Elec4621 S1 2018
Lab2, Odd Week

March 23, 2018

1. A digital oscillator is to be implemented using the following recursive relationship:
$$y[n] = \alpha y[n-1] - \beta y[n-2]$$

   (a) Determine suitable values of β and α such that the system will oscillate at a frequency of $f = \frac{\omega_0}{2\pi}$ when excited with an impulse at time $n = 0$, for various values of ω_0.

   (b) Implement the oscillator in Matlab and verify that it is working as expected. Plot the output, $y[n]$, for the case, $\omega_0 = \frac{\pi}{4}$, assuming zero initial conditions, $y[n] = 0$ for $n < 0$.

   (c) By driving the system
$$y[n] = x[n] + \alpha y[n-1] - \beta y[n-2]$$
   with an input, $x[n]$, having two non-zero sample values, $x[0]$ and $x[-1]$, get the oscillator to produce the signal
$$y[n] = \sin\left(\omega_0 n + \frac{\pi}{4}\right)$$
   exactly.

2. A fifth order all-zero filter has the following transfer function,
$$H(z) = \left(\frac{z - r_1}{z}\right)\left(\frac{z^2 - 2r_2 z\cos\theta_2 + r_2^2}{z^2}\right)\left(\frac{z^2 - 2r_3 z\cos\theta_3 + r_3^2}{z^2}\right)$$
   where
$$r_1 = e^{-\frac{1}{8}}, \quad r_2 = r_3 = 0.925, \quad \theta_2 = 0.45\pi, \quad \theta_3 = 0.8\pi$$

(a) Find the impulse response of the filter.

Hint: There are a number of ways to do this: One way is to build a vector, **v**, containing all of the zeroes, and then use the Matlab function, 'poly', to find the coefficients of the polynomial with these roots – the coefficients of the polynomial are the filter taps. Another way is to proceed from the second order sections of the transfer function and use polynomial multiplication to build 5-th order numerator. Then you will have the filter taps.

(b) Using Matlab code which you write yourself (as you did in Lab 1), find and plot the magnitude and phase responses of the filter.

Hint: You should remember that the frequency response is obtained by substituting $z = e^{j\omega}$. Therefore you can think of this as the evaluation of the transfer function on a fine grid of ω. Thus, you can construct a vector **w** of angular frequencies and evaluate the transfer function for this vector, then plot the amplitude and phase of the result.

(c) By selectively taking reciprocals of the filter's zeros (i.e., replacing r_k by $\frac{1}{r_k}$), you can generate seven other FIR filters, all having 6 taps and exactly the same magnitude response as the filter given above. Write Matlab code to walk through these filters, plotting their magnitude and phase responses in turn. What do you notice about the phase responses, in comparison to that found in part (b) above?

Hint: The implementation of this part is quite simple if you use the code of the last part as a function that takes the poles and zeroes as input.

(d) Examine the impulse responses of all 8 filters (the original and the extra 7 found in part (c)). What special characteristic is exhibited by the impulse response of the original filter?

(e) Now assume that the filter coefficients are to be rounded to some number of decimal places, N_{digits}, write code that plots the location of the zeroes and the resulting magnitude and frequency responses for different values of N_{digits}.