Assignment No.7

Mr.Ganesh Yadav

Download all python codes from

```
svn co https://github.com/Ganeshyadav712/
    Assignment−7.git
```

Question taken from

```
https://github.com/gadepall/ncert/blob/main/linalg/
    optimization/gvv_ncert_opt.pdf question 2.7
```

1 Question No 1

A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10 metres. Find the dimensions of the rectangle so as to admit maximum light through the whole opening.

2 Solution

Let x and y be the length and width of rectangle part of window respectively.
Let A be the opening area of the window which admits Light. Obviously, for admitting the maximum light through the opening, A must be maximum.
Now
A = Area of rectangle + Area of the semi-circle.

$$\mathbf{A} = xy + \frac{1}{2}\pi.\frac{x^2}{4} \tag{2.0.1}$$

$$\mathbf{A} = xy + \frac{\pi.x^2}{8} \tag{2.0.2}$$

$$\mathbf{A} = x\frac{5 - x(\pi+2)}{4} + \frac{\pi.x^2}{8} \tag{2.0.3}$$

$$\mathbf{A} = 5x - \frac{(\pi+2)x^2}{4} + \frac{\pi.x^2}{8} \tag{2.0.4}$$

$$\mathbf{A} = 5x - \frac{(\pi+2)}{4} - \frac{\pi}{8}x^2 \tag{2.0.5}$$

$$\mathbf{A} = 5x - \frac{(\pi+4)}{8}x^2 \tag{2.0.6}$$

$$\frac{dA}{dX} = 5 - \frac{\pi+4}{8}2X \tag{2.0.7}$$

$$\tag{2.0.8}$$

from question

$$\therefore x + 2y + \pi\frac{x}{2} = 10 \tag{2.0.9}$$

$$x\frac{\pi}{2} + 1 + 2y = 0 \tag{2.0.10}$$

$$2y = 10 - x\frac{\pi+2}{2} \tag{2.0.11}$$

$$y = 5 - \frac{x(\pi+2)}{4} \tag{2.0.12}$$

for maximum or minimum value of A,

$$\frac{dA}{dX} = 0 \tag{2.0.13}$$

$$5 - \frac{\pi+4}{8}.2X = 0 \tag{2.0.14}$$

$$\mathbf{X} = \frac{20}{\pi+4} \tag{2.0.15}$$

$$\left.\frac{d^2A}{dx^2}\right]\mathbf{X} = \frac{20}{\pi+4} \tag{2.0.16}$$

for $\mathbf{X} = \frac{20}{\pi+4}$, $\mathbf{A}$ is maximum
and hence

$$\mathbf{Y} = 5 - \frac{20}{\pi+4}.\frac{\pi+2}{4} \tag{2.0.17}$$

$$5 - \frac{5(\pi+2)}{\pi+4} \tag{2.0.18}$$

$$\frac{5\pi + 20 - 5\pi - 10}{\pi+4} \tag{2.0.19}$$

$$\frac{10}{\pi+4} \tag{2.0.20}$$

$\therefore$ for maximum $\mathbf{A}$ for admitting the maximum light
length of rectangle = $\mathbf{X}$= $\frac{20}{\pi+4}$
breadth of rectangle = $\mathbf{Y}$= $\frac{10}{\pi+4}$