A Comparison of Timber Market Models: Static Simulation and Optimal Control Approaches

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ABSTRACT. In this paper, we compare and contrast two types of timber models that have been used to analyze the market impacts of policy proposals or exogenous forces that affect timber markets. The framework and theory for static simulation and optimal control models are presented and discussed. We then compare single region, empirical versions of the models across six scenarios of exogenous economic shocks. The models are found to predict different outcomes for timber market behavior when demand changes, or when young timber is affected by a supply shock. Similar outcomes between the models are obtained when older timber stocks are affected by shocks, or when the exogenous forces impact timber markets gradually over time. For. Sci. 44(1):24–36.
Additional Key Words: Forest sector models, optimal control, timber market.

In recent years, policy analysis in timber markets has become more complex as the list of human induced influences on forested ecosystems has grown. Issues such as population growth, forest conversion to agricultural lands in the tropics (see, for example, Skole and Tucker 1993), sustainable harvest practices, old growth harvests, acid rain and climate change (Haynes and Kaiser 1991, Winnett et al. 1993, Joyce 1995, Sohngen 1996), waste disposal and paper recycling (Ince 1994), and forest health (Sampson and Adams 1994) are among the growing list of exogenous factors that are (or may be) affecting timber markets. For analysts to make informed policy decisions, they must understand how these factors affect timber markets.

To date, forest sector models often have been employed to provide information on how markets will respond to these forces. In this paper, forest sector models refer to a class of models that capture the analytical relationship between the growth of timber and the demand for wood products at a national or global level. Nonmarket uses of forests are not considered within the objective functions of the forest sector models considered in this paper. The first forest sector models relied on "gap" analysis, which attempted to determine likely demands and likely supplies. Because population was expected to continue increasing, gaps would occur between demand and supply. Recently, models have been more closely tied to economic theory. Some examples include TAMM (Adams and Haynes 1980), the Center For International Trade in Forest Products Global Trade Model (CGTM, described in Kallio et al. 1987), and the global Timber Supply Model (TSM, Sedjo and Lyon, 1990).

While there are numerous differences between these models, two irreconcilable differences relate to how the models are solved and how they treat timber supply. Static simulation models, such as TAMM and CGTM, solve for annual harvests and prices by maximizing each period's consumer plus producer surplus. Supply is modeled through a function based on prices and total timber inventory. Optimal control models, TSM for example, solve for the maximum net present value of consumer's plus producer's surplus. Timber supply is determined as a result of a complex intertemporal adjustment.

These differences are important in at least two different situations. Harvesting forests as they transition from old growth to steady state, Faustmann type forests is one (Faustmann forests are defined as those that are maintained in equal age classes and harvested at the same optimally chosen rotation age). The optimal control model provides a framework for handling this transition directly (Lyon 1981). A second situation arises when Faustmann forests are exposed to exogenous factors such as those discussed above. Demand adjustments, or large fires, for example, may instantaneously

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Acknowledgments: This research was undertaken while Sohngen was Gilbert White Postdoctoral Fellow at Resources For the Future. The authors thank Richard Pierson and two anonymous reviewers for many helpful comments, although they remain responsible for any errors.

Manuscript received March 25, 1996. Accepted November 19, 1996.

change the age distribution of timber, while acid rain or climate change may slowly affect both the age distribution of timber and the volume on each hectare. Basic differences between the static simulation and the optimal control models may lead to disparate predictions about market behavior in response to exogenous forces.

In this paper, we compare and contrast static simulation and optimal control approaches to timber modeling. Our main emphasis is on comparing how the approaches model timber supply. We begin by tracing the theoretical development of the models, and by comparing analytical results. We then present an empirical analysis that compares both models under six scenarios of exogenous forces. While the scenarios are implemented in a stylized fashion, they are chosen to represent changes that may currently be affecting timber markets.

Theoretical Components of Forest Sector Models

The emphasis in this section is to examine the theoretical background of static simulation and optimal control forest sector models. It is important to note, however, that the forestry economics literature has a long tradition of articles that explore issues of timber supply and demand. Perhaps the most noted example is the seminal piece written by Martin Faustmann (1849). More recently, several good reviews of the timber supply literature are contained in Samuelson (1976), Newman (1988), and Ovaskainen (1992). Two articles with thorough reviews of the timber demand literature include Adams and Haynes (1980) and Newman (1987).

Static Simulation Timber Market Models

The static simulation timber market models have their roots in the literature on spatial market structures pioneered by Samuelson (1952). One of the first of these, TAMM (Adams and Haynes 1980), models the spatial structure of markets in the United States by considering separate demand and supply regions. That model recognizes the importance of the transportation costs necessary to move products from manufacturing facilities to demand centers. Because the most productive forests are located remotely from the urban centers where most wood is demanded, transportation costs are an important component in the overall value of wood.

A spatial market model attempts to maximize consumer's plus producer's surplus minus the costs of transporting products to other markets. In any period, then, the objective is to maximize

$$\sum_i \int_0^{Q_i^D} D(Q_1 \ldots Q_I)dq_i - \sum_j \int_0^{Q_j^S} S(Q_1 \ldots Q_J)dq_j - \sum_i \sum_j C_{i,j}Q_{i,j} \tag{1}$$

subject to

$$Q_i^D \leq \sum_j Q_{i,j} \tag{2}$$

$$Q_j^S \geq \sum_i Q_{i,j} \tag{3}$$

Q_i^D is the quantity demanded, Q_j^S is the quantity supplied, $D(\cdot)$ is the demand function, $S(\cdot)$ is the supply function, i is the demand region, j is the supply region, and $C_{i,j}$ is the cost of transporting from region i to j. As suggested by Adams and Haynes (1987), this type of model does not need to have a social welfare interpretation, although it implicitly maximizes the yearly value of net market welfare, minus transportation costs.

Although this spatial representation concentrates solely on the end-product markets, the TAMM model recognizes that both end-product and timber markets must simultaneously be in equilibrium. Derived demand curves for timber can be developed from the production function (Adams and Haynes 1980, Abt 1987, and Newman 1987). Timber supply can be estimated based on prices and total timber inventory (Adams and Haynes 1980, Newman 1987, and Newman and Wear 1993).

The following specification for the timber stumpage market is often used in forest sector models:

$$Q^D(t) = f(P_s(t), P_z(t), k(t)) \tag{4}$$

$$Q^S(t) = g(P_s(t), Inv(t)) \tag{5}$$

where $P_s(t)$ is the price of stumpage, $P_z(t)$ price of final products, $k(t)$ is the capacity to produce lumber and plywood, and $Inv(t)$ is the total timber inventory. The set of prices and harvests that equates (4) and (5) in any period can be solved if $P_z(t)$ and $Inv(t)$ are known. TAMM solves $P_z(t)$ simultaneously with additional equations that characterize end product markets. Although studying end product markets is interesting, we consider only the stumpage market level, represented by (4) and (5), in the rest of this paper. $Inv(t)$ is determined in any period with the following growth-drain equation:

$$Inv(t) = Inv(t-1) - Harvest(t) + Growth(t) \tag{6}$$

With equations (4), (5), and (6), future timber market behavior can be predicted by equating supply and demand in each period. A demand projection can be used to control $P_z(t)$ and shift the demand function. Supply shifts according to $Inv(t)$. Policy analysis involves either adjusting demand or supply conditions in the initial period, or gradually over time.

Optimal Control Timber Market Models

Optimal control timber market models are rooted in the theory of renewable and nonrenewable resources (Hotelling 1931, Solow 1974). Historically, timber has been harvested from old-growth stocks, which are essentially nonrenewable resources. Optimal control models show how benevolent social planners utilize stocks of natural resources over time. If timber markets operate efficiently, the social planner's solution will be the same as that achieved in a competitive market. Berck (1979, 1981), Lyon (1981), and Lyon and Sedjo (1983) showed how these dynamic models could be tied directly to timber markets. They showed how prices would adjust over time in a way that maximizes the net present value of the net surplus (consumer's plus producer's) in the market, thereby reflecting the scarcity of the remaining old growth stock.

In this section we present a continuous time, optimal control model of the forest sector. The objective of this model posits that a benevolent social planner attempts to maximize the net present value of net surplus in timber markets. Net surplus is defined as the difference between the area under a timber stumpage demand function and the costs of managing and regenerating timberland, as well as land rental costs associated with holding land in timber. It is defined only over marketed values of timber products, without considering the many non-market values associated with forests.

The social planner's problem is thus

$$\underset{H(t)}{Max}\int_0^{\infty} e^{-rt} \left\{ \int_0^{Q^*(t)} \{D(Q(H(t),V(a)))\}dQ - bG(t) - R(t)X(t) \right\} dt \tag{7}$$

$D(\cdot)$ is a downward sloping demand function for timber stumpage. Because we are considering timber stumpage markets directly, harvesting and transportation costs are included directly in the stumpage price. $H(t)$ is the number of hectares harvested, $V(a)$ is the yield function, a is the age of the timber harvested, $Q(H(t),V(a))$ is the total quantity harvested. The costs of management are assumed to be known at the moment of regeneration, and are capitalized into the costs of regeneration, b (Lyon and Sedjo 1983). Although it is possible to optimize over regeneration and management costs (as in Lyon and Sedjo 1983), in this model, we assume that they are constant. $G(t)$ is the number of hectares replanted, and $X(t)$ is the total number of hectares in the forest. While $X(t)$ formally represents the total size of the forest, the yield function associated with each hectare of timberland allows us to differentiate age classes. $R(t)$ represents land rent, or the capital cost of maintaining land in timber. It is the opportunity cost associated with holding land in timber rather than allowing it to flow to some other use.

The total size of the forest will vary over time according to:

$$\dot{X} = -H(t) + G(t) \tag{8}$$

While land that is harvested for timber purposes is often replanted, Equation (8) allows for the possibility that some land will not be, and it allows for the possibility that additional land will enter forests. Replanting decisions for a new rotation are separated from the harvesting decision in this model because of the long time lags involved before the next harvest.

The decision to regenerate timberland is determined through an additional condition placed on the system. Assuming that the social planner uses rational expectations, she will replant when the present value of future timber rotations on a piece of land are greater than the opportunity cost of doing something else with the land, $LV_{alt}(t)$. This occurs when

$$\sum_{n=1}^{\infty}\left\{P_S(t_n)V_i(t_{n-1})e^{-r(t_n-t_{n-1})} - b\right\}e^{-r(t_{n-1}-t_0)} \geq LV_{alt}(t) \tag{9}$$

$LV_{alt}(t)$ is the value of land in the next best alternative to forestry, $P_s(t)$ is the stumpage price, n is the rotation number, and the difference $t_n - t_{n-1}$ is the length of the rotation in question. If land is most profitable in forestry, it will remain in forestry; if it is more profitable elsewhere, it will convert to something else. In this model, we assume that land supply is inelastic, and that the value of land initially in forests remains higher than the alternative uses, so that Equation (9) is not binding.

Equations (8) and (9) constrain the maximization given in (7). Two additional pieces of information must be used to solve the problem First, we must be given a yield function that is concave, initial values for $X(0)$, $P_s(0)$, b, and r, and an initial age distribution over $X(0)$. Second, we must assume that $H(t)$, $G(t)$, *and* $P_s(t)$ are greater than or equal to 0 in every period. With this, the problem can be defined in terms of a current value Hamiltonian (Kamien and Schwartz 1981):
Letting

$$W(H(t),V(a)) = \int_0^{Q(t)} \{D(Q(H(t),V(a)))\}dQ$$

$$\mathcal{H} = W(H(t),V(a)) - bG(t) - R(t)X(t) + \mu(t)[-H(t)+G(t)] \tag{10}$$

Utilizing the maximum principle (Pontryagin et al. 1962), the following first-order conditions are derived:

$$W_H(t) - \mu(t) = 0 \tag{11}$$

$$\dot{\mu} - r\mu(t) = -H_X \tag{12}$$

$$\lim_{t\to\infty} \mu(t)H(t) = 0 \tag{13}$$

where the final equation is the transversality condition. With $-H_x = R(t)$, Equations (11) and (12) can be combined and rewritten to obtain

$$\frac{\dot{W}_H}{W_H(t)} = r + \frac{R(t)}{W_H(t)} \quad (14)$$

The social planner will harvest so that the marginal benefit of harvesting rises faster than the rate of interest. This is due to the final term, which is a stock effect. Because there is an opportunity cost (land rent) involved with holding land in timber, the marginal benefits from harvesting additional timberland must increase faster than the rate of interest. The marginal benefits depend on the yield of timber and the demand function.

A more intuitive representation of (14) can be obtained by proposing a specific functional form for demand:

$$P_S(t) = D(Q(H(t), V(a))) = \alpha - 2\beta Q(H(t), V(a)) \quad (15)$$

α and β are estimated parameters. Taking the integral of (15) over timber harvests, and noting in the result that $Q(\cdot)=H(t)V(a)$, we determine

$$W(H(t), V(a)) = k + \alpha(H(t)V(a)) - \beta(H(t)V(a))^2 \quad (16)$$

where k is the constant of integration. Equation (16) corresponds to (10) above, as it is the total area under the demand curve, and it is measured in terms of the choice variable $H(t)$ and the yield function. Differentiating (16) with respect to $H(t)$ determines

$$W_H = \alpha V(a) - 2\beta H(t)V(a)^2 = (\alpha - 2\beta H(t)V(a))V(a) \quad (17)$$

which is the marginal value of harvesting an additional hectare of forestland. Differentiating (17) with respect to $V(a)$ gives the marginal value of a unit of timber. In a competitive marketplace, the marginal value of a unit of timber is equal to the stumpage price:

$$P_S(t) = \alpha - 2\beta(H(t)V(a)) \quad (18)$$

Substituting (18) into (17), and then placing the result in (14), we find the following condition:

$$\dot{P}_S V(a) + P_S(t)\dot{V} = rP_S(t)V(a) + R(t) \quad (19)$$

Equation (19) must be satisfied over all time if the social maximization is to be achieved. It expresses Equation (14) in terms of the price of timber and the yield function. The first term on the left hand side of (19) is the marginal benefit of an increase in price (it is a marginal cost if prices are decreasing), and the second term is the marginal benefit of another moment of timber growth. The first term on the right-hand side is the opportunity cost of not harvesting this moment, and the second term is the marginal cost of holding land in timber for one more moment. A hectare of timber will be harvested as long as the marginal benefits of waiting an extra moment to harvest are just equal to the marginal costs.

This equation has several interesting properties. First, when both the demand function and the stock of land are constant (i.e., steady state), the system will resolve to the same rotation as if timberland managers were all acting like Faustmann entrepreneurs given price (Brazee and Mendelsohn 1990). Assuming prices and the age of the marginal tree harvested stabilize at $\overline{P}$ and $\overline{a}$, the steady state is expressed in terms of:

$$\frac{\dot{V}}{\overline{V}} = r + \frac{\overline{R}}{\overline{P_S}\overline{V}} \quad (20)$$

Trees are thus harvested when they are growing at the rate of interest plus the stock effect.

In transition, Equation (19) can be used to model the harvesting of old growth, as well as the transition around shocks to a steady state system. The old-growth condition is met when tree stands no longer accumulate harvestable timber (Oliver and Larson 1990). Some trees will continue to grow, others will stop growing, and still others may die altogether. Mathematically, this occurs when the net growth of timber on each hectare approaches 0, or when $\dot{V} \approx 0$. Equation (19), then, can be rewritten as

$$\frac{\dot{P}_S}{P_S(t)} = r + \frac{R(t)}{P_S(t)V(a)} \quad (21)$$

Prices will rise faster than the rate of interest if there is land rent. If, as was the case when the settlers first arrived on the North American continent, there is no land rent, $R(t) = 0$, and prices rise exactly at the rate of interest. Over time, the stock of timber will decline, and competition for land will increase, thereby increasing land rent, which also signals landowners to replant. The depletion of the old growth stock will continue as we harvest successively younger trees. $\dot{V}$ then becomes greater than 0, and ultimately, prices will begin to follow,

$$\frac{\dot{P}_S}{P_S(t)} = r + \frac{R(t)}{P_S(t)V(a)} - \frac{\dot{V}}{V(a)} \quad (22)$$

Assuming demand is constant, prices will rise, but at a slower and slower rate until they have achieved the steady state. Thus, the old-growth case is really just a special case of Equation (19).

Equation (19) suggests that only the oldest timber stocks will be harvested, a result shown by Heaps (1984). Intuitively, this makes sense, because the marginal costs of waiting an extra moment are greatest in the older stocks due to the high volume per hectare. The marginal benefits, on the other hand, are the lowest because the stock is growing slowly (due to concavity in the yield function).

Within a broader scope, the continuous time model described here differs from the discrete time model of Lyon and Sedjo (1983) and Sedjo and Lyon (1990). One difference is that their model assumes that harvests and regeneration occur jointly. Their result balances the net marginal benefits of harvesting and replanting today with the marginal costs of waiting an extra year to harvest and replant (including the effects on all future rotations). In the model above, harvesting and regenerating decisions are separated, and land rent is determined exogenously. While landowners weigh the marginal costs and benefits of waiting an additional moment to harvest, the exact specification of those values differs from Lyon and Sedjo's model. Marginal benefits are determined by price and volume growth, while marginal costs are determined by the opportunity costs of foregoing harvests today, and land rental costs associated with holding that land in timber for another moment.

In this model, we have not attempted to optimize over regeneration investment, as in Lyon and Sedjo (1983), or silvicultural regimes, as in Johnson and Scheurman (1977). The capitalized costs of management activities over the life of a stand are added to the costs of regeneration, and they are included in b. Stand treatments in the intervening period before harvest are assumed to add no additional volume to the total quantity harvested in any year. While considering investment alternatives is important for timber analysis, the main focus in this paper is to compare the basic behavior of supply between the static simulation and the optimal control models.

There are at least two other differences between the resulting model of timber management in this section and that of others. First, we account only for the marketed benefits, and second, we provide no mechanism for harvesting multi-cohort stands. Alternative formulations of the problem (such as described in Johnson and Scheurman 1977, Calish et al. 1978, and Hartman 1976), or additional constraints, would allow for different harvesting schedules and multicohort stands. Alternative models or formulations, while important, represent strategies that are not the focus of this paper.

Similarities and Differences Between the Models

The most apparent difference between the two types of models is that the static simulation models have been developed with multiple market layers, that is, they describe the vertical market for forest products, and simultaneously solve for equilibrium between supply and demand at each market level. Although Lyon et al. (1987) developed an optimal control model that solved both market levels simultaneously (by solving endogenously for both capital investment and timber harvest), other modelers have considered only the timber stumpage or log markets. One reason may lie in the computational burden of solving all time periods simultaneously.

An equally important difference is how the models treat timber supply. The optimal control models adjust according to the amount of timber in economically mature age classes. Harvest begins with the oldest stock and continues until the marginal opportunity cost of waiting an additional moment to harvest an additional hectare just equals the marginal benefit. By solving all time periods simultaneously, optimal control models will adjust inventories in a forward-looking manner according to a rational expectation's derived path of price and harvest.

A similar harvest rule does not result from the algorithms used to simulate behavior in static simulation models. Instead, static simulation models determine the total timber quantity harvested, without reference to which age classes are harvested. The modeler must then choose a harvest rule which may include only the oldest trees, or which may include a host of trees from different ages. Depending on the particular harvest rule chosen, the static simulation models may produce a very different adjustment over time, as this will affect the inventory adjustment.

Suppose, for example, that an exogenous supply shock kills younger timber inventories. This impact will affect the early phase of the transition in the static simulation model as the supply curve shifts inward. Because plenty of timber exists in merchantable age classes immediately following the supply shock, in the optimal control model, consumers and producers will react at the moment of the shock, but their reaction will appear muted compared to the static simulation model.

A Comparison of Behavior

In this section, we compare the behavior of static simulation and optimal control models over six different scenarios of external forces affecting timber markets. Both static simulation and optimal control forest sector models have been used in the literature for this type of policy analysis (see, for example, Montgomery et al. 1994 and Sedjo and Lyon 1990). Rather than using the large-scale forest sector models for this analysis, we instead develop and compare single region models, based on the theoretical models described above. This allows us to eliminate many differences between the forest sector models, such as differing demand elasticities or regional definitions, and it allows us to focus on how the models characterize timber supply.

This section is concerned specifically with how the models project the transition from an initial to a final steady state after an exogenous shock has affected the market. Six scenarios are considered (Table 1). While these scenarios are implemented in a highly stylized fashion, they are broadly representative of forces that currently are affecting timber markets, or may be expected to affect timber markets in the future. Table 1 presents only a few examples of many potential shocks.

The models are compared based on how they predict the market transition to the exogenous forces. A stumpage market is considered, and in each case it is assumed to be closed (i.e., there is no trade with other regions) and competitive. The future paths of stumpage price, harvest, and inventory are determined endogenously for each model, although only price and inventory are shown in the results (harvest paths can be inferred from those of price). Inventory is measured as the sum of all merchantable timber within the forest; it is reported as an aggregate number. Both the static simulation and the optimal control models keep track of inventories by age class, but the age class distributions are suppressed in this analysis.

We begin by characterizing the initial conditions. Both models and model parameters are calibrated so that they are resting at the same initial steady state. The same demand

Table 1. Description and examples of the six scenarios.

Scenario 1	
Instantaneous demand shock	• Demand increases 20% in the first yr and stabilizes. • Energy shock or opening new markets. • New substitutes or recycling would cause a decrease in demand.
Scenario 2	
Young timber dieback	• 20% of timber aged 10–20 yr dies back in the first yr. • All damaged land is replanted instantly to the same species. • Fire or insect damage in young timber.
Scenario 3	
Old timber dieback	• Disturbance destroys all timber 27 yr and older in the first yr. • All disturbed land is replanted instantly to the same species. • Wind, fire, or insect damage in old timber.
Scenario 4	
Slow demand increase	• Demand increases 0.5% each yr for 40 yr and then stabilizes. • Population or economic growth.
Scenario 5	
Slow yield increase	• Timber growth rates increase 0.5% each yr for 50 yr and then stabilize. • Climate change or acid rain (may cause increase or decrease in growth).
Scenario 6	
Slow increase in timber area	• Timberland increases 50,000 ha/yr for 50 yr and then stabilizes. • New timberland is added to the first age class. • Land use change, such as reforestation from agricultural lands. • Suburbanization (athough it would cause a decrease in area).

function is used for both models, and in the initial steady state, it is assumed to be constant over time. There are 16 million ha of timberland, and the forest stock is maintained in 32 equal age classes of 500,000 ha each. Interest rates are assumed to be 4%, and land supply is assumed to be inelastic in the initial steady state.

The demand function for each model is

$$Q^D(t) = 476.69 - 11.92 * P(t) \tag{23}$$

Annual harvests are 238.35 million cubic meters (MMm^3), prices are \$20 per cubic meter (m^3), and the initial demand elasticity is 1.0. Since we consider only a timber stumpage market, this is a derived demand function, and other factors of production are assumed constant. The yield function is derived from a typical stand of southern pine, with the following functional form:

$$\ln(V(a)) = 7.82 - (52.9 / \alpha) \tag{24}$$

where a is the age of the timber. $V(a)$ is the yield of merchantable timber in m^3 per hectare.

The models are distinguished by their specification of timber supply. The supply function for the static simulation model is given as a function of price, $P(t)$, and total timber inventory (I_t),

$$Q^S(t) = -85.52 + 2.21 * P(t) + 0.112 * I(t) \tag{25}$$

Under the steady state conditions, this implies an initial price elasticity of supply of 0.186, and an inventory elasticity of 1.18. The parameter values in Equation (25) were determined by approximating elasticity values found in the literature (Adams and Haynes 1980, and Newman 1987) in order to approximate behavior currently observed in timber markets. Given an initial inventory, $P(t)$ and $Q(t)$ can be determined in the first period by solving (23) and (25). This maximizes consumer's plus producer's surplus in the initial period. Equation (6) then provides a new inventory level for the next period. While no generalized derivation of harvesting behavior exists for the static simulation model, we assume that harvests begin with the oldest timber first, and continue until the demand and supply functions are equilibrated.

The demand function [Equation (23)] is used directly in the optimal control model described in the last section. It conforms to the specification given in Equation (15). The resulting empirical model is a dynamic system that involves harvesting the oldest timber until prices and harvest quantities in each period determined by (19) and (23) are equated. Land is assumed to be regenerated immediately following harvest. The resulting time path maximizes the net present value of net surplus. Annual supply is therefore determined endogenously. The initial steady state in this model produces the same price and harvest conditions as the static simulation model.

Scenario 1: Instantaneous Demand Shock

In the first scenario, we assume that demand instantly increases 20%. Such a demand increase may correspond to energy shocks or opening new markets (for example, finding new uses for timber). The introduction of new technologies or substitute resources (such as new building materials or a paperless society) would cause the opposite adjustment in demand. This case is implemented by adjusting the constant in the demand function to 572.03 in the initial period, and holding it constant at this level throughout the model run. No other parameters in either model were adjusted. Prices, inventories, and harvests consequently adjust to their new steady state levels. The adjustment period is defined as the length of time it takes for prices and inventories to stabilize at a new steady state level.

Figure 1 shows the price and inventory schedules for both models under this shock. Prices initially are \$20/m3, and in both cases they jump upwards at the time of the shock, t^*. In the static simulation model, the initial price increase is lower, but prices continue rising for some time, and they eventually stabilize at a higher level (\$0.64) than in the optimal control model. In the optimal control model, prices increase to their new steady state level rapidly, and are maintained there.

Forests are harvested sooner in the static simulation model than in the optimal control model for the instantaneous demand increase. This results in declining inventories in the static simulation model, but fairly stable inventories in the optimal control model. The steady state rotation age in the optimal control model drops to 31.65 yr. This is not a large change in a, and it affects Equation (22) only slightly, which explains the smaller adjustments in both price and inventory relative to the static simulation model. In that model, the new steady state harvest age is 30.26 yr. Both of these harvest levels are sustainable over the long run, but only the optimal control model enters a steady state consistent with rotations predicted by the Faustmann formula [Equation (22)] at the given price. In the static simulation model, net present value of the forest is far from optimal because landowners are harvesting too soon.

Scenario 2: Young Timber Dieback

Here, a discrete shock to the supply of timber occurs where 20% of the timberland in age classes 10 to 20 instantaneously dies back. We assume that this is a one-time, unexpected event; that all timber on the land affected dies back; that all land with dieback is replanted instantly into the same timber type; and that there is no salvage associated with the dieback. The event occurs in the first year and stocks subsequently adjust toward the steady state level. Such a shock may be caused by fires or insect infestations that selectively impact these younger to middle age classes.

Although both models experience an instantaneous jump in prices, the initial price adjustment is greater in the static simulation model (Figure 2). The supply function in the static simulation model immediately reflects the inventory loss and shifts inward. Prices continue to increase for 10 yr as the proportion of older age classes with the full 500,000 ha declines and inventory declines. After peaking, prices decline rapidly as the land that was regenerated after the impact ages and increases in volume.

In the optimal control model, prices jump up initially because consumers and producers foresee both the coming

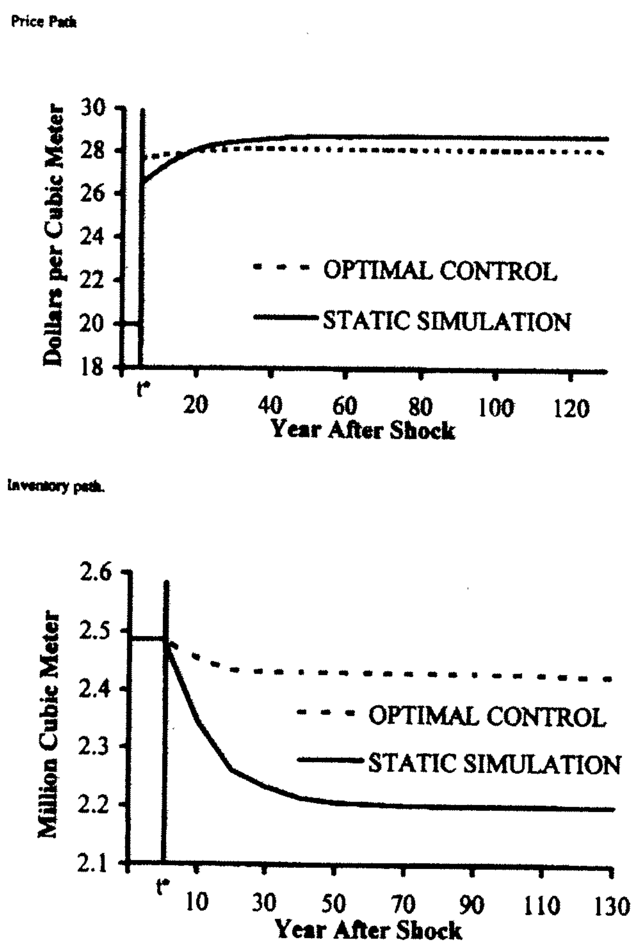


Figure 1. Comparison of price and inventory paths for the instantaneous demand shock scenario.

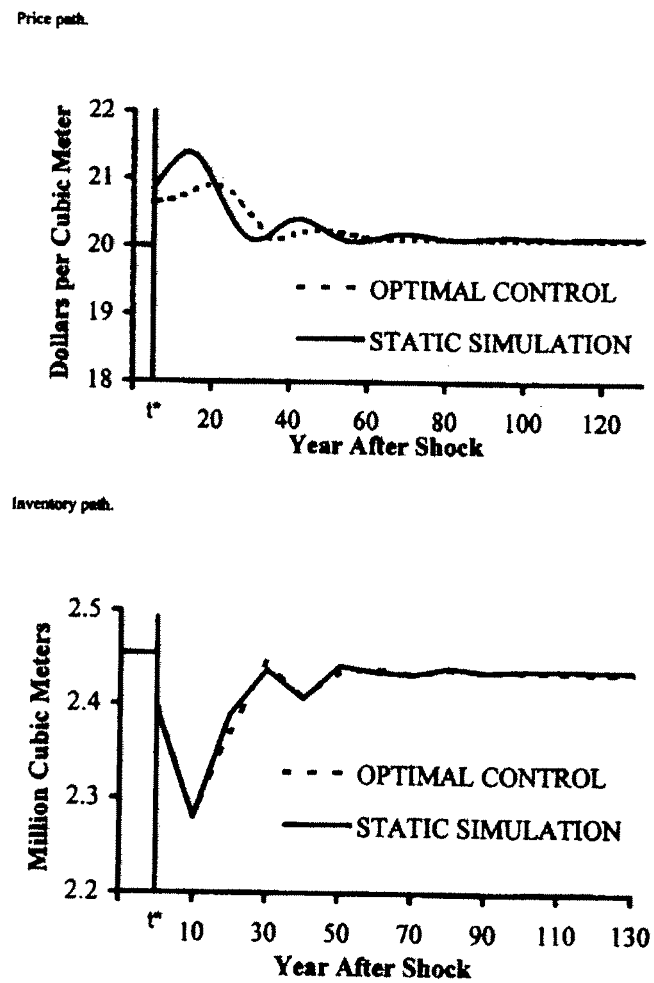


Figure 2. Comparison of price and inventory paths for the young timber dieback scenario.

shortage of timber in merchantable age classes, as well as the surplus of timber that will be available as the regenerated hectares mature. Prices, however, do not peak as they do in the static simulation model. Harvests are heavier in early periods in the optimal control model, partly because there is plenty of timber available in merchantable age classes. Landowners are making marginal adjustments each year that effectively move stocks from lower valued periods to higher valued periods.

Producers, for example, know that prices will be higher after 10 yr when timber becomes relatively scarce, so they harvest less than in the initial steady state in the first few periods. This pushes timber inventories into future, more highly valued periods, which smooths the transition across the shortfall in inventories that occurs when the affected age classes are mature. Consumers benefit because they do not encounter a price "spike" of the size shown in the static simulation model. When the inventory shortfall has been crossed, supply increases and prices decline. As the damaged hectares approach maturity, producers begin to harvest more heavily to take advantage of prices that are higher than prices in the new steady state. The optimal control model captures efficient intertemporal allocation of resources, as producers and consumers arbitrage between current and future periods using the interest rate to assess the marginal value of timber stocks in each period.

Scenario 3: Old Timber Dieback

In the third scenario, an exogenous environmental damage kills the oldest five age classes of timber. Such an event may be caused by large storms, such as hurricanes or other windstorms, fires, or insect infestations. Here, in contrast to scenario 2, it is the oldest, most mature stands that are affected. The following assumptions are made: all timber in age classes older than 27 yr dies back, landowners do not salvage, no one expects this event, and landowners replant the land immediately to the same timber type.

The price and inventory paths (Figure 3) are similar in both models. Prices jump initially, and then decline to their steady state level (which is the same in both models). Inventories jump down initially, but return to their original steady state level. Although not great, the differences become more pronounced after year 20, when age classes regenerated after the dieback event have accumulated enough merchantable timber to affect the supply function in the static simulation model. As these age classes mature, they push the supply function out, and prices begin to decrease more rapidly than in the optimal control model.

Scenario 4: Slow Demand Increase

The slow demand increase scenario incorporates a demand function that shifts out for 40 yr at 0.5% annually, and then stabilizes. Demand is thus 22% greater in the final steady state than in the initial. Such a change may be caused by population or economic growth. Stabilization may result from increased substitution possibilities that exist with higher price levels.

An interesting difference in the price paths occurs for the two models (Figure 4). The static simulation model suggests

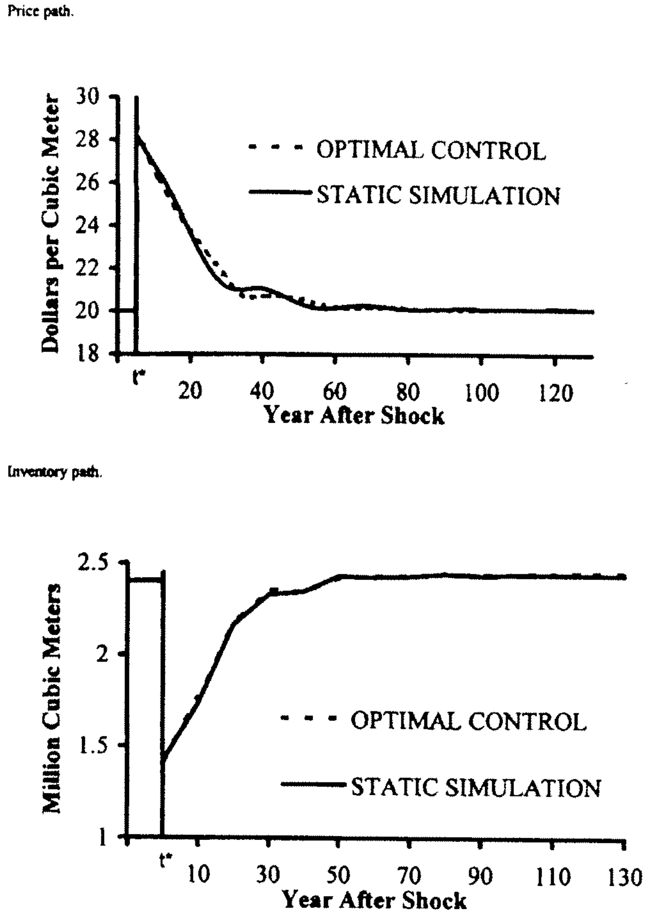


Figure 3. Comparison of price and inventory paths for the old timber dieback scenario.

a slight initial price decrease, and then a gradual increase until about year 40, when prices begin to stabilize rapidly. The optimal control model, on the other hand, predicts a larger initial increase in prices, but a more gradual adjustment after that. Prices ultimately stabilize at a level lower than in the static simulation model (for reasons outlined in scenario 1 above).

In the optimal control model, producers hold on to timber for a little longer when prices are rising. Timber rotations are extended during the transition period, a result also found by Berck (1981) and Newman et al. (1985). This reduces yearly harvests in the first period, so prices jump up initially. Older inventories, however, ultimately increase the yearly supply of timber, so that prices rise slower than in the static simulation model. In this way, optimal control models generally will predict slower price growth during periods of demand growth.

By comparing the models around the year 40, when demand stabilizes, one can also see the effect of perfect foresight on the optimal control model. In the static simulation model, consumers and producers do not recognize in advance that demand will stabilize, so they are caught by surprise and a big shift in price growth occurs in year 40. In the optimal control model, however, future demand conditions are considered in earlier periods, so that the transition to stable prices is smoother.

There are large differences in the inventory transition and the steady state inventory between the two types of models

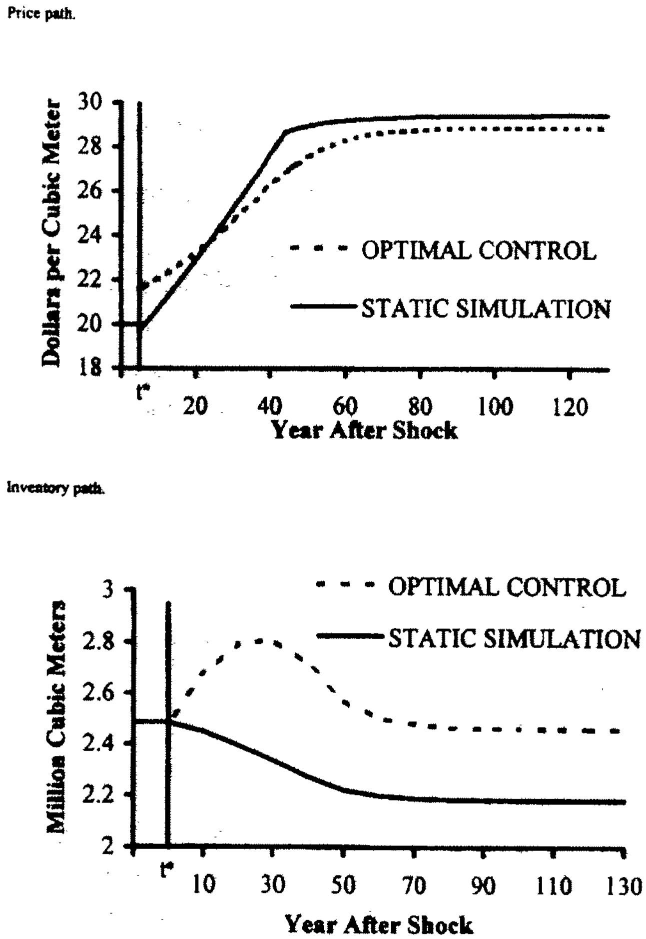


Figure 4. Comparison of price and inventory paths for the slow demand increase scenario.

(Figure 4). Inventory levels decrease to a lower steady state level throughout the adjustment in the static simulation model. This is consistent with the results of scenario 1 where demand instantaneously increased 20%. As consumers demand more and more timber, and prices rise, the model does not allow producers to adjust by limiting the drawdown of inventories.

In the optimal control model, however, inventory levels increase initially, and then decline. The increase is caused by the higher rotation ages that occur during periods of increasing prices. Producers realize that prices are rising, so there is some advantage associated with holding timber longer. Inventories begin declining to their initial steady state level before demand stabilizes due to the perfect foresight conditions. Landowners realize that future prices will stabilize, so they begin taking advantage of higher prices earlier, rather than waiting until the last moment.

Scenario 5: Slow Yield Increase

The slow yield increase involves a gradual 1.0% annual increase in the growth rate of timber. Gradual adjustments like this may result from exogenous forces such as global warming, where many ecological models predict increased timber growth over time (Joyce 1995 and Sohngen 1996). A slow decrease in yield, although not considered as an example in this paper, could be caused by acid precipitation (Haynes and Kaiser 1991). This change is implemented as an adjustment to the current yearly increment to growth. At t years after the change in growth occurs, the yearly increment is:

$$\dot{V}(a)adjusted = (1 + 0.01t)\dot{V}(a)_{initial} \qquad (24)$$

This adjustment continues for 50 yr, and then the yearly growth increment stabilizes at a level 50% greater than initially. The inventory, however, will continue to adjust because any timber older than the first age class in year 50 does not incorporate the full effect of the new steady state growth rate until it has been harvested and regenerated.

As yield increases, timber supply expands and prices decline in both the static simulation and the optimal control models (Figure 5). Several interesting differences in behavior occur, however, although the price paths do not differ greatly. In the static simulation model, prices decline slowly at first from their original level of \$20/m^3, but then more rapidly after about 30 yr. The more rapid decline in later periods results from the combined influence of increased growth rates, and the impact of those growth rates on younger age classes. Increased growth in younger age classes have a direct effect on current price and harvest through the supply function. Prices continue to fall until about year 70, well after the gradual increment in growth has stopped (year 50). This

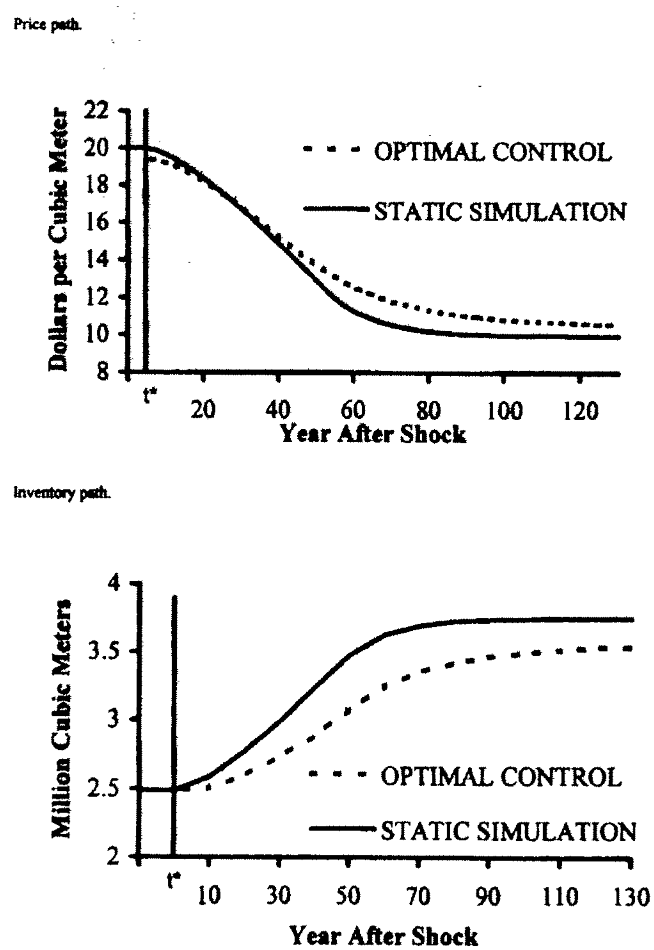


Figure 5. Comparison of price and inventory paths for the slow yield increase scenario.

occurs because only trees planted after year 50 will have the full change in yield associated with the adjustment.

For the optimal control model, prices jump downward at the start of the transition period. Harvests are heavier in early periods for two reasons. First, consumers and producers realize that the new steady state Faustmann rotation age is younger than previously. Second, they realize that additional timber will be available in future periods because yield functions are increasing. Higher yields in future periods will increase annual supply and depress prices. Owners therefore attempt to take advantage of the relatively higher prices today by harvesting more heavily today. Over the long term, prices decline steadily, but at a slower rate than in the static simulation model.

The inventory adjusts upward in both models, but it increases more rapidly and to a higher level in the static simulation model (Figure 5). A difference in steady state rotation ages causes the disparity in the steady state level of inventory. The static simulation model holds timber above the optimal Faustmann rotation age for the final steady state price level, which increases the standing stock of timber. The slower trend in the change in inventory for the optimal control model arises because that model holds the oldest age class near its optimal rotation age throughout the transition.

Scenario 6: Slow Increase in Timber Area

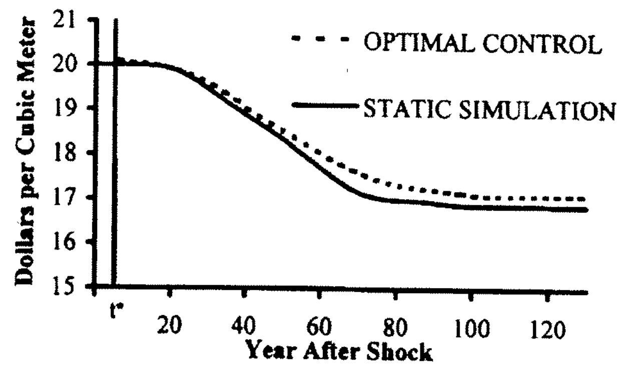
In this final example, we consider potential increases in the area of forests due to replanting or natural regeneration. Such changes occurred throughout the United States in the last century as land in the northeastern and southeastern United States converted from agriculture back to forestland (Powell et al. 1993). These types of changes are important in a global sense today, as plantation forests continue to arise in different regions (Sedjo 1995). We implement this scenario by allowing 50,000 additional hectares to regenerate for each of the first 50 yr of the simulation. No additional land enters after that. This increases the total land base by 2.5 million ha.

Prices in both models decline throughout the transition, and ultimately enter a lower steady state level (Figure 6). In this scenario, the price paths are similar, with a couple of notable exceptions. In the static simulation model, for example, prices remain stable for the first 25 yr before they begin to decline. Although new land is added at the start of the simulation, it has no immediate impact on timber inventory. Only after this area begins to accumulate merchantable timber does it impact total inventory, and therefore, the price path.

In contrast, prices in the optimal control model initially increase, before they begin to decrease to the new steady state. This result may seem odd because supply ultimately increases, but it stems from the long-term changes in timber inventories that occur. In the long term, the optimal rotation period will be slightly extended, so markets begin to adjust instantly by reducing harvests and increasing prices in the initial period.

Inventories follow similar paths during the transition (Figure 6), but they resolve to different steady state levels. The adjustment in the optimal control model is slower, however, as that model holds harvested hectares near their Faustmann rotation age throughout the transition. The static simulation model harvests land at older rotation ages, resulting in a larger steady state inventory.

Price path.



Inventory path.

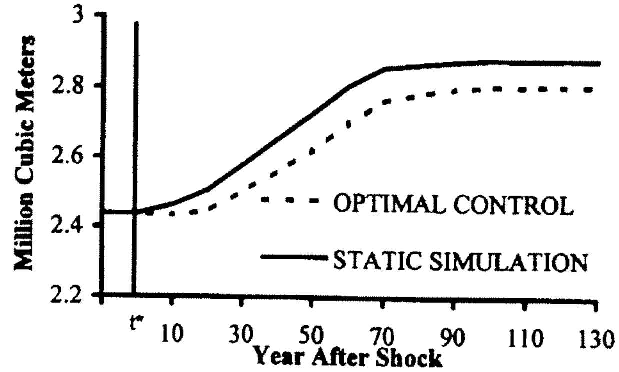


Figure 6. Comparison of price and inventory paths for the slow increase in timber area scenario.

Discussion and Conclusion

This study began with a description of the theory behind two types of forest sector models. The paper then compared two empirical versions of these models in a single region, closed timber market. Price and inventory paths were compared over six potential exogenous forces that either have affected or are (or possibly will be) affecting timber markets.

The comparisons in Table 2 characterize broad differences among the models. The theoretical section examined the first three points. Both model types maximize net surplus. The optimal control model, however, maximizes the net present value of all period's net surplus, whereas the static simulation model solves for the maximum net surplus only one period at a time. Solving all periods simultaneously introduces perfect foresight into optimal control models, while solving static simulation models in a period-by-period fashion uses no foresight at all. As shown in the empirical comparison above, these can lead to differences in timber market behavior.

The last four points summarize the results of the empirical comparison. Point (4) shows the difference in how timber supply is characterized in each model. The timber

Table 2. Points of comparison.

	Optimal control	Static simulation
(1) Objective function	Maximize net present value of welfare in all periods	Maximize welfare in each period independently
(2) Solution technique	All periods simultaneously	Period-by-period
(3) Foresight	Perfect	None
(4) Inventory important to current timber supply	Mature timber	Arbitrary subset of all age classes
(5) Harvest age	Optimally determined by model	Determined by assumption
(6) Final steady state	Even-aged Faustmann rotations	Even-aged rotations (not necessarily Faustmann)
(7) Sustainable long term harvests	Yes	Yes

supply function in the static simulation model shifts in and out over time, depending on the arbitrarily chosen set of age classes used to determine total timber supply. The optimal control model, on the other hand, looks forward and determines the entire inventory adjustment at once, in response to a timber shock. Currently mature timber classes are most important for determining current harvests and prices. As point (5) suggests, the optimal control model limits how deeply timber owners are willing to harvest into their existing stock. In some cases, the harvest mechanism will limit the welfare benefits of harvesting in the current period if that action reduces future benefits and the net present value of all period's benefits. The static simulation model, on the other hand, will harvest as much timber stock as necessary to maximize welfare in a particular period, unless the modelers introduce other constraints. Static simulation models do not account for the effects of today's harvests on future prices.

The optimal control model always leads to a Faustmann forest in the final steady state, while the static simulation model does not necessarily do so (Table 2, point 6). Both models will lead to even-aged, sustainable forests (Table 2, point 7), but the forests will have different rotation lengths. This last point may be closely related to the small-scale models utilized in the empirical section, however. In large-scale models, a static simulation model may adjust to a Faustmann forest by allowing for additional land to flow to or from forests, or by shifting harvests from one region to another. Faustmann forests in this case are not the result of optimizing behavior.

Differences between the model predictions of price and harvest range from distinct to nearly imperceptible. The most distinct differences occur in the two demand cases, both in the transition path and the final steady state. The differences result mainly from the choice of harvest age for timber. The inventory adjustment for the slowly increasing demand case may be of particular interest. A static simulation model suggests that forest inventories are depleted as demand continues to increase, whereas an optimal control model would have the opposite prediction.

Differences are also apparent when exogenous forces affect younger age classes. In the static simulation model, behavior is affected by the supply function, which relies on inventory that is measured over the entire distribution of timber ages. Although behavior in the optimal control model changes immediately when only young timber is affected, a different mechanism is responsible: perfect foresight. In addition, the forward looking nature of the optimal control model tends to smooth the transition from the initial conditions to the final steady state. In contrast, very little difference between the model responses is observed in the gradual changes of scenarios (5) and (6), and in the old timber dieback case.

Several implications of these results may be of interest to modelers and policy makers alike. First, differences in price paths resulting from the large-scale, forest sector models described in the introduction may be more distinct, or more limited, depending on the exact specification of the exogenous forces. For example, most forest sector models have many regions and timber types; it is likely that the differences resulting from supply adjustments may be limited by the inclusion of additional regions or timber types. Another example occurs when policy applications include a combination of the slowly increasing demand and some other exogenous force (many modelers incorporate the idea that increases in population and economic growth will continue to increase demand at least for some time to come). Since the differences in the slow demand scenario are so distinct, this may enhance other differences noted above.

Second, policy makers should pay careful attention to the specific demand projections utilized and the terminal conditions proposed for the optimal control models. While optimal control models may be expected to have lower price growth than static simulation models in general, another factor that limits the rate of price growth in the optimal control model is the assumption that demand stabilizes after year 40. If instead, demand was projected to increase continuously (still at 0.5% annually), prices would grow more quickly. Under this different demand projection, decisions to replant, or other policy decisions, may be altered. This issue is specific mainly to the optimal control models. Because the static simulation model is not forward looking, the price trend for the first 40 yr would not be different under a scenario where demand increased past 40 yr.

Third, policy makers should also carefully consider the inventory projections. Differences are the most apparent in the two demand scenarios. Under the slowly increasing demand scenario, the optimal control model suggests that landowners will harvest at greater ages than initially if prices are expected to increase, while the static simulation model predicts the opposite. The result is a different trend in inventory entirely. While this paper does not attempt to prove which path is correct, understanding landowner responses to changing prices should be an important focus in future research. In particular, additional emphasis should be placed on empirical models that determine how inventories in general have behaved in response to historical demand (accounting for other factors such as old growth drawdown and land use change).

Finally, researchers and policy makers should be clear on how they implement policy scenarios. They should put significant effort into determining exactly how the exogenous force will impact the market, and implement it appropriately. Our analysis has concentrated mainly on the differences between the two types of models, but another look at the results reveals that there are distinct differences within each model type depending on how the exogenous force was implemented. For example, scenarios (2) and (3) can be caused by the same type of disturbance, such as a fire, but the impacts predicted for each are quite different, for either the static simulation or the optimal control models. Incorrect scenario specification may cause large errors in the resulting policy analysis.

Clearly, each model type (and model types yet developed) will have its place in the forestry research and policy making community. Neither is superior to the other. This paper has attempted to show how one basic difference between the models, the way they model timber supply, may affect timber market projections under different scenarios. In some cases, the models behave similarly, while they behave differently in others. Modelers and policy makers alike must carefully consider the models they choose and how they implement scenarios for policy analysis on timber market behavior.

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