On The Neoclassical Version of The Dual Economy [1, 2]

ANTHONY M. MARINO
University of Kansas

1. INTRODUCTION

The neoclassical models of a dual economy, in contrast to the classical models, assume that the marginal product of agricultural labour is everywhere positive (cf. [1], [4], [5] and [9]). Here, a necessary condition for the release of agricultural labour to industry is the creation of a food [3] surplus beyond agrarian consumption needs.

Dale Jorgenson, in his celebrated neoclassical model of a dual economy [4], studies purely agrarian growth, the emergence of a viable industrial sector and the asymptotic properties of the two-sector system. The Jorgenson system is purely agrarian if the average product in agriculture is not greater than some critical level. Over this range, all food is consumed by the agricultural sector and the price and income elasticities of demand for food are unity. If agricultural average product exceeds the critical level, an agrarian food surplus is created, and industrial employment is possible. Over this range, the price and income elasticities of demand for food (by all workers) are zero.

Avinash Dixit [1] goes beyond the Jorgenson asymptotic results by characterizing the qualitative features of capital growth at finite time periods. Paul Zarembka [9] generalizes the demand side of the Jorgenson model, by assuming non-zero price and income elasticities of demand for food. Concerning himself mainly with the viability of an existing industrial sector, Zarembka shows that, given a constant industrial capital-output ratio (this is analogous to considering asymptotic behaviour), industrialization is less rapid relative to the Jorgenson model and that the Jorgenson viability condition is unaltered.[4]

A significant drawback of the above models is the assumption of specific production functions (Cobb-Douglas) to describe technological possibilities. The present paper introduces a model of the dual economy which places only the usual neoclassical restrictions on productive relations and considers the implications of this generalization on purely agrarian growth, the emergence of industry, and both the finite and asymptotic properties of the relevant growth paths for two-sector growth. Section 2 presents the basic model, where, for tractability, the Jorgenson demand assumptions are retained.[5] Section 3 studies existence, uniqueness and stability of balanced growth equilibria in an agrarian economy, with and without a positive rate of technical progress. Section 4 gives necessary and sufficient conditions for the possibility of the emergence of a viable industrial sector. Section 5 completely characterizes non-steady-state and steady-state behaviour of the relevant growth paths for two-sector growth. Section 6 contains some conclusions.

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2 The author thanks Robert R. Russell who provided valuable advice in the preparation of an earlier version of this paper. Benefit was also derived from Dale Jorgenson and the referee.

3 The words " food " and " agricultural commodity " are used interchangeably.

4 The latter result is not surprising, since viability depends on the existence of an agrarian food surplus, rather than its size. The size of this surplus then determines the rate of release of labour to industry.

5 Here we are primarily interested in the generation, rather than the size of a food surplus. This assumption allows a more rapid industrialization process and greatly simplifies analytical matters.

2. THE MODEL

Agricultural output is given by

$$Y_1 = e^{\alpha t} f_1(L_1, \bar{N}), \qquad \text{...(1)}$$

where L_1 denotes labour, $\bar{N} > 0$ denotes some fixed amount of land, and $e^{\alpha t}$, $\alpha \geqq 0$, allows for autonomous shifts in output corresponding to technical progress. The function f_1 satisfies $df_1/dL_1 > 0$, $d^2 f_1/dL^2 < 0$ and $f_1(L_1, \bar{N}) = 0$, if $L_1 = 0$. That is, the marginal product of labour is positive, there are diminishing returns to labour on fixed land, and labour is an essential factor.

Industrial production is characterized by a neoclassical production function in the labour augmenting form. Let Y_2, L_2 and K denote industrial output, labour and capital, respectively. Effective industrial labour is given by $E = e^{\beta t} L_2$, $\beta > 0$. Production is described by

$$Y_2 = f_2(E, K). \qquad \text{...(2)}$$

For given t, f_2 is homogeneous of degree one, $f_2(E, K) = 0$, if $E = 0$ or $K = 0$, and $\partial f_2/\partial E$, $\partial f_2/\partial K > 0$ and $\partial^2 f_2/\partial E^2$, $\partial^2 f_2/\partial K^2 < 0$. Note that technical progress is assumed to be Harrod neutral in industry, while it is assumed to appear as an autonomous shift parameter in agriculture.

Industrial producers maximize profits, while for the agricultural sector the profit maximization motive is not assumed. Each agricultural worker receives an average share of total income which includes both wages and rent. For agricultural workers to seek industrial employment, some differential between their average income and the industrial wage may be required. If such a differential exists, it is assumed to be proportional to the industrial wage. Let w, r and p denote the industrial wage, the price of capital, and the price of the agricultural good, respectively, where all are in terms of the industrial good. Define $y_1 = Y_1/L_1$. We have

$$w = e^{\beta t}(\partial f_2/\partial E), \; r = \partial f_2/\partial K, \text{ and } \sigma w = p y_1, \; \sigma \in (0, 1]. \qquad \text{...(3)}$$

Capitalists save all earnings, while workers consume all earnings. Define $k = K/E$ and let g be the intensive form of f_2. The growth of capital is given by

$$\hat{K} = g'(k) - \mu, \qquad \text{...(4)}$$

where $\hat{K} = d(\ln K)/dt$ and μ is the depreciation rate. Since capital is assumed to be an essential factor, a positive capital stock is required for positive industrial production. We assume that at the time of the inception of industry there is a $K > 0$ which is available from outside the system.

Let the working population equal total population and measure the latter in man-units. Define $L = L_1 + L_2$ and $l_i = L_i/L$, $i = 1, 2$. Denote agricultural output per man-unit, Y_1/L, as y, and term $y^* > 0$ the " critical level ". Population growth is given by

$$\hat{L} = \begin{cases} \delta(y) & \text{if } y < y^*, \\ n & \text{if } y \geqq y^*. \end{cases} \qquad \text{...(5)}$$

The function δ is increasing, continuous, one-to-one, and onto, mapping $(0, y^*)$ into (c, n), where $c \in (-\infty, 0)$.

For $y < y^*$, $\hat{L}$ is an increasing function of food per man-unit, while for $y \geqq y^*$, $\hat{L}$ is constant at rate n. As $y \to 0$, $\hat{L} \to c$, where c may be thought of as a constant force of mortality.

Following Jorgenson [4], if $y_1 \leqq y^*$, all labour remains on the land and all agricultural output is consumed by the agricultural sector. If $y_1 > y^*$, industrial employment is possible and, given aggregate balance, consumption and production of food per man-unit remain constant at $y = y^*$. Once y_1 reaches y^*, any increases in agrarian consumption are in terms of the industrial good. The relation governing labour distribution is then given by

$$l_1 = \min(y^*/y_1, 1). \qquad \ldots(6)$$

From (6), $y<y^*$ implies $y_1 = y$. Moreover, $y \geqq y^*$ if and only if $y_1 \geqq y^*$. Hence in (5) (population growth), food per man-unit, y, may be replaced by agricultural average product, y_1.

This completes the specification of the basic model. We turn next to agriculture in isolation.

3. GROWTH PROCESSES IN A PURELY AGRARIAN ECONOMY

If the economy is purely agrarian, $l_1 = 1$, and, from (6), $y_1 \in (0, y^*]$. Since f_1 is strictly concave and twice differentiable in L_1, there exists a function $\eta(y_1) = (dY_1/dL_1)(1/y_1)$ which maps from each average product into a value of the elasticity of agricultural output with respect to labour. Clearly, $\eta(y_1) \in (0, 1)$, $y_1 > 0$, and the growth of y_1 can be written as a continuous function of y_1, for $y_1 \in (0, y^*]$. The growth of output per worker in agriculture is given by

$$\hat{y}_1 = \begin{cases} \alpha+[\eta(y_1)-1]\delta(y_1), & \text{if } y_1 \in (0, y^*), \\ \alpha+[\eta(y_1)-1]n, & \text{if } y_1 = y^*. \end{cases} \qquad \ldots(7)$$

The purely agrarian economy has a balanced growth equilibrium (BGE) if there exists a $\bar{y}_1 \in (0, y^*]$ such that $\hat{y}_1 = 0$. A ratio $\bar{y}_1$ with the latter property is termed " balanced ", and the growth rates of L_1 and Y_1 secured by $\bar{y}_1$ are termed " balanced growth rates ". We will study the characteristics of such equilibria with and without the assumption of a positive rate of technical progress in agriculture.

First, consider the case where the rate of technical progress is zero. From the assumptions on the population growth function δ, there is a unique $y_1^0 \in (0, y^*)$ such that $\delta(y_1^0) = 0$. Since η is a positive fraction, we have from (7) that the growth of output and population are zero, if and only if $y_1 = y_1^0$. Moreover, $y_1 \in (0, y_1^0)$ implies $\hat{y}_1 > 0$, while $y_1 \in (y_1^0, y^*]$ implies $\hat{y}_1 < 0$. Hence, with no technical progress, the agrarian economy has a unique Malthusian BGE which is locally asymptotically stable over $(0, y^*]$. Path I, Figure 1, illustrates the growth of output for this case.

In the case where the rate of technical progress is positive, a pure Malthusian BGE is not possible, since at any balanced $\bar{y}_1$, balanced growth rates given by

$$\hat{Y}_1 = \hat{L}_1 = \alpha/[1-n(\bar{y}_1)]$$

are strictly positive. Given that $\alpha > 0$, a sufficient condition for the existence of a BGE is given by

(C.1) $$\alpha+[\eta(y_1)-1]n \leqq 0, \quad \text{for } y_1 = y^*.$$

Condition (C.1) states that the rate of technical progress is no greater than a positive fraction (determined by $\eta(y^*)$) of the maximum rate of population growth. In terms of (7), (C.1) states that $\hat{y}_1 \leqq 0$, for $y_1 = y^*$. If $\alpha > 0$, from above, we have that $\hat{y}_1 > 0$, for $y_1 \in (0, y_1^0]$. Since $\hat{y}_1$ is continuous in $y_1 \in (0, y^*]$, (C.1) guarantees the existence of a $y_1 \in (y_1^0, y^*]$ such that $\hat{y}_1 = 0$. Path II, Figure 1, illustrates an example of output growth for this case.

By employing the mean-value theorem for derivatives, it is easily shown that condition (C.1) implies the existence of a balanced y_1 which is asymptotically stable over some region contained in $(0, y^*]$, although other balanced y_1 may be unstable. Since $\hat{y}_1$ is not necessarily a monotone function of y_1, (C.1) does not ensure uniqueness and stability of a balanced y_1 over the entire interval $(0, y^*]$. However, if η is non-increasing in y_1, then, from (7), $\hat{y}_1$ is monotonically decreasing y_1. In this case, it is clear that (C.1) is necessary and

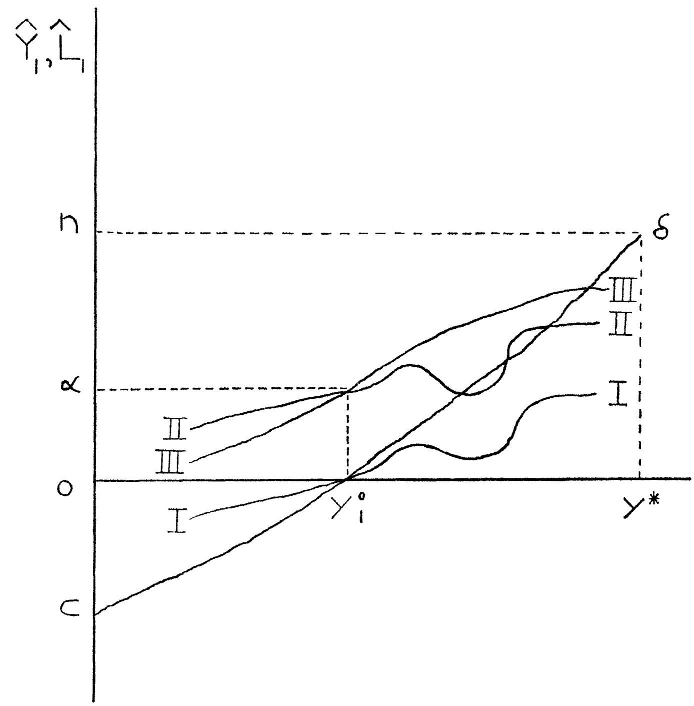

FIGURE 1
Balanced growth in a purely agrarian economy.

sufficient for the existence of a unique balanced $y_1 \in (0, y^*]$ which is asymptotically stable over $(0, y^*]$. Path III, Figure 1, illustrates output growth for this case.

Note that if η is a constant function, we have the Jorgenson-Dixit-Zarembka special case where f_1 is Cobb-Douglas. As another example, if f_1 is derived from a CES technology and the elasticity of substitution exceeds one, η is decreasing in y_1.

The results of this section point out that high rates of population growth relative to the rate of technical progress and diminishing returns to labour can trap an agrarian economy at a lower level balanced growth equilibrium.

4. THE EMERGENCE OF AN INDUSTRIAL SECTOR

Since the basic economy is purely agrarian if and only if $y_1 \in (0, y^*]$, here we wish to consider how it would be possible for y_1 to rise above y^* and remain above that level. More precisely, we say that an industrial sector *can emerge* if for some initial $y_1(0) \in (0, y^*]$, $\lim_{t \to \tau} y_1(t) > y^*$, for $\tau > 0$. From Section 3 it is clear that agriculture must have a positive rate of technical progress for emergence to be possible.

Consider the condition

$$\text{(C.2)} \qquad \alpha + [\eta(y_1) - 1]n > 0, \quad \text{for } y_1 \geqq y^*, \; y_1 \in u(y^*),$$

where $u(\cdot)$ denotes a neighbourhood. Condition (C.2) says that the rate of technical progress in agriculture is greater than a positive fraction (determined by η for $y_1 \geqq y^*$, $y_1 \in u(y^*)$) of the maximum rate of population growth. From (6), agricultural output is just $Y_1 = Ly^*$, for $y_1 > y^*$. Hence, for $y_1 > y^*$, the growth of agricultural output and labour are given by $\hat{Y}_1 = n$ and $\hat{L}_1 = (n - \alpha)/\eta(y_1)$. It follows that (C.2) holds if and only if $\hat{y}_1 > 0$ for $y_1 \geqq y^*$, $y_1 \in u(y^*)$. Utilizing this result, it is easily shown that (C.2) is necessary and sufficient for the possibility of the emergence of an industrial sector. Given that an industrial sector emerges from an agrarian base, it is also evident that (C.2) guarantees its sustenance over time.

Since (C.2) is a local condition, it admits the situation where $\hat{y}_1 \leqq 0$, or, equivalently, $n<(n-\alpha)/\eta(y_1)$, for some $y_1 \in (y^*, +\infty)$. Since $L = L_1+L_2$, we have that $\hat{L}_2 \leqq n$ and $dl_2/dt \leqq 0$. That is, the proportion of labour employed in industry does not rise over time. Since emergence entails sustenance, l_2 cannot decline to zero (or y_1 cannot fall to y^*), except, possibly, if the system is disturbed.

The most interesting case for the study of industrialization is where

$$\text{(C.3)} \qquad \alpha+[\eta(y_1)-1]n>0, \quad \text{for } y_1 \geqq y^*,$$

holds. Since (C.3) implies (C.2), emergence is possible. Moreover, (C.3) holds if and only if $\hat{y}_1>0$, for $y_1 \geqq y^*$. Thus, (C.3) is necessary and sufficient for the possibility of emergence and that the proportion of labour in industry rises over time.[1]

The results of this section suggest a familiar policy prescription. If industrialization is desired, an agrarian economy should take steps to introduce population control measures (reduce n) and increase the rate at which new techniques are introduced (increase α).

5. GROWTH PROCESSES IN A TWO SECTOR ECONOMY

Two-sector growth will be studied only for the case where an industrial sector emerges and employs an increasing share of the labour force. Hence, in what follows, we assume that (C.3) holds.

Choose $t = 0$ such that $y_1(0) = y^*$. From (5), population growth is given by $\hat{L} = n$ and, from (6), agricultural output also grows at rate n. From Section 4, the growth of output per worker in agriculture is given by

$$\hat{y}_1 = \alpha+[n(y_1)-1]\hat{L}_1, \qquad \ldots(8)$$

and agricultural labour growth is given by

$$\hat{L}_1 = (n-\alpha)/\eta(y_1). \qquad \ldots(9)$$

By (C.3) $\hat{y}_1>0$. From (9), $\hat{L}_1 \gtreqless 0$ if and only if $n \gtreqless \alpha$. Moreover, the qualitative movement of $\hat{L}_1$ satisfies $d\hat{L}_1/dt \geqq 0$ if $n \geqq \alpha$, where $d\hat{L}_1/dt = -\hat{\eta}\hat{L}_1$.[2] In the case where $\alpha>n$, the sign of $d\hat{L}_1/dt$ is ambiguous.

Consider the case where $n \geqq \alpha$. From the above results, $\hat{L}_1$, $d\hat{L}_1/dt \geqq 0$ and $\hat{\eta}$, $d\eta/dt \leqq 0$. Since, by (C.3), $\hat{y}_1>0$, it follows that $d\eta/dy_1 \leqq 0$. Thus, η is non-increasing in y_1, and from (8), $\hat{y}_1$ is non-increasing in y_1, for $y_1>y^*$. If, in addition, η is non-increasing in y_1 over $(0, y^*]$, the results of Section 3 point out that (C.3), which implies the converse of (C.1), guarantees that there does not exist a balanced $y_1 \in (0, y^*]$ for a purely agrarian economy.

From (6), the proportion of labour in industry is given by

$$l_2 = (1-y^*/y_1). \qquad \ldots(10)$$

Since $y_1 = y^*$ for $t = 0$, $\lim_{t\to 0} l_2 = 0$ and $\lim_{t\to 0} l_1 = 1$. By (C.3), $\hat{y}_1>0$, for all $y_1 \geqq y^*$ and all $t \geqq 0$, so that $\lim_{t\to\infty} y_1 = +\infty$. From (10), we have that $\lim_{t\to\infty} l_2 = 1$ and $\lim_{t\to\infty} l_1 = 0$. Initially the economy is purely agrarian and over time the proportion of labour in industry increases steadily. In the limit the economy approximates an industrial economy of the neoclassical variety.

[1] Note that if η is a constant function of y_1, (C.2), (C.3) and the converse of (C.1) are equivalent. This is the Jorgenson-Dixit-Zarembka special case where f_1 is Cobb-Douglas. See [1, p. 299], [4, p. 324] and [9, p. 109].

[2] We have that $d\hat{L}_1/dt = -\hat{\eta}\hat{L}_1$, where $\hat{\eta} = [(d^2Y_1/dL_1^2)(dL_1/dt)/(dY_1/dL_1)]-\hat{y}_1$. By (C.3) $\hat{y}_1>0$ and from the assumptions on f_1, $d^2Y_1/dL_1^2<0$ and $dY_1/dL_1>0$. Hence, the assertion holds.

The growth of industrial labour, as calculated from (10), is given by

$$\hat{L}_2 = n+(l_1/l_2)(\hat{y}_1). \qquad \text{...(11)}$$

Routine calculation yields $d\hat{L}_2/dt<0$.[1] From the analysis of (10), we have that

$$\lim_{t\to 0} \hat{L}_2 = +\infty \text{ and } \lim_{t\to\infty} \hat{L}_2 = n.$$

Initially the growth of industrial labour is arbitrarily large and over time it is monotonically decreasing. In the limit, it approaches the rate of population growth. The growth of industrial labour in efficiency units, $\hat{E} = \hat{L}_2+\beta$, is illustrated in Figure 2.

From the above results, no balanced growth equilibrium exists for the two-sector economy at finite time periods. However, as $t\to\infty$, the possibility for an asymptotic BGE presents itself. The growth of capital, from (4), is given by $g'(k)-\mu$, where, from the assumptions on f_2, $g'(k)$ is monotonically decreasing in k. Define $\gamma = n+\beta+\mu$. Since $\lim_{t\to\infty} \hat{E} = n+\beta$, we have the usual neoclassical problem where

(C.4) $$\lim_{k\to 0} g'(k)>\gamma> \lim_{k\to\infty} g'(k)$$

is sufficient for a unique asymptotic BGE which is locally asymptotically stable over $k\in(0, +\infty)$. At such an equilibrium, Y_2, E and K grow at rate $n+\beta$, while L_2 grows at rate n.

Next, we turn attention to the basic differential equation for the analysis of capital growth. Time differentiation of (4) yields

$$d\hat{K}/dt = [-kg''(k)][\hat{E}-\hat{K}]. \qquad \text{...(12)}$$

Since $-kg''(k)>0$, $t>0$, we have from (12) that

$$d\hat{K}/dt \gtreqless 0 \text{ if and only if } \hat{E} \gtreqless \hat{K}. \qquad \text{...(13)}$$

For very small t, $\hat{K}$ is finite so that $\hat{E}>\hat{K}$ and, from (13), $\hat{K}$ is increasing, unless it reaches $\hat{E}$. If $\hat{K}(t^*) = \hat{E}(t^*)$, then $\hat{K}$ has a zero derivative at t^*. Since $\hat{E}$ monotonically decreases from plus infinity to approach $n+\beta$ in the limit, $\hat{E}$ intersects $\hat{K}$ from above at any such critical value, $\hat{K}(t^*)$. Utilizing the monotonicity of $\hat{E}$ and (13), it follows that $\hat{K}$ has a unique maximum at t^*. Such a process is called *finite maturity* and $\hat{K}(t^*) = \hat{E}(t^*)$ is called the *finite maturity value* of $\hat{K}$.

The asymptotic properties of capital growth depend in part on whether the economy becomes finitely mature. In the finite maturity case, $\hat{K}(t^*)$ is an upper bound and $n+\beta$ a lower bound for capital growth in the interval $(t^*, +\infty)$. From (13), it follows that $\lim_{t\to\infty} \hat{K} = d$, where $d\in[n+\beta, \hat{K}(t^*))$. If the economy does not become finitely mature, $n+\beta$ is an upper bound and $-\mu$ is a lower bound for capital growth in the interval $(0, +\infty)$. Again, from (13), $\lim_{t\to\infty} \hat{K} = d'$, where $d'\in(-\mu, n+\beta]$.

Figure 2 illustrates four possible patterns of capital growth. In Cases I and II, the economy does not become finitely mature and, for the sake of example, we assume that $\lim_{k\to\infty} g'(k) = \lim_{t\to 0} g'(k) = 0$ and $\lim_{t\to\infty} \hat{K} = d'>0$. In Case I, capital growth does not reach efficiency labour growth at finite t or as t tends to infinity. Here, $d'<n+\beta$ and there is long-run labour deepening. Case II illustrates the situation where condition (C.4) is satisfied ($d' = n+\beta$), and the economy becomes mature only asymptotically.[2]

[1] Differentiating,

$$d\hat{L}_2/dt = (l_1/l_2)[c\hat{L}_1/\eta(y_1)-\hat{L}_2\hat{y}_1], \text{ where } c = [(d^2Y_1/dL_1^2)(dL_1/dt)]/y_1.$$

Clearly, $c \geqq 0$ if $dL_1/dt \leqq 0$, and $c<0$ if $dL_1/dt>0$. By (C.3), $-\hat{L}_2\hat{y}_1<0$, and the result holds.

[2] This is the only case which Dale Jorgenson considers. Since, for Jorgenson, f_2 is Cobb-Douglas, condition (C.4) is satisfied and there is an asymptotic BGE. However, Jorgenson neglects to consider the case of finite maturity.

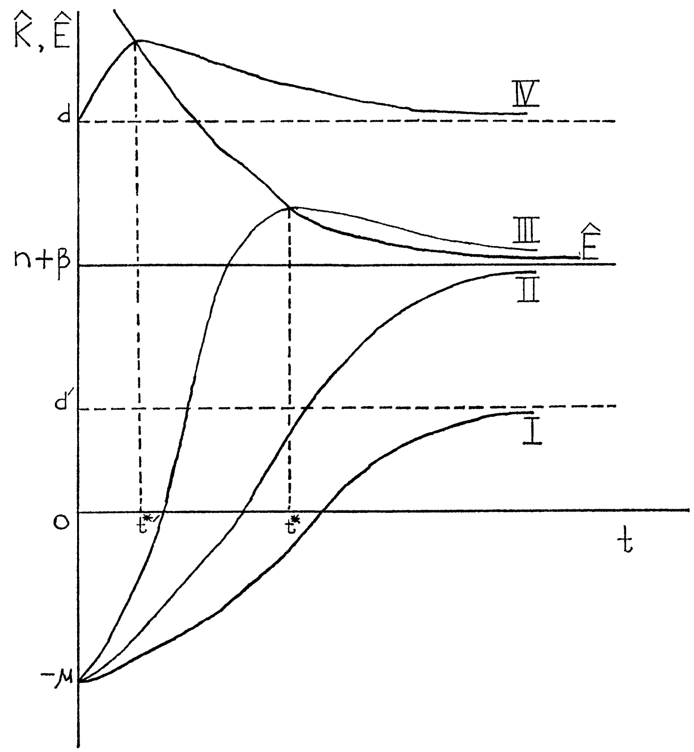

FIGURE 2
Patterns of growth for industrial capital.

Cases III and IV illustrate alternative paths in the finite maturity case. First, suppose that condition (C.4) is met and, for simplicity, that $\lim_{k\to\infty} g'(k) = \lim_{t\to 0} g'(k) = 0$ (Case III). Here, capital growth increases to $\hat{K}(t^*) = \hat{E}(t^*)$ and decreases thereafter, remaining above $\hat{E}$. As $t\to\infty$, $\hat{K}\to n+\beta$, and we have finite as well as asymptotic maturity. Second, let there be no asymptotic BGE (Case IV). In this case, condition (C.4) fails, and

$$\lim_{k\to\infty} [g'(k)-\mu] = d,$$

where $d>n+\beta$. Here, capital growth initiates above $n+\beta$, increases to its finite maturity value, $\hat{K}(t^{*\prime})$, and decreases thereafter, remaining above $\hat{E}$. As $t\to\infty$, $\hat{K}\to d$, and there is long-run capital deepening.[1]

Finally, it is of interest to specify the conditions under which the economy becomes finitely mature. From (13) and Figure 2, it is clear that the failure of condition (C.4) in the sense of Case IV entails finite maturity, while the failure of (C.4) in the sense of Case I makes finite maturity an impossibility. Hence, we wish to specify sufficient conditions for finite maturity, under the assumption that (C.4) holds. Invoking a slightly stronger form of (C.4), we assume that $\lim_{k\to\infty} g'(k) = 0$ and $\lim_{k\to 0} g'(k) > \gamma$.

Avinash Dixit [1] shows that $n>\alpha$ is sufficient for finite maturity, for the case where production functions are Cobb-Douglas. This condition says that the maximum rate of population growth is greater than the rate of technical progress in agriculture or, from

[1] Avinash Dixit [1] discovers the possibility of Case III in the Jorgenson model. However, since production functions are Cobb-Douglas, non-steady state asymptotic capital growth (Cases I and IV) cannot be analysed.

(9), that the growth of agricultural labour is positive.[1] In the present model, we find that the condition $n \geqq \alpha$ is sufficient for finite maturity, given the assumptions: (*a*) η is a constant function of y_1, for $y_1 \geqq y^*$, $y_1 \in u(y^*)$; and (*b*)

$$\lim_{E\to 0} g''(k)k = \lim_{E\to 0} (\partial^2 f_2/\partial K^2)K = 0.$$

Given (C.3), assumption (*a*) implies that the agricultural production function is Cobb-Douglas at the time of the inception of industry. Assumption (*b*) is satisfied if f_2 is CES and the elasticity of substitution is in the interval (0, 1]. Moreover, it is easily shown that (*b*) implies that $\lim_{t\to 0} l_2(d\hat{K}/dt) = 0$.[2]

To prove the above assertion suppose, on the contrary, that the economy only becomes asymptotically mature. Then we have that

$$d(l_2\hat{E} - l_2\hat{K})/dt < 0, \qquad \ldots(14)$$

for finite t. Using the results

$$dl_2/dt = -dl_1/dt, \hat{l}_1 = -\hat{y}, d\hat{L}_1/dt = -\hat{\eta}\hat{L}_1 \text{ and } l_2\hat{E} = n + l_2\beta - l_1\hat{L}_1,$$

we can, from (14), write

$$-\hat{y}_1[\hat{L}_1 - (\hat{K} - \beta)] - \hat{\eta}\hat{L}_1 + (l_2/l_1)d\hat{K}/dt > 0. \qquad \ldots(15)$$

Utilizing the result $\lim_{t\to 0} l_1 = 1$ and the assumptions (*a*) and (*b*), the limit of (15) as t tends to zero results in the inequality

$$\hat{L}_1(0) < -(\mu + \beta). \qquad \ldots(16)$$

Thus, $n \geqq \alpha$ is sufficient for finite maturity, under the above assumptions.

6. CONCLUDING REMARKS

The results on agrarian growth and the emergence of industry readily suggest that to avoid a low-level equilibrium trap and attain sustained industrial growth, an economy must take explicit account of its agricultural sector. Under the assumption that an industrial sector emerges and condition (C.3) holds, the results on two-sector growth point out that any $K(0) > 0$ can carry the system into sustained growth of industrial output. Hence, the critical minimum effort of capital accumulation is not a necessary ingredient of the present model. If the converse of condition (C.3) holds, then the two-sector economy will degenerate to its agrarian base. In this case, $y_1 \to y^*$, $l_2 \to 0$ and capital decays without replacement.

We have been able to provide sufficient conditions for the process of finite maturity under fairly restrictive assumptions. However, we have attained a marginal improvement on the Dixit result.

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[1] Satisfaction of (C.3) and $n > \alpha$ in the present model prohibits η from declining to zero as y_1 tends to infinity. Thus, we rule out the class of agricultural production functions with labour as a non-essential factor and everywhere finite marginal products. The assumption that labour is essential, taken by itself, does not appear to rule out the above limiting behaviour of η.

[2] The expression $l_2(d\hat{K}/dt)$ is given by $l_2K(\partial^2 f_2/\partial K^2)\hat{K} - l_2\beta(\partial^2 f_2/\partial K^2)K - (dL_2/dt)(\partial^2 f_2/\partial K^2)K/L$. Clearly $\lim_{t\to 0} l_2 = \lim_{t\to 0} E = 0$ and, by assumption, $\lim_{E\to 0} g' = \lim_{E\to 0} (\partial^2 f_2/\partial K^2)K = 0$. Thus, the result holds.

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