

Assignment - 1

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Abstract—This is a simple document to learn about writing vectors and matrices using latex, draw figures using Python, Latex.

Download all and latex-tikz codes from

svn co <https://github.com/Ganeshyadav712/Assignment-1.git>

1 VECTORS (CBSE/MATH/10/2006 SET1 - Q11)

1.1. Draw the graphs of the following equations:

$$4x - y - 8 = 0 \quad (1.1.1)$$

$$\text{or } \begin{pmatrix} 4 & -1 \end{pmatrix} \mathbf{x} = 8 \quad (1.1.2)$$

$$2x - 3y + 6 = 0 \quad (1.1.3)$$

$$\text{or } \begin{pmatrix} 2 & -3 \end{pmatrix} \mathbf{x} = -6 \quad (1.1.4)$$

Also determine the vertices of the triangle formed by the lines and the x axis.

Solution:

a) We have equations of two lines: Which is written in vector form:

$$\begin{pmatrix} 4 & -1 \end{pmatrix} \mathbf{x} = 8 \quad (1.1.5)$$

and

$$\begin{pmatrix} 2 & -3 \end{pmatrix} \mathbf{x} = -6 \quad (1.1.6)$$

where

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (1.1.7)$$

Both equations are written together in matrix form as:

$$\begin{pmatrix} 4 & -1 \\ 2 & -3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 8 \\ -6 \end{pmatrix} \quad (1.1.8)$$

Augmented matrix for above is:

$$\begin{pmatrix} 4 & -1 & 8 \\ 2 & -3 & -6 \end{pmatrix} \quad (1.1.9)$$

This can be reduced as follows:

$$\begin{pmatrix} 4 & -1 & 8 \\ 2 & -3 & -6 \end{pmatrix} \xrightarrow{R_1 \leftarrow \frac{R_1}{4}} \begin{pmatrix} 1 & -\frac{1}{4} & 2 \\ 2 & -3 & -6 \end{pmatrix} \quad (1.1.10)$$

$$\xrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 1 & -\frac{1}{4} & 2 \\ 0 & -\frac{5}{2} & -10 \end{pmatrix} \quad (1.1.11)$$

$$\xrightarrow{R_2 \leftarrow -\frac{2}{5}R_2} \begin{pmatrix} 1 & -\frac{1}{4} & 2 \\ 0 & 1 & 4 \end{pmatrix} \quad (1.1.12)$$

$$\xrightarrow{R_1 \leftarrow R_1 + \frac{1}{4}R_2} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \end{pmatrix} \quad (1.1.13)$$

$$\therefore \mathbf{P} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad (1.1.14)$$

is the point of intersection of the lines and the vertex of the triangle formed by the two lines with x-axis as base.

b) To find out intersection of (1.1.5) with the x axis:

equation of x axis is

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 0 \quad (1.1.15)$$

we have 2 equations:

$$\begin{pmatrix} 4 & -1 \end{pmatrix} \mathbf{x} = 8 \quad (1.1.16)$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 0 \quad (1.1.17)$$

Augmented matrix for above is:

$$\begin{pmatrix} 4 & -1 & 8 \\ 0 & 1 & 0 \end{pmatrix} \quad (1.1.18)$$

This can be reduced as follows:

$$\begin{pmatrix} 4 & -1 & 8 \\ 0 & 1 & 0 \end{pmatrix} \xrightarrow{R_1 \leftarrow -\frac{1}{4}R_1} \begin{pmatrix} 1 & -\frac{1}{4} & 2 \\ 0 & 1 & 0 \end{pmatrix} \quad (1.1.19)$$

$$\xrightarrow{R_1 \leftarrow R_1 + \frac{1}{4}R_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \end{pmatrix} \quad (1.1.20)$$

$$(1.1.21)$$

$$\therefore \mathbf{Q} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (1.1.22)$$

is the point of intersection of the line (1.1.5) with the x axis.

- c) To find out intersection of (1.1.6) with the x axis:

equation of x axis is

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 0 \quad (1.1.23)$$

we have 2 equations:

$$\begin{pmatrix} 2 & -3 \end{pmatrix} \mathbf{x} = -6 \quad (1.1.24)$$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 0 \quad (1.1.25)$$

Augmented matrix for above is:

$$\begin{pmatrix} 2 & -3 & -6 \\ 0 & 1 & 0 \end{pmatrix} \quad (1.1.26)$$

This can be reduced as follows:

$$\begin{pmatrix} 2 & -3 & -6 \\ 0 & 1 & 0 \end{pmatrix} \xrightarrow{R_1 \leftarrow -\frac{1}{2}R_1} \begin{pmatrix} 1 & -\frac{3}{2} & -3 \\ 0 & 1 & 0 \end{pmatrix} \quad (1.1.27)$$

$$\xrightarrow{R_1 \leftarrow R_1 + (-\frac{3}{2})R_2} \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \end{pmatrix} \quad (1.1.28)$$

$$(1.1.29)$$

$$\mathbf{R} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \quad (1.1.30)$$

is the point of intersection of the line (1.1.6) with the x axis.

$$\mathbf{P} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad (1.1.31)$$

$$\mathbf{Q} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (1.1.32)$$

$$\mathbf{R} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \quad (1.1.33)$$

$$(1.1.34)$$

represent the vertices of the triangle formed by the lines (1.1.5) & (1.1.6) with the X-axis.

P is the vertex of the triangle. Q is the point at which $4x - y - 8 = 0$ meets the X-axis.

R is the point at which $2x - 3y + 6 = 0$ meets the X-axis.

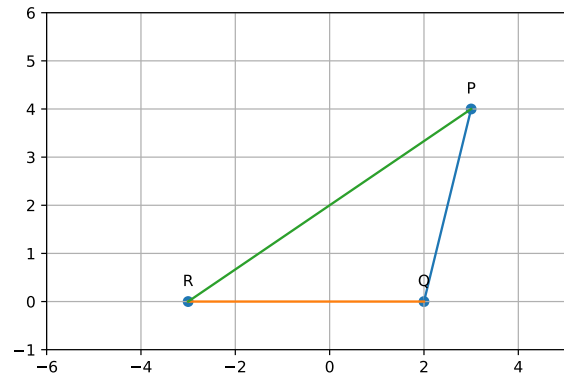


Fig. 1.1. Two lines representing given equations meet at point $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$