An LM Test Based on Generalized Residuals for Random Effects in a Nonlinear Model

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Abstract

We derive a Lagrange Multiplier test for variance components in the random effects probit model. In the natural parameterization of the model the derivatives needed for the test are identically zero at the restricted estimate. Using a reparameterized model, the now feasible LM test is based on generalized residuals. The result will extend to other nonlinear single index models. The technique is illustrated with an application.

Keywords: Lagrange multiplier test, panel data, probit model, random effects

JEL Codes: C23, C25, I11

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1. Introduction

With the growing availability of panel data sets, empirical analyses involving limited dependent variables and individual effects have become quite common. When it is assumed that the number of time periods is finite, consistent estimates of the parameters when fixed individual effects are assumed can only be obtained in a limited number of cases because of the incidental parameters problem. As a result, random individual effects are commonly assumed. After the linear regression model, by far the leading application of the more general class of random effects models is the random effects probit model.

In the presence of random effects, it is important to understand why we might want to estimate a random effects probit model rather than just estimating a pooled probit model. As Robinson (1982) shows, in the presence of correlation across time periods, estimates of the parameters using standard probit maximum likelihood on the pooled data will be consistent, but inefficient. However, a robust estimator of covariance matrix of the estimated parameters will need to be used for hypothesis testing. It is important to recognize that the estimated coefficients from random effects probit models and pooled probit models are quite different because of the different normalization assumptions that popular software use, but as Arulampalam (1999) discusses, it is relatively easy to adjust these estimates and the estimates of the marginal effects so that they are comparable.

Our interest here is in testing for random effects in the random effects probit model using the Lagrange Multiplier (LM) test. In analyses of health outcomes, for example controlling for unobserved person specific heterogeneity is argued to be important because the propensity to seek health care might differ systematically across individuals (see Riphahn et al. ((2003)). The LM test has provided a standard means of testing parametric restrictions for a variety of models. Its primary advantage among the trinity of tests (LM, Likelihood Ratio (LR) , Wald) generally used in likelihood-based inference is that the LM statistic is computed using only the results of the null, restricted model, which is usually simpler to estimate than the alternative, unrestricted model. The random effects linear regression model is a prominent example where the LM test is used (Greene, 2012, p. 376). Breusch and Pagan's (1980) LM test for random effects in a linear model is based on pooled OLS residuals, while estimation of the alternative model involves generalized least squares either based on a two-step procedure or maximum likelihood (ML) estimation.

Testing for random effects in the probit model is an example of a problem that emerges when the parametric restriction in the null hypothesis puts the value of a variance parameter on the boundary of the parameter space. In the random effects model, the restriction is that the standard deviation of the random effect equals zero. When random effect probit models are estimated, popular computer packages like STATA and LIMDEP automatically produce LR and Wald-type tests of the null hypothesis of no random effects, but would appear to use the $\chi^2_{(1)}$ distribution (or the standard normal distribution) to compute the p-values for these tests. If, under the null hypothesis, the parameter being tested lies on the boundary of the parameter space, an additional advantage of the LM test is that it will still have standard distributional properties, whereas the LR and Wald tests will not (see Andrews (2001)). In fact, in testing for random effects in the probit model, the LR and Wald tests will be distributed as a (1/2) $\chi^2_{(1)}$ distribution under the null hypothesis (see Gourieroux et al. (1988)). This means the correct critical values for these two tests at the 5% and 10% significance level are 5.02 and 3.84 respectively, rather than the commonly used values of 3.84 and 2.71 taken from the $\chi^2_{(1)}$ distribution.

Our interest here is testing for random effects in the random effects probit model using the LM test. This model is, after the linear regression model, by far the leading application of the more general class of random effects models. But, despite the obvious simplicity of the restricted model, the standard probit model, the LM test for this model does not appear in the existing literature. One reason for this is that the usual parameterization of the model has the inconvenient feature that the score vector is identically zero at the restricted ML estimates. In the received literature, there are a handful of other cases in which the score vector needed to compute the LM statistic is identically zero at the restricted estimates, which would seem to preclude using the LM test. [See Chesher (1984), Lee and Chesher (1986) and Kiefer (1982).]

While Chesher (1984), Lee and Chesher (1986) and Kiefer (1982) discuss a general theory of how to deal with score vectors that are zero under the null hypothesis, and despite what would seem to be its broad application, we have not been able to locate any applications in the subsequent 30+ years of literature. In section 2, we will provide what we expect to be some useful analytical expressions for the LM test for random effects in the random effects probit model. We illustrate its use in section 3 with an empirical application on hospitalization behavior.

2. The Random Effects Probit Model

The random effects probit model is

$$y_{it}^* = \boldsymbol{\beta}'\mathbf{x}_{it} + u_i + \varepsilon_{it};\ i=1,...,n;\ t=1,...,\mathrm{T}_i, \tag{1}$$

$$y_{it} = \mathbb{1}[y_{it}^* > 0],$$

$$\varepsilon_{it} \sim N[0,1^2],$$

$$E[\varepsilon_{it}\varepsilon_{js}] = 0,\ i \neq j, t \neq s,$$

$$E[u_i u_j] = 0,\ i \neq j,$$

where $\boldsymbol{\beta}$ and $\mathbf{x}_{it}$ are both $K\times 1$ vectors. It is assumed that ε_{it} and u_j are independent $\forall j,t,s,$ and that conditional on $\mathbf{x}_{i1},..,\mathbf{x}_{iT}$, $u_i \sim N[0,\sigma_u^2]$. Letting $T=\Sigma i\ T_i$, then the log likelihood for a sample of T observations, conditioned on the unobserved heterogeneity, $u_1,u_2,\ldots,u_n$, is

$$\log L(\boldsymbol{\beta} \mid u_1,...,u_n) = \sum_{i=1}^{n} \log \prod_{t=1}^{T_i} \Phi[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it} + u_i)], \tag{2}$$

where $\Phi(t)$ is the cumulative density function (cdf) of the standard normal distribution, and $q_{it} = 2y_{it} - 1$. Maximum likelihood estimation is based on the unconditional log likelihood given by

$$\log L(\boldsymbol{\beta},\sigma_u) = \sum_{i=1}^{n} \log \int_{-\infty}^{\infty} \left\{ \prod_{t=1}^{T_i} \Phi[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it} + u_i)] \right\} \frac{1}{\sigma_u} \phi\left(\frac{u_i}{\sigma_u}\right) du_i, \tag{3}$$

where $\phi(t)$ is the standard normal density. The computation is simplified by making the change of variable from u_i to $v_i = u_i/\sigma_u$; the resulting log likelihood is

$$\log L(\boldsymbol{\beta},\sigma_u) = \sum_{i=1}^{n} \log L_i(\boldsymbol{\beta},\sigma_u) = \sum_{i=1}^{n} \log \int_{-\infty}^{\infty} \left\{ \prod_{t=1}^{T_i} \Phi[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it} + \sigma_u v_i)] \right\} \phi(v_i)\, dv_i. \tag{4}$$

Butler and Moffitt (1982) developed the estimation method based on Hermite quadrature generally used in contemporary applications of this model.

2.1 LM Test for Random Effects

To form the LM statistic for the test of the null hypothesis of no random effects, $\sigma_u = 0$, we require the derivative of log $L_i(\boldsymbol{\beta},\sigma_u)$ with respect to $\sigma_{u.}$ of each term in the sum:

$$\frac{\partial \log L_i(\boldsymbol{\beta},\sigma_u)}{\partial \sigma_u} = \frac{\int_{-\infty}^{\infty} \left\{ \prod_{t=1}^{T_i} \Phi[q_{it}\, \mathrm{a}_{it}] \right\} \left\{ \sum_{t=1}^{T_i} \frac{\phi[q_{it}\, \mathrm{a}_{it}]}{\Phi[q_{it}\, \mathrm{a}_{it}]} \right\} q_{it} v_i \phi(v_i)\, dv_i}{\int_{-\infty}^{\infty} \left\{ \prod_{t=1}^{T_i} \Phi[q_{it}\, \mathrm{a}_{it}] \right\} \phi(v_i)\, dv_i}, \tag{5}$$

where $\mathrm{a}_{it} = \boldsymbol{\beta}'\mathbf{x}_{it} + \sigma_u v_i$. In order to compute the LM statistic, we need to evaluate this expression at $\sigma_u = 0$. Moving all terms not involving v_i outside the integrals produces simple results in both the numerator and denominator.

$$\frac{\partial \log L_i(\boldsymbol{\beta},0)}{\partial \sigma_u} = \frac{\left\{\prod_{t=1}^{T_i} \Phi[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it})]\right\}\left\{\sum_{t=1}^{T_i} \frac{\phi[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it})]}{\Phi[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it})]} q_{it}\right\}\int_{-\infty}^{\infty} v_i\phi(v_i)\,dv_i}{\left\{\prod_{t=1}^{T_i} \Phi[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it})]\right\}\int_{-\infty}^{\infty} \phi(v_i)\,dv_i} \tag{6}$$

Given the assumed standard normal distribution for v_i, the integral in the numerator is $E[v_i] = 0$ and that in the denominator is $\int_{v_i} \phi(v_i)dv_i = 1$ by definition. It follows that regardless of the value of $\boldsymbol{\beta}$ and the values of the data, each term in the derivative of the log likelihood with respect to σ_u is identically zero. The derivatives of $\log L_i(\boldsymbol{\beta},\sigma_u)$ with respect to $\boldsymbol{\beta}$ evaluated at the restricted maximum likelihood estimates (MLE) of $\boldsymbol{\beta}$ and σ_u are also zero by the same construction. Hence, the score vector under the null hypothesis is identically zero. It also follows that the information matrix will be singular with the row and column corresponding to σ_u being identically zero. These results will still hold if the normal distribution underlying the probit model is replaced by another symmetric distribution, and if the assumption for normality of the random effects is replaced by another continuous distribution with mean zero.

2.2 LM Test Based on a Reparameterization

In this situation, Chesher (1984), Lee and Chesher (1986) and Cox and Hinley (1974) suggest reparameterization of the model as a possible strategy for obtaining the LM test. For the probit model, we use $\gamma = \sigma_u^2$, so that the log likelihood in the parameter space $(\boldsymbol{\beta},\gamma)$ becomes

$$\begin{aligned}\log L(\boldsymbol{\beta},\gamma) &= \sum_{i=1}^{n} \log L_i(\boldsymbol{\beta},\gamma)\\ &= \sum_{i=1}^{n} \log\int_{-\infty}^{\infty} \left\{\prod_{t=1}^{T_i} \Phi\left[q_{it}(\boldsymbol{\beta}'\mathbf{x}_{it} + v_i\sqrt{\gamma})\right]\right\}\phi(v_i)\,dv_i.\end{aligned} \tag{7}$$

The necessary derivative becomes

$$\frac{\partial \log L_i(\boldsymbol{\beta},\gamma)}{\partial \gamma} = \frac{\frac{1}{2\sqrt{\gamma}}\int_{-\infty}^{\infty} \left\{\prod_{t=1}^{T_i} \Phi_{it}\right\}\left\{\sum_{t=1}^{T_i} g_{it}\right\} v_i\phi(v_i)\,dv_i}{\int_{-\infty}^{\infty} \left\{\prod_{t=1}^{T_i} \Phi_{it}\right\}\phi(v_i)\,dv_i} \tag{8}$$

where $b_{it} = \boldsymbol{\beta}'\mathbf{x}_{it} + v_i\sqrt{\gamma}$, $\phi_{it} = \phi(q_{it}b_{it})$, $\Phi_{it} = \Phi(q_{it}b_{it})$ and $g_{it} = q_{it}\phi_{it}/\Phi_{it}$. Note that $g_{it}v_{it}$ is the first derivative of $\log\Phi_{it}$ with respect to $\sqrt{\gamma}$. Evaluated at $\gamma = 0$ using the same approach as earlier, the numerator now takes the form 0/0. We use L'Hopital's rule to evaluate the numerator, taking the limits as γ approaches zero from above. Then,

$$\frac{\partial \log L_i(\boldsymbol{\beta},0)}{\partial\gamma} = \frac{\lim\limits_{\gamma\downarrow 0}\dfrac{1}{2\dfrac{1}{2\sqrt{\gamma}}}\displaystyle\int_{-\infty}^{\infty} L_i\left[\begin{array}{l}\sum_{t=1}^{T_i}\left\{-\left(\dfrac{(q_{it}a_{it})\phi[q_{it}b_{it}]}{\Phi[q_{it}b_{it}]}\right)-\left(\dfrac{q_{it}\phi[q_{it}b_{it}]}{\Phi[q_{it}b_{it}]}\right)^2\right\}\\ +\left(\sum_{t=1}^{T_i} g_{it}\right)^2\end{array}\right]\dfrac{1}{2\sqrt{\gamma}}v_i^2\phi(v_i)\,dv_i}{\displaystyle\int_{-\infty}^{\infty} L_i\phi(v_i)\,dv_i}$$

$$= \frac{\lim\limits_{\gamma\downarrow 0}\dfrac{1}{2\dfrac{1}{2\sqrt{\gamma}}}\displaystyle\int_{-\infty}^{\infty} L_i\left[\left(\sum_{t=1}^{T_i} h_{it}\right)+\left(\sum_{t=1}^{T_i} g_{it}\right)^2\right]\dfrac{1}{2\sqrt{\gamma}}v_i^2\phi(v_i)\,dv_i}{\displaystyle\int_{-\infty}^{\infty} L_i\phi(v_i)\,dv_i}, \tag{9}$$

where $L_i = \prod_{t=1}^{T_i}\Phi_{it}$ and h_{it} is the second derivative of $\log\Phi_{it}$ with respect to its argument. The two occurrences of $1/(2\sqrt{\gamma})$ in (9) cancel. The integral in the numerator now involves $E[v_i^2] = 1$. Moving the now invariant (with respect to v_i) terms out of the integrals as before, the product terms, L_i, in the numerator and denominator cancel and we now have

$$\frac{\partial \log L_i(\boldsymbol{\beta},0)}{\partial\gamma} = \frac{1}{2}\sum_{t=1}^{T_i}\left\{-\left(\frac{(q_{it}\boldsymbol{\beta}'\mathbf{x}_{it})\phi[q_{it}\boldsymbol{\beta}'\mathbf{x}_{it}]}{\Phi[q_{it}\boldsymbol{\beta}'\mathbf{x}_{it}]}\right)-\left(\frac{q_{it}\phi[q_{it}\boldsymbol{\beta}'\mathbf{x}_{it}]}{\Phi[q_{it}\boldsymbol{\beta}'\mathbf{x}_{it}]}\right)^2\right\}+\frac{1}{2}\left[\sum_{t=1}^{T_i}\left(\frac{q_{it}\phi[q_{it}\boldsymbol{\beta}'\mathbf{x}_{it}]}{\Phi[q_{it}\boldsymbol{\beta}'\mathbf{x}_{it}]}\right)\right]^2$$

$$= \frac{1}{2}\left[\left(\sum_{t=1}^{T_i} h_{it}^0\right)+\left(\sum_{t=1}^{T_i} g_{it}^0\right)^2\right], \tag{10}$$

where the superscripts on h_{it} and g_{it} indicate they are evaluated at $\gamma = 0$. Under the null hypothesis, as T_i goes to infinity, each term (i) above would converge to zero by virtue of the information matrix inequality. The remainder of the score vector at the restricted estimates is

$$\frac{\partial \log L(\boldsymbol{\beta},0)}{\partial\boldsymbol{\beta}} = \sum_{i=1}^{n}\left(\sum_{t=1}^{T_i} g_{it}^0\mathbf{x}_{it}\right). \tag{11}$$

Finally, collecting all K+1 terms, we denote the score vector as

$$\frac{\partial \log L(\boldsymbol{\beta},0)}{\partial\begin{pmatrix}\boldsymbol{\beta}\\ \gamma\end{pmatrix}} = \sum_{i=1}^{n}\mathbf{g}_i(\boldsymbol{\beta},0) = \sum_{i=1}^{n}\mathbf{g}_{i0}. \tag{12}$$

(A result to be used later is that this part of the score vector remains identically zero at $\gamma = 0$.)

2.3 LM Test Based on a Generalized Residuals

Let $w_{it} = g_{it}^0 = \left(\dfrac{q_{it}\phi[q_{it}\boldsymbol{\beta}'\mathbf{x}_{it}]}{\Phi[q_{it}\boldsymbol{\beta}'\mathbf{x}_{it}]}\right)$. Note that w_{it} is the generalized residual for the probit model under the null hypothesis (see Chesher and Irish (1987) and Gourieroux et al. (1987)). Then, the first line of equation (10) can be written as

$$\frac{\partial \log L_i(\boldsymbol{\beta},0)}{\partial \gamma} = \frac{1}{2}\sum_{t=1}^{T_i} \left\{-\boldsymbol{\beta}'\mathbf{x}_{it}\mathbf{w}_{it} - \mathbf{w}_{it}{}^2\right\} + \frac{1}{2}\left[\sum_{t=1}^{T_i} \mathbf{w}_{it}\right]^2. \tag{10a}$$

Accumulating all n terms, we now have

$$\begin{aligned}
\frac{\partial \log L\ (\boldsymbol{\beta},0)}{\partial \gamma} &= \frac{1}{2}\sum_{i=1}^{N} \{\sum_{t=1}^{T_i} \left\{-\boldsymbol{\beta}'\mathbf{x}_{it}\mathbf{w}_{it} - \mathbf{w}_{it}{}^2\right\} + \frac{1}{2}\left[\sum_{t=1}^{T_i} \mathbf{w}_{it}\right]^2 \} \\
&= -\frac{1}{2}\sum_{i=1}^{N} \sum_{t=1}^{T_i} \boldsymbol{\beta}'\mathbf{x}_{it}\mathbf{w}_{it} + \frac{1}{2}\sum_{i=1}^{N} \sum_{s=1,s\neq t}^{T_i} \sum_{t=1}^{T_i} \mathbf{w}_{it}\mathbf{w}_{is} \qquad (10b) \\
&= \frac{1}{2}\sum_{i=1}^{N} \sum_{s=1,s\neq t}^{T_i} \sum_{t=1}^{T_i} \mathbf{w}_{it}\mathbf{w}_{is}.
\end{aligned}$$

We obtain the third line of (10b) from the second line by using equation (11). Equation (10b) essentially looks at the correlation of generalized residuals for each i! In Wooldridge (2010), there is an equation similar to (10b) when he seeks to create a test for random effects in the linear regression model from first principals instead of the LM test.

Conditional on $\mathbf{x}_{it}$ under the null hypothesis, y_{it} will be identically and independently distributed. This means that w_{it} will also be identically and independently distributed with $E(w_{it} \mid \mathbf{x}_{it}) = 0$, so it is possible to apply a central limit theorem very easily. Under the null hypothesis, using the outer product form of the information matrix, it is relatively easy to show that the information matrix will be block diagonal between $\boldsymbol{\beta}$ and γ. From (10a) and (11), the key element of

$$\frac{\partial \log L_i(\boldsymbol{\beta},0)}{\partial \gamma}\frac{\partial \log L(\boldsymbol{\beta},0)}{\partial \boldsymbol{\beta}} = \frac{1}{2}\sum_{t=1}^{T_i} \boldsymbol{\beta}'\mathbf{x}_{it}\mathbf{w}_{it}\sum_{s=1,s\neq t}^{T_i} \sum_{t=1}^{T_i} \mathbf{w}_{it}\mathbf{w}_{is}, \tag{13}$$

Given $E(w_{it} \mid \mathbf{x}_{it}) = 0$ and the IID nature of w_{is}, it is easy to show that $E(w_{is}w_{it}w_{iu}) = 0$ if $s \neq t \neq v$ or $s = t \neq$ v. Hence, the expected value of the right hand side of (13) is zero, and the information matrix will be block diagonal.

The first K elements of the score vector equal zero when evaluated at the restricted (pooled probit) MLE of $\boldsymbol{\beta}$. Denote by $\mathbf{G}$ the $n\times(K+1)$ matrix with ith row equal to $\mathbf{g}_{i0}{}'$ evaluated at the restricted maximum likelihood estimates, and let $\mathbf{i}$ denote an $n\times1$ column vector of ones. Then, taking advantage of the information matrix equality to estimate the covariance matrix of the score vector, we compute the LM statistic using

$$LM = (\mathbf{i}'\mathbf{G})(\mathbf{G}'\mathbf{G})^{-1}(\mathbf{G}'\mathbf{i}) = (g_\gamma)^2(\mathbf{G}'\mathbf{G})^{(K+1),(K+1)}, \tag{14}$$

where g_γ is the last element of the score evaluated at the restricted maximum likelihood estimates, and $(\mathbf{G}'\mathbf{G})^{(K+1),(K+1)}$ is the $(K+1),(K+1)$ element of $(\mathbf{G}'\mathbf{G})^{-1}$ in which the rows of $\mathbf{G}$ are the elements in (12) and (11), respectively.

Given the well-known invariance of the LM test to re-parameterization (see Dagenais and Dufour (1981)), it might seem peculiar that a re-parameterization can change the properties of the LM test. However, their proof of invariance requires that the matrix containing the derivatives of one set of parameters with respect to the other set of parameters be non-singular at the restricted parameter values. Since $\partial\gamma/\partial\sigma_u = 2\sigma_u$, this non-singularity condition will not be satisfied here at $\sigma_u = 0$. Given the results for the parameterization using γ, it is easy to show that the parameterization using σ_u will lead to a zero score since

$$\frac{\partial \log L_i(\boldsymbol{\beta}, \sigma_u)}{\partial \sigma_u} = \frac{\partial \log L_i(\boldsymbol{\beta}, \gamma)}{\partial \gamma} \frac{\partial \gamma}{\partial \sigma_u}. \tag{15}$$

3. Application

Riphahn, Wambach and Million (2003) use data from the German Socioeconomic Panel Survey over the period 1984-95 to model jointly the number of times a patient visits a doctor and the number of times a patient is hospitalized in a year (see also Geil et al. (1997)), and determine whether public and/or private insurance significantly affects the demand for health care. The authors conduct separate analyses for male and female patients. Here, instead of analyzing the number of hospitalizations, we restrict our analysis to whether or not male patients are hospitalized in the relevant year. We use an unbalanced panel that contains seven years of data on 3,691 households for a total of 14,243 observations.

To model the number of hospital visits, we follow the model specification used by Riphahn et al. (2003). The variables used to model the decision of whether or not to visit a hospital in a calendar year (Y) are age (AGE), age squared (AGE^2), health satisfaction (HSAT), a dummy for whether or not the person is handicapped (HANDDUM), the degree of the handicap (HANDPER), marital status (MARRIED), the years of schooling (EDUC), household income (HHINC), a dummy variable for whether or not there are children under the age of 16 in the household (HHKIDS), dummies for self-employment (SELF), civil servants (BEAMT), blue collar employees (BLUEC) and employed people (WORKING) , and dummies for public health insurance (PUBLIC) and add-on insurance (ADDON).

The estimates of the pooled probit model and the random effects probit model are reported in Table 1. To account for the panel nature of the data, in the pooled probit model the t-statistics are computed using an estimated covariance matrix of the estimated coefficients that is corrected for clustering. It is worth nothing that significance of the impact of public insurance depends on whether the pooled probit estimates or the random effects probit model is used. The computed value of the LM test is 129.441 (0.00) which clearly rejects the null hypothesis of no random effects (p-value in brackets). The values of the Wald and LR tests are 162.69 (0.00) and 262.417 (0.00), respectively (p-values computed using their non-standard distribution in brackets, see Andrews (2001)). All three tests clearly reject the null hypothesis of no random effects, so that public insurance does not affect hospitalizations, a conclusion consistent with Riphahn et al. (2003).

4. Conclusion

The strategy used here appears in Lee and Chesher (1986) and Chesher (1984). The reformulation in terms of generalized residuals is new. The latter result implies that the test should be generalizable to other single index models, such as the tobit and Poisson regression models. We find it surprising that despite its simplicity, the LM test for random effects in a probit model has not been used routinely, in spite of the fact that the null hypothesis being tested is typically part of empirical analysis using the probit model with panel data.

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probit;for[female=0];lhs=hospital;rhs=one,age,age^2,hsat,handdum,handper,
       married,educ,hhninc,hhkids,self,beamt,bluec,working,public,addon;cluster=id$
probit;for[female=0];lhs=hospital;rhs=one,age,age^2,hsat,handdum,handper,
       married,educ,hhninc,hhkids,self,beamt,bluec,working,public,addon;panel$
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Normal exit:   6 iterations. Status=0, F=    3674.921
+----------------------------------------------------------------------+
| Covariance matrix for the model is adjusted for data clustering.     |
| Sample of  14243 observations contained   3691 clusters defined by   |
| variable ID       which identifies by a value a cluster ID.          |
+----------------------------------------------------------------------+
-----------------------------------------------------------------------------
Binomial Probit Model
Dependent variable              HOSPITAL
Log likelihood function     -3674.92068
Restricted log likelihood   -3898.19278
Chi squared [ 15](P= .000)    446.54419
Significance level               .00000
McFadden Pseudo R-squared      .0572758
Estimation based on N =  14243, K =  16
Inf.Cr.AIC  =   7381.8 AIC/N =     .518
--------+--------------------------------------------------------------------
        |                  Standard            Prob.      95% Confidence
HOSPITAL|  Coefficient       Error       z    |z|>Z*         Interval
--------+--------------------------------------------------------------------
        |Index function for probability
Constant|      .24112         .36002       .67  .5030     -.46450     .94675
     AGE|    -.02948*         .01633     -1.80  .0711     -.06149     .00254
 AGE^2.0|      .00035*        .00019      1.89  .0594     -.00001     .00072
    HSAT|    -.11341***       .00845    -13.42  .0000     -.12997    -.09685
 HANDDUM|    -.03022          .04485      -.67  .5004     -.11813     .05768
 HANDPER|      .00335***      .00116      2.88  .0040      .00107     .00564
 MARRIED|    -.04625          .05304      -.87  .3832     -.15019     .05770
    EDUC|    -.02423**        .00997     -2.43  .0152     -.04378    -.00468
  HHNINC|      .17338         .10978      1.58  .1142     -.04177     .38854
  HHKIDS|      .03060         .04566       .67  .5028     -.05890     .12010
    SELF|    -.05199          .08559      -.61  .5436     -.21975     .11577
   BEAMT|    -.04447          .08093      -.55  .5826     -.20309     .11415
   BLUEC|      .07262         .05008      1.45  .1470     -.02553     .17076
 WORKING|    -.06951          .06558     -1.06  .2892     -.19806     .05903
  PUBLIC|    -.12616*         .07466     -1.69  .0911     -.27248     .02017
   ADDON|      .26152**       .11883      2.20  .0277      .02862     .49442
--------+--------------------------------------------------------------------
***, **, * ==>  Significance at 1%, 5%, 10% level.
-----------------------------------------------------------------------------
Normal exit:   6 iterations. Status=0, F=    3674.921
-----------------------------------------------------------------------------
Binomial Probit Model
Dependent variable              HOSPITAL
Log likelihood function     -3674.92068
Restricted log likelihood   -3898.19278
Chi squared [ 15](P= .000)    446.54419
Significance level               .00000
McFadden Pseudo R-squared      .0572758
Estimation based on N =  14243, K =  16
Inf.Cr.AIC  =   7381.8 AIC/N =     .518
------ LM test for Random Effects -----
ChiSqd.[1]  129.441     P value   .00000
--------+--------------------------------------------------------------------
        |                  Standard            Prob.      95% Confidence
HOSPITAL|  Coefficient       Error       z    |z|>Z*         Interval
--------+--------------------------------------------------------------------
        |Index function for probability
Constant|      .24112         .30232       .80  .4251     -.35142     .83367
     AGE|    -.02948**        .01341     -2.20  .0279     -.05575    -.00320
 AGE^2.0|      .00035**       .00015      2.28  .0223      .00005     .00065
    HSAT|    -.11341***       .00704    -16.11  .0000     -.12720    -.09961
 HANDDUM|    -.03022          .04761      -.63  .5256     -.12355     .06310
 HANDPER|      .00335***      .00096      3.50  .0005      .00148     .00523
 MARRIED|    -.04625          .04513     -1.02  .3055     -.13471     .04222
    EDUC|    -.02423***       .00816     -2.97  .0030     -.04022    -.00823
```

```
  HHNINC|     .17338*        .09750     1.78  .0754     -.01772     .36449
  HHKIDS|     .03060         .03987      .77  .4428     -.04754     .10874
    SELF|    -.05199         .06606     -.79  .4313     -.18146     .07749
   BEAMT|    -.04447         .07234     -.61  .5387     -.18626     .09731
   BLUEC|     .07262*        .04287     1.69  .0903     -.01141     .15664
 WORKING|    -.06951         .05545    -1.25  .2100     -.17820     .03917
  PUBLIC|    -.12616*        .06582    -1.92  .0553     -.25517     .00285
   ADDON|     .26152**       .11247     2.33  .0201      .04109     .48195
--------+--------------------------------------------------------------------
***, **, * ==>  Significance at 1%, 5%, 10% level.
-----------------------------------------------------------------------------
Normal exit:  29 iterations. Status=0, F=    3542.612
-----------------------------------------------------------------------------
Random Effects Binary Probit Model
Dependent variable               HOSPITAL
Log likelihood function     -3542.61224
Restricted log likelihood   -3674.92068
Chi squared [   1](P= .000)    264.61689
Significance level               .00000
(Cannot compute pseudo R2.  Use RHS=one
to obtain the required restricted logL)
Estimation based on N =  14243, K =  17
Inf.Cr.AIC  =    7119.2 AIC/N =     .500
Unbalanced panel has   3691 individuals
- ChiSqd[1] tests for random effects  -
LM   ChiSqd  129.441   P value   .00000
LR   ChiSqd  264.617   P value   .00000
Wald ChiSqd  162.690   P value   .00000
--------+--------------------------------------------------------------------
        |                  Standard            Prob.      95% Confidence
HOSPITAL|  Coefficient       Error       z    |z|>Z*         Interval
--------+--------------------------------------------------------------------
Constant|     .28196         .38470      .73  .4636     -.47205    1.03596
     AGE|    -.04537***      .01717    -2.64  .0082     -.07903    -.01172
 AGE^2.0|     .00055***      .00020     2.77  .0056      .00016     .00094
    HSAT|    -.12433***      .00844   -14.74  .0000     -.14087    -.10780
 HANDDUM|    -.04927         .06441     -.76  .4443     -.17552     .07697
 HANDPER|     .00371***      .00127     2.93  .0034      .00123     .00619
 MARRIED|    -.05337         .05924     -.90  .3676     -.16948     .06274
    EDUC|    -.02900**       .01137    -2.55  .0108     -.05128    -.00671
  HHNINC|     .22703*        .11751     1.93  .0534     -.00329     .45736
  HHKIDS|     .05239         .05119     1.02  .3061     -.04795     .15273
    SELF|    -.11428         .08943    -1.28  .2013     -.28955     .06100
   BEAMT|    -.04848         .09966     -.49  .6266     -.24382     .14685
   BLUEC|     .08981         .05759     1.56  .1189     -.02308     .20269
 WORKING|    -.06517         .06930     -.94  .3470     -.20100     .07066
  PUBLIC|    -.09987         .08712    -1.15  .2517     -.27063     .07089
   ADDON|     .24998*        .14339     1.74  .0813     -.03106     .53103
     Rho|     .33613***      .02635    12.76  .0000      .28448     .38779
--------+--------------------------------------------------------------------
***, **, * ==>  Significance at 1%, 5%, 10% level.
-----------------------------------------------------------------------------
```