高级宏观经济学 第一次作业 Life cycle consumption problem

This part involves solving a model of household consumption and savings. Consider a household that lives for periods $t = 0, \ldots, T$, and values consumption in each period, c_t, and has income y_t. His initial assets are $a_0 > 0$, and he can borrow or save, $a_t \gtreqless 0$ for periods $t = 0, \ldots, T$. However he cannot leave debt, $a_{T+1} \geq 0$. There is a common real interest rate that applies to savings or loans, and the real interest rate between time t and $t+1$ is $(1 + r_{t+1})$.

The household has time-separable period utility, and his lifetime welfare from a given vector of consumption, $\{c_t\}_{t=0}^{T}$ is

$$\sum_{t=0}^{T} \beta^t u(c_t), \tag{1}$$

where u is twice-continuously differentiable, strictly increasing and concave. In addition, we assume that $\lim_{c \to 0} u'(c) = \infty$ and $\beta \in (0,1)$. The time t budget constraint is

$$c_t + a_{t+1} \leq (1 + r_t) a_t + y_t,\ t = 0, \ldots, T. \tag{2}$$

1) Define the real interest factor between period t and $t+s$ as $R_{t,t+s} \equiv (1 + r_{t+1}) \cdots (1 + r_{t+s})$ where $t + s > t$. Divide each period t budget constraint by $R_{0,t}$ and sum to derive the lifetime budget constraint,

$$\sum_{t=0}^{T} \frac{c_t}{R_{0,t}} \leq \sum_{t=0}^{T} \frac{y_t}{R_{0,t}} + (1 + r_0) a_0. \tag{3}$$

2) The lifetime budget constraint exists because the consumer can borrow or save across periods. It allows for a solution to the household's problem that involves only two constraints, equation (3) and the non-negativity constraint on a_{T+1} (there are no other necessary non-negativity multipliers). Let Λ and μ be the LaGrange multiplier for these two constraints. Derive the first-order conditions for c_t, a_{T+1} and the Euler equation:

$$u'(c_t) = \beta (1 + r_{t+1}) u'(c_{t+1}). \tag{4}$$

3) Assume that the period utility function is iso-elastic, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ where $\sigma > 0$. Using (4), solve for consumption in each period, c_t, as a function of $R_{0,t}$ and c_0. Label your result (A1) and substitute it into (3) to solve for c_0. Label your result (A2).

4) Simplify the problem by assuming that $y_t = 0$ for all t. When $\sigma > 1$, how does a rise in r_1 affect c_0? Explain your result.

5) Now assume that $y_t > 0$ and $r_t = r$ for $t = 0, \ldots, T$. Consider a temporary tax rebate which increases y_0 but leaves income unchanged in all other periods. If y_0 rises by ε, what is the resultant rise in c_0? This is an illustration of consumption smoothing; convexity of preferences leads the household to spread the rise in consumption over its lifetime.