Assignment No.6

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Download all python codes from

svn co https://github.com/Ganeshyadav712/ Assignment-7.git

Question taken from

https://github.com/gadepall/ncert/blob/main/linalg/ optimization/gvv ncert opt.pdf question 2.7

1 Ouestion No 1

A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10 metres. Find the dimensions of the rectangle so as to admit maximum light through the whole opening.

2 Solution

Let x and y be the length and width of rectangle part of window respectively.

Let A be the opening area of the window which admits Light. Obviously, for admitting the maximum light through the opening, A must be maximum. Now

A = Area of rectangle + Area of the semi-circle.

$$\mathbf{A} = xy + \frac{1}{2}\pi \cdot \frac{x^2}{4} \tag{2.0.1}$$

$$\mathbf{A} = xy + \frac{\pi \cdot x^2}{8} \tag{2.0.2}$$

$$\mathbf{A} = x \frac{5 - x(\pi + 2)}{4} + \frac{\pi \cdot x^2}{8}$$
 (2.0.3)

$$\mathbf{A} = 5x - \frac{(\pi + 2)x^2}{4} + \frac{\pi \cdot x^2}{8}$$
 (2.0.4)

$$\mathbf{A} = 5x - \frac{(\pi+2)}{4} - \frac{\pi}{8}x^2 \tag{2.0.5}$$

$$\mathbf{A} = 5x - \frac{(\pi + 4)}{8}x^2 \tag{2.0.6}$$

$$\frac{dA}{dX} = 5 - \frac{\pi + 4}{8} 2X \tag{2.0.7}$$

(2.0.8)

from question

$$\therefore x + 2y + \pi \frac{x}{2} = 10 \tag{2.0.9}$$

$$x\frac{\pi}{2} + 1 + 2y = 0 \tag{2.0.10}$$

$$2y = 10 - x \frac{\pi + 2}{2} \tag{2.0.11}$$

$$y = 5 - \frac{x(\pi + 2)}{4} \tag{2.0.12}$$

for maximum or minimum value of A,

$$\frac{dA}{dX} = 0 (2.0.13)$$

$$5 - \frac{\pi + 4}{8}.2X = 0 \tag{2.0.14}$$

$$\mathbf{X} = \frac{20}{\pi + 4} \tag{2.0.15}$$

$$\frac{d^2A}{dx^2} \left| \mathbf{X} = \frac{20}{\pi + 4} \right| \tag{2.0.16}$$

for $\mathbf{X} = \frac{20}{\pi + 4}$, **A** is maximum and hence

$$\mathbf{Y} = 5 - \frac{20}{\pi + 4} \cdot \frac{\pi + 2}{4} \tag{2.0.17}$$

$$5 - \frac{5(\pi+2)}{\pi+4} \tag{2.0.18}$$

$$5 - \frac{5(\pi + 2)}{\pi + 4}$$

$$\frac{5\pi + 20 - 5\pi - 10}{\pi + 4}$$
(2.0.18)

$$\frac{10}{\pi + 4} \tag{2.0.20}$$

:. for maximum A for admitting the maximum light

length of rectangle =
$$\mathbf{X} = \frac{20}{\pi + 4}$$

breadth of rectangle = $\mathbf{Y} = \frac{10}{\pi + 4}$

breadth of rectangle =
$$\mathbf{Y} = \frac{10}{\pi + 4}$$