# Elec4621 Lab4 - S1 2018

You are reminded that you must upload your working to Moodle using the submission box.

This lab is the start of a series of labs on statistical signal processing. We will look at the Fourier transform, filtering, detection, estimation and the effect of noise.

1. Consider a signal consisting of a single exponential:

$$x(t) = Ae^{j\omega t} = Ae^{j2\pi ft},$$

   where $f = 50$Hz. As you may recall, the signal of the power system in Australia is $A\sin(2\pi ft)$, where $A$ is approximately 339V. Therefore, the imaginary part of $x(t)$ could represent a measurement of the power signal. Although the signal in a power system is real, a complex phasor can be obtained using in-phase and quadrature measurements.

   Suppose that the signal, $x(t)$ is sampled at $f_s = 500$Hz to give $x[n]$.

   (a) What is the Fourier Transform of $x(t)$? What is the DTFT of $x[n]$?

   (b) Suppose we have $N = 64$ samples. Plot the real and imaginary parts of the signal versus time.

   (c) Using the DFT formula, calculate the expression of the DFT of the signal for $N$. Plot the resulting magnitude of the DFT versus frequency for a frequency spacing of 0.1Hz (this is called the periodogram). Comment on the result.

   (d) Using the FFT command in Matlab, obtain the DFT of $x[n]$ and plot the mangitudes of the resulting DFT samples over the plot as that in (1c). Comment on the result.

   (e) Now obtain the FFT in Matlab using a zero-padding to a length of 512 (Hint: using the command FFT(x,L) calculates the DFT of x, zero-padded to L). Again plot the result on the same plot as that of (1c). What do you observe?

(f) Estimating the frequency of the power system is extremely important to control it in order to ensure its stability. As you may have observed, the maximum of the periodogram can be used to find the frequency of the signal. This is written as

$$\hat{f} = \arg\max_{\lambda} \left| \sum_{n=0}^{N-1} x[n] e^{-j2\pi\lambda} \right|^2 . \tag{1}$$

Note: This equation means calculate the DFT coefficients for all $\lambda$ in the interval $[-\frac{f_s}{2}, \frac{f_s}{2}]$ and then find maximum value of the magnitude squared of these coefficients (which is essentially the periodogram) and then return the value of $\lambda$ that corresponds to this frequency.

One way to implement the periodogram is using the approach of (1e), where an estimate of the periodogram is obtained on a grid with spacing $\frac{f_s}{L}$. Write Matlab code to calculate the f the frequency using a periodogram of length $L$ for $L = 64 : 64 : 1024$. Plot the frequency error against $L$.

**Bonus part: adding noise and simulating the estimtor performance.**

(g) In reality, the signal is always contaminated by noise, which we assume is Gaussian distributed with mean 0 and variance $\sigma^2$. The signal to noise ratio (in dB) is defined as $\rho = 10 \log_{10} \frac{A^2}{\sigma^2}$. Use the function randn to generate Gaussian distributed noise, then add the noise to $x[n]$. For each SNR value, use 200 statistical runs to obtain the mean squared error (MSE) of the frequency estimates. Plot the MSE versus $\rho$ for $L = N$, $2N$, $4N$, and $8N$.

**Hint:** Since the signal record contains $N$ complex samples, we need to add complex noise to it. This is done by generating noise using the command $\frac{\sigma}{\sqrt{2}}(\text{randn}(N, 1) + j\text{randn}(N, 1))$.