Check the following system described with difference equation for linearity, shift invariance, memory and causality

(i) $y(n)+y(n+1)=nx(n)$

(ii) $y(n)=x(n)+x(n-1)+x(n-2)$

Linearity:

(i) $nx(n)=y(n)+y(n+1)$

$a(nx_1(n))=(y_1(n)+y_1(n+1))a$

$b(nx_2(n))=(y_2(n)+y_2(n+1))b$

$anx_1(n)=ay_1(n)+ay_1(n+1) \rightarrow ①$

$bnx_2(n)=by_2(n)+by_2(n+1) \rightarrow ②$

$anx_1(n)+bnx_2(n)=(ay_1(n)+by_2(n))$
$+(ay_1(n+1)+by_2(n+1))$

$n(ax_1(n)+bx_2(n)) \neq (ay_1(n)+by_2(n))+$
$(ay_1(n+1)+by_2(n+1))$

The given is non linear system.

(ii) $y(n)=x(n)+x(n-1)+x(n-2)$

$(y_1(n)=x_1(n)+x_1(n-1)+x_1(n-2))\times a$

$(y_2(n)=x_2(n)+x_2(n-1)+x_2(n-2))\times b$

$ay_1(n)=ax_1(n)+ax_1(n-1)+ax_1(n-2) \rightarrow ①$

$by_2(n)=bx_2(n)+bx_2(n-1)+bx_2(n-2) \rightarrow ②$

Summing eq → ① & ②

$ay_1(n)+by_2(n)=[ax_1(n)+bx_2(n)]+(ax_1(n-1)$
$+bx_2(n-1))+(ax_1(n-2)+bx_2(n-2))$

L.H.S ≠ R.H.S

The given system is non-linear

Shift invariance

(i) $nx(n)=y(n)+y(n+1)$

$y(t-t_0)=b[x(t-t_0)]$.

Put $n=n-n_0$

$(n-n_0)x(n-n_0)=y(n-n_0)+y(n-n_0+1)$.

It is shift invariant

(ii) $y(n)=x(n)+x(n-1)+x(n-2)$

$y(n,k)=T[x(n-k)]$

$y(n-k)=x(n-k)+x(n-k-1)+x(n-k-2)$

Put $n=n-k$

$y(n,k)=y(n-k)$

It is Invariant

1 Causality :-

(i) $y(n) + y(n+1) = nx(n)$

Put $n=0$

$y(0) + y(1) = (0)x(0)$

Put $n=1$

$y(1) + y(1+1) = (1)x(1)$

Put $n=2$

$y(2) + y(2+1) = (2)x(2)$

It is non-causal system

depends on future value.

(ii) $y(n) = x(n) + x(n-1) + x(n-2)$

Put $n=0$

$y(0) = x(0) + x(0-1) + x(0-2)$

Put $n=1$

$y(1) = x(1) + x(1-1) + x(1-2)$

$y(1) = x(1) + x(0) + x(-1)$

Put $n=2$

$y(2) = x(2) + x(2-1) + x(2-2)$

It is a causal system. depends on present & past values

Memory :- Memory less $\Rightarrow$ O/P $\Rightarrow$ present i/p (state)

Memory $\rightarrow$ O/P $\Rightarrow$ future i/p (or) past (dynamic)

(i)

~~$y(t) = x(t)$~~

$y(n) + y(n+1) = nx(n)$

Put $n=0$

$0 x(0) = y(0) + y(0+1)$

Put $n=1$

$1x(1) = y(1) + y(1+1)$

$x(1) = y(1) + y(2)$

Put $n=2$

$2x(2) = y(2) + y(2+1)$

$2x(2) = y(2) + y(3)$

dynamic.

(ii) $y(n) = x(n) + x(n-1) + x(n-2)$

Put $n=0$

$y(0) = x(0) + x(0-1) + x(0-2)$

Put $n=1$

$y(1) = x(1) + x(1-1) + x(1-2)$

Put $n=2$

$y(2) = x(2) + x(2-1) + x(2-2)$

dynamic.

Q4) Determine the linear convolution of following two sequences

$x(n) = [3, 2, 1, 2]$; $h(n) = [1, 2, 1, 2]$.
$\uparrow$

Given

$x(n) = [3, 2, 1, 2] = L$

$h(n) = [1, 2, 1, 2] = M$

$N = L + M - 1 = 4 + 4 - 1 = 7$

$x(k) = \{3, 2, 1, 2\} = n = n_1 = -1$
$\uparrow$

$h(k) = \{1, 2, 1, 2\} = n = n_2 = 0$

$n = n_1 + n_2 = -1 + 0 = -1$

k	-4	-3	-2	-1	0	1	2	3	4	5	6
x(k)				-3	2	1	2				
h(k)					1	2	1	2			
h(-k)		2	1	2	1						
h(1-k)			2	1	2	1					
h(2-k)				2	1	2	1				
h(3-k)					2	1	2	1			
h(4-k)						2	1	2	1		
h(5-k)						~~2~~	2	~~1~~	2	1	

$n = -1 \Rightarrow y(-1) = 3$

$n = 0 \Rightarrow y(0) = 6 + 2 = 8$

$n = 1 \Rightarrow y(1) = x(k) \cdot h(1-k) = 3 + 4 + 1 = 8$

$n = 2 \Rightarrow y(2) = x(k)\, h(2-k) = 6 + 2 + 2 + 2 = 12$

$n = 3 \Rightarrow y(3) = x(k)\, h(3-k) = 4 + 1 + 4 = 9$.

$n = 4 \Rightarrow y(4) = x(k)\, h(4-k) = 2 + 2 = 4$

$n = 5 \Rightarrow y(5) = x(k)\, h(5-k) = 2 \times 2 = 4$.

$\therefore y(n) = \{3, 8, 8, 12, 9, 4, 4\}$
$\uparrow$

Explain ~~how to manage time~~ ... the system described by the different equation

3 b) Explain the power signal & Energy signal?

The Energy 'E' is defined for a discrete time signal is

$$E = \sum_{n=\infty}^{\infty} |x(n)|^2$$

The average power of an discrete time signal x(n) is defined as

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{N=-N}^{N} |x(n)|^2$$

A signal is an energy signal, if and only if the total Energy of the signal is finite. for an energy signal P=0.

Similarly the signal is said to be power signal if the average power of the signal is finite

for a power signal $E=\infty$. The signal that do not satisfy above properties are neither Energy nor power signal.

4. Check for causality and stability of the following system?

(i) $y(n) = x(n) + x(n-1) + x(n-2)$

(ii) $y(n) - 2y(n-1) = x(n)$

Causality:

(i) $y(n) = x(n) + x(n-1) + x(n-2)$

Put n=0

$y(0) = x(0) + x(0-1) + x(0-2)$

Put n=1

$y(1) = x(1) + x(1-1) + x(1-2)$

Put n=2

$y(2) = x(2) + x(2-1) + x(2-2)$

It is a causal system. depends on present & past

(ii) $x(n) = y(n) - 2y(n-1)$

Put n=0

$x(0) = y(0) - 2y(0-1)$

Put n=1

$x(1) = y(1) - 2y(1-1)$

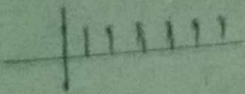
Put n=2

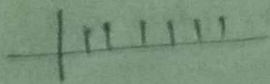
$x(2) = y(2) - 2y(2-1)$

It is a causal system depends on present & past value

Stability

(i) $y(n) = x(n) + x(n-1) + x(n-2)$

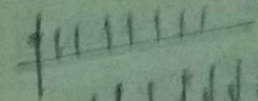
$x(n) =$ 

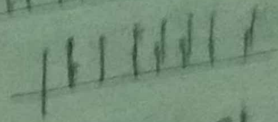
$x(n-1) =$ 

$x(n-2) =$ 

* Here the system is stable system it has a finite value.
* As we can trace the amplitude value at infinity so the system is stable
* at infinity the amplitude value is finite that is '3'

(ii) $x(n) = y(n) - 2y(n-1)$

$y(n) =$ 

$2y(n-1) =$ 

* Here the amplitude at infinity is finite
Here the system is stable
* at infinity the amplitude value is finite that is $= 1$

Say find 8-point DFT of the sequence $x(n) = \{1, 2, 1, 0, 2, 3, 0, 1\}$

Given $x(n) = \{1, 2, 1, 0, 2, 3, 0, 1\}$

we know
$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi nk}{N}} \quad ; \quad K = 0, 1, \cdots N-1$$

For $N = 8$
$$X(k) = \sum_{n=0}^{7} x(n)\, e^{-j\frac{2\pi nk}{8}} \quad ; \quad k = 0, 1, \cdots 7$$
$$X(k) = \sum_{n=0}^{7} x(n)\, e^{\frac{-j\pi nk}{4}} \quad ; \quad k = 0, 1, 2 \cdots 7$$

For $k = 0$,
$$X(0) = \sum_{n=0}^{7} x(n)$$
$$X(0) = x(0) + x(1) + x(2) + x(3) + x(4) + x(5) + x(6) + x(7)$$
$$= 1 + 2 + 1 + 0 + 2 + 3 + 0 + 1$$
$$X(0) = 10$$

For $k = 1$
$$X(1) = \sum_{n=0}^{7} x(n)\, e^{-j\pi n/4}$$
$$= x(0) + x(1)\, e^{-j\pi/4} + x(2)\, e^{-j\pi/2} + x(3)\, e^{-j3\pi/4} + x(4)\, e^{-j\pi} +$$
$$x(5)\, e^{-j5\pi/4} + x(6)\, e^{-j3\pi/2} + x(7)\, e^{-j7\pi/4}$$
$$= 1 + 2(0.707 - j0.707) + 1(-j) + 0 + 2(-1) + 3(-0.707 + j0.707)$$
$$+ 0 + 1(0.707 + j0.707)$$

$= 1 + 1.414 + j1.414 - j = 2 - 2.12 + j2.12 + 0.707 + j0.707$

$X(1) = -1 + 0.414j$

For $k = 2$, $X(2) = \sum_{n=0}^{7} x(n) e^{-j\pi n/2}$

$= x(0) + x(1) e^{-j\pi/2} + x(2) e^{-j\pi} + x(3) e^{-j3\pi/2} + x(4) e^{-j2\pi} + x(5) e^{-j5\pi/2}$
$+ x(6) e^{-3\pi j} + x(7) e^{-7\pi j/2}$

$= 1 + 2(-1) + 1(-1) + 0 + 2(1) + 3(-1) + 0 + 1(1)$

$X(2) = -2$

For $k = 3$;

$X(3) = \sum_{n=0}^{7} x(n) e^{-j3\pi n/4}$

$= x(0) + x(1) e^{-j3\pi/4} + x(2) e^{-j3\pi/2} + x(3) e^{-j9\pi/4} + x(4) e^{-j3\pi} +$
$x(5) e^{-j15\pi/4} + x(6) e^{-j9\pi/2} + x(7) e^{-j21\pi/4}$

$= 1 + 2(0.707 - j0.707) + 1(1) + 0 + 2(-1) + 3(0.707 + j0.707)$
$+ 0 + 1(0.707 + j0.707)$

$= 1 - 1.414 - j1.414 + j - 2 + 2.12 + j2.121 - 0.707 - j0.707$

$X(3) = -1 + j$

For $k = 4$;

$X(4) = \sum_{n=0}^{7} x(n) e^{-j\pi n}$

$= x(0) + x(1) e^{-j\pi n} + x(2) e^{-j2\pi} + x(3) e^{-j3\pi} + x(4) e^{-j4\pi} + x(5) e^{-j5\pi}$
$+ x(6) e^{-j6\pi} + x(7) e^{-j7\pi}$

$= 1 + 2(-1) + 1(1) + 0 + 2(1) + 3(-1) + 0 + 1(-1)$

$= 1 - 2 + 1 + 2 - 3 - 1$

$X(4) = -2$

For $k = 5$;

$X(5) = \sum_{n=0}^{7} x(n) e^{-j5\pi n/4}$

$= x(0) + x(1) e^{-j5\pi/4} + x(2) e^{-j5\pi/2} + x(3) e^{-j15\pi/4} + x(4) e^{-j5\pi}$
$+ x(5) e^{-j25\pi/4} + x(6) e^{-j15\pi/2} + x(7) e^{-j35\pi/4}$

$= 1 + 2(-0.707 + j0.707) + 1(-1) + 0 + 2(-1) + 3(0.707 - j0.707)$
$+ 0 + 1(-0.707 - j0.707)$

$X(5) = -1 - j2.414$

for k = 6

$$X(6) = \sum_{n=0}^{7} x(n) e^{-j3\pi n/2}$$

$$= x(0) + x(1) e^{-j3\pi/2} + x(2) e^{-j3\pi} + x(3) e^{-j9\pi/2} + x(4) e^{-j6\pi} + x(5) e^{-j15\pi/2}$$
$$+ x(6) e^{-j9\pi} + x(7) e^{-j21\pi/2}$$

$$= 1 + 2(j) + 1(-1) + 0 + 2(1) + 3(j) + 0 + 1(-j)$$

$$= 1 + 2j - 1 + 2 + 3j - j$$

$$X(6) = 2 + 4j$$

for k = 7

$$X(7) = \sum_{n=0}^{7} x(n) e^{-j7\pi n/4}$$

$$= x(0) + x(1) e^{-j7\pi/4} + x(2) e^{-j7\pi/2} + x(3) e^{-j21\pi/4} + x(4) e^{-j7\pi} +$$
$$x(5) e^{-j35\pi/4} + x(6) e^{-j42\pi/4} \text{ (or) } x(6) e^{-j21\pi/2} + x(7) e^{-j49\pi/4}$$

$$= 1 + 2(0.707 + j0.707) + 1(j) + 0 + 2(-1) + 3(-0.707 - j0.707)$$
$$+ 0 + 1(0.707 - j0.707)$$

$$X(7) = -1 - j0.414$$

$$X(k) = \{10, -1+j0.414, -2, -1+j, -2, -1-j2.414, 2+4j, -1-j0.414\}$$

by Relationship between DFT to Z-Transform.

Let, A sequence $x(n)$ of finite duration N with Z-transform

$$X(z) = \sum_{n=0}^{N-1} x(n) z^{-n} \longrightarrow ①$$

we have

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N} \longrightarrow ②$$

Substituting eq→② in eq→①

$$X(z) = \sum_{n=0}^{N-1} \left[\frac{1}{N} \sum_{k=0}^{N-1} X(k) \cdot e^{j2\pi kn/N} \right] z^{-n}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X(k) \sum_{n=0}^{N-1} \left(e^{j2\pi k/N} z^{-1} \right)^{-n}$$

$$\boxed{X(z) = \frac{1 - z^{-N}}{N} \sum_{k=0}^{N-1} \frac{X(k)}{1 - e^{j2\pi k/N} z^{-1}}}$$

Relationship between DFT & Fourier Series

The Fourier series $X_p(k)$ on a finite duration sequence $x(n)$ having length N is given by

$$X_p(k) = \sum_{n=0}^{N-1} x_p(n) e^{-j2\pi kn/N} \rightarrow ① \quad k=0,1,2 \ldots N-1 \rightarrow ①$$

The discrete fourier transform of $x(n)$ is given by

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N} \rightarrow ② \quad k=0,1,2 \ldots N-1 \rightarrow ②$$

From eq ① & ② we find that the DFT of $x(n)$ and the function series are same and it's given by

$$X(k) = X_p(k) \Big| x(n) = x_p(n) \quad k=0,1,2,\ldots N-1.$$

6a) State and prove following properties of DFT
(i) Linearity (ii) Circular shift

(i) Statement:
The DFT of the linear combination of two (or) more signals is the sum of linear combination of DFT of individual signal

Proof:
$$a_1x_1(n) + a_2x_2(n) \xrightarrow{DFT} a_1X_1(k) + a_2X_2(k)$$

We have the formula to calculate DFT:

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk} \quad \text{where } k=0,1,2,\ldots N-1$$

Here $x(n) = a_1x_1(n) + a_2x_2(n)$

Therefore

$$X(k) = \sum_{n=0}^{N-1} \left[a_1x_1(n) + a_2x_2(n)\right] W_N^{nk}$$

$$= \sum_{n=0}^{N-1} a_1x_1(n) W_N^{nk} + \sum_{n=0}^{N-1} a_2x_2(n) W_N^{nk}$$

a_1 and a_2 are constants and can be separated, therefore

$$= a_1X_1(k) + a_2X_2(k)$$

Hence proved

(ii) circular time shift

Shifting the sequence in time domain by 'l' samples is equivalent to multiplying the sequence in frequency domain by the twiddle factor

$$x(n-l) \xleftrightarrow{DFT} X(k)\, e^{-2\pi j l k/N}$$

According to DFT

$$DFT\{x(n)\} = \sum_{n=0}^{N-1} x(n)\, e^{-2\pi j k n/N}$$

$$DFT[x(n-l)] = \sum_{n=0}^{N-1} x(n-l)\, e^{-2\pi j k n/N}$$

let $n-l = p$

$$DFT[x(p)] = \sum_{p=0}^{N-1} x(p)\, e^{-2\pi j k(p+l)/N}$$

$$= e^{-2\pi j k l/N} \sum_{p=0}^{N-1} x(p)\, e^{-2\pi j k p/N}$$

$$= X(k)\, e^{-2\pi j k/N}$$

Hence proved.

6b) Compute the 4 point DFT of the sequence, $x(n) = \{1,1,-1,-1\}$?

$x(n) = [1,1,-1,-1]$

We know

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-\frac{j2\pi nk}{N}};\quad k = 0,1,2,3.$$

For $N=4$ $$X(k) = \sum_{n=0}^{3} x(n)\, e^{-\frac{j2\pi nk}{4}} \quad; k=0,1,2,3$$

$$X(k) = \sum_{n=0}^{3} x(n)\, e^{-\frac{j\pi nk}{2}} \quad; k=0,1,2,3$$

For $k=0$ $$X(0) = \sum_{n=0}^{4} x(0)\, e^{-\frac{j2\pi nk}{2}}$$

$$= x(0)+x(1)+x(2)+x(3)$$

$$= 1+1-1-1$$

$$= 0$$

For $k=1$ $$X(1) = \sum_{n=0}^{4} x(n)\, e^{-\frac{j2\pi nk}{2}}$$

$$= x(0) + x(1)\, e^{j\pi k/2} + x(2)\, e^{-j\pi} + x(3)\, e^{-j3\pi/2}$$

$$= 1 + 1(1).$$