Advanced Microeconomics

Lecture 3: Utility and decision theory under risk and uncertainty

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Outline

1. Rational choice under uncertainty
   - Risk and uncertainty
   - Expected utility theory
   - Money lotteries and risk aversion
   - Arrow-Pratt measures

2. Example
   - Village-level risk insurance model
   - Intertemporal consumption smoothing model

Outline

1. Rational choice under uncertainty
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2. Example
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1. Risk and uncertainty

- So far: Decision makers act in a world of absolute certainty.
- New: Economic decisions most often involve uncertainties.
- Question: How do individuals decide when the consequences of their decisions are uncertain?
- Examples:
  - Studies (what will career prospects look like in 5-year's time)
  - Purchase of goods of unknown quality (drugs)
  - Investment
  - Research...
- Virtually no decisions are made under certainty.

1. Risk and uncertainty

(1) Definition: Simple gamble

- The finite set $A = \{a_1, \dots, a_N\}$ includes N elements, indexed as $n = \{1, \dots, N\}$.
- A lottery (or gamble) describes the probabilities with which these outcomes occur.
- A set of simple lotteries is defined as a probability distribution

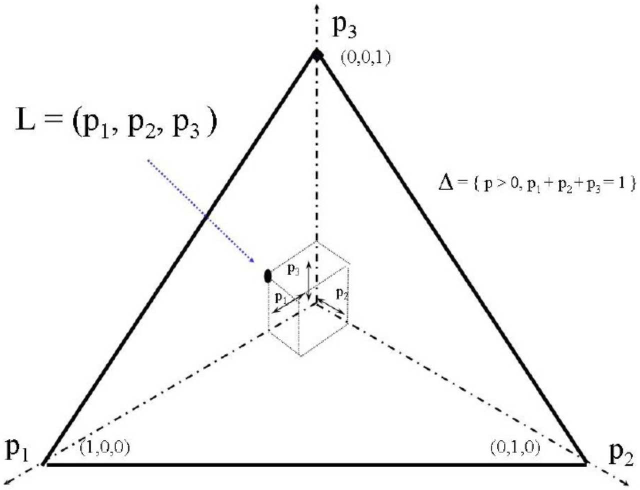
$$G_s = (p_1 \circ a_1, \dots, p_N \circ a_N) \quad \text{with } p_N \geq 0 \quad \text{and} \sum_{i=1}^{N} p_i = 1$$

  where $\mathbf{p}$=probability vector; $p_i \circ a_i \equiv$'a_i occurs with probability p_i'.
- G_s contains A. Why?
  - For each i, it contains $1 \circ a_i \equiv a_i$, the gamble yields a_i with certainty.

1. Risk and uncertainty

(1) Definition: Simple gamble

n-dimensional unit simplex: $\triangle^{n-1}$.



1. Risk and uncertainty

(1) Definition: Simple gamble

- Example 1: Uncertain investments
  - Investment I: cost RMB 1000; 80% return RMB 2000, 20% return 0.
  - Investment II: cost RMB 500; 50% return RMB 2000, 50% return 0.
- Example 2: Dice game
  - Throw I: You receive RMB 1000 if the die is cast yielding either 6 or 1; RMB 0 for all other numbers.
  - Throw II: You receive nothing if the die is cast yielding 6; RMB 400 for all other numbers.

Describe the set of all outcomes $\mathbf{X}$, the two lotteries g_1 and g_2, and calculate the expected values for both lotteries.

1. Risk and uncertainty

(1) Definition: Compound gamble

- Gambles whose prizes are themselves gambles (can be infinitely nested).
- Example: effective probability of realising a_1

$$\text{A } \textbf{compound gamble} \text{ yields: } \begin{cases} a_1, & \text{with prob. } \alpha \\ \text{a lottery ticket}, & \text{with prob. } 1-\alpha \end{cases}$$

$$\textbf{Lottery ticket}\text{: a simple gamble yielding } \begin{cases} a_1, & \text{with prob. } \beta \\ a_2, & \text{with prob. } 1-\beta \end{cases}$$

Effective probability of a_1 is:

$$\overbrace{\alpha}^{\text{directly}} + \overbrace{\underbrace{(1-\alpha)}_{\text{prob. lottery}}\ \beta}^{\text{from lottery ticket}}$$

Effective probability of realizing a_2 is $(1-\alpha)(1-\beta)$

1. Risk and uncertainty

(1) Definition: Compound gamble

- Assumption: Compound gambles result in some a_j after $N < \infty$ randomisations.
- Notation:

$$\begin{aligned} g = p_1 \circ g_1 + p_2 \circ g_2 &= (p_1, p_2) \\ \text{where } g_1 &= (p_{11}, p_{12}) \\ g_2 &= (p_{21}, p_{22}) \end{aligned}$$

1. Risk and uncertainty

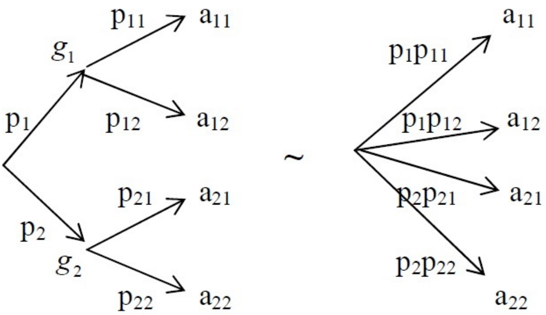
(1) Definition: Compound gamble

Combined lottery

$$g = (p_1 \circ g_1, p_2 \circ g_2)$$

Reduced lottery

$$p_1 \circ g_1 \oplus p_2 \circ g_2 =$$
$$(p_1 p_{11}, p_1 p_{12}, p_2 p_{21}, p_2 p_{22})$$



Recursively define *G*?

1. Risk and uncertainty
(3) Preferences over lotteries

- Individuals have a preference relation $\succeq$ over gambles G_s.
- $\sim$ & $\succ$ denote indifference & strict preference relations induced by $\succeq$.
- We assume that the agents have a preference ordering $\succeq$ on G_s, which has the same characteristics as the preference ordering for ordinal utility theory.
- If a preference ordering on G_s satisfies the von Neumann-Morgenstern Axioms 1-4, then a continuous utility function $v : G_s \to \mathbb{R}$ exists such that

$$g \prec g' \Leftrightarrow v(g) < v(g')$$

2. Expected utility theory

(1) The St. Petersburg Paradox

We will now look at a lottery game using different amounts of money y_n; i.e., all the elements in the result set A are amounts of money y_n, $n = 1, \ldots, N$.

- Throw the coin until heads comes up.
- If heads comes up on the n-th throw, the prize is RMB 2^n.
- What RMB amount would you pay to participate in this lottery?

2. Expected utility theory

(1) The St. Petersburg Paradox

- The expected value of this lottery is:

$$\sum_{n=1}^{\infty} p_n y_n = \sum_{n=1}^{\infty} \frac{1}{2^n} 2^n = \sum_{n=1}^{\infty} 1 = \infty$$

- Although the expected payoff of this lottery is infinitely large, gamblers were not willing to pay a large sum of money to participate in it (St. Petersburg Paradox).
- This indicates that individuals do not assess a lottery solely on the basis of the expected payoff.
- The Basler mathematician Daniel Bernoulli proposed a solution to the St. Petersburg Paradox in the 18th century.
- He postulated that money has a diminishing marginal utility.
- He suggested a utility function with the following characteristics: $u(y)$ where $u'(y) > 0$ and $u''(y) < 0$.

2. Expected utility theory

(1) The St. Petersburg Paradox

- The value that individuals assign to this money lottery is then

$$\sum_{n=1}^{\infty} p_n u(y_n)$$

- This expression is called **expected utility.**
- If one calculates the expected utility of this lottery for the utility function $u(y) = y^{0.5}$, one arrives at

$$\sum_{n=1}^{\infty} \frac{1}{2^n}(2^n)^{0.5} \approx 2.414$$

- What if different entry fees?

2. Expected utility theory

(2) von Neumann-Morgenstern utility

- Bernoullis' suggestion that individuals maximise not the *expected value* of a monetary payoff but instead its *expected utility* corresponds to the following expected utility theory:
- The preferences of *rational* individuals over lotteries can be described by the utility function $u : G \to \mathbb{R}$, such that:

$$g \succeq g' \Leftrightarrow \sum_{i=1}^{N} p_i u(x_i) \geq \sum_{i=1}^{N} p'_i u(x_i)$$

- Often a utility function that possesses an expected utility form is called a 'von Neumann-Morgenstern' utility function (vNM).
- As we have seen, the expected utility form was already suggested in the 18th century by Daniel Bernoulli.
- We shall return to Bernoulli later.

2. Expected utility theory

(3) Axioms of choice under uncertainty

- We shall discuss assumptions that allow us to represent a preference ordering $\succeq$ on G using the von Neumann-Morgenstern utility function.
- These axioms were formulated by John Lewis von Neumann and Oskar Morgenstern (1944) in the classic, 'Theory of Games and Economic Behavior'.
- G1: Completeness: For any two gambles $g, g' \in G$, either $g \succeq g'$ or $g' \succeq g$.
- G2: Transitivity: For any three gambles $g, g', g'' \in G$, if $g \succeq g'$ and $g' \succeq g''$, then $g \succeq g''$. (Can rank all elements of A.)
- Note: We now consider preferences over the set G (the set of all lotteries on A), and not preferences over the set A. A is a subset of G.

2. Expected utility theory
(3) Axioms of choice under uncertainty

- G3: Continuity. For all $g, g', g'' \in G$ with $g \succeq g' \succeq g''$, there are some probabilities $\alpha, \beta \in [0, 1]$ and $\alpha \geq \beta$ such that

$$\alpha \circ g + (1 - \alpha) \circ g'' \succeq g' \succeq \beta \circ g' + (1 - \beta) \circ g''$$

  - Another loose form of Continuity?
  - Intuition: There is no lottery that a gambler would avoid at all costs.

2. Expected utility theory

(3) Axioms of choice under uncertainty

Example: Safari

The death of the hunter is result x''; the slain lion in the Serengeti is result x; The dead duck in the bathtub is result x'.

- Given $x \succeq x' \succeq x''$, Axiom G3 demands that α and β exist such that

$$\alpha \circ x + (1 - \alpha) \circ x'' \succeq x' \succeq \beta \circ x' + (1 - \beta) \circ x''$$

- A safari with only a very small mortality risk is preferred to a safari in the bathtub with no mortality risk.
- True?

2. Expected utility theory

(3) Axioms of choice under uncertainty

- G4: Independence. For all $g, g', g'' \in G$ and all probabilities $\alpha \in [0, 1]$,

$$\alpha \circ g + (1-\alpha) \circ g'' \succeq \alpha \circ g' + (1-\alpha) \circ g''$$

- The term $(1-\alpha) \circ g''$ should not influence the decision as it is identical for both lotteries.
- Recall IIA?
- True?

2. Expected utility theory

(3) Axioms of choice under uncertainty

- G5: Substitution: If
  - $g = (p_1 \circ g^1, \ldots, p_k \circ g^k)$, $h = (p_1 \circ h^1, \ldots, p_k \circ h^k) \in G$, and
  - $h^i \equiv g^i$ for every i;
  - then $h \equiv g$.
- Intuition: Fixing p, indifference over outcomes implies indifference over gambles.
- Remark: If $h \equiv g$, then since $g \equiv g$, we have

$$(\alpha \circ g, (1-\alpha) \circ h) \equiv (\alpha \circ g, (1-\alpha) \circ g) = g$$

- True?

2. Expected utility theory

(3) Axioms of choice under uncertainty

- G6: Reduction to Simple Gambles: If $g \in G$ induces $g_s \in G_s$, then

$$g_s = (p_1 \circ a_1, \ldots, p_N \circ a_N) \equiv g$$

- Intuition: It does not matter how the gamble is presented (simple or more complex).
- Act of gambling is irrelevant: An individual is indifferent between a combined lottery and its reduced lottery (see the previous Figure).
- Recall IIA?
- True?

2. Expected utility theory

(4) Allais Paradox

- A choice problem designed by Maurice Allais (1953) to show an inconsistency of actual observed choices with the predictions of expected utility theory.
  - Allais, M. (1953) Le comportement de l'homme rationnel devant le risque: Critique des postulats et axiomes de l'école Américaine. *Econometrica* 21 (4): 503-546.
- One can construct experiments that reveal that real people do not always satisfy these assumptions. And yet, despite this experimental failure, expected utility theory is used everywhere in Economics.
- How to construct/utilise counterfactuals?

2. Expected utility theory

(5) Ellsberg Paradox

- People's choices violate the postulates of subjective expected utility.
- It is generally taken to be evidence for ambiguity aversion.
- The paradox was popularised by Daniel Ellsberg (1931 -), although a version of it was noted considerably earlier by John Maynard Keynes.

2. Expected utility theory

(6) Definition of expected utility

- Axioms G1-G6 allow us to represent a the preference ordering of $\succeq$ on G using a continuous utility function that is linear in probabilities:

$$u(g) = \sum_{i=1}^{n} p_i u(x_i)$$

- We say that a utility function of this kind possesses the characteristics of expected utility.

2. Expected utility theory

(6) Definition of expected utility

Definition

The utility function $u\colon G \rightarrow \mathbb{R}$ has the expected utility property if

$$u(g) = \sum_{i=1}^{n} p_i u(x_i) \quad \text{for every} \quad g \in G,$$

where $g_s = (p_1 \circ a_1, \ldots p_n \circ a_n)$ is the simple gamble induced by g.

Remarks:

- $u(a_i) \equiv u(1 \circ a_i)$, i.e., the utility of outcomes $a_i \in A$.
- $u(\cdot)$ is continuous and linear with respect to the probabilities p_i.
- $u(g_s) = u(g)$ because both g & g_s induce the simple gamble g_s.

2. Expected utility theory

(7) Proposition of expected utility

1. Proposition (existence of vNM): If a preference ordering $\succeq$ on G satisfies Axioms $G1 - G6$, then a utility function $u : G \rightarrow \mathbb{R}$ exists, such that

$$g \succeq g' \Leftrightarrow u(g) = \sum_{i=1}^{n} p_i u(x_i) > u(g') = \sum_{i=1}^{n} p_i' u(x_i)$$

2. Proposition (uniqueness): The vNM-utility function $u(.)$ represents the preference ordering $\succeq$ on G. The vNM-utility function $v(.)$ represents the same preferences on the strict condition that constants α and $\beta > 0$ exist such that for all $v(g) = \alpha + \beta u(g)$.
    - Recall homothetic preferences & homothetic utility functions.

2. Expected utility theory

(8) Example: Constructing a vNM utility

Let $A = \{a_1, a_2, a_3\} = \{\$10, \$4, -\$2\}$, with obvious ranking.

- Based on the existence of vNM utility, we ask the individual
  - What probability distribution makes you indifferent between (i) a best-worst gamble and (ii) the outcome a_i with certainty?
  - Must find the probability $u(a_i)$ of the best-worst gamble.
- Consider $i = 1$: probability of best outcome **must be** 1, i.e., $u(\$10) = 1$.
- Consider $i = 3$: probability of worst outcome **must be** 0, i.e., $u(-\$2) = 0$.
- Consider $i = 2$: suppose we find the probability of best outcome is 0.6, i.e., $u(\$4) = 0.6$.

2. Expected utility theory

(8) Example: Constructing a vNM utility

$$\text{Summary:} \begin{cases} \$10 \sim (1 \circ \$10, 0 \circ -\$2), & \text{with } u(\$10) = 1, \\ \$4 \sim (.6 \circ \$10, 0.4 \circ -\$2), & \text{with } u(\$4) = .6, \\ -\$2 \sim (0 \circ \$10, 1 \circ -\$2), & \text{with } u(-\$2) = 0. \end{cases}$$

- Now can rank all gambles involving A. For instance,
  - $g_1 = (.2 \circ \$4, .8 \circ \$10)$
  - $g_2 = (.07 \circ -\$2, .03 \circ \$4, .9 \circ \$10)$
- Since $u(g) = \sum_{i=1}^{n} p_i u(a_i)$, we have
  - $u(g_1) = .2u(\$4) + .8u(\$10) = .92$
  - $u(g_2) = .07u(-\$2) + .03u(\$4) + .9u(\$10) = .918 < u(g_1)$
  - g_1 is the preferred gamble.

3. Money lotteries & risk aversion

Question

How can we measure an individual's willingness to assuming risk?

1. Certainty equivalent.
2. Risk premium.
3. Arrow-Pratt measure of risk aversion.

3. Money lotteries & risk aversion

- We will now consider lotteries for amounts of money x_i, $i = \{1, \ldots, n\}$.
- A vNM-utility function $u(x)$ measures the utility that x units of money yield.
- Monotony: It holds for $x, x' \in X$ with $x > x'$, that $u(x) > u(x')$.
- Let $E(g)$ be the expected value of the lottery g: $E(g) = \sum_{i=1}^{n} p_i x_i$.
- Let $u(g)$ be the expected utility of the lottery g: $u(g) = \sum_{i=1}^{n} p_i u(a_i)$.

3. Money lotteries & risk aversion

- An agent is risk-neutral if

$$u(E(g)) = u(g)$$

- An agent is risk-averse if

$$u(E(g)) > u(g)$$

- An agent is risk-loving if

$$u(E(g)) < u(g)$$

3. Money lotteries & risk aversion

Example 1:

- Consider $\$4 \sim (.6 \circ \$10; .4 \circ -\$2)$. Average or expected payoffs are $E(\$4) = \$4 < E(.6 \circ \$10; .4 \circ -\$2) = .6 \times \$10 + .4 \times (-\$2) = \$5.2$.
- Similarly
  - $E(g_1) = E(.2 \circ \$4, .8 \circ \$10) = .2 \times \$4 + .8 \times \$10 = \$8.8$.
  - $E(g_2) = E(.07 \circ \$2, .03 \circ \$4, .9 \circ \$10) = \$8.98$.
  - g_1 is preferred to g_2 even if $E(g_2) > E(g_1)$.
- We say that the individual prefers to avoid risk.
- Risk aversion implies $u(E(g_1)) = u(\$8.8) > u(g_1) = 0.92$ and $u(E(g_2)) = u(\$8.98) > u(g_2) = 0.918$.

3. Money lotteries & risk aversion

Example 2: Consider that the government issues a new fair-game lottery. You could buy this *$*10 lottery & you have 50%-50% game with payoffs of *$*0 and *$*20. Will you buy this lottery? Note: There is no utility derived from the states of world *per se*; all that matters is the payoff in each state.

- You are risk-averse if you won't buy this instant
  - $U(\$10) > EU(invst) = 0.5 \times \$0 + 0.5 \times \$20 = \$10$
  - $U'' < 0$, concave utility function
- You are risk-loving if you buy this lottery
  - $U(\$10) < EU(invst)$
  - $U'' > 0$, convex utility function
- You risk-neutral if you are indifferent whether to buy or not
  - $U(\$10) = EU(invst)$
  - $U'' = 0$

3. Money lotteries & risk aversion

- Certainty equivalent: Let u be a vNM utility function for X and g a lottery with the expected value $E(g)$. The certainty equivalent $CE(g)$ of the lottery g is defined as

$$u(CE(g)) = u(g) = Eu(X)$$

- $CE(g)$ is the assured monetary amount which makes the individual indifferent between lottery g and $CE(g)$.

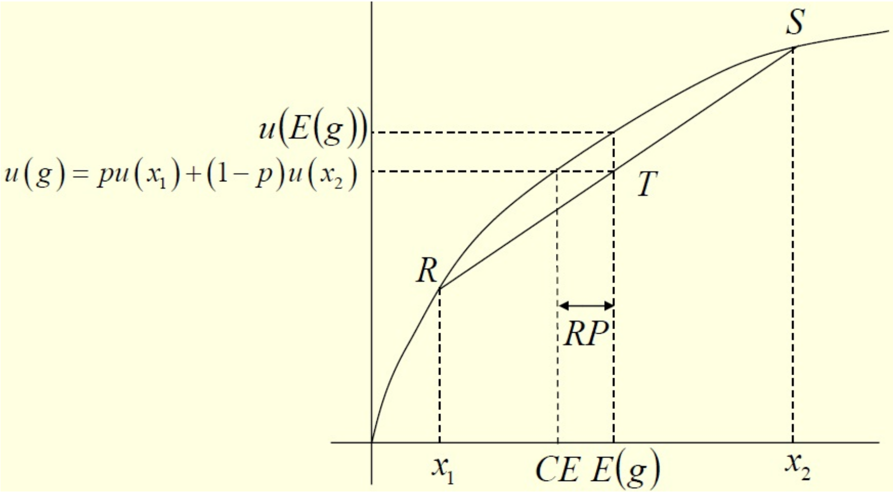
3. Money lotteries & risk aversion

- Risk premium: The risk premium $RP(g)$ is the difference between the expected value of the lottery $E(g)$ and the certainty equivalent $CE(g)$:

$$RP(g) = E(g) - CE(g)$$

- The risk premium is that monetary amount which an individual is willing to pay so as to certainly gain the expected value of the lottery instead of the lottery.
- Risk aversion implies that the risk premium is positive; i.e., $RP(g) = E(g) - CE(g) > 0$.
- Let us assume that for all lotteries g, $E(g) \succ g$ holds. This is exactly the case when the utility function is strictly concave.
- This can be shown graphically for lotteries with two prices x_1 and x_2.

3. Money lotteries & risk aversion



4. Arrow-Pratt measures

(1) Absolute risk aversion

- Let $u(\cdot)$ be strictly monotonically increasing, concave and twice possess continuous derivations.
- Absolute risk aversion is a measure for a small additive risk.
- The Arrow-Pratt measure of absolute risk aversion is:

$$R_A(x) = -\frac{u''(x)}{u'(x)}$$

- Proof?
- $R_A(x)$ is also called the coefficient of absolute risk aversion.
- The greater the absolute risk aversion, the smaller the certainty equivalent.

4. Arrow-Pratt measures

(1) Absolute risk aversion

1. Constant absolute risk aversion (CARA)

$$R_A'(x) = 0$$

2. Diminishing absolute risk aversion (DARA)

$$R_A'(x) < 0$$

3. Increasing absolute risk aversion (IARA)

$$R_A'(x) > 0$$

4. Arrow-Pratt measures

(1) Absolute risk aversion

Examples

- CARA:

$$u(x) = -e^{rx}; u'(x) = re^{-rx}; u''(x) = -r^2 e^{-rx}$$
$$R_A(x) = -\frac{u''(x)}{u'(x)} = r$$

- DARA:

$$u(x) = r\ln(x); u'(x) = \frac{r}{x}; u''(x) = -\frac{r}{x^2}$$
$$R_A(x) = -\frac{u''(x)}{u'(x)} = \frac{1}{x}$$

4. Arrow-Pratt measures

(2) Relative risk aversion

- The coefficient of relative risk aversion is

$$R_R(x) = -\frac{xu''(x)}{u'(x)} = xR_A(x)$$

- If an agent has a constant relative risk aversion and we double the prices of a lottery, then the agent's risk premium also doubles.

4. Arrow-Pratt measures

(2) Relative risk aversion

Examples:

- CRRA:

$$u(x) = Ax^{1-\rho}; u'(x) = (1-\rho)Ax^{-\rho}; u''(x) = -\rho(1-\rho)Ax^{-\rho-1}$$

$$R_R(x) = -\frac{xu''(x)}{u'(x)} = \rho$$

- CRRA:

$$u(x) = r\ln(x); u'(x) = \frac{r}{x}; u''(x) = -\frac{r}{x^2}$$

$$R_R(x) = -\frac{xu''(x)}{u'(x)} = 1$$

Outline

1. Village-level risk insurance model

(1) Model set-up

Assumptions

- Villages in rural areas: A village typically consists of several or even one small and informal communities. Information flows are rich and a common shock is known to all villagers.
- A village in which Pareto-efficient allocation of risk is achieved, but no access to credit or storage (no saving).
    - $i \in (1, 2, \ldots, N)$ households;
    - $t \in (1, 2, \ldots, T)$ periods;
    - $s \in (1, 2, \ldots, S)$ states of nature with prob. of occurrence π_s;
    - in state s, each household receives an income $y_{ist} > 0$;
    - household consumption at time t and state s is c_{ist}.

1. Village-level risk insurance model

(1) Model set-up

Household problem

- Suppose each household has a separable and additive utility function:

$$U_{it} = \sum_{t=1}^{T} \beta^t \sum_{s=1}^{S} \pi_s u_i \left(c_{ist} \right) \text{ where } u' \left(\cdot \right) > 0, u'' \left(\cdot \right) < 0$$

- A Pareto-efficient allocation of risk within the village can be found by maximising the weighted sum of utilities of each of the N households, where the weight of i in the Pareto programme is λ_i satisfying $0 < \lambda_i < 1$ and $\sum \lambda_i = 1$.

1. Village-level risk insurance model

(1) Model set-up

$$\max_{c_{ist}} \left(\sum_{i=1}^{N} \lambda_i \sum_{t=1}^{T} \beta^t \sum_{s=1}^{S} \pi_s u_i \left(c_{ist} \right) \right)$$

$$\text{s.t.} \quad \sum_{i=1}^{N} \sum_{t=1}^{T} \beta^t \sum_{s=1}^{S} \pi_s c_{ist} = \sum_{i=1}^{N} \sum_{t=1}^{T} \beta^t \sum_{s=1}^{S} \pi_s y_{ist}$$

$$c_{ist} \geq 0, \ \forall i, s, t.$$

$$(\text{s.t.} \quad \sum_{i=1}^{N} c_{ist} = \sum_{i=1}^{N} y_{ist}, \ \forall s, t.)$$

1. Village-level risk insurance model

(2) Solution

- Lagrangian multipliers μ_i, $i \in (1, 2, \ldots, N)$:

$$\mathcal{L} = \sum_{i=1}^{N} \lambda_i \sum_{t=1}^{T} \beta^t \sum_{s=1}^{S} \pi_s u_i \left(c_{ist}\right) + \sum_{i=1}^{N} \mu_i \sum_{t=1}^{T} \beta^t \sum_{s=1}^{S} \pi_s \left(y_{ist} - c_{ist}\right)$$

- FOCs for any i, j at any t:

$$\frac{d\mathcal{L}_i}{dc_{ist}} = \lambda_i \beta^t \pi_s \frac{du_i\left(c_{ist}\right)}{dc_{ist}} - \mu_i \beta^t \pi_s = 0$$
$$\frac{d\mathcal{L}_j}{dc_{jst}} = \lambda_j \beta^t \pi_s \frac{du_j\left(c_{jst}\right)}{dc_{jst}} - \mu_j \beta^t \pi_s = 0$$

- Hence:

$$\mu_i = \lambda_i \frac{du_i\left(c_{ist}\right)}{dc_{ist}}, \ \mu_j = \lambda_j \frac{du_j\left(c_{jst}\right)}{dc_{jst}}$$

1. Village-level risk insurance model

(2) Solution

- $\mu_i = \mu_j$ (?)
- This leads to:

$$\frac{u_i'(c_{ist})}{u_j'(c_{jst})} = \frac{\lambda_j}{\lambda_i}, \ \forall i, j, s, t.$$

- This holds across N households in the village.
- Higher MU (lower consumption) -> smaller weight.
- The marginal utilities and therefore consumption levels of all households in the village move together.
  - Consumption of an individual household co-moves with the village-level consumption.

1. Village-level risk insurance model

(3) Pareto-efficient allocation

- MU is a monotonically increasing function of the average MU of households in any state.

$$u_j'\left(c_{jst}\right) = \frac{\lambda_i}{\lambda j} u_i'\left(c_{ist}\right),\ \forall i, j, s, t.$$

$$\sum_{j=1}^{N} u_j'\left(c_{jst}\right) = \lambda_i u_i'\left(c_{ist}\right) \sum_{j=1}^{N} \frac{1}{\lambda_j}$$

$$\lambda_i u_i'\left(c_{ist}\right) = \frac{1}{\sum_{j=1}^{N} \frac{1}{\lambda_j}} \cdot \frac{\sum_{j=1}^{N} u_j'\left(c_{jst}\right)}{N} \cdot N$$

$$= \frac{1}{\sum_{j=1}^{N} \frac{1}{\lambda_j}} \cdot \overline{u_j'\left(c_{jst}\right)} \cdot N$$

- Transient income shocks are fully pooled at the community level. Also, households' consumption is not affected by idiosyncratic shocks.

1. Village-level risk insurance model

(4) An illustration

- Suppose that everyone in the village has an identical constant absolute risk aversion utility function

$$u_i(c) = -\frac{1}{\sigma} e^{-\alpha c}$$

- Thus (In Townsend (1994), $\alpha = \sigma$)

$$u_i'(c) = \frac{\alpha}{\sigma} e^{-\alpha c}, \quad u_i''(c) = -\frac{\alpha^2}{\sigma} e^{-\alpha c}$$

$$R_A(c) = -\frac{u''}{u'} = \alpha, \quad R_A'(c) = 0$$

$$\frac{u_i'}{u_j'} = \frac{e^{-\alpha c_{ist}}}{e^{-\alpha c_{jst}}} = \frac{\lambda_j}{\lambda_i}, \ \forall i, j, s, t. \Rightarrow \ln \frac{e^{-\alpha c_{ist}}}{e^{-\alpha c_{jst}}} = \ln \frac{\lambda_j}{\lambda_i}$$

$$-\alpha c_{ist} + \alpha c_{jst} = \ln \lambda_j - \ln \lambda_i$$

$$c_{ist} = c_{jst} + \frac{1}{\alpha}\left(\ln \lambda_i - \ln \lambda_j\right)$$

1. Village-level risk insurance model

(4) An illustration

- It holds across N households in the village at t:

$$c_{ist} = c_{1st} + \frac{1}{\alpha}(\ln\lambda_i - \ln\lambda_1)$$

$$c_{ist} = c_{2st} + \frac{1}{\alpha}(\ln\lambda_i - \ln\lambda_2)$$

$$\vdots$$

$$c_{ist} = c_{Nst} + \frac{1}{\alpha}(\ln\lambda_i - \ln\lambda_N)$$

- Average:

$$c_{ist} = \overline{c}_{st} + \frac{1}{\alpha}\left(\ln\lambda_i - \frac{1}{N}\sum_{j=1}^{N}\ln\lambda_j\right) \text{ where } \overline{c}_{st} = \frac{1}{N}\sum_{j=1}^{N} c_{jst}$$

1. Village-level risk insurance model

(4) An illustration

- Household consumption equals the *average* level of consumption in the village $\overline{c}_{st}$ plus a time-invariant household fixed effect reflecting its relative weight in the village in the Pareto-efficient programme. It is not affected by income!
- 1st difference:

$$dc_{ist} = d\overline{c}_{st}$$

  - change of the household consumption equals change of the *average* consumption.
- A general utility form?
  - change of the household consumption monotonically increases in the change of the *average* consumption.

1. Village-level risk insurance model

(5) Empirics

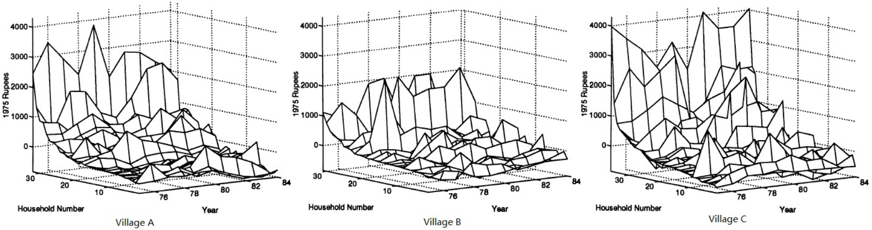
- Townsend (1994) and Ravallion and Chaudhuri (1997) (cited by B-U on pp. 99-100) have tested the risk-insurance hypothesis using the ICRISAT data (village-level household panel data) in India.
- If using the regression:

$$\triangle \ln c_{itv} = \alpha + \beta \ln y_{itv} + \gamma \triangle (\ln \overline{y_{vt}}) + \delta X_{itv} + \triangle \epsilon_{itv}$$

  - Model specification? (average consumption? log then average, or the reverse? 1st order time differential?)
  - Null: $\beta = 0$ (γ=?)
  - Rejected the null.
  - Partial risk sharing.
  - What else or other modification can you do?

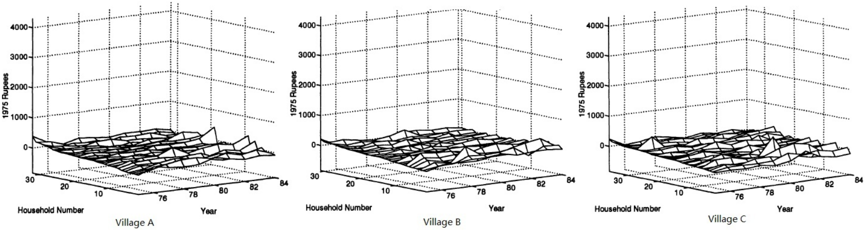
1. Village-level risk insurance model
(5) Empirics

Comovement of household incomes (deviation from village average):



1. Village-level risk insurance model
(5) Empirics

Comovement of household consumption (grain only) (deviation from village average):



1. Village-level risk insurance model

(6) Further discussion

- How to (can we) achieve Pareto-efficient risk allocation within the village?
  - The 2nd Welfare Theorem.
  - However, information? *ex post* vs. *ex ante*? which community? ...
- Scaling up (macro)? Du *et al.* (2010).

2. Intertemporal consumption smoothing model

(1) Model set-up

Assumptions

- Consider a household with no opportunity for cross-sectional risk pooling.
- Expected utility (in the remaining life-cycle):

$$U_t = \sum_{t=1}^{T} \beta^t \sum_{s=1}^{S} \pi_s u_i \left(c_{ist}\right)$$

$$U_t = E_t \sum_{t=\tau}^{T} \beta^{t-\tau} u\left(c_t\right)$$

- Suppress the subscript i.

2. Intertemporal consumption smoothing model

(1) Model set-up

Assumptions

1. In any period the household can borrow or lend on a credit market with r_t.
2. The household asset stock at the start of t is A_t.
3. The household receives a random income y_t & decides how to allocate its resources between consumption and net saving for the next period.

$$A_{t+1} = (1 + r_t)\,(A_t + y_t - c_t)$$

   Cash-in-hand: $A_t + y_t$; $A_t + y_t - c_t > 0$ as in Deaton (1991, cited by B-U, pp. 102).

4. $c_t > 0$.

2. Intertemporal consumption smoothing model

(1) Model set-up

Household problem

$$\max E_t \sum_{t=\tau}^{T} \beta^{t-\tau} u(c_t) \text{ s.t. } A_{t+1} = (1 + r_t)(A_t + y_t - c_t)$$

The period t value function for the household's problem satisfies a Bellman equation:

$$V_t(A_t + y_t) = \max_{c_t} \left\{ \begin{array}{c} u(c_t) + \\ \beta E_t V_{t+1}\left[(1 + r_t)(A_t + y_t - c_t) + y_{t+1}\right] \end{array} \right\}$$

Solving it by ET gives the Euler equation.

2. Intertemporal consumption smoothing model

(2) Solution

$$u'(c_t) = \beta(1+r_t)E_t u'(c_{t+1})$$

- LHS: MU at t; RHS: discounted expected MU at $t+1$.
- Suppose $\beta(1+r_t) = 1$ (intuition?), then $u'(c_t) = E_t u'(c_{t+1})$.
- Suppose a quadratic utility function: $c_t = E_t c_{t+1}$.
- Based on the budget constraint, we have the *permanent income hypothesis*:

$$c_t = \frac{r}{1+r}\left(A_t + E_t \sum_{t=\tau}^{\infty} (1+r)^{\tau-t} y_t\right)$$

2. Intertemporal consumption smoothing model

(3) Permanent income hypothesis

- Implications:
  1. If the income shock is transitory, there is little change in the expected future income stream & consumption would change little.
  2. If income shock is deemed permanent, consumption will change dramatically.
- Consumers have concave utility functions and therefore prefer smooth paths of consumption (over time and across states of the world) over variable ones. Only unanticipated changes in income that are perceived as permanent induce substantive changes in consumption. Expected and temporary changes to income should not induce a strong change in consumption.
- Empirics? Paxon (1992): deviations of rainfall in Thai, save 75%-80% of transitory income changes.

2. Intertemporal consumption smoothing model

(3) Permanent income hypothesis

- How does it come out?
  - 'The fundamental psychological law ... is that men are disposed, as a rule and on the average, to increase their consumption as their income increases, but not by as much as the increase in their income.' (Keynes)
  - This sentence was the basis for the static 'Keynesian' consumption function: $C = a + bY + error$.
  - But: on macro data, the marginal propensity to consume is lower in the short run than in the long run (Kuznets paradox).
  - Also: saving rates change systematically with income (e.g., Friedman, Katona, 1949).
  - Permanent income model offers a coherent explanation for the stylised facts.

2. Intertemporal consumption smoothing model

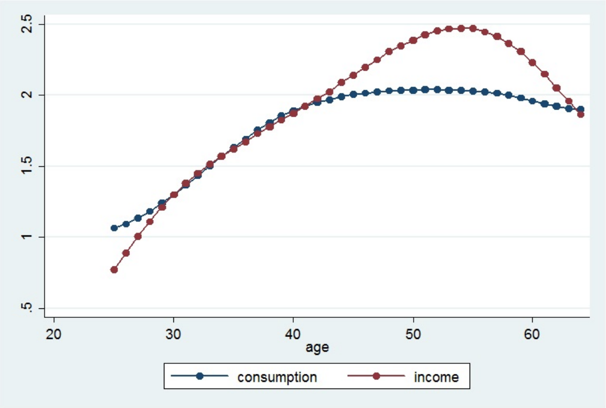
(3) Permanent income hypothesis

- The explanation of the stylized facts above boils down to the observation that a large fraction of the changes in income considered are temporary.
  - Kuznets paradox: short run fluctuations in disposable income are more likely to be dominated by the variance of temporary shocks that would be averaged out in the long run. That is why consumption is more responsive to income in the long run than in the short run.
  - The saving rates of blacks is higher than that of whites, at any income level, because the permanent income of blacks is lower and therefore, conditioning on a common income level, one selects the blacks with higher level of temporary shocks that should, according to the permanent income/life-cycle model, be saved.
  - Individuals with income increases are more likely to be affected by positive transitory shocks, that should be saved according to the model.

2. Intertemporal consumption smoothing model

(4) Criticisms of the life cycle model

Attanasio *et al.* (1999):



2. Intertemporal consumption smoothing model

(4) Criticisms of the life cycle model

Retirement consumption puzzle: Those that consider short run fluctuations linked to changes in earnings and income.

- The recent literature has focused on estimating how consumption levels change around retirement. The existence of a consumption fall around retirement is documented for the UK, the US and Italy - puzzle.
- Retirement implies an increase in leisure time, with improved shopping and home production opportunities - a drop in expenditure may not translate into a drop in utility (Aguiar and Hurst, 2005; 2007). Also, work-related expenses are no longer needed.
- Retirement may be the result of adverse health shocks - a drop in utility may not violate the life-cycle model predictions (Smith, 2006; Blau, 2008).
- Retirement may also induce grown children to leave home (Battistin *et al.*, 2009).

2. Intertemporal consumption smoothing model
(4) Criticisms of the life cycle model

Retirement consumption puzzle

- Once preferences are correctly modelled, home production is taken into account, and attention is focussed on those who retire at the expected age, then the drop in (per-capita) food spending and total spending around retirement does not imply a violation of the model prediction that consumers smooth marginal utility over time.
- As Hurst (2008) recently put it, we should no longer talk about the retirement consumption puzzle, rather about 'the retirement of a consumption puzzle'.
- This literature suggests that we should pay attention to the composition in consumption: consumption of work-related goods falls with age, consumption of leisure-intensive goods increases.

2. Intertemporal consumption smoothing model

(4) Criticisms of the life cycle model

Excess sensitivity to transitory receipts: Those that refer to short run fluctuations that are linked to *ad-hoc* payments non-necessarily related to labour supply behaviour.

- Recent papers have estimated non-zero effects of predictable tax changes (such as tax rebates, social security withholding tax) (Parker, 1999; Souleles, 1999; Shapiro & Slemrod, 2003; Johnson *et al*, 2004).
- However, consumption does not appear to react to other anticipated income changes (Browning & Collado, 2003; Hsieh, 2003).
- Cons. reacts to predicted changes in disposable inc., if these changes are relatively small (Browning & Crossley, 2007). These small changes tend to be tax rebates or other predictable changes in tax-related payments - maybe consumers deep down distrust the government, and are surprised when they receive what they are due.

2. Intertemporal consumption smoothing model

(4) Criticisms of the life cycle model

Excess sensitivity to transitory receipts:

- Recent papers report evidence in favour of a liquidity constraints interpretation.
- Stephens (2008) shows that consumption reacts to the repayment of vehicle loans, more for young individuals, who are more likely to be liquidity constrained. Agarwal, Liu and Souleles (2007) investigate credit card holders' response to the 2001 tax rebates: most people first increased repayments, but then the young and those initially close to their credit card limit started spending more. The eventual rise in spending could be attributed to the operation of liquidity constraints.
- Hsieh, Shimizutani and Hori (2008) find that Japanese consumers' response to a spending coupon program tailored to families with children and the elderly was highest among those with low wealth.