Advanced Microeconomics

Lecture 6: Information economics

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Outline

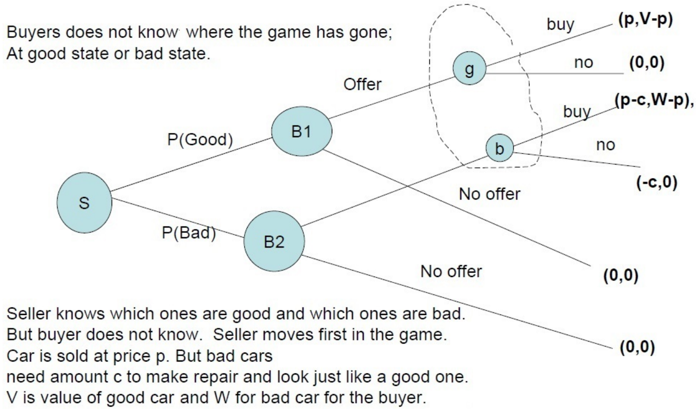
Outline

1. Market situations

- Complete market failure
  - pooling equilibrium (same price for good and bad cars; good cars disappear from the market)
- Complete market success
  - Separating equilibrium where players act as they should according to the signal (prices according to quality)
- Partial market success
  - Both good and bad cars are bought, some feel cheated.
- Near Market failure (mixed strategies)
  - Bayesian updating mechanism at work

2. Markets for used cars (Akerlof)

Informed player moves first in a signalling game (Akerlof)



The buyer is uncertain about where the game is going - need to resolve this - then the game can be solved by backward induction.

2. Markets for used cars (Akerlof)

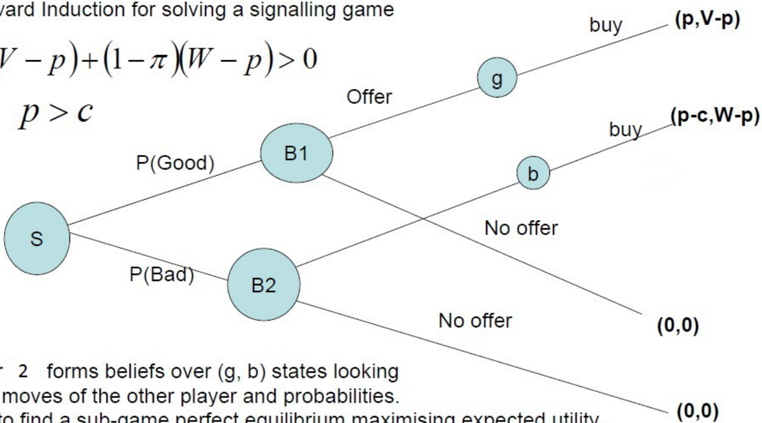
Partial success of market with asymmetric information:

Signals need to be credible

Backward Induction for solving a signalling game

$$\pi(V-p)+(1-\pi)(W-p)>0$$

$$p>c$$

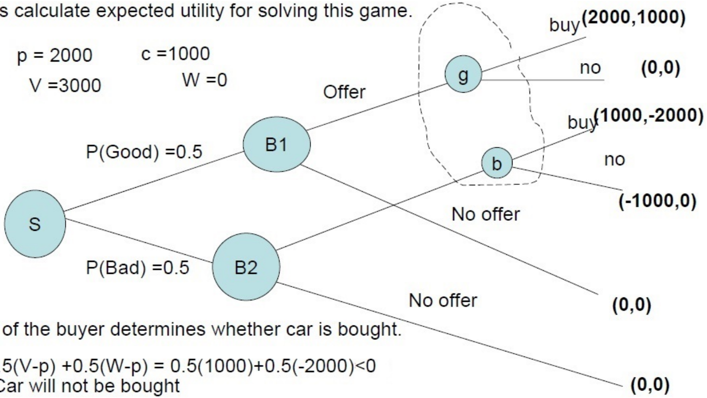


Player 2 forms beliefs over (g, b) states looking at the moves of the other player and probabilities.
Tries to find a sub-game perfect equilibrium maximising expected utility.
Both good and bad cars will be sold; Some buyers who get bad cars feel cheated

2. Markets for used cars (Akerlof)

Mixed strategy sequential equilibrium:

Buyers calculate expected utility for solving this game.



Belief of the buyer determines whether car is bought.

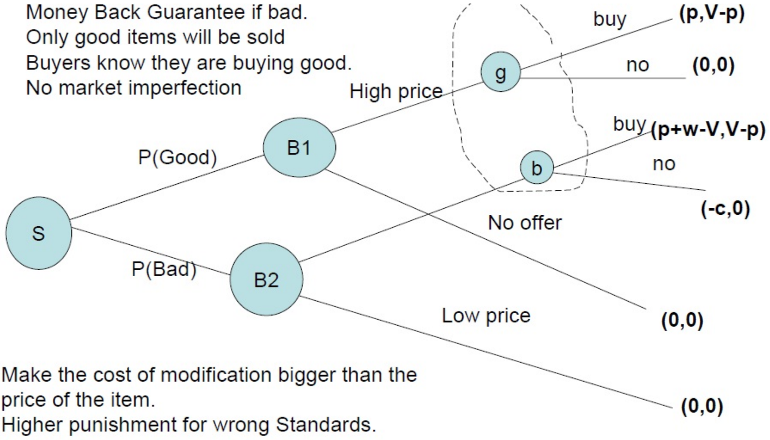
$0.5(V\text{-}p) + 0.5(W\text{-}p) = 0.5(1000)+0.5(-2000)<0$
Car will not be bought

$0.66(V\text{-}p)+0.33(W\text{-}p) = 0.66(1000)+0.33(-2000)=0$ Indifferent

EU(good) = 0.5(2000)+0.5(0) = 1000>0
EU(bad) = 0.5(1000)+0.5(-1000) = 0
Indifferent

2. Markets for used cars (Akerlof)

Costly commitment for separating equilibrium: Complete market success:



Credit Rationing (Stiglitz): Asian Economic Crises

3. Adverse selection

(1) Market failures

- used cars, even if they are like new, sell far below their dealership price.
- laid-off workers experience longer spells of unemployment than workers for different reasons without a job (e.g. military).
- private health care for the elderly is essentially unavailable.
- young drivers pay very expensive insurance premiums.

3. Adverse selection

(2) Productivity uncertainty in labour markets

- 2 identical firms F, 1 representative worker W.
- The worker's productivity is $\theta \in [\theta_L, \theta_H]$, private information of W, c.d.f. $F(\theta)$ is continuous and has full support.
- Utilities: $u_W = wq + r(\theta)(1-q)$, $u_F = (\theta - w)q$ where $q = 1$ if W works and $q = 0$ otherwise; $w \in R$ (wage paid by F); $r(\theta)$ is W's reservation utility with $r' > 0$.
- Timing:
  1. $t = 0$: W observes θ.
  2. $t = 1$: Fs offer fixed-wage employment contracts.
  3. $t = 2$: W accepts/rejects $\Rightarrow$ payoffs.

3. Adverse selection

(2) Productivity uncertainty in labour markets

- Equilibrium under full information:
  - Both worker and firms know $\theta \Rightarrow$ firms can offer productivity dependent wages $w(\theta)$.
  - Utility of worker of type θ who accepts wage offer of w is $u_W = w$.
  - Comparing this with outside utility $r(\theta)$ gives *labour supply*: $\Theta(w) = \{\theta | r(\theta) \leq w\}$.
  - *labour demand* is $z(w) = 0$ if $\mu < w$; $z(w) = [0, \infty]$ if $\mu = w$; $z(w) = \infty$ if $\mu > w$, where $\mu = E[\theta | \theta \in \Theta^*(w)]$ is the firms' beliefs about the average productivity of workers who will accept employment.

3. Adverse selection

(2) Productivity uncertainty in labour markets

- Equilibrium under full information (cont.):
  - Expected profit of F is $E[\theta|\theta \in \Theta^*(w)] - w$.
  - Competition gives 0 profits $\Rightarrow$ necessary equilibrium condition is (market clearing): $\Theta^*(w) = \{\theta : r(\theta) \leq w\}$ and $w^* = E[\theta|\theta \in \Theta^*(w)] = E[\theta : r(\theta) \leq w^*]$. We have the CE under unobservable worker productivity levels.
  - However, this CE is generally non-existed, which is of course due to the incapability for firms to distinguish workers' type.

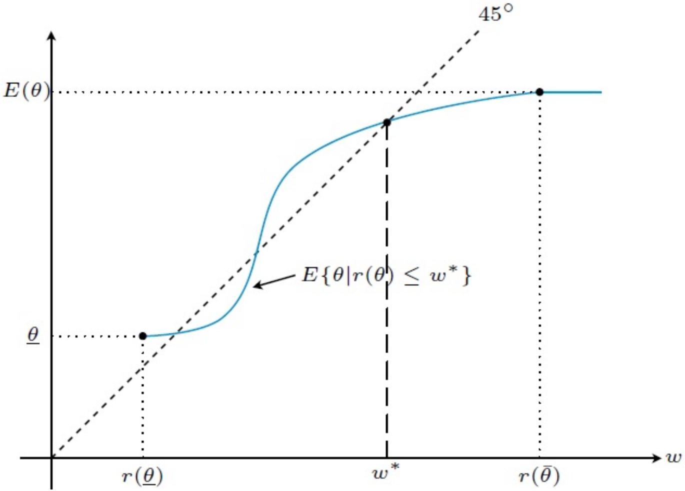
3. Adverse selection

(2) Productivity uncertainty in labour markets

- Equilibrium under asymmetric information:
  - Only W knows θ. Fs do not know θ, only know $F(\theta) \Rightarrow$ offer (same) fixed wage contract w in equilibrium.
  - Assume $r(\theta_L) \leq \theta$ and $r(\theta) < \theta$, $\forall \theta > \theta_L \Rightarrow$ efficient employment has $q(\theta) = 1\ \forall \theta$.
  - Equilibrium exists and is generically unique: if there are multiple solutions to equation $w^* = E[\theta|\theta \in \Theta^*(w)] = E[\theta : r(\theta) \leq w^*]$. equilibrium is the highest w satisfying the condition.
  - Welfare properties:
    1. if $r(\theta_H) \leq E(\theta)$, equilibrium is *efficient*;
    2. if $r(\theta_H) > E(\theta)$, equilibrium is *inefficient*;
    3. if $r(\theta_L) = \theta_L$, can have complete market breakdown.
- Intuition: F cannot break even at wage $w = E(\theta) < r(\theta_H)$. w falls $\Rightarrow$ even more high-productivity workers drop out of market $\Rightarrow$ wage must drop further. Alternatively: Market participation of individual worker introduces externality.

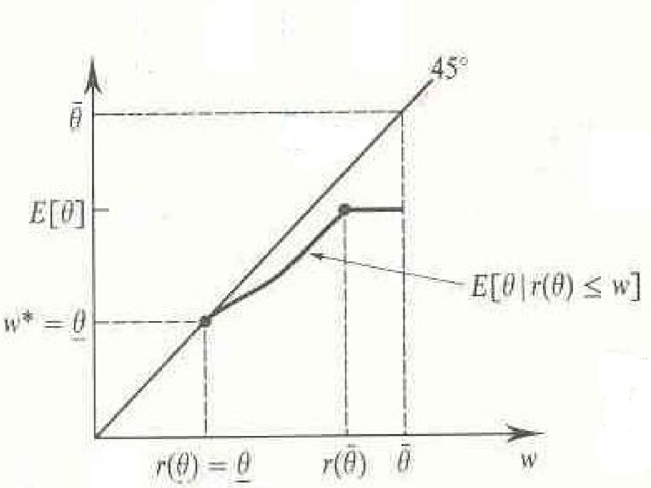
3. Adverse selection

(2) Productivity uncertainty in labour markets



3. Adverse selection

(2) Productivity uncertainty in labour markets



3. Adverse selection
(3) Other examples

Formal model translates into goods markets and insurance markets:

labor		consumption good (cars)	insurance
workers	$\rightarrow$	sellers	insuree
firms	$\rightarrow$	buyers	insurer
θ	$\rightarrow$	buyer's value	-(exp. payments to insuree)
$r(\theta)$	$\rightarrow$	seller's value	-(inverse CE of risk)
w	$\rightarrow$	price	-(insurance premium)

- Credit market (θ = default risk of debtor)
- Insurance market (θ = risk of having been ill)
- Dating and marriage market (θ = attractiveness of partner)
- Stock market and corporate equity market (IPO's) (θ = firm value)

3. Adverse selection
(4) Remarks

Adverse selection can lead to total market failure ⇒ if trade occurs, it will be less than efficient

- In markets with adverse selection (asymmetric information).
- Prices are correlated with quality. (Prices affect CE.)
- Prices serve dual role of info transmission and market clearing.
- Insitutional/market responses against market failure caused by adverse selection.
  - Signaling and screening devices, e.g. warrantees
  - Reputation (brand names and chains)
  - Experts, inspections, standards, licensing
  - Mandatory insurance (health, automobile)
  - Liability laws

Outline

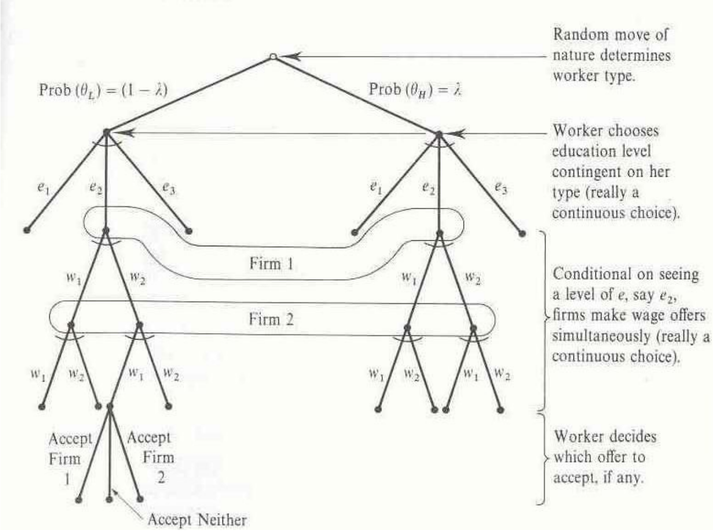
1. The model set-up

- Asymmetric information causes market failure $\Rightarrow$ participants have incentives to develop ways to reduce informational asymmetries.
  - Signaling: informed market participants move first to convey info.
  - Screening: uninformed market participants move first to elicit info.
- Signaling
  - Some market participants may be worse off as a result of their privately held information (sellers in lemons market, consumers in insurance market) $\Rightarrow$ would want to reveal this information to others.
  - Problem: information revealed must be credible $\Rightarrow$ use of signals.
  - Examples: warranties, lineups, peacock tail.
  - But for the signal to work (be credible), it must be costly to fake.

1. The model set-up

- Players consisting of workers, firms and nature.
- There are two types of workers $\theta \in [\theta_L, \theta_H]$. Type L is less productive, while H is more productive.
- Firms do not know which one is low or high quality worker but sees level of education.
- Firms are risk-neutral, seek to maximise their expected profits, and act as price takers.
- Nature decides whether a worker is high or low productivity type.
- Level of education *signals* the quality of worker.

1. The model set-up



1. The model set-up

- 2 identical firms F, 1 representative worker W
- The worker's productivity is $\theta \in [\theta_L, \theta_H]$, private information of W with $p = \Pr(\theta = \theta_H)$.
- Set $r(\theta_H) = r(\theta_L) = 0$ for simplicity.
- W can invest in edu. $e \in \mathbb{R}_0^+$:
  - Marginal cost of edu. of θ_i-type is c_i with $0 < c_H < c_L$.
  - Edu. does NOT improve productivity θ.
- Utilities: $u_W = u(w) - c(e, \theta) = w - c(e, \theta)$, $u_F = \theta_i - w$.
- Timing:
  1. $t = 1$: W learns θ_i, chooses e.
  2. $t = 2$: Fs observe e, form beliefs $\mu(e) = \Pr(\theta = \theta_H | e)$.
  3. $t = 3$: Fs offer (same) wage w.

2. General structure of signaling (& cheap talk) game

- Dynamic game of incomplete information with sender S and receiver R.
- Timing:
  1. $t = 0$: Nature draws type $\theta_i \in \Theta = \{\theta_1, \ldots, \theta_n\}$ for sender; with $p_i = p(\theta_i) = \Pr(\theta = \theta_i) > 0$, $\forall \theta_i \in \Theta$.
  2. $t = 1$: Sender observes own type θ_i, chooses message $m_j \in M = \{m_1, \ldots, m_n\}$.
  3. $t = 2$: Receiver observes message m_j (but not θ_i), and chooses action $x \in X$.
  4. $t = 3$: Payoffs $u_S(x, m_j, \theta_i)$ and $u_R(x, m_j, \theta_i)$.
- Signaling game: u_S depends on m_j.
- Cheap talk game: u_S (and u_R) is independent of m_j.

3. Perfect Bayesian Nash equilibrium

- A Perfect Bayesian Equilibrium (PBE) is a pair of strategies $m^*(\theta_i)$ and $x^*(m_j)$ and a belief $\mu^*(\theta_i|m_j)$ such that
  1. $m^*(\theta_i)$ maximises S's utility given R's strategy:

$$m^*(\theta_i) = \arg\max_{m_j \in M} u_S(x, m_j, \theta_i), \forall \theta_i \in \Theta$$

  2. $x^*(m_j)$ maximises R's utility given beliefs about S's type:

$$x^*(m_j) = \arg\max_{x \in X} \sum_{\theta_i} \mu(\theta_i|m_j) u_R(x, m_j, \theta_i)$$

  3. R forms consistent beliefs that are calculated by Bayes Rule whenever possible:

$$\sum_{\theta_i \in \Theta} \mu(\theta_i|m_j) = 1, \forall m_j \quad \mu(\theta_i|m_j) = \frac{p(\theta_i)}{\sum_{\theta_i \in \Theta(m_j)} p(\theta_i)}$$

where $\Theta(m_j) \equiv \{\theta_i \in \Theta | m^*(\theta_i) = m_j\}$.

4. Equilibrium analysis

(1) Benchmark cases

- Equilibrium under full information
  - Both W and Fs observe ability θ_i, $i = \{L, H\}$.
  - Fs offer wage $w_i = \theta$ *independent* of e and earn zero profit. $\Rightarrow$ W chooses $e_H = e_L = 0$. $\Rightarrow$ *efficient*.
- Imperfect but symmetric information
  - Neither W nor Fs observe θ_i, $i = \{L, H\}$ $\Rightarrow$ wage w can no longer depend on θ.
  - Fs offer $w(e = 1) = w(e = 0) = p\theta_H + (1 - p)\theta_L$ independent of e $\Rightarrow$ worker chooses $e_H = e_L = 0$ $\Rightarrow$ *efficient*.

4. Equilibrium analysis

(2) Imperfect and asymmetric Information

Only W not Fs observe θ_i, $i = \{L, H\}$.

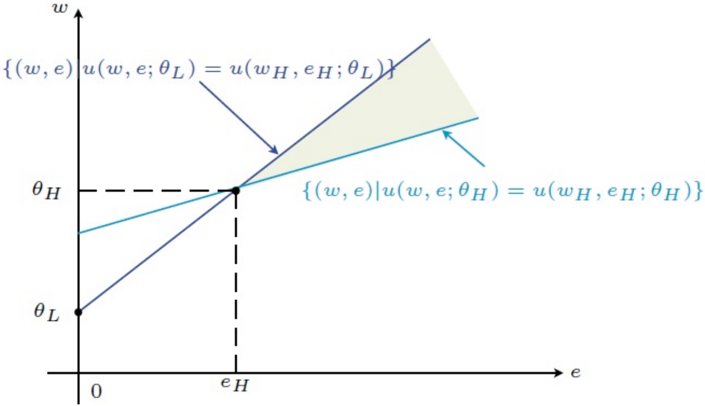
- Separating equilibrium
  - 2 types of workers choose different education levels $e_H \neq e_L$
    - Fs have beliefs $\mu(e_H) = \theta_H$ and $\mu(e_L) = \theta_L$.
    - Fs offer wages $w(e_H) = \theta_H$ and $w(e_L) = \theta_L$.
  - $e_H \neq e_L$ optimal for W requires (from (IC)'s) $e_L = 0$ and $e_H > 0$ where

$$c(e_H, \theta_H) \leq \theta_H - \theta_L \leq c(e_H, \theta_L)$$

  - Off equilibrium beliefs? Let $\mu(e) = 0$ for $e < e_H$, $\mu(e) = 1$ for $e \geq e_H$.

4. Equilibrium analysis

(3) Graphic illustration



Note: W's preferences satisfy single crossing property: $\frac{dw}{de}|_{\overline{u}} \uparrow$ in θ or $u_e(w,e;\theta_L) - u_e(w,e;\theta_H) = c_L - c_H > 0$

4. Equilibrium analysis

(4) Remarks

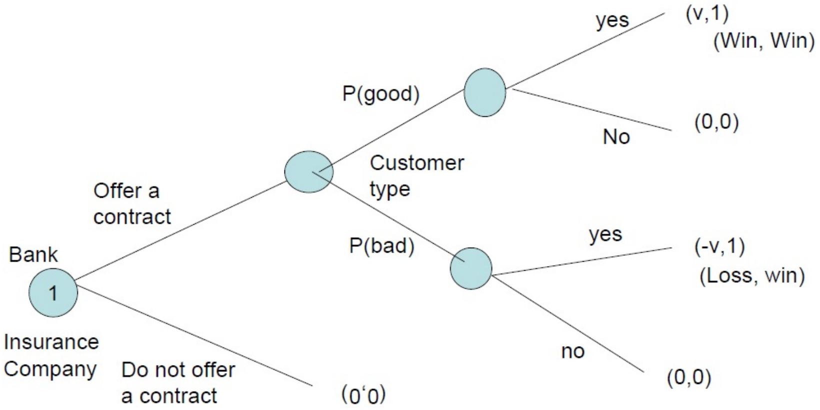
Signaling can lead to wasteful resource allocation and the market outcome may thus be inefficient.

- In markets with signaling
  - Privately informed individuals use signal to reveal their information.
  - Signal only works (is credible) if sending the same signal is too costly for other individuals.
- Other markets where wasteful signaling is relevant
  - consumer products (signal = warranty, advertisements, price)
  - corporate equity and start-ups (signal = equity/own money invested)
  - legal disputes (signal = pre-trial settlement demands)
  - bargaining (signal = rejection of offer/delay)
  - live entertainment and restaurants (signal = lineups)
  - marriage and dating (signal = fancy car)
  - poker (signal = stakes)

Outline

1. A screening game (Stiglitz)

Uninformed players moves first in: Credit rationing or insurance market



2. Competitive screening

(1) The model set-up

Screening = contractual arrangements originating from uniformed side of market to elicit information from informed market participants

- 2 identical firms Fs, 1 representative worker W.
- W with ability(=productivity) $\theta \in [\theta_L, \theta_H]$, private information of W with $p = \Pr(\theta = \theta_H)$, $r(\theta_i) = 0$.
- Fs can set task difficulty level $t_i \in \mathbb{R}_0^+$.
- Task difficulty does not influence productivity and costs workers effort c_i with $0 < c_H < c_L$.
- Utilities: $u_W = u(w) - c(t, \theta)$, $u_F = \theta_i - w$.
- Timing:
  1. $t = 1$: W learns θ_i.
  2. $t = 2$: Fs offer menu of contracts (w, t).
  3. $t = 3$: W picks firm/contract.

2. Competitive screening

(2) Equilibrium analysis

- Equilibrium under full information
    - Both W and Fs observe ability θ_i, $i = \{L, H\}$.
    - Fs offer wage/task contracts $(w_i, t_i) = (\theta_i, 0)$ and earn zero profit. $\Rightarrow$ W accepts employment. $\Rightarrow$ SPE is *efficient*.
- Imperfect but symmetric information
    - Neither W nor Fs observe θ_i, $i = \{L, H\}$ $\Rightarrow$ wage w can no longer depend on θ.
    - Fs offer wage/task contract $(w, t) = (p\theta_H + (1-p)\theta_L, 0) \Rightarrow$ W accepts $\Rightarrow$ *efficient*.
- Note: W's preferences satisfy single-crossing property.

2. Competitive screening

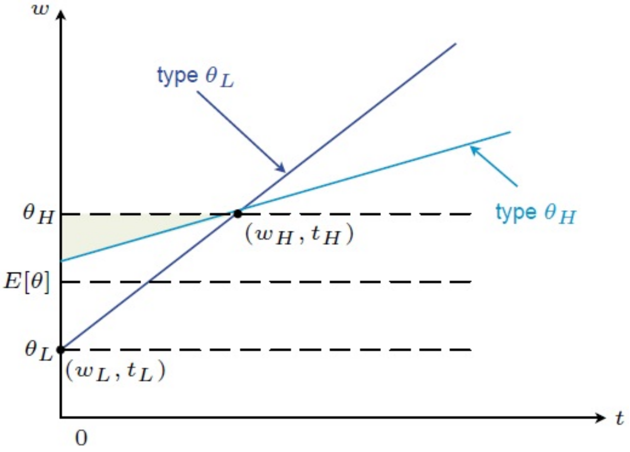
(2) Equilibrium analysis

Only W not Fs observe θ_i, $i = \{L, H\}$. Find equilibrium in steps:

- Let (w_H, t_H) and (w_L, t_L) be the lowest-wage contracts accepted by workers of type i in equilibrium.
- Step 1: in any SPE, Fs earn zero profit.
- Step 2: there is no SPE in which the high-type W accepts a wage $w_H < \theta_H$ with positive probability. $\Rightarrow$ all low types must earn $w_L = \theta_L$ and the equilibrium must be separating.
- In a separating SPE, must have $t_L = 0$ and $\theta_H - c(t_H, \theta_L) = \theta_L - c(0, \theta_L)$, i.e., the only candidate for equilibrium is the least-cost separating one.

3. Screening with task assignments

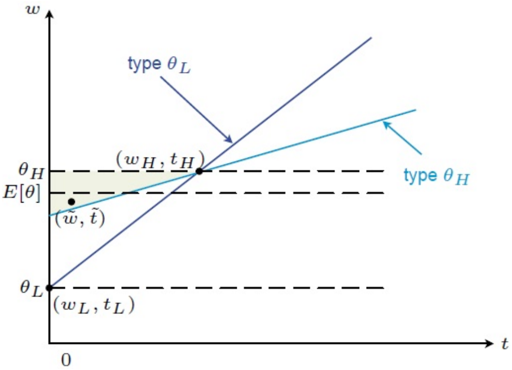
(1) Graphic illustration of the separating equil.



Note: no deviation profitable; in particular, any contract attracting both types of workers lies above break even line.

3. Screening with task assignments

(1) Graphic illustration of the separating equil.



- Note: deviation profitable; a contract $(\tilde{w}, \tilde{t})$ attracting both types of workers lies below break even line.
- If least-cost separating equilibrium is Pareto dominated by pooling, then no equilibrium exists.

3. Screening with task assignments
(2) Remarks

Sceening can lead to wasteful resource allocation and the market outcome may thus be inefficient

- In markets with screening
  - Uninformed individuals use screening devices to make informed individuals reveal their information by choice of (preferred) contract from menu.
  - Screening device only works (separates types) if accepting same contract is undesirable for other individuals.
- Other markets where wasteful screening is relevant
  - consumer products (screening device = warranty, price)
  - credit (screening device = collateral, incomes)
  - marriage and dating (screening device = household chores)
  - poker (screening device = stakes)

Outline

1. Basics

Principal	Agent	Action
Plantation owner	Share cropper	Labour input
Insurance company	Policy holder	Careful behaviour
Patient	Doctor	Intervention
Owner	Renter	Maintenance
Firms	Workers	Work effort

- Both principal and agents have their objective functions they like to maximise.
- The principal is interested in maximising profit from the business.
- Agents aim to maximise utility(payoff) choosing the best contract available from the principal with proper allowances for its efforts.
- Rasmusen (2007) has interesting examples on this topic.

1. Basics

Two kinds of informational asymmetries can arise:

- Hidden information: the agent obtains better information about aspects relevant to the principal. For example, the manager in charge of sales in a particular region is likely to acquire a better knowledge of the market than his superiors.
- Hidden action (moral hazard): the agent undertakes an action that affects the principal's utility and this action is difficult to monitor for the principal. For example, the amount of effort or the quality of work put in by an employee is difficult to monitor directly and the principal has to rely on results of the employee's work (e.g, output produced, the number of breakdowns/repairs on manufacturing equipment etc.). Another example: the bank handing out credit may not be able to assess the riskiness of the project chosen by the creditor.

1. Basics

Basic problem in corporate finance: separation of ownership and control:

- The owners of the firm are typically not those who manage it on a daily basis.
- Owners (principal) delegate tasks to managers (agent).
- Yet, managers have their own objective function. They may not exert much effort, for example, because it is costly for them.
- The way to solve the problem would be to write a contract that compensates the manager on the basis of his effort. Yet, the effort is typically unobservable (*hidden action*).
- Hence, we write contracts that compensate the manager based on performance, which is a noisy signal of the manager's effort.
- This might be costly when the manager is risk averse, since extra compensation is needed for the risk taken.

2. The model

(1) Technology & effort

- π denotes the observable profit, e denotes the manager's effort.
- For simplicity, the manager has two possible efforts: $e_H > e_L$.
- The distribution of profits $f(\pi|e)$ depends on the level of effort:
  - The distribution conditional on e_H first-order stochastically dominates the one conditional on e_L : $F(\pi|e_H) \leq F(\pi|e_L)$ at all $\pi \in [\underline{\pi}, \overline{\pi}]$ with strict inequality on some open set.
  - As a result: $\int \pi f(\pi|e_H)d\pi > \int \pi f(\pi|e_L)d\pi$.

2. The model
(2) Preferences

- Manager maximises utility function $u(w, e)$ over wage and effort

$$u'(w, e) > 0; u''(w, e) \leq 0; u(w, e_H) < u(w, e_L)$$

- Concentrate on: $u(w, e) = v(w) - g(e)$.

$$v'(w) > 0; v''(w) \leq 0; g(e_H) > g(e_L)$$

- The owner receives the profit minus the wage. We assume here that he is *risk neutral*, and thus tries to maximise his expected payoff.
- Assumption that manager is *risk averse* and owner is *risk neutral* can be justified by patterns of diversification.

2. The model

(2) Optimal contract with observable effort

- A contract specifies effort level (e_H or e_L) and wage function $w(\pi)$.
- Owner solves following problem:

$$\max_{e \in \{e_L, e_H\}, w(\pi)} \int [\pi - w(\pi)] f(\pi|e) d\pi$$

s.t. $\int v(w(\pi)) f(\pi|e) d\pi - g(e) \geq \bar{u}$ (*participation constraint (PC)* always binding).

- Find $w(\pi)$ for a given e, and then find optimal e.
- Solution often referred to as *first-best* choice.

2. The model

(2) Optimal contract with observable effort

- Given e, owner's problem is equivalent to:

$$\max_{w(\pi)} \int [\pi - w(\pi)] \, f(\pi|e) d\pi$$

s.t. $\int v(w(\pi)) f(\pi|e) d\pi - g(e) \geq \overline{u}$ (PC).

- Denoting the multiplier on the constraint as γ:

$$f(\pi|e) - \gamma v'(w(\pi)) f(\pi|e) = 0$$
$$\frac{1}{v'(w(\pi))} = \gamma$$

Why 1 in the numerator?

- Hence, when the manager is strictly risk averse, the owner offers a fixed compensation: *Risk sharing* (why?).

2. The model

(2) Optimal contract with observable effort

- For effort level e, the owner offers w_e^*, such that $v(w_e^*) - g(e) = \overline{u}$.
- Then, the owner chooses e that maximises:

$$\int \pi f(\pi|e)d\pi - v^{-1}(\overline{u} + g(e))$$

- If $\int \pi f(\pi|e_H)d\pi - v^{-1}(\overline{u} + g(e_H)) > \int \pi f(\pi|e_L)d\pi - v^{-1}(\overline{u} + g(e_L))$
  - Then, $e = e_H$ and $w = v^{-1}(\overline{u} + g(e_H))$.
  - Otherwise, $e = e_L$ and $w = v^{-1}(\overline{u} + g(e_L))$.
- With risk neutrality, same spirit, but fixed wage is not necessary.

2. The model

(3) Optimal contract with unobservable effort

Risk neutral manager

- Suppose that $v(w) = w$.
- With observable effort, owner solves:

$$\max_{e \in \{e_L, e_H\}} \int \pi f(\pi|e) d\pi - \overline{u} - g(e)$$

- A basic result is that the owner can achieve the same value with a compensation contract when effort is unobservable.
- This contract then must be optimal because the owner cannot do better under unobservable effort than under observable effort.

2. The model

(3) Optimal contract with unobservable effort

Risk neutral manager

- Consider a compensation schedule of the form: $w(\pi) = \pi - \alpha$.
    - This is effectively like selling the project to the manager for α.
- The manager then chooses e to maximise:

$$\int \pi f(\pi|e)d\pi - \alpha - g(e)$$

  And thus chooses the same e^* as in the first-best solution.
- Set $\alpha = \int \pi f(\pi|e^*)d\pi - \overline{u} - g(e^*)$, will then give the owner the same value as in the first-best solution.
- When risk is not a problem, it is easy to incentivise the manager.

2. The model

(3) Optimal contract with unobservable effort

Risk averse manager

- As before, find $w(\pi)$ for the level e that we choose to implement, and then find optimal e.
- The owner solves:

$$\max_{w(\pi)} \int [\pi - w(\pi)]\, f(\pi|e) d\pi$$

$$\text{s.t. (1)} \int v(w(\pi)) f(\pi|e) d\pi - g(e) \geq \overline{u}$$

$$\text{(2) } e \text{ solves } \max_{\tilde{e}} \int v(w(\pi)) f(\pi|\tilde{e}) d\pi - g(\tilde{e})$$

where (2) is the *incentive compatibility constraint* (IC), ensuring that the manager chooses the right level of effort.

2. The model

(3) Optimal contract with unobservable effort

Risk averse manager

- Suppose that the first-best level of effort (achieved under observable effort) is e_L:

$$\int \pi f(\pi|e_H)d\pi - v^{-1}(\overline{u}+g(e_H)) < \int \pi f(\pi|e_L)d\pi - v^{-1}(\overline{u}+g(e_L))$$

- The owner can implement e_L in exactly the same way as he did when effort was observable, i.e., by paying the manager a fixed wage: $w = v^{-1}(\overline{u} + g(e_L))$.
- Since the manager's wage does not depend on his performance, he always chooses the low effort, and e_L is implemented.
- So, when e_L is first-best, non-observability of effort is costless.

2. The model

(3) Optimal contract with unobservable effort

Risk averse manager

- Suppose that the first-best level of effort is e_H:

$$\int \pi f(\pi|e_H)d\pi - v^{-1}(\overline{u}+g(e_H)) > \int \pi f(\pi|e_L)d\pi - v^{-1}(\overline{u}+g(e_L))$$

- Implementing e_H implies IC:

$$\int v(w(\pi))f(\pi|e_H)d\pi - g(e_H) \geq \int v(w(\pi))f(\pi|e_L)d\pi - g(e_L)$$

- Denoting the multipliers on the participation and incentive constraints as γ and μ, respectively:

$$f(\pi|e_H) - \gamma v'(w(\pi))f(\pi|e_H) - \mu[f(\pi|e_H) - f(\pi|e_L)]v'(w(\pi)) = 0$$

2. The model

(3) Optimal contract with unobservable effort

Risk averse manager

- Hence, the condition becomes:

$$\frac{1}{v'(w(\pi))} = \gamma + \mu \left[1 - \frac{f(\pi|e_L)}{f(\pi|e_H)} \right]$$

- It is straightforward to show that γ and μ must be strictly positive in any solution, and thus both the participation constraint and the incentive constraint are binding.
  - If $\gamma = 0$, condition is violated for π where $\frac{f(\pi|e_L)}{f(\pi|e_H)} > 1$.
  - If $\mu = 0$, wage must be constant, but then there is no way to implement e_H.

2. The model

(4) Implementing high effort with risk aversion

- Consider fixed $\hat{w} : \frac{1}{v'(\hat{w})} = \gamma$ (optimal without incentive constraints).
  - $w(\pi) > \hat{w}$ if $\frac{f(\pi|e_L)}{f(\pi|e_H)} < 1$.
  - $w(\pi) < \hat{w}$ if $\frac{f(\pi|e_L)}{f(\pi|e_H)} > 1$.
  - The optimal contract compensates the manager more at profit realisations that are statistically more likely with high effort.
  - The gap between $f(\pi|e_L)$ and $f(\pi|e_H)$ determines the extent of deviation from fixed wage.

2. The model

(4) Implementing high effort with risk aversion

- For the compensation contract to be monotonically increasing in π, $\frac{f(\pi|e_L)}{f(\pi|e_H)}$ has to be decreasing in π.
  - This condition is called the *monotone likelihood ratio property (MLRP)*. It implies that high profits are relatively more likely with high effort than low profits.
  - It is not guaranteed by first-order stochastic dominance.
  - Hence, non-monotone compensation contracts are possible in this model. Compensation contracts here are complicated.
- Given the variability in wages, the expected wage here is higher than under observable effort (where wage is fixed). Formally,
  - Under observable effort, the wage is $v^{-1}(\overline{u} + g(e_H))$.
  - Here, $E[v(w(\pi))] = \overline{u} + g(e_H)$.
  - By Jensen's inequality, because $v'' < 0$, $v(E[w(\pi)]) > E[v(w(\pi))] = \overline{u} + g(e_H) \Rightarrow$ $E[w(\pi)] > v^{-1}(\overline{u} + g(e_H))$.

2. The model

(4) Implementing high effort with risk aversion

- The main conclusion is that providing incentives to a risk averse manager to choose high effort when effort is unobservable is costly.
  - If the owner chooses to implement the high effort when effort is unobservable, he pays more than when effort is observable.
  - Given that the manager always gets his reservation utility, the solution under unobservable effort is always inferior to that under observable effort.
    - The owner gets a lower utility, and the manager gets the same utility.

2. The model

(4) Implementing high effort with risk aversion

- In some cases, moving to unobservable effort will be so costly that it will lead to a shift from high to low effort.
  - The owner picks the level of effort to implement by comparing the difference in expected profits between high effort and low effort with the difference in the associated compensation cost.
  - Relative to the case of observable effort, nothing is changed except that the cost of wage to implement high effort increases.
  - Hence, it is possible that due to the non-observability of effort, there will be a shift from high to low effort.

2. The model

(4) Implementing high effort with risk aversion

- Important:
  - Non-observability of effort is a problem even though in equilibrium the principal knows exactly what effort the agent is choosing.
    - He designs a contract that ensures that the agent is choosing a particular level of effort.
  - Yet, the level of effort is not observable and cannot be contracted upon.
  - The contract has to ensure it will be desirable and this is costly.

2. The model

(5) Additional information

- The analysis demonstrates that non-observability of effort is costly.
- Since effort is generally believed to be impossible to observe in most settings, the analysis goes on to consider other signals and their ability to improve the allocation of profits and risks.
- Suppose that in addition to π, both parties observe a signal y. The condition becomes:

$$\frac{1}{v'(w(\pi, y))} = \gamma + \mu \left[1 - \frac{f(\pi, y|e_L)}{f(\pi, y|e_H)}\right]$$

2. The model

(5) Additional information

- $\frac{f(\pi, y|e_L)}{f(\pi, y|e_H)}$ may now change with y. Hence, for the same level of profit π, the agent may receive different wages for different levels of y.
  - The observation of y provides more information about whether the agent chose the desired action, and thus conditioning the wage on y helps provide incentive without harming risk sharing.
  - For example, y can represent average profits in the industry which generate changes in π that are beyond the control of the manager. Thus change in π that are associated with changes in y should not affect wages much.

2. The model

(5) Additional information

- When exactly will wages depend on y in addition to π?
    - When π is not a *sufficient statistic* for y with respect to e.
    - Formally, we can write:

$$f(\pi, y|e) = f_1(\pi|e)f_2(y|\pi, e)$$

    - When $f_2(y|\pi, e)$ does not depend on e (π is a sufficient statistic) it will cancel out, and the wage will not depend on y.
    - When y doesn't add information on the effort (e.g., it is pure noise), there is no reason to condition w on it and add variation.