Linear Algebra

06 July 2020 04:01 PM

Starting @ 4:05 PM

Linear Algebra

Matrices

$$A = [\quad]_{m \times n}$$

$m \rightarrow$ no. of rows
$n \rightarrow$ " " cols

Unit Matrix (Identity)

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$$

$$\cancel{I' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}_{2\times 3}}$$

{why?}

1. $m = n \rightarrow$ It should be a square matrix
2. Diagonal should be $= 1$

$$\Rightarrow I_{ij} = \begin{cases} 1, & \text{if } i == j \\ 0, & \text{otherwise} \end{cases}$$

$\rightarrow \quad AI = A = IA$

Matrix Addition

$$\underbrace{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}}_{} + \underbrace{\begin{bmatrix} 5 \\ 6 \end{bmatrix}}_{} = \quad ?$$

$\rightarrow \begin{bmatrix} 1 & 2 \end{bmatrix} \quad + \begin{bmatrix} 5 & 7 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} + \begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix}_{2\times 2} =$$

$$= \begin{bmatrix} 6 & 9 \\ 9 & 12 \end{bmatrix}$$ { Elementwise addition

Matrix Multiplication :

↳ Col of 1st matrix should be same as the row of 2nd matrix.

$$\begin{bmatrix} & A & \end{bmatrix}_{m\times n} * \begin{bmatrix} & B & \end{bmatrix}_{n\times l}$$

$$= \begin{bmatrix} & & \end{bmatrix}_{\underbrace{m * l}}$$

→ Transpose of Matrix :

$$(A_{ij})^T = \underbrace{A_{ji}}$$

$$\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}_{2\times 1} \right)^T = \left(\begin{bmatrix} 1 & 3 \end{bmatrix}_{1\times 2} \right)^T = \begin{bmatrix} 1 \\ 3 \end{bmatrix}_{2\times 1}$$

→ $(A^T)^T = A$

→ $(A_{m\times n} + B_{m\times n})^T = A^T + B^T$

→ $(A * B)^T = \cancel{A^T B^T}$ ×

$B^T * A^T$

$B' * A'$

$$(A_{m\times n})^T = A_{n*m}$$

$$(B_{n\times p})^T = B_{p*n}$$

A B

L.H.S

$$(A_{m*n} * B_{n*p})^T = (B_{n*p})^T * (A_{m*n})^T$$

$$= B_{p*n} * A_{n*m}$$

→ Linear Algebra (why)?

- Matrices ✓
- Coordinate Geometry
- Vector Algebra

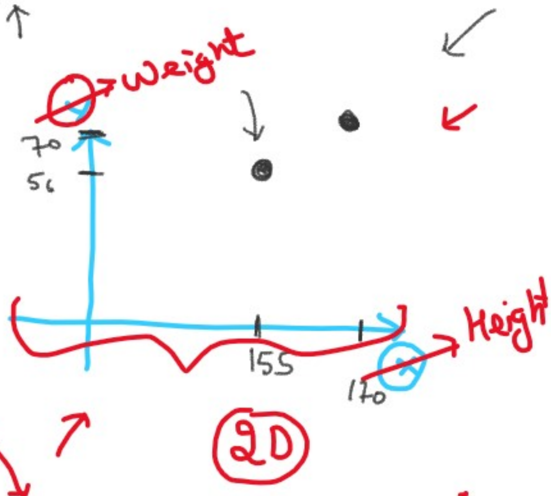
Machine Learning

FN	LN	Age	Gender
Karan	Bansal	—	Male

→ 1 person | 1 Data point

Property | Attributes

	kg	cm
S.No	Weight	Height
1	56	155
2	70	170
3	⋮	⋮
4		



2D

4D

3 columns

3D

W

4 columns → 3D

100 columns

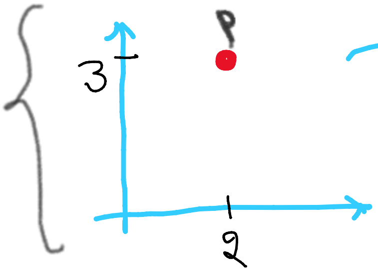
NLP ⇝ 1000's → 60,000, 90,000

Solution

→ Linear Algebra ✓

↳ Viz & operate on high-Dimensional data

→ **Point Representation using a Vector**



① Row Vector

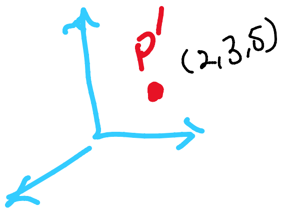
$[2 \quad 3]_{1\times 2}$

Default

② Column Vector

$\begin{bmatrix} 2 \\ 3 \end{bmatrix}_{2\times 1}$

$P = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$



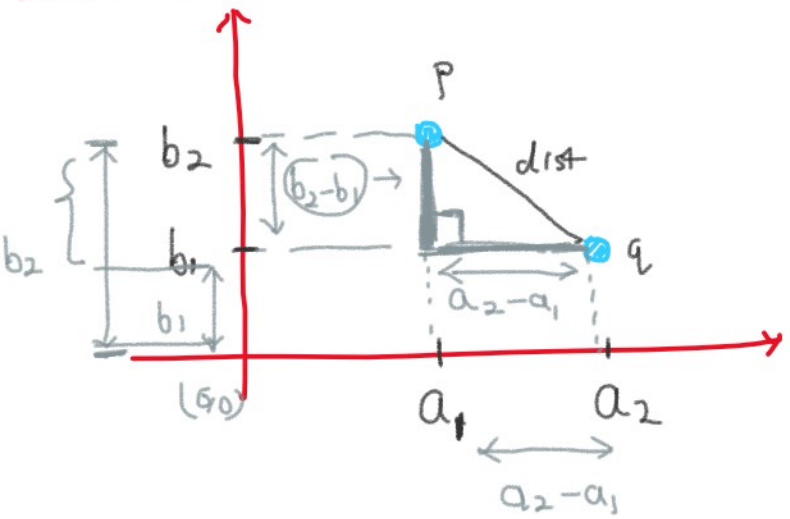
$p' = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$ OR $p' = [2 \quad 3 \quad 5]^{T}$

4D → ?

$p'' = \begin{bmatrix} 2 \\ 3 \\ 5 \\ 7 \end{bmatrix}$ ✓ → 4 Properties

Vector Rep

nD $\rightarrow$ $p''' = \begin{bmatrix} 2 \\ 3 \\ 5 \\ \vdots \end{bmatrix}$ ✓



→ Distance b/w two points (Dissimilarity)

$p = \begin{bmatrix} a_1 \\ b_2 \end{bmatrix}$ $q = \begin{bmatrix} a_2 \\ b_1 \end{bmatrix}$

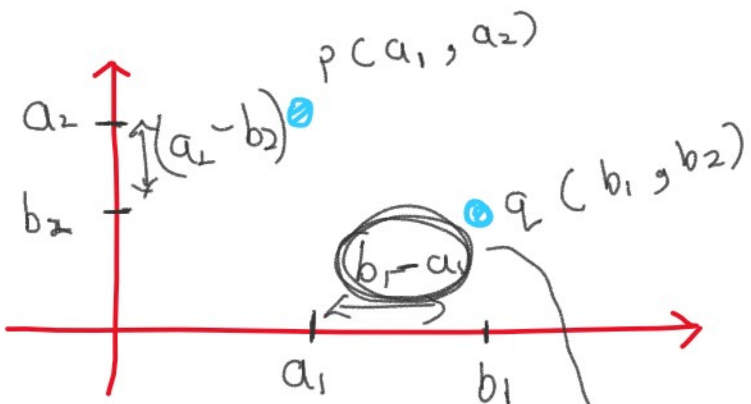
$$dist^2 = (b_2 - b_1)^2 + (a_2 - a_1)^2$$

2-D

$$dist = \sqrt{(b_2 - b_1)^2 + (a_2 - a_1)^2}$$

3D $\rightarrow$ $p = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}$ $q = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$

$$dist = \sqrt{(a_2 - a_1)^2 + (b_2 - b_1)^2 + (c_2 - c_1)^2}$$



$p = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$

$q = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

2D distance $= \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$

2D distance $= \sqrt{(a_1-b_1)^2 + (a_2-b_2)^2}$

3-D $\rightarrow$ $p = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ $q = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

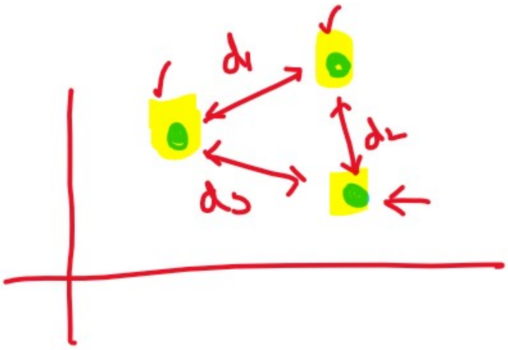
3D dist $= \sqrt{(a_1-b_1)^2 + (a_2-b_2)^2 + (a_3-b_3)^2}$

n-D $\rightarrow$ $p = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix}$ $q = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$

$$\text{dist} = \sqrt{(a_1-b_1)^2 + (a_2-b_2)^2 + \cdots + (a_n-b_n)^2}$$

n-D $\rightarrow$

$$\text{dist} = \sqrt{\sum_{i=1}^{n} (a_i - b_i)^2}$$

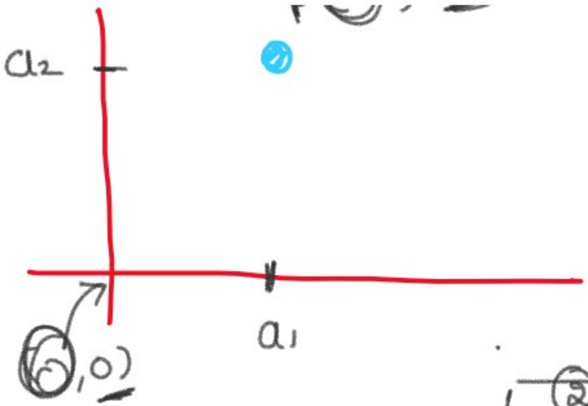


$d_1 < d_3 < d_2$ ✓

→ Distance of point from origin ⇁

$p(a_1, a_2)$

2D



2D

$$d = \sqrt{\sum_{i=1}^{2} (a_i - b_i)^2}$$

$$d = \sqrt{(a_1 - 0)^2 + (a_2 - 0)^2}$$

2-D $\quad d = \sqrt{a_1^2 + a_2^2}$

3-D $\quad P = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \qquad \boxed{d = \sqrt{\sum_{i=1}^{3} (a_i)^2}}$

5 min → 5:22 to 5:28

Distance → Dissimilarity b/w two pts ✓

Multiplication of two vectors]
↳ Dot Product

→ $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix}_{n \times 1}$ → $b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}_{n \times 1}$

Dot Prod]

$$\boxed{a \cdot b} = a_1 * b_1 + a_2 * b_2 + \cdots + a_n * b_n$$ ✓

$$\boxed{a \cdot b} = a_1 \cdot b_1 \quad \ldots$$

$$= \begin{bmatrix} a_1 & a_2 & a_3 & \cdots & a_n \end{bmatrix}_{1 \times n} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}_{n \times 1}$$

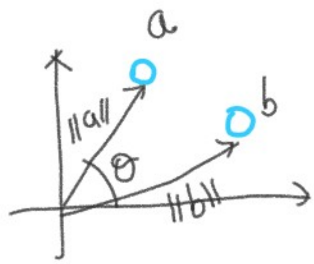
$$= \boxed{a^T * b}$$

$$a \cdot b = a^T * b$$

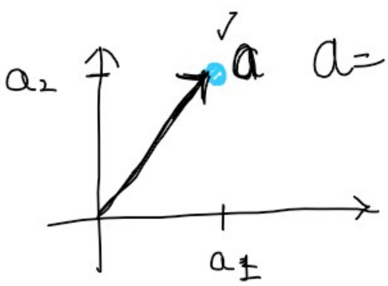
$$= a_1 * b_1 + a_2 * b_3 + \cdots + a_n * b_n$$

$$= \sum_{i=1}^{n} a_i * b_i$$

$$= \|a\| \, \|b\| \cos\theta_{a,b}$$



$$= \sqrt{a_1^2 + a_2^2} * \sqrt{b_1^2 + b_2^2} * \cos\theta_{a,b}$$



$$a = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$d = \sqrt{a_1^2 + a_2^2}$$

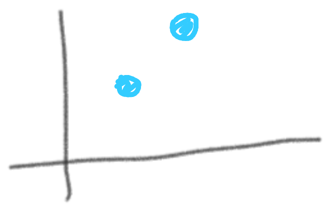
$$\underbrace{\|a\|}_{} = \sqrt{a_1^2 + a_2^2}$$

↳ Magnitude / Length

$$\boxed{a \cdot b} = a^T * b = \underbrace{\sum_{i=1}^{n} a_i * b_i}_{} = \boxed{\|a\| \, \|b\| \cos\theta_{a,b}}$$

Can you find the angle b/w two given points?

$$a = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$



$$\hookrightarrow a \cdot b = \sqrt{1^2+1^2}\,\sqrt{2^2+2^2}\,\cos\theta$$

$$\boxed{a \cdot b = \|a\|\,\|b\|\cos\theta}$$

$$1*2 + 1*2 = \sqrt{2}\,\sqrt{8}\,\cos\theta$$

$$4 = \sqrt{2}\,\sqrt{8}\,\cos\theta$$

$$\cos\theta = \frac{4}{\sqrt{2}\,\sqrt{8}} \Rightarrow \theta = \cos^{-1}\left\{\frac{4}{\sqrt{2}\sqrt{8}}\right\}$$

$$\Rightarrow \theta = \cos^{-1}\left\{\frac{4}{\sqrt{2}\;2\sqrt{2}}\right\}$$

$$\theta = \cos^{-1}\{1\}$$

$$\theta = 0^\circ$$

Angle b/w two Points

$$\|a\|\,\|b\|\cos\theta = a \cdot b \qquad \sum_{i=1}^{n} a_i b_i$$

$$\cos\theta = \frac{a \cdot b}{\|a\|\,\|b\|} \checkmark$$

$$\theta = \cos^{-1}\left\{\frac{\sum_{i=1}^{n} a_i b_i}{\|a\|\,\|b\|}\right\} \checkmark$$

Lets say $\vec{a} \cdot \vec{b} = 0$

angle b/w $\vec{a}$ & $\vec{b}$?

$$\cos^{-1}\{0\}$$

angle b/w a & b

$$\theta = \cos^{-1}\left\{\frac{a \cdot b}{\|a\| \ \|b\|}\right\} = \cos^{-1}\{0\}$$

$\theta = 90^\circ$

$$\vec{a} \cdot \vec{b} = \|a\| \ \|b\| \cos\theta$$

$$0 = \underset{+ve}{\|a\|} \ \underset{+ve}{\|b\|} \ \underset{0}{\cos\theta}$$

$$\left\{\begin{array}{l} \cos\theta = 0 \Rightarrow \theta = 90^\circ \\ \cos 90^\circ = 0 \end{array}\right.$$

$$\left\{\cos\theta = \frac{Adj}{Hyp}\right\}$$

$$\vec{a} \cdot \vec{b} = 0$$

$\boxed{\theta = 90^\circ}$ → Orthogonal

Starting @ 4:10 PM

→ Linear Algebra Cont...

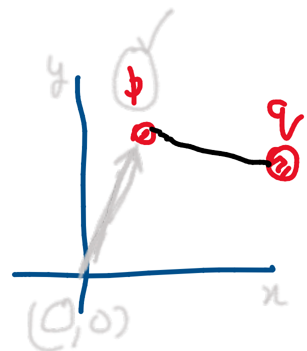
For $\boxed{5 *}$ → 90% attendance
→ No late Submissions
→ 85% score :-

For $\boxed{4*}$ → 80% attendance
→ 1 late submission (allowed)
→ 70% score

→ Matrices

→ Vectors → (Point Represent)

→ Distance (Dissimilarity)



$$d_{pq} = \sqrt{\sum_{i=1}^{n} (a_i - b_i)^2}$$

Euclidean Distance

→ Magnitude of a vector (Distance of a point from origin)

$$\|p\| = \sqrt{\sum_{i=1}^{n} (a_i)^2}$$

→ Dot Product

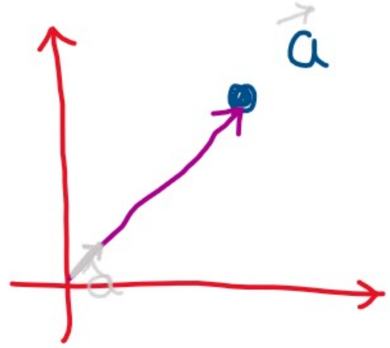
$$\vec{a} \cdot \vec{b} = a^T * b = \sum_{i=1}^{n} a_i * b_i$$

$$= \boxed{\|\vec{a}\| * \|\vec{b}\| * \cos\theta_{a,b}}$$

→ $\vec{a} \cdot \vec{b} = 0$ iff $\theta_{a,b} = 90°$

→ Unit Vector]

$$\hat{a} = \frac{\vec{a}}{\| a \|}$$

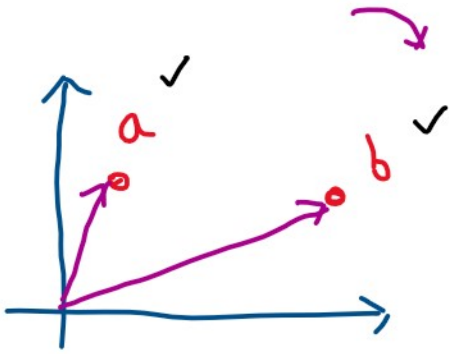


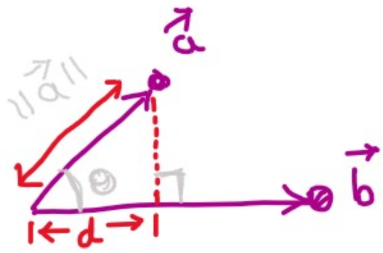
→ $\|\hat{a}\| = 1$

→ $\hat{a}$ is in same direction of $\vec{a}$

→ PROJECTION]

{ d → length of projection





$$\cos\theta = \frac{Adj}{Hyp} = \frac{d}{\|\vec{a}\|}$$

a = 10

$$a = \frac{2 * 10}{2}$$

$$d = \|\vec{a}\| \cos\theta_{a,b}$$

→ Mult & divide $\|\vec{b}\|$

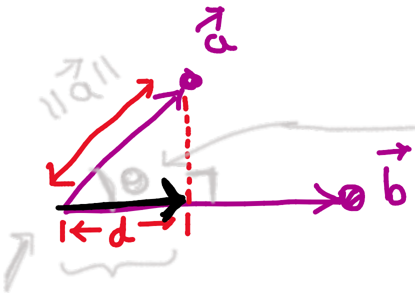
$$\left\{ d = \frac{\|\vec{a}\| \, \|\vec{b}\| * \cos\theta_{a,b}}{\|\vec{b}\|} \right.$$

$\hat{b}$

$$\left\{ d = \frac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|} \right.$$

$\vec{a}$

Projection of $\vec{a}$ on $\vec{b}$



→ direction
→ magnitude

Projection of $\vec{a}$ on $\vec{b}$

$$\text{Proj}_{\vec{b}}\,\vec{a} = \text{mag} * \text{direction}$$

$$= d * \hat{b}$$

$$\boxed{\text{Proj}_{\vec{b}}\,\vec{a} = \frac{\vec{a}\cdot\vec{b}}{\|\vec{b}\|} * \frac{\vec{b}}{\|\vec{b}\|}}$$