$

* Next we will see how to compute these probabilities with the help of 'FREQUENCY TABLE'.

Consider this binary classification data set:

Day	Outlook	Temperature	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No

Frequency Table

For Outlook vs PlayTennis

Joint Freq:

Consider this binary classification data set:

Day	Outlook	Temperature	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

For Outlook vs PlayTennis

Joint Freq:

	Yes	No	
Sunny	2	3	5
Overcast	4	0	4
Rainy	3	2	5
	9	5	

Marginal Frequencies

Marginal Prob →

$P(\text{PlayTennis} = \text{Yes}) = \frac{9}{14}$

$P(\text{PlayTennis} = \text{No}) = \frac{5}{14}$

$P(\text{Outlook} = \text{Sunny}) = \frac{5}{14}$

Joint Prob →

$P(\text{Outlook} = \text{Sunny} \cap \text{PlayTennis} = \text{Yes}) = \frac{2}{14}$

$P(\text{Outlook} = \text{Overcast} \cap \text{PlayTennis} = \text{No}) = \frac{0}{14}$

$P(\text{Outlook} = \text{Rainy} \cap \text{PlayTennis} = \text{No}) = \frac{2}{14}$

Conditional Prob →

$$P(\text{Outlook} = \text{Sunny} \mid \text{PlayTennis} = \text{Yes}) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{2/14}{9/14} = \frac{2}{9}$$

Bayes Theorem →

Accⁿ to Conditional Prob → $P(A|B) = \frac{P(A \cap B)}{P(B)}$ —(1)

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$\because B \cap A = A \cap B$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(B|A)\,P(A) \quad \text{—(2)}$$

Substituting (2) in eqⁿ (1):

Likelyhood

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

$P(A)$ ← Prior

$P(B)$ ← Marginal

Posteriori ↑ $P(A|B)$

Independent Events ⇁

If given events A & B are know to be independent:

$$P(A|B) = P(A)$$
$$P(B|A) = P(B)$$

Similarly:

$$P(A \cap B) = P(A) * P(B)$$