ECEN620: Network Theory
Broadband Circuit Design
Fall 2025

Lecture 7: Voltage-Controlled Oscillators

Sam Palermo
Analog & Mixed-Signal Center
Texas A&M University

Announcements

- HW2 due Oct 7
  - Requires transistor-level design
  - For 90nm CMOS device models, see https://people.engr.tamu.edu/spalermo/ecen689/cadence_90nm_2024.pdf
  - Can use other technology models if they are a 90nm or more advanced CMOS node

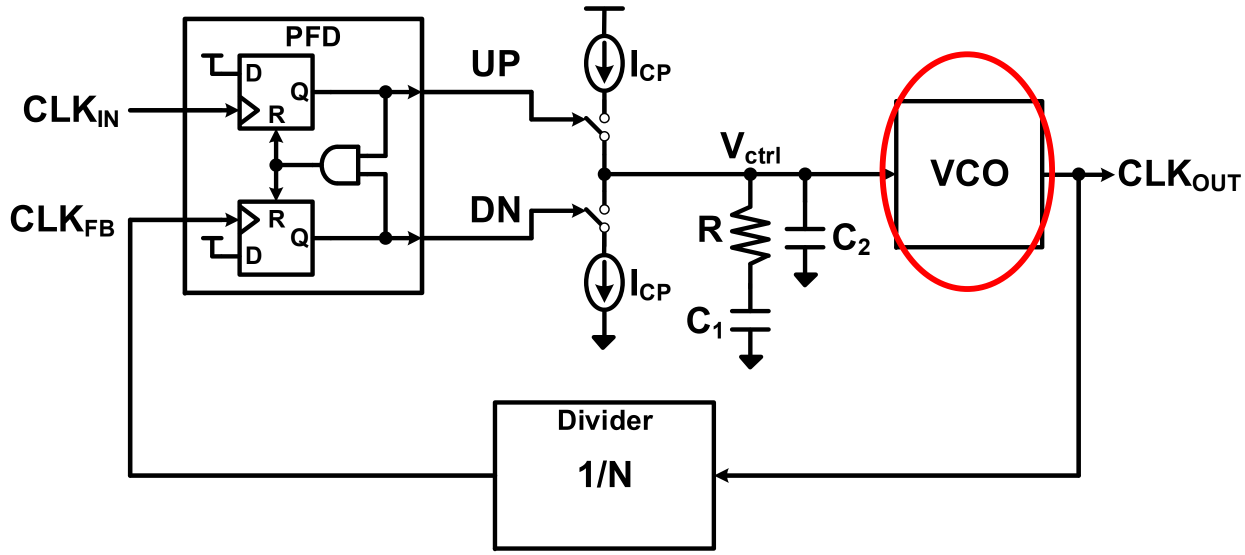
- Exam 1 Thursday Oct 9
  - One double-sided 8.5x11 notes page allowed
  - Bring your calculator
  - Covers through Lecture 7

Agenda

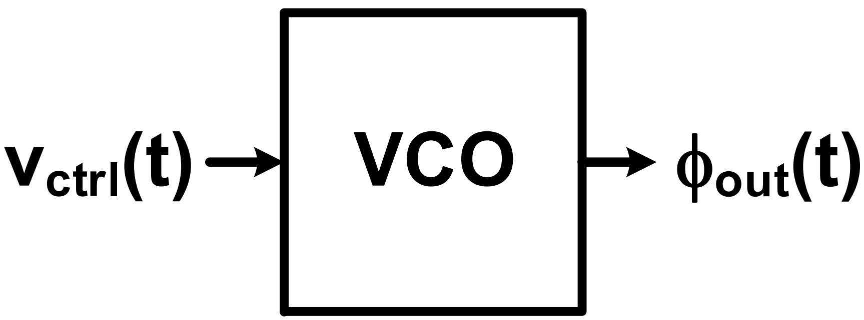
- VCO Fundamentals
- VCO Examples
- VCO Phase Noise
  - Phase Noise Definition and Impact
  - Ideal Oscillator Phase Noise
  - Leeson Model
  - Hajimiri Model
  - LC-VCO Phase Noise Sources
- VCO Jitter

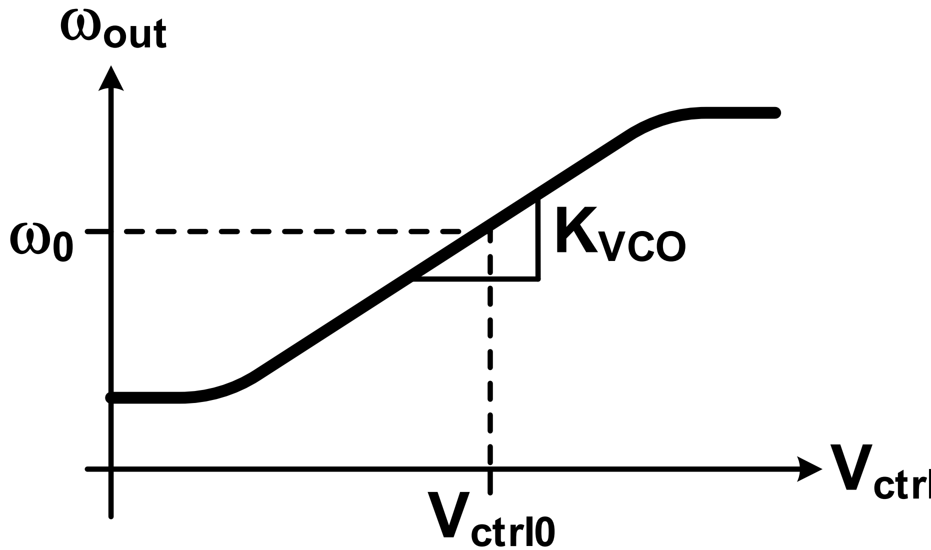
Analog Charge-Pump PLL Circuits

- Phase Detector
- Charge-Pump
- Loop Filter
- VCO
- Divider



Voltage-Controlled Oscillator





$$\omega_{out}(t) = \omega_0 + \Delta\omega_{out}(t) = \omega_0 + K_{VCO} v_{ctrl}(t)$$

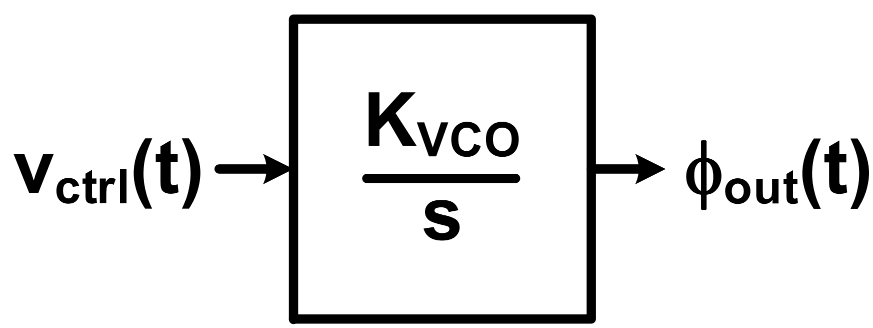
- Time-domain phase relationship

$$\phi_{out}(t) = \int \Delta\omega_{out}(t)dt = \int K_{VCO} v_{ctrl}(t)dt$$

$$K_{VCO} \text{ units are } \frac{\text{rad}}{\text{s} \cdot \text{V}}$$

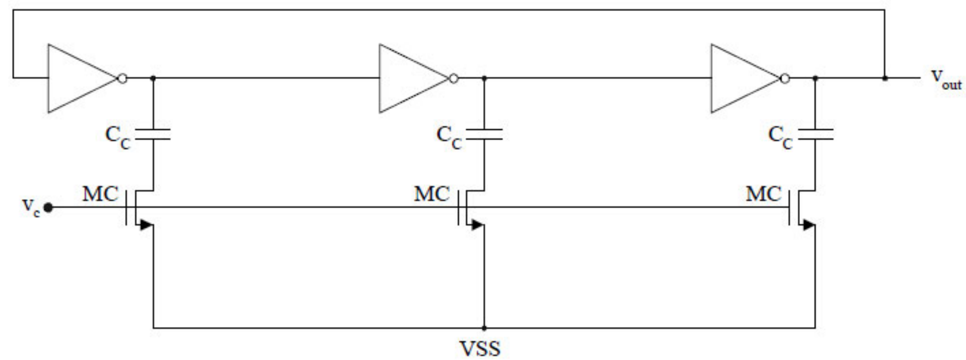
$$\frac{2\pi(1\text{MHz})}{\text{V}} = 2\pi \times 10^6 \frac{\text{rad}}{\text{s} \cdot \text{V}}$$

Laplace Domain Model

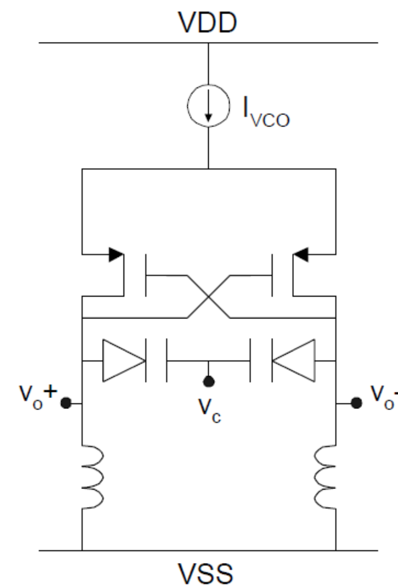


Voltage-Controlled Oscillators (VCO)

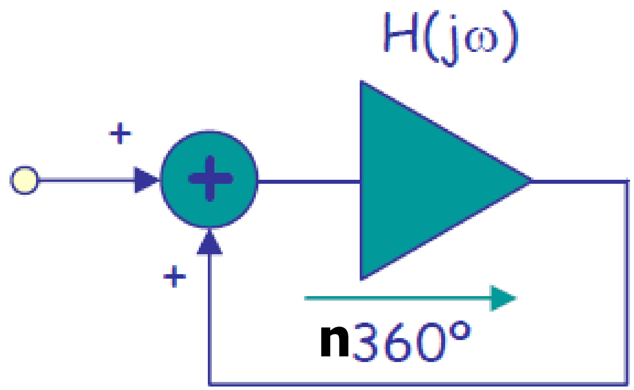
- Ring Oscillator
  - Easy to integrate
  - Wide tuning range (5x)
  - Higher phase noise



- LC Oscillator
  - Large area
  - Narrow tuning range (20-30%)
  - Lower phase noise



Barkhausen's Oscillation Criteria



[**Sanchez**]

Closed-loop transfer function: $$\frac{H(j\omega)}{1-H(j\omega)}$$

- Sustained oscillation occurs if $H(j\omega)=1$
- 2 conditions:
  - Gain = 1 at oscillation frequency ω_0
  - Total phase shift around loop is n360° at oscillation frequency ω_0

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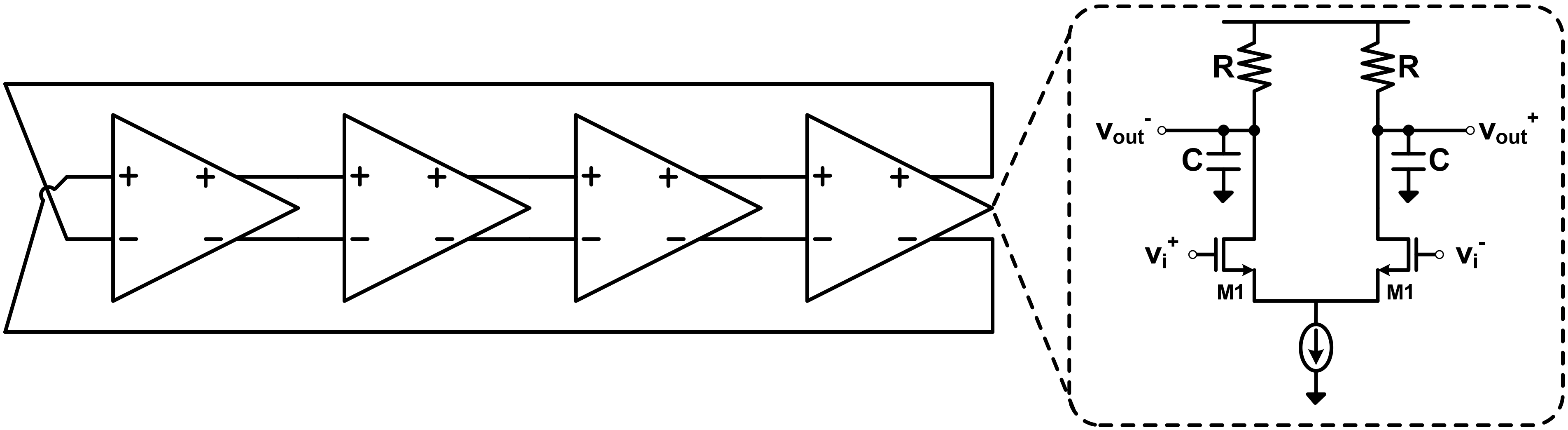
Ring Oscillator Example

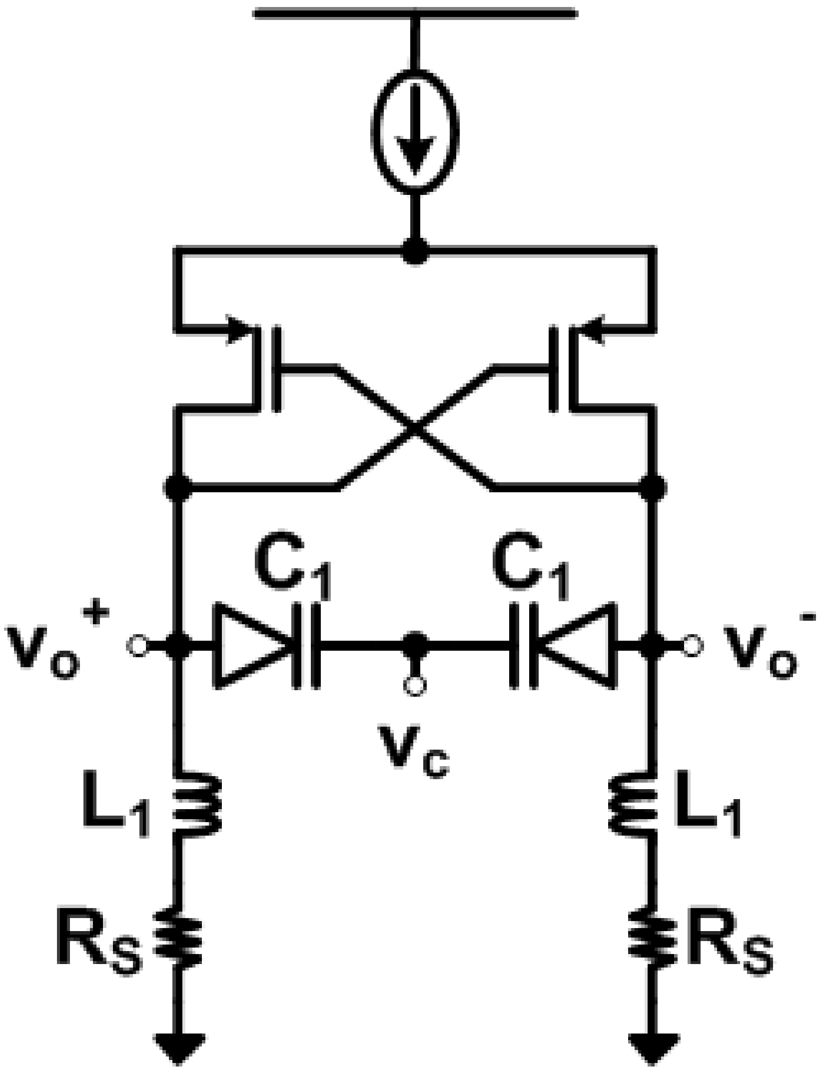


$$H(s) = -\frac{A_0^4}{\left(1+\frac{s}{\omega_o}\right)^4}$$

Phase Condition: $\tan^{-1}\left(\frac{\omega_{osc}}{\omega_o}\right) = 45° \rightarrow \boxed{\omega_{osc} = \omega_o = \frac{1}{RC}}$

Gain Condition: $\frac{A_0^4}{\left[\sqrt{1+\left(\frac{\omega_{osc}}{\omega_o}\right)^2}\right]^4} = 1 \rightarrow \boxed{A_0 = \sqrt{2} = g_{m1}R}$

LC Oscillator Example



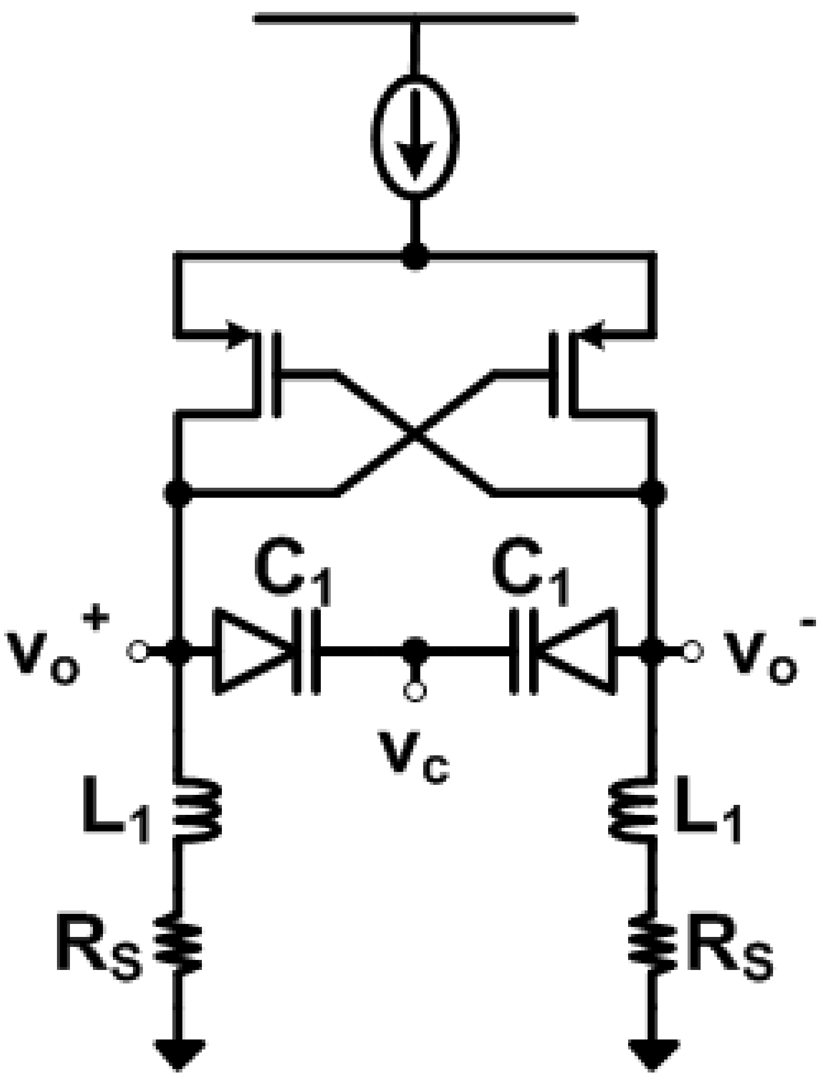
- Oscillation phase shift condition satisfied at the frequency when the LC (and R) tank load displays a purely real impedance, i.e. 0° phase shift

LC tank impedance

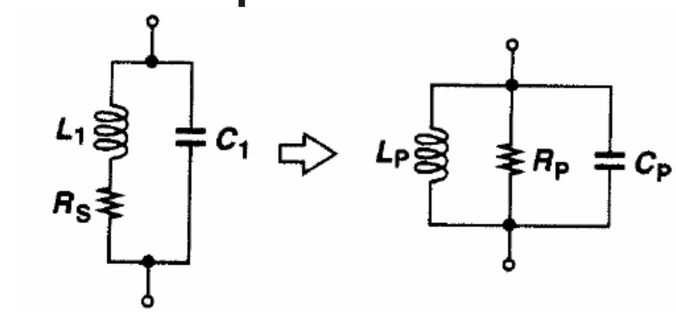
$$Z_{eq}(s) = \frac{R_S + L_1 s}{1 + L_1 C_1 s^2 + R_S C_1 s}$$

$$\left|Z_{eq}(s = j\omega)\right|^2 = \frac{R_S^2 + L_1^2 \omega^2}{\left(1 - L_1 C_1 \omega^2\right)^2 + R_S^2 C_1^2 \omega^2}$$

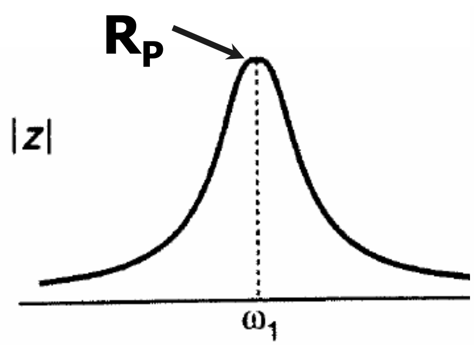
LC Oscillator Example

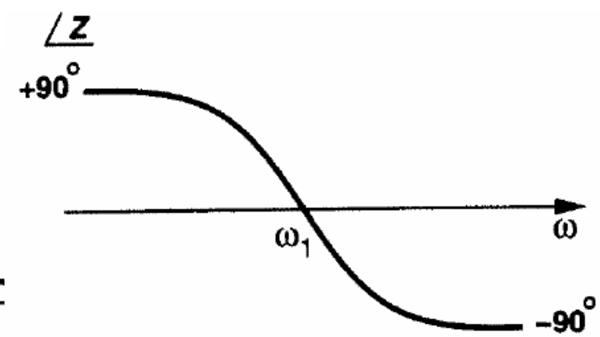


- Transforming the series loss resistor of the inductor to an equivalent parallel resistance



$$L_P = L_1\left(1+\frac{R_S^2}{L_1^2\omega^2}\right),\quad C_P = C_1,\quad R_P \approx \frac{L_1^2\omega^2}{R_S}$$

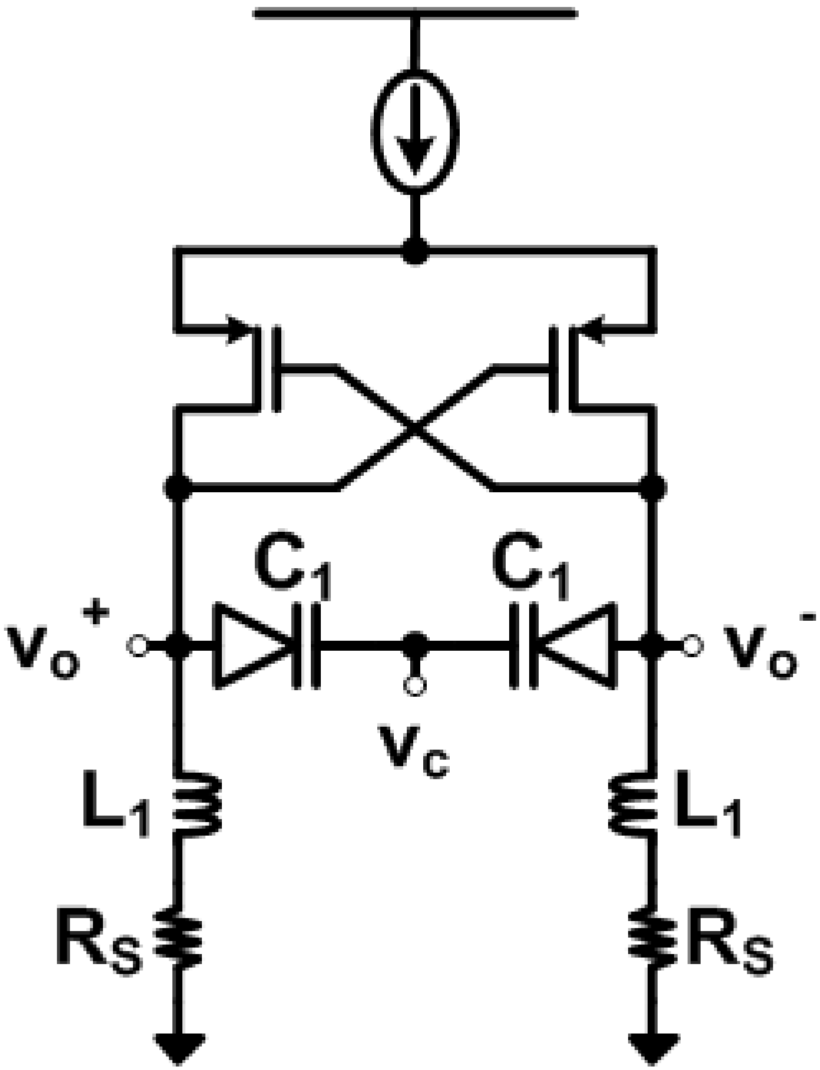




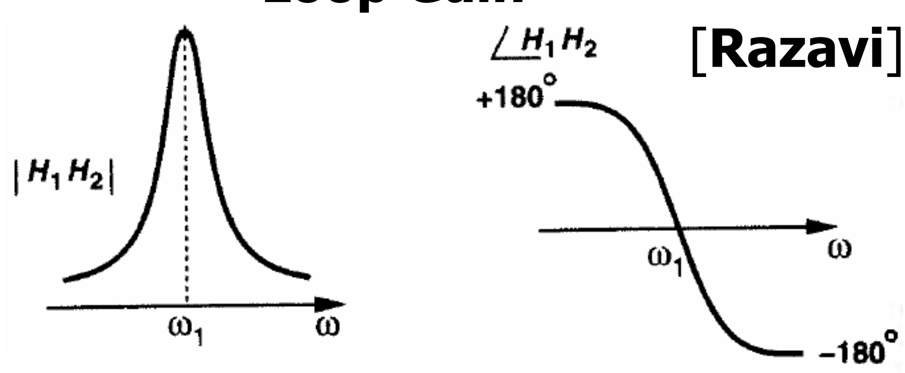
$$\omega_1 = \frac{1}{\sqrt{L_P C_P}}$$

[Razavi]

LC Oscillator Example



Loop Gain



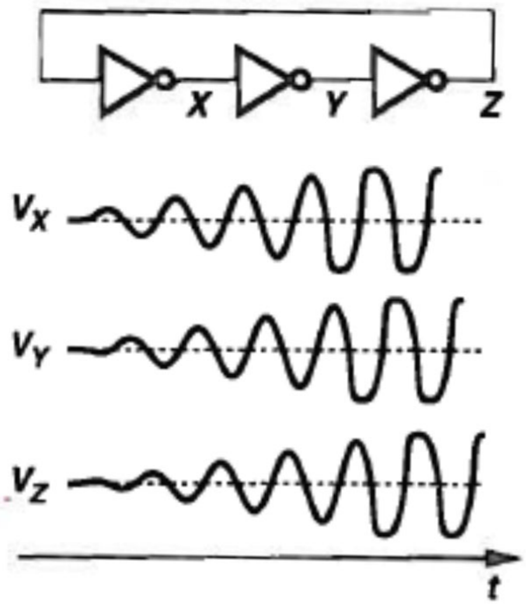
[Razavi]

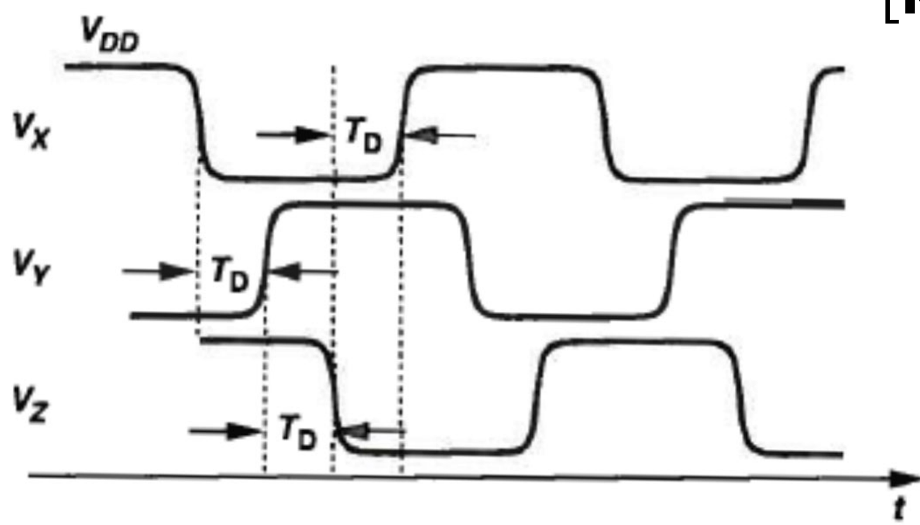
- Phase condition satisfied at $\omega_1 = \dfrac{1}{\sqrt{L_P C_P}}$
- Gain condition satisfied when $(g_m R_P)^2 \geq 1$
- Can also view this circuit as a parallel combination of a tank with loss resistance $2R_P$ and negative resistance of $2/g_m$
- Oscillation is satisfied when

$$\frac{1}{g_m} \leq R_P$$

CMOS Inverter Ring Oscillator

[**Razavi**]

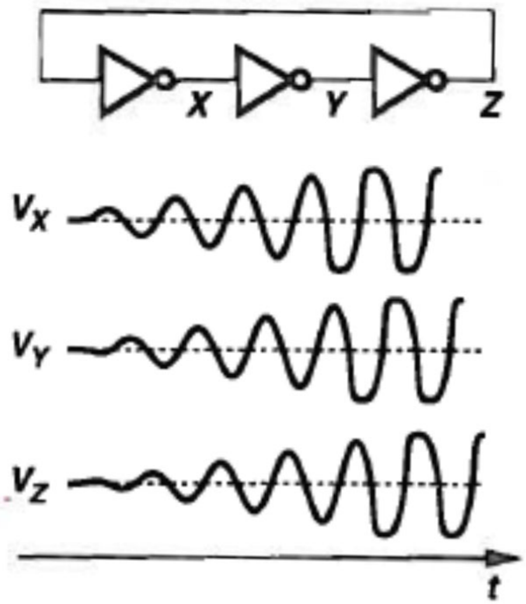


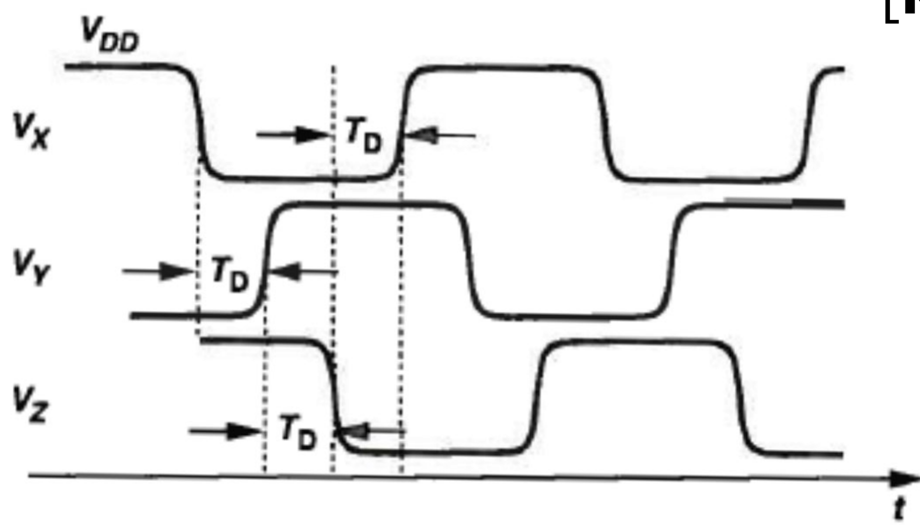


- Noise in the system will initiate oscillation, with the signals eventually exhibiting rail-to-rail swings
- While the small-signal transistor parameters (g_m, g_o, C_g, etc…) can be used to predict the initial oscillations during small-signal start-up, these parameters can vary dramatically during large-signal operation

CMOS Inverter Ring Oscillator

[**Razavi**]

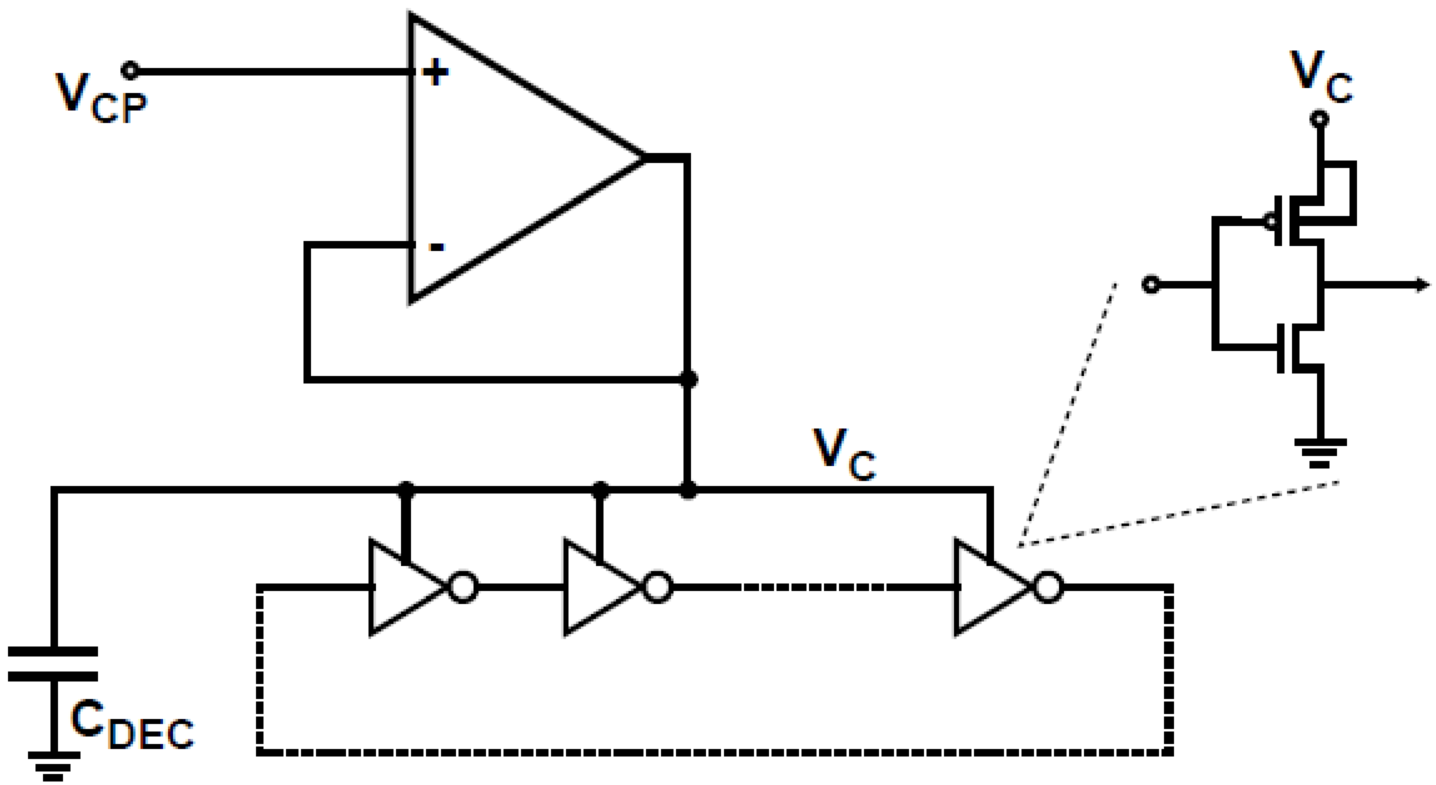




- For this large-signal oscillator, the frequency is set by the stage delay, T_D
- T_D is a function of the nonlinear current drive and capacitances of each stage
- As an "edge" has to propagate twice around the loop

$$f_{osc} = \frac{1}{6T_D}, \text{ or } \frac{1}{2NT_D} \text{ where N is the oscillator stage number}$$

Supply-Tuned Ring Oscillator

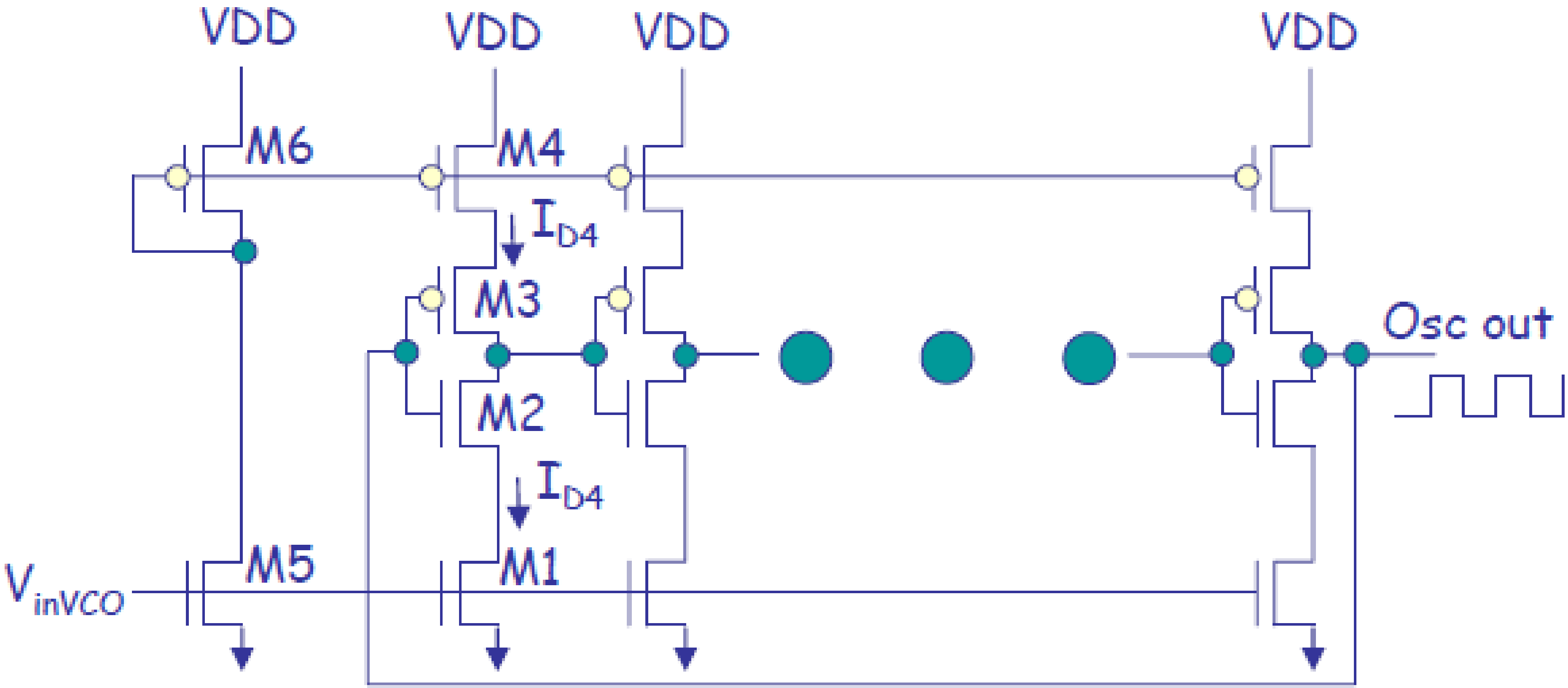


[**Sidiropoulos VLSI 2000**]

$$T_{VCO} = 2nT_D \approx \frac{2nC_{stage}}{\beta\left(V_c - V_{th}\right)}$$

$$K_{VCO} = \frac{\partial f_{VCO}}{\partial V_c} = \frac{\beta}{2nC_{stage}}$$

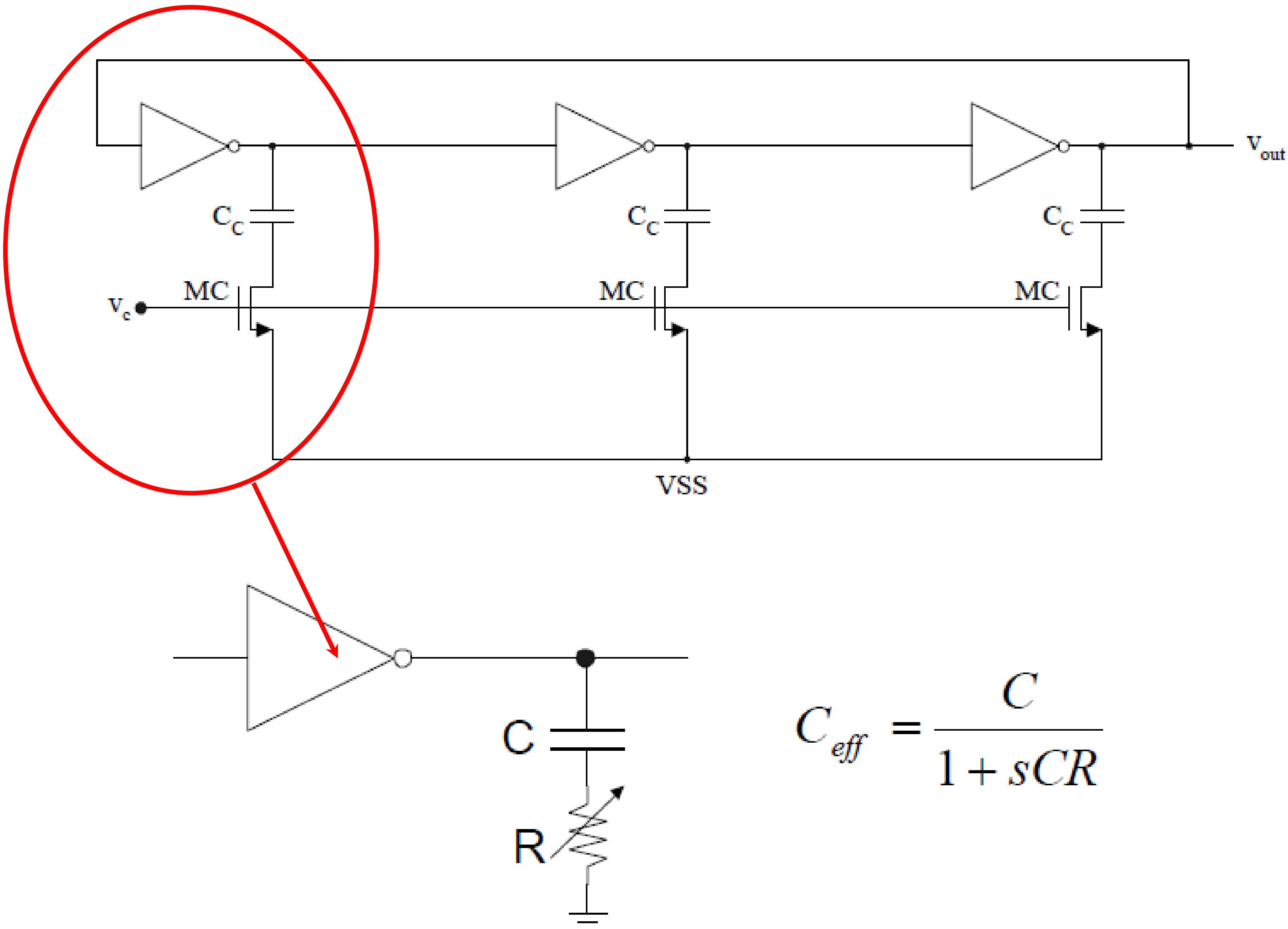
Current-Starved Ring Oscillator



[**Sanchez**]

Current - starved VCO.

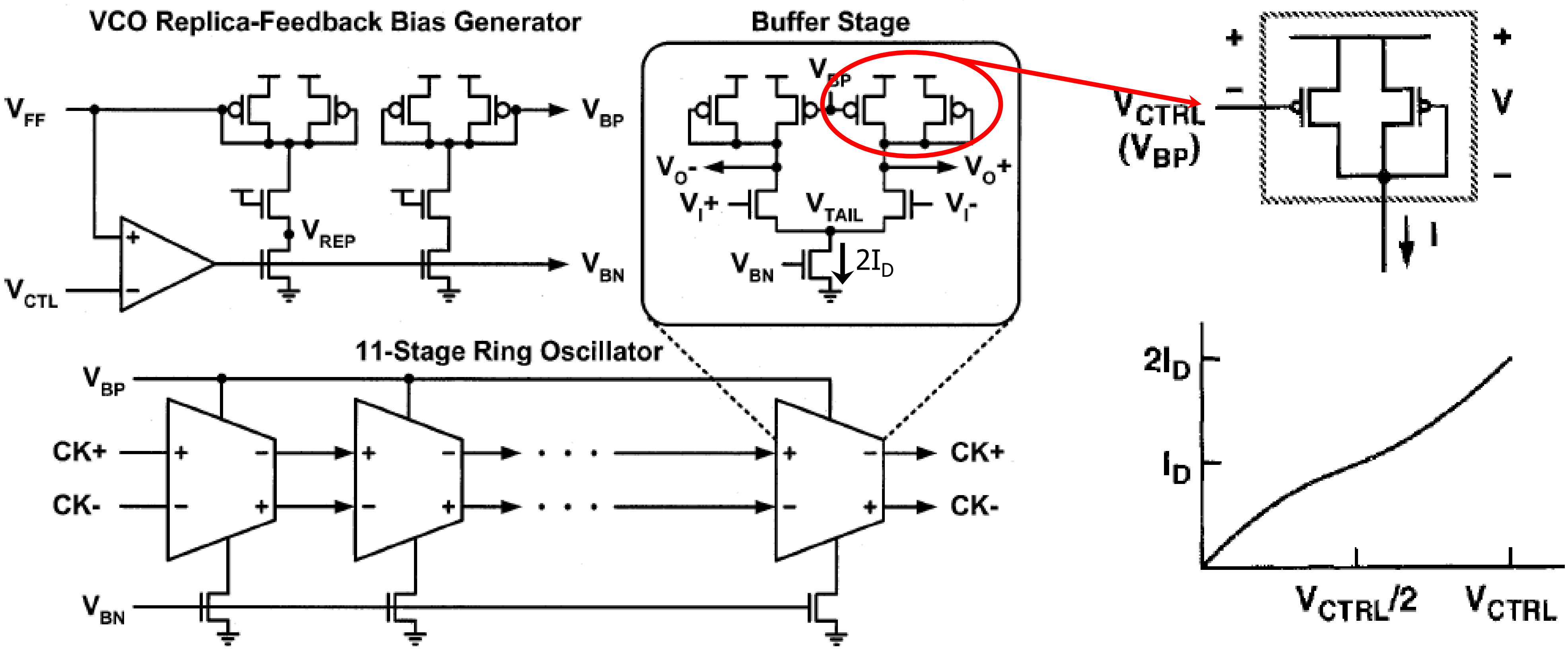
Capacitive-Tuned Ring Oscillator



$$C_{eff} = \frac{C}{1 + sCR}$$

Symmetric Load Ring Oscillator

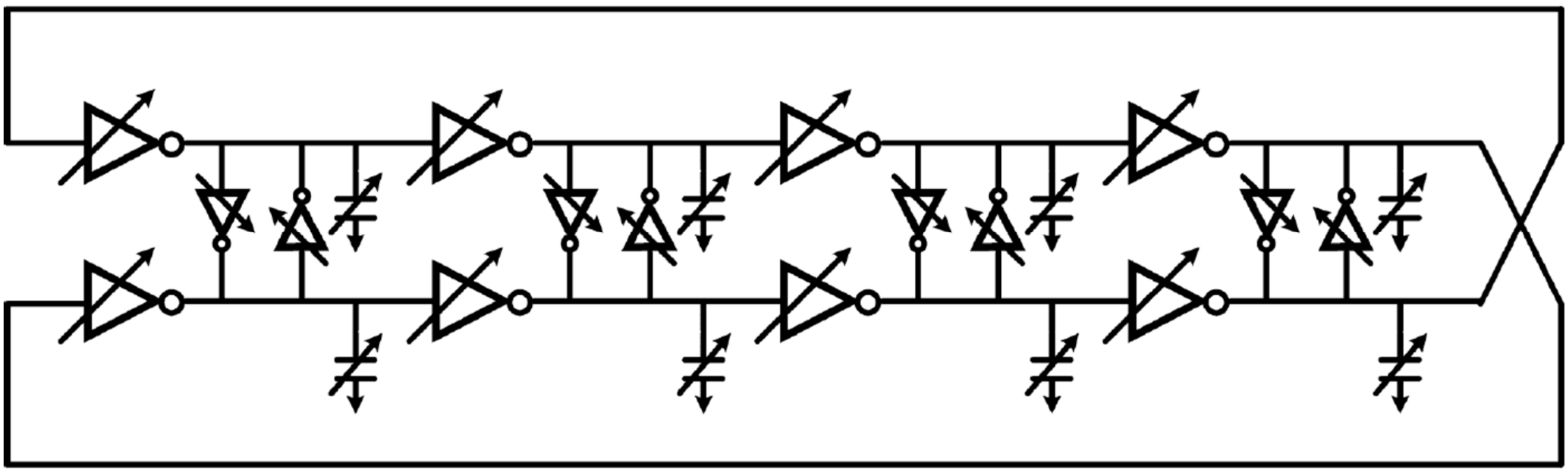
[Maneatis JSSC 1996 & 2003]



- Symmetric load provides frequency tuning at excellent supply noise rejection
- See Maneatis papers for self-biased techniques to obtain constant damping factor and loop bandwidth (% of ref clk)

Pseudo-Differential Ring Oscillators

[**Frans JSSC 2015**]



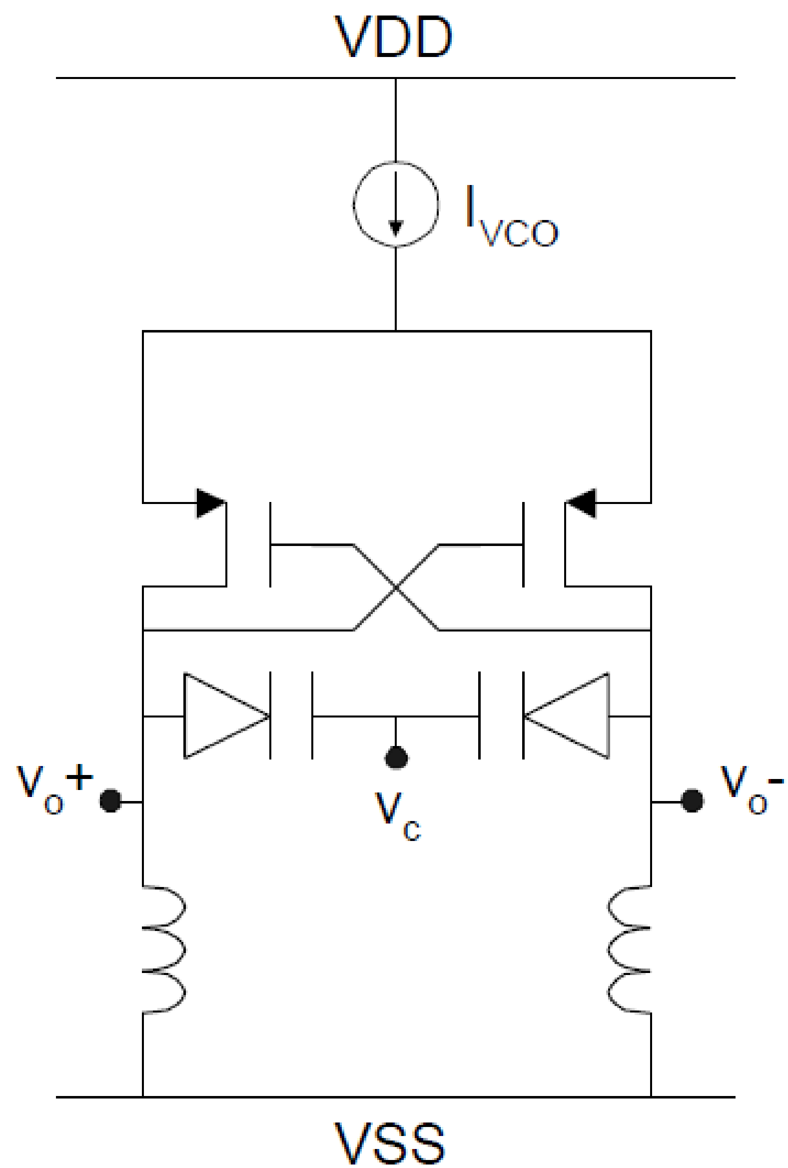
- Often supply-tuned with a regulator
- Band select is possible with adjustable capacitors and main path inverter strength
- Due to even stages, there is a potential for the oscillator to be stuck in a non-differential mode
- Adjustable ratio between cross-coupled and main inverters, initially large, ensures startup in differential mode. This can be reduced during normal operation.

LC Oscillator

- A variable capacitor (varactor) is often used to adjust oscillation frequency

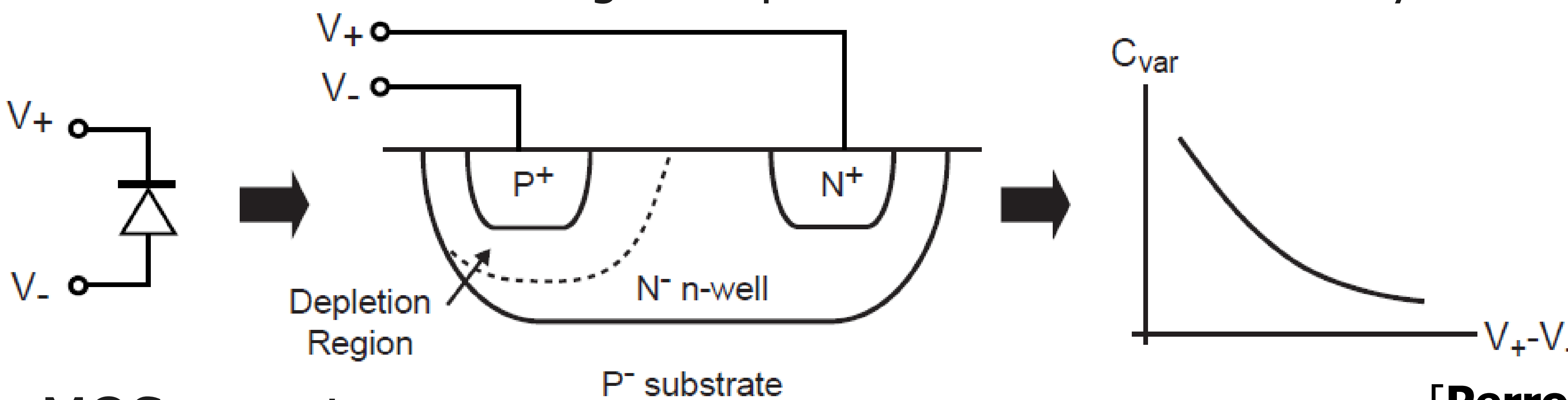
- Total capacitance includes both tuning capacitance and fixed capacitances which reduce the tuning range

$$\omega_{osc} = \frac{1}{\sqrt{L_P C_P}} = \frac{1}{\sqrt{L_P\left(C_{tune} + C_{fixed}\right)}}$$



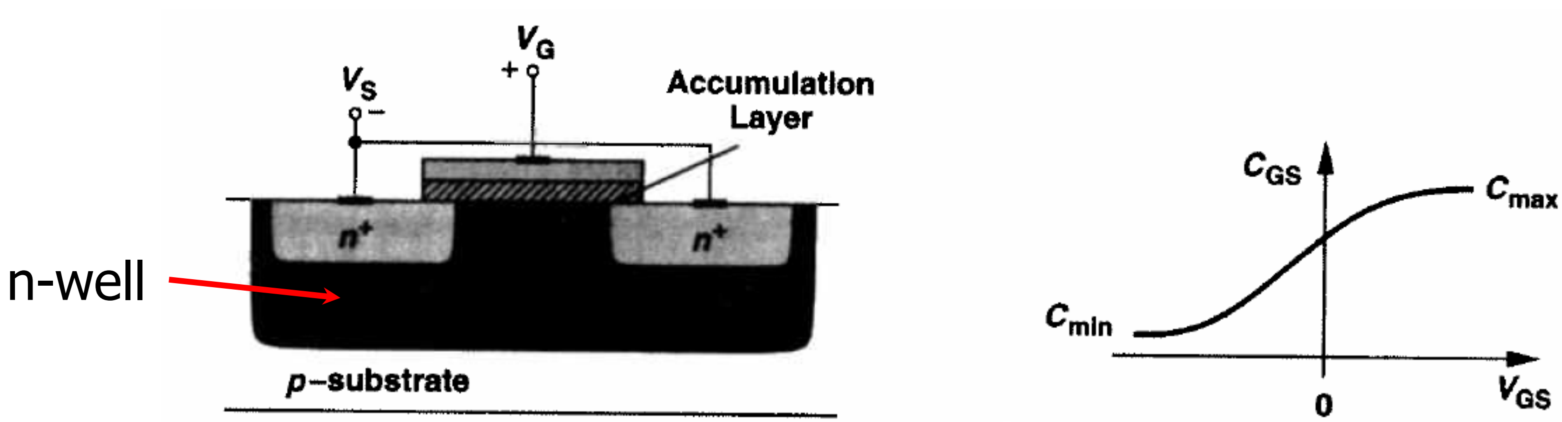
Varactors

- pn junction varactor
  - Avoid forward bias region to prevent oscillator nonlinearity



[Perrott]

- MOS varactor
  - Accumulation-mode devices have better Q than inversion-mode



[Razavi]

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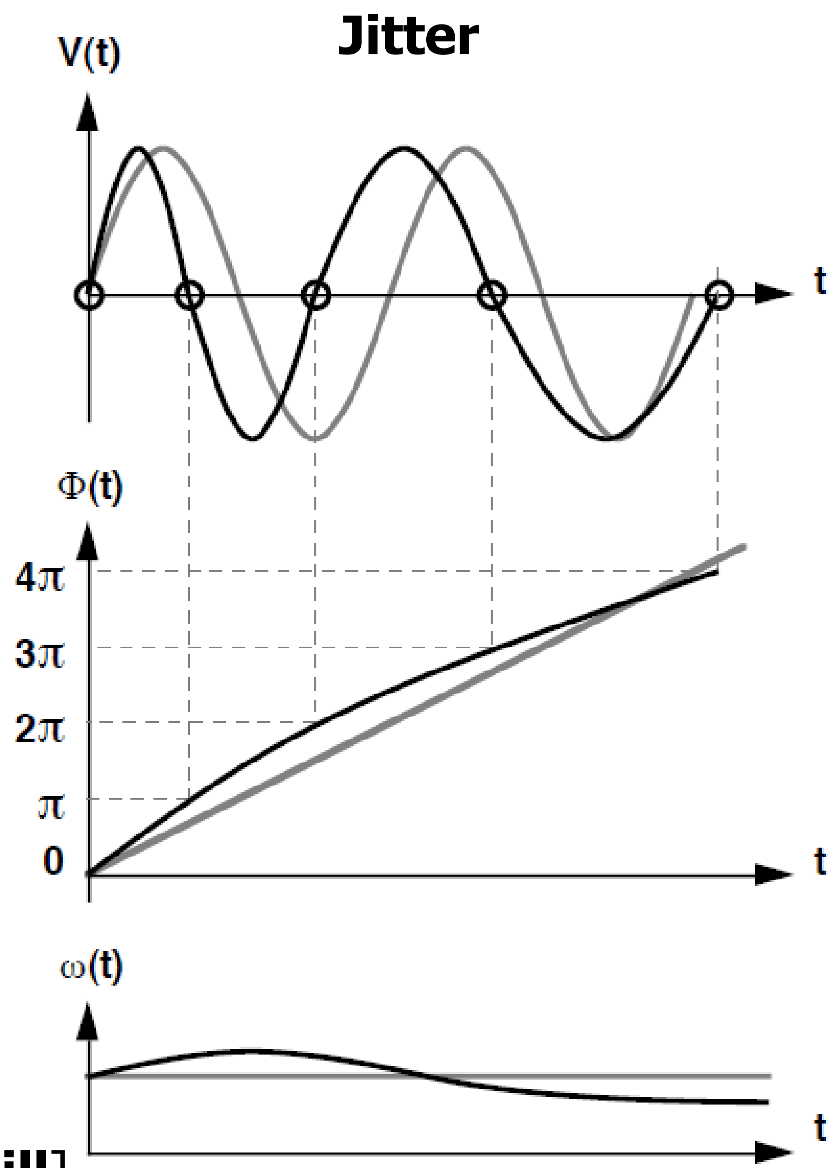
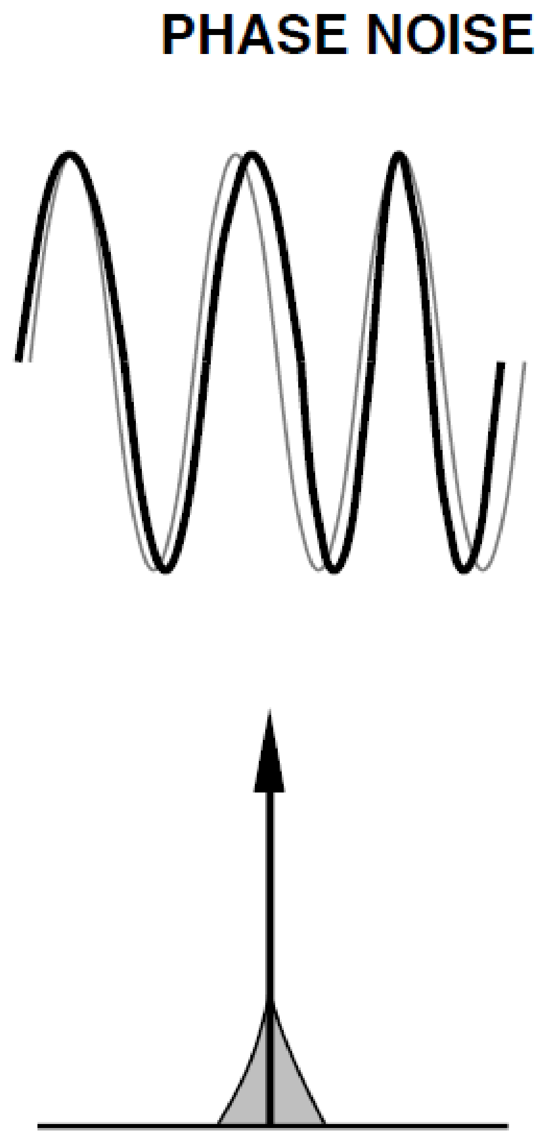
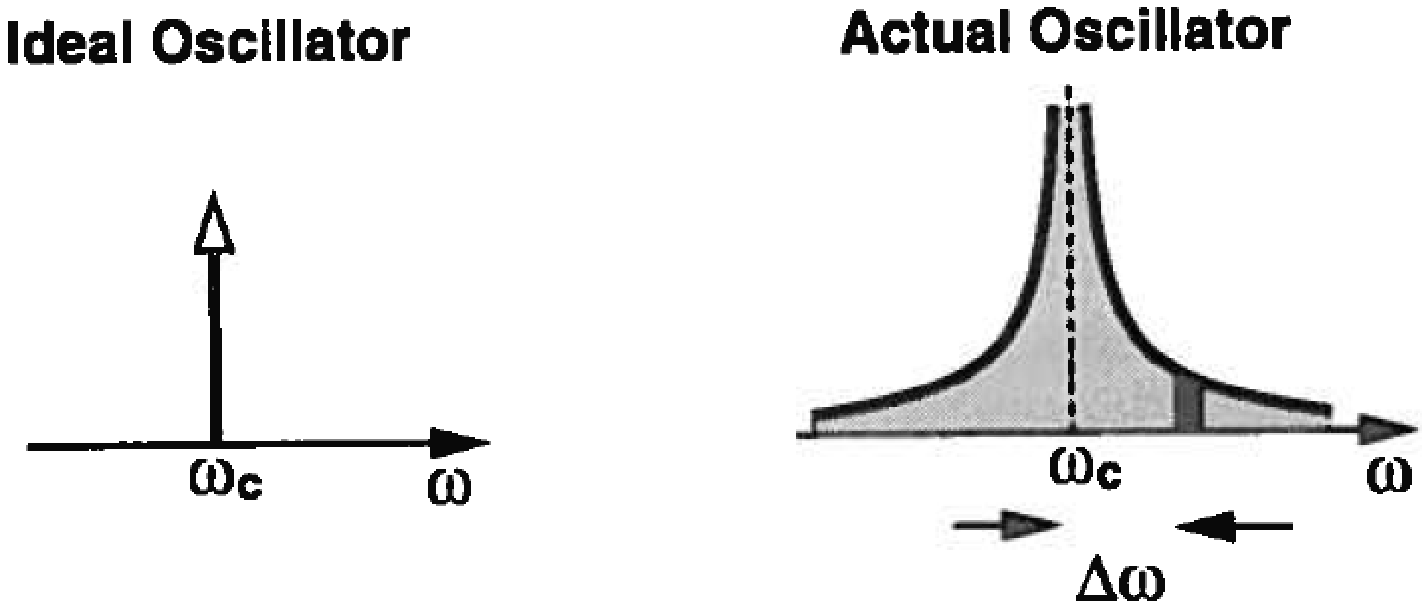
Oscillator Noise





[McNeill]

Phase Noise Definition

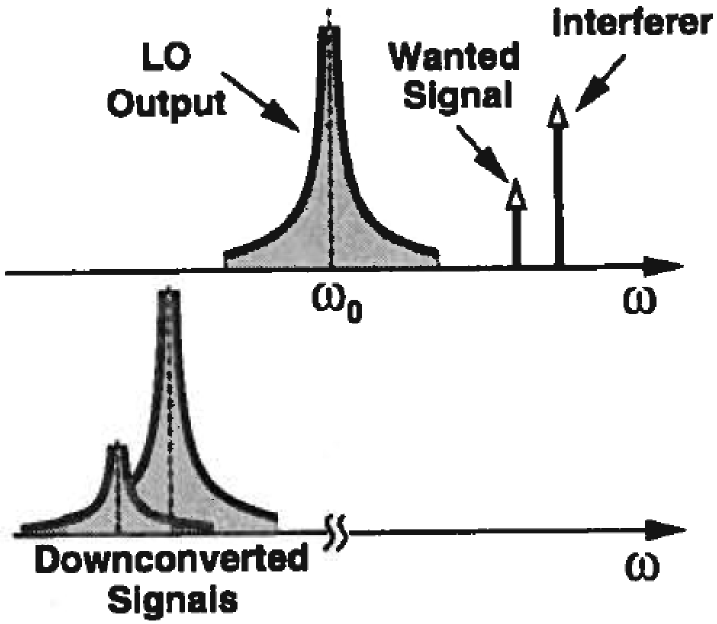


- An ideal oscillator has an impulse shape in the frequency domain
- A real oscillator has phase noise "skirts" centered at the carrier frequency
- Phase noise is quantified as the normalized noise power in a 1Hz bandwidth at a frequency offset $\Delta\omega$ from the carrier

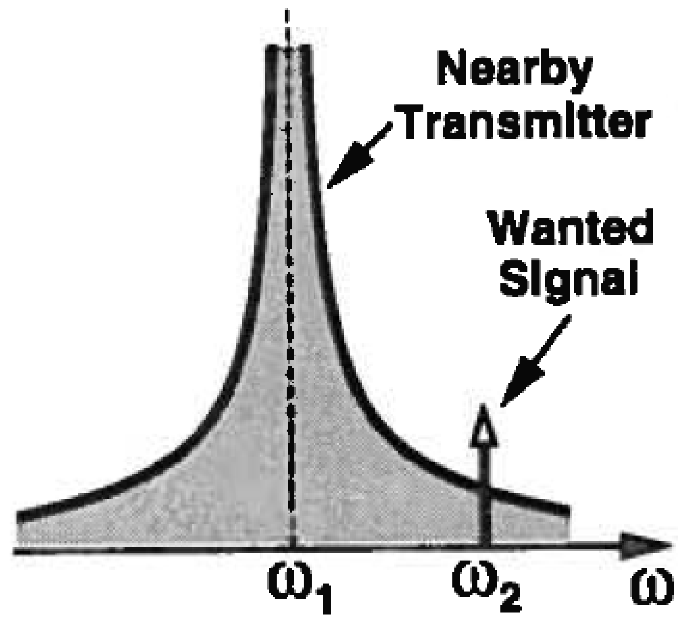
$$L(\Delta\omega) = 10\log\left(\frac{P_{\text{sideband}}(\omega_o + \Delta\omega, 1\text{Hz})}{P_{\text{carrier}}}\right) \ (\text{dBc/Hz})$$

Phase Noise Impact in RF Communication

RX Reciprocal Mixing

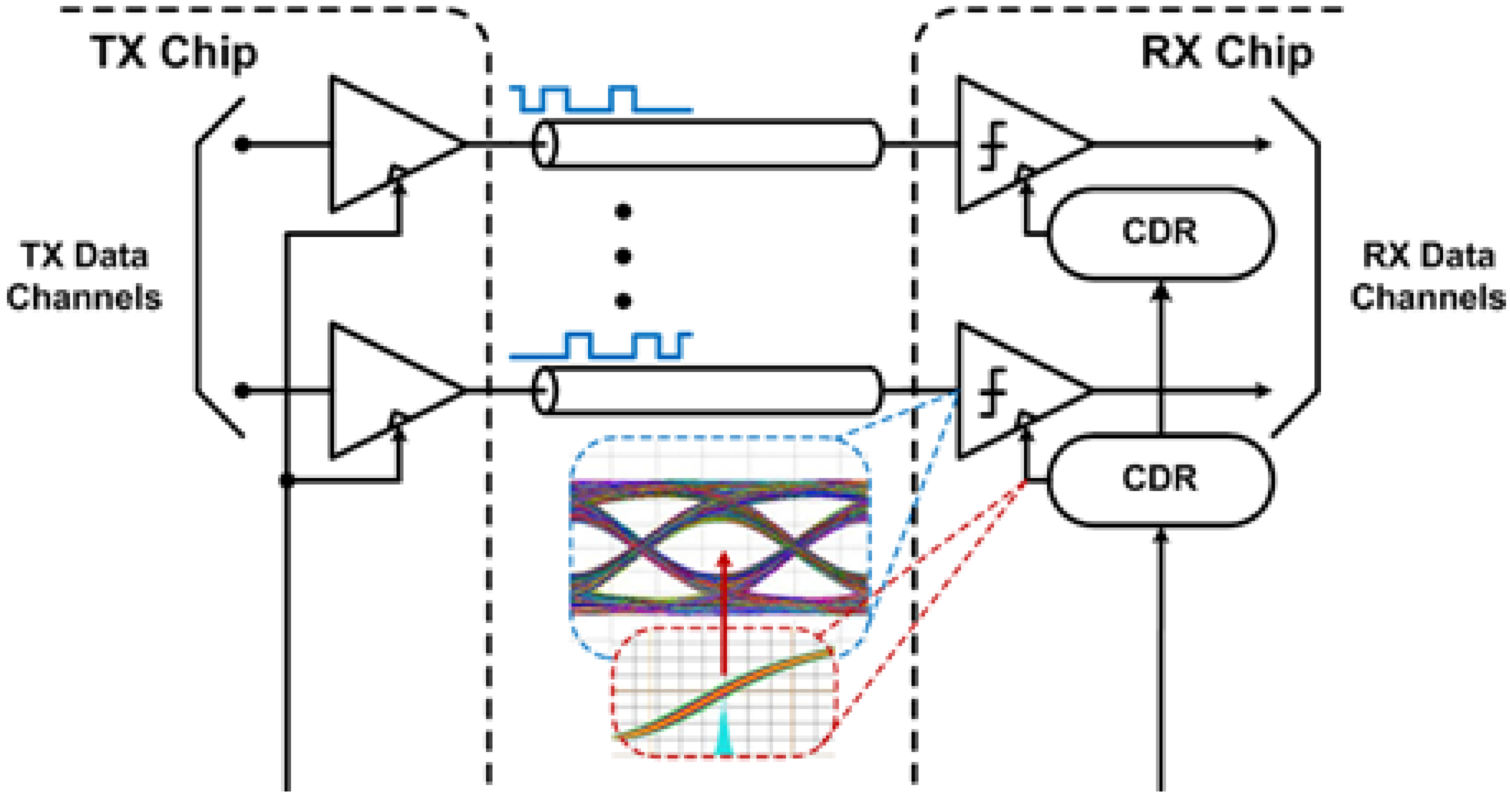


Strong Noisy TX Interfering with RX



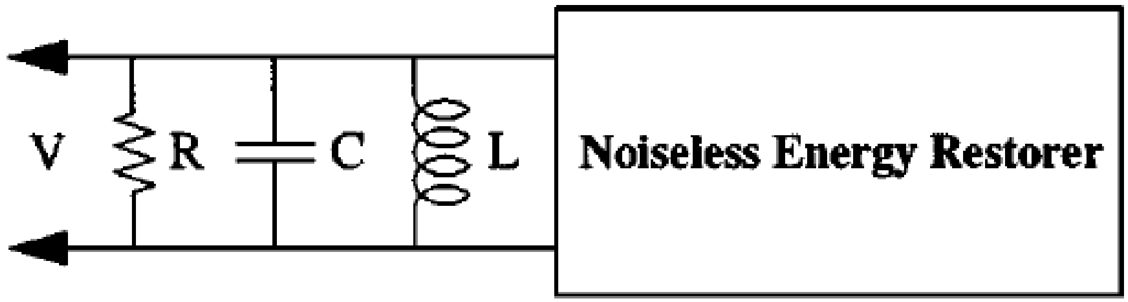
- At the RX, a large interferer can degrade the SNR of the wanted signal due to "reciprocal mixing" caused by the LO phase noise
- Having large phase noise at the TX can degrade the performance of a nearby RX

Jitter Impact in HS Links



- RX sample clock jitter reduces the timing margin of the system for a given bit-error-rate
- TX jitter also reduces timing margin, and can be amplified by low-pass channels

Ideal Oscillator Phase Noise



The tank resistance will introduce thermal noise

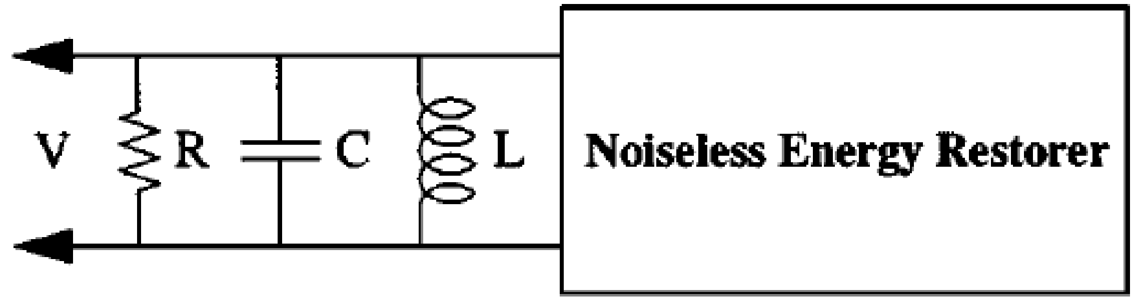
$$\frac{\overline{i_n^2}}{\Delta f} = \frac{4kT}{R}$$

The spectral density of the mean - squared noise voltage is

$$\frac{\overline{v_n^2}}{\Delta f} = \frac{\overline{i_n^2}}{\Delta f} \left| Z_{\text{tank}} \right|^2$$

Tank Impedance Near Resonance

$$Z_{\text{tank}}(\omega) = \frac{1}{j\omega C} \Bigg\| j\omega L = \frac{j\omega L}{1-\omega^2 LC}$$



$$\text{Resonance Frequency: } \omega_o = \frac{1}{\sqrt{LC}}$$

Consider frequencies close to resonance $\omega = \omega_o + \Delta\omega$

$$Z_{\text{tank}}(\Delta\omega) = \frac{j(\omega_o + \Delta\omega)L}{1-\omega_o^2 LC - 2\omega_o \Delta\omega LC - \Delta\omega^2 LC} \approx -\frac{j\omega_o L}{-2\omega_o \Delta\omega LC} = -\frac{j}{2}\frac{1}{\omega_o C}\left(\frac{\omega_o}{\Delta\omega}\right)$$

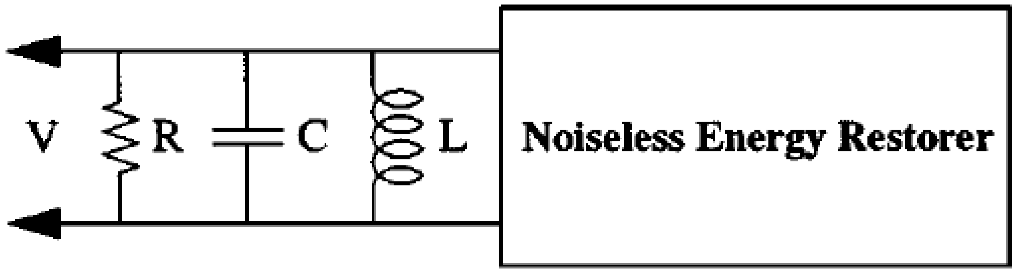
$$\text{Tank } Q = R\omega_o C \ \Rightarrow \ \frac{1}{\omega_o C} = \frac{R}{Q}$$

$$Z_{\text{tank}}(\Delta\omega) \approx -\frac{j}{2}\frac{R}{Q}\left(\frac{\omega_o}{\Delta\omega}\right)$$

$$\left|Z_{\text{tank}}(\Delta\omega)\right|^2 = \left(\frac{R\omega_o}{2Q\Delta\omega}\right)^2$$

- Tank impedance at $\Delta\omega$ is inversely proportional to Q^2 and $\Delta\omega^2$

Ideal Oscillator Phase Noise



$$\frac{\overline{v_n^2}}{\Delta f} = \frac{\overline{i_n^2}}{\Delta f}\left|Z_{\text{tank}}\right|^2 = \left(\frac{4kT}{R}\right)\left(\frac{R\omega_o}{2Q\Delta\omega}\right)^2 = 4kTR\left(\frac{\omega_o}{2Q\Delta\omega}\right)^2$$

The Equipartition Theorem [Lee JSSC 2000] states that, in equilibrium, amplitude and phase - noise power are equal. Therefore, this noise power is split evenly $\left(\frac{1}{2}\right)$ between amplitude and phase.

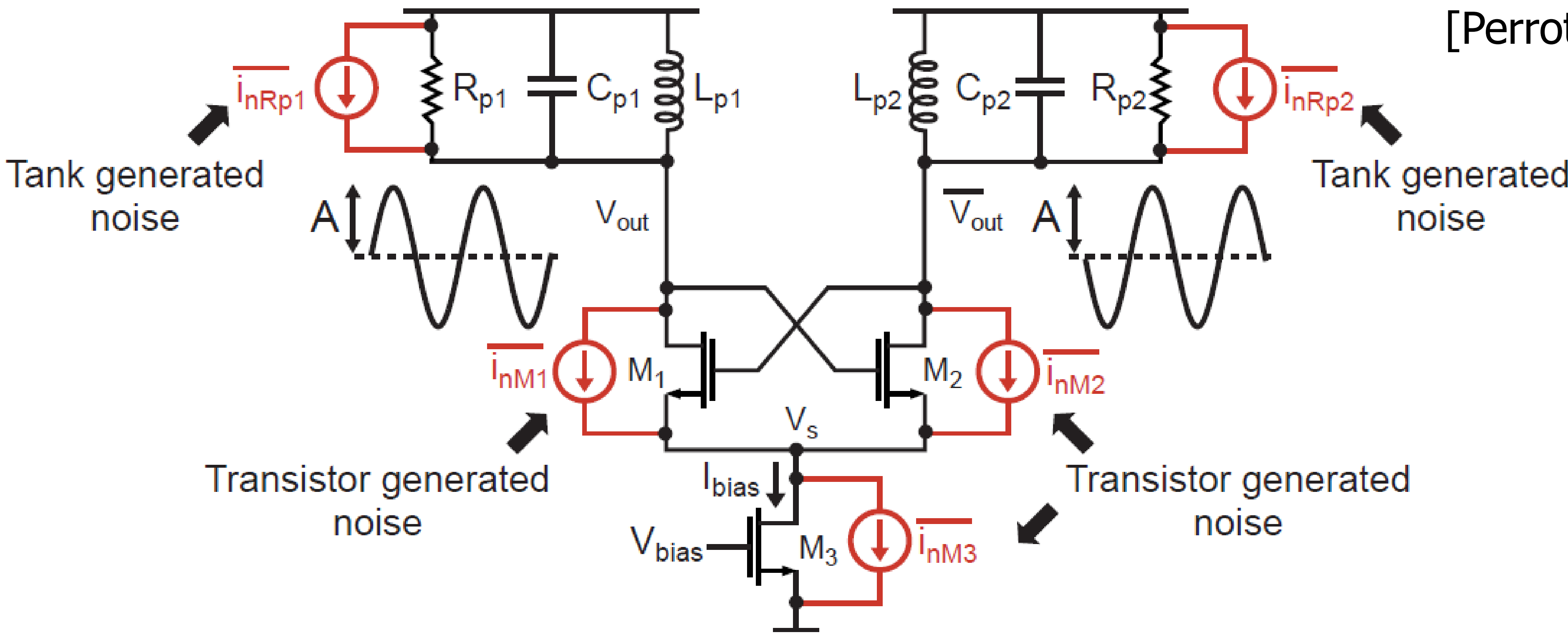
$$L\{\Delta\omega\} = 10\log\left[\frac{\left(\frac{1}{2}\right)\frac{\overline{v_n^2}}{\Delta f}}{v_{sig}^2}\right] = 10\log\left[\frac{\left(\frac{1}{2}\right)4kTR}{v_{sig}^2}\left(\frac{\omega_o}{2Q\Delta\omega}\right)^2\right] = 10\log\left[\frac{2kT}{P_{sig}}\left(\frac{\omega_o}{2Q\Delta\omega}\right)^2\right] \quad (\text{dBc/Hz})$$

Phase noise due to thermal noise will display a - 20dB/dec slope away from the carrier

- Phase noise improves as both the carrier power and Q increase

Other Phase Noise Sources

[Perrott]



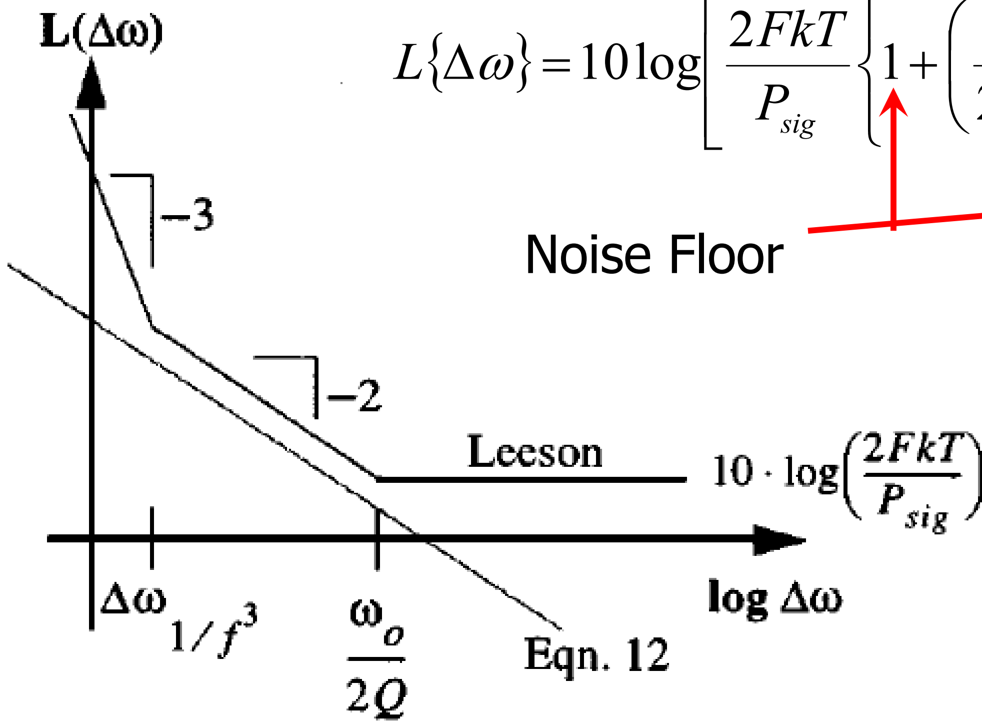
- Tank thermal noise is only one piece of the phase noise puzzle
- Oscillator transistors introduce their own thermal noise and also flicker (1/f) noise

Leeson Phase Noise Model

$$L\{\Delta\omega\} = 10\log\left[\frac{2FkT}{P_{sig}}\left\{1+\left(\frac{\omega_o}{2Q\Delta\omega}\right)^2\right\}\left(1+\frac{\Delta\omega_{1/f^3}}{|\Delta\omega|}\right)\right] \quad (\text{dBc/Hz})$$

1/f Noise

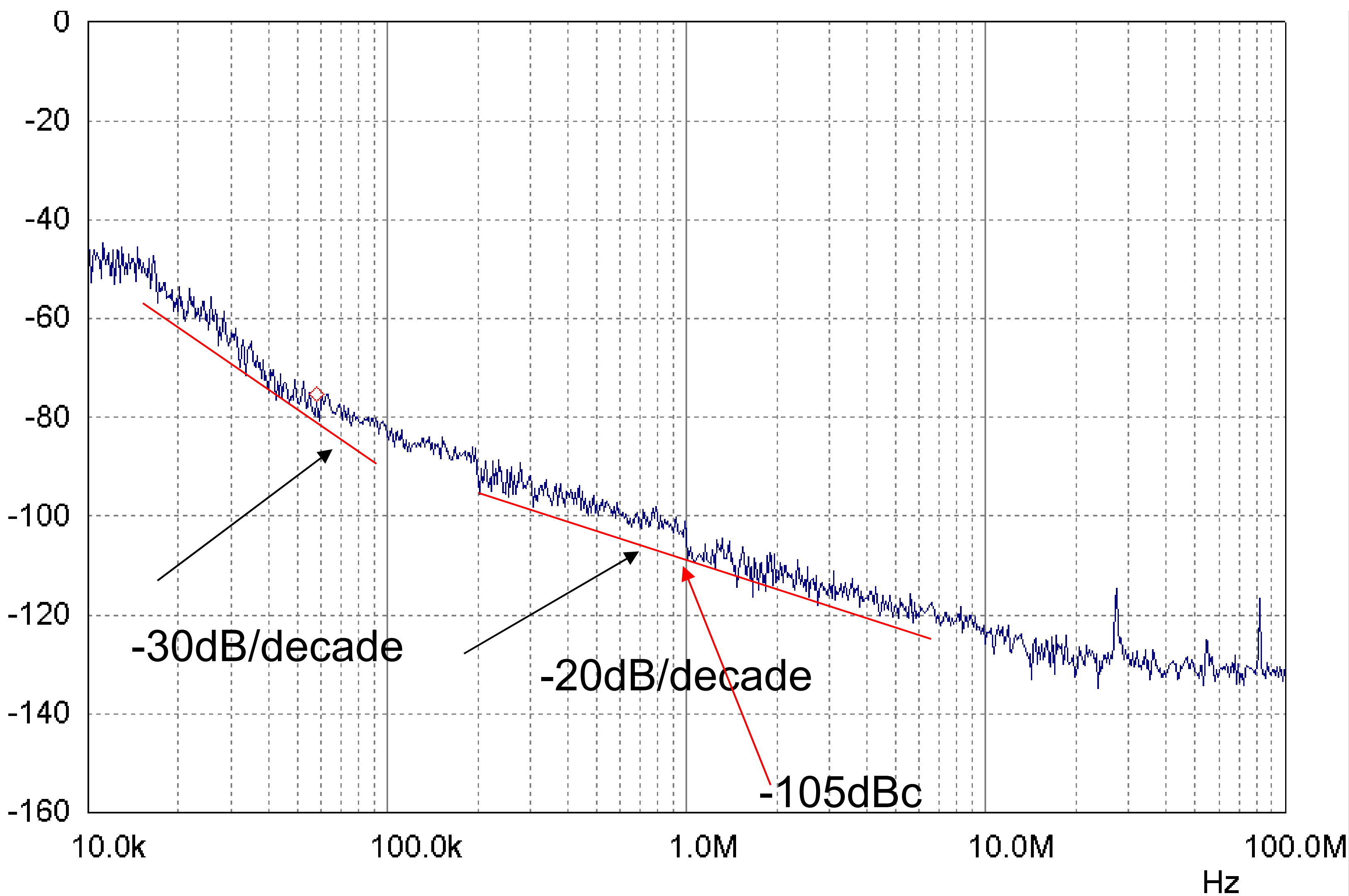
Noise Floor



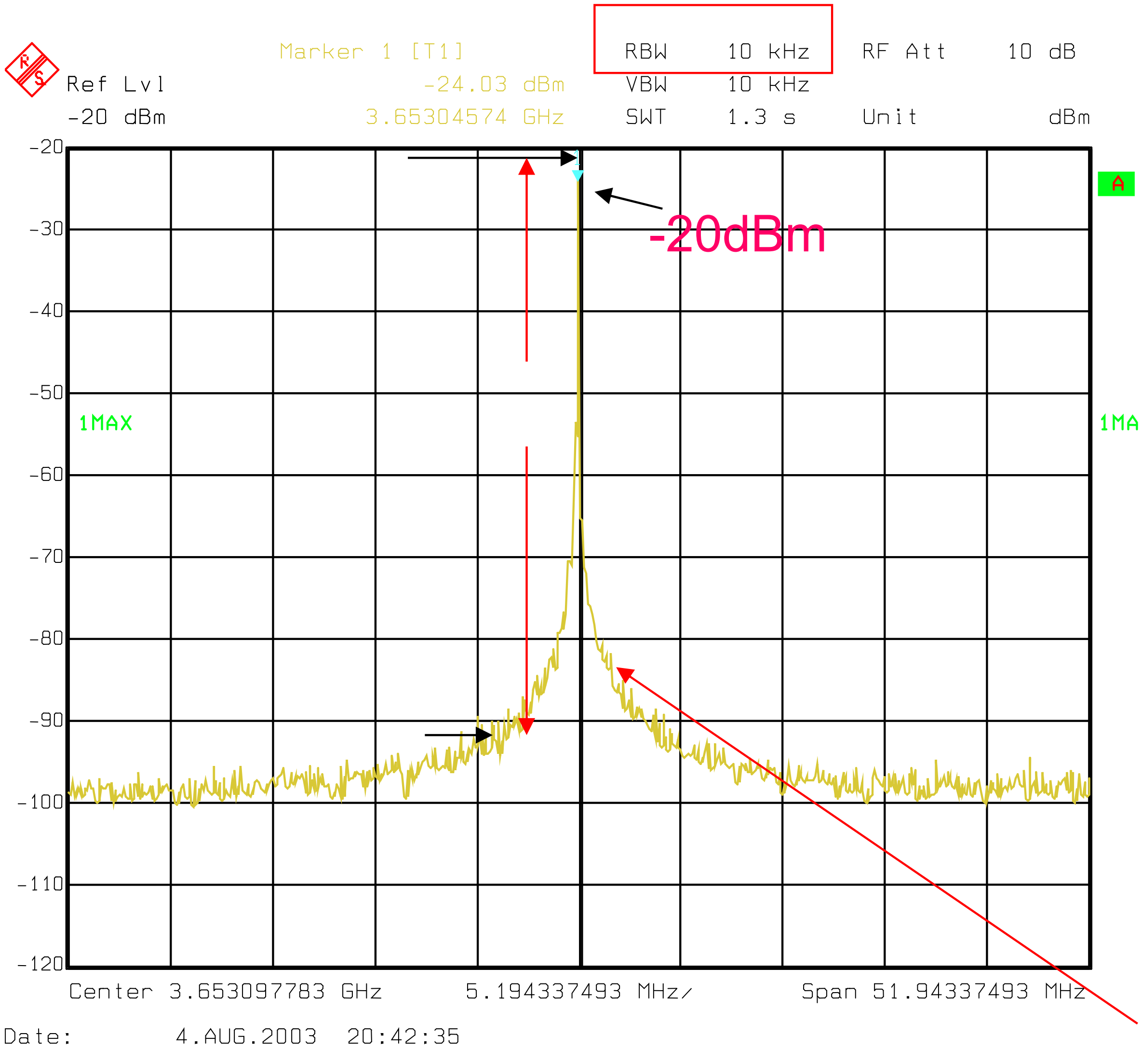
- Leeson's model modifies the previously derived expression to account for the high frequency noise floor and 1/f noise upconversion
- A empirical fitting parameter F is introduced to account for increased thermal noise
- Model predicts that the $(1/\Delta\omega)^3$ region boundary is equal to the 1/f corner of device noise and the oscillator noise flattens at half the resonator bandwidth

A 3.5GHz LC-Tank VCO Phase Noise

Measured Phase Noise



VCO Output Spectrum Example



Make sure to account for the spectrum analyzer resolution bandwidth

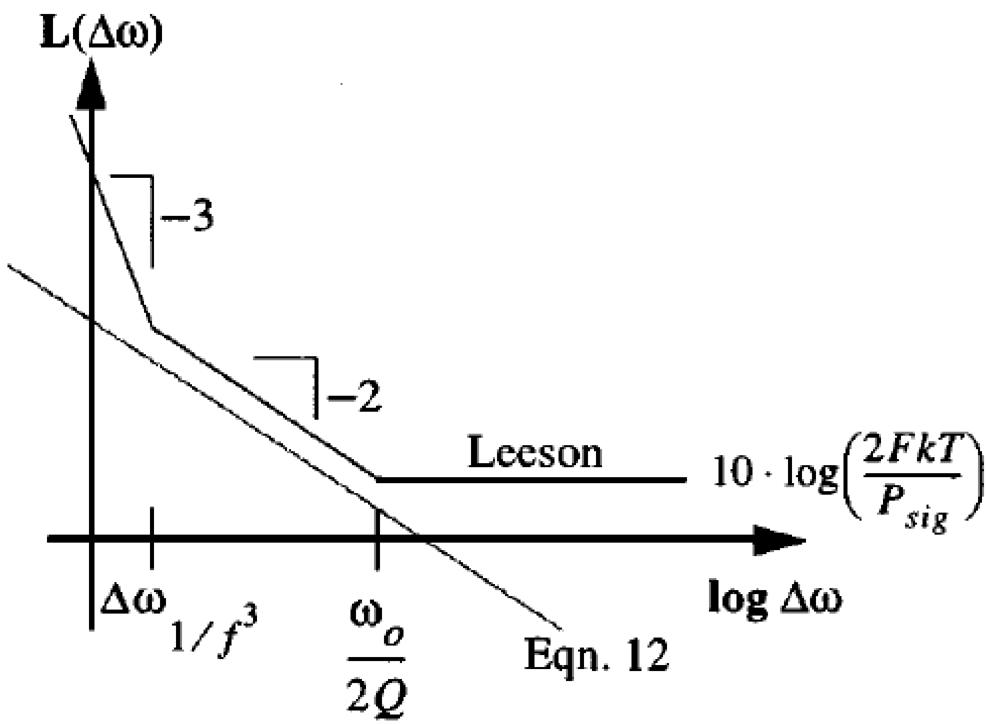
RBW=10K

PN=-85dBm-(-20dBm)-$10\log_{10}(10e3)$

=-105dBc

dBc---in dB with respect to carrier

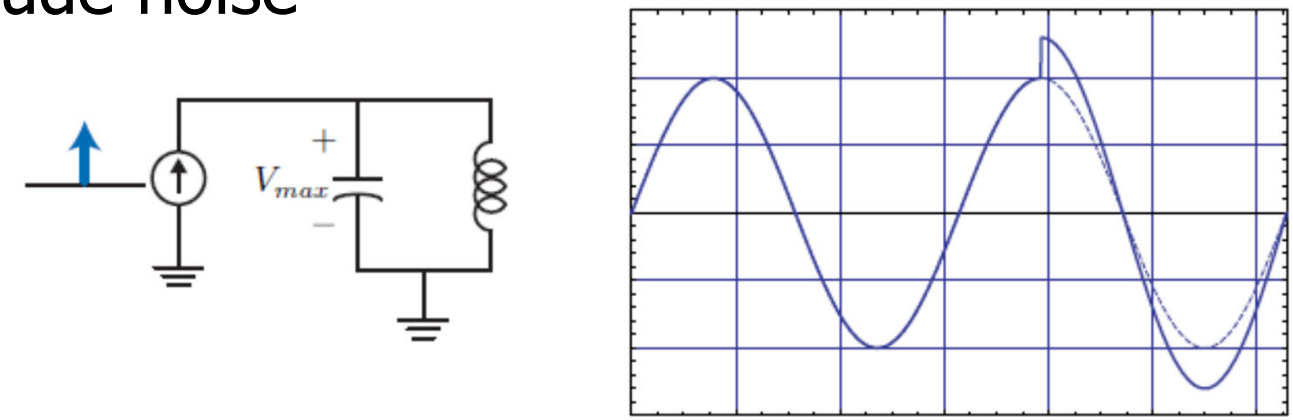
Leeson Model Issues



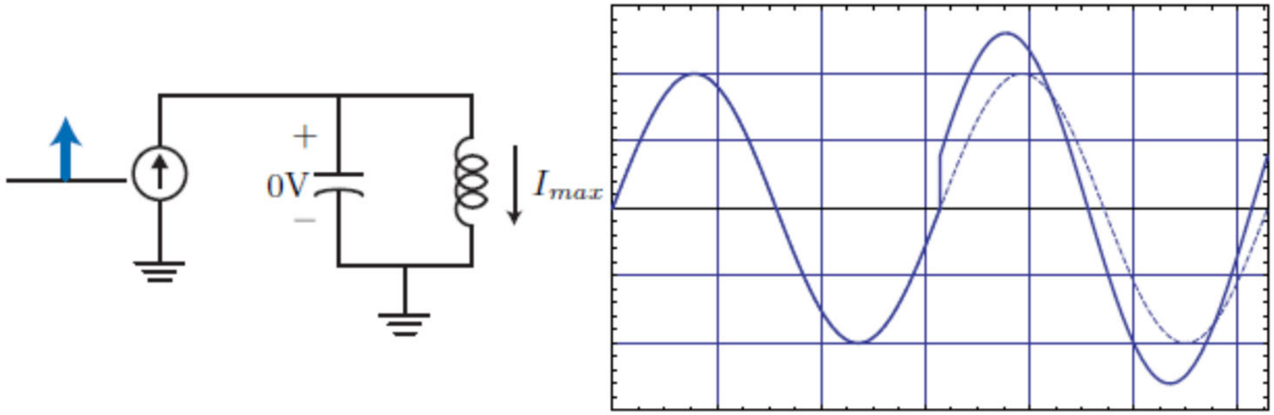
- The empirical fitting parameter F is not known in advance and can vary with different process technologies and oscillator topologies
- The actual transition frequencies predicted by the Leeson model does not always match measured data

Time-Varying Phase Noise Model: Hajimiri-Lee Model

- Noise injection at oscillator peak output time introduces only amplitude noise

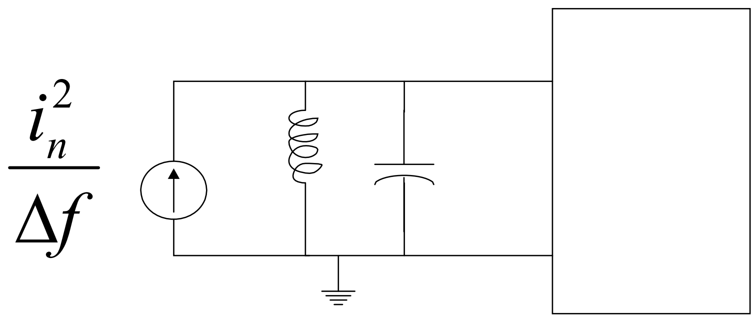


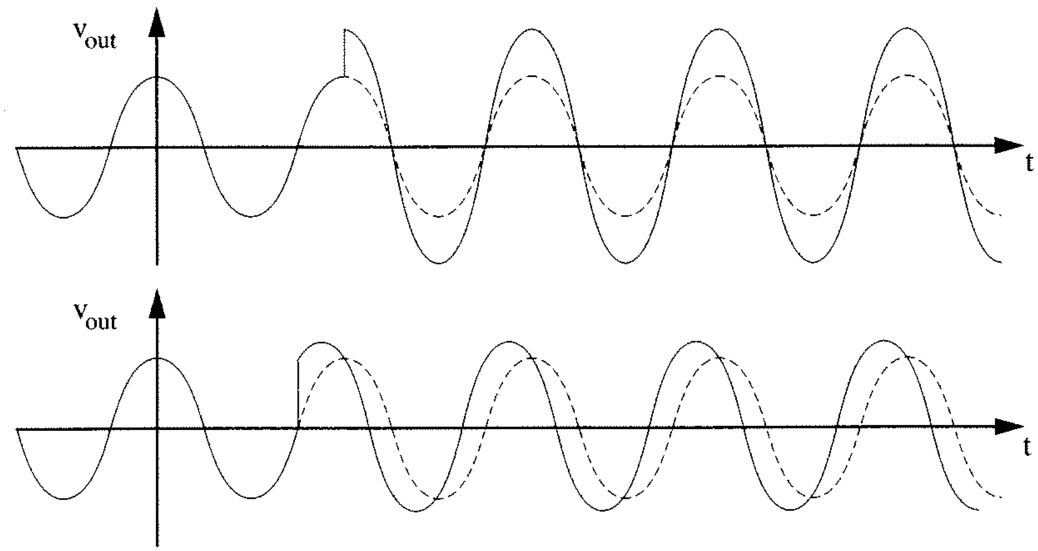
- Noise injection at oscillator zero crossing time introduces maximum phase noise



Time-Varying Phase Noise Model: Hajimiri-Lee Model

- Impulse current applied to the tank to measure its sensitivity function





- The impulse response for the phase variation can be represented as

$$h_\phi(t,\tau) = \frac{\Gamma(\omega_0 \tau)}{q_{\max}} u(t-\tau),$$

Γ is the impulse sensitivity function (ISF)

q_{max}, the maximum charge displacement across the capacitor, is a normalizing factor

Impulse Sensitivity Function (ISF) Model

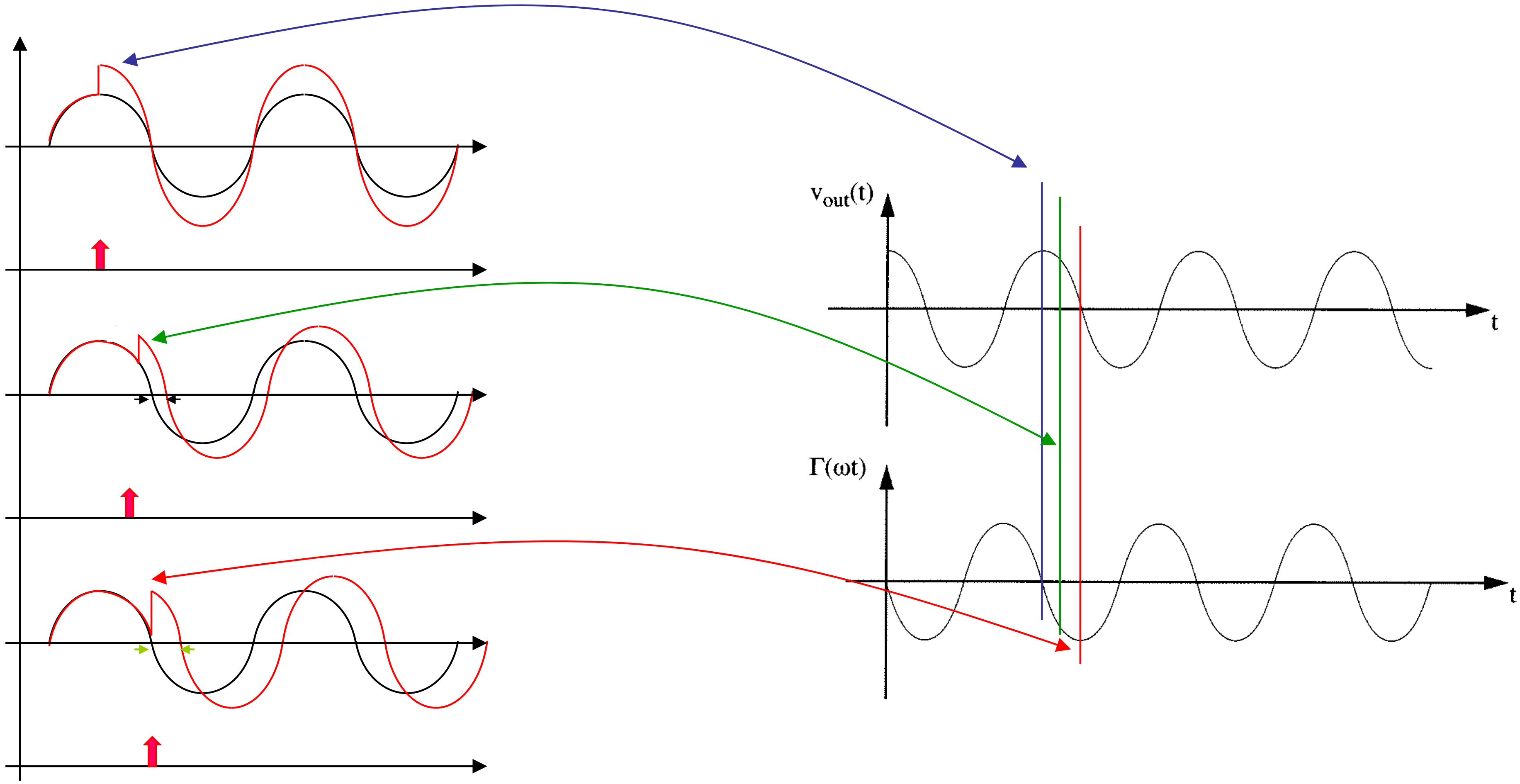
- The phase variation due to injected noise can be modeled as

$$\Delta\phi = \Gamma(\omega_0\tau)\frac{\Delta V}{V_{max}} = \Gamma(\omega_0\tau)\frac{\Delta q}{q_{max}} \qquad \Delta q \ll q_{max}$$

- The function $\Gamma(\omega_o\tau)$ is a time-varying proportionality factor called the "impulse sensitivity function"
  - Encodes information about the sensitivity of the oscillator to an impulse injected at phase $\omega_o\tau$ (0 to 2π)
  - Phase shift is assumed linear to charge injection
  - ISF has the same oscillation period as the oscillator
- The phase impulse response can be written as

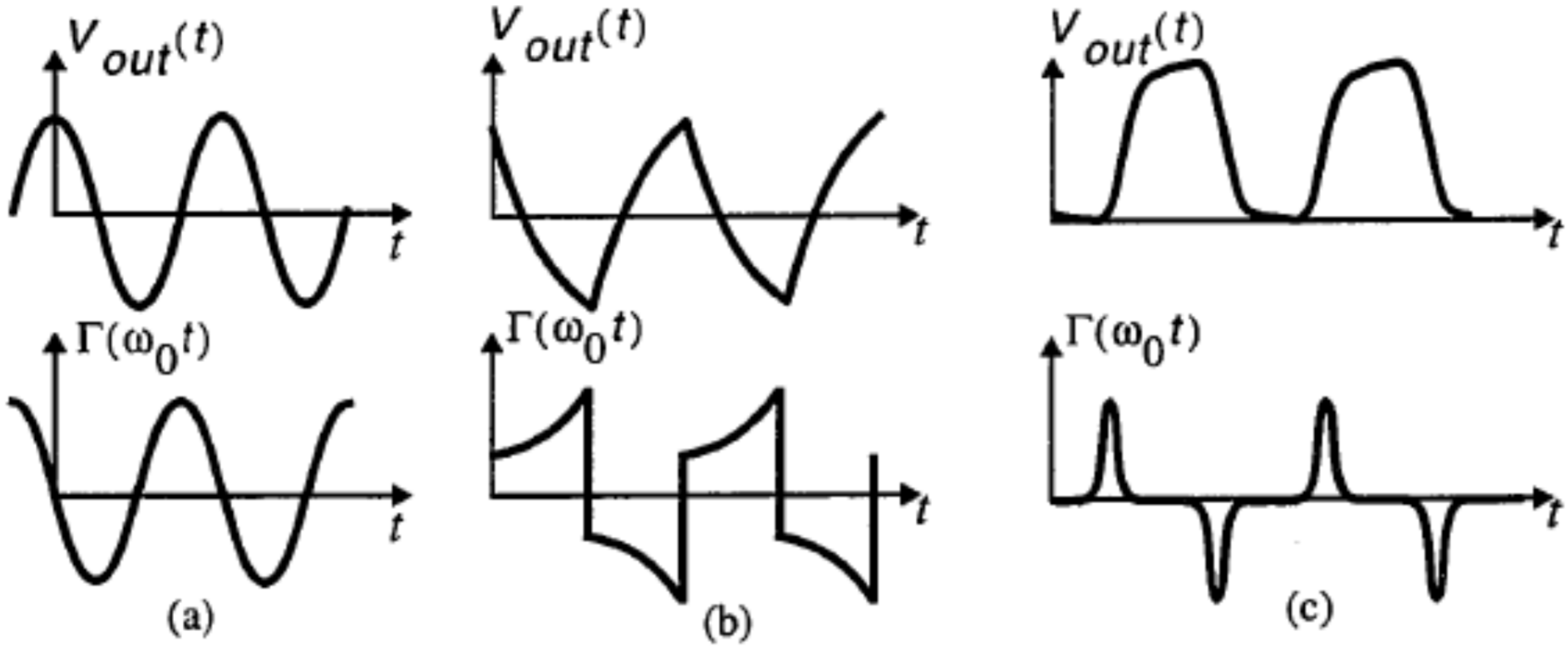
$$h_\phi(t,\tau) = \frac{\Gamma(\omega_0\tau)}{q_{max}}u(t-\tau)$$

Obtaining the ISF



- $\Gamma(\omega\tau)$ can be obtained using Cadence for each oscillator noise source

Typical Oscillator ISFs



Typical ISF for (a) LC, (b) Bose and (c) ring oscillators.

- ISF estimated analytically or calculated from simulation
- ISF peaks during zero crossing time and is zero at the signal peak time for typical LC and ring oscillators

Phase Noise Computation

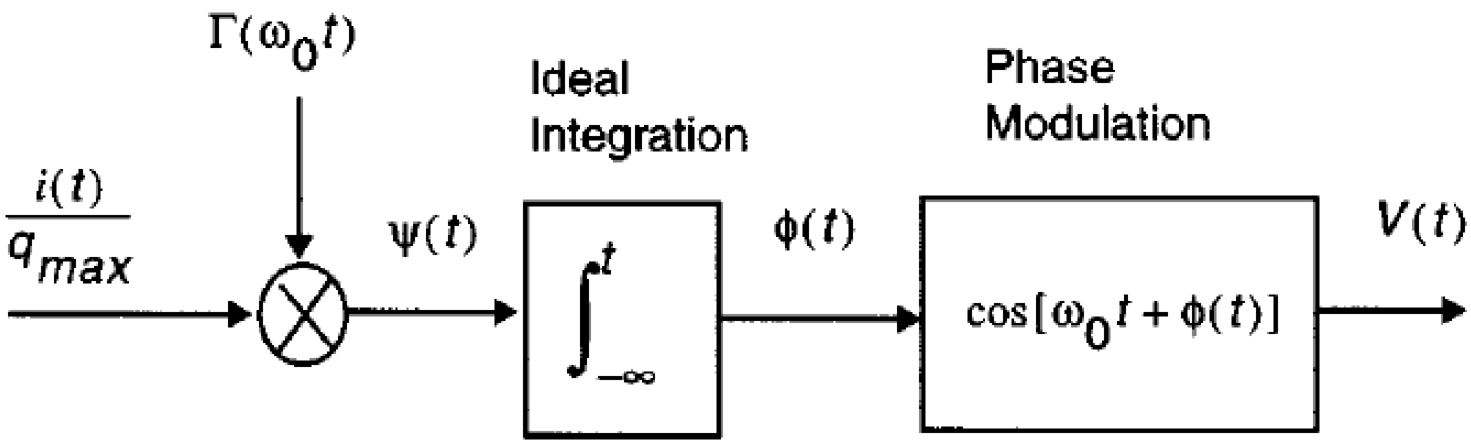
The impulse sensitivity function is used to obtain the phase noise impulse function

$$h_\phi(t,\tau) = \frac{\Gamma(\omega_o \tau)}{q_{\max}} u(t-\tau)$$

The phase noise can then be computed by the superposition (convolution) integral of the

any arbitrary noise current with the phase noise impulse function

$$\phi(\mathrm{t}) = \int_{-\infty}^{\infty} h_\phi(t,\tau) i(\tau) d\tau = \frac{1}{q_{\max}} \int_{-\infty}^{t} \Gamma(\omega_o \tau) i(\tau) d\tau$$



ISF Decomposition w/ Fourier Series

In order to gain further insight, and because the ISF is periodic, it may be expressed as a Fourier series

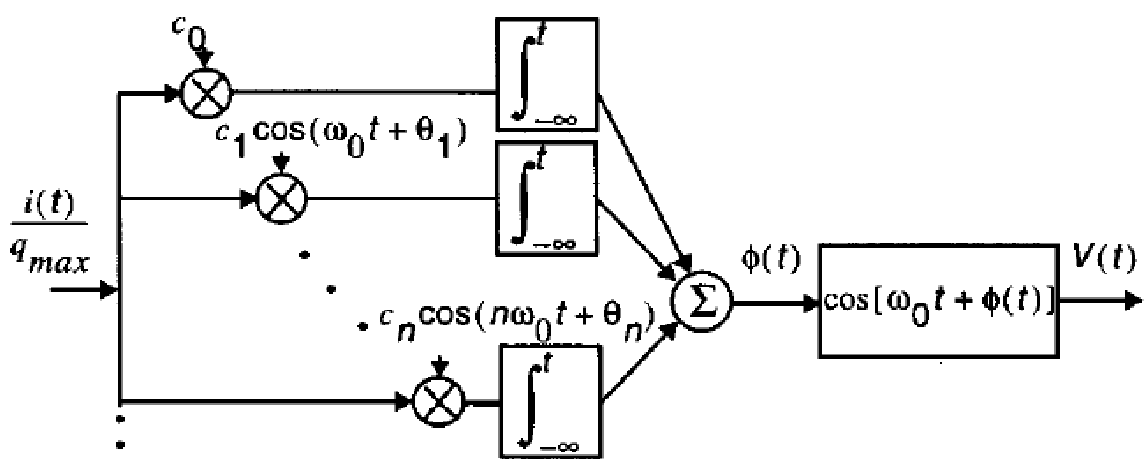
$$\Gamma(\omega_o\tau) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o\tau + \theta_n)$$

where the coefficients c_n are real and θ_n is the phase of the nth ISF harmonic. Note, θ_n is typically ignored, as it is assumed that the noise components are uncorrelated, and their relative phase is irrelevant.

The phase noise can then be computed by

$$\phi(\mathrm{t}) = \frac{1}{q_{\max}}\left[\frac{c_0}{2}\int_{-\infty}^{t} i(\tau)d\tau + \sum_{n=1}^{\infty} c_n \int_{-\infty}^{t} i(\tau)\cos(n\omega_o\tau)d\tau\right]$$

This allows the excess phase from an arbitrary noise source to be computed once the ISF Fourier coefficients are deteremined. Essentially, the current noise is mixed down from different frequency bands and scaled according to the ISF coefficients.



Phase Noise Frequency Conversion

First consider a simple case where we have a sinusoidal noise current whose frequency is near an integer multiple m of the oscillation frequency

$$i(t) = I_m \cos[(m\omega_o + \Delta\omega)t]$$

When performing the phase noise computation integral, there will be a negligible contribution from all terms other than $n = m$

$$\varphi(t) \approx \frac{I_m c_m \sin(\Delta\omega t)}{2q_{\max}\Delta\omega}$$

The resulting frequency spectrum will show two equal sidebands at $\pm\Delta\omega$. Assuming a sinusoidal waveform $v_{out}(t) = \cos[\omega_o t + \phi(t)]$, there will be two equally weighted sidebands symmetric about the carrier with power

$$P_{SBC}(\Delta\omega) \approx 10\log\left(\frac{I_m c_m}{4q_{\max}\Delta\omega}\right)^2$$

Note that this power is proportional to $\left(\frac{1}{\Delta\omega}\right)^2$.

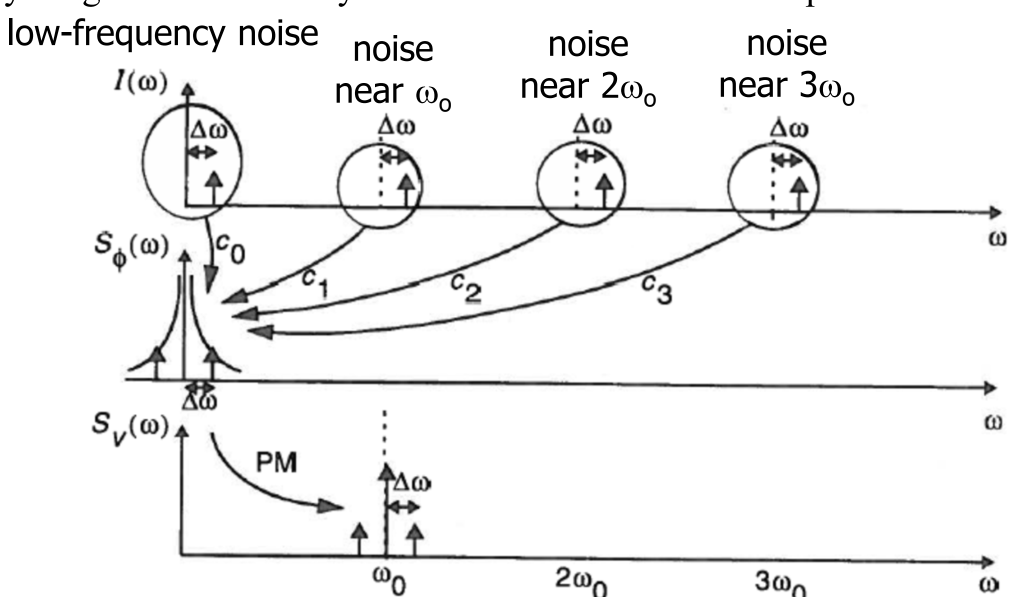


Figure 4.18: The conversion of tones in the vicinity of integer multiples of ω_0.

Phase Noise Due to White & 1/f Sources

Extending the previous analysis to the general case of a white noise source results in

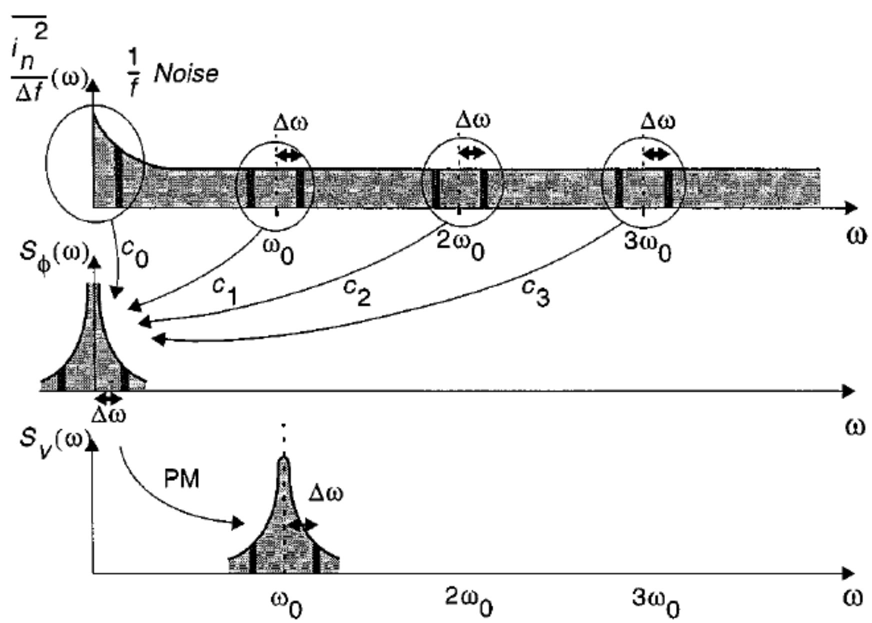
$$P_{SBC}(\Delta\omega) \approx 10\log\left(\frac{\frac{\overline{i_n^2}}{\Delta f}\sum_{m=0}^{\infty} c_m^2}{4q_{\max}^2 \Delta\omega^2}\right)$$

Here noise components near integer multiples of the carrier frequency all fold near the carrier itself and are weighted by $\left(\frac{1}{\Delta\omega}\right)^2$.

Noise near dc gets upconverted, weighted by coefficient c_0, so $1/f$ noise becomes $1/f^3$ noise near the carrier.

White noise near the carrier is weighted by c_1 and $1/f^2$ and stays near the carrier.

White noise near higher integer multiples of the carrier gets downconverted and weighted by c_m and $1/f^2$.



How to Minimize Phase Noise?

In order to minimize phase noise, the ISF coefficients c_n should be minimized. Using Parseval's theorem

$$\sum_{m=0}^{\infty} c_m^2 = \frac{1}{\pi}\int_0^{2\pi} \left|\Gamma(x)\right|^2 dx = 2\Gamma_{rms}^2$$

The spectrum in the $1/f^2$ region can be expressed as

$$L(\Delta\omega) = 10\log\left(\frac{\frac{\overline{i_n^2}}{\Delta f}\Gamma_{rms}^2}{2q_{\max}^2\Delta\omega^2}\right)$$

Thus, reducing Γ_{rms} will reduce the phase noise at all frequencies.

1/f Corner Frequency

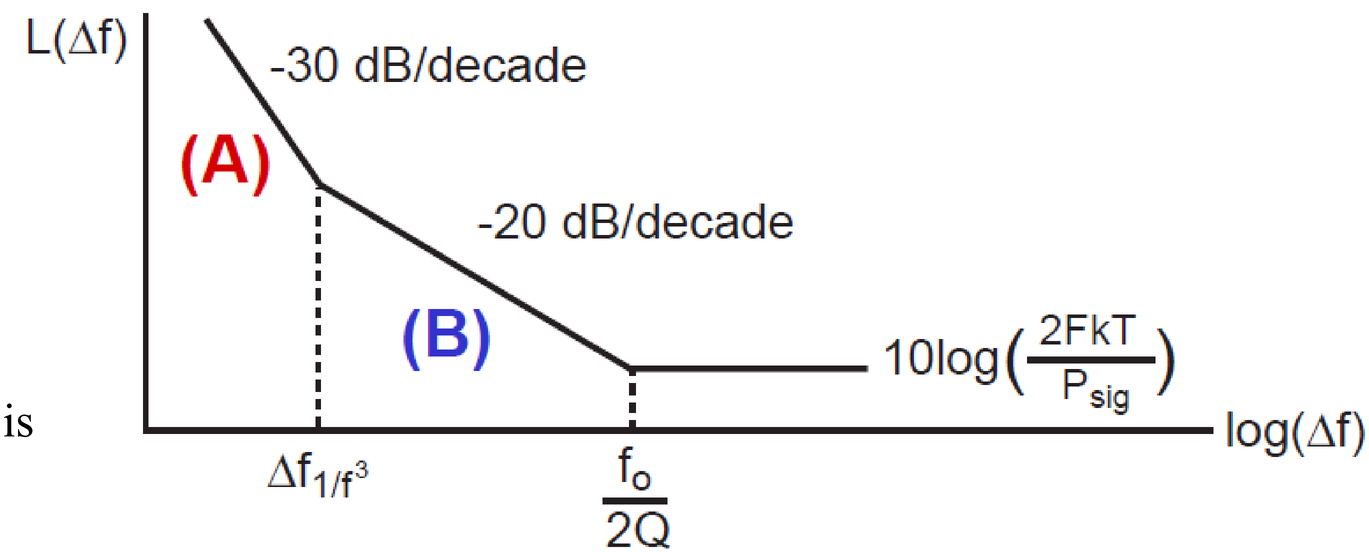
Consider current noise which includes $1/f$ content

$$\overline{i_{n,1/f}^2} = \overline{i_n^2}\,\frac{\omega_{1/f}}{\Delta\omega}$$

where $\omega_{1/f}$ is the $1/f$ corner frequency

From the previous slide

$$L(\Delta\omega) = 10\log\left(\frac{\frac{\overline{i_n^2}}{\Delta f}c_0^2}{8q_{\max}^2\Delta\omega^2}\,\frac{\omega_{1/f}}{\Delta\omega}\right)$$



Thus, the $1/f^3$ corner frequency is

$$\Delta\omega_{1/f^3} = \omega_{1/f}\,\frac{c_0^2}{4\Gamma_{rms}^2} = \omega_{1/f}\left(\frac{\Gamma_{dc}}{\Gamma_{rms}}\right)^2$$

This is generally lower than the $1/f$ device/circuit noise corner. If Γ_{dc} is minimized

through rise - and fall - time symmetry, then there is the potential for dramatic reductions in $1/f$ noise.

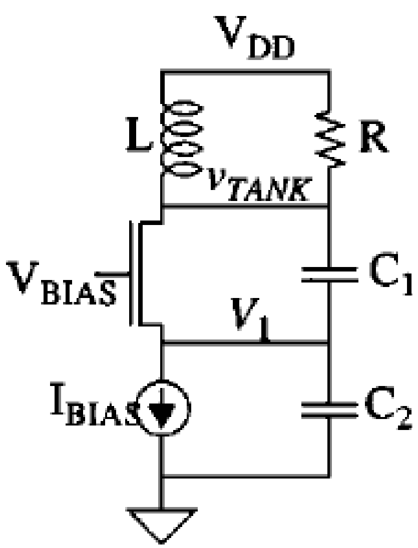
Cyclostationary Noise Treatment

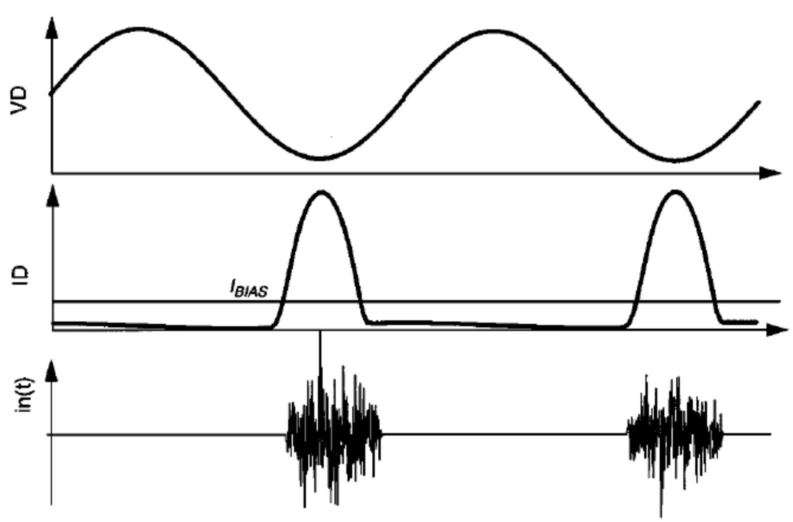
Transistor drain current, and thus noise, can change dramatically over an oscillator cycle. The LTV model can easily handle this by treating it as the product of stationary white noise and a periodic function.

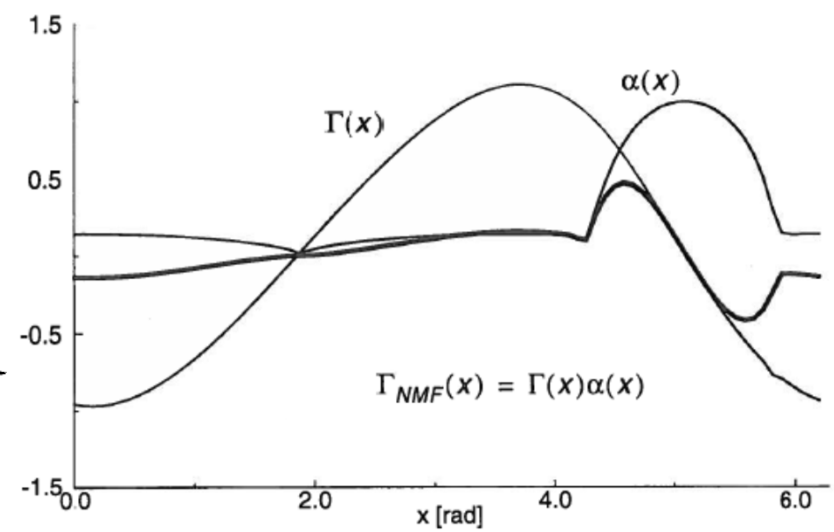
$$i_n(t) = i_{n0}(t)\alpha(\omega_0 t)$$

Here i_{n0} is a stationary white noise source whose peak value is equal to that of the cyclostationary noise source, and $\alpha(x)$ is a periodic unitless function with a peak value of unity. Using this, we can formulate an effective ISF

$$\Gamma_{\mathrm{NMF}}(x) = \Gamma(x)\alpha(x)$$

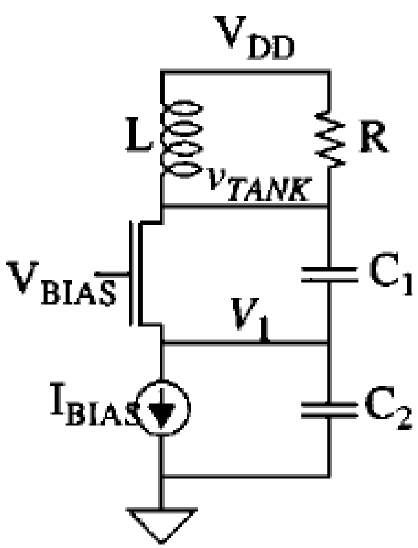


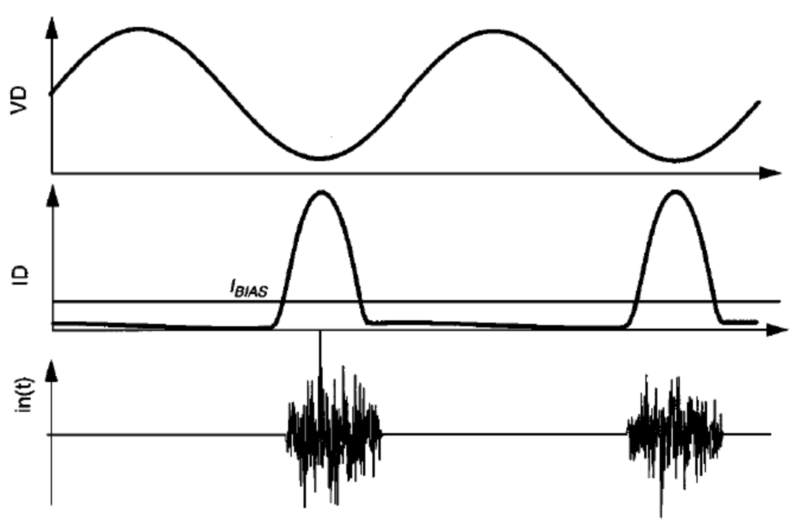


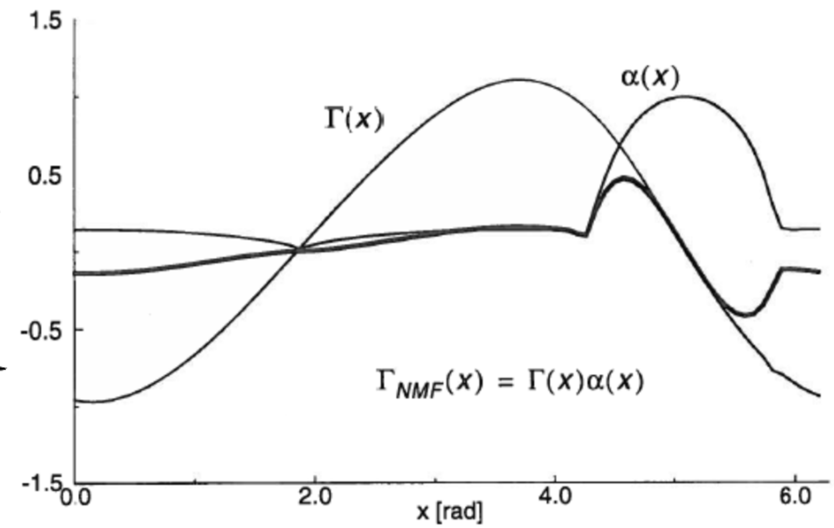


Key Oscillator Design Points from Hajimiri Model

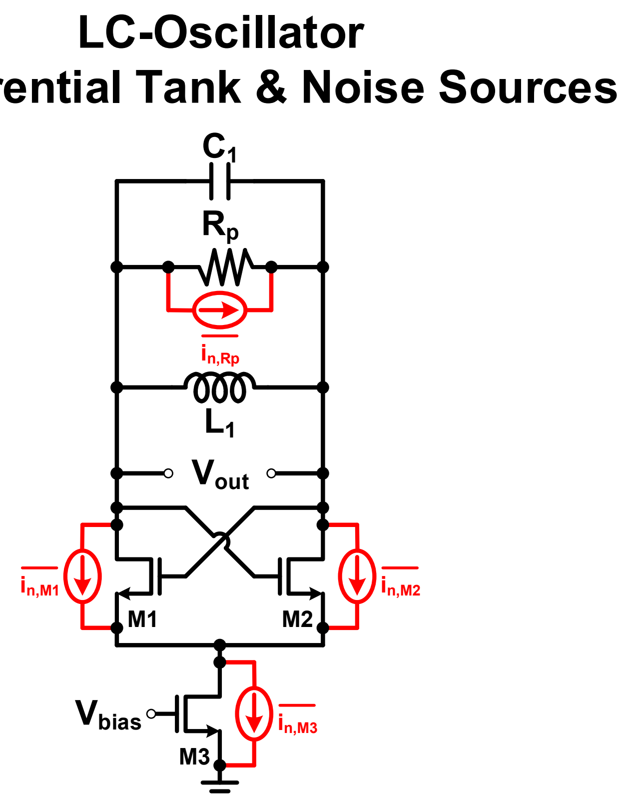
- As the LTI model predicts, oscillator signal power and Q should be maximized
- Ideally, the energy returned to the tank should be delivered all at once when the ISF is minimum
- Oscillators with symmetry properties that have small Γ_{dc} will provide minimum 1/f noise upconversion

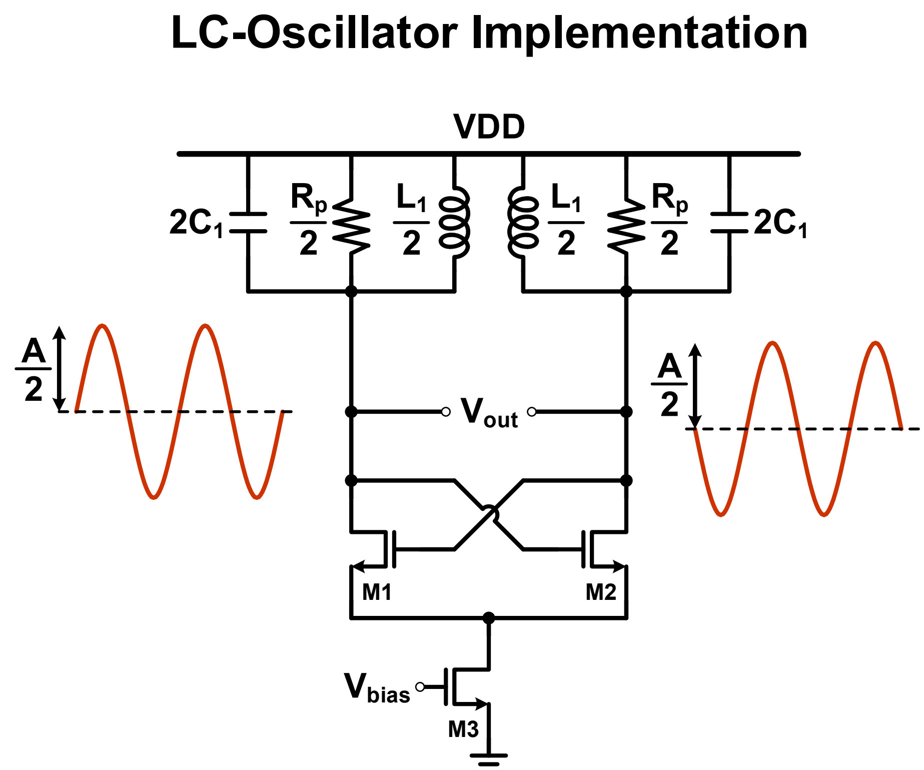






LC-VCO Phase Noise Sources

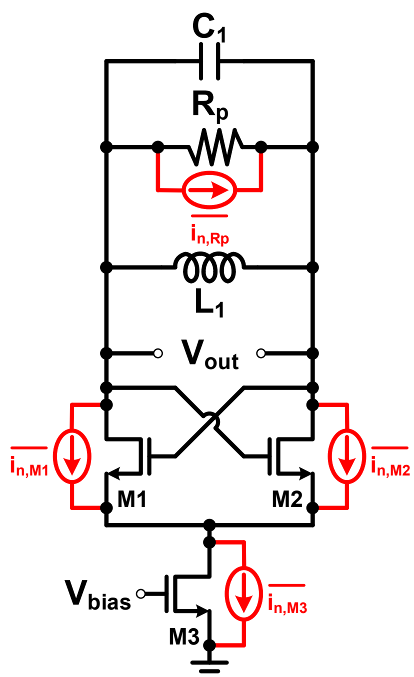




- Finite tank quality factor (R_p)
- Cross-coupled pair (M_1 & M_2)
- Tail current source (M_3)

Tank Noise (R_p)

LC-Oscillator
w/ Differential Tank & Noise Sources



- Two-Sided R_p Noise Spectral Density

$$\overline{I_{n,R_p}^2} = \frac{4kT}{R_p}\left(\frac{1}{2}\right) = \frac{2kT}{R_p}$$

- This gets filtered by the tank impedance near resonance frequency

$$|Z_{tank}(\Delta\omega)|^2 \approx \frac{1}{4C_1^2\Delta\omega^2} = \frac{\omega_0^2 R_p^2}{4Q^2\Delta\omega^2}$$

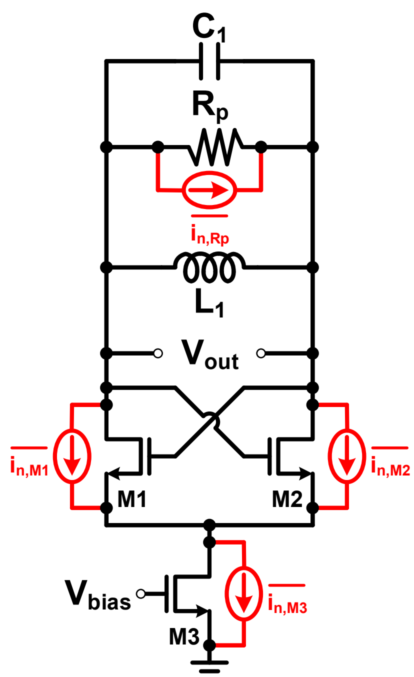
w/ tank $Q = \omega_0 R_p C_1 = \dfrac{R_p}{\omega_0 L_1}$

- Differential Signal Amplitude

$$A = \frac{4}{\pi}\left(\frac{I_{tail}R_p}{2}\right) = \frac{2I_{tail}R_p}{\pi}$$

Tank Noise (R_p)

LC-Oscillator
w/ Differential Tank & Noise Sources



- One-Sided Normalized Phase Noise Spectral Density

Convert to 1-Sided

Equipartition Theorem

$$S_{\phi n}(\Delta\omega) = \frac{\overline{I_n^2}(2)\left(\frac{1}{2}\right)|Z_{tank}(\Delta\omega)|^2}{(Differential\ RMS\ Voltage)^2}$$

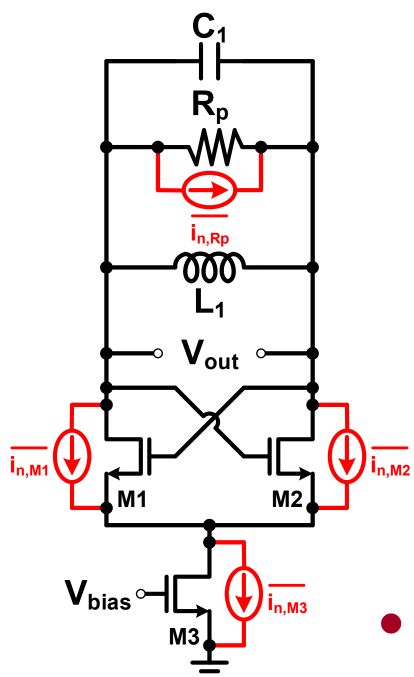
- Tank Noise Phase Noise Contribution

$$\boxed{S_{\phi n,R_p}(\Delta\omega) = \frac{\pi^2 kT}{I_{tail}^2 R_p}\left(\frac{\omega_o^2}{4Q^2\Delta\omega^2}\right)}$$

- Tank phase noise is filtered by the tank impedance and falls off $\propto \frac{1}{\Delta\omega^2}$ (-20dB/dec)
- Phase noise improves $\propto$ Q^2 and signal power

Cross-Coupled Pair Noise (M1 + M2)

LC-Oscillator w/ Differential Tank & Noise Sources



- Two-Sided M1+M2 Noise Density w/ 3/8 Cyclostationary Factor (Thermal Noise Only)

$$\overline{I_{n,M1+M2}^2} = \frac{3}{8} kT\gamma g_m$$

- Cross-Coupled Pair Contribution

$$\boxed{S_{\phi n,M1+M2}(\Delta\omega) = \frac{\pi^2}{2I_{tail}^2}\left(\frac{3}{8} kT\gamma g_m\right)\left(\frac{\omega_o^2}{4Q^2\Delta\omega^2}\right)}$$

- Thermal noise falls off $\propto \frac{1}{\Delta\omega^2}$ (-20dB/dec)

Cross-Coupled Pair Flicker Noise

- If we use a similar approach for the cross-coupled pair flicker noise

$$\overline{I_{n,M1+M2}^2} = \frac{3}{8}\left(\frac{\pi K_F g_m^2}{2WLC_{ox}\Delta\omega}\right)$$

$$\text{Flicker Noise } S_{\phi n,M1+M2}(\Delta\omega) = \frac{\pi^2}{2I_{tail}^2}\left(\frac{3}{8}\right)\left(\frac{\pi K_F g_m^2}{2WLC_{ox}}\right)\left(\frac{\omega_o^2}{4Q^2\Delta\omega^3}\right)$$

- Predicts that flicker noise falls off $\propto \frac{1}{\Delta\omega^3}$ (-30dB/dec)

- However, the expression is not very accurate and is only a conservative estimate that does not capture the oscillator's linear time varying nature

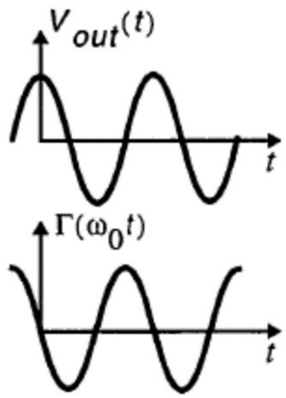
- A more accurate approach extracts the oscillator's time varying impulse sensitivity function (Hajimiri model)

Linear Time Varying Model [Lee JSSC 2000]

- Models output phase noise as convolution with a time-varying impulse response h(t,τ) and the noise source n(t)

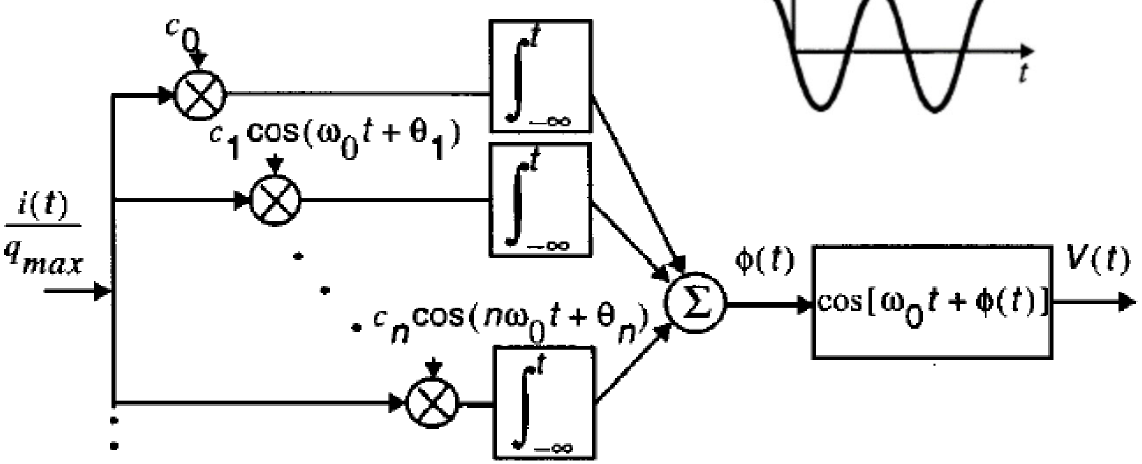
$$\phi(t) = h_\phi(t,\tau) * n(t)$$

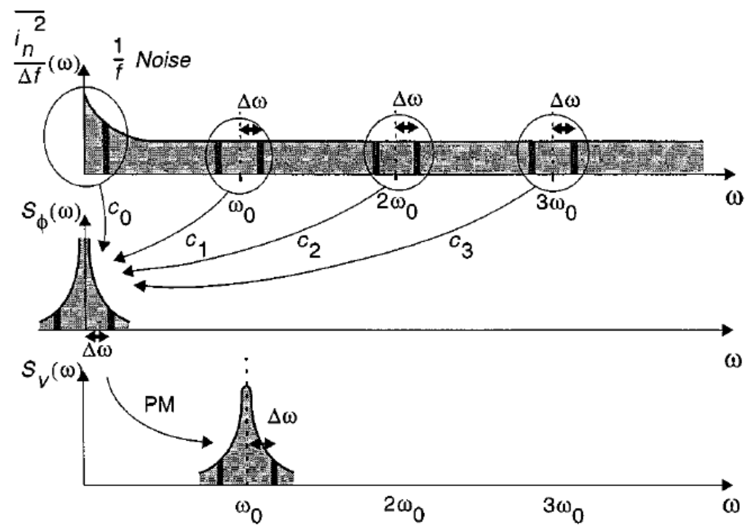
$$h_\phi(t,\tau) = \frac{\Gamma(\omega_o \tau)}{q_{max}} u(t-\tau)$$



Impulse Sensitivity Function

$$\Gamma(\omega_o \tau) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o \tau + \theta_n)$$





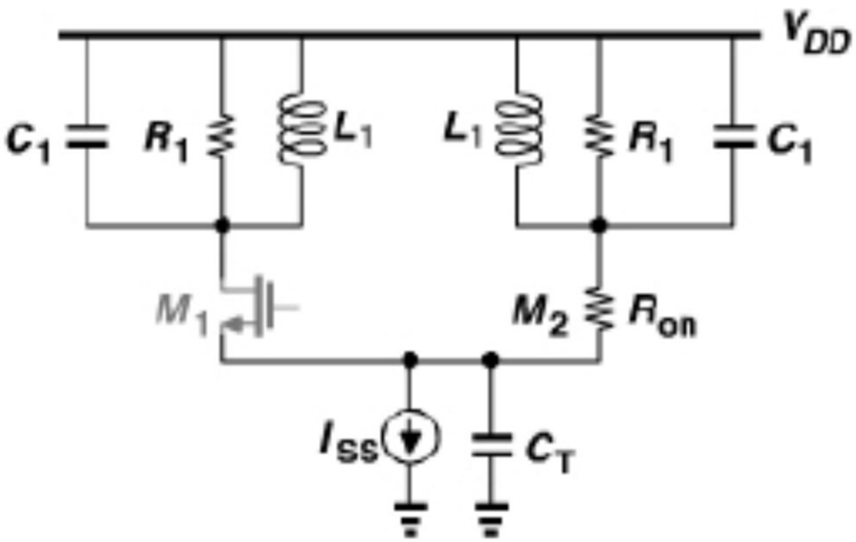
- Making the oscillator's ISF odd-symmetric can minimize c_0 and the flicker noise contribution

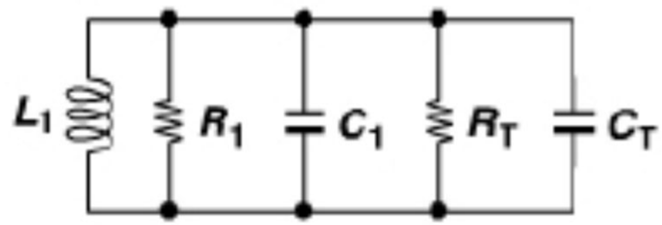
Hajimiri Model Flicker Noise

$$S_{\phi n,M1+M2}(\Delta\omega) = \frac{c_0^2 \pi K_F g_m^2}{4 q_{max} W L C_{ox} \Delta\omega^3}$$

Bias Current Source Noise #1: Tail Capacitance Loading

[Razavi]



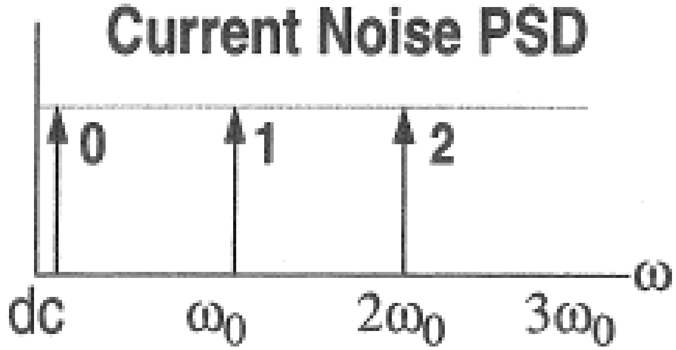


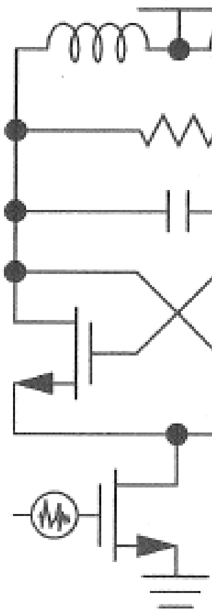
$$R_T = \frac{1}{R_{on} C_T^2 \omega_o^2}$$

- If the oscillator's swing is large the cross-coupled transistors can enter triode and have a small R_{on}
- The tank is then loaded by the series combination of the transistor R_{on} and tail current source capacitance
- If the tail current source capacitance is large due to parasitics or in an attempt to filter the tail current noise, then the tank Q and phase noise degrades

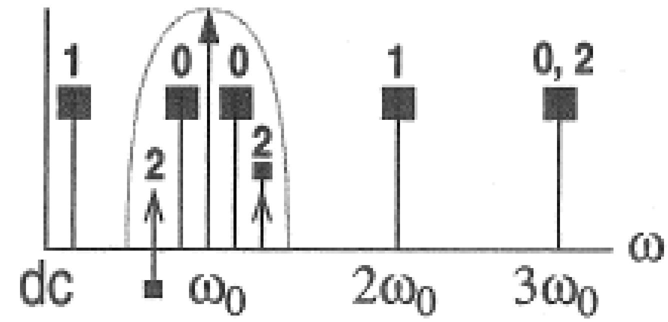
Bias Current Source Noise #2: $2\omega_0$ Noise

[**Hegazi JSSC 2001**]





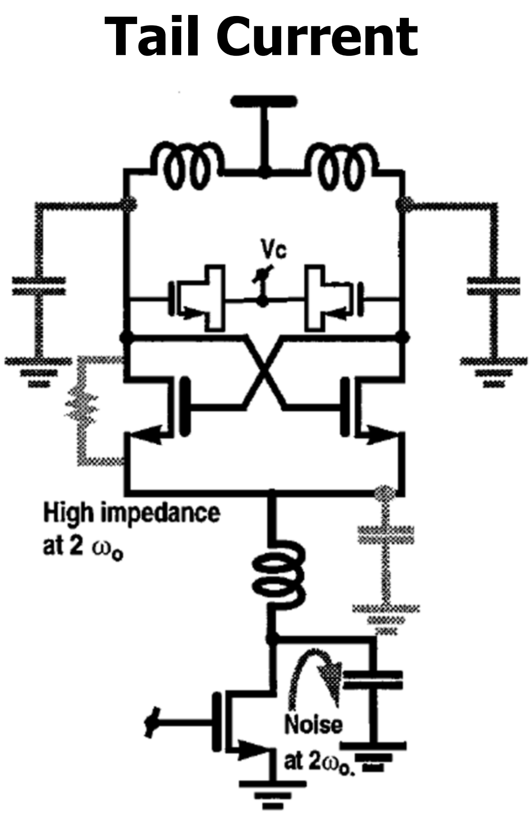
Output PSD

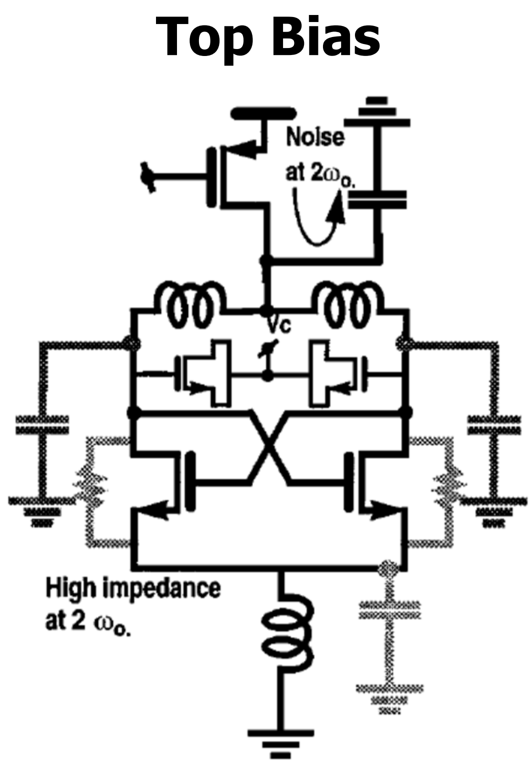


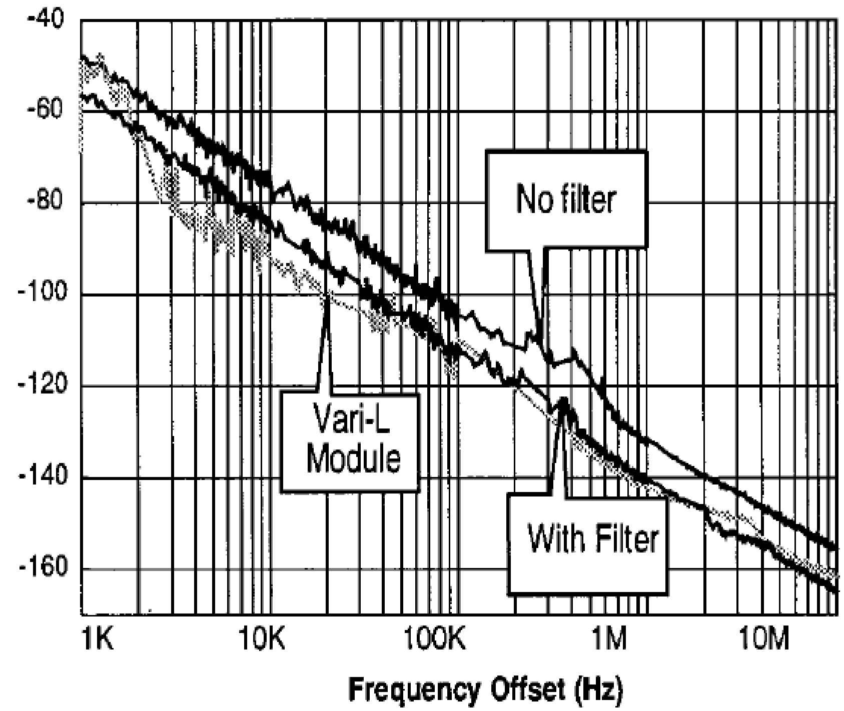
- The switching differential pair can be modeled as a mixer for current source noise
- Low-frequency noise is up-converted near the carrier, but produce components that add in a parallel manner and only produce AM
- Noise near $2\omega_0$ is down-converted near the carrier and produces components that are both parallel (AM) and orthogonal (PM)
- Using a 3rd-order switching model and considering only $2\omega_0$ noise

$$\boxed{S_{\phi n,M3}(\Delta\omega) = \frac{4\overline{I_{n,M3}^2}}{9I_{tail}^2}\left(\frac{\omega_o^2}{4Q^2\Delta\omega^2}\right)}$$

Tail Current Filter to Reduce $2\omega_0$ Noise





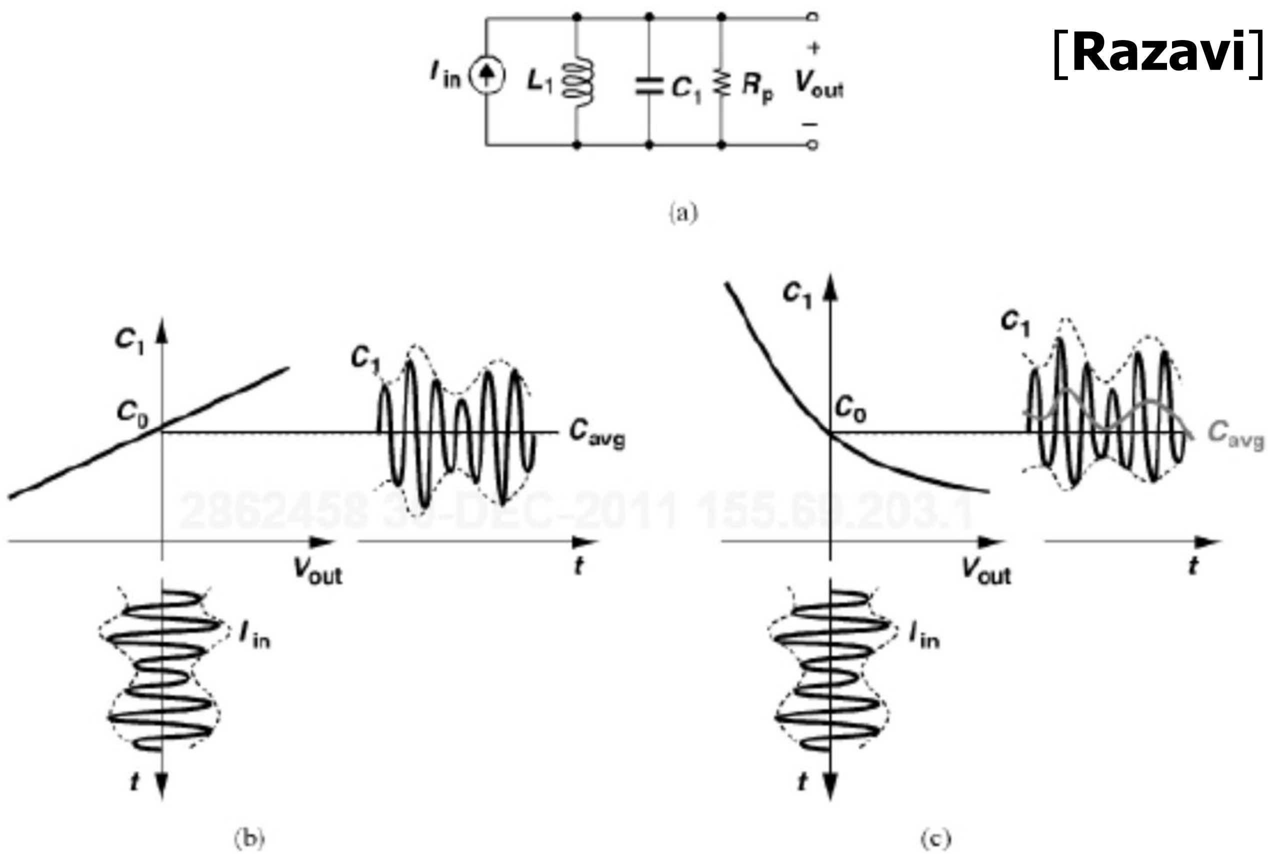


[**Hegazi JSSC 2001**]

- Introducing a tail-current filter to attenuate this second-order harmonic can improve phase noise performance
- Large capacitor in parallel with current source shorts high-frequency noise
- Series inductor resonating with differential transistors' source capacitors at $2\omega_0$ provides a high impedance and reduces tank loading
- Provides near 7dB phase noise improvement

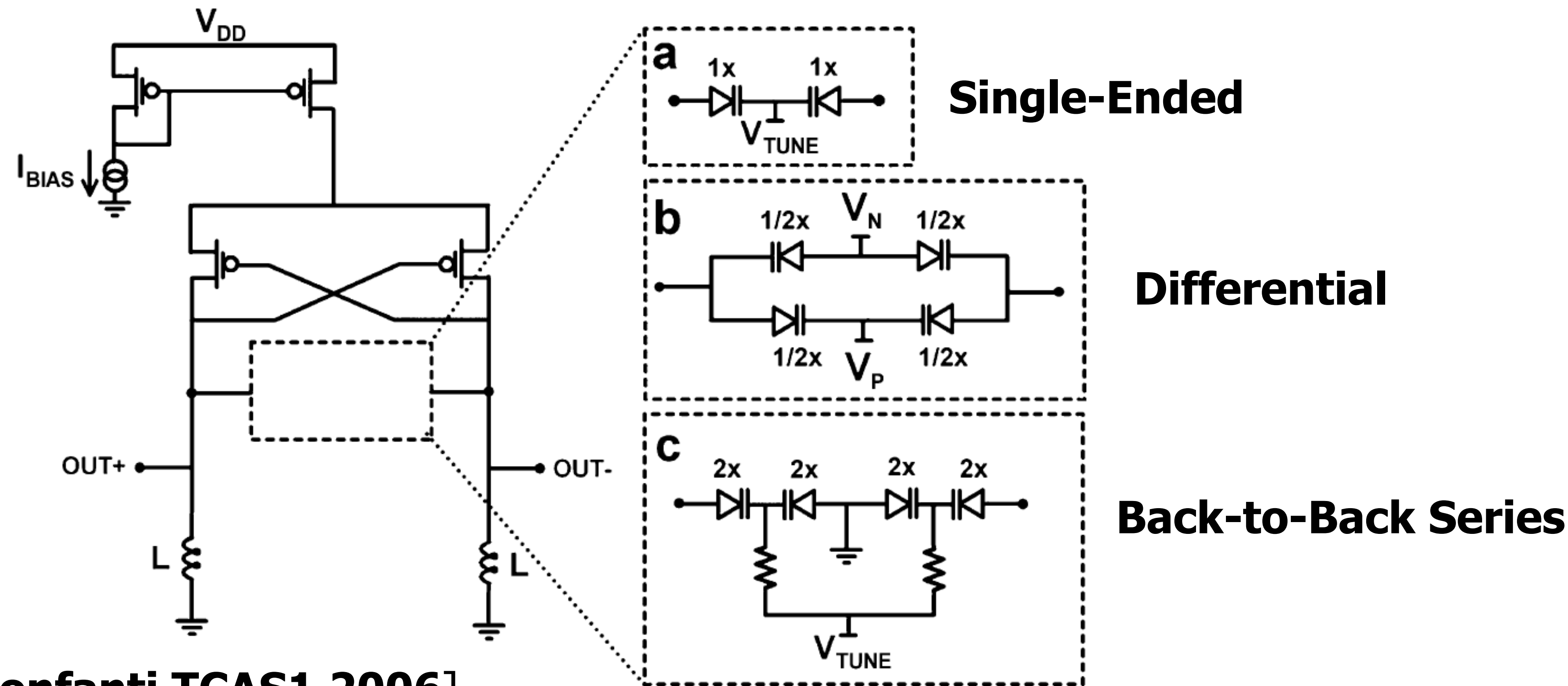
Bias Current Source Noise #3: Flicker Noise AM/PM Conversion Through Varactors

[**Razavi**]



- Tail current flicker noise can produce AM on the carrier
- AM/PM conversion can result if the varactors are not symmetric about the voltage axis (even-order voltage dependence)

Varactor Configurations to Minimize AM/PM

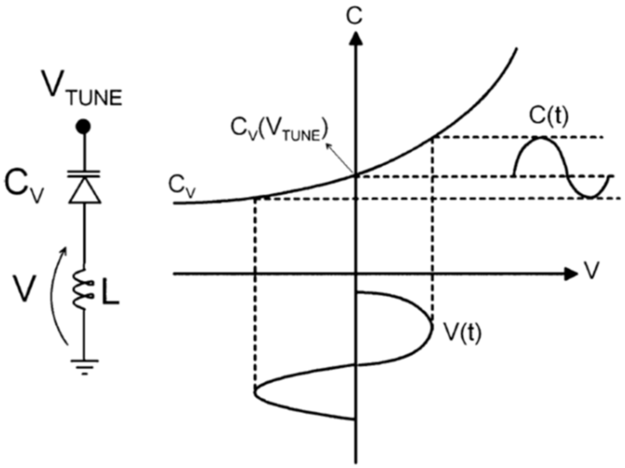


[**Bonfanti TCAS1 2006**]

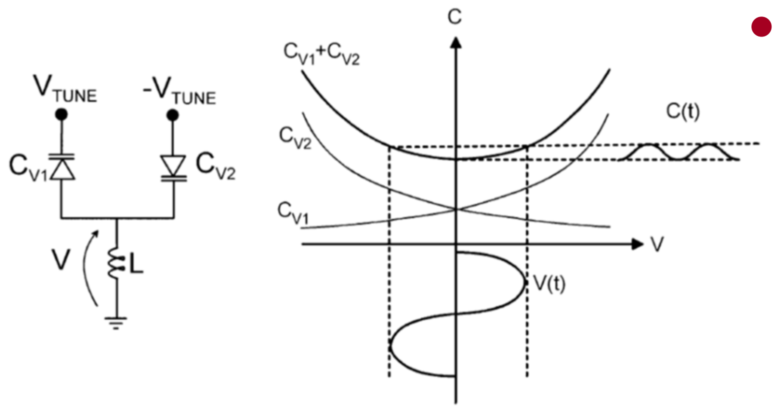
- Goal is to make the varactor more symmetric with changes in oscillator amplitude due to flicker noise

Varactor Configurations to Minimize AM/PM

Single-Ended

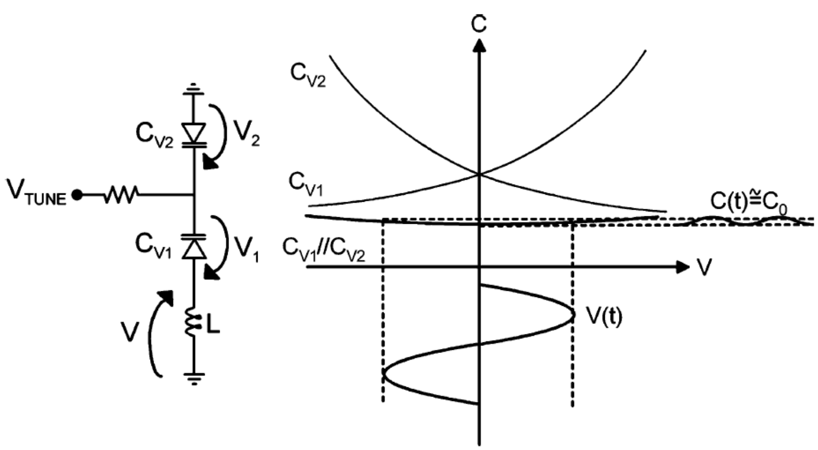


Differential



- Differential is worse due to the always increasing value (even-order dependence)

Back-to-Back Series

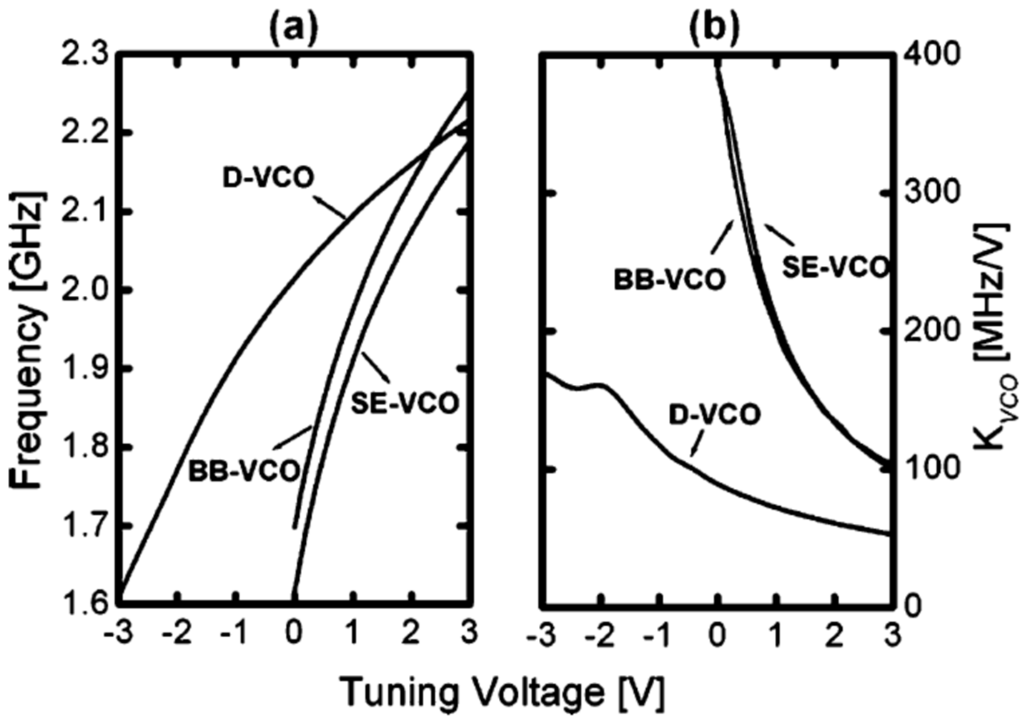


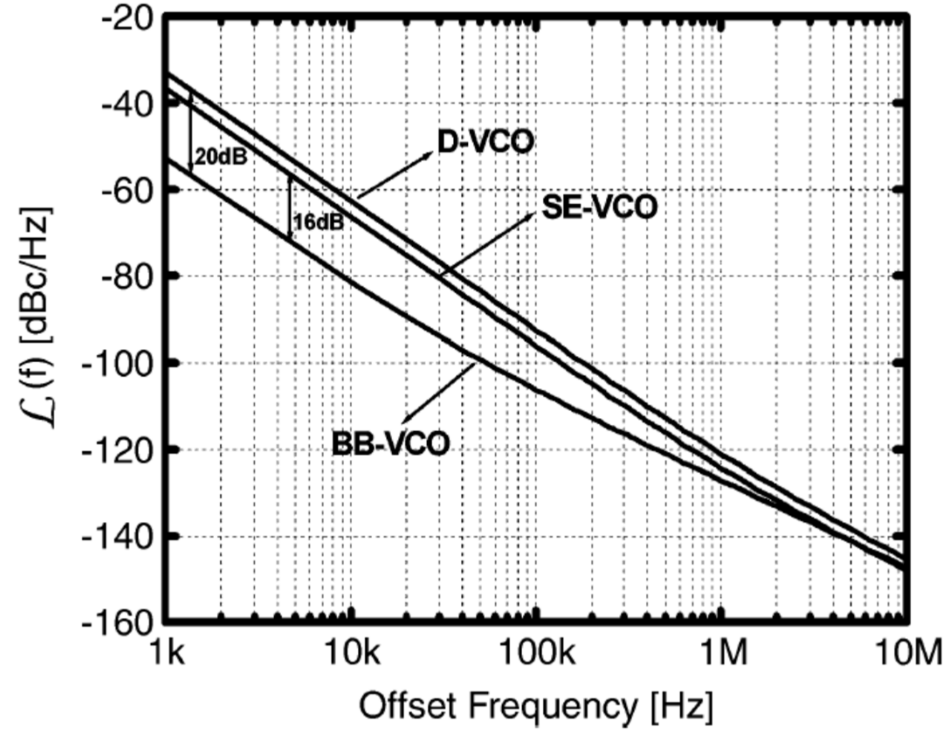
- Tuning voltage is applied with through a resistor that appears as an open at ω_0
- Back-to-back series has the smallest deviation from the average value

[**Bonfanti TCAS1 2006**]

2GHz LC-VCO Simulation Results

[**Bonfanti TCAS1 2006**]

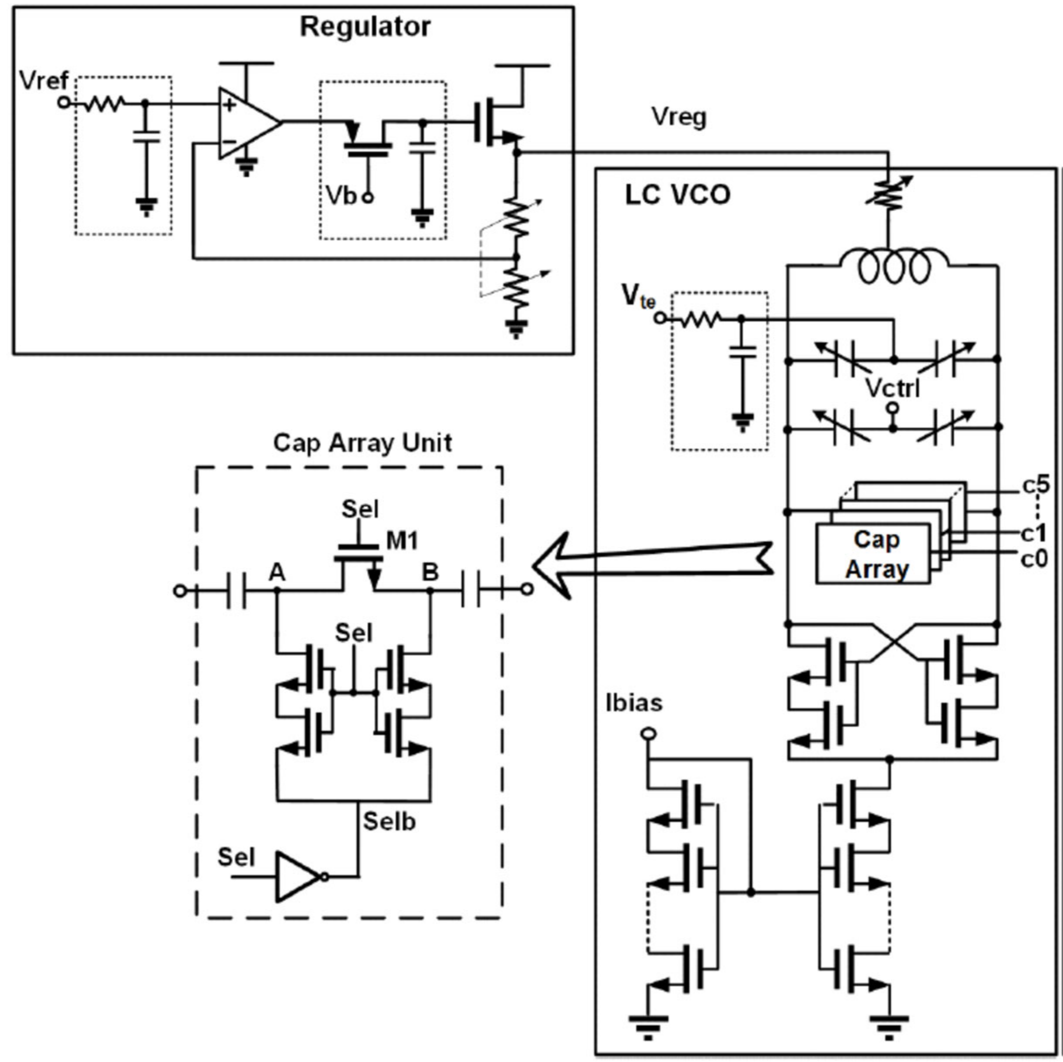




- Back-to-back series has comparable tuning range and KVCO as the default single-ended configuration
- 16dB improvement in low-frequency phase noise
- Main trade-off is increased varactor area (4X)

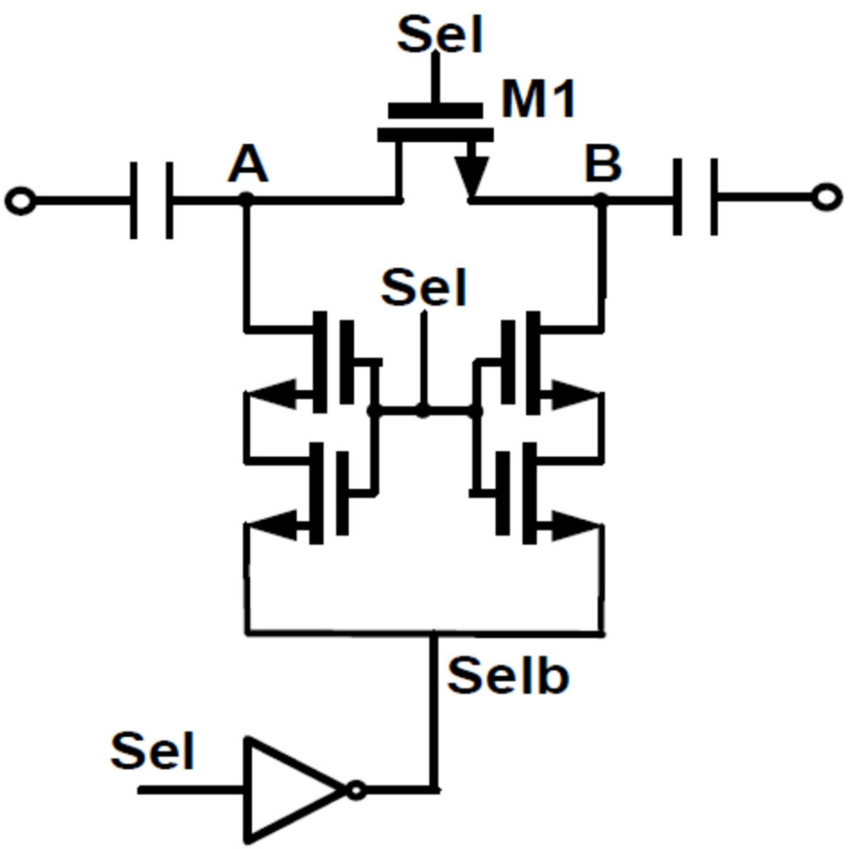
Reducing LC-VCO Noise in FinFET Processes

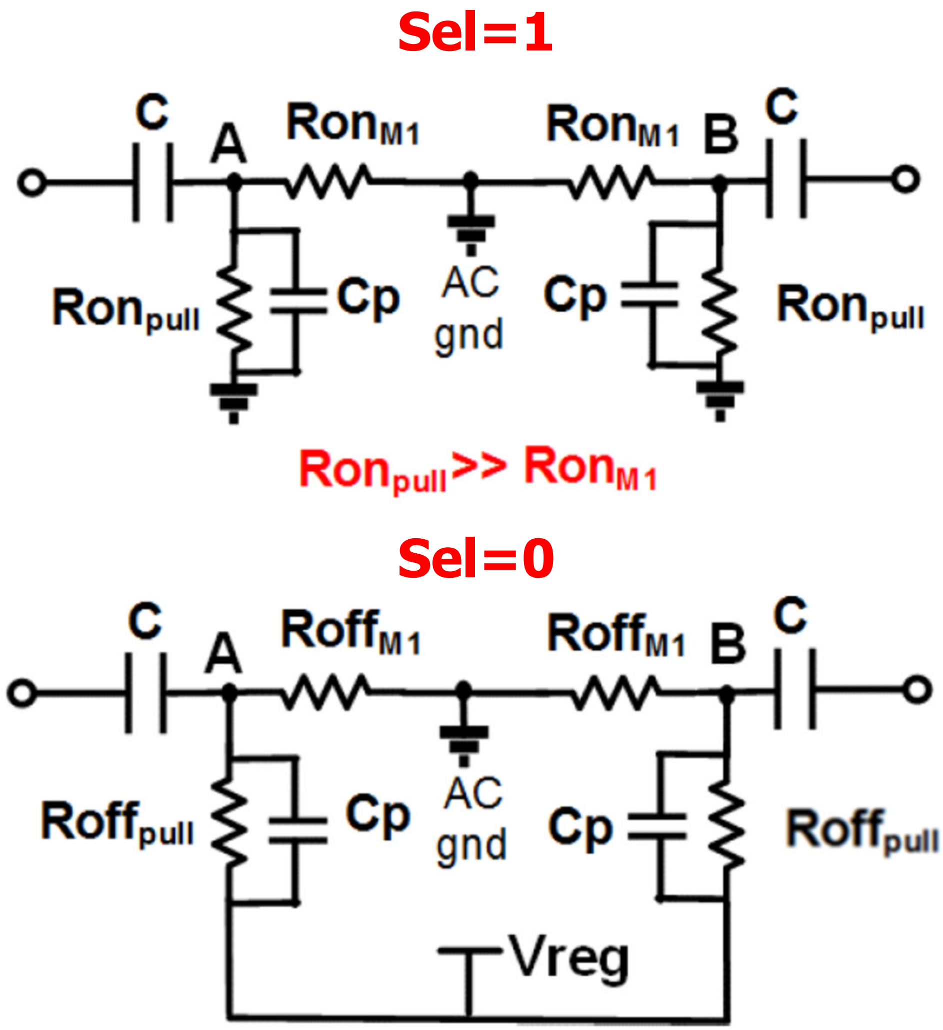
- NMOS only current-limited LC-VCO avoids the high flicker noise present in PMOS transistors
- Stacked devices further reduced flicker noise
- Regulator noise reduced with RC filtering
- Low current multiplication factor to minimize tail current noise
- MOM cap switches designed to improve tank Q



[**Turker ISSCC 2018**]

LC-VCO Capacitor Switch

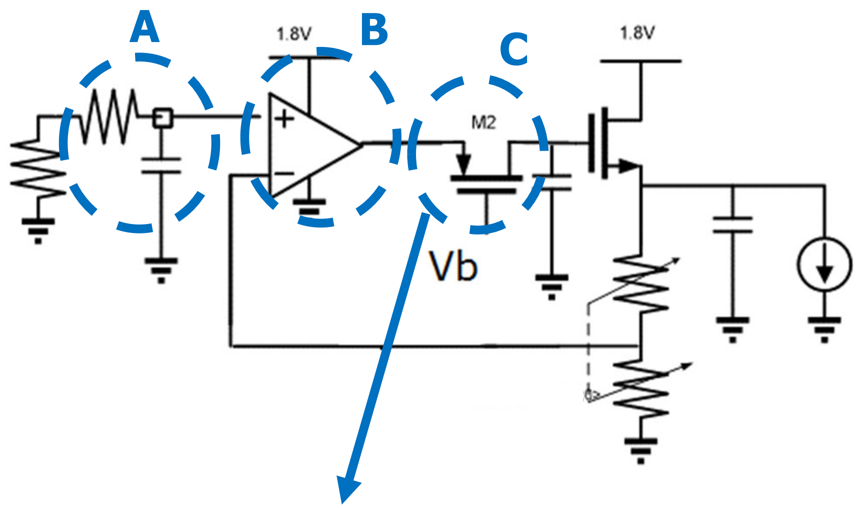




- When sel=0, high impedance stacked transistors pull nodes A & B higher to minimize Q degradation due to M1 leakage

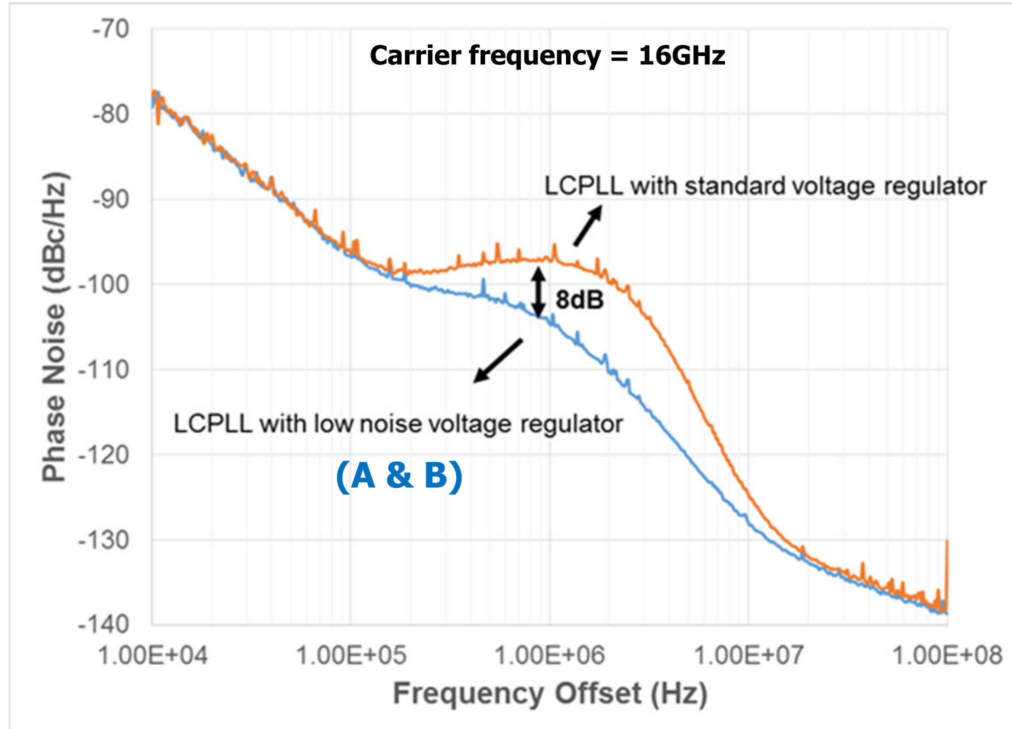
[Turker ISSCC 2018]

Filtering LC-VCO Regulator Flicker Noise



Sub-threshold FET R~= 1M-ohm to 50M-ohm

(C) Further improves noise by ~3.5dB



- RC filter at reference input
- Regulator designed with transistor stacking, double gate contacts, and large width & lengths
- Programmable sub-threshold FET resistor to realize kHz range filter

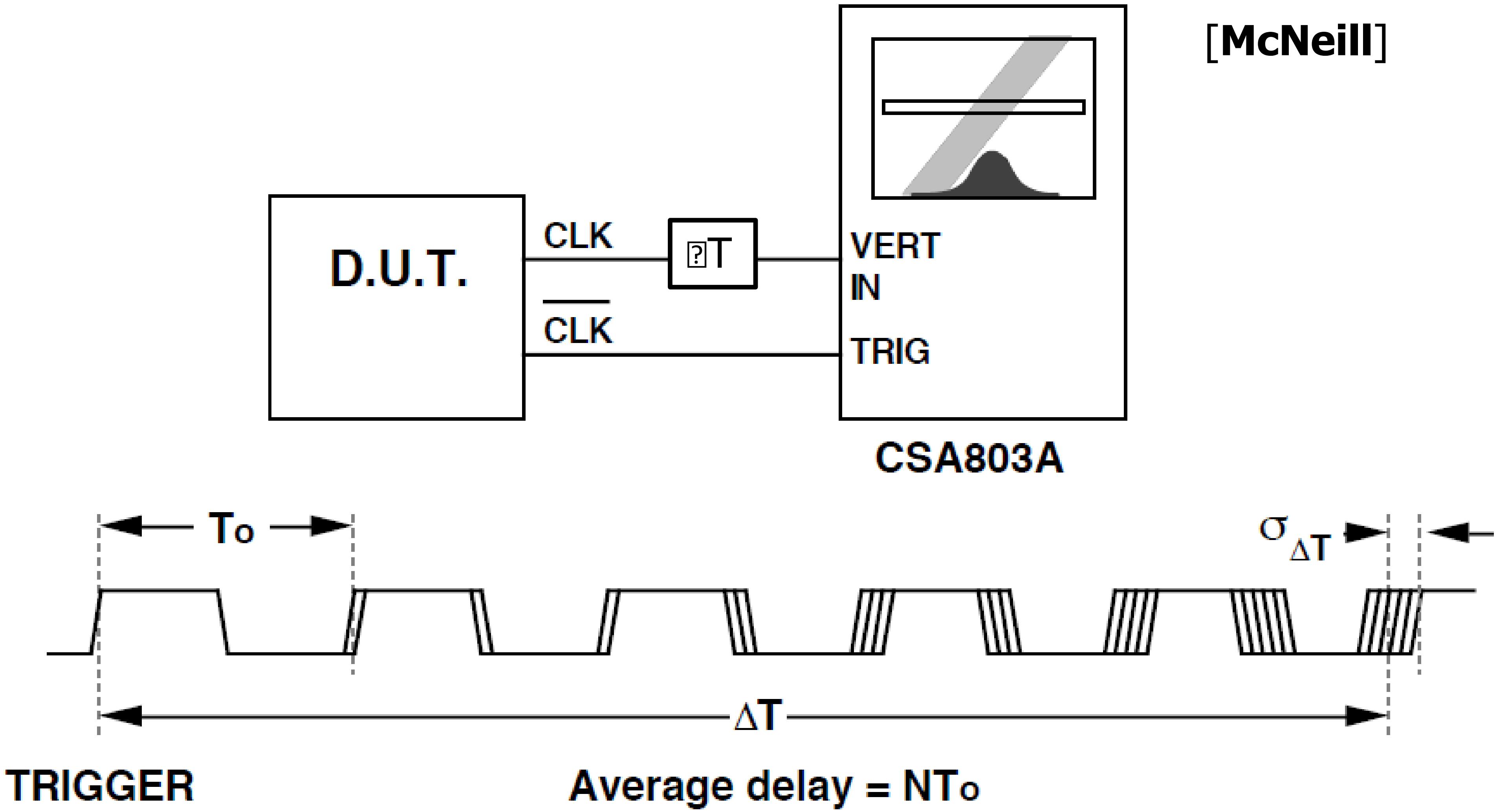
[**Turker ISSCC 2018**]

Agenda

- VCO Fundamentals
- VCO Examples
- VCO Phase Noise
  - Phase Noise Definition and Impact
  - Ideal Oscillator Phase Noise
  - Leeson Model
  - Hajimiri Model
  - LC-VCO Phase Noise Sources
- VCO Jitter

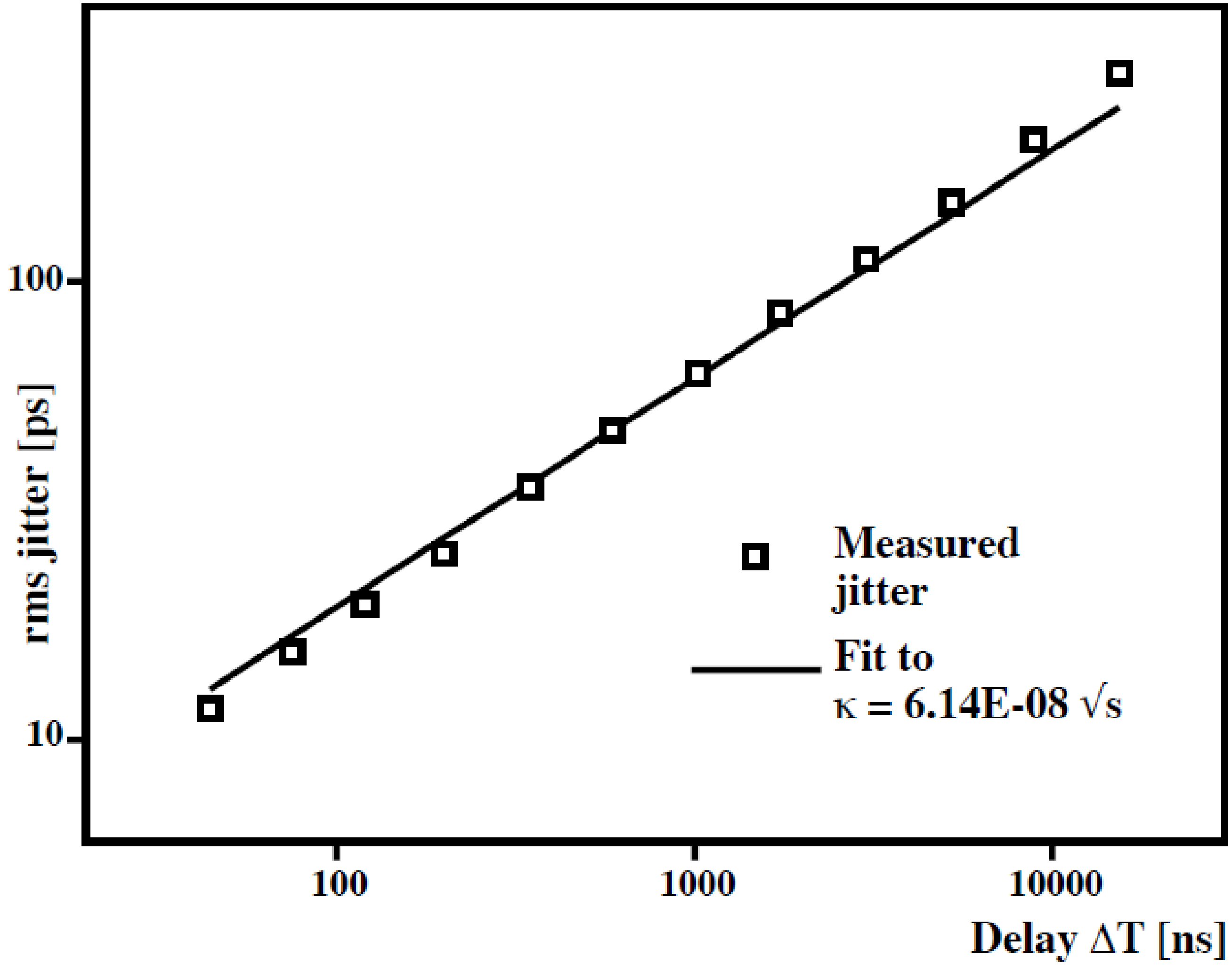
Open-Loop VCO Jitter

[**McNeill**]



- Measure distribution of clock threshold crossings
- Plot σ as a function of delay ΔT

Open-Loop VCO Jitter

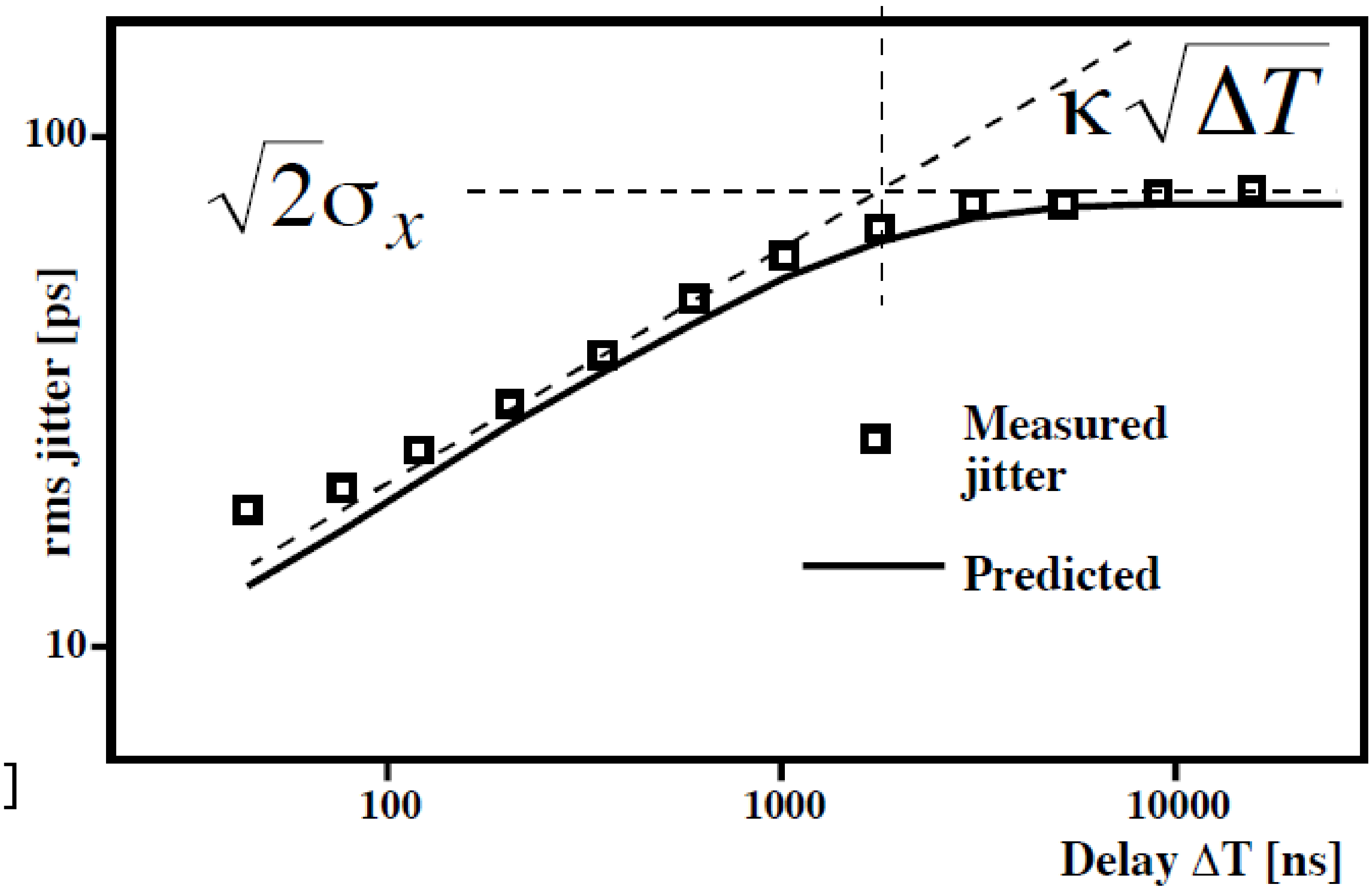


[**McNeill**]

$$\sigma_{\Delta T(OL)}\left(\Delta T\right) \approx \kappa\sqrt{\Delta T}$$

- Jitter σ is proportional to sqrt(ΔT)
- K is VCO time domain figure of merit

VCO in Closed-Loop PLL Jitter



[**McNeill**]

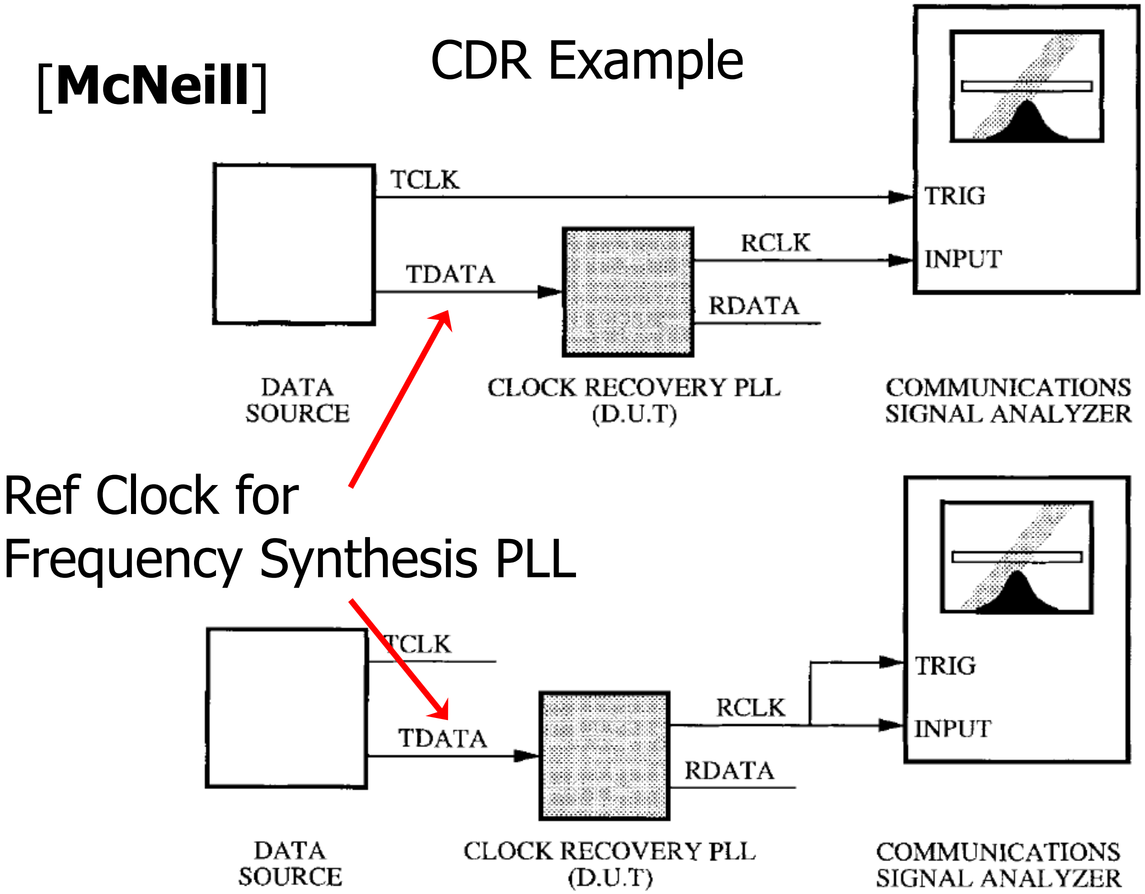
- PLL limits σ for delays longer than loop bandwidth τ_L

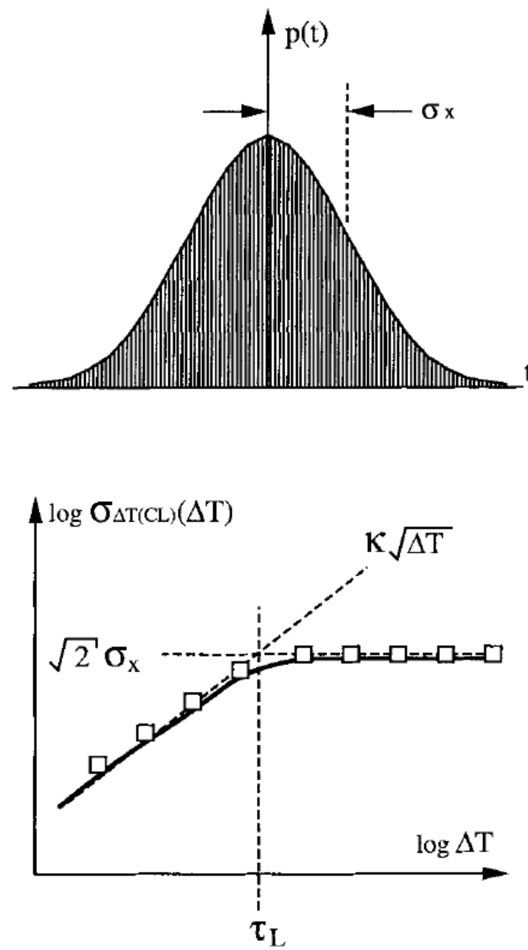
$$\tau_L = 1/2\pi f_L$$

Ref Clk-Referenced vs Self-Referenced

[**McNeill**]

CDR Example

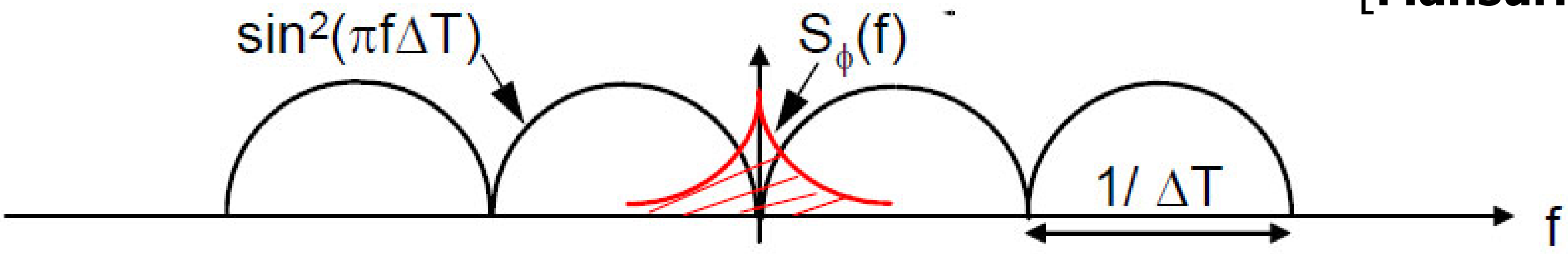




- Generally, we care about the jitter w.r.t. the ref. clock (σ_x)
- However, may be easier to measure w.r.t. delayed version of output clk
  - Due to noise on both edges, this will be increased by a sqrt(2) factor relative to the reference clock-referred jitter

Converting Phase Noise to Jitter

[**Mansuri**]



- RMS jitter for ΔT accumulation $$\sigma_{\Delta T}^2 = \frac{4}{\omega_o^2}\int_0^{\infty} S_\phi(f)\sin^2(\pi f \Delta T)df$$
- As ΔT goes to ∞ $$\sigma_T^2 = \frac{2}{\omega_o^2}R_\phi(0) = \frac{2}{\omega_o^2}\int_0^{\infty} S_\phi(f)df$$
- Actual integration range depends on application bandwidth
  - f_{min} set by asumed CDR tracking bandwidth
  - f_{max} set by Nyquist frequency ($f_0/2$)
- Most exact approach $$\sigma_T^2 = \frac{2}{\omega_o^2}\int_0^{f_0/2} S_\phi(f)\left|H_{sys}(f)\right|^2 df$$

where $\left|H_{sys}(f)\right|^2$ is the system jitter transfer function

Next Time

- Divider Circuits