Unit-1

Signal :

A signal is a physical quantity which varies with respect to time, space, temperature like any independent variable.

Types of signal:

Continuous Time and Discrete time signal:

It is defined for all instead of time is called continuous time signal.

Only at a discrete instant of Time.

Deterministic and Determinstic Signal:

It is an exact mathematical formulae.

Even and odd signal:

Even ⇒ $x(t) = x(-t)$

Odd ⇒ $x(t) = -x(-t)$

Periodic and Aperiodic signal :

Satisfies the condition $x(t) = x(t+T)$ is called period signal.

Energy and Power signal:

Energy is finite power is zero is zero is called Energy signa

Power is finite, Energy is zero is called Power signal.

Real and imaginary signal:

* $x(t) = x^*(t)$ = real
* $x(t) = -x^*(t)$ = imaginary

System :

A system is the one which responds to particular signal by producing other signal.

(or)

A system is any physical set of components that takes a signal and produces a signal.

Types of systems:

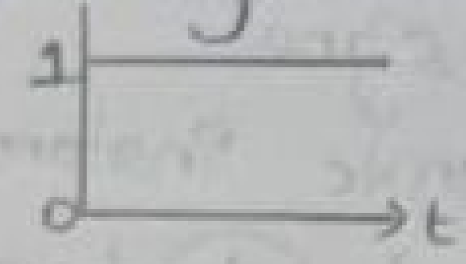
* Linear and Non linear
* Time variant and Time invarient
* Linear time variant and Linear Time invariant
* Static and Dynamic
* Causal and Non causal
* Invertiable and Non Invertiable
* Stable and unstable

Basic signals:

Unit step function:

Unit step function is denoted by $u(t)$

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



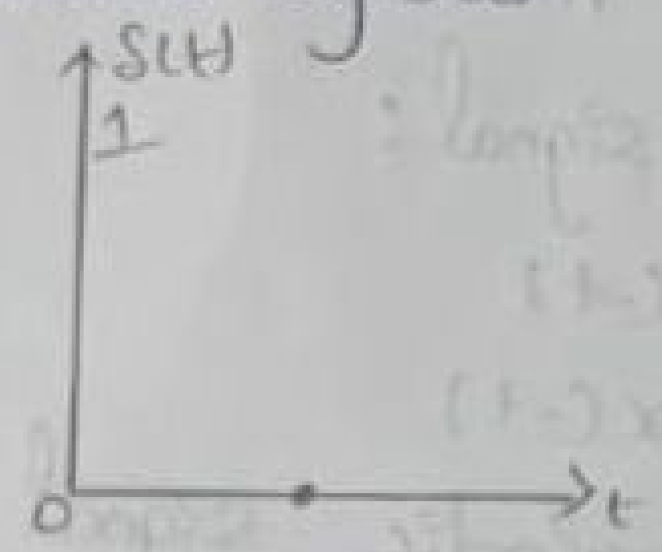
Unit impulse function:

Impulse function is denoted by $\delta(t)$

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)dt = u(t)$$

$$\delta(t) = \frac{d\,u(t)}{dt}$$

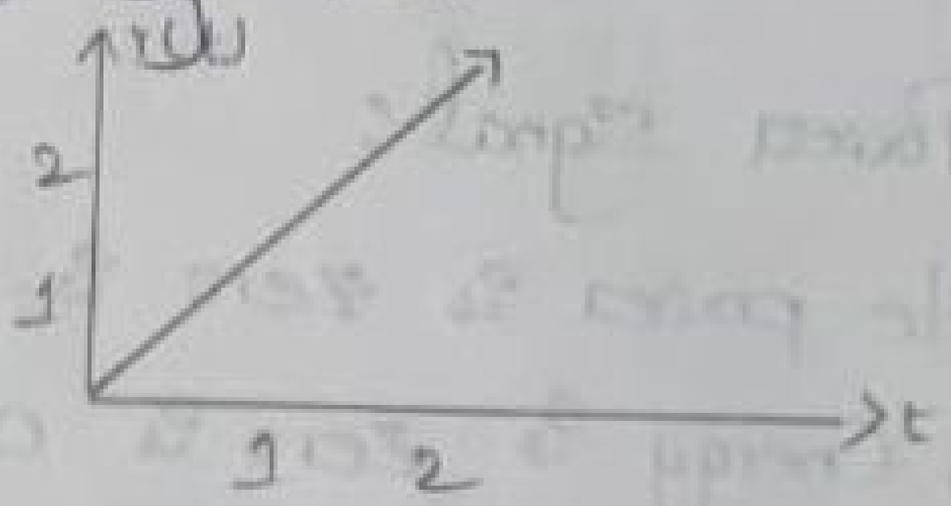


Ramp signal:

Ramp signal is denoted by $r(t)$

$$r(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$u(t) = \frac{dr(t)}{dt}$$

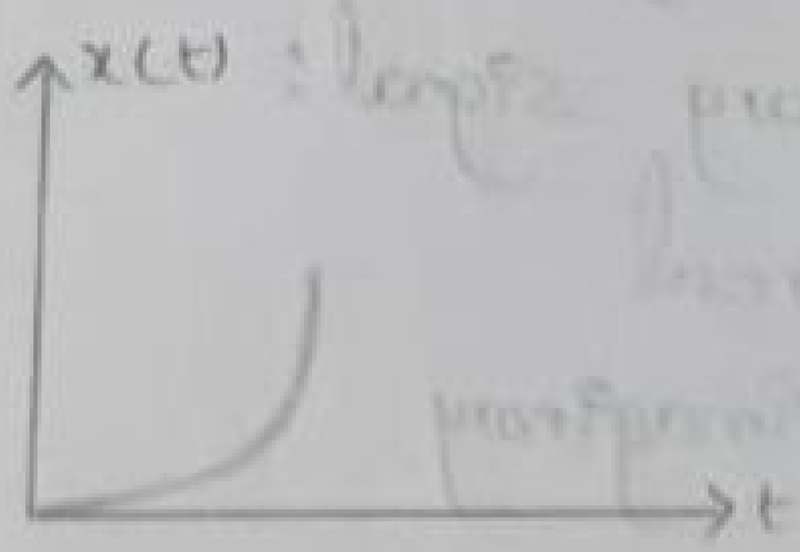


Parabolic signal:

Defined by $x(t)$

$$x(t) = \begin{cases} t^2/2 & t = 0 \\ 0 & t < 0 \end{cases}$$

$$u(t) = \frac{d^2x(t)}{dt^2}$$



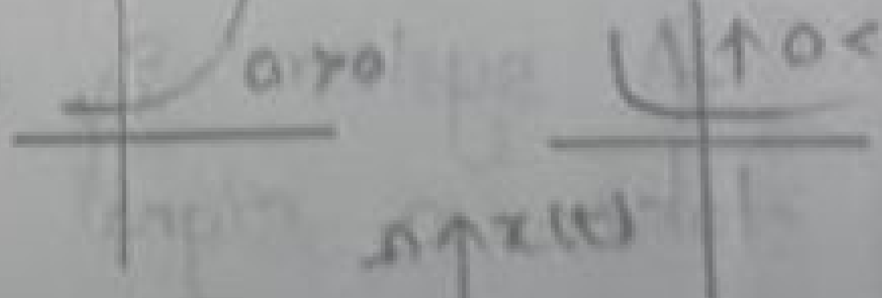
Signum function:

Signum function is denoted as $Sgn(t)$

$$Sgn = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$

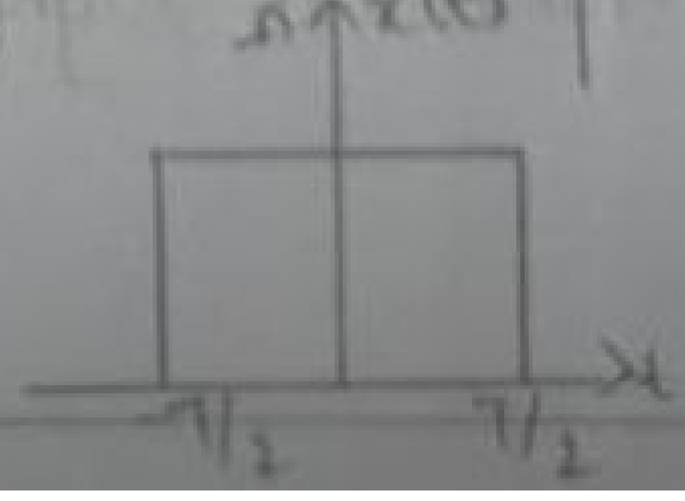
Exponential signal:

It is in the form of $x(t) = e^{at}$



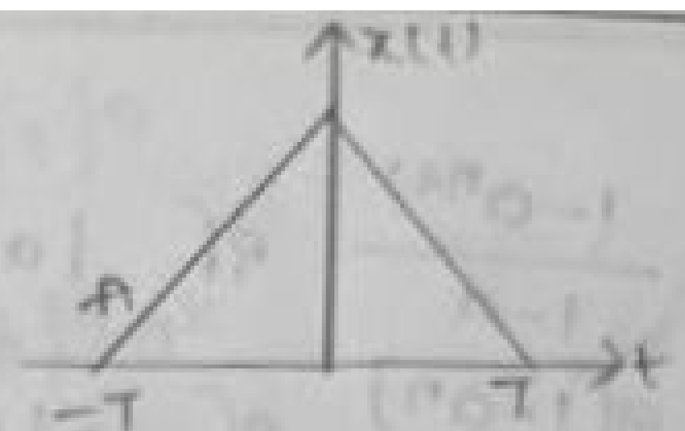
Rectangular signal:

$x(t) = A\,rect(t/T)$



Triangular signal:

$x(t) = A\left[1 - \frac{|t|}{T}\right]$



Sinusoidal signal:

$x(t) = A\cos(\omega_0 \pm \phi)$ /

$x(t) = A\sin(\omega_0 \pm \phi)$

Sinc function:

$t = \frac{\sin \pi t}{\pi t}$

Sampling Function:

$Sa(t) = \frac{\sin t}{t}$

=> Sampling of Quantization is used for A/D converter

* Quantization is two types

1. Trunkation - Cut the sample
2. Round off - the value not inpoints

Sampling:

The continuous signal is converts into discrete sample signal with instant of time

* Digital is the best according to analog
* Because of accuracy is very high in output.
* Output response is high.
* All values all sequence has same finite.

Periodic signal:

$x(n) = x(n+N)$ Discrete periodic signal

N = Fundamental period

Causual signal:

$x(n) \geq 0$ ex: step, ramp

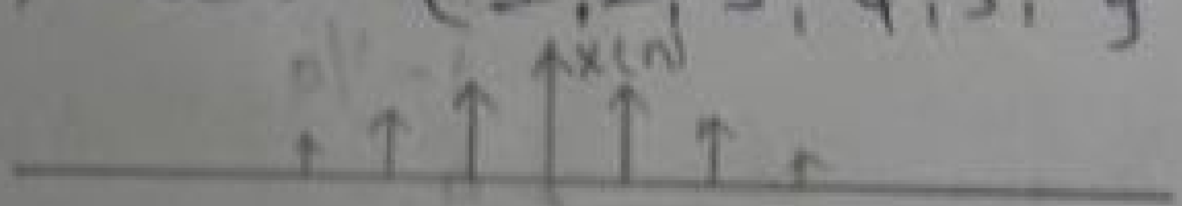
Causual signal is also called as Right handed signal

Non Causual signal:

$x(n) < 0$ Ex:

Non Causal signal is also called as left handed signal.

* Functional representation => $u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$
* Sequence representation => $x(n) = \{1, 2, 3, 4, 5, \}$
* Graphical representation



* $x(n) = \{1, 2, 3, 4, 5\}$

$x(n) = \{1, 1, 2, 3\}$

=> represents origin

Before the origin the numbers represent negative values

Formula:

$$S_n = \sum_{k=0}^{n} a^k = \frac{1-a^{n+1}}{1-a} \quad \text{if } |a|<1$$

$$S_n = \sum_{k=0}^{n} a^k = \frac{a[1-a^n]}{1-a} \quad \text{if } |a|<1$$

$$S_n = \sum_{k=1}^{n+1} a^k = \frac{a[1-a^{n+1}]}{1-a} \quad \text{if } |a|<1$$

$$S_n = \sum_{k=0}^{\infty} a^k = \begin{cases} \frac{1}{1-a} & \text{if } |a|<1 \\ \infty & \text{if } |a|>1 \end{cases}$$

$$S_n = \sum_{k=1}^{\infty} a^k = \begin{cases} \frac{a}{1-a} & \text{if } |a|<1 \\ \infty & \text{if } |a|>1 \end{cases}$$

Discrete Time signal:

Energy and Power signal:

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

* A signal is an energ signal if and only if the total energy of the signal is finite for energy signal power is zero.
* A signal is power signal the total power is finite and the energy is finite.
* When the signal doesn't satisfy the property then the signal is Energy nor power signal.

$x(n) = (1/3)^n u(n)$ find the signal is Energy or Power

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^{\infty} |(1/3)^n|^2$$

$$= \sum_{n=0}^{\infty} (1/9)^n$$

$$= \sum_{n=0}^{\infty} a^n$$

$$a^n = S_n = \sum_{k=0}^{\infty} a^k = \left\{ \frac{1}{1-a} \text{ if } |a|<1 \right.$$

$$= \frac{1^{\infty}}{1-1/9}$$

$$= \frac{9}{8}$$

Energy is finite

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} |(1/3)^n|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} (1/a)^n$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} (1/a)^n a^k$$

$$\sum_{n=0}^{N} a^k = \frac{1-a^{n+1}}{1-a} \text{ if } |a|<1$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} \frac{1-1/a^{n+1}}{1/a}$$

$x(n) = e^{2n} u(n)$ find the signal is Energy or Power

$$x(n) = e^{2n} u(n)$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} |e^{2n}|^2$$

$$= \sum_{n=0}^{\infty} e^{4n}$$

$$= e^{4(0)} + e^{4(1)} + e^{4(2)} + \dots + e^{4(\infty)}$$

$$= \infty$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |e^{2n}|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \cdot \sum_{n=0}^{\infty} e^{4n}$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} [1 + e^{4(1)} + e^{4(2)} + \dots + e^{4N}]$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \left[\frac{(e^4)^{N+1} - 1}{e^4 - 1}\right] \cdot \infty$$

The GP factor is greater than '1' than it can be ∞

$x(n) = e^{j(\pi/2 n + \pi/4)}$ find the signal is Energy or power

$$x(n) = e^{j(\pi/2 n + \pi/4)}$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-\infty}^{\infty} |e^{j(\pi/2 n + \pi/4)}|^2$$

$$= \sum_{n=-\infty}^{\infty} 1 = \infty$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(t)|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |e^{j(\pi/2 n + \pi/4)}|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \cdot \sum_{n=-N}^{N} 1$$

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$$= \lim_{N\to\infty} \frac{1}{2N+1} \times 2N+1$$

$$= 1$$

Power is finite

$x(n) = \sin \pi/_4 n$ find the signal is Energy or Power.

$$x(n) = \sin \pi/_4 n$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$E = \sum_{n=-\infty}^{\infty} |\sin \pi/_4 n|^2$$

$$= \sum_{n=-\infty}^{\infty} \sin^2 \frac{\pi n}{4}$$

$$= \sum_{n=-\infty}^{\infty} \frac{1 - \cos^2 2\pi n/_4}{2}$$

$$= \sum_{n=-\infty}^{\infty} \frac{(1 - \cos n\pi/_2)}{2}$$

$$E = 1/_2 \sum_{n=-\infty}^{\infty} (1 - \cos n\pi/_2)$$

$$= 1/_2 \left[\sum_{n=-\infty}^{\infty} 1 - \sum_{n=-\infty}^{\infty} \cos \frac{\pi n}{2}\right]$$

$$= 1/_2 (1 - \infty)$$

Energy is infinite

$$P_{avg} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left[|x(n)|^2\right]$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left| \left[\sin \frac{\pi n}{4}\right] \right|^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|\sin^2 \frac{n\pi}{4}\right|$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left[\frac{1 - \cos \frac{2\pi n}{4}}{2}\right]$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left[\frac{1 - \cos n\pi/_2}{2}\right]$$

$$= 1/_2 \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} (1 - \cos n\pi/_2)$$

$$= 1/_2 \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{n=-N}^{N} 1 - \sum_{n=-N}^{N} \cos \frac{n\pi}{2}\right)$$

$$= 1/_2 \left[\lim_{N\to\infty} \frac{1}{2N+1} \times 2N+1 - \frac{1}{2N+1} \times 0\right]$$

$$= 1/_2 [1 - 0]$$

$= \frac{1}{2}$

Power is finite

Discrete time system:

Static and Dynamic system:

Discrete time system is called static or memoryless system if its output is at any instant 'n' depends on the input samples at the same time but not on past or future samples.

$x(n) \rightarrow$ input $\quad y(n) \rightarrow$ output

$y(n) = a\,x(n)$

$y(n) = x^2(n)$

Dynamic system:

$y(n) = x(n) + x(n-1) + x(n+1)$

$n=0$

$y(0) = x(0) + x(-1) + x(1)$

Time delay shifting

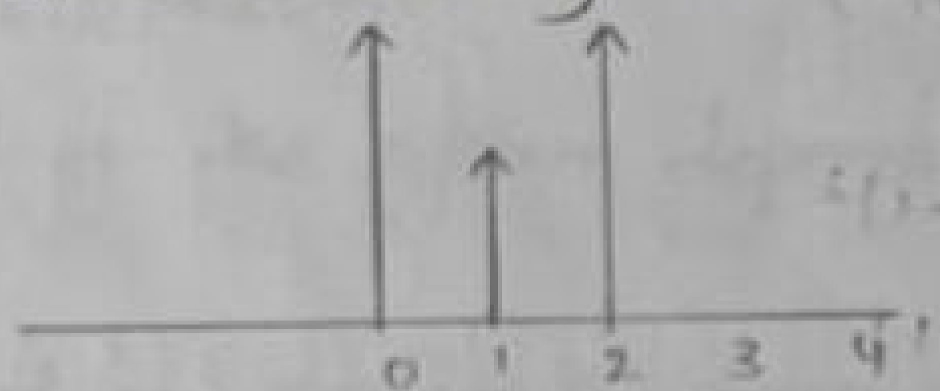
advance delay shifting

$y(n) = y(n-k)$

$y(n) = x(n+k)$

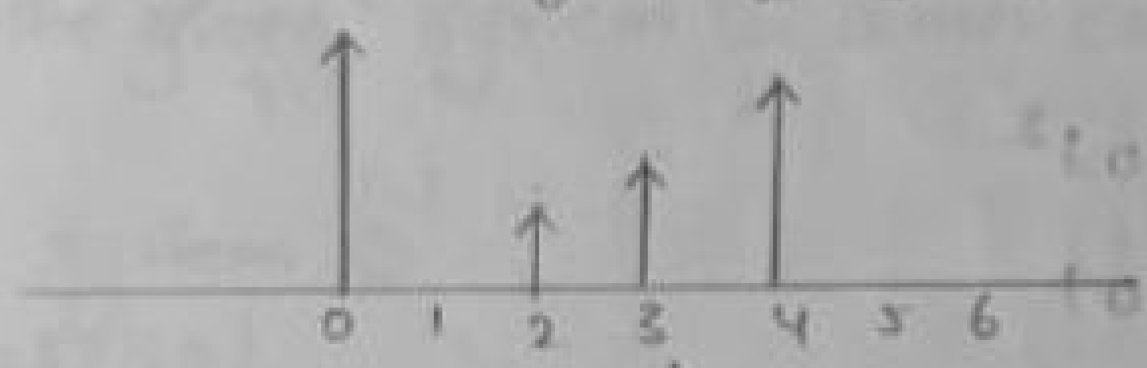
$y(n) = x(n)$

$y(n) = (0)$



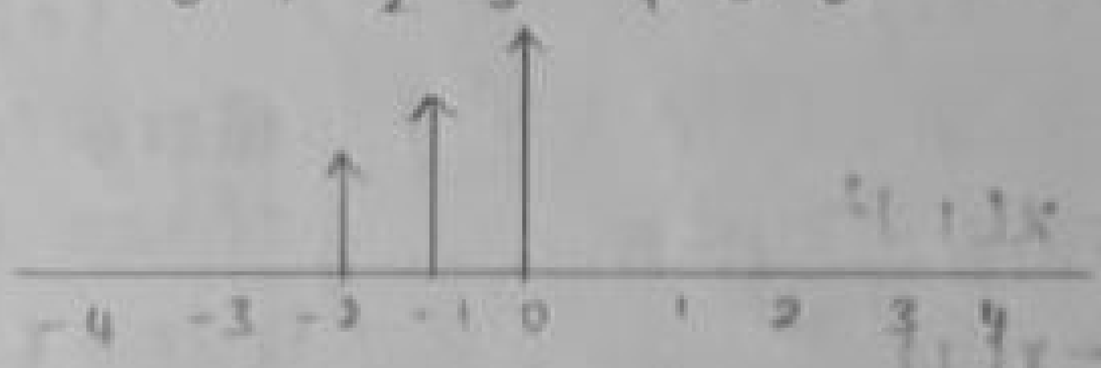
$y(0) = x(0-3)$

$y(0) = x(-3)$



$y(n) = x(n+k)$

$y(0) = x(0+3)$



Time variant and Time invariant:

$\rightarrow y(n) = x(n) + x(n-1)$ find the system is time Variant / Invariant system

$y(n,k) = y(n-k)$

$= T[x(n-k)]$

k delay until depends on input

$y(n-k) = T[x(n-k) + x(n-k-1)]$

Causal and Non causal system:

A system is said to be causal if the output of the system at any time (n) depends only on present and past inputs but not dependen on future inputs. If a system depends not only on present and past inputs but also on future inputs then it is said to be non causal system.

$$y(n) = x(n) + \frac{1}{x(n-1)}$$

$n = -1$

$$y(-1) = x(-1) + \frac{1}{x(-1-1)}$$

$$= x(-1) + \frac{1}{x(-2)}$$

$n = 0$

$$y(0) = x(0) + \frac{1}{x(0-1)}$$

$$= x(0) + \frac{1}{x(-1)}$$

$n = 1$

$$y(1) = x(1) + \frac{1}{x(1-1)}$$

$$= x(1) + \frac{1}{x(0)}$$

It is a causal

$$y(n) = x(n^2)$$

$n = -1$

$$y(-1) = x(-1)^2$$

$$= x(1)$$

$n = 0$

$$y(0) = x(0)^2$$

$$= x(0)$$

$n = 1$

$$y(1) = x(1)^2$$

$$= x(1)$$

It is a non causal

Stability:

Stable and Unstable system:

An orbitary relaxed system is said to be bounded input and bounded output if every bounded input produces bounded output.

Sufficient condition for stability

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

The output response must be finite

* System function or Transfer function.

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Linear and Non linear system:

A system that satisfies the super position principle is said to be a linear system.

Super position principle states that the response of the system to a weighted sum of signals be equal to the corresponding weighted sum of outputs of the system to each of the individual input signals.

* A relaxed system that doesnot satisfy the super position principle is called Non linear system.

$$T[a_1x_1(n)+a_2x_2(n)] = a_1T[x_1(n)] + a_2T[x_2(n)]$$

Causal and Non causal:

* $y(t) = x(3-t)+x(t-2)$

Given system is

$y(t) = x(3-t)+x(t-2)$

$t=0 \Rightarrow y(0) = x(3-0)+x(t-0)$

$t=-1 \Rightarrow y(-1) = x(4)+x(-3)$

$t=1 \Rightarrow y(1) = x(2)+x(-1)$

The output of the system depends on the future values of the inputs.

Hence the given system is a non causal system

* $y(n) = x(3n)$

Given system is

$y(n) = x(3n)$

$n=0 \rightarrow y(0) = x(0)$

$n=-1 \rightarrow y(-1) = x(-3)$

$n=1 \rightarrow y(1) = x(3)$

$n=2 \rightarrow y(2) = x(6)$

It is a non causal system

* $y(n) = \sin[x(n)]$

$n=0 \Rightarrow y(0) = \sin[x(0)]$

$n=-3 \Rightarrow y(-3) = \sin(x(-3))$

$n=3 \Rightarrow y(3) = \sin(x(3))$

The system is a causal system

Linear / Non linear

* $y(t) = ax(t)+b$

$y_1(t)+y_2(t) = ax_1(t)+b+ax_2(t)+b$

$y(t) = ax_1(t)+bx_2(t)+b$

$y(t) \neq y_1(t)+y_2(t)$

The system is non linear

* $y(t) = x\sin(t)$

$y_1(t)+y_2(t) = x_1\sin(t)+x_2\sin(t)$

$y(t) = x_1\sin(t)+x_2\sin(t)$

$y(t) = y_1(t)+y_2(t)$

$ky(t) = kx\sin(t)$

$kx(t) = kx\sin(t)$

$ky(t) = kx(t)$

System is linear.

Time Variant / Time invariant:

* $y(t) = 2t^2x(t)$

$y(t) = T[x(t)] = 2t^2x(t)$

$y(t_1-t_0) = T[x(t-t_0)] = y(t)/x(t) = x(t-t_0) = 2t^2 x(t-t_0)$

The output of the system delayed by t_0 see

$y(t-t_0) = y(t)/t = t-t_0 = 2(t-t_0)^2x(t-t_0)$

$y(t-t_0) \neq y(t-t_0)$

Given system is a Time varying system

* $y(t) = T[x(t)] = 3e^{3x(t)}$

Output of the system for the input delayed by t_0 see

$y(t_1-t_0) = T[x(t-t_0)] = y(t)/x(t) = x(t-t_0) = 3e^{3x}(t-t_0)$

Output of the system delayed t_0 see

$y(t-t_0) = y(t)/t = t-t_0 = 3e^{3x(t-t_0)}$

$y(t,t_0) = y(t-t_0)$

Given system is a time variant system

Static and Dynamic system:

$y(t) = x(3t)$

put $t=1$

$y(1) = x(3)$

System is Dynamic as output depends on future

→ $y(t) = 5x(t)$

put $t=1$

$y(1) = 5x(1)$

System is static as output depends on present

→ $x(\cos t)$

put $t=0$

$y(0) = x(\cos 0)$

$y(0) = x(1)$

System is dynamic as output depends on future

Convolution :

* Choose an initial values of n the starting time for evaluating the output sequence of y(n)

  If x(n) states at n ie $n=n_1$, h(n) start at $n=n_2$ then $n=n_1+n_2$ is a good choice.
* Express both sequence in terms of the index k.
* Fold h(k) about k=0 to obtain h(-k) and shift by n to the right n is +ve and left n is -ve obtain h(n-k)
* Multiply the two sequence x(k) h(n-k) element by element sum of the product to get y(n).

  input: x(k); impulse response: h(n-k); folded and shifting sequence.
* Increment the index 'n' shift the sequence h(n-k) to right by one sample and do step4.
* Repeat steps until the sum of product is zero for all remaining values of 'n'

Determine the Convolution sum of two Sequence

$x(n) = \{\underset{\uparrow}{3}, 2, 1, 2\}$ and $h(n) = \{1, \underset{\uparrow}{2}, 1, 1\}$

Given $x(n) = \{\underset{\uparrow}{3}, 2, 1, 2\}$ & $h(n) = \{1, \underset{\uparrow}{2}, 1, 1\}$

$n = n_1 + n_2$

$n = -1$

$x(k) = \{\underset{\uparrow}{3}, 2, 1, 2\}$ & $h(k) = \{1, \underset{\uparrow}{2}, 1, 1\}$

Tabular Representation :

K	-3	-2	-1	0	1	2	3	4	5	6	7
x(k)				3	2	1	2				
h(k)			1	2	1	2					
h(-k)		2	1	2	1						
n=-1 h(-1-k)	2	1	2	1							
n=0 h(-k)		2	1	2	1						
n=1 h(1-k)			2	1	2	1					
n=2 h(2-k)				2	1	2	1				
n=3 h(3-k)					2	1	2	1			
n=4 h(4-k)						2	1	2	1		
n=5 h(5-k)							2	1	2	1	
n=6 h(6-k)								2	1	2	1

$y(-1) = x(k) * h(-1-k)$

$y(-1) = 3$

$y(1) = 8$

$y(0) = 8$

$y(2) = 12$

$y(3) = 9$

$y(4) = 4$

$y(5) = 4$

$y(6) = 0$

$y(n) = \{3, 8, 8, 12, 9, 4, 4, 0\}$

* Diagonal method

	3	2	1	2
1	3	2	1	2
2	6	4	2	4
1	3	2	1	2
2	6	4	2	4

$y(n) = \{3, 8, 8, 12, 9, 4, 4, 0\}$

Find the convolution of sequence

$$x(n) = \begin{cases} 1 & n = -2, 0, 1 \\ 2 & n = -1 \\ 0 & \text{else where} \end{cases}$$

$h(n) = \delta(n) - \delta(x-1) + \delta(x-2) + \delta(x-3)$

Impulse $\delta(n) = 1$ for $n > 0$

0 for $n < 1$

$x(n) = 1 \;\; 2 \;\; 1 \;\; 1$

$h(n) = 1 \;\; -1 \;\; 1 \;\; -1$

$x(n) = \{-2 \;\; -1 \;\; 0 \;\; 1\}$

$h(n) = \{1 \;\; 2 \;\; \underset{\uparrow}{1} \;\; 1\}$

$h(n) = \delta(n) - \delta(n-1) + \delta(n-2) - \delta(n-3)$

$h(0) = 1 - 0 + 0 - 0 = 1$

$h(1) = 0 - 1 + 0 - 0 = -1$

$h(2) = 0 - 0 + 1 + 0 = 1$

$h(3) = 0 - 0 + 0 - 1 = -1$

$x(n) = \{1 \;\; 2 \;\; \underset{\uparrow}{1} \;\; 1\}$ $n_1 = 2$

$h(n) = \{1 \;\; -1 \;\; 1 \;\; -1\}$ $n_2 = 0$

$n = n_1 + n_2$

$= -2 + 0$

$= -2$

$x(k) = \{1 \;\; 2 \;\; 1 \;\; 1\}$

$h(k) = \{1 \;\; -1 \;\; 1 \;\; -1\}$

K	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
x(k)				1	2	1	2						
h(k)						1	-1	1	-1				
h(-k)			-1	1	-1	1							
h(-2-k)	-1	1	-1	1									
n=-1 (-1-k)		-1	1	-1	1								
n=0 (0-k)			-1	1	-1	1							
n=1 (1-k)				-1	1	-1	1						
n=2 (2-k)					-1	1	-1	1					
n=3 (3-k)						-1	1	-1	1				
n=4 (4-k)							-1	1	-1	1			
n=5 (5-k)								-1	1	-1	1		
n=6 (6-k)									-1	1	-1	1	
n=7 (7-k)										-1	1	-1	1

$y(-2) = 1$
$y(-1) = -1+2 = 1$
$y(0) = 1-2+1 = 0$
$y(1) = -1+2-1+1 = 1$
$y(2) = -2+1-1 = -2$
$y(3) = -1+1 = 0$
$y(4) = -1 = -1$
$y(5) = 0$
$y(n) = \{-1, 3, -2, 3, -2, 0, -1\}$

h(n) \ x(n)	1	2	1	1
1	1	2	1	1
-1	-1	-2	-1	-1
1	1	2	1	1
-1	-1	-2	-1	-1

$y(n) = \{1, 1, 0, 1, -2, 0, -1\}$

Circular Convolution :

Long duration sequence we use circular convolution

$N = \max(L, M)$

Both sequence length must be equal

$x(n) = \{1, 2, 3, 4\}$

zero padding $h(n) = \{1\ 2\ 3\ 0\}$

adding of zeros

*Concentric Circular convolution

*Matrix method

$\rightarrow x_2(n) = \{2\ 3\ 4\ 0\}$

$x_1(n) = \{2\ 3\ 4\ 3\}$

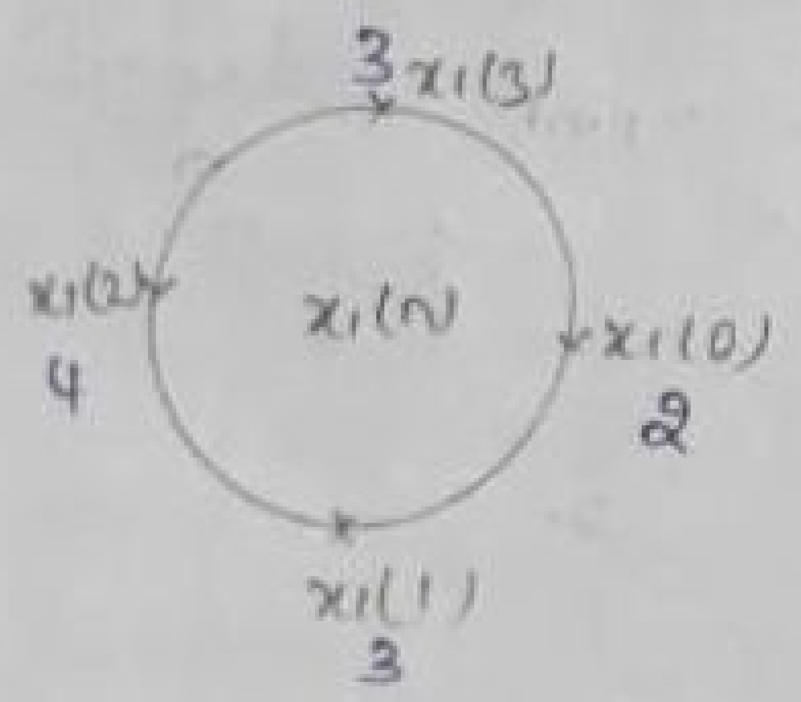
Matrix method:

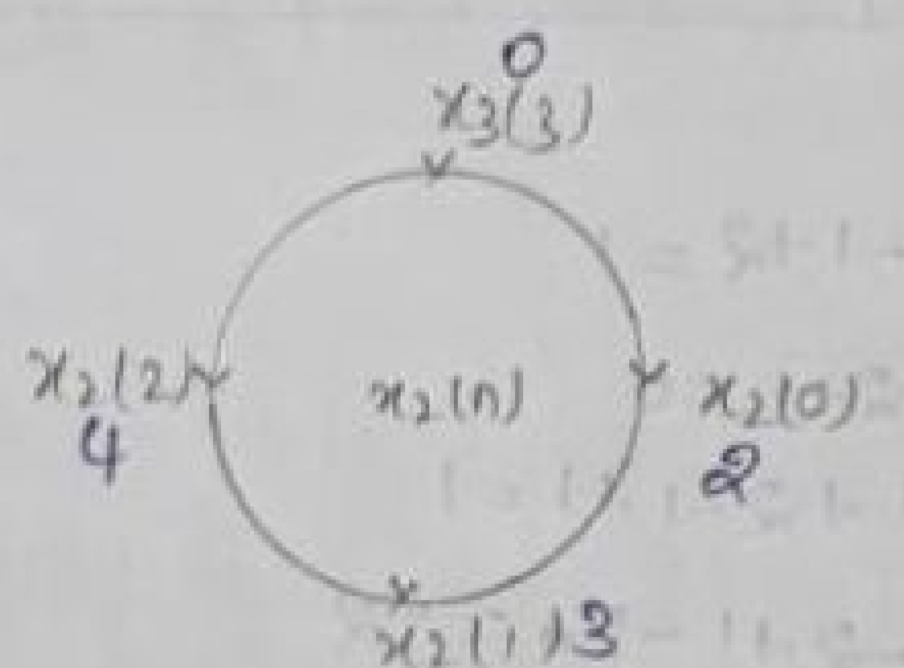
$$\begin{bmatrix} 2 & 3 & 4 & 3 \\ 3 & 2 & 3 & 4 \\ 4 & 3 & 2 & 3 \\ 3 & 4 & 3 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 4+9+16+0 \\ 6+6+12+0 \\ 8+9+8+0 \\ 6+12+12+0 \end{bmatrix} = \begin{bmatrix} 29 \\ 24 \\ 25 \\ 30 \end{bmatrix}$$

Concentric Circular Convolution

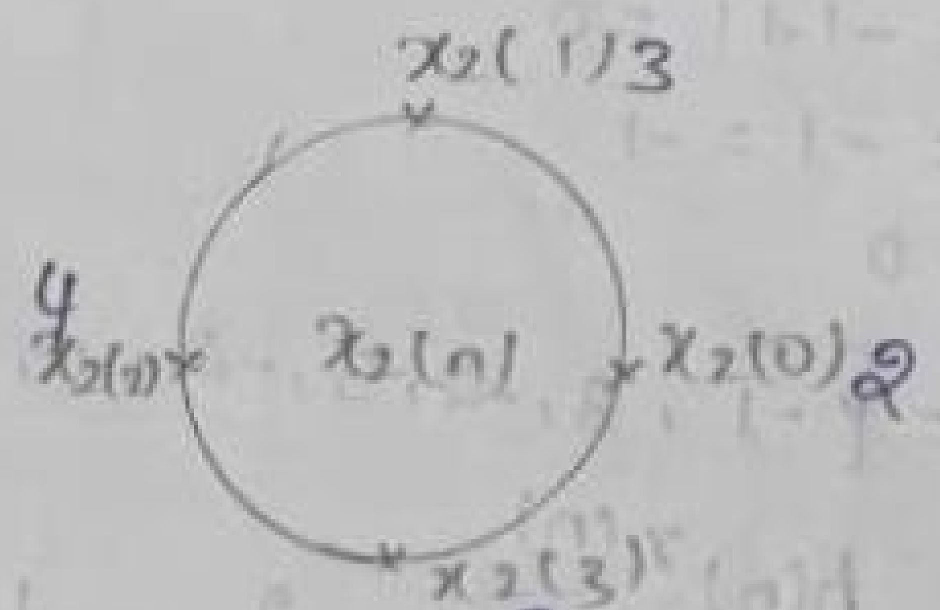
$x_1(n) = \{\underset{\uparrow}{2}\ 3\ 4\ 3\}$

$x_2(n) = \{\underset{\uparrow}{2}\ 3\ 4\ 0\}$

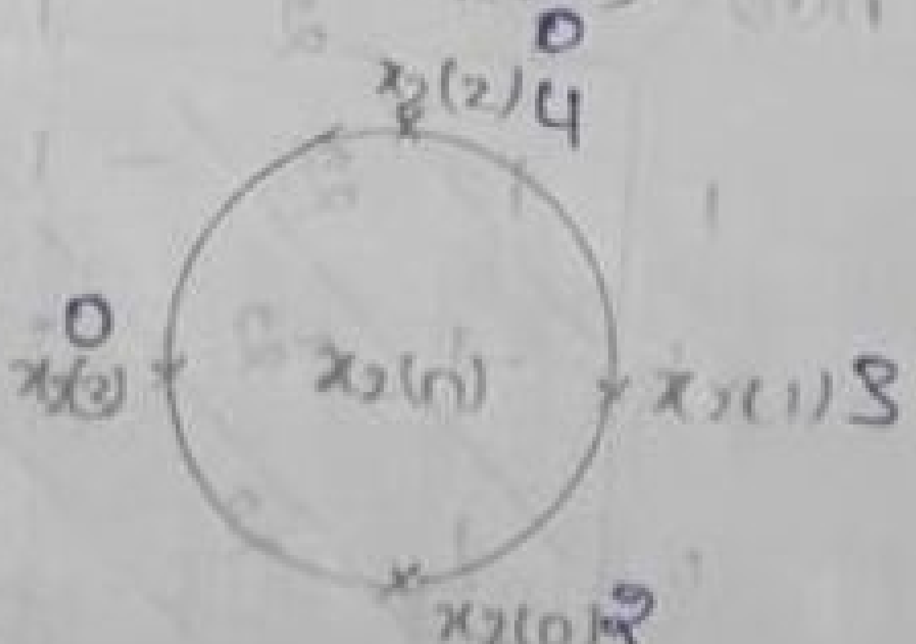




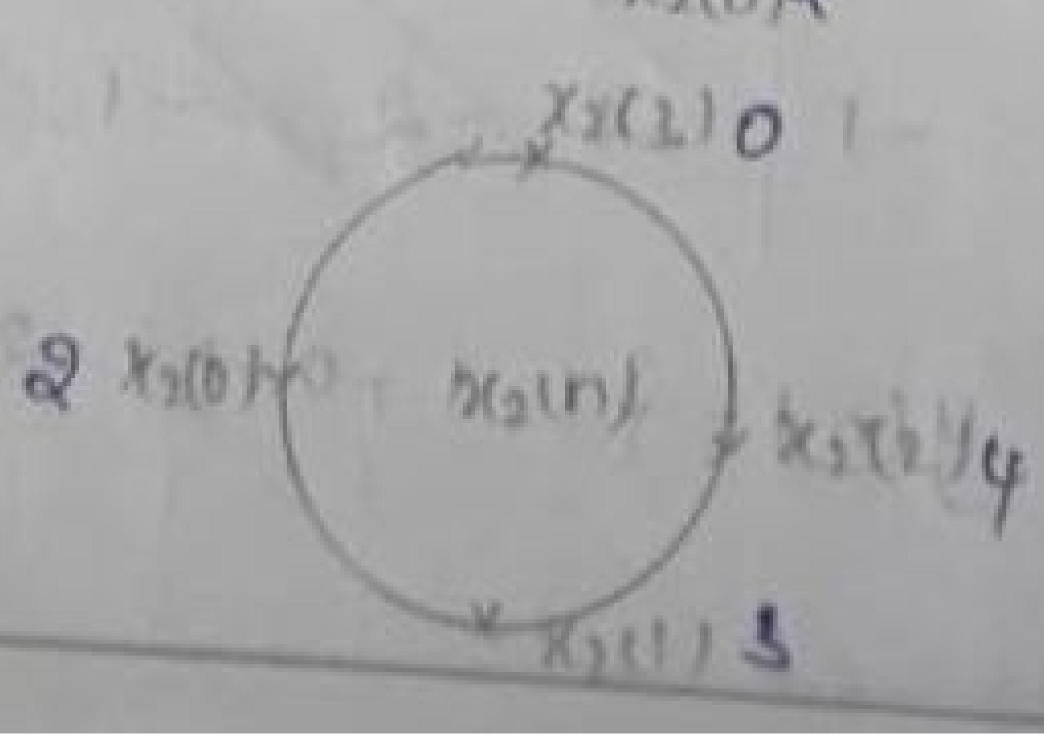
$4+9+16+0 = 29$



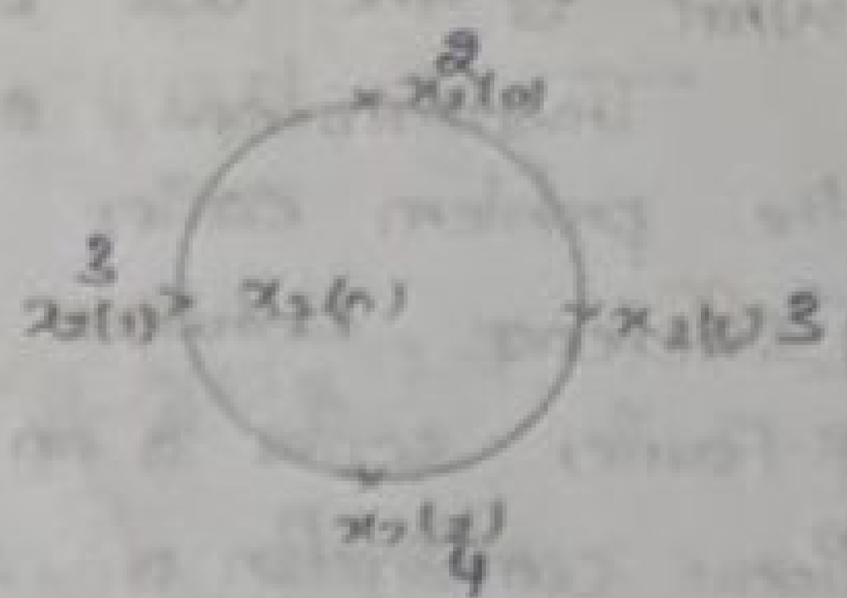
$6+12+0+6 = 24$



$8+0+8+9 = 25$



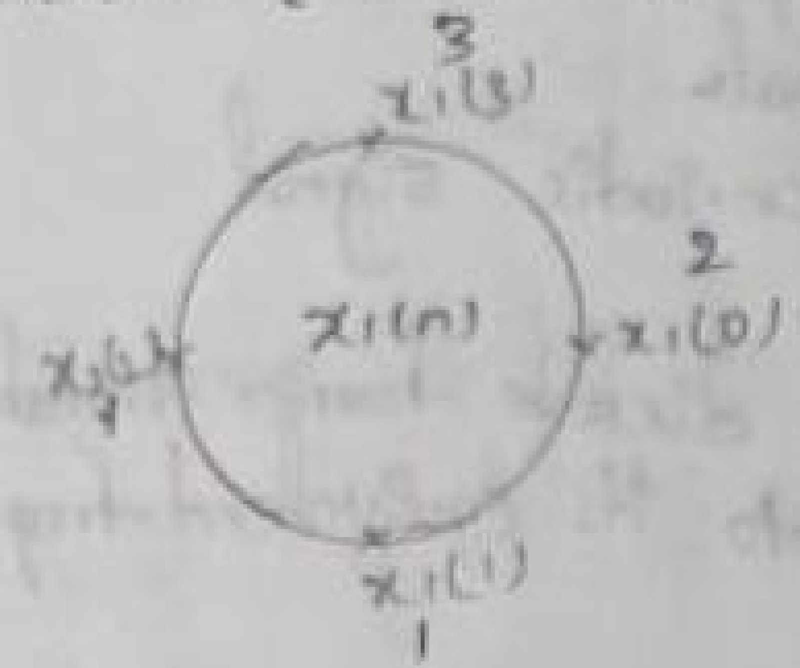
$0+16+12+12 = 30$

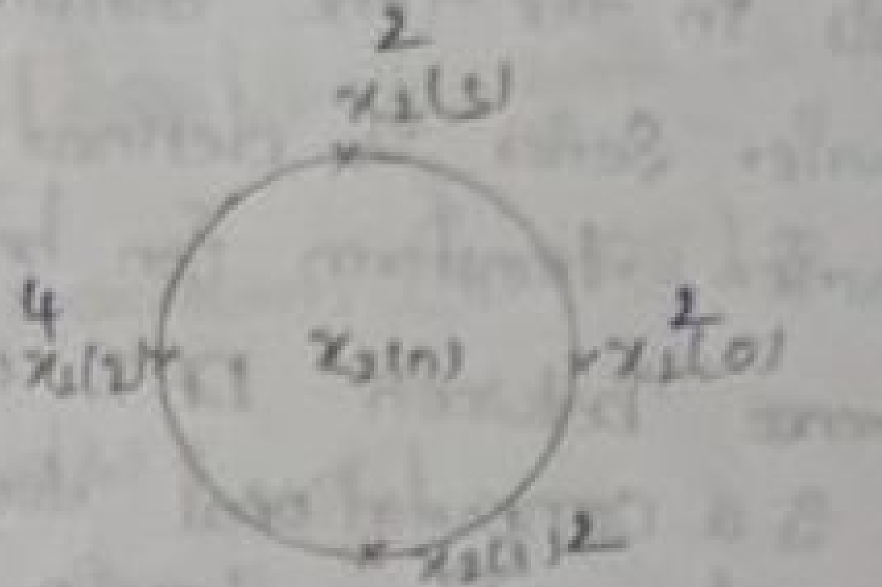


$y(n) = \{29, 24, 25, 30\}$

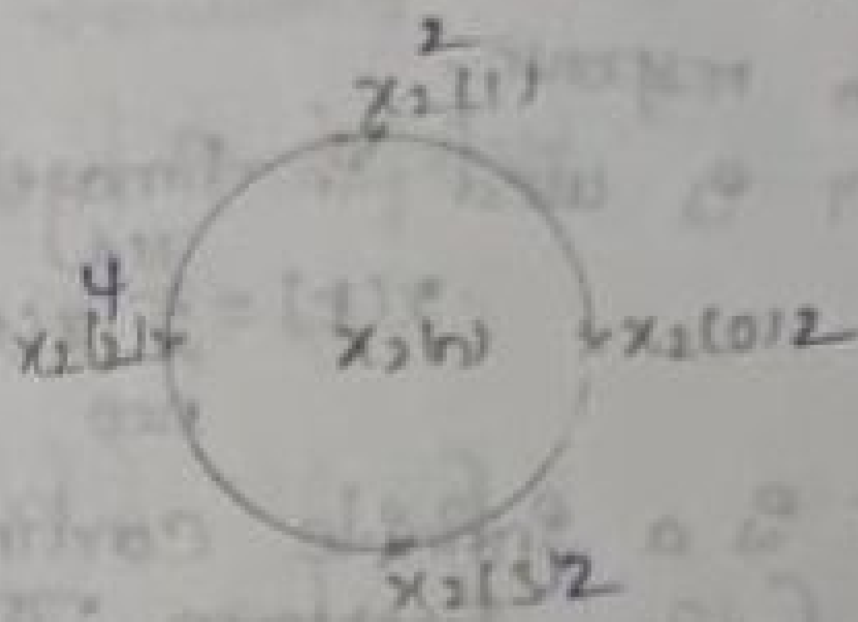
$x_1(n) = \{2 \quad 1 \quad 1 \quad 3\}$

$x_2(n) = \{2 \quad 2 \quad 4 \quad 2\}$

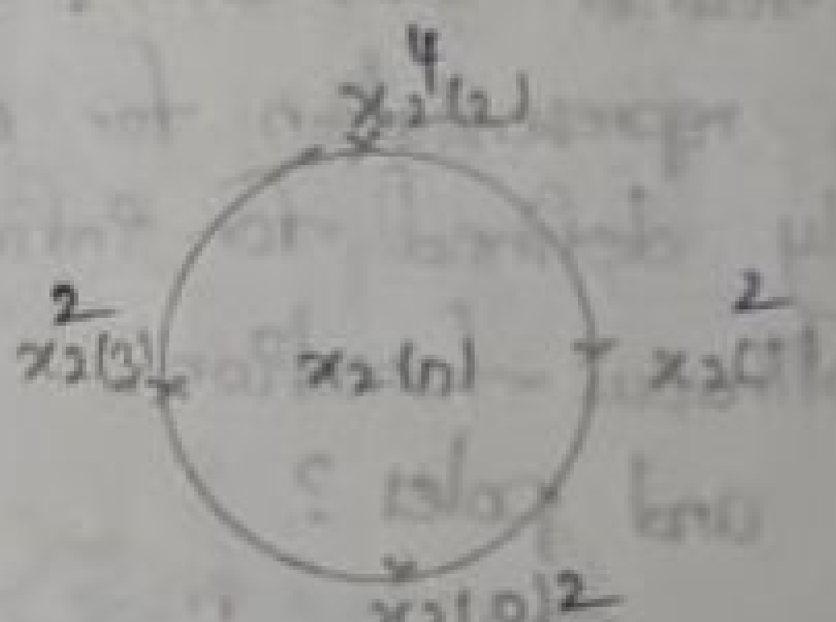




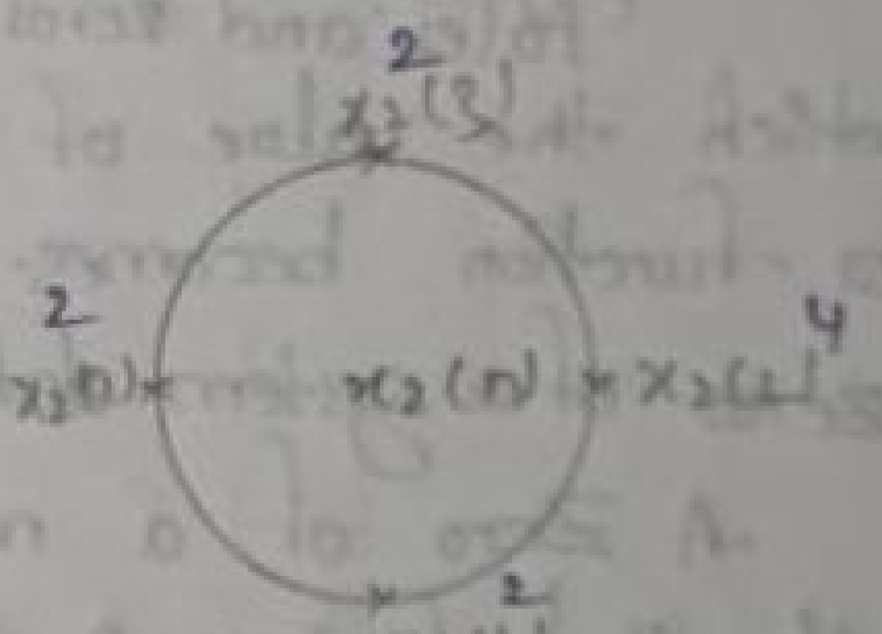
$4+6+4+2$
$= 16$



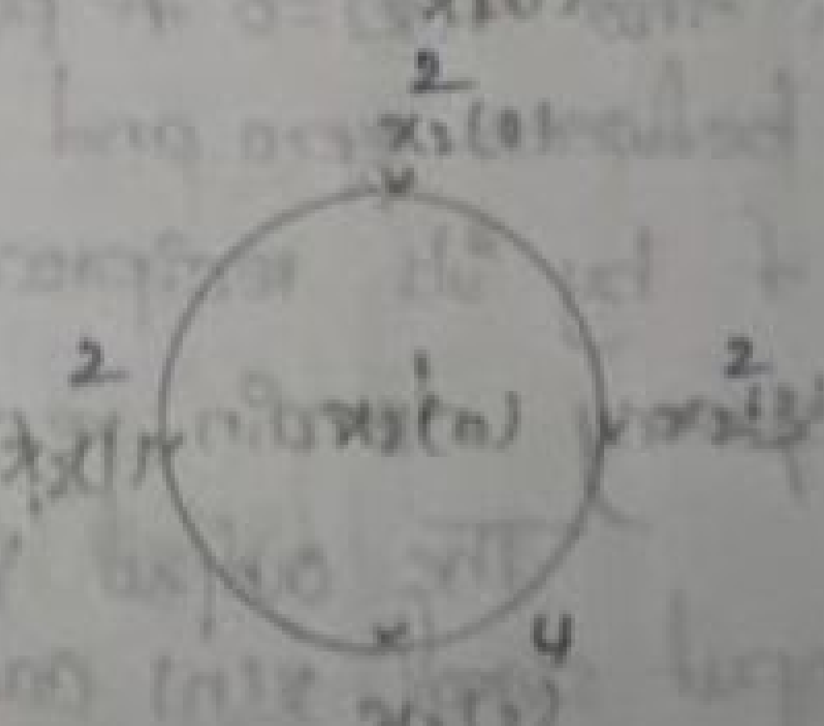
$4+12+2+2$
$= 20$



$8+6+2+2 = 18$



$4+6+2+4 = 16$



$$\begin{bmatrix} 2 & 3 & 1 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 3 \\ 3 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 4+6+4+2 \\ 2+4+12+2 \\ 2+2+8+6 \\ 6+2+4+4 \end{bmatrix} = \begin{bmatrix} 16 \\ 20 \\ 18 \\ 16 \end{bmatrix}$$

* Fourier transform is applicable for non-Periodic system

What is the use of transform.

Transformations are useful because it makes understanding the problem easier in one domain than in another.

Difference between fourier series and Fourier transform

* Fourier series is an extension of the periodic signal as a linear combination of sine and cosine.

While the fourier transform is a process or function used to convert signals in the time domain to the frequency domain

→ Fourier Series is defined for periodic signals

→ Fourier transform can be applied to a periodic signal

Difference between DFT and DTFT

DFT is a computational tool that stands for discrete fourier transform to convert a time domain discrete signal to its equivalent frequency domain response.

→ DFT is used in image processing

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi nk}{N}}$$

DTFT is a infinite continuous sequence that stands for discrete time fourier transform. The DTFT sequence provided the frequency domain representation for absolutely summable signals. This transformation is only defined for infinite length signals that are functions of a continuous function.

Zeros and poles ?

Poles and zeros of a transfer function are the frequencies for which the value of the denominator and numerator of transfer function becomes zero respectively. The values of the poles and zeros of a system determine whether the system performs.

A Zero of a meromorphic function f is a complex number z such that $f(z)=0$. A pole of f is zero of $1/f$. This includes a duality between zero and poles that is obtained by replacing the function f by its reciprocal $1/f$.

The Frequency domain representation of

The output $y(n)$ of any linear time invariant system to an input signal $x(n)$ can be obtained using convolution sum

$$x(n) \rightarrow \boxed{h(n)} \rightarrow y(n) \qquad y(n) = x(n) * h(n)$$

So let us consider a complex exponential signal

$$x(n) = e^{j\omega n}$$

Substitute this input signal in below equation

$$y(n) = \sum_{n=-\infty}^{\infty} h(n) x(n-k)$$

$$= \sum_{n=-\infty}^{\infty} h(k) e^{j}$$

$$= e^{j\omega n} \sum_{n=-\infty}^{\infty} h(k) e^{-j\omega k}$$

$$= e^{j\omega n} H(e^{j\omega}) \qquad H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h(k) e^{-j\omega k}$$

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x(k) e^{-j\omega k}$$

$$h(n) = \frac{y(n)}{x(n)} \qquad Y(e^{j\omega}) = \sum_{k=-\infty}^{\infty} y(k) e^{j\omega k}$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} \rightarrow \text{It is applicable for both finite and infinite}$$

Inverse Discrete Time Fourier transform (DTFT):

$$h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} h(e^{j\omega}) e^{j\omega} \, d\omega$$

It is called as inverse transform

Suppose $h(n) = (1/2)^n u(n)$ Find system response

Given $h(n) = (1/2)^n u(n)$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} (1/2)^n e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} (1/2)^n e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} (1/2 e^{-j\omega})^n \qquad \left\langle \sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \right\rangle$$

$$= \frac{1}{1 - 1/2 e^{-j\omega}}$$

$$= \frac{1}{1 - 0.5 e^{-j\omega}}$$

$$= \frac{1}{1 - 0.5(\cos\omega - j\sin\omega)}$$

$$= \frac{1}{1 - 0.5\cos\omega \, 0.5 \sin\omega}$$

$$|1 + e^{j\omega}| = \frac{1}{\sqrt{(1 - 0.5\cos\omega)^2 - (0.5)^2(\sin^2\omega)}}$$

$$= \frac{1}{\sqrt{1.25 - \cos\omega}}$$

$$\theta = \tan^{-1}(b/a)$$

$$= \tan^{-1}\left(\frac{0.5\sin\omega}{1 - 0.5\cos\omega}\right)$$

Suppose $h(n) = (0.9)(e^{j\pi/2})^n \; u(n)$ Find the system response

Given $h(n) = (0.9)^n (e^{j\pi/2})^n u(n)$

$$h|e^{j\omega}| = \sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} (0.9)^n (e^{j\pi/2})^n e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} (0.9)^n (\cos \pi/2 + j\sin \pi/2)^n e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} (0.9)^n (0 + j(1)) e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} (0.9) j e^{-j\omega n}$$

$$= \frac{1.}{1 - 0.9 (\cos\omega - j\sin\omega)}$$

$$= \frac{1}{1 - 0.9\cos\omega - j\, 0.9\sin\omega} = \frac{1}{1 - 0.9(-j)(\cos\omega - j\sin\omega)}$$

$$= \frac{1}{\sqrt{(1 - 0.9\sin\omega)^2 + 0.9\cos\omega j}}$$

$$\theta = \tan^{-1}(b/a)$$

$$= \tan^{-1}\left(\frac{0.9\cos\omega}{1 - 0.9j}\right)$$

$h(x) = \delta(n) - \delta(n-1)$

Given $h(x) = \delta(n) - \delta(n-1)$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} [\delta(n) - \delta(n-1)] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \delta(n) e^{-j\omega n} - \sum_{n=-\infty}^{\infty} \delta(n-1) e^{-j\omega n}$$

$$= 1 - e^{-j\omega n} \quad \langle e^{-j\theta} = \cos\theta - j\sin\theta \rangle$$

$$= 1 - e^{-j\omega}$$

$$H(e^{j\omega}) = 1 - (\cos\omega - j\sin\omega)$$

$$|H e^{j\omega}| = \sqrt{(1 - \cos\omega - j\sin\omega)^2}$$

$$= \sqrt{(1 - \cos\omega)^2 - (j\sin\omega)^2}$$

$$= \sqrt{1 + \cos^2\omega - 2\cos\omega + \sin^2\omega}$$

$$= \sqrt{1 + 1 - 2\cos\omega}$$

$$= \sqrt{2(1 - \cos\omega)}$$

$$= \sqrt{2 \cdot 2\sin^2\frac{\omega}{2}}$$

$$= \sqrt{4\sin^2\frac{\omega}{2}} = 2\sin\frac{\omega}{2}$$

$$\angle H(e^{j\omega}) = \tan^{-1}\left(\frac{\sin\omega}{1 - \cos\omega}\right)$$

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$= \tan^{-1} \dfrac{2\sin \omega/2 \cos \omega/2}{2\sin^2 \omega/2}$

$= \tan^{-1} \dfrac{\cos \omega/2}{\sin \omega/2}$

$= \tan^{-1} (\cot \omega/2)$

$= \dfrac{1}{\cot} (\cot \omega/2)$

$= \omega/2$

$\omega = -\pi$ to π

$|1+e^{j\omega}| = 2\sin \omega/2$

$= 2\sin(2\pi/2) = 0.11$

$\omega \Rightarrow 3\pi$

$\omega = 0 \Rightarrow 2\sin(0/2) = 0$

$\omega = \pi/2 \Rightarrow 2\sin\left(\frac{\pi/2}{2}\right) = \sqrt{2}$

$\omega = \pi/4 \Rightarrow 2\sin\left(\frac{\pi/4}{2}\right) = 0.76$

$\omega = \pi/6 \Rightarrow 2\sin\left(\frac{\pi/6}{2}\right) = 0.518$

$\omega = \pi/8 \Rightarrow 2\sin\left(\frac{\pi/8}{2}\right) = 0.39$

$\angle 1+e^{j\omega} = \omega/2$

$= 0/2, \frac{\pi/2}{2}, \frac{\pi/4}{2}, \frac{\pi/6}{2}, \frac{\pi/8}{2}$

Trignometry Formulas :

* $\sin(A+B) = \sin A \sin B + \cos A \cos B$
* $\sin(A-B) = \sin A \cos B - \cos A \sin B$
* $\cos(A+B) = \cos A \cdot \cos B - \sin A \sin B$
* $\cos(A-B) = \cos A \cdot \cos B + \sin A \sin B$
* $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
* $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$
* $\cot(A+B) = \dfrac{\cot B \cot A - 1}{\cot B + \cot A}$
* $\cot(A-B) = \dfrac{\cot B \cot A + 1}{\cot B - \cot A}$
* $\sin 2x = 2\sin x \cdot \cos x = \dfrac{2\tan x}{1+\tan^2 x}$
* $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$
  $= 1 - 2\sin^2 x$.
  $= \dfrac{1-\tan^2 x}{1+\tan^2 x}$
* $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$
* $\sin 3x = 3\sin x - 4\sin^3 x$
* $\cos 3x = 4\cos^3 x - 3\cos x$
* $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$
* $\sin\alpha + \sin B = 2\sin\left(\frac{\alpha+B}{2}\right)\cos\left(\frac{\alpha-B}{2}\right)$
* $a\sin x + b\cos x \Rightarrow$
  Max Valye $= +\sqrt{a^2+b^2}$
  Min value $= -\sqrt{a^2+b^2}$
* $\sin\alpha - \sin B = 2\cos\left(\frac{\alpha+B}{2}\right)\sin\left(\frac{\alpha-B}{2}\right)$
* $\cos\alpha + \cos B = 2\cos\left(\frac{\alpha+B}{2}\right)\cos\left(\frac{\alpha-B}{2}\right)$
* $\cos\alpha - \cos B = -2\sin\left(\frac{\alpha+B}{2}\right)\sin\left(\frac{\alpha-B}{2}\right)$ (or) $2\sin\left(\frac{\alpha+B}{2}\right)\sin\left(\frac{B-\alpha}{2}\right)$

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* Odd $F(x) \Rightarrow F(-x) = -F(x)$
* Even $F(x) \Rightarrow F(-x) = +F(x)$
* $\cos(-x) = \cos x$
* $\sin(-x) = \sin x$
* $\tan(-x) = -\tan x$
* $\sec(-x) = \sec x$
* $\operatorname{cosec}(-x) = -\operatorname{cosec} x$
* $\cot(-x) = -\cot x$
* $2\sin A\cos B = \sin(A+B) + \sin(A-B)$
* $2\cos A\sin B = \sin(A+B) - \sin(A-B)$
* $2\cos A\sin B = \cos(A+B) + \cos(A-B)$
* $-2\sin A\sin B = \cos(A+B) - \cos(A-B)$
* $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$
* $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$
* $\sin^2\theta + \cos^2\theta = 1$
* $\sec^2\theta - \tan^2\theta = 1$
* $\operatorname{cosec}^2\theta - \cot^2\theta = 1$
* $1 + \cos 2x = 2\cos^2 x$
* $1 - \cos 2x = 2\sin^2 x$
* $\tan x = \frac{1-\cos 2x}{\sin 2x}$

General solutions of trignometry

* $\sin x = 0 \Rightarrow x = n\pi$
* $\cos x = 0 \Rightarrow x = (2n+1)\pi/2$
* $\tan x = 0 \Rightarrow x = n\pi$
* $\sin x = \sin y \Rightarrow x = n\pi + (-1)^n y$
* $\cos x = \cos y \Rightarrow x = 2n\pi \pm y$
* $\tan x = \tan y \Rightarrow x = n\pi + y$

$$\left.\begin{array}{l} \sin^2 x = \sin^2 y \\ \cos^2 x = \cos^2 y \\ \tan^2 x = \tan^2 y \end{array}\right\} \Rightarrow x = n\pi \pm y,\ n \in Z$$

Integration Formulas:

* $\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad n \neq 1$
* $\int dx = x + c$
* $\int \cos x \, dx = \sin x + c$
* $\int \sin x \, dx = -\cos x + c$
* $\int \sec^2 x \, dx = \tan x + c$
* $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$
* $\int \sec x \tan x \, dx = \sec x + c$
* $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$
* $\int \frac{dx}{\sqrt{1+x^2}} = \sin^{-1} x + c$
* $\int \frac{dx}{\sqrt{1-x^2}} = \cos^{-1} x + c$
* $\int \frac{dx}{1+x^2} = \tan^{-1} x + c$
* $\int \frac{dx}{1+x^2} = \cot^{-1} x + c$
* $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c$
* $\int \frac{dx}{x\sqrt{x^2-1}} = -\operatorname{cosec}^{-1} x + c$
* $\int e^x dx = e^x + c$
* $\int \frac{dx}{x} = \log|x| + c$
* $\int a^x dx = \frac{a^x}{\ln a} + c$

Differentiation Formulas :

+ $\frac{dk}{dx} = 0$

+ $\frac{dx}{dx} = 1$

+ $\frac{d(kx)}{dx} = k$

+ $\frac{d(x^n)}{dx} = nx^{n-1}$

+ $\frac{d}{dx}(\cos x) = -\sin x$

+ $\frac{d}{dx}(\sin x) = \cos x$

+ $\frac{d}{dx}(\tan x) = \sec^2 x$

+ $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

+ $\frac{d}{dx}(\sec x) = \sec x \tan x$

+ $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x(\cot x)$

+ $\frac{d}{dx}(\ln x) = \frac{1}{x}$

+ $\frac{d}{dx}(e^x) = e^x$

+ $\frac{d}{dx}(a^x) = (\ln a)a^x$

+ $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

+ $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

+ $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$

+ $\frac{d}{dx}(\sinh x) = \cosh x$

+ $\frac{d}{dx}(\cosh x) = \sinh x$

+ $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

+ $\frac{d}{dx}(\coth x) = -\operatorname{cosech}^2 x$

+ $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

+ $\frac{d}{dx}(\operatorname{cosech} x) = -\operatorname{cosech} x \coth x$

DIF

$x(n) = \{1, 1, 2, 2\}$

Given $x(n) = \{1, 1, 2, 2\}$

$N = 4$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi kn}{N}}$$

$$= \sum_{n=0}^{4-1} x(n) e^{-j\frac{2\pi kn}{4}}$$

$$= \sum_{n=0}^{3} x(n) e^{-j\frac{\pi kn}{2}}$$

Where $k = 0$ to $N-1$

$k=0$

$$X(0) = \sum_{n=0}^{3} x(n) e^{-j\frac{\pi nk}{2}}$$

$$= x(0) e^{-\frac{j\pi(0)(0)}{2}} + x(1) e^{-\frac{j\pi(0)(1)}{2}} + x(2) e^{-\frac{j\pi(0)(2)}{2}} + x(3) e^{-\frac{j\pi(0)(3)}{2}}$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1 + 1 + 2 + 2$$

$$= 6$$

$k=1$

$$X(1) = \sum_{n=0}^{3} x(n) e^{-j\frac{\pi(1)n}{2}}$$

$$= \sum_{n=0}^{3} x(n) e^{-j\frac{\pi n}{2}}$$

$$= x(0) e^{-j\frac{\pi(0)}{2}} + x(1) e^{-j\frac{\pi(1)}{2}} + x(2) e^{-j\frac{\pi(2)}{2}} + x(3) e^{-j3\pi/2}$$

$$= x(0) e^{-0} + x(1)(\cos\pi/2 - j\sin\pi/2) + x(2)(\cos\pi - j\sin\pi) + x(3)(\cos 3\pi/2 - j\sin 3\pi/2)$$

$$= 1 + 1(0 - j(1)) + 2(-1 - j(0)) + 2(0 - j(-1))$$

$$= 1 - j - 2 + 2j$$

$$= -1 + j$$

$k=2$

$$X(2) = \sum_{n=0}^{3} x(n) e^{-j\frac{\pi(2)n}{2}}$$

$$= \sum_{n=0}^{3} x(n) e^{-j\pi n}$$

$$= x(0) e^{-j\pi(0)} + x(1) e^{-j\pi(1)} + x(2) e^{-j\pi(2)} + x(3) e^{-j3\pi}$$

$$= 1(1) + (1)(\cos\pi - j\sin\pi) + 2(\cos 2\pi - j\sin 2\pi) + 2(\cos 3\pi - j\sin 3\pi)$$

$$= 1 + 1(-1 - j(0)) + 2(1 - 0) + 2(-1 - 0)$$

$$= 1 - 1 + 2 - 2$$

$$= 0$$

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If $k=3$

$$x(3)=\sum_{n=0}^{3}x(n)e^{-\frac{j\pi n3}{2}}$$

$$=x(0)e^{-\frac{j\pi(0)3}{2}}+x(1)e^{-\frac{j\pi(1)3}{2}}+x(2)e^{-\frac{j\pi(2)3}{2}}+x(3)e^{-\frac{j\pi(3)3}{2}}$$

$$=x(0)e^{-0}+x(1)e^{-j3\pi/2}+x(2)e^{-j3\pi}+x(3)e^{-j9\pi/2}$$

$$=1+1\left(\cos\frac{3\pi}{2}-j\sin\frac{3\pi}{2}\right)+2\left(\cos3\pi-j\sin3\pi\right)+2\left(\cos9\pi/2-j\sin9\pi/2\right)$$

$$=1+1(0-j(-1))+2(-1-0)+2(0-j)$$

$$=1+j-2-2j$$

$$=-1-j$$

$$x(k)=\{6,-1+j,0,-1-j\}$$

Time Domain Analysis of Discrete Time signals and systems:

The General form of differential equation is

$$y(n)=-\sum_{k=1}^{N}a_k y(n-k)+\sum_{k=0}^{y}b_k x(n-k)$$

The solution of the Difference equation consists of two parts

$$y(n)=y_h(n)+y_p(n)$$

Where

$y_h(n)$ = Natural response is known as the Homogeneous solution

$y_p(n)$ = Forced Response is called Particular solution

The Homogeneous solution is ~~known as~~ the obtained by the input $x(n)$ to zero.

$$\sum_{k=0}^{N}a_k y(b-k)=0 \longrightarrow (2)$$

To solve the Equation (2) Assume

$$y_h(n)=\lambda^n \longrightarrow (3)$$

The subscript of h on $y(n)$ is used to denote the solution the Homogeneous difference equation

Substitute equation (3) in equation (2)

$$\sum_{k=0}^{N} a_k \lambda^{n-k} = 0$$

$$\lambda^N + a_1\lambda^{N-1} + \ldots\ldots + a_{N-1}\lambda + a_N \longrightarrow (4)$$

The Equation (4) is known as characteristic equation and has N roots which we denoted as $\lambda_1, \lambda_2 \ldots \lambda_N$

The General solution of in this form

$$y_h(n) = C_1\lambda_1^n + C_2\lambda_2^n + \ldots + C_N\lambda_N^n \longrightarrow (5)$$

If the roots are $\lambda_1 = 2 \quad \lambda_2 = 3$

$$y_h(n) = C_1(2)^n + C_2(3)^n \longrightarrow (6)$$

If the roots of the characterisation are repeated for M times the general solution of $y_h(n)$

$$y_h(n) = (\lambda_1)^n (C_1 + C_2 n + C_3 n^2 + \ldots + C_m n^{m-1}) \longrightarrow (7)$$

$\lambda_1 = 1 \qquad \lambda_2 = 2$

$2^n(C_1 + C_2)$

The characteristics of equation has complex roots $\lambda_1 \,\&\, \lambda_2$ is $a \pm ib$ then the solution is

$$y_h(n) = r^n (A_1 \cos n\theta + A_2 \sin n\theta)$$

$r = \sqrt{a^2+b^2}$

$\theta = \tan^{-1}(b/a)$

$A_1 \,\&\, A_2$ = constant

Similarly,

The particular solution of $y_p(n)$ is to satisfy the difference equation for the specific input signal $x(n)$

$n \geq 0$

The General form of particular solution

$x(n)$ input signal	$y_p(n)$ Particular solution
A (step)	K
AM^n	KM^n
$A^n N^M$	$A^n(k_0 n^M + k_1 n^{M-1} + \dots k_M)$
An^M	$k_0 n^M + k_1 n^{m-1} + \dots k_M$
$A\cos\omega_0 n$, $A\sin\omega_0 n$ $\longrightarrow$	$k_1\cos\omega_0 n + k_2\sin\omega_0 n$

Determine the Response $y(n)$ $n\geq 0$ of the system described by the second order difference equation $y(n)-4y(n-1)+4y(n-2) = x(n)-x(n-1)$ when the input is $x(n)=(-1)^n u(n)$ and the initial conditions are $y(-1)=y(-2)=1$

Particular solution:

$$y_p(n) = k(-1)^n u(n)$$

Substituting the values of $x(n)$ and $y_p(n)$ in the difference equation we have

$$k(-1)^n u(n) - 4k(-1)^{n-1}u(n-1) + 4k(-1)^{n-2}u(n-2) = (-1)^n u(n) - (-1)^{n-1}u(n-1)$$

Now substitute $n=2$

$$k(-1)^2 u(2) - 4k(-1)^{2-1}u(2-1) + 4k(-1)^{2-2}u(2-2) = (-1)^2 u(2) - (-1)^{2-1}u(2-1)$$

$$ku(2) + 4ku(1) + ku(0) = u(2) + u(1)$$

$$k + 4k + 4k = 1+1$$

$$9k = 2$$

$$k = 2/9$$

Now the particular solution is

$$y_p(n) = 2/9\,(-1)^n u(n)$$

Homogeneous solution:

$$y_h(n) = \lambda^n$$

$$y(n) - 4y(n-1) + 4y(n-2) = 0$$

$$\lambda^n - 4\lambda^{n-1} + 4\lambda^{n-2} = 0$$

Substitute $n=2$

$\lambda^2 - 4\lambda^{2-1} + 4\lambda^{2-2} = 0$

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda^2 - 2\lambda - 2\lambda + 4 = 0$

$\lambda(\lambda-2) - 2(\lambda-2) = 0$

$\lambda - 2 = 0 \qquad \lambda - 2 = 0$

$\lambda = 2, 2$

$y_h(n) = 2^n (C_1 + nC_2) \longrightarrow$ Homogeneous solution

$y_p(n) = 2/9 (-1)^n u(n) \longrightarrow$ Particular solution

$y(n) = 2^n (C_1 + nC_2) + 2/9 (-1)^n u(n)$

It is Total Response

In initial condition n=0 in above equation

$y(0) = 2^0 (C_1 + 0C_2) + 2/9 (-1)^0 u(0)$

$y(0) = C_1 + 2/9 \longrightarrow$ (1a)

$n=1 \Rightarrow$

$y(1) = 2^1 (C_1 + C_2) - 2/9 \longrightarrow$ (1b)

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

Put n=0 in above equation

$y(0) - 4y(0-1) + 4y(0-2) = x(0) - x(0-1)$

$y(0) - 4y(-1) + 4y(-2) = x(0) - x(-1)$

$y(0) - 4 + 4 = 1$

$y(0) = 1 \longrightarrow$ (2)

Put n=1

$y(1) - 4y(1-1) + 4y(1-2) = x(1) - x(1-1)$

$y(1) - 4y(0) + 4y(-1) = x(1) - x(0)$

$y(1) - 4 + 4 = -1 - 1$

$y(1) = -2$

Substitute y(0)=1 in equation (1a)

$1 = C_1 + 2/9 \Rightarrow$

$C_1 = 1 - 2/9$

$C_1 = 7/9$

Put $y_1(1)$ in equation 1b

$-2 = 2(C_1+C_2) - 2/9$

$-2 = 2(7/9 + C_2) - 2/9$

$-2 = \frac{14}{9} + \frac{18C_2}{9} - 2/9$

$-2 = \frac{14 + 18C_2 - 2}{9}$

$14 + 18C_2 - 2 = -18$

$12 + 18C_2 = -18$

$18C_2 = -30$

$C_2 = \frac{-30}{18}$

$C_2 = \frac{-5}{3}$

The second order differente equation $y(n) - 1/6\, y(n-1) - 1/6\, (y(n-2))$ $= x(n)$. Determine the impulse response $h(n)$ for the system.

$y(n) - 1/6\, y(n-1) - 1/6\, (y(n-2)) = x(n)$ input

$y_p(n) = k$

$\delta(n) = \begin{cases} 1 & \text{for } n=0 \\ 0 & \text{for } n \neq 0 \end{cases}$

Homogeneous solution :

$\lambda^n - 1/6\, \lambda^{n-1} - 1/6\, \lambda^{n-2} = 0$

$n=2$

$\lambda^2 - 1/6\, \lambda^{2-1} - 1/6\, \lambda^{2-2} = 0$

$6\lambda^2 - \lambda - 1 = 0$

$6\lambda^2 - 3\lambda + 2\lambda - 1 = 0$

$(3\lambda+1)(2\lambda-1) = 0$

$\lambda = -1/3,\ 1/2$

$C_1 (1/2)^n - C_2 (1/3)^n = y_h(n)$

Impulse response :

$y(0) - 1/6\, y(-1) - 1/6\, y(-2) = x(0)$

$y(0) - 1/6 - 1/3 = 1$

$y(0) - 1/3 = 1$

$n=0$

$y(0) = C_1 + C_2 + 1 \cdot (-1a)$

$n=1$

$y(1) = C_1(1/2)^1 + C_2(1/3)^1 \quad (-1/6$

$y(n) - 1/6(n-1) - 1/6(n-2) = x(n)$

$n=0$

$y(0) - 1/6\, y(-1) - 1/6\, y(-2) = 1$

$y(0) - 1/6 - 1/6 = 1$

$y(0) - \frac{2}{6} = 1$

$y(0) = 1 + 2/6$

$y(0) = 8/6$

$y(0) = 4/3$

$n=1$

$y(1) - 1/6\, y(0) - 1/6\, y(-1) = x(1)$

$y(1) - 2/6 = 1/3$

Correlation :

It quantities the similarly of two spatial or time dependent signal x & y

The main property of correlation is that both signals do not have to depend on each other.

Ex : Radar

$$\gamma_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)\, y(t-\tau)\, dt$$

Where

τ = temporal distance between both signals

Correlation	Convolution
* Measurement of the similarity between two signals $y(t) = x(t) * h(t)$ $= \int_{-\infty}^{\infty} x(t)\, h(t-\tau)\, dt$	* Measurement of effect of one signal on the other signal $y(t) = \int_{-\infty}^{\tau} x_1(t)\, x_2(t-\tau)\, dt$
* Not Commutative	* Commutative
* Our main aim is to measure the degree to which into sgn are similar and thus to extract some information that depends to a large extent on the application	* Our main aim is calculate the response given by the system

Types of Correlation:

It is classified into two types

(i) Auto correlation

(ii) Cross correlation

Auto Correlation:

$x_1(n) = \{2, 3, 1, 2\}$

$x_2(n) = \{1, 5, 3, 7\}$

$\gamma_{xx}(n) = x_1(n) * x_2(-n)$

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$x_1(-n)$ \ $x_1(n)$	2	3	1	2
2	4	6	2	4
1	2	3	1	2
3	6	9	3	6
2	4	6	2	4

$r_{xy}(l) = \{4, 8, 11, 18, 11, 8, 4\}$

Cross correlation :

$x_1(n) = \{2, 3, 1, 2\}$

$x_2(n) = \{1, 5, 3, 7\}$ $\quad S_{xy}(l) = x_1(n) * x_2(-n)$

$x_2(-n)$ \ $x(n)$	2	3	1	2
7	14	21	7	14
3	6	9	3	6
1	10	15	5	10
5	2	3	1	2

$S_{xy}(l) = \{14, 27, 26, 34, 14, 11, 2\}$

Z - Transform :

$$x(z) = \sum_{-\infty}^{\infty} x(n) z^{-n}$$

$$z = Ae^{j\theta}$$

A = Magnitude Response

Example :

$x(n) = \{2, 3, 2, 4, 5\}$

Given $x(n) = \{2, 3, 2, 4, 5\}$

$= x(0)z^0 + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} + x(4)z^{-4}$

$= 2z^0 + 3z^{-1} + 2z^{-2} + 4z^{-3} + 5z^{-4}$

$= 2 + 3z^{-1} + 2z^{-2} + 4z^{-3} + 5z^{-4}$

Radar :

Radio Detection and Ranging

It is an electronic device that provides microwave segment or Ultra high frequency of the radio spectrum to identify obstacles to control the area of the spot or range of an object.

Types of Radar :

- Radar
  - First classification
    - Primary radar
      - Pulsed radar
        - Short pulses
        - Pulses compression
      - continuous wave radar
        - unmodulated
        - modulated
    - secondar Radar
  - Second classification
    - Military
      - Air defuse
        - surveillance
        - Air policing
        - missile control
      - Battlefield
        - surveillance
        - Nevigation
        - Missile control
        - Weapon control
    - civil
  - Air traffic control
    - Air surve-illance radar
    - Precision approach radar
    - Airport surface Defection Equipment
    - wheather radar
    - En Route Radar
  - Miscellaneous
    - Wheater radar
    - speed gauges
    - Radar Controller
    - Nondestructive material test
    - Ground Penetrating radar

Stability and Roc :

The stability of the system can be form from ROC using following theorem. The theorem is linear time invariant system the system function $h(z)$ is BIBO stable. If and only If the Roc for $h(z)$ contains unit circle.

The properties of Region of Convergence :

1. The ROC is ring and Disc in zplane the center at the region.
2. The ROC cannot contain any poles i.e.,
3. If $x(n)$ in causual sequence than the Roc is the entire z - plane except at $z=0$
4. $x(n)$ is non causual when the Roc in the entire the z-plane except at $z=\infty$

5. If $x(n)$ is finite duration two sided sequence the ROC is entire z plane except $z=0$ and $z=\infty$

6. If $x(n)$ is infinite duration two sided sequence ROC is consists of a ring in z the plane the bounded on interio and exterior by a pole not contoing any poles.

7. The ROC of a LTI stable system contain the unit circle.

8. The ROC must be in connected Region.

Determine the pole zero plote for the system dscribed by different equations $y(n) - 3/4\, y(n-1) + 1/8\,(n-2) = x(n) - x(n-1)$

Given $y(n) - 3/4\, y(n-1) + 1/8\, y(n-2) = x(n) - x(n-1)$

$$y(z) - 3/4\, y(z) z^{-1} + 1/8\, y(z) z^{-2} = x(z) - x(z)(z^{-1})$$

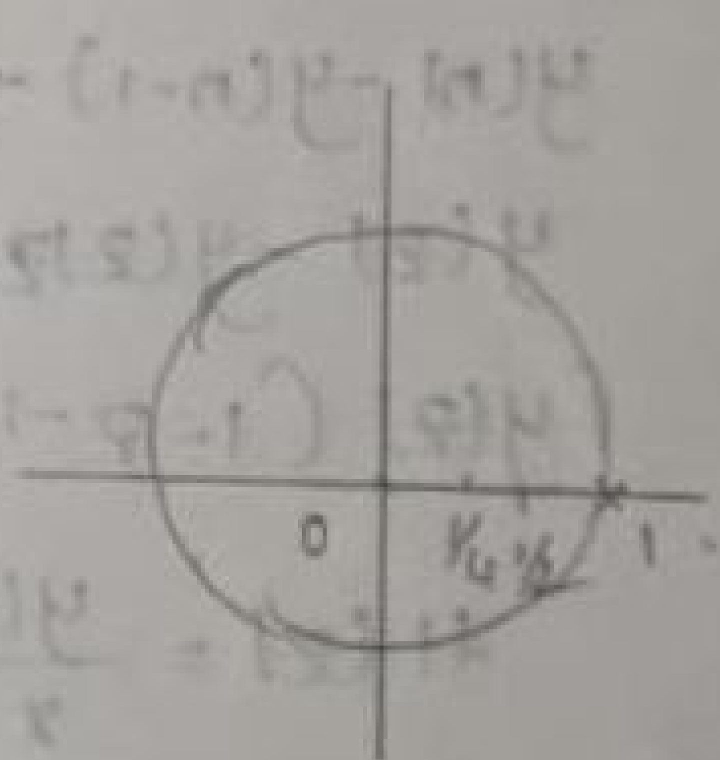
$$y(z)\left[1 - 3/4\, z^{-1} + 1/8\, z^{-2}\right] = x(z)\,(1 - z^{-1})$$

$$\therefore H(z) = \frac{y(z)}{x(z)}$$

$$= \frac{1 - z^{-1}}{1 - 3/4\, z^{-1} + 1/8\, z^{-2}} \times \frac{z^2}{z^2}$$

$$= \frac{z^2 - z'}{(z - 1/2)(z - 1/4)}$$

$$= \frac{z(z-1)}{(z-1/2)(z-1/4)} \begin{array}{l} \rightarrow \text{zeros} \\ \rightarrow \text{poles} \end{array}$$



Zeros :
$z = 0$
$z = 1$

Poles
$z = 1/2$
$z = 1/4$

$y(n) = 5/6\, y(n-1) - 1/6\, y(n-2) + x(n) - x(n-1)$

Given

$$y(n) = 5/6\, y(n-1) - 1/6\,(n-2) + x(n) - x(n-1)$$

$$y(n) - 5/6\, y(n-1) + 1/6\,(n-2) = x(n) - x(n-1)$$

$$y(z) - 5/6\, y(z) z^{-1} + 1/6\, y(z) z^{-2} = x(z) - x(z) z^{-1}$$

$$Y(z)\left[1-\tfrac{5}{6}z^{-1}+\tfrac{1}{6}z^{-2}\right] = X(z)\left\{1-z^{-1}\right\}$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$= \frac{1-z^{-1}}{1-\tfrac{5}{6}z^{-1}+\tfrac{1}{6}z^{-2}} \times \frac{z^2}{z^2}$$

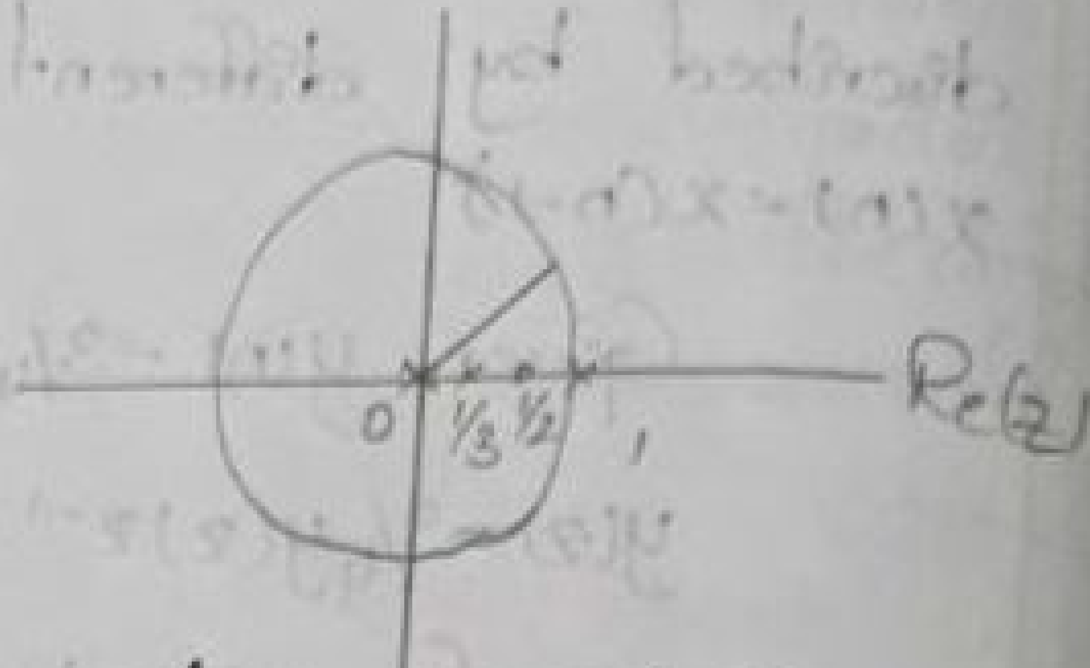
$$= \frac{z(z-1)}{z^2-\tfrac{5}{6}z+\tfrac{1}{6}}$$

$$= \frac{z(z-1)}{(z+\tfrac{1}{2})(z+\tfrac{1}{3})}$$

Zeros   Poles

$z=0$ $z=-\tfrac{1}{2}$

$z=1$ $z-\tfrac{1}{3}$



A causual LTI system described by the difference equation $y(n) = y(n-1)+y(n-2)+x(n-1)$ Find the system function (ii) Find unit sample response of the system (iii) Is the system is stable or unstable

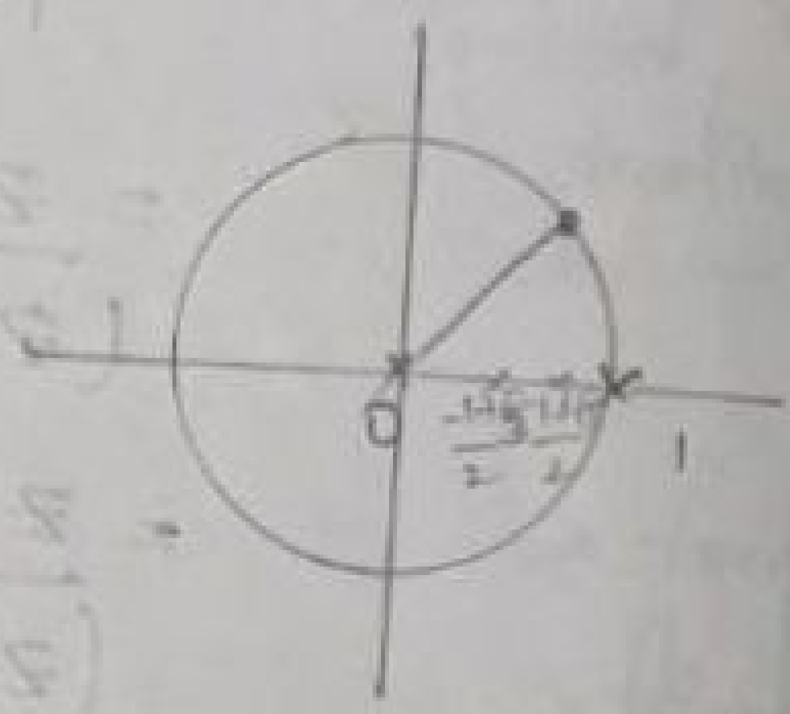
Given

$$y(n) = y(n-1)+y(n-2)+x(n-1)$$

$$y(n)-y(n-1)-y(n-2) = x(n-1)$$

$$Y(z)-Y(z)z^{-1}-Y(z)z^{-2} = X(z)z^{-1}$$

$$Y(z)\left(1-z^{-1}-z^{-2}\right) = X(z)z^{-1}$$



$$H(z) = \frac{Y(z)}{X(z)}$$

$$= \frac{z^{-1}}{1-z^{-1}-z^{-2}} \times \frac{z^2}{z^2}$$

$$= \frac{z}{z^2-z-1}$$

$$= \frac{z}{\left(z-\frac{1+\sqrt{3}}{2}\right)\left(z+\frac{1+\sqrt{5}}{2}\right)}$$

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Zeros : Poles

0 $\frac{-1+\sqrt{5}}{2}$, $\frac{1+\sqrt{5}}{2}$

- This system is unstable

The relation between z transform and Fourier transform :

$$H(z) = \sum_{n=-\infty}^{\infty} h(n)\, z^{-n} \longrightarrow ①$$

$$z = re^{j\omega}$$

$$= \sum_{n=-\infty}^{\infty} h(n)(re^{j\omega})^{-n}$$

Fourier transform

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-j\omega n}$$

$$H(e^{j\omega}) = H(z)$$

Evaluate the frequency response of the system described by $h(z) = \frac{1}{1-0.5z^{-1}}$

Given

$$h(z) = \frac{1}{1-0.5z^{-1}}$$

$$= \frac{1}{1-\frac{0.5}{z}}$$

$$= \frac{z}{z-0.5}$$

substitute $z = e^{j\omega}$

$$h(e^{j\omega}) = \frac{e^{j\omega}}{e^{j\omega}-0.5}$$

$$= \frac{\cos\omega + j\sin\omega}{(\cos\omega + j\sin\omega)-0.5}$$

$$|h(e^{j\omega})| = \sqrt{\left[\frac{(\cos\omega + j\sin\omega)}{((\cos\omega + j\sin\omega)-0.5)}\right]^2}$$