UNIT - 1

(7)

a) $H(z) = \left[\frac{1}{2} + z^{-1} + \frac{1}{2^2} z^{-2}\right]\left(1 + \frac{1}{3} z^{-1} + z^{-2}\right)$

Sol:-

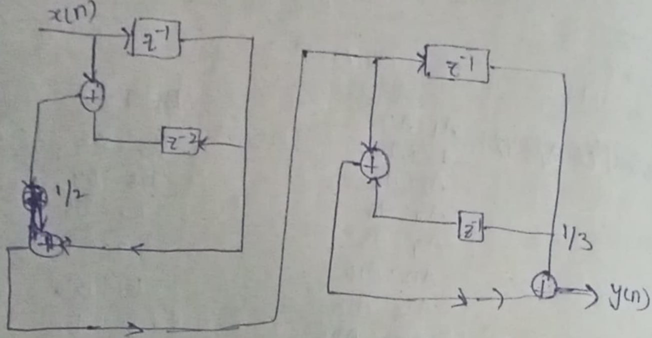


fig:- cascade realization of ex:-

(6)

$H(z) = \frac{1}{2} + \frac{1}{3} z^{-1} + z^{-2} + \frac{1}{4} z^{-3} + z^{-4} + \frac{1}{3} z^{-5} + \frac{1}{2} z^{-6}$

solution:- By inspection we find that the system function $H(z)$ is that of a linear phase FIR filter $h(n) = h(N-1-n)$

∴ we can realize the system function is as shown in fig

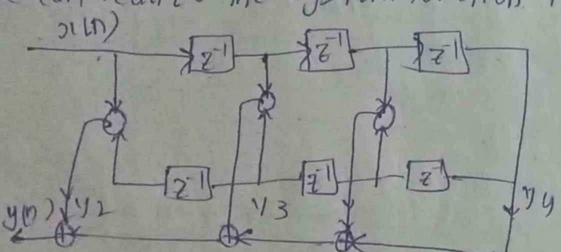


fig:- cascade realization of example.

(8) Explain about transposed, cascade form and linear phase realizations?

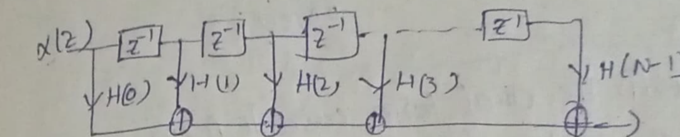
Sol:- Direct structure of transversal structure:

The system function of a FIR filter is given by

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$\frac{Y(z)}{X(z)} = h(0) + h(1) z^{-1} + h(2) z^{-2} + \cdots + h(N-1) z^{N-1}$$

$$y(z) = h(0) X(z) + h(1) z^{-1} X(z) + h(2) z^{-2} + \cdots + h(N-1) z^{N-1} x(n)$$

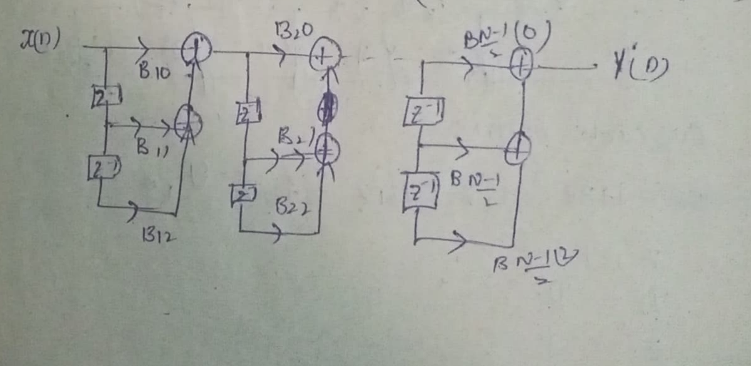


Cascade form Realizations:-

The cascade form of realization of $H(z)$ is in the form of factored terms and is given by $\frac{N-1}{2}$

$$H(z) = \prod_{k=1}^{\frac{N-1}{2}} (b_{k0} + b_{k1} z^{-1} + b_{k2} z^{-2})$$

$$= (b_{10} + b_{11} z^{-1} + b_{12} z^{-2})(b_{20} + b_{21} z^{-1} + b_{22} z^{-2})$$
$$(b_{30} + b_{31} z^{-1} + b_{32} z^{-2}) \cdots (b_{\frac{N-1}{2}}(0) + b_{\frac{N-1}{2}}(1) z^{-1} + b_{\frac{N-1}{2}}(2) z^{-2})$$

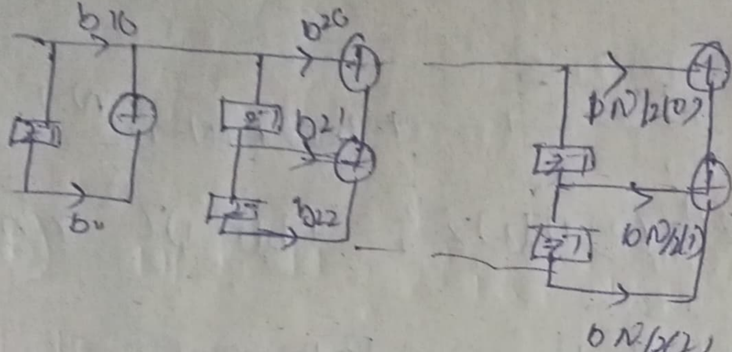


for n → even

$$H(z) = (b_{10} + b_{11}) \prod_{k=2}^{N/2} (b_{k0} + b_{k1} z^{-1} + b_{k2} z^{-2})$$

$$H(z) = (b_{10} + b_{11})(b_{20} + b_{21} z^{-1} + b_{22} z^{-2})(b_{30} + b_{31} z^{-1} + b_{32} z^{-2})$$

$$\cdots\ b_{N/2}(0) + b_{N/2}(1) z^{-1} + b_{N/2}(2) z^{-2})$$



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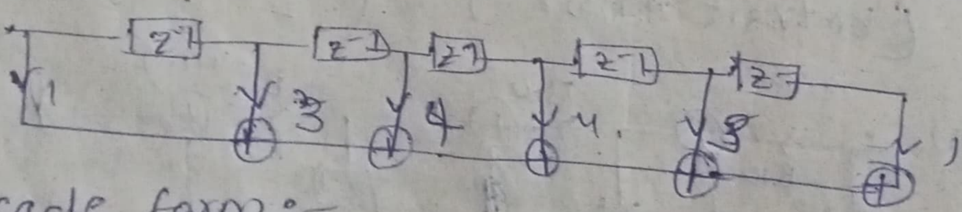
(10) obtain the direct form, cascade form realization for the system function $H(z) = 1 + 3z^{-1} + 4z^{-2} + 4z^{-3} + 3z^{-4} + z^{-5}$

Sol:- Direct form:-

given $H(z) = 1 + 3z^{-1} + 4z^{-2} + 4z^{-3} + 3z^{-4} + z^{-5}$

$$\frac{Y(z)}{X(z)} = 1 + 3z^{-1} + 4z^{-2} + 4z^{-3} + 3z^{-4} + z^{-5}$$

$$Y(z) = 1X(z) + 3z^{-1}X(z) + 4z^{-2}X(z) + 3z^{-4}X(z) + z^{-5}X(z).$$



Cascade form:-

$$H(z) = 1 + 3z^{-1} + 4z^{-2} + 4z^{-3} + 3z^{-4} + z^{-5}$$

$z = -1$

$1/z = -1/1$

$z^{-1} = -1$

$(1+z^{-1}) = 0$

$$1+z^{-1} \overline{\big) 1+3z^{-1}+4z^{-2}+4z^{-3}+3z^{-4}+z^{-5}}$$

$$\begin{array}{l} \quad z^{-4}+2z^{-3}+2z^{-2}+2z^{-1}+1 \\ 1+z^{-1} \overline{\big) \cancel{z^{-5}}+3z^{-4}+4z^{-3}+4z^{-2}+3z^{-1}+1} \\ \quad \cancel{z^{-5}} + z^{-4} \\ \quad - \quad - \\ \hline \quad 2\cancel{z^{-4}}+4z^{-3}+4z^{-2}+3z^{-1}+1 \\ \quad -2\cancel{z^{-4}} + z^{-4} \\ \hline \quad 2\cancel{z^{-3}}+4z^{-2}+3z^{-1}+1 \\ \quad 2\cancel{z^{-3}}+2z^{-2} \\ \quad - \quad - \\ \hline \quad 2\cancel{z^{-2}}+3z^{-1}+1 \\ \quad 2\cancel{z^{-2}}+2z^{-1} \\ \quad \quad - \\ \hline \quad \cancel{z^{-1}}+\cancel{1} \\ \quad \cancel{z^{-1}}+\cancel{1} \\ \quad - \quad - \\ \hline \quad (0) \end{array}$$

$(1+z^{-1})(z^{-4}+2z^{-3}+2z^{-2}+2z^{-1}+1)$

$H_1(z) = 1+z^{-1}$

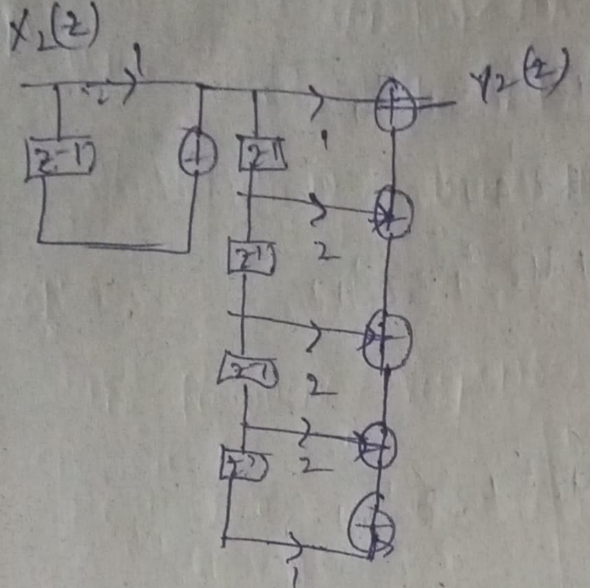
$H_2(z) = z^{-4}+2z^{-3}+2z^{-2}+2z^{-1}+1$

$\frac{Y_1(z)}{X_1(z)} = 1+z^{-1}$

$Y_1(z) = 1(X_1(z)+z^{-1}(X_1(z)))$

$\frac{Y_2(z)}{X_2(z)} = z^{-4}+2z^{-3}+2z^{-2}+2z^{-1}+1$

$Y_2(z) = z^{-4}X_2(z)+2z^{-3}X_2(z)+2z^{-2}X_2(z)+2z^{-1}X_2(z)+X_2(z)$



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6 b) $H(z) = 1 + \frac{5}{2} z^{-1} + 2z^{-2} + 2z^{-3}$

Sol:

given $H(z) = 1 + \frac{5}{2} z^{-1} + 2z^{-2} + 2z^{-3}$

$$= \left(1 + 2z^{-1}\right)\left(1 + \tfrac{1}{2} z^{-1} + z^{-2}\right)$$

∴ The above eq can be realized in cascade form as shown in fig:-

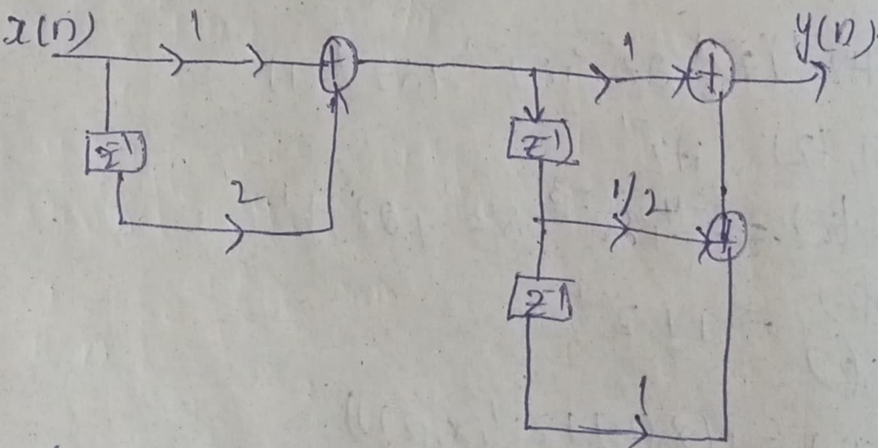


fig:- cascade relization of example

6a) Discuss the realization of FIR filter structures?

Sol:-

FIR systems are represented in four different ways-

1) Direct form structure
2) Cascade form
3) Frequency - sampling
4) Lattice

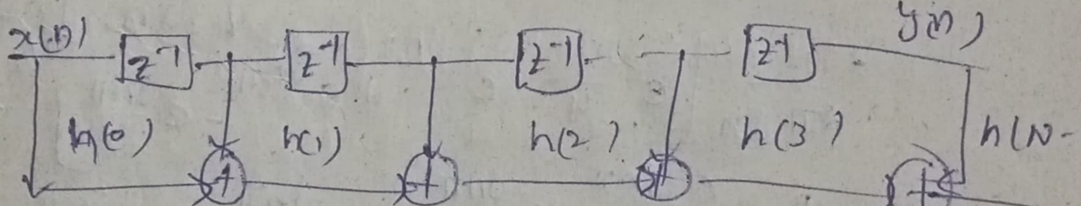
① Direct form:-

→ The convolution of $h(n)$ and $x(n)$ for FIR systems can be written.

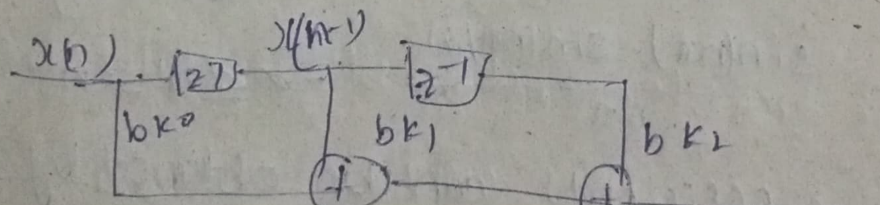
$$y(n) = \sum_{k=0}^{M-1} h(k)\, x(n-k)$$

The above eq can be expanded as

$$y(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \text{---} + h(M-1)\,x(n-M+1)$$



② Cascade form



$$H(z) = \sum_{k=0}^{m-1} b_k z^{-k}$$

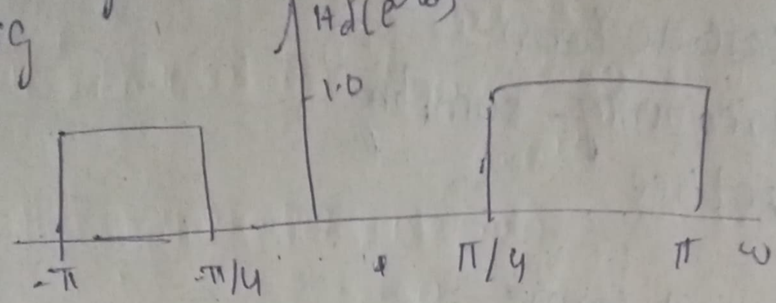
$$H(z) = H_1(z) \cdot H_2(z) \text{ ---- } H_k(z)$$

(9) $H_d(e^{j\omega}) = \begin{cases} 1 & \pi/4 < |\omega| \leq \pi/4 \\ 0 & |\omega| \leq \pi/4 \end{cases}$

of length $N=9$ using Hamming window.

sol:-

The frequency response of High pass filter is show in fig



step:- find the desired impulse response

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\pi/4} 1 - e^{j\omega n} d\omega + \int_{\pi/4}^{\pi} 1 - e^{j\omega n} d\omega\right] = \frac{1}{2\pi}\left\{\left[\frac{e^{j\omega n}}{jn}\right]_{-\pi}^{-\pi/4} + \left[\frac{e^{j\omega n}}{jn}\right]_{\pi/4}^{\pi}\right\}$$

$$= \frac{1}{2\pi \cdot jn}\left[e^{\frac{-jn\pi}{4}} - e^{-jn\pi} + e^{jn\pi} - e^{\frac{jn\pi}{4}}\right]$$

$$= \frac{1}{\pi \cdot n}\left[\frac{e^{jn\pi} - e^{-jn\pi}}{2j} - \frac{e^{\frac{jn\pi}{4}} - e^{\frac{-jn\pi}{4}}}{2j}\right]$$

$$h_d(n) = \frac{\sin(n\pi) - \sin(n\pi/4)}{n\pi} \quad -\infty \leq n \leq n$$

The filter coefficients can be obtained by

for $n=0$; $h_d(0)$ = indeterminate. so

$$h_d(0) = \lim_{n \to 0} \frac{\sin(n\pi)}{n\pi} - \lim_{n \to 0} \frac{\sin(n\pi/4)}{n\pi}$$

$= 1 - 1/4 \lim_{n \to 0} \frac{\sin n\pi/4}{n\pi/4} = 1 - 1/4 = 0.75$

for $n = 1$; $h_d(1) = h_d(-1) = \frac{\sin(\pi) - \sin(\pi/4)}{\pi} = -0.225$

$n = 2$; $h_d(2) = h_d(-2) = \frac{\sin(2\pi) - \sin(\pi/2)}{2\pi} = -0.159$

$n = 3$; $h_d(3) = h_d(-3) = \frac{\sin(3\pi) - \sin 3\pi/4}{3\pi} = -0.075$

$n = 4$; $h_d(4) = h_d(-4) = \frac{\sin(4\pi) - \sin \pi}{4\pi} = 0$

$n = 5$; $h_d(5) = h_d(-5) = \frac{\sin 5\pi - \sin\frac{5\pi}{4}}{5\pi} = 0.045$

step-2:

The Hamming window seq is given by

$$w_H(n) = \begin{cases} 0.54 + 0.46 \cos\frac{2\pi n}{N-1} & |n| \le \left|\frac{N-1}{2}\right| \\ 0 & \text{otherwise} \end{cases}$$

The Hamming window seq for $N = 9$ is given by

$$w_H(n) = \begin{cases} 0.54 + 0.46 \cos\frac{\pi n}{4} & -4 \le n \le 4 \\ 0 & \text{otherwise} \end{cases}$$

$w_H(0) = 0.54 + 0.46 \cos\frac{\pi(0)}{4} = 0.54 + 0.46 = 1$

$w_H(1) = 0.54 + 0.46 \cos\frac{\pi(1)}{4} = 0.865$

$w_H(2) = 0.54 + 0.46 \cos\frac{\pi(2)}{4} = 0.540$

$w_H(3) = 0.54 + 0.46 \cos\frac{\pi(3)}{4} = 0.215$

$w_H(4) = 0.54 + 0.46 \cos\frac{\pi(4)}{4} = 0.080$

$w_H(5) = 0.54 + 0.46 \cos\frac{\pi(5)}{4} = 0.215$

step-3: The filter coefficients using Hamming window seq are

$h(n) = h_d(n) w(n) \quad |n| \le \frac{N-1}{2} = h_d(n) \cdot w(n) \quad -4 \le n \le 4$

$h(0) = h_d(0) \cdot w(0) = 0.75 \times 1$
$= 0.75.$

$h(1) = h_d(1) = h_d(-1) \cdot w(1) = -0.225 \times 0.865$
$= -0.195$

$h(2) = h_d(2) = h_d(-2) \cdot w(2) = -0.159 \times 0.540$
$= -0.086$

$h(3) = h_d(3) - h_d(-3)\, w(3) = -0.075 \times 0.215$
$= -0.016$

$h(4) = h_d(4) - h_d(-4)\, w(4) = 0 \times 0.080$
$= 0$

$h(5) = h_d(5) - h_d(-5)\, w(5) = 0.045 \times 0.215$
$= 0.010$

Step-4 :- The transfer function of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{-n} + z^{n}\right] = h(0) + \sum_{n=1}^{4} h(n)\left[z^{-n} + z^{n}\right]$$

$$= 0.75 + \sum_{n=1}^{5} h(n)\left[z^{-n} + z^{n}\right]$$

$$= 0.75 + h(1)\left(z^{-1} + z^{1}\right) + h(2)\left[z^{-2} + z^{2}\right] + h(3)\left[z^{-3} + z^{3}\right]$$
$$+ h(4)\left(z^{4} + z^{-4}\right) + h(5)\left[z^{5} + z^{-5}\right]$$

$$= 0.75 - 0.195\left[z^{-1} + z^{1}\right] - 0.086\left(z^{-2} + z^{2}\right) - 0.016\left[z^{-3} + z^{3}\right]$$
$$+ 0.010\left[z^{-5} + z^{5}\right]$$

Step-5 The transfer function of the realizable filter is

$$H(z) = H(z)\, z^{-(N-1)/2} = H(z)\, z^{-4}$$

$$H(z) = z^{-4}\left\{0.75 - 0.195\left[z^{-1} + z^{1}\right] - 0.086\left(z^{-2} + z^{2}\right) - 0.016\right.$$
$$\left.\left(z^{-3} + z^{3}\right) + 0.010\left[z^{-5} + z^{5}\right]\right\}$$

$= 0.75z^{-4} - 0.195(z^{-5} + z^{-3}) - 0.086(z^{-6} + z^{-2}) - 0.016(z^{-7} + z^{-1}) + 0.010(z^{-9} + z^{1})$

$= 0.75z^{-4} - 0.195z^{-5} - 0.195z^{-3} - 0.086z^{-6} - 0.086z^{-2} - 0.016z^{-7} - 0.016z^{-1} + 0.010z^{-9} + 0.010z^{1}$

$H(z) = 0.010z^{1} - 0.010z^{-1} - 0.086z^{-2} - 0.195z^{-3} - 0.75z^{-4} - 0.195z^{-5} - 0.086z^{-6} - 0.016z^{-7} + 0.010z^{-9}$

$h(1) = h(7) = -0.016z^{-7}$ $\quad h(3) = h(5) = -0.195$

$h(2) = h(6) = 0.086$ $\quad h(9) = 0.010$

$h(0) = h(8) = 0$ $\quad h(4) = 0.75$