2.(a):
$$V(k) = \max_{0 \le k' \le Ak} \left(\frac{[Ak-k']^{1-\sigma}}{1-\sigma} + \beta V(k') \right)$$

$$= \max_{0 \le k' \le Ak} \left(\frac{[Ak-k']^{1-\sigma}}{1-\sigma} + \beta \cdot E \cdot \frac{(k')^{1-\sigma}}{1-\sigma} \right)$$

$$FOC:[k']: [Ak-k']^{-\sigma} \cdot (-1) + \beta \cdot E \cdot (k')^{-\sigma} = 0$$

$$Ak-k' = \beta^{-\frac{1}{\sigma}} \cdot E^{-\frac{1}{\sigma}} \cdot k'$$

$$\Rightarrow Ak = [1+(\beta E)^{-\frac{1}{\sigma}}] \cdot k'$$

$$\Rightarrow k' = \frac{1}{1+(\beta E)^{-\frac{1}{\sigma}}} \cdot Ak = g(k;E) \qquad (A3)$$

$$\Rightarrow E \cdot \frac{k^{1-\sigma}}{\cancel{1-\sigma}} = \frac{[AK - \frac{1}{1+(\beta E)^{-\frac{1}{\sigma}}} \cdot Ak]^{1-\sigma}}{\cancel{1-\sigma}} + \beta \cdot E \cdot \frac{[\frac{1}{1+(\beta E)^{-\frac{1}{\sigma}}} \cdot Ak]^{1-\sigma}}{\cancel{1-\sigma}}$$

$$= \left[\frac{(\beta E)^{-\frac{1}{\sigma}}}{1+(\beta E)^{-\frac{1}{\sigma}}} \right]^{1-\sigma} (Ak)^{1-\sigma} + \beta E \cdot \left(\frac{Ak}{1+(\beta E)^{-\frac{1}{\sigma}}} \right)^{1-\sigma}$$

$$\Rightarrow E = \left[\frac{(\beta E)^{-\frac{1}{\sigma}}}{1+(\beta E)^{-\frac{1}{\sigma}}} \right]^{1-\sigma} \cdot A^{1-\sigma} + \beta E \cdot \left[\frac{A}{1+(\beta E)^{-\frac{1}{\sigma}}} \right]^{1-\sigma}$$

$$E = A^{1-\sigma} \cdot \frac{\beta E + (\beta E)^{-\frac{1-\sigma}{\sigma}}}{[1+(\beta E)^{-\frac{1}{\sigma}}]^{1-\sigma}}$$

$$E = A^{1-\sigma} \cdot \beta E \cdot \frac{1+(\beta E)^{-\frac{1}{\sigma}}}{[1+(\beta E)^{-\frac{1}{\sigma}}]^{1-\sigma}}$$

$$\cancel{E} = A^{1-\sigma} \cdot \beta \cancel{E} \cdot [1+(\beta E)^{-\frac{1}{\sigma}}]^{\sigma}$$

$$1 = A^{1-\sigma} \cdot \beta \cdot [1+(\beta E)^{-\frac{1}{\sigma}}]^{\sigma} \Rightarrow \left(\frac{1}{A^{1-\sigma}\beta} \right)^{\frac{1}{\sigma}} = 1 + (\beta E)^{-\frac{1}{\sigma}}$$

(a). $(\beta E)^{-\frac{1}{\sigma}} = \left(\frac{1}{A^{1-\sigma}\cdot\beta}\right)^{\frac{1}{\sigma}} - 1$

$$\beta E = \left[\left(\frac{1}{A^{1-\sigma}\cdot\beta}\right)^{\frac{1}{\sigma}} - 1\right]^{-\sigma}$$

$$E = \frac{1}{\beta}\left[\left(\frac{1}{A^{1-\sigma}\cdot\beta}\right)^{\frac{1}{\sigma}} - 1\right]^{-\sigma}$$

(b). $g(k,E) = \frac{1}{1+(\beta E)^{-\frac{1}{\sigma}}}\cdot Ak = (A^{1-\sigma}\cdot\beta)^{\frac{1}{\sigma}}\cdot Ak$

$$= A^{\frac{1}{\sigma}}\cdot\beta^{\frac{1}{\sigma}}\cdot k$$

$$\cancel{\Rightarrow s = (A\beta)^{\frac{1}{\sigma}} \Rightarrow A\nearrow \Rightarrow s\nearrow}$$

output growth:

$$r_y = \frac{A\cdot A^{\frac{1}{\sigma}}\cdot\beta^{\frac{1}{\sigma}}\cdot k}{Ak} = (A\beta)^{\frac{1}{\sigma}}$$

$$\Rightarrow A\nearrow \Rightarrow r_y\nearrow$$

saving rate:

$$s = \frac{(A\beta)^{\frac{1}{\sigma}}\cdot k}{Ak} = A^{\frac{1}{\sigma}-1}\cdot\beta^{\frac{1}{\sigma}} = A^{\frac{1-\sigma}{\sigma}}\cdot\beta^{\frac{1}{\sigma}}$$

because $\sigma<1$, $\frac{1-\sigma}{\sigma} > 0$

$$\Rightarrow \frac{\partial s}{\partial A} = \frac{1-\sigma}{\sigma}\cdot A^{\frac{1}{\sigma}-2}\cdot\beta^{\frac{1}{\sigma}} > 0$$

$$\Rightarrow A\nearrow \Rightarrow s\nearrow$$