Elec4621 S1 2018
Lab1, Odd Week

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Note: Your work will be inspected by the lab demonstrators at the end of your laboratory session and your mark will be recorded. Be sure to prepare ahead of time. You will also need to upload your lab work to Moodle (You should zip all of your working, whether it is lab notes or matlab code, and upload them using the lab 1 submission link. The file should be named ZID_Lab1, where ZID is your student number starting with z).

1. Consider the signal $x(t) = cos(2\pi ft)$, where $f = 2200$Hz.

   (a) What is the minimum sampling frequency, f_{min} that we should use for this signal?

   (b) Using Matlab, plot 5 periods of the continuous signal $x(t)$. We simulate a continuous signal by taking a very fine sampling time. That is set $T_s = 10^{-6}$.

   (c) Let the actual sampling frequency be $f_s = 1.6f_{min}$. Plot the sampled signal (the samples) on top of the "continuous" signal. How many samples per period of the signal do we get? What is the minimum frequency cosine (or sine) that can fit those points? **Hint:** Use the function stem to plot the sampled signal.

   (d) Now let the sampling frequency be $f_s = 0.6f_{min}$. Plot the sampled signal (the samples) on top of the "continuous" signal. How many samples per period of the signal do we get? What is the minimum frequency cosine (or sine) that can fit those points?

   (e) Calculate and plot the spectra of the original and two sampled signals for $-15,000 \leq f \leq 15,000$. Use a step of 1Hz for the plot. **Hint:** You should calculate the DFT of the signal. The FFT function does not give you all the frequencies you want.

2. A discrete FIR filter has impulse response $h[n]$ given by

$$h[n] = \begin{cases} -0.0625 & n = 0 \\ 0.25 & n = 1 \\ 0.625 & n = 2 \\ 0.25 & n = 3 \\ -0.0625 & n = 4 \end{cases}$$

(a) Write Matlab code to apply this filter to the sequence

$$x[n] = \{1, 2, 3, 4, 5, 6, 7, 6, 5, 4, 3, 2, 1\}, \quad n = 0, 1, \ldots, 12$$

Your code should directly implement the convolution equation

$$\mathbf{y} = \sum_n x[n]\, \mathbf{h}_n \tag{1}$$

That is, you should start by creating an output vector $\mathbf{y}$, large enough to accommodate the filtered output, setting all entries to 0. Your code should then add $x[n]\, \mathbf{h}_n$ into $\mathbf{y}$, for each n in turn. Do not use any special Matlab functions to do this; the idea is to learn what the equation means by implementing it directly.

(b) Equation (1) and the code you wrote in part (a) to implement it, represent an *input-based perspective* for filtering. Specifically, each input sample serves as a separate excitation source for the filter, and the output is the sum of the filter outputs from each separate excitation source. In this second part, you should implement exactly the same filter, adopting an *output-based perspective* instead. In this case, each individual output sample is written as

$$y[n] = \left\langle \mathbf{x}, \tilde{\mathbf{h}}_n \right\rangle$$

where $\tilde{\mathbf{h}} \equiv h[-n]$. As for part (a), your code should directly implement this equation, finding the output at each location (or at any given location), by taking the dot product between the translated, time-reversed impulse response, and the input signal.

(c) Write Matlab code to evaluate $\hat{h}(\omega)$ over a range of frequencies from $-\pi$ to π. Again, you should do this directly using primitive functions. Inspect the magnitude and phase of $\hat{h}(\omega)$. Do either of these possess any special properties? Is there a sense in which the filter can be said to shift (or delay) the input sequence? If so, by how much? Is this property evident from the filtered output, $y[n]$?

(d) Repeat parts (a) and (c) for the impulse response shown below

$$h[n] = \begin{cases} -0.1 & n = 0 \\ 0.6 & n = 1 \\ 0.6 & n = 2 \\ -0.1 & n = 3 \end{cases}$$

(e) Repeat parts (a) and (c) for the impulse response shown below, paying particular attention to the phase response

$$h[n] = \begin{cases} 0.2 & n = 0 \\ 0.5 & n = 1 \\ 0.2 & n = 2 \\ 0.1 & n = 3 \end{cases}$$

3. Two first order digital filters are given by their z-transform transfer functions as:

$$H(z) = 1 - az^{-1}$$

and

$$G(z) = \frac{1}{1 - az^{-1}}$$

In each case, the parameter a satisfies $|a| < 1$.

(a) Write Matlab code to compute the impulse responses $h[n]$ and $g[n]$. Using $a = \frac{1}{2}$ and $a = -\frac{1}{2}$, compare the results obtained and explain their similarities and differences.

(b) Starting with the sequence $x[n]$, given in question 1, filter first with $H(z)$ to obtain a sequence $y[n]$. Now apply the second filter $G(z)$ to $y[n]$, recovering a new sequence $w[n]$. In doing this, you should use the fact that $w[n] = 0$ for all $n < 0$. Note that you will need to implement $G(z)$ second filter recursively, which may require you to recall some of the things you learned in Elec3004. Ask the lab demonstrators to help you if there is anything you do not understand – we will be reviewing this material also in lectures, but your lab session may come before the revision material is covered. **In any event, you should not be using Matlab built-in functions to do the filtering for you here.** What is the relationship between $x[n]$ and $w[n]$?

(c) Write the code necessary to calculate and plot the magnitude responses, $\left|\hat{h}(\omega)\right|$ and $|\hat{g}(\omega)|$.