Assignment 3

1. There are two types of households, in equal proportion, indexed as $h = 0, 1$. Households of type 0 have a $l > 0$ units of time endowed in even periods, and no time endowment in odd periods. Households of type 1 have $l > 0$ units of time endowed in odd periods, and no time endowment in even periods. Let $c_{h,t}$ be the consumption of household h, and $n_{h,t}$ its hours of work, in period t.[1] All households have the same preferences $U^h(\{c_{h,t}, 1 - n_{h,t}\}) \equiv \sum_{t=0}^{\infty} \beta^t u(c_{h,t})$ where $\beta \in (0,1)$ and u is strictly increasing, concave and twice-continuously differentiable. Let $K_{h,0}$ represent the initial capital stock of households of type h. A *representative* firm uses a strictly increasing, concave and constant returns to scale production function, $F(K, L)$ where K is the capital stock and L is labour. Let p_t be the relative price of consumption at time t, in units of consumption at time 0. Let r_t be the competitive rental rate for capital, and w_t the real wage for labour, at time t, in units of c_t.

   (a) Define date-0 trading competitive equilibrium for this model. Please notice that there will be two separate household conditions. Furthermore, be careful to present conditions that relate the aggregate stock of capital and labour, demanded by the firm, to individual factor endowments, held by households of each type.

   (b) Prove that $\frac{u'(c^*_{0,t})}{u'(c^*_{0,t+1})} = \frac{u'(c^*_{1,t})}{u'(c^*_{1,t+1})} = \beta\left[D_1 F\left(K^*_{t+1}, l\right) + 1 - \delta\right]$ for each $\forall t$. Does aggregate consumption satisfy an Euler equation in this economy?

   (c) Define sequential trading competitive equilibrium for this model. Show that the sequence of budget constraints under sequential trading and the date-0 budget constraint in (a) are equivalent when $\frac{p_t}{p_{t+1}} = \frac{1}{R_{t+1}}$, where R_{t+1} is the date $t+1$ gross return on capital for the household.

   (d) Define recursive competitive equilibrium for this model. Note that value functions, policy functions, and aggregate laws of motion will be functions of e, an indicator variable that is 0 during odd periods and 1 in even dates.

   (e) (Stationary competitive equilibrium) Assume that the economy is always in its steady state. Using the date-0 competitive equilibrium, derive an expression for p_t and solve for the time-invariant levels of household consumption, c^*_h, $h = 0, 1$. What is the marginal propensity to consume from wealth?

2. Consider a representative household with a time-varying endowment, $\omega_t = \omega_h$ when $t = 0, 2, \ldots$, and $\omega_t = \omega_l$ for $t = 1, 2, \ldots$, where $\omega_h > \omega_l$. There is no production and the household's preferences are given by $U^h(\{c_{h,t}\}) \equiv \sum_{t=0}^{\infty} \beta^t u(c_{h,t})$ where $\beta \in (0,1)$ and u is strictly increasing, concave and twice-continuously differentiable.

   (a) Define Recursive Competitive Equilibrium.

   (b) Derive the real interest rate R_{t+1} between period t and $t+1$. Show that for any utility function satisfying the conditions above, the real interest rate cycles between two values, R_h and R_l. Defining R_h to be the gross return on bonds in periods with ω_h, show that $R_h < R_l$. Explain why.

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[1] To be explicit, $n_{0,t} = 0$ for $t = 1, 3, 5, \ldots$, and $n_{0,t} \in [0, l]$ for $t = 0, 2, 4$. In contrast, $n_{1,t} \in [0, l]$ when $t = 1, 3, 5, \ldots$, and $n_{1,t} = 0$ for $t = 0, 2, 4, \ldots$