23-03-

REALIZATION OF FIR:

1) DIRECT FORM REALIZATION / TRANSVERSAL STRUCTURE:

$$\boxed{H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}}$$

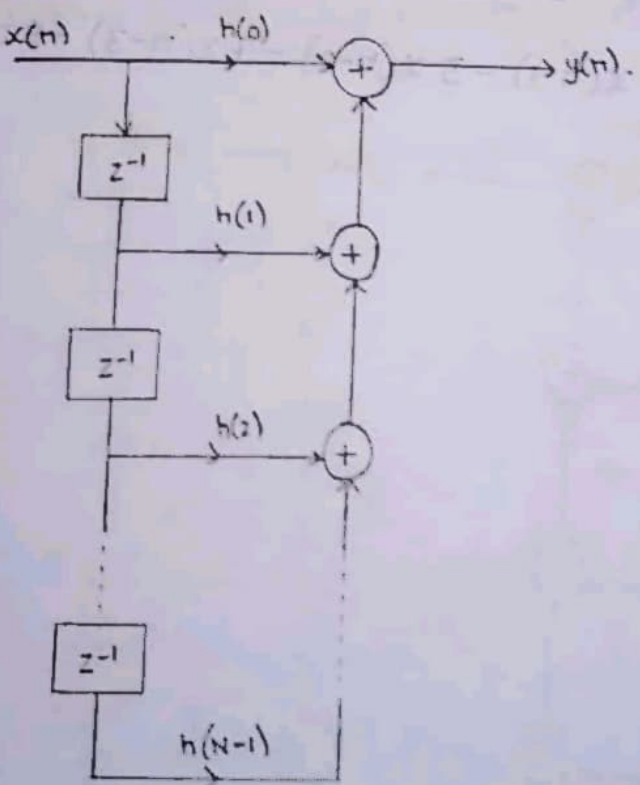
$$\frac{Y(z)}{X(z)} = h(0) + h(1) z^{-1} + h(2) z^{-2} + \cdots + h(N-1) z^{-(N-1)}$$

$$Y(z) = h(0) X(z) + h(1) z^{-1} X(z) + h(2) z^{-2} X(z) + \cdots + h(N-1) z^{-(N-1)} X(z)$$

Applying Inverse z-transform

$$y(n) = h(0) x(n) + h(1) x(n-1) + h(2) x(n-2) + \cdots + h(N-1) x(n-N+1)$$

REALIZATION STRUCTURE:



(OR)

REALIZATION OF FIR:

1) DIRECT FORM REALIZATION / TRANSVERSAL STRUCTURE:

$$\boxed{H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}}$$

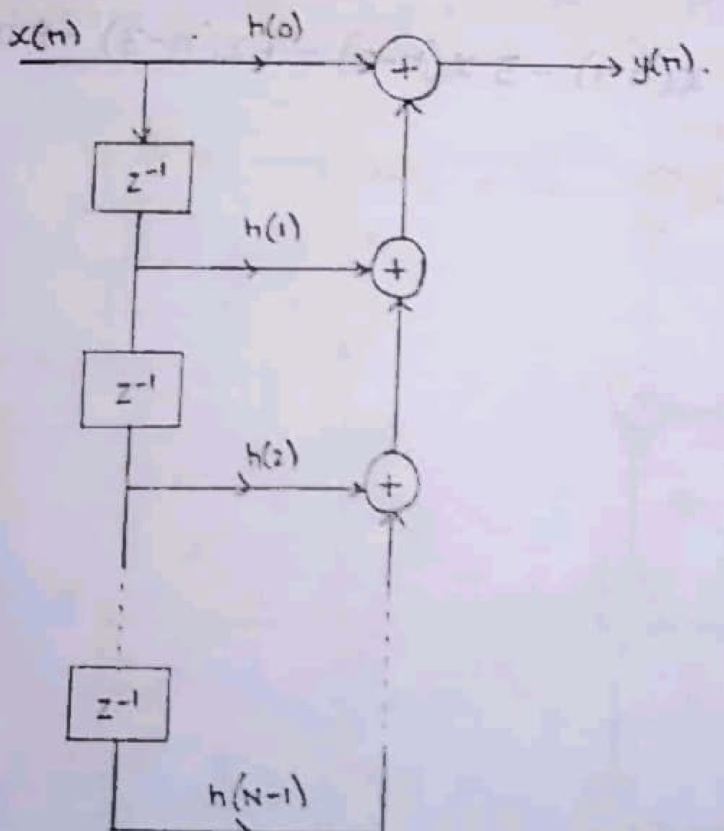
$$\frac{Y(z)}{X(z)} = h(0) + h(1) z^{-1} + h(2) z^{-2} + \cdots + h(N-1) z^{-(N-1)}$$

$$Y(z) = h(0)\, X(z) + h(1) z^{-1} X(z) + h(2) z^{-2} X(z) + \cdots$$
$$\cdots + h(N-1)\, z^{-(N-1)} X(z)$$

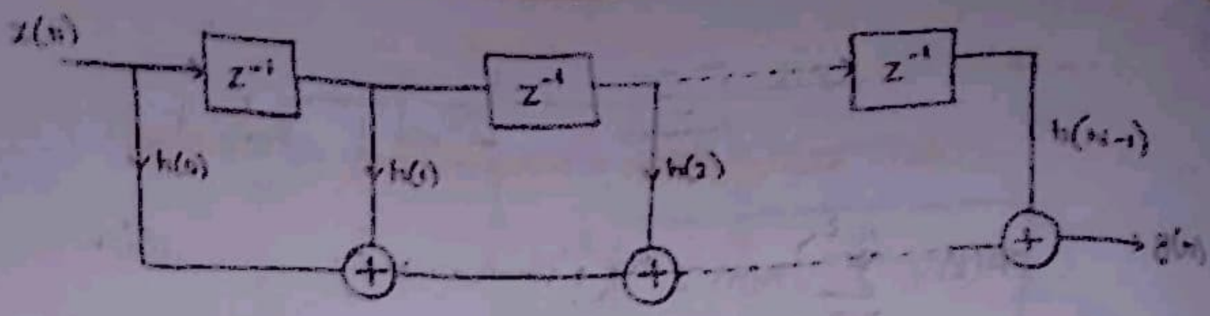
Applying Inverse z-transform

$$y(n) = h(0)\, x(n) + h(1)\, x(n-1) + h(2)\, x(n-2) + \cdots + h(N-1)\, x(n-N+1)$$

REALIZATION STRUCTURE:



(OR)



1) Determine the direct form Realization of the System function $H(z) = 1+2z^{-1}-3z^{-2}-4z^{-3}+8z^{-4}$.

$$H(z) = 1+2z^{-1}-3z^{-2}-4z^{-3}+8z^{-4}$$

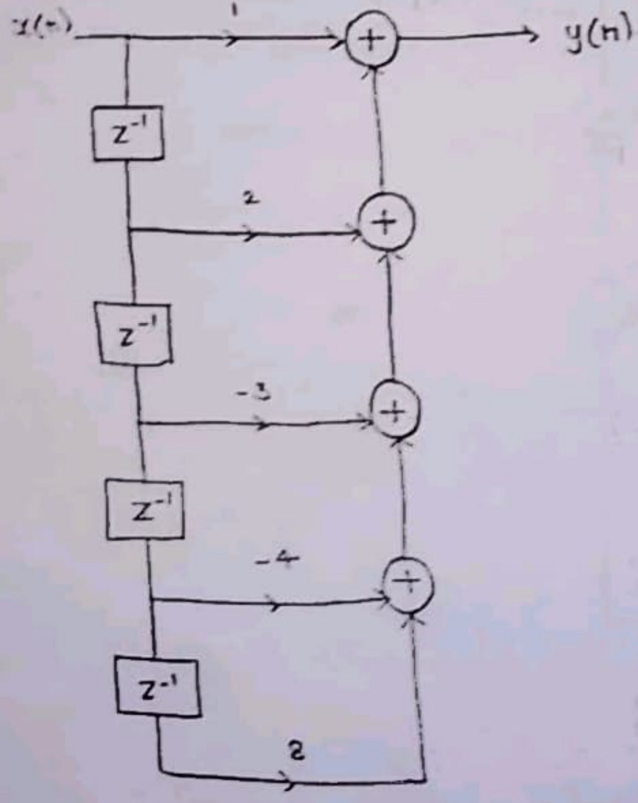
$$\frac{Y(z)}{X(z)} = 1+2z^{-1}-3z^{-2}-4z^{-3}+8z^{-4}$$

$$Y(z) = X(z)+2z^{-1}X(z)-3z^{-2}X(z)-4z^{-3}X(z)+8z^{-4}X(z)$$

Applying Inverse Z-transform on both sides

$$y(n) = x(n)+2x(n-1)-3x(n-2)-4x(n-3)+8x(n-4)$$

REALIZATION STRUCTURE :



2> Determine Direct form Realization of the system function $H(z) = 1 + \frac{1}{5}z^{-1} + \frac{1}{8}z^{-2} + \frac{1}{3}z^{-4} + \frac{1}{5}z^{-5}$

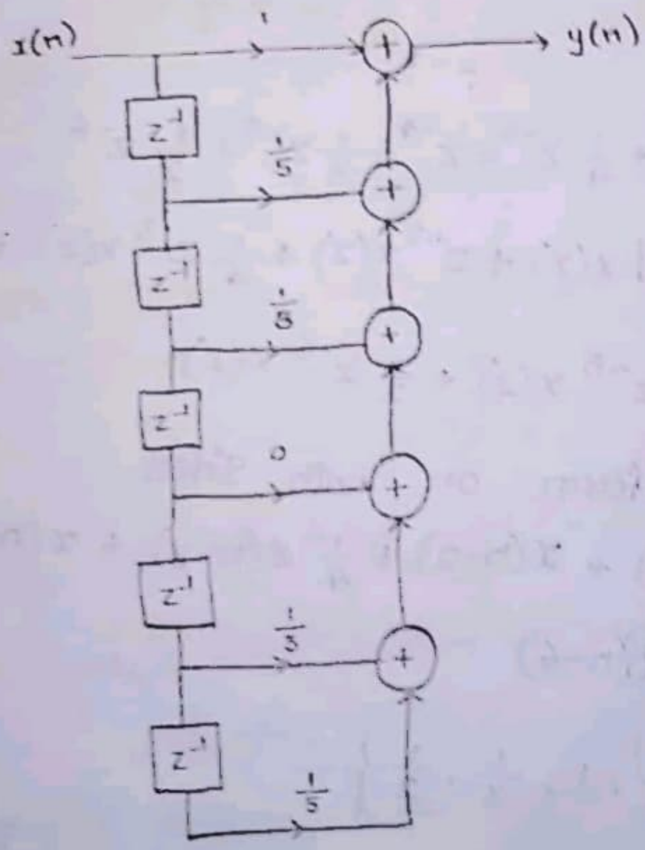
$$H(z) = 1 + \frac{1}{5}z^{-1} + \frac{1}{8}z^{-2} + \frac{1}{3}z^{-4} + \frac{1}{5}z^{-5}$$

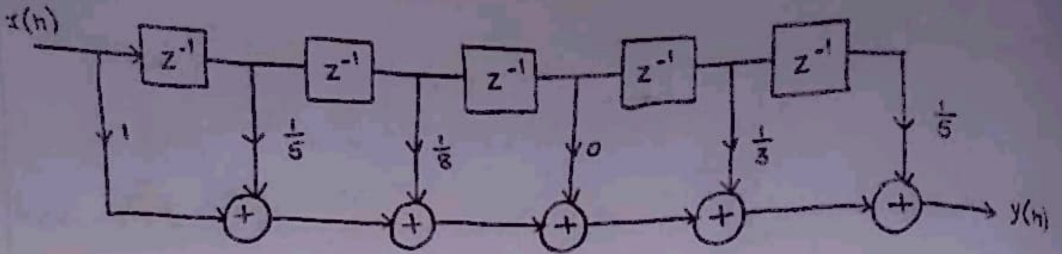
$$\frac{Y(z)}{X(z)} = 1 + \frac{1}{5}z^{-1} + \frac{1}{8}z^{-2} + \frac{1}{3}z^{-4} + \frac{1}{5}z^{-5}$$

$$Y(z) = X(z) + \frac{1}{5}z^{-1}X(z) + \frac{1}{8}z^{-2}X(z) + \frac{1}{3}z^{-4}X(z) + \frac{1}{5}z^{-5}X(z)$$

Applying inverse z-transform on both sides

$$y(n) = x(n) + \frac{1}{5}x(n-1) + \frac{1}{8}x(n-2) + \frac{1}{3}x(n-4) + \frac{1}{5}x(n-5)$$





25-03-19

LINEAR PHASE REALIZATION OR MINIMUM NUMBER OF MULTIPLIER'S :

For Linear phase FIR filter

$$h(n) = h(N-1-n)$$

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

1) obtain Direct form Realization with min. no. of multipliers for the system transfer function

$H(z) = \frac{1}{2} + \frac{1}{3} z^{-1} + z^{-2} + \frac{1}{4} z^{-3} + z^{-4} + \frac{1}{3} z^{-5} + \frac{1}{2} z^{-6}$

$$\frac{Y(z)}{X(z)} = \frac{1}{2} + \frac{1}{3} z^{-1} + z^{-2} + \frac{1}{4} z^{-3} + z^{-4} + \frac{1}{3} z^{-5} + \frac{1}{2} z^{-6}$$

$$Y(z) = \frac{1}{2} X(z) + \frac{1}{3} z^{-1} X(z) + z^{-2} X(z) + \frac{1}{4} z^{-3} X(z) + z^{-4} X(z) + \frac{1}{3} z^{-5} X(z) + \frac{1}{2} z^{-6} X(z)$$

Apply Inverse z-transform on both sides

$$y(n) = \frac{1}{2} x(n) + \frac{1}{3} x(n-1) + x(n-2) + \frac{1}{4} x(n-3) + x(n-4) + \frac{1}{3} x(n-5) + \frac{1}{2} x(n-6)$$

$$h(n) = \left\{ \frac{1}{2}, \frac{1}{3}, 1, \frac{1}{4}, 1, \frac{1}{3}, \frac{1}{2} \right\}$$

$$h(n) = h(N-1-n)$$

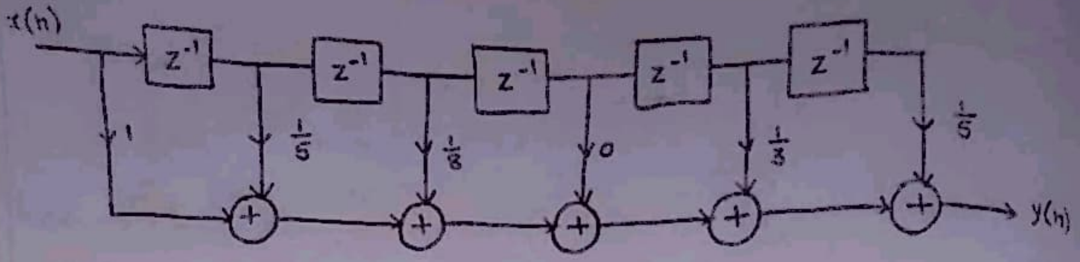
$N = 7$

$n=0 \longrightarrow h(0) = h(7-1-0) = h(6)$

$n=1 \longrightarrow h(1) = h(7-1-1) = h(5)$

$n=2 \longrightarrow h(2) = h(7-1-2) = h(4)$

$h(3)$



25-03-19

LINEAR PHASE REALIZATION OR MINIMUM NUMBER OF MULTIPLIER'S :

For Linear phase FIR filter

$$h(n) = h(N-1-n)$$

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

1) obtain Direct form Realization with min. no. of multipliers for the system transfer function

$$H(z) = \frac{1}{2} + \frac{1}{3} z^{-1} + z^{-2} + \frac{1}{4} z^{-3} + z^{-4} + \frac{1}{3} z^{-5} + \frac{1}{2} z^{-6}$$

$$\frac{Y(z)}{X(z)} = \frac{1}{2} + \frac{1}{3} z^{-1} + z^{-2} + \frac{1}{4} z^{-3} + z^{-4} + \frac{1}{3} z^{-5} + \frac{1}{2} z^{-6}$$

$$Y(z) = \frac{1}{2} X(z) + \frac{1}{3} z^{-1} X(z) + z^{-2} X(z) + \frac{1}{4} z^{-3} X(z) + z^{-4} X(z) + \frac{1}{3} z^{-5} X(z) + \frac{1}{2} z^{-6} X(z)$$

Apply Inverse z-transform on both sides

$$y(n) = \frac{1}{2} x(n) + \frac{1}{3} x(n-1) + x(n-2) + \frac{1}{4} x(n-3) + x(n-4) + \frac{1}{3} x(n-5) + \frac{1}{2} x(n-6)$$

$$h(n) = \left\{ \frac{1}{2}, \frac{1}{3}, 1, \frac{1}{4}, 1, \frac{1}{3}, \frac{1}{2} \right\}$$

$$h(n) = h(N-1-n)$$

$N = 7$

$n=0 \longrightarrow h(0) = h(7-1-0) = h(6)$

$n=1 \longrightarrow h(1) = h(7-1-1) = h(5)$

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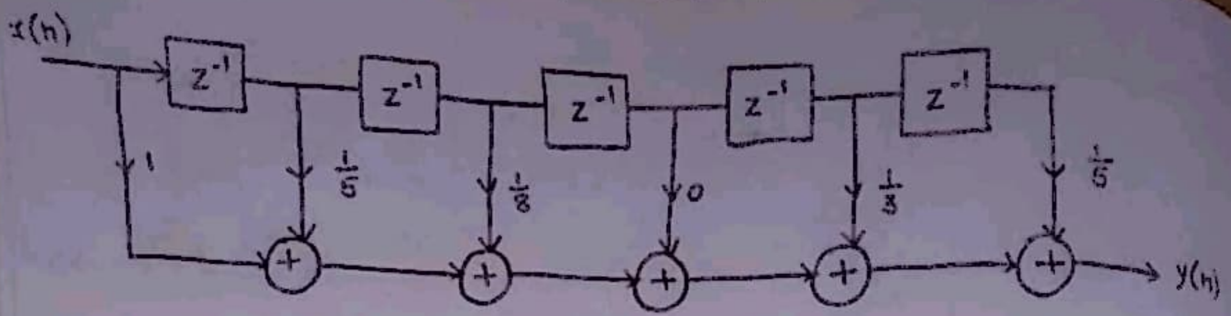
$h(3)$



25-03-19

LINEAR PHASE REALIZATION OR MINIMUM NUMBER OF MULTIPLIER'S :

For Linear phase FIR filter

$$h(n) = h(N-1-n)$$

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}$$

1) Obtain Direct form Realization with min. no. of multipliers for the system transfer function

$$H(z) = \frac{1}{2} + \frac{1}{3}z^{-1} + z^{-2} + \frac{1}{4}z^{-3} + z^{-4} + \frac{1}{3}z^{-5} + \frac{1}{2}z^{-6}$$

$$\frac{Y(z)}{X(z)} = \frac{1}{2} + \frac{1}{3}z^{-1} + z^{-2} + \frac{1}{4}z^{-3} + z^{-4} + \frac{1}{3}z^{-5} + \frac{1}{2}z^{-6}$$

$$Y(z) = \frac{1}{2}X(z) + \frac{1}{3}z^{-1}X(z) + z^{-2}X(z) + \frac{1}{4}z^{-3}X(z)$$
$$z^{-4}X(z) + \frac{1}{3}z^{-5}X(z) + \frac{1}{2}z^{-6}X(z)$$

Apply Inverse z-transform on both sides

$$y(n) = \frac{1}{2}x(n) + \frac{1}{3}x(n-1) + x(n-2) + \frac{1}{4}x(n-3) + x(n-4)$$
$$\frac{1}{3}x(n-5) + \frac{1}{2}x(n-6)$$

$$h(n) = \left\{ \frac{1}{2}, \frac{1}{3}, 1, \frac{1}{4}, 1, \frac{1}{3}, \frac{1}{2} \right\}$$

$$h(n) = h(N-1-n)$$

$N = 7$

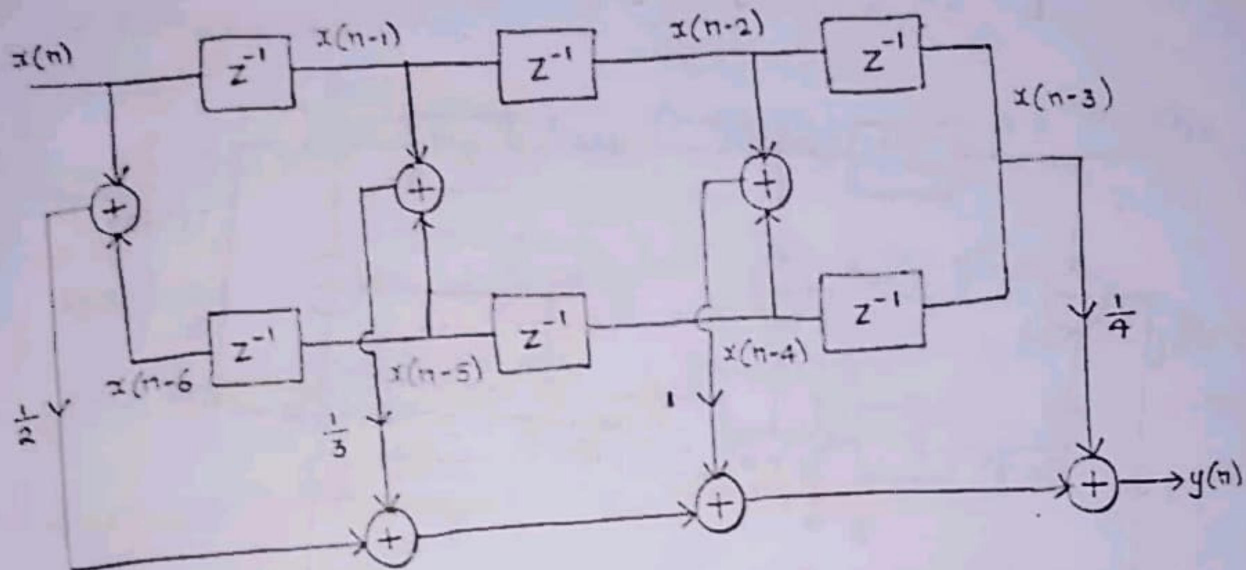
$n=0 \longrightarrow h(0) = h(7-1-0) = h(6)$

$n=1 \longrightarrow h(1) = h(7-1-1) = h(5)$

$n=2 \longrightarrow h(2) = h(7-1-2) = h(4)$

$n=3 \longrightarrow h(3) = h(7-1-3) = h(3)$

$$y(n) = \frac{1}{2}\left[x(n) + x(n-6)\right] + \frac{1}{3}\left[x(n-1) + x(n-5)\right] + 1\left[x(n-2) + x(n-4)\right] + \frac{1}{4}x(n-3)$$



* Linear phase Realization requires $\frac{N+1}{2}$ multipliers.

2) Obtain Direct form Realization with minimum number of multipliers for the system transfer function

$$H(z) = 1 + 3z^{-1} + 4z^{-2} + 4z^{-3} + 3z^{-4} + z^{-5}$$

$$\frac{Y(z)}{X(z)} = 1 + 3z^{-1} + 4z^{-2} + 4z^{-3} + 3z^{-4} + z^{-5}$$

$$Y(z) = X(z) + 3z^{-1}X(z) + 4z^{-2}X(z) + 4z^{-3}X(z) + 3z^{-4}X(z) + z^{-5}X(z)$$

Applying Inverse z-transform on both sides

$$y(n) = x(n) + 3x(n-1) + 4x(n-2) + 4x(n-3) + 3x(n-4) + x(n-5)$$

$h(n) = \{1, 3, 4, 4, 3, 1\}$

$h(n) = h(N-1-n)$

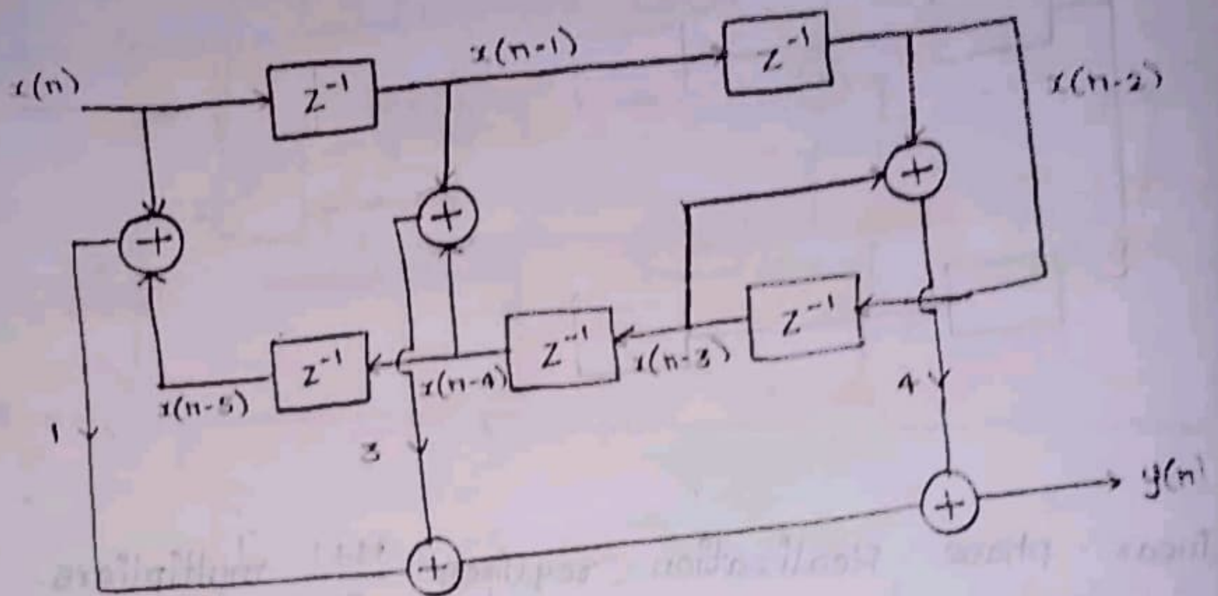
$N = 6$

$n=0 \longrightarrow h(0) = h(6-1-0) = h(5)$

$n=1 \longrightarrow h(1) = h(6-1-1) = h(4)$

$n=2 \longrightarrow h(2) = h(6-1-2) = h(3)$

$y(n) = 1[x(n) + x(n-5)] + 3[x(n-1) + x(n-4)] + 4[x(n-2) + x(n-3)]$



NOTE :

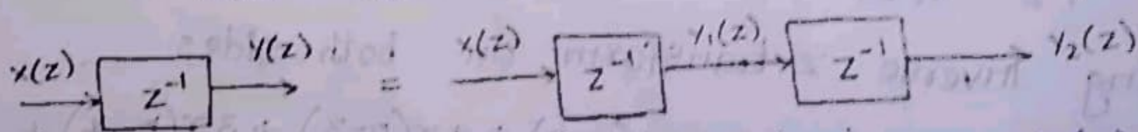
In Linear phase Realization, If

$N = \text{Odd} \longrightarrow$ it requires $\frac{N+1}{2}$ Multipliers

$N = \text{even} \longrightarrow$ it requires $\frac{N}{2}$ Multipliers

CASCADE FORM REALIZATION :

$H(z) = H_1(z) \cdot H_2(z)$



$H(z) = \frac{Y(z)}{X(z)}$ $\quad H_1(z) = \frac{Y_1(z)}{X(z)}$. $H_2(z) = \frac{Y_2(z)}{Y_1(z)}$

1) Find the cascade form realization for the system transfer function $H(z) = (1+2z^{-1}-z^{-2})(1+z^{-1}-z^{-2})$

$$H(z) = (1+2z^{-1}-z^{-2})(1+z^{-1}-z^{-2})$$

$$H(z) = H_1(z) \cdot H_2(z)$$

$$\frac{Y(z)}{X(z)} = \frac{Y_1(z)}{X(z)} \cdot \frac{Y_2(z)}{Y_1(z)}$$

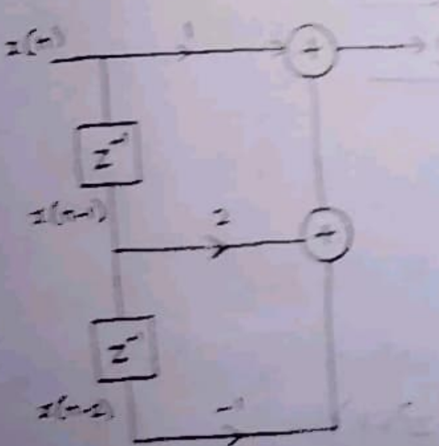
$$\frac{Y_1(z)}{X(z)} = 1+2z^{-1}-z^{-2} \qquad \frac{Y_2(z)}{Y_1(z)} = 1+z^{-1}-z^{-2}$$

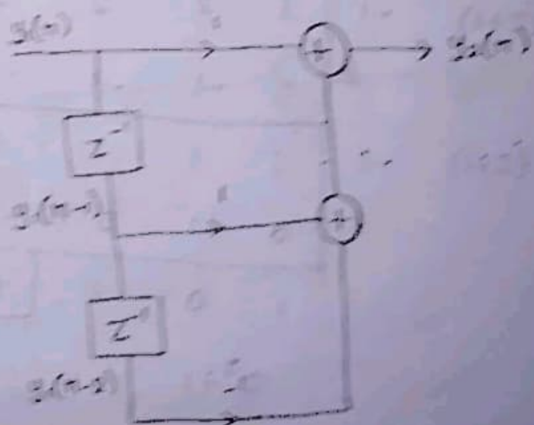
$$Y_1(z) = X(z) + 2z^{-1}X(z) - z^{-2}X(z) \qquad Y_2(z) = Y_1(z) + z^{-1}Y_1(z) - z^{-2}Y_1(z)$$

Applying inverse z-transform on both sides

$$y_1(n) = x(n) + 2x(n-1) - x(n-2) \rightarrow (2) \qquad y_2(n) = y_1(n) + y_1(n-1) - y_1(n-2) \rightarrow (3)$$

REALIZATION STRUCTURES FOR EQN (2) & (3)





COMBINATION OF BOTH REALIZATION STRUCTURES :

$X(z)$ $X(z)$ $Y_1(z)$

nd the Cascade form Realization for the System
sfer function $H(z) = (1+2z^{-1}-z^{-2})(1+z^{-1}-z^{-2})$

$$H(z) = (1+2z^{-1}-z^{-2})(1+z^{-1}-z^{-2})$$

$$H(z) = H_1(z) \cdot H_2(z)$$

$$\frac{Y(z)}{X(z)} = \frac{Y_1(z)}{X(z)} \cdot \frac{Y_2(z)}{Y_1(z)}$$

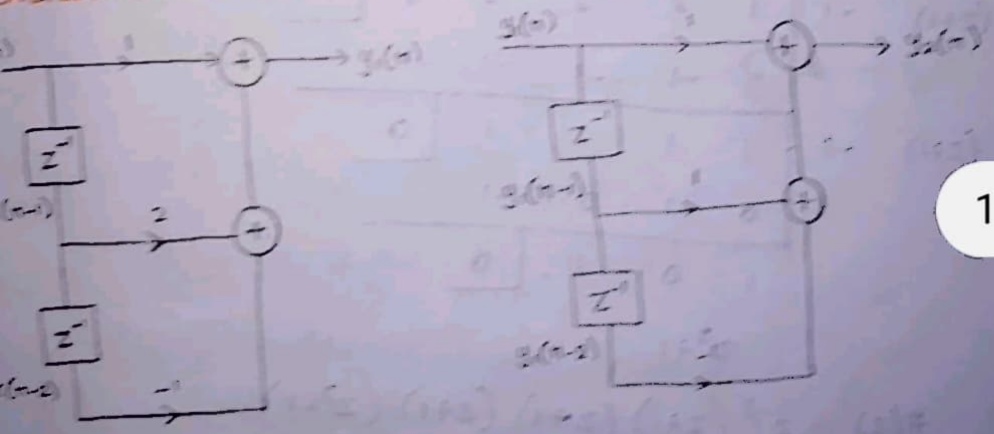
$$\frac{Y_1(z)}{X(z)} = 1+2z^{-1}-z^{-2} \qquad \frac{Y_2(z)}{Y_1(z)} = 1+z^{-1}-z^{-2}$$

$$Y_1(z) = X(z)+2z^{-1}X(z)-z^{-2}X(z) \qquad Y_2(z) = Y_1(z)+z^{-1}Y_1(z)-z^{-2}Y_1(z)$$

pplying inverse z-transform on both sides

$$y_1(n) = x(n)+2x(n-1)-x(n-2) \rightarrow (2) \qquad y_2(n) = y_1(n)+y_1(n-1)-y_1(n-2) \rightarrow (3)$$

EALIZATION STRUCTURES FOR EQN (2) & (3)



OMBINATION OF BOTH REALIZATION STRUCTURES:

1) Find the cascade form Realization for the system Transfer function $H(z) = (1+2z^{-1}-z^{-2})(1+z^{-1}-z^{-2})$

$$H(z) = (1+2z^{-1}-z^{-2})(1+z^{-1}-z^{-2})$$

$$H(z) = H_1(z) \cdot H_2(z)$$

$$\frac{Y(z)}{X(z)} = \frac{Y_1(z)}{X(z)} \cdot \frac{Y_2(z)}{Y_1(z)}$$

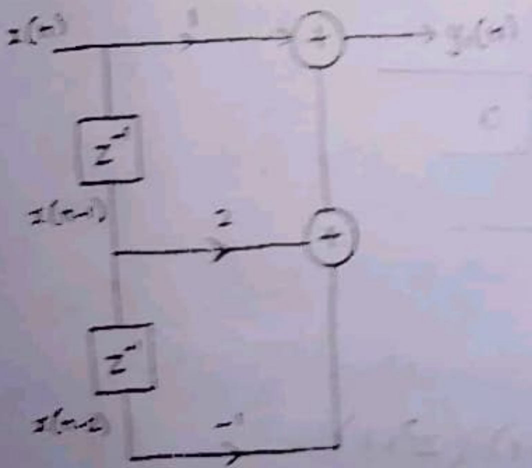
$$\frac{Y_1(z)}{X(z)} = 1+2z^{-1}-z^{-2} \qquad \frac{Y_2(z)}{Y_1(z)} = 1+z^{-1}-z^{-2}$$

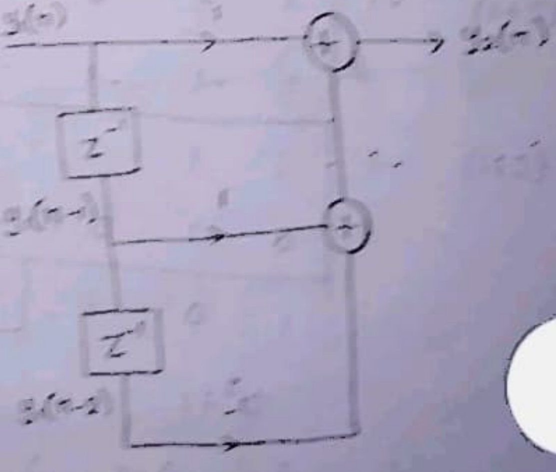
$$Y_1(z) = X(z) + 2z^{-1}X(z) - z^{-2}X(z) \qquad Y_2(z) = Y_1(z) + z^{-1}Y_1(z) - z^{-2}Y_1(z)$$

Applying inverse z-transform on both sides

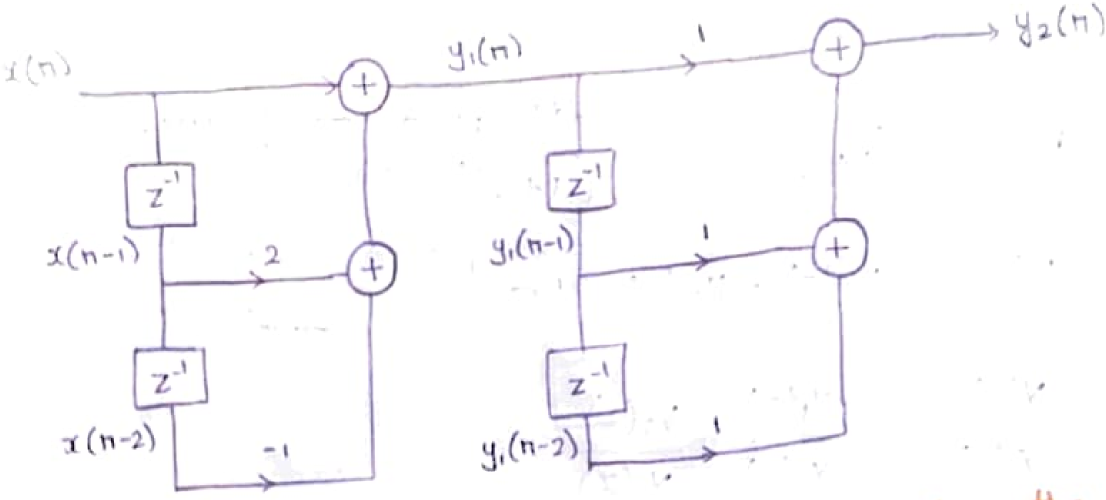
$$y_1(n) = x(n) + 2x(n-1) - x(n-2) \rightarrow (2) \qquad y_2(n) = y_1(n) + y_1(n-1) - y_1(n-2) \rightarrow (3)$$

REALIZATION STRUCTURES FOR EQN (2) & (3)





COMBINATION OF BOTH REALIZATION STRUCTURES :



2> Obtain the cascade form Realization for the system transfer function. $H(z) = 1+3z^{-1}+4z^{-2}+4z^{-3}+3z^{-4}+z$

$$H(z) = 1+3z^{-1}+4z^{-2}+4z^{-3}+3z^{-4}+z^{-5} \longrightarrow (1)$$

$$= z^{-5}\left[z^5+3z^4+4z^3+4z^2+3z^1+1\right]$$

By using Remainder value theorem

		1	3	4	4	3	1	
$(z+1)$	-1	0	-1	-2	-2	-2	-1	
		1	2	2	2	1		0
$(z+1)$	-1	0	-1	-1	-1	-1		
		1	1	1	1		0	
$(z+1)$	-1	0	-1	0	-1			
		1	0	1	0			

z^2+1

$$H(z) = z^{-5}(z+1)(z+1)(z+1)(z^2+1)$$

$$H(z) = z^{-1}(z+1)\, z^{-1}(z+1)\, z^{-1}(z+1)\, z^{-2}(z^2+1)$$

26-03-19

$$H(z) = H_1(z)\,H_2(z)\,H_3(z)\,H_4(z)$$

$$\frac{Y(z)}{X(z)} = \frac{Y_1(z)}{X(z)} \cdot \frac{Y_2(z)}{Y_1(z)} \cdot \frac{Y_3(z)}{Y_2(z)} \cdot \frac{Y_4(z)}{Y_3(z)}$$

$$\frac{Y_1(z)}{X(z)} = z^{-1}(z+1)$$

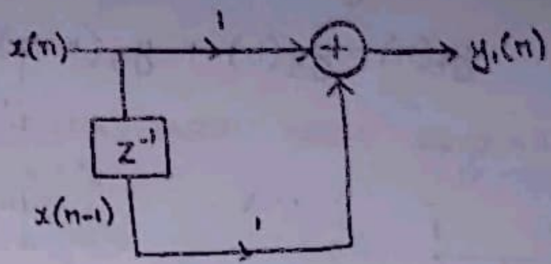
$$\frac{Y_1(z)}{X(z)} = 1+z^{-1}$$

$$Y_1(z) = X(z) + z^{-1}X(z)$$

Apply inverse z-transform

$$y_1(n) = x(n) + x(n-1) \longrightarrow (2)$$

REALIZATION STRUCTURE FOR EQN (2)



$$\frac{Y_2(z)}{Y_1(z)} = z^{-1}(z+1)$$

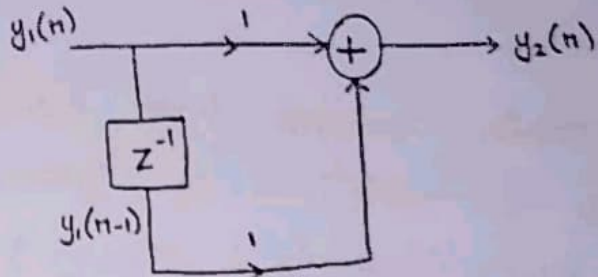
$$\frac{Y_2(z)}{Y_1(z)} = 1+z^{-1}$$

$$Y_2(z) = Y_1(z) + z^{-1}Y_1(z)$$

Apply inverse z-transform

$$y_2(n) = y_1(n) + y_1(n-1) \longrightarrow (3)$$

REALIZATION STRUCTURE FOR EQN (3)



$$\frac{Y_3(z)}{Y_2(z)} = z^{-1}(z+1)$$

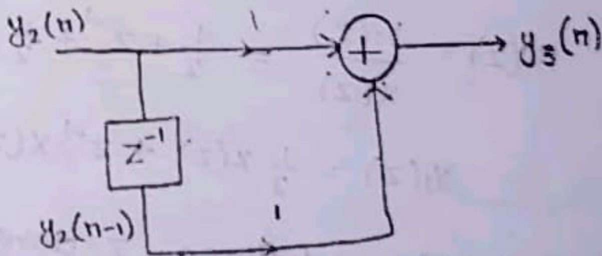
$$\frac{Y_3(z)}{Y_2(z)} = 1+z^{-1}$$

$$Y_3(z) = Y_2(z) + z^{-1}Y_2(z)$$

Apply inverse z-transform

$$y_3(n) = y_2(n) + y_2(n-1) \longrightarrow (4)$$

REALIZATION STRUCTURE FOR EQN (4)

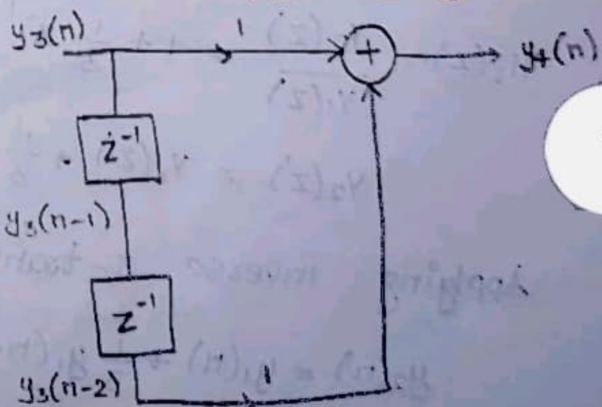


$$\frac{Y_4(z)}{Y_3(z)} = z^{-2}(z^2+1)$$

$$\frac{Y_4(z)}{Y_3(z)} = 1+z^{-2}$$

$$Y_4(z) = Y_3(z) + z^{-2}Y_3(z)$$

REALIZATION STRUCTURE FOR EQN (5)



Apply Inverse Z-transform

$$y_4(n) = y_3(n) + y_3(n-2) \longrightarrow (5)$$

CASCADE FORM REALIZATION STRUCTURE: