$P_0 = 0.596\ (-0.5) + j\ 2.0869\ (0.866)$

$P_0 = -0.298 + j\ 1.8072$

$K=1 \Rightarrow P_1 = \sigma_1 + j\Omega_1$

$P_1 = a \cos \phi_1 + j\, b \sin \phi_1$

$P_1 = 0.596 \cos \pi + j\ 2.0869 \sin \pi$

$P_1 = 0.596\,(-1) + j\ 2.0869\,(0)$

$P_1 = -0.596$

$K=2 \Longrightarrow P_2 = \sigma_2 + j\Omega_2$

$P_2 = a \cos \phi_2 + j\, b \sin \phi_2$

$P_2 = 0.596 \cos\left(\frac{4\pi}{3}\right) + j\ 2.0869 \sin\left(\frac{4\pi}{3}\right)$

$P_2 = 0.596\,(-0.5) + j\,(2.0869)\,(-0.866)$

$P_2 = -0.298 - j\ 1.8072$

3) Transfer function

$$H_a(s) = \frac{K}{(s-P_0)\,(s-P_1)\,(s-P_2)}$$

$$= \frac{K}{(s+0.298-j\ 1.8072)\,(s+0.596)\,(s+0.298+j\ 1.8072)}$$

$$H_a(s) = \frac{K}{(s+0.596)\,(s^2+0.596s+0.0888+3.2659)}$$

$$H_a(s) = \frac{K}{(s+0.596)\,(s^2+0.596s+3.3547}$$

$$H_a(s) = \frac{K}{s^3 + 0.596s^2 + 3.3547s + 0.596s^2 + 0.3552s + \ldots}$$

$$H_a(s) = \frac{K}{s^3 + 1.192s^2 + 3.7099s + 1.9994}$$

For the given Specifications of $\alpha_p = 3\,dB$, $\alpha_s = 16\,dB$, $f_p = 1\,KHz$, $f_s = 2\,KHz$. Determine the filter order and design analog lowpass chebyshev filter.

$$\alpha_p = 3\,dB$$

$$\Omega_p = 2\pi \times f_p = 2\pi \cdot 1000 = 2000\pi \text{ rad/sec}$$

$$\alpha_s = 16\,dB$$

$$\Omega_s = 2\pi \times f_s = 2\pi \cdot 2000 = 4000\pi \text{ rad/sec}$$

→ Order of the filter

$$N \geq \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\cosh^{-1}\left(\dfrac{\Omega_s}{\Omega_p}\right)} \qquad \cosh^{-1} x = \ln\left(x + \sqrt{x^2 \ldots}\right.$$

$$N \geq \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1(16)} - 1}{10^{0.1(3)} - 1}}}{\cosh^{-1}\left(\dfrac{4000\pi}{2000\pi}\right)}$$

$$N \geq \frac{\cosh^{-1} 6.2446}{\cosh^{-1} 2}$$

As N ...
then $K = b_0$
$\rightarrow K = 1.9994$

$$G(s) = \frac{1.9994}{s^3 + 1.1928s^2 + 3.70995s + 1.9994}$$

$$N \geq \frac{\ln\left(6.2446 + \sqrt{6.2446^2 + 1}\right)}{\ln\left(2 + \sqrt{2^2 + 1}\right)}$$

$$N \geq \frac{2.5312}{1.4436}$$

$$N \geq 1.7533$$

$$N \cong 2$$

2) POLES:

$$\alpha_p = \frac{1}{\sqrt{1+\varepsilon^2}} = 3$$

$$\sqrt{1+\varepsilon^2} = \frac{1}{3}$$

$$1+\varepsilon^2 = \frac{1}{9} \Longrightarrow \varepsilon^2 = \frac{1}{9} - 1$$

$$\varepsilon^2 =$$

$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1}$$

$$= \sqrt{10^{0.1(3)} - 1}$$

$$\varepsilon = 0.9976$$

$$\mu = \frac{1 + \sqrt{1+\varepsilon^2}}{\varepsilon} = \frac{1 + \sqrt{1 + 0.9976^2}}{0.9976} = 2.418$$

$$a = \Omega_p\left(\frac{\mu^{\frac{1}{N}} - \mu^{-\frac{1}{N}}}{2}\right) = 2000\pi\left(\frac{2.418^{\frac{1}{2}} - 2.418^{-\frac{1}{2}}}{2}\right)$$

$$= 1000\pi\,(0.9119)$$

$$a = 911.9\pi$$

$$b = \Omega_p\left(\frac{\mu^{\frac{1}{N}} + \mu^{-\frac{1}{N}}}{2}\right) = 2000\pi\left(\frac{2.418^{\frac{1}{2}} + 2.418^{-\frac{1}{2}}}{2}\right)$$

$$= 1000\pi\,(2.198)$$

$$\phi_K = \frac{(2K+N+1)\pi}{2N} \qquad K = 0, 1, \dots N-1$$
$$= 0, 1, \dots 2-1$$
$$K = 0, 1$$

$$K=0 \Rightarrow \phi_0 = \frac{(0+2+1)\pi}{2 \cdot 2} = \frac{3\pi}{4}$$

$$K=1 \Rightarrow \phi_1 = \frac{(2+2+1)\pi}{2 \cdot 2} = \frac{5\pi}{4}$$

$$K=0 \Longrightarrow P_0 = \sigma_0 + j\Omega_0$$
$$= a\cos\phi_0 + j\, b\sin\phi_0$$
$$= 911.9\pi \cos\left(\frac{3\pi}{4}\right) + j\ 2198\pi \sin\left(\frac{3\pi}{4}\right)$$
$$= 911.9\pi(-0.7071) + j\ 2198\pi(0.7071)$$
$$P_0 = -644.804\pi + j\ 1554.205\pi$$

$$K=1 \Longrightarrow P_1 = \sigma_1 + j\Omega_1$$
$$= a\cos\phi_1 + j\, b\sin\phi_1$$
$$= 911.9\pi\cos\left(\frac{5\pi}{4}\right) + j\ 2198\pi \sin\left(\frac{5\pi}{4}\right)$$
$$= 911.9\pi(-0.7071) + j\ 2198\pi(-0.7071)$$
$$P_1 = -644.804\pi - j\ 1554.205\pi$$

3> Transfer function

$$H_a(s) = \frac{K}{(s-P_0)(s-P_1)}$$

$$= \frac{K}{(s+644.804\pi - j\,1554.205\pi)(s+644.804\pi + j\,1554.205\pi)}$$

$$= \frac{K}{S^2 + 415772.1984\pi^2 + 1289.608\pi S + 2415553.182\pi^2}$$

$$H_a(S) = \frac{K}{S^2 + 1289.608\pi S + 2831325.38\pi^2}$$

AS $N = \text{even}$ $(N=2)$ then $K = \frac{b_0}{\sqrt{1+\varepsilon^2}} = \frac{2831325.38\pi^2}{\sqrt{1+0.9976^2}}$

$K = 2004453.275\pi^2$

$$H_a(S) = \frac{2004453.275\pi^2}{S^2 + 1289.608\pi S + 2831325.38\pi^2}$$

24-04-19

DESIGN OF IIR FILTER FROM ANALOG FILTER :

$$\boxed{\begin{matrix}\text{Analog filter} \\ [H_a(S)] \\ S\text{-domain}\end{matrix}} \xrightarrow[\text{Transformation}]{\text{S to z plane}} \boxed{\begin{matrix}\text{Digital filter} \\ H(Z) \\ z\text{-domain}\end{matrix}}$$

1. Approximation derivate method or Backward difference method.
2. Impulse Invariance
3. Bilinear Transformation

BACKWARD DIFFERENCE METHOD :

$$S = \frac{1-Z^{-1}}{T} \qquad \{T = 1 \text{ sec}$$

Use the backward difference method, convert analog filter to digital filter. The system function $H(S) = \frac{1}{S+2}$

$$H(S) = \frac{1}{S+2} = \frac{1}{\frac{1-Z^{-1}}{T}+2} = \frac{1}{\frac{1-Z^{-1}}{1}+2}$$

$$H(S) = \frac{1}{3-Z^{-1}}$$

Use the backward difference method, convert ... filter to digital filter. The system function $H(s) = \frac{1}{(s+0.1)^2 + ...}$

$$H(s) = \frac{1}{(s+0.1)^2+9}$$

$$H(z) = \frac{1}{\left(\frac{1-z^{-1}}{T}+0.1\right)^2+9}$$

$$= \frac{1}{(1-z^{-1}+0.1)^2+9}$$

$$= \frac{1}{(1.1-z^{-1})^2+9}$$

$$= \frac{1}{1.21+z^{-2}-2.2z^{-1}+9}$$

$$H(z) = \frac{1}{z^{-2}-2.2z^{-1}+10.21}$$

IMPULSE INVARINCE METHOD:

let $H_a(s)$ is the system function of analog f...

This can be expressed in partial fraction.

$$H_a(s) = \sum_{K=1}^{N} \frac{C_K}{s-P_K}$$

where $P_K \longrightarrow$ poles of an analog filter

$C_K \longrightarrow$ coefficients in the partial fraction

Inverse Laplace Transform

$$h_a(t) = \sum_{K=1}^{N} C_K e^{P_K t}$$

$h_a(t)$ periodically at $t = nT$

$$h(n) = h_a(nT)$$

$$h(n) = \sum_{K=1}^{N} C_K \, e^{P_K nT}$$

$$H(z) = \sum_{n=0}^{\infty} h(n) \, z^{-n}$$

$$= \sum_{n=0}^{\infty} \sum_{K=1}^{N} C_K \, e^{P_K nT} \, z^{-n}$$

$$= \sum_{K=1}^{N} C_K \sum_{n=0}^{\infty} \left(e^{P_K T} z^{-1}\right)^{n}$$

$$H(z) = \sum_{K=1}^{N} \frac{C_K}{1 - e^{P_K T} \cdot z^{-1}}$$

STEPS TO DESIGN A DIGITAL FILTER

STEP 1 : For the given specifications, find $H_a(s)$

STEP 2 : Select the sampling rate of the digital filter T sec/samples.

STEP 3 : Express the analog filter transfer function

$$H_a(s) = \sum_{K=1}^{N} \frac{C_K}{s - P_K}$$

STEP 4 : Compute the z-transform of digital filter

$$H(z) = \sum_{K=1}^{N} \frac{C_K}{1 - e^{P_K T} \cdot z^{-1}}$$

$$S = -1 \implies 2 = A(-1+2) + B(-1+1)$$

$$2 = A(1) + B(0)$$

$$2 = A + 0$$

$$A = 2$$

$$S = -2 \implies 2 = A(-2+2) + B(-2+1)$$

$$2 = 0 + B(-1)$$

$$2 = -B$$

$$B = -2$$

$$H(s) = \frac{2}{s+1} - \frac{2}{s+2}$$

$$= \frac{2}{s-(-1)} - \frac{2}{s-(-2)} \quad \left\{ \begin{array}{l} c_1 = A \\ c_2 = B \end{array} \right.$$

There are two poles

$$P_1 = -1 \qquad P_2 = -2$$

$$H(z) = \sum_{K=1}^{N} \frac{c_K}{1 - e^{P_K T} \cdot z^{-1}}$$

$$= \sum_{K=1}^{2} \frac{c_K}{1 - e^{P_K T} \cdot z^{-1}}$$

$$= \frac{c_1}{1 - e^{P_1 T} \cdot z^{-1}} + \frac{c_2}{1 - e^{P_2 T} \cdot z^{-1}}$$

$$= \frac{2}{1 - e^{-T} \cdot z^{-1}} + \frac{-2}{1 - e^{-2T} \cdot z^{-1}}$$

$$= \frac{2}{1 - e^{-1} \cdot z^{-1}} + \frac{-2}{1 - e^{-2} z^{-1}}$$

$$= \frac{2}{1 - 0.3678 z^{-1}} + \frac{-2}{1 - 0.1353 z^{-1}}$$

$$= 2 \left[\frac{1}{1 - 0.3678 z^{-1}} - \frac{1}{1 - 0.1353 z^{-1}} \right]$$

$$= 2 \left[\frac{\cancel{1} - 0.1353 z^{-1} - \cancel{1} + 0.3678 z^{-1}}{1 - 0.1353 z^{-1} - 0.3678 z^{-1} + 0.0497 z^{-2}} \right]$$

$$= 2 \left[\frac{0.2325 z^{-1}}{1 - 0.5031 z^{-1} + 0.0497 z^{-2}} \right]$$

$$H(z) = \frac{0.465 z^{-1}}{1 - 0.5031 z^{-1} + 0.0497 z^{-2}}$$

For the analog Transfer function $H(s) = \frac{10}{s^2 + 7s + 10}$ Determine $H(z)$ by using Impulse Invariance method assuming $T = 0.2$ sec.

$$\frac{10}{s^2+7s+10} = \frac{c_1}{s+2} + \frac{c_2}{s+5}$$

$$10 = c_1(s+5) + c_2(s+2)$$

$$s = -2 \Longrightarrow 10 = c_1(-2+5) + c_2(-2+2)$$

$$10 = c_1 \cdot 3 + 0$$

$$c_1 = \frac{10}{3}$$

$$s = -5 \Longrightarrow 10 = c_1(-5+5) + c_2(-5+2)$$

$$10 = 0 + c_2 \cdot -3$$

$$c_2 = \frac{-10}{3}$$

$$H(s) = \frac{10/3}{s+2} + \frac{-10/3}{s+5}$$

$$H(s) = \frac{10/3}{s-(-2)} - \frac{10/3}{s-(-5)}$$

$\therefore$ There are two poles

$P_1 = -2 \qquad P_2 = -5$

$$= \frac{c_1}{1-e^{P_1 T}\cdot z^{-1}} + \frac{c_2}{1-e^{P_2 T}\cdot z^{-1}}$$

$$= \frac{10/3}{1-e^{-2(0.2)}\cdot z^{-1}} + \frac{-10/3}{1-e^{-5(0.2)}\cdot z^{-1}}$$

$$= \frac{10/3}{1-e^{-0.4}z^{-1}} - \frac{10/3}{1-e^{-1}\cdot z^{-1}}$$

$$= \frac{10}{3}\left[\frac{1}{1-0.6703z^{-1}} - \frac{1}{1-0.3678z^{-1}}\right]$$

$$= \frac{10}{3}\left[\frac{1-0.3678z^{-1}-1+0.6703z^{-1}}{1-0.3678z^{-1}-0.6703z^{-1}+0.2465z^{-2}}\right]$$

$$= \frac{10}{3}\left[\frac{0.3025z^{-1}}{1-1.0381z^{-1}+0.2465z^{-2}}\right]$$

$$H(z) = \frac{3.025z^{-1}}{3-3.1143z^{-1}+0.7395z^{-2}}$$

DESIGN OF IIR FILTER USING BILINEAR TRANSFORMATIO

Let us consider a analog filter with system func

$$H(s) = \frac{b}{s+a} \longrightarrow ①$$

$$\frac{Y(s)}{X(s)} = \frac{b}{s+a}$$

$$s\,Y(s) + a\,Y(s) = b\,x(s)$$

This can be characterized by differential equation

$$\frac{d}{dt} y(t) + a\,y(t) = b\,x(t) \longrightarrow ②$$

$y(t)$ can be treated by trapezoidal formula

$$y(t) = \int_{t_0}^{t} y'(\tau)\,d\tau + y(t_0)$$

where $y'(\tau)$ is the derivative of $y(t)$

The approximation of the integral by the trapezoid formula at $t = nT$ and $t_0 = nT - T$

$$\left\{ \int_a^b f(x)\,dx = \frac{b-a}{2}\left[f(b) + f(a)\right] \right.$$

$$y(nT) = \frac{T}{2}\left[y'(nT) + y'(nT-T)\right] + y(nT-T) \longrightarrow ③$$

from ②

$$y'(nT) = -a\,y(nT) + b\,x(nT) \longrightarrow ④$$

$$y'(nT-T) = -a\,y(nT-T) + b\,x(nT-T) \longrightarrow ⑤$$

sub eqn 4 and 5 in eqn ③

$$y(nT) = \frac{T}{2}\left[-a\,y(nT) + b\,x(nT) - a\,y(nT-T) + b\,x(nT-T)\right] + y(nT-T)$$

$$= \frac{-aT}{2}\,y(nT) + \frac{bT}{2}\,x(nT) - \frac{aT}{2}\,y(nT-T) + \frac{bT}{2}\,x(nT-T) + y(nT-T)$$

$$y(nT) + \frac{aT}{2}\,y(nT) + \frac{aT}{2}\,y(nT-T) - y(nT-T) = \frac{bT}{2}\left[x(nT) + x(nT-T)\right]$$

with $y(n) = y(nT)$ and $x(n) = x(nT)$

$$y(n) + \frac{aT}{2}\,y(n) + \frac{aT}{2}\,y(n-1) - y(n-1) = \frac{bT}{2}\left[x(n) + x(n-1)\right] \longrightarrow ⑥$$

Apply Z-Transform for eqn ⑥

$$Y(z) + \frac{aT}{2}\,Y(z) + \frac{aT}{2}\,z^{-1}Y(z) - z^{-1}Y(z) = \frac{bT}{2}\left[X(z) + z^{-1}X(z)\right]$$

$$Y(z)\left[1 + \frac{aT}{2} + \frac{aT}{2}z^{-1} - z^{-1}\right] = \frac{bT}{2}\left[1 + z^{-1}\right]X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{\frac{bT}{2}(1+z^{-1})}{1 + \frac{aT}{2} + \frac{aT}{2}z^{-1} - z^{-1}}$$

$$H(z) = \frac{\frac{bT}{2}(1+z^{-1})}{1 - z^{-1} + \frac{aT}{2}(1+z^{-1})}$$

$$H(z) = \frac{b}{\dfrac{1-z^{-1}}{\frac{T}{2}(1+z^{-1})} + \dfrac{\frac{aT}{2}\cancel{(1+z^{-1})}}{\frac{T}{2}\cancel{(1+z^{-1})}}}$$

$$H(z) = \frac{b}{\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] + a} \longrightarrow (7)$$

$$\boxed{s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

The relationship between s and z is known Bilinear transformation.

Let $z = re^{j\omega}$ and $s = \sigma + j\Omega$

$$s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$$

$$= \frac{2}{T}\left[\frac{z-1}{z+1}\right]$$

$$= \frac{2}{T}\left[\frac{re^{j\omega}-1}{re^{j\omega}+1}\right]$$

$$= \frac{2}{T}\left[\frac{r\cos\omega - 1 + jr\sin\omega}{r\cos\omega + 1 + jr\sin\omega}\right]\left[\frac{r\cos\omega + 1 - j}{r\cos\omega + 1 - j}\right]$$

$$= \frac{2}{T}\left[\frac{r^2\cos^2\omega + r\cos\omega - jr^2\sin\omega\cos\omega - r\cos\omega - 1 + jrs + jr^2\sin\omega\cos\omega + jr\sin\omega + r^2\sin^2\omega}{r^2\cos^2\omega + 1 + 2r\cos\omega + r^2\sin^2\omega}\right]$$

$$= \frac{2}{T}\left[\frac{r^2(\cos^2\omega + \sin^2\omega) - 1 + 2jr\sin\omega}{r^2(\cos^2\omega + \sin^2\omega) + 1 + 2r\cos\omega}\right]$$

$$= \frac{2}{T}\left[\frac{\gamma^2 - 1 + 2j\gamma\sin\omega}{\gamma^2 + 1 + 2\gamma\cos\omega}\right]$$

$$s = \frac{2}{T}\left[\frac{\gamma^2 - 1}{1+\gamma^2 + 2\gamma\cos\omega} + j\,\frac{2\gamma\sin\omega}{1+\gamma^2+2\gamma\cos\omega}\right]$$

comparing with $s = \sigma + j\Omega$

$$\sigma = \frac{2}{T}\left[\frac{\gamma^2 - 1}{1+\gamma^2+2\gamma\cos\omega}\right]$$

$$\Omega = \frac{2}{T}\left[\frac{2\gamma\sin\omega}{1+\gamma^2+2\gamma\cos\omega}\right]$$

Filter stability

$\gamma = 1$ $\quad \sigma = 0$

$$\Omega = \frac{2}{T}\left[\frac{2\cdot 1\sin\omega}{1+1+2\cos\omega}\right]$$

$$= \frac{2}{T}\left[\frac{2\sin\omega}{2+2\cos\omega}\right]$$

$$= \frac{2}{T}\left[\frac{\cancel{2}\sin\omega}{\cancel{2}(1+\cos\omega)}\right]$$

$$= \frac{2}{T}\left[\frac{\cancel{2}\sin\frac{\omega}{2}\cos\frac{\omega}{2}}{\cancel{2}\cos^2\frac{\omega}{2}}\right]$$

$$= \frac{2}{T}\left[\frac{\sin\frac{\omega}{2}}{\cos\frac{\omega}{2}}\right]$$

$$\boxed{\Omega = \frac{2}{T}\tan\frac{\omega}{2}} \longrightarrow (8)$$

$$\tan\frac{\omega}{2} = \frac{\Omega T}{2}$$

$$\frac{\omega}{2} = \tan^{-1}\frac{\Omega T}{2}$$

$$\boxed{\omega = 2\tan^{-1}\frac{\Omega T}{2}} \longrightarrow (9)$$

for small value of θ

$$\tan\theta = \theta$$

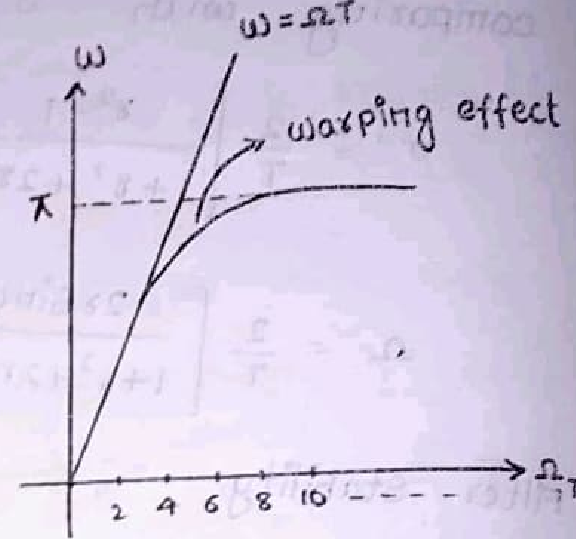
$\{\tan 0 = 0\}$

Eqn (8)

$$\Omega = \frac{2}{T}\cdot\frac{\omega}{2}$$

$$\Omega = \frac{\omega}{T}$$

$$\boxed{\omega = \Omega T}$$



RELATIONSHIP BETWEEN Ω AND ω

STEPS

1. For the given Specifications, find analog filter $H_a(s)$
2. Select the Sampling rate of the digital filter T sec/s
3. Substitute $S = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$

Applying Bilinear transformation $H(s) = \frac{2}{(s+1)(s+2)}$ with T= 1 sec.

$$H(s) = \frac{2}{(s+1)(s+2)}$$

$$H(z) = \frac{2}{\left[\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+1\right]\left[\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+2\right]}$$

$$= \frac{2}{\left[2\left(\frac{z-1}{z+1}\right)+1\right]\left[2\left(\frac{z-1}{z+1}\right)+2\right]}$$

$$= \frac{2}{\frac{2z-2+z+1}{z+1} \cdot \frac{2z-2+2z+2}{z+1}}$$

$$= \frac{\cancel{2}(z+1)^2}{(3z-1)(\cancel{4}^{\,2}z)}$$

$$= \frac{(z+1)^2}{2z(3z-1)}$$

$$= \frac{z^2+2z+1}{6z^2-2z}$$

$$= \frac{\cancel{z^2}\left(1+\frac{2}{z}+\frac{1}{z^2}\right)}{\cancel{z^2}\left(6-\frac{2}{z}\right)}$$

$$H(z) = \frac{1+2z^{-1}+z^{-2}}{6-2z^{-1}}$$

Applying Bilinear Transformation $H(s) = \frac{s^2+4.525}{s^2+0.692s+0.504}$ with T = 1 sec

$$H(S) = \frac{s^2+4.525}{s^2+0.692s+0.504}$$

$$H(z) = \frac{\left[\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + 4.525}{\left[\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + 0.692\left[\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right] + 0.504}$$

$$= \frac{\frac{4}{1}\cdot\frac{1-2z^{-1}+z^{-2}}{1+2z^{-1}+z^{-2}} + 4.525}{\frac{4}{1}\cdot\frac{1-2z^{-1}+z^{-2}}{1+2z^{-1}+z^{-2}} + 0.692\left(\frac{2}{1}\cdot\frac{1-z^{-1}}{1+z^{-1}}\right) + 0.504}$$

$$= \frac{\frac{4-8z^{-1}+4z^{-2}+4.525+9.05z^{-1}+4.525z^{-2}}{1+2z^{-1}+z^{-2}}}{\frac{4-8z^{-1}+4z^{-2}+0.692(2-2z^{-1})(1+z^{-1}) + 0.504+1.008z^{-1}+0.504z^{-2}}{1+2z^{-1}+z^{-2}}}$$

$$= \frac{8.525+1.05z^{-1}+8.525z^{-2}}{4.504-6.992z^{-1}+4.504z^{-2}+0.692(2+\cancel{2z^{-1}}-\cancel{2z^{-1}}-2z^{-2})}$$

$$= \frac{8.525+1.05z^{-1}+8.525z^{-2}}{4.504-6.992z^{-1}+4.504z^{-2}+1.384-1.384z^{-2}}$$

$$H(z) = \frac{8.525+1.05z^{-1}+8.525z^{-2}}{5.888-6.992z^{-1}+3.12z^{-2}}$$