A Theory of Commodity Price Fluctuations

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This paper studies the price fluctuations of storable commodities that are traded in open markets and are subject to random shocks to demand or, more particularly, to supply. It relaxes the common assumption that the shocks are identically and independently distributed in favor of temporally dependent and periodic disturbances. The existence of a unique stationary rational expectations equilibrium is demonstrated for each of the models analyzed, and testable implications of the models are derived. An illustrative empirical investigation is then undertaken for the model with periodic disturbances using monthly time-series observations for seven commodities over the period 1960–93.

I. Introduction

In models of price determination for storable commodities, knowledge of the quantities produced and stored obviously forms an important ingredient in the derivation of testable predictions about the time path of prices. This paper explores the extent to which predictions can be obtained when statistical inferences, from practical necessity, must be made from prices alone, in the absence of data on quantities. That predictions are difficult to obtain in models of speculative price equilibrium is clear from the important early paper of Samuelson (1957), which concentrated, almost entirely, on circumstances of perfect foresight in the absence of uncertainty. When uncertainty

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is allowed, the assumption of rational expectations pioneered by Muth (1961) remains the most attractive way of ruling out arbitrary behavior, and this approach, together with the further contributions of Samuelson (1971, 1972), provides the analytical background for this paper.

The theory developed below builds on these early contributions, and later work,[1] by investigating the implications of different assumptions defining the stochastic process that governs the "harvest" disturbances about which economic agents form their expectations. Although very general processes can be allowed when the intention is to obtain fundamental properties such as the existence of equilibrium, more definite predictions are typically derived only after assuming that the random shocks are independently and identically distributed (i.i.d.) (see, e.g., Danthine 1977; Scheinkman and Schechtman 1983). In many applications it is desirable to allow the disturbances to be drawn from less restrictive probability distributions that relax the i.i.d. assumption and permit either time dependence or heterogeneity in the disturbances. In achieving this goal the analysis below follows the lead given by the innovative paper by Deaton and Laroque (1992), which, however, like its predecessors, restricted attention to the i.i.d. case.

The temporal dependence studied below is based on a simple, but fairly general, Markov process that could account for a wide range of fluctuations, in which the shocks persist from one period to another. Such persistence could be occasioned by the physical effects of crop variation from one year to the next as soil nutrients are dissipated after an abundant harvest or are replenished after a niggardly season. Similarly, the vagaries of the weather could induce serial dependence especially over short spells of time. The principal difficulty that emerges immediately, however, is that the equilibrium price function in such circumstances has as an argument the current output of the commodity or, at the very least, some measure of the proximate source of the output fluctuations. Given that one of the motivations here is to study price fluctuations when quantity information is absent, the assumption of temporal dependence has limited value unless it can be embodied in a framework in which rather more information than price alone is available.

The assumption of heterogeneity poses different, and potentially even more severe, problems. Complete generality would allow any pattern of (independent) disturbances and thus rule out virtually nothing. Some restriction must be introduced to permit empirical

[1] For a comprehensive survey of the literature on commodity markets, see Williams and Wright (1991).

testing, and it is assumed here that heterogeneity can be represented by periodic disturbance distributions. In the periodic disturbances model, time is divided into epochs, each epoch being composed of a fixed number of primitive time periods. The epochs studied in this paper are interpreted as years, each of which comprises 12 months. Obviously, this is but one example among many, but it serves well enough for the purpose of outlining the theory and also forms the basis of the empirical work reported here. The way in which this approach allows for the introduction of heterogeneous disturbances while permitting empirical estimation and testing is explained in detail in Section III.

Section II begins with a brief overview of commodity price determination and notes how the approach adopted here is related to standard models of futures markets. This serves also to highlight the role of storage and the ways in which inferences can be drawn about the effects of inventories on commodity prices. In Section III the underlying stochastic assumptions, which govern the sample paths of prices, are studied in detail beginning with the case of i.i.d. disturbances. We build on the concept of *stationary rational expectations equilibrium* to obtain existence theorems for (unique) equilibrium price functions for models of time-dependent and periodic disturbances.[2] Also, it will be demonstrated that testable predictions can be derived if additional assumptions are introduced with respect to the patterns of disturbances. Despite the restrictiveness of the assumptions, the models can account for a wide range of empirical fluctuations, and it will be shown that there is ample scope for the inclusion of information that would be specific to any particular commodity under investigation. A different, and complementary, approach that focuses on the autocorrelation properties of observed commodity prices is followed by Deaton and Laroque (1996).

Section IV presents an empirical illustration using monthly price data for the period 1960–93 for seven commodities: coffee, sugar, wheat, maize, rice, soybeans, and cotton. The section begins with a statement of the empirical formulation of the periodic disturbances model. The generalized method of moments estimation technique is then applied to periodic disturbance models for the price data on each of these commodities. Evidence that tends to support the periodic disturbances assumption over an alternative autoregressive model of prices is found despite the tentative and illustrative nature of the empirical models. The sense in which the Deaton and Laroque

[2] The extended version of this paper (Chambers and Bailey 1994), available from the authors, includes a technical appendix containing proofs of all the results stated in Sec. III.

(1992) model can be interpreted as a special case of the periodic disturbances assumption will become clear in Section III, but it is argued in Section IV that, empirically, pairwise comparisons with a common alternative, the simple autoregressive model, provide a straightforward basis for testing among the models.

The empirical experiments discussed in Section IV cannot pretend to describe the operation of the markets for the commodities in question and do not purport to capture the peculiar, and possibly unique, patterns of heterogeneity that would be applicable in each market. Given that a common parametric specification is imposed for each commodity, it is, perhaps, rather surprising that the specifications fare as well as they do. It should become clear how additional information about such features as the timing of harvests could be employed to enhance the capacity of the models to account for the observed fluctuations of commodity prices.[3]

Section V concludes with a summary of the findings of the paper.

II. Fundamentals of the Theory

The type of commodity for which the following analysis is appropriate is one in which, during each discrete interval of time, t, the quantity available for trade comprises the "harvest," a random variable with realization denoted by z_t, plus stocks held over from the previous period net of any deterioration (to represent storage costs) that has taken place, $(1 - \delta)S_{t-1}$, where S_{t-1} denotes the amount stored in $t - 1$ and δ is the (exogenous and fixed) rate of wastage. The demand for the commodity consists of two elements: (*a*) the final demand and (*b*) the amount, S_t, held in store by traders. It is not assumed that inventories of the commodity provide any benefits other than the revenue to be generated by subsequent sale, but an implicit "convenience yield" could emerge under circumstances that are noted below.

The final demand for the commodity is denoted by $D(p_t) - v_t$, where $D(p_t)$ is the systematic component of demand as a function of market price, p_t, and v_t denotes the random, unsystematic component of demand. The simplest interpretation of final demand is that it is demand for consumption, but an alternative interpretation, which will be exploited below, allows for trade flows to and from the market so that $D(p_t) - v_t$ represents consumption plus net exports.

[3] A substantive application of the periodic disturbances model is made in Chambers and Bailey (1995), which shows how monthly wheat price fluctuations in England from 1685 to 1850 can be interpreted in the context of the framework developed below.

The spot market clearing condition is given by

$$z_t + (1 - \delta)S_{t-1} = D(p_t) - v_t + S_t. \tag{1}$$

Agents are assumed to know the function $D(\cdot)$, and before trade takes place each period, they observe the harvest, z_t, and the demand shock, v_t, together with the net (of storage costs) stock, $(1 - \delta)S_{t-1}$. Even if output measurements are available, it is not always feasible to distinguish empirically between z_t and v_t (and thus it is certainly not possible to do so here), but they are included in equation (1) to aid interpretation later on. Another aid for later interpretation is to note that systematic supply responses to price can be absorbed into $D(\cdot)$, which then represents *net* demand, a function with the same qualitative properties (provided, of course, that supply responds positively to price).

It is convenient to rewrite equation (1) as

$$x_t \equiv w_t + y_t = D(p_t) + S_t, \tag{2}$$

where $w_t \equiv z_t + v_t$ and $y_t \equiv (1 - \delta)S_{t-1}$, the carryover of stocks from the previous period net of storage costs. Ignoring the demand shock, v_t, we can interpret the value of x_t as the quantity of the commodity available to satisfy final demand and storage in period t, an amount that is known to agents before trading commences. In two of the models to be introduced, x_t can be used to represent the state variable of the system, representing the information available to the agents at time t. However, in at least one case (that of time-dependent disturbances), the value of w_t conveys additional information and can, in principle, be observed by agents at time t. In this case there are two state variables, y_t and w_t (or, equivalently, x_t and w_t).

The properties of the final demand function and its inverse, $P(\cdot) \equiv D^{-1}(\cdot)$, are expressed in the following assumption.

ASSUMPTION 1. (i) The demand function D: $(p_0, p_1) \rightarrow \mathbf{R}$ is continuous and strictly decreasing on its domain with $0 \leq p_0 < p_1$ and $\lim_{p \rightarrow p_0} D(p) = +\infty$. (ii) $0 < P(w^*) < p_1$, where w^* denotes the infimum of the support for the disturbance, w.

Assumption 1 asserts simply that the final demand function has a conventional negative relationship with p_t and ensures that, in the models of Section III, the equilibrium price is unique and bounded away from zero from below and has a finite upper bound.

The demand to hold stocks is assumed to originate from the activities of agents be they arbitragers, hedgers, or speculators. For the moment it is necessary to assume only that every agent can borrow or lend funds at a known and fixed interest rate, r, per period. Formally, the following assumption is required throughout.

Assumption 2. $0 < \theta \equiv (1 - \delta)/(1 + r) < 1$.

The demand for inventories is assumed to be the outcome of price-taking, profit-maximizing behavior on the part of risk-neutral inventory holders. Their objective is, then, to maximize the expected value of profit, $(\theta E_t p_{t+1} - p_t)S_t$. In view of the linear storage technology and risk-neutral behavior, profit maximization is characterized by

$$(\theta E_t p_{t+1} - p_t)S_t = 0, \quad S_t \geq 0, \theta E_t p_{t+1} - p_t \leq 0. \tag{3}$$

The interpretation of (3) is simply that if expected profit per unit of inventory, $\theta E_t p_{t+1} - p_t$, is negative, zero stocks are held; otherwise market forces drive the price to the level at which expected profit is zero, the volume of inventories being equal to the quantity available for sale minus the final demand.[4]

The remainder of the paper studies the determination of the spot price p_t by making strong, testable assumptions about the source of uncertainty, that is, about the stochastic process generating the disturbances, w_t. Following Deaton and Laroque (1992), we express the market price, p_t, as a function of x_t, the quantity of the commodity available to satisfy final demand or storage at date t. Under certain conditions, such an equilibrium function can be shown to exist, the equilibrium function representing a stationary rational expectations equilibrium (SREE). It is common (see, e.g., Danthine 1977; Deaton and Laroque 1992) to confine the analysis to i.i.d. disturbances across time. The relaxation of this assumption is the focus of this paper.

The most general stochastic process studied here can be expressed by a transition function, $Q^s(w, w')$, which can be interpreted as the conditional probability of next period's disturbance, "harvest," given the current period's harvest. The superscript s indicates that the distribution could differ across periods. More formally, $Q^s(a, A) = \Pr\{w' \in A | w = a\}$ is the probability that the harvest, w', in the next period (i.e., w_{t+1}) is a member of the set, A, given that the harvest, w, in the current period (i.e., w_t) is equal to a. It is also assumed that the random variable is drawn from a distribution with a compact support; that is, each harvest w' is assumed to be a member of a set

$$W^s = \{w' \in \mathbf{R} | -\infty < \underline{w}^s \leq w' \leq \overline{w}^s < +\infty\}. \tag{4}$$

In what follows the set $\mathcal{W}^s$ denotes the Borel algebra of sets for W^s. Details are contained in the technical appendix to Chambers and Bailey (1994).

[4] An alternative derivation of (3) is to note that if $E_t p_{t+1}$ is replaced by the forward price at time t for delivery at $t + 1$, $F_{t,t+1}$, the resulting condition simply defines the absence of arbitrage opportunities in the forward and spot markets for the commodity. Then (3) is obtained by assuming informational market efficiency, i.e., $F_{t,t+1} = E_t p_{t+1}$.

Time dependency of the disturbances is captured by the familiar first-order Markov process in which the realized value of the harvest affects the probability distribution of the next period's harvest.[5] Periodicity of the disturbances is captured by allowing the transition functions to differ across time periods: this is the purpose of the s superscript in the definition of $Q^s(\cdot, \cdot)$. Evidently, the most general case, in which the transition function could differ for *every* time period, is empirically vacuous, and a method is proposed below to allow for a tractable but testable degree of heterogeneity.

The main implications of this section can be summed up with reference to (3). In the event that stocks are positive ($S_t > 0$), the equilibrium market price is such that the pure profits from storage are zero so that $p_t = \theta E_t p_{t+1}$. Alternatively, if stocks are zero,[6] final demand equals the total output plus carryover from the previous period, x_t, so that market price is given by $p_t = P(x_t)$. More compactly,

$$p_t = \max[\theta E_t p_{t+1}, P(x_t)]. \tag{5}$$

Together with a stochastic specification of the w_t disturbances, equation (5) provides the basis for an equilibrium price function that, under certain circumstances, will permit characterization of $E_t p_{t+1}$ and enable empirical estimation and testing using price data alone.

III. Alternative Stochastic Specifications

This section provides substance to the framework developed in Section II by studying the predictions implied by various forms of the probability law for the underlying "harvest" disturbances. The three categories—i.i.d., time-dependent, and periodic disturbances—are considered in turn, beginning with a statement of results for the i.i.d. case studied in detail by Deaton and Laroque (1992).

[5] The assumption of a single lag is not restrictive. Stokey and Lucas (1989, p. 237) show how higher-order systems can be studied with a suitable reinterpretation of the state space.

[6] In the literature on commodity markets (particularly futures markets), it is widely argued that $\theta E_t p_{t+1} - p_t < 0$ even if inventories are positive (but low). For world markets of the sort studied in Sec. IV, the probability of a stock-out in which aggregate stocks are strictly zero is infinitesimal. Wright and Williams (1989) (see also Williams and Wright 1991, pp. 248–49) have shown, however, that what matters is the spatial disposition of stocks, not necessarily their total amount. Moreover, the casual assertion that inventories are not strictly zero even when world stocks are generally accepted to be low could reflect the fact that such stocks that do exist are held by final users and are not available for the transactions that determine the observed spread of futures and spot prices. For these reasons, the concept of a "stock-out" as used here should not be understood to mean that stocks are literally zero but, rather, circumstances in which the observed spot price is determined by the demands of final users. The only remaining ambiguity is that it may not be possible to draw a firm line between final users and other inventory holders.

Identical and Independent Disturbances

In this case the transition function is the same for each period and does not depend on any previous outcome of the disturbance.

ASSUMPTION 3a. *i.i.d. disturbances.*—The disturbances w are identically and independently distributed with compact support

$$W = \{w \in \mathbf{R} \mid -\infty < \underline{w} \leq w \leq \overline{w} < +\infty\}. \tag{6}$$

Given the i.i.d. assumption, the equilibrium price function takes the form $p_t = f(x_t)$ such that price is a function of time only insofar as the state variable x_t varies across time, and the function itself, $f(\cdot)$, is stationary with $w^* = \underline{w}$ (see assumption 1). This is stated more formally in the following theorem.

THEOREM 1. Under assumptions 1, 2, and 3a, there exists a unique price function f: $X \rightarrow \mathbf{R}$, where $X = \{x \mid x \in \mathbf{R}, x \geq \underline{w}\}$, such that $f(\cdot)$ is continuous, nonnegative, and nonincreasing and satisfies

$$f(x) = \max\left\{\theta \int_W f\{w' + (1 - \delta)[x - D(f(x))]\}Q(dw'), P(x)\right\}. \tag{7}$$

Proof. See Deaton and Laroque (1992). Q.E.D.

It follows from theorem 1 and (5) that $p^* \equiv \theta \int_W f(w')Q(dw')$ provides a convenient reference point for dividing the two ranges of the equilibrium price function, for this threshold price is the minimum such that the carryover of stocks is zero. At $p_t = p^*$, $x_t = D(f(x_t))$ so that $p_t = \theta E_t p_{t+1}$. In view of the monotonicity of $P(x)$,

$$p_t = P(x_t) \qquad \text{for } p_t \geq p^*,$$

$$p_t = \theta E_t p_{t+1} \quad \text{for } p_t < p^*.$$

Alternatively, in terms of the equilibrium price function, $f(x) = P(x)$ for $P(x) \geq p^*$ and $f(x) > P(x)$ for $P(x) < p^*$ for all x such that price is in the domain of $P(\cdot)$.

The empirical implementation is based on the following corollary.

COROLLARY TO THEOREM 1. $E_t p_{t+1} = \theta^{-1} \min(p^*, p_t)$.

Proof. See Deaton and Laroque (1992). Q.E.D.

A notable feature of the i.i.d. case is that the threshold price is a constant and can be estimated, along with θ^{-1}, in a generalized autoregression of prices with a sample path that switches between regimes when price crosses the threshold value.

Time-Dependent Disturbances

It should come as no surprise that almost any relaxation of the assumption that the disturbances follow an i.i.d. process will result in the implication that the threshold price, p^*, is no longer a constant, independent of time. The main aim of the analysis of this and the following subsection is to characterize the required modifications to the equilibrium price functions under time dependency or periodicity and, thereby, to obtain predictions about the pattern of threshold prices.

The case of time-dependent disturbances involves assuming that the random shock in one period is correlated with that in the previous period. It is convenient to discuss the disturbances in terms of harvest fluctuations, although, as noted above, the random variable incorporates demand shocks that may also exhibit patterns of time dependency.

Time-dependent disturbances are modeled here by assuming that the transition function takes the form introduced above except that the conditional probability distribution does not change across time. That is, the transition function can be written as $Q(w, w')$, for every time period. Formally, the disturbances are assumed to be drawn from a set W of the form given by (6). Assumption 3a above is now replaced by the following assumption.

ASSUMPTION 3b. *Time-dependent disturbances.*—The transition function Q defined on $W \times \mathcal{W}$ (where $\mathcal{W}$ denotes the Borel algebra of sets for W) has the following properties: (i) For any continuous function h: $(W \times W) \rightarrow \mathbf{R}$, which is bounded (with respect to the Euclidean metric), the operator J defined by

$$(Jh)(w) = \int_W h(w, w')Q(w, dw') \quad \text{for each } w \in W \tag{8}$$

is such that (Jh): $W \rightarrow \mathbf{R}$ is a continuous, bounded function. (ii) $Q(w, A)$ is continuous in $w \in W$ for each $A \in \mathcal{W}$.

Assumption 3b serves to provide mild regularity conditions needed for the existence of an equilibrium price function and implies that it takes the form $p_t = f(y_t, w_t)$, where w_t appears as an argument because the probability distribution of next period's disturbance (relevant for price expectations and hence inventory holdings) is a function of the present disturbance.[7] Also, $w^* = \underline{w}$ (see assumption 1). Formally, it can be proved that an SREE price function, $f(y, w)$, exists and is unique.

[7] The methods used in Chambers and Bailey (1994) to characterize the equilibrium price function focus directly on the speculative storage condition. An alternative approach, which is based on the idea that the market as a whole solves a dynamic programming problem, appears in Danthine (1977) and Stokey and Lucas (1989). The more recent contribution of Hopenhayn and Prescott (1992), which obtains more general results, is also motivated by, though not restricted to, the dynamic optimization approach.

THEOREM 2. Under assumptions 1, 2, and 3b, there exists a unique price function $f: \Lambda \rightarrow \mathbf{R}$, where $\Lambda = Y \times W$ and $Y = \{y | y \in \mathbf{R}, y \geq 0\}$, such that $f(\cdot, \cdot)$ is continuous, nonnegative, and nonincreasing in its first argument and satisfies

$$f(y, w) =$$

$$\max\left\{\theta \int_W f\{(1-\delta)[y + w - D(f(y, w))], w'\}Q(w, dw'), P(y + w)\right\}. \quad (9)$$

Proof. See the technical appendix in Chambers and Bailey (1994). Q.E.D.

An exactly analogous result to the corollary to theorem 1 is the following corollary.

COROLLARY TO THEOREM 2. $E_t p_{t+1} = \theta^{-1} \min[p^*(w_t), p_t]$, where

$$p^*(w) = \theta \int_W f(0, w')Q(w, dw').$$

Proof. See the technical appendix in Chambers and Bailey (1994). Q.E.D.

In contrast to the case for i.i.d. disturbances, the autoregression of prices no longer involves a constant p^* but, instead, one that varies systematically with the current, observed harvest. In order to examine the properties of this dependence, it is helpful to study how the equilibrium price function, $f(y, w)$, depends on w.

The role of w in $f(y, w)$ is apparent from an examination of equation (9), where w appears as an argument of the transition function for the disturbance in the following period (in addition to its rather obvious contribution to the total amount available for sale, $y + w$, in the current period). Thus, rather imprecisely, if disturbances are positively autocorrelated, an abundant harvest in the current period provides information that the harvest in the following period will also be abundant. In this case, it would seem plausible to suggest that $f(y, w)$ is nonincreasing in w: the "high expected value" of the harvest next period informs inventory holders that the equilibrium price will be relatively "low" next period, making storage less profitable, decreasing inventory demand, and thus leading to a lower market price in the current period. Furthermore, as already noted, a high value of w in the current period increases the total amount available for sale, thus creating additional downward pressure on the current equilibrium price.

Figure 1 illustrates a typical configuration that is shown below to be relevant for the case of positive autocorrelation. In interpreting the figure, one should note that the equilibrium price functions for prices exceeding the threshold levels coincide with the (inverse of the)

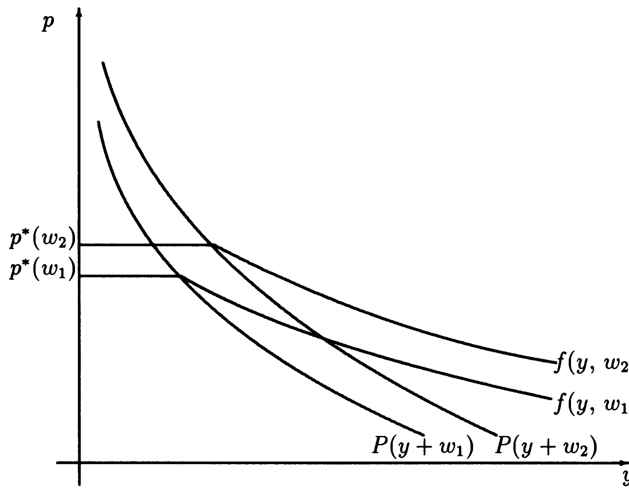

FIG. 1.—Positively autocorrelated disturbances with $w_1 > w_2$

final demand function, $P(\cdot)$. Figure 1 depicts an example in which a plentiful harvest, w_1 (in contrast with a meager harvest, w_2), provides information that the following period's harvest will also be relatively plentiful. Heuristically, inventory demand will be lower than otherwise, and the equilibrium price function shifts downward. The corollary to theorem 3 (see below) demonstrates that the threshold prices have the ranking shown, that is, $p^*(w_1) \leq p^*(w_2)$. Another very general property (which holds irrespectively of any assumption about autocorrelation) is that the equilibrium price function is bounded above by $P(\underline{w})$ for all values of y and w, that is, $f(\cdot, \cdot) \leq P(\underline{w})$.

The rigorous analysis of autocorrelated disturbances is examined below in terms of assumptions 4a and 4b. Assumption 4a (positive autocorrelation) leads directly to theorem 3, which supports the intuition that $f(y, w)$ varies inversely with w. Formally, under positive autocorrelation, the influence of w on $Q(w, w')$ is captured by assumption 4a.

ASSUMPTION 4a. *Positive autocorrelation.*—For any function $h(w')$ that is nonincreasing (respectively, nondecreasing) in w', the transition function, $Q(w, w')$, is such that

$$\int_W h(w')Q(w_1, dw') \leq (\text{resp.} \geq) \int_W h(w')Q(w_2, dw') \tag{10}$$

for all $w_1 > w_2$ such that $w_1, w_2 \in W$.

Assumption 4a asserts roughly that, for any two harvests in the present period ($w_1 > w_2$), the probability distribution in the next period for the higher present harvest dominates (in the sense of first-order stochastic dominance) that for the lower harvest. More precisely, define the set $A(x) \equiv [\underline{w}, x]$ and postulate that

$$Q(w_1, A(x)) \leq Q(w_2, A(x)) \\ \text{for all } x \geq \underline{w}, w_1, w_2 \in W \text{ such that } w_1 > w_2. \tag{11}$$

Now the theorem of first-order stochastic dominance (see, e.g., Hanoch and Levy 1969) can be invoked to show that (11) implies and should help to justify assumption 4a.

Assumption 4a, together with the earlier assumptions, implies the following theorem.

THEOREM 3. Let $f(y, w)$ denote the unique SREE defined in expression (9). Then, under assumptions 1, 2, 3b, and 4a, for each $y \in Y$, $f(y, \cdot)$: $W \to \mathbf{R}$ is nonincreasing.

Proof. See the technical appendix in Chambers and Bailey (1994). Q.E.D.

Direct application of assumption 4a to $p^*(w) = \theta \int_W f(0, w')Q(w, dw')$ shows immediately the following corollary.

COROLLARY TO THEOREM 3. *Positive autocorrelation.*—Under assumption 4a, $p^*(w)$ is monotonic nonincreasing in w.

The converse case of negative autocorrelation, although straightforward to characterize by simple reversal of the assertion of assumption 4a, does not yield such definite predictions as for positive autocorrelation. By analogy with assumption 4a, we make the following assumption.

ASSUMPTION 4b. *Negative autocorrelation.*—For any function $h(w')$ that is nonincreasing (respectively, nondecreasing) in w', the transition function, $Q(w, w')$, is such that

$$\int_W h(w')Q(w_1, dw') \geq (\text{resp.} \leq) \int_W h(w')Q(w_2, dw') \tag{12}$$

for all $w_1 > w_2$ such that $w_1, w_2 \in W$.

It has *not* been possible to demonstrate that $f(y, w)$ is nondecreasing in w when assumption 4b holds. The reason is that the effects of w on price tend to pull in two contrary directions. An abundant harvest in the current period tends to drive down price but conveys information that the harvest in the following period will be unusually meager, thus raising the demand to hold stocks in the present. The equilibrium price function is then such that $f(y, \cdot)$ could be nonincreasing or nondecreasing in its second argument, the current period's harvest.[8]

It should be pointed out that, at the level of generality considered

[8] This being so, it is evident that no result analogous to the corollary to theorem 3 holds for the case of negative autocorrelation.

here, nothing of substance is changed if the state variables of the system are chosen to be (x_t, w_t) instead of (y_t, w_t). The equilibrium price function takes the form $\tilde{f}(x_t, w_t)$ with exactly the same qualitative implications as those discussed above (including positive and negative autocorrelation).

In summary, it has been shown that, in contrast to the case for i.i.d. disturbances, time dependence among the disturbances implies a threshold price that is not constant but, instead, varies systematically with the current, observed, harvest. The corollary to theorem 3 shows clearly that the empirical implementation of the model with time-dependent shocks requires the measurement of, or at least some proxy (such as current output)[9] for, the values of the disturbances. Moreover, specification of the functional relationship $p^*(\cdot)$ requires an explicit form for the equilibrium price function or, at least, some a priori information about its properties, information that may well prove awkward to obtain. For these reasons, empirical study for the case of time-dependent disturbances is likely to present formidable problems that are more severe than for periodic disturbances.

Periodic Disturbances

The case of periodic disturbances offers a plausible and empirically implementable way of introducing heterogeneity[10] among successive disturbances. For analytical and empirical tractability, periodic disturbances are studied under the assumption that the disturbances are independent across time. Once again, the threshold price, p^*, is no longer constant from one period to the next, although its variation is driven by forces that are distinct from those of time dependence. In order to motivate the study of periodic disturbances, suppose that the primitive time period in question is a month rather than a year. The harvest for an agricultural commodity such as wheat occurs in a narrow range of months (essentially August and September for wheat in the northern temperate latitudes) so that the harvest is identically zero for the remaining months of the year. Even for the harvest months, the distributions could differ from one month to the next. For other commodities, harvests might occur throughout the year, but the pattern of randomness is such that the assumption of identical probability distributions is likely to be unsatisfactory.

Similarly, it may be appropriate to model the demand shocks, v_t, to allow for different distributions across time periods. This could be

[9] In practice, such data are often unavailable, especially for short time intervals between observation points.

[10] The extended version of this paper (Chambers and Bailey 1994) uses the term heterogeneous, instead of periodic, disturbances.

appropriate because consumption demand displays such fluctuations or, alternatively, because the probability distributions of the random components in trade flows differ across periods. It would be plausible to allow for the latter effect when flows of imports or exports vary with random harvests in other geographical locations that may occur at different times of the year.

As in Gladýsev (1961), periodic disturbances are represented by grouping together individual time periods into "epochs" (say, years), each of which comprises an equal number of time periods (say, months). Suppose that there are n time periods per epoch. The notation introduced above is retained except that w is now replaced by sets of n random variables of the form $\{w_t^1, w_{t+1}^2, \ldots, w_{t+n-1}^n\}$ for all $t = 0, \pm 1, \pm 2, \ldots$. Each w_t^i is assumed to be identically distributed to, and independent of, w_{t+nr}^i for every $r \neq 0$ and every t; the distributions of w_t^i, $w_{t+(j-i)r}^j$ for $i \neq j$, $r \neq 0$ may differ, although independence must be preserved. The assumption implies that the random variables for every epoch can be written as $\{w^1, w^2, \ldots, w^n\}$, and from this point, subscripts are used to represent realizations in any time period.

In the example of months as primitive time periods and years as epochs, the assumption asserts that the random shock for each August, say, is drawn from a probability distribution that is identical to the shock in every other August. Similarly, the disturbance for every other month of the year is assumed to be independent of that for all other months and to have a probability distribution that is identical for its corresponding month in different epochs. The disturbance, w^k, includes, of course, the random components of demand as well as those of supply; they also are assumed to be drawn from distributions that are periodic across epochs.

It is clearly not necessary, or even sensible, to confine the interpretation of epochs to groups of time periods shorter than a year. At least with respect to the subset of commodities particularly sensitive to climatic conditions, it may be appropriate to group *years* into epochs. For example, insofar as crops are influenced by climatic fluctuations induced by sunspot cycles, periodicity is likely to be present. Evidently, the independence of the shocks across time might also be brought into question in this example, although some such assumption is required to permit the derivation of an equilibrium price function.

The assumption of periodicity cannot pretend to cope with every conceivable pattern of heterogeneity but allows for greater flexibility in the specification of the empirical model of price determination without rendering the model empirically vacuous. Rather than present the theory in the generality outlined above, the analysis can be developed most transparently when each epoch consists of exactly two periods, labeled "even" and "odd." The generalization to epochs of any (finite) length is then straightforward.

Most of the notation retains the meaning given above except that w is replaced by w^e and w^o for *even* and *odd* periods, with the assumption that w^e is i.i.d. and w^o is i.i.d., each w^e is independent of each w^o, and each has its own (generally different) compact support; that is, each harvest w' is assumed to be a member of one of

$$W^e = \{w' \in \mathbf{R} \mid -\infty < \underline{w}^e \leq w' \leq \overline{w}^e < +\infty\} \tag{13}$$

and

$$W^o = \{w' \in \mathbf{R} \mid -\infty < \underline{w}^o \leq w' \leq \overline{w}^o < +\infty\}. \tag{14}$$

The sequences of realizations take the form $\{w^e_{2t}, w^o_{2t+1}\}$ for $t = 0, \pm 1, \pm 2, \ldots$. The crucial difference between the periodic disturbances model and that with i.i.d. disturbances is that the equilibrium price function $f(\cdot)$ must be replaced by two functions, denoted by $f^e(\cdot)$ and $f^o(\cdot)$ for the even and odd periods, respectively. In principle, the systematic component of net demand (represented by $D(p)$ or $P(x)$) could be allowed to differ between periods, but this would not add any substantive generality to the equilibria studied here and, hence, has been omitted. With this in mind, assumption 3a becomes assumption 3c.

ASSUMPTION 3c. *Periodic disturbances.*—The disturbances w^o and w^e are independently distributed. That is, the functions Q^k: $\mathcal{W}^k \rightarrow [0, 1]$ are probability measures on $\mathcal{W}^k$ for $k = e, o$, respectively.

The lower bound, w^*, in assumption 1 now becomes $w^* = \min(\underline{w}^e, \underline{w}^o)$, and assumption 2 remains unchanged.

The following theorem justifies the use of price functions $f^e(\cdot)$ and $f^o(\cdot)$.

THEOREM 4. Under assumptions 1, 2, and 3c, there exists a unique pair of price functions f^e: $X \rightarrow \mathbf{R}$ and f^o: $X \rightarrow \mathbf{R}$ such that $f^e(\cdot)$ and $f^o(\cdot)$ are continuous, nonnegative, and nonincreasing and satisfy

$$f^e(x) = \max\left\{\theta \int_{W^o} f^o\{w' + (1 - \delta)[x - D(f^e(x))]\}Q^o(dw'), P(x)\right\} \tag{15}$$

and

$$f^o(x) = \max\left\{\theta \int_{W^e} f^e\{w' + (1 - \delta)[x - D(f^o(x))]\}Q^e(dw'), P(x)\right\}. \tag{16}$$

Proof. See the technical appendix in Chambers and Bailey (1994). Q.E.D.

The main operational implication of theorem 4 is that it allows p^*

to differ across periods (and in a way that is distinct from that of time-dependent disturbances).

COROLLARY 1 TO THEOREM 4. For t odd,

$$E_t p_{t+1} = \theta^{-1} \min(p^{*e}, p_t), \quad p^{*e} \equiv \theta \int_{W^e} f^e(w') Q^e(dw'). \tag{17}$$

For t even,

$$E_t p_{t+1} = \theta^{-1} \min(p^{*o}, p_t), \quad p^{*o} \equiv \theta \int_{W^o} f^o(w') Q^o(dw'). \tag{18}$$

Proof. To demonstrate (17), let t correspond to an odd period and $t + 1$ to an even period. Note, first, that in each *odd* period the expression corresponding to equation (5) can be written as

$$p_t = \max\left\{\theta \int_{W^e} f^e\{w' + (1 - \delta)[x_t - D(f^o(x_t))]\} Q^e(dw'), P(x_t)\right\}. \tag{19}$$

Consider the two cases that can occur: (i) $p_t \geq p^{*e}$: In this case, zero inventories are held from period t to $t + 1$ and

$$p_{t+1} = f^e(w_{t+1}). \tag{20}$$

(ii) $p_t < p^{*e}$: Now,

$$p_t = \theta \int_{W^e} f^e\{w' + (1 - \delta)[x_t - D(f^o(x_t))]\} Q^e(dw')$$

and

$$p_{t+1} = f^e(x_{t+1}). \tag{21}$$

Now define $\eta_{t+1} \equiv f^e(x_{t+1}) - \theta^{-1} p_t$ so that equation (21) can be written as

$$p_{t+1} = \theta^{-1} p_t + \eta_{t+1}, \tag{22}$$

where the definition of η_{t+1} implies that $E_t \eta_{t+1} = 0$. Combining cases i and ii yields

$$\begin{aligned} E_t p_{t+1} &= \begin{cases} \theta^{-1} \int_{W^e} f(w') Q^e(dw') & \text{if } p_t \geq p^{*e} \\ \theta^{-1} p_t & \text{if } p_t < p^{*e} \end{cases} \\ &= \theta^{-1} \min(p^{*e}, p_t) \end{aligned} \tag{23}$$

as required for equation (17). An identical argument yields equation (18). Q.E.D.

In the general case of epochs comprising n time periods, it is easy to see that an SREE requires the definition of n equilibrium price functions $\{f^1(\cdot), f^2(\cdot), \ldots, f^n(\cdot)\}$. That a unique SREE exists is a consequence of the theorem for the two-period case. Formally, the extension of assumption 3c is given by assumption 3d.

ASSUMPTION 3d. *Periodic disturbances.*—The disturbances w^k, $k = 1, 2, \ldots, n$, are independently distributed. That is, the functions Q^k: $\mathcal{W}^k \rightarrow [0, 1]$ are probability measures on $\mathcal{W}^k$.

The lower bound, w^*, in assumption 1 now becomes $w^* = \min(\underline{w}^1, \underline{w}^2, \ldots, \underline{w}^n)$, and assumption 2 remains unchanged. The existence theorem can be stated as follows.

THEOREM 4′. Under assumptions 1, 2, and 3d, a unique SREE in the n-period case exists.

Proof. A routine extension of the methods used to prove theorem 4 establishes the result.[11] Q.E.D.

Finally, n conditions of the form (17) with n threshold prices denoted by the vector $\{p^{*1}, p^{*2}, \ldots, p^{*n}\}$ such that $p^{*j} = \theta \int_{W^j} f^j(w')Q(dw'), j = 1, 2, \ldots, n$, can be defined. If t denotes chronological time and j a time period within any epoch, corollary 1 to theorem 4 can be generalized as follows.

COROLLARY TO THEOREM 4′. For t a "j period,"

$$E_t p_{t+1} = \theta^{-1} \min[p^{*(j+1)}, p_t], \quad j = t - \mathrm{int}\left(\frac{t-1}{n}\right)n, \tag{24}$$

where the notation int(x) is shorthand for the largest integer no greater than x and n is the number of periods per epoch.

Proof. The proof is exactly analogous to the proof of corollary 1 to theorem 4. Q.E.D.

It is obviously not necessary that there are n distinct p^* values in any particular application. For example, if the disturbances are dominated by harvest fluctuations and if there is a single harvest period each epoch, then it may be that there are just two distinct p^* values, one corresponding to the harvest months and the other corresponding to the nonharvest months. The nature of such patterns is a matter for empirical evaluation and is examined in the following section.

Although an SREE has been shown to exist under the assumption of periodic disturbances, all but the most basic properties of such an equilibrium (as expressed by the corollaries to theorems 4 and 4′) have yet to be established. It should be clear at the outset that the relationships among the equilibrium price functions, $f^j(\cdot)$, are generally far from simple, given that only assumptions that are needed to ensure the existence of an equilibrium have been made. Much more structure must be imposed on the mechanism generating the distur-

[11] Theorems 4 and 4′ can be interpreted as providing a nonlinear counterpart to the periodic correlated processes of Gladẏsev (1961). We are grateful to an editor for this point and, more generally, for drawing our attention to the work of Gladẏsev.

bances (and, perhaps, also the final demand function, $D(\cdot)$, though this is not attempted here) before any definite predictions can be obtained.

In exploring the properties of the equilibrium price functions, we focus our attention on the associated threshold prices, for it is these prices that appear most obviously in the empirical applications (on the basis of the corollaries to theorems 4 and 4′). Also, the threshold prices express the simplest differences among the price functions: if definite implications cannot be derived for them, they are most unlikely to be obtained for any other properties of the functions. A further simplification, though one that will be relaxed below, is to begin with the case of epochs that can be divided into the familiar "odd" and "even" periods.

The most natural restriction on the probability distributions is the one that associates one of the periods with a "large" disturbance and the other with a "small" disturbance. Thus, without loss of generality, suppose that the odd periods correspond to the times in which the harvest is gathered and the even periods correspond to the "nonharvest" times.[12] This assumption can be made more precise by asserting that the distribution of w_t for odd periods dominates (in the sense of first-order stochastic dominance) that of the even periods. More formally, first extend the domains of the transition functions $Q^o(\cdot)$ and $Q^e(\cdot)$ to the whole of the real line by defining them to take on the value zero outside their supports, W^o and W^e, respectively. Now define the set $A = (-\infty, x]$ for any $x \in \mathbf{R}$.[13] The assumption that the odd periods are those with the "large" disturbances can be expressed as

$$Q^o(A(x)) \leq Q^e(A(x)) \quad \text{for all } x \in \mathbf{R},$$

$$Q^o(A(x)) < Q^e(A(x)) \quad \text{for some } x \in \mathbf{R}.$$

Strong though the assumption may appear, it is hardly surprising that even stochastic dominance is not restrictive enough to rank the threshold prices, p^{*o} and p^{*e}. For conflicting pressures are at play. In any odd period the harvest is likely to be very high, thus suggesting a low price. But the demand to hold inventories is likely to be high in anticipation of a low supply in the following, even, period, thus mitigating against a low price. Note, also, from corollary 1 to theorem 4 that the threshold price in the *odd* period is given by $p^{*e} = \int_{W^e} f^e(w')Q^e(dw')$ and in the *even* period is $p^{*o} = \int_{W^o} f^o(w')Q^o(dw')$. Thus, to the extent that prices are likely to be relatively low in the odd periods (with plentiful supplies), the price at which a stock-out occurs,

[12] Recall that, strictly, the disturbances, w_t, include both supply and demand elements so that, even in the nonharvest periods, the probability distribution is nondegenerate.

[13] Clearly, for most applications it would suffice to restrict the disturbances to the nonnegative real line. Nothing is gained by this restriction, and hence it is not made.

p^{*e}, is likely to be relatively high (but observed with a low probability). This is not to suggest that p^{*e} always exceeds p^{*o} but that stochastic dominance on its own is not sufficient to guarantee a definite result.

A restriction of stochastic dominance that enables a definite result is stated as follows.

ASSUMPTION 5. *Nonoverlapping supports.*—The supports of W^e and W^o satisfy $-\infty < \underline{w}^e < \overline{w}^e < \underline{w}^o < \overline{w}^o < \infty$.

Assumption 5 asserts that the worst harvest in odd periods always exceeds the best harvest in even periods, clearly a sufficient but not a necessary condition for stochastic dominance. The following corollary straightforwardly demonstrates this.

COROLLARY 2 TO THEOREM 4. Under assumption 5, $p^{*e} > p^{*o}$.

Proof. Note first that, since $f^o(\cdot)$ is nonincreasing,

$$p^{*o} \equiv \theta \int_{\underline{w}^o}^{\overline{w}^o} f^o(w')Q^o(dw') \leq \theta f^o(\underline{w}^o). \tag{25}$$

From theorem 4, equation (16), with $x = \underline{w}^o$, two cases are possible: (i) $f^o(\underline{w}^o) = P(\underline{w}^o)$; hence,

$$\begin{aligned}
\theta f^o(\underline{w}^o) &= \theta P(\underline{w}^o) \\
&< \theta P(\overline{w}^e) \quad \text{since } P(\cdot) \text{ is decreasing and } \overline{w}^e < \underline{w}^o \\
&\leq \theta \int_{\underline{w}^e}^{\overline{w}^e} P(w')Q^e(dw') \quad \text{since } P(\cdot) \text{ is decreasing} \\
&\leq \theta \int_{\underline{w}^e}^{\overline{w}^e} f^e(w')Q^e(dw') \quad \text{since } f^e(w') \geq P(w') \text{ for all } w' \\
&= p^{*e} \quad \text{by definition.}
\end{aligned}$$

Thus $p^{*o} \leq \theta f^o(\underline{w}^o) < p^{*e}$. (ii)

$$f^o(\underline{w}^o) = \theta \int_{\underline{w}^e}^{\overline{w}^e} f^e\{w' + (1-\delta)[\underline{w}^o - D(f^o(\underline{w}^o))]\}Q^e(dw').$$

Hence,

$$\begin{aligned}
\theta f^o(\underline{w}^o) &= \theta^2 \int_{\underline{w}^e}^{\overline{w}^e} f^e\{w' + (1-\delta)[\underline{w}^o - D(f^o(\underline{w}^o))]\}Q^e(dw') \\
&\leq \theta^2 \int_{\underline{w}^e}^{\overline{w}^e} f^e(w')Q^e(dw') \quad \text{since } f^e(\cdot) \text{ is nonincreasing and} \\
&\qquad\qquad \underline{w}^o - D(f^o(\underline{w}^o)) \geq 0 \\
&\leq \theta \int_{\underline{w}^e}^{\overline{w}^e} f^e(w')Q^e(dw') \quad \text{since } 0 < \theta < 1 \\
&= p^{*e} \quad \text{by definition.}
\end{aligned}$$

Thus, again, $p^{*o} \leq \theta f^o(\underline{w}^o) < p^{*e}$.

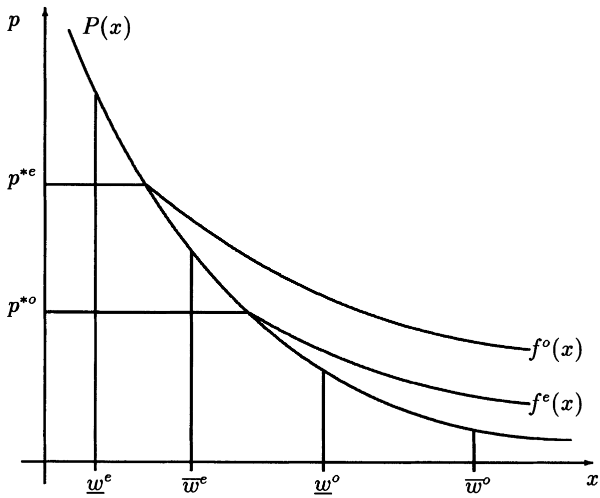

FIG. 2.—Periodic disturbances: the nonoverlapping case

The result asserted by the corollary is illustrated in figure 2.[14] Contrary to what figure 2 suggests, there should be no presumption that prices in odd (plentiful harvest) periods exceed those of even (meager harvest) periods. It would not be unreasonable to expect that, although the equilibrium function for odd periods tends to lie above the function for even periods, the harvests are sufficiently plentiful that observed prices are normally lower in the odd periods. Clearly, the model is compatible with a wide variety of outcomes.

It is trivial to generalize corollary 2 to theorem 4 to multiperiod epochs and thereby to rank the threshold prices for adjacent time periods, for which the supports for the random disturbances do not overlap.

Summary

Apart from the simplest case of i.i.d. disturbances, the two classes of models studied in this section both have potentially important implications for empirical work in that the market equilibrium price as a function of the currently available amount of the commodity, x_t, differs from one time period to the next. In the case of time-dependent disturbances of the simple Markovian variety assumed here, the price function has an additional argument, the current shock (harvest), w_t, which provides information about the probability distribution of the

[14] It should be pointed out that (in common with the interpretation of fig. 1) the lines that show the two equilibrium price functions may well intersect.

disturbance in the next period. In the case of periodic disturbances, the equilibrium price function itself differs according to the period in question but in a precisely defined and testable way.

It is easy to see how, in principle, the two extensions could be combined to generate equilibrium price functions that differ across time (because of periodic disturbances) and are functions of the currently observed shock (because of the temporal correlation of the disturbances). Whether such a generalization has any operational value depends on how much information is available and how much structure can be imposed on the price functions themselves. In some applications there may be evidence, perhaps indirect, of the variation in the disturbances even when data on quantities of the commodity are not available. In other applications such evidence is not available or would be costly to obtain. In these circumstances the predictions derived from the assumption of periodic disturbances are easier to test. The following section shows how this can be done.

IV. An Empirical Illustration

A fully parameterized representation of the equilibrium price function would require the specification not only of the probability law governing the underlying disturbance process but also of the net demand function, $D(p)$, embodying the systematic features of demand and supply. Such an approach is adopted by Deaton and Laroque (1996) in which the inverse demand function is assumed to be linear and the disturbance process (w_t in the notation used here) is defined by its first two moments. Even with these restrictive assumptions, only partial identification of the model's parameters is possible given that prices alone are used in the estimation.[15] In view of the obstacles to the identification of the equilibrium price function, the empirical analysis below uses the generalized method of moments (GMM) technique to test the implications of the model rather than to characterize the price function itself. In particular, given the absence of data on the realizations of the disturbances and with several time periods allowed per epoch, it is convenient to base the empirical investigation on the corollary to theorem 4′, discussed in Section III.

For empirical purposes the general model expressed by the corollary to theorem 4′ can be written as follows:

$$p_t = \gamma \min(p^{*j}, p_{t-1}) + u_t, \quad t = 1, 2, \ldots, T, \quad j = t - \mathrm{int}\left(\frac{t-1}{n}\right) n, \quad (26)$$

[15] This point was also made in Deaton and Laroque (1992, p. 15).

where $\gamma \equiv \theta^{-1}$ and u_t is a random error (innovation) such that $E(u_t|\Omega_t) = 0$, $E(u_t u_{t-s}|\Omega_t) = 0$ for $s \neq 0$, and $E(u_t^2|\Omega_t) = \sigma_t^2$. As in the previous section, Ω_t denotes the set of information available to market agents at time t, which for the purposes of this section can be represented by the set of all past prices: $\Omega_t = \{p_{t-1}, p_{t-2}, \ldots\}$. Thus it is assumed that the innovations have zero expectation and are serially uncorrelated but may display heteroscedasticity conditional on the information set, Ω_t, available at time t.

In the form expressed by (26), the sample is implicitly set to begin at the first period of an epoch, though, trivially, this is no restriction in practice. Also, in this most general representation, the number of p^{*j} parameters is the same as the number of periods per epoch: $j = 1, 2, \ldots, n$. This turns out to allow too many free parameters in most applications, at least with monthly data, and some restriction on the number of p^{*j} parameters is often necessary to obtain reliable estimates.

Many different restrictions on the p^{*j} could be introduced. One of the simplest, and perhaps most plausible, is one in which the p^{*j} takes on one value for a sequence of months followed by a different value for the next sequence. Each epoch (year) could then be divided into two, or more, of these sequences. For example, it is possible to define six p^{*j} values, one for each 2-month period in the year.[16] A general way of writing this sort of restriction is as follows:

$$\begin{gathered} p_t = \gamma \min(p^{*k}, p_{t-1}) + u_t, \quad t = 1, 2, \ldots, T, \\ j = t - \operatorname{int}\left(\frac{t-1}{n}\right)n, \quad k = \operatorname{int}\left[\frac{r(j+1)}{n}\right], \end{gathered} \tag{27}$$

where r is the number of different p^{*k} parameters per epoch, each p^{*k} being constant for n/r consecutive time periods in each epoch. Thus, in the experiments studied here, with monthly data for annual epochs, $r = 1$ allows a single p^*; $r = 2$ provides $\{p^{*1}, p^{*2}\}$; and $r = 6$ provides $\{p^{*1}, p^{*2}, \ldots, p^{*6}\}$. These three models are referred to below as PD_1, PD_2, and PD_6, respectively, where the subscript denotes the number of threshold prices estimated in each case.[17] While these restrictions are fairly arbitrary, only a more detailed knowledge of the factors influencing the individual markets would generate more plausible patterns, and to incorporate this would involve going beyond the scope of the present illustrative exercise.

The parameter vector to be estimated is denoted by $\boldsymbol{\beta} = (\gamma, \mathbf{p}^*)'$,

[16] There remains the question of whether to begin the sequence on an odd or even numbered month.

[17] Obviously, PD_1 corresponds to the model studied by Deaton and Laroque (1992).

where $\mathbf{p}^*$ is a vector with the number of elements determined by the model under investigation. The dimension of $\boldsymbol{\beta}$ is given by $m \times 1$, where $m = r + 1$. It should be noted that the data of any particular application determine whether individual elements of $\mathbf{p}^*$ are identified. If $p^{*k} \geq \max p_{t-1}$, where the maximum is taken over all sample prices for the relevant preceding period in (27), then for that period within each epoch the model reduces to a first-order autoregression of prices. That is, for some month, or group of months, it may be the case that supplies of the commodity are sufficiently plentiful that the p^{*k} for that month is never reached at any observed sample point. In such circumstances, it is not possible to estimate the relevant element of the $\mathbf{p}^*$ vector, and its dimension is reduced accordingly.

The parameter vector $\boldsymbol{\beta}$ is estimated using the GMM technique of Hansen (1982) with criterion function

$$\mathbf{u}(\boldsymbol{\beta})'\mathbf{Z}\boldsymbol{\Phi}^{-1}\mathbf{Z}'\mathbf{u}(\boldsymbol{\beta}), \tag{28}$$

where $\mathbf{u}(\boldsymbol{\beta}) = (u_1(\boldsymbol{\beta}), \ldots, u_t(\boldsymbol{\beta}), \ldots, u_T(\boldsymbol{\beta}))'$, with $u_t(\boldsymbol{\beta}) = p_t - \gamma \min(p^{*k}, p_{t-1})$; $\mathbf{Z} = \{z_{ij}\}$ is a $T \times l$ matrix of instruments, described below; and $\boldsymbol{\Phi}$ is an $l \times l$ positive definite matrix. The weighting matrix, $\boldsymbol{\Phi}$, is chosen to allow for possible heteroscedasticity of the innovations in the model, its elements being estimated by $\hat{\boldsymbol{\Phi}} = \{\hat{\phi}_{ij}\}$:

$$\hat{\phi}_{ij} = T^{-1}\sum_{t=1}^{T} z_{ti}z_{tj}\hat{u}_t^2, \tag{29}$$

with $\hat{u}_t = u_t(\hat{\boldsymbol{\beta}})$.

The matrix $\mathbf{Z}$ is constructed using a constant and lagged prices as instruments: $(1, p_{t-1}, \ldots, p_{t-6})'$ for the PD_1 and PD_2 models and $(1, p_{t-1}, \ldots, p_{t-12})'$ for the PD_6 model. Finally, the estimate $\hat{\boldsymbol{\beta}}$ is chosen to minimize (28), a choice that results in asymptotically normally distributed estimators under certain regularity conditions.[18]

The criteria used to select commodities for study were that each commodity should be traded on markets for which prices reflect the forces of demand and supply; that the production of the commodity should be subject to regular but stochastic harvest fluctuations for

[18] It is obvious that there is a kink in $u_t(\boldsymbol{\beta})$ at $p^{*j} = p_{t-1}$, although the function remains continuous at such points. This type of complication is studied by Laroque and Salanié (1994) and Hansen, Heaton, and Luttmer (1995, app. C), who develop the relevant asymptotic theory. These authors demonstrate that $T^{1/2}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})$ converges in distribution to $N(0, (\mathbf{D}'\boldsymbol{\Phi}^{-1}\mathbf{D})^{-1})$, where the elements of the $l \times m$ matrix $\mathbf{D} = \{d_{ij}\}$ are estimated by

$$\hat{d}_{ij} = T^{-1}\sum_{t=1}^{T} z_{ti}\frac{\partial \hat{u}_t}{\partial \beta_j}.$$

which epochs of a year or less would be appropriate; and that the commodity be storable from one month to the next. With these conditions in mind, monthly price data were obtained for seven commodities: coffee, sugar, wheat, rice, maize, soybeans, and cotton.

Not surprisingly, the price data for these commodities display sharp positive "spikes" at certain times, presumably reflecting shortages, or the prospect of perceived shortages in the coming months. Table A2 in the Data Appendix shows that there is strong evidence of skewness and kurtosis in the price data. These characteristics of the data lend further weight to the use of the GMM estimation method, which, unlike, for example, maximum likelihood techniques, imposes no distributional assumptions on the random errors.

It is also worth noting that the estimated autocorrelation functions for the prices (not reproduced here) show a pattern of steadily decaying coefficients with increasing time lags: there is no evidence of seasonality in the price fluctuations. This evidence is consistent with the activities of well-informed traders, the operations of which smooth out systematic seasonal components of price fluctuations as the theory predicts.

The results of the empirical estimations for the three models are summarized in tables 1, 2, and 3, which share a common structure. Caution should be exercised in attempting to draw inferences about the magnitudes of the estimated $\mathbf{p}^*$ without detailed knowledge (of a degree that goes far beyond the illustrative exercise attempted here) about the markets in question. It was not possible to estimate the p^{*k} coefficient for some of the time periods. For these cases, tables 2 and 3 contain entries, in brackets, of the maximum price observed during the relevant months of the sample.

The inability to estimate some of the threshold parameters reflects circumstances in which the estimation algorithm fails to find any observed prices in excess of the candidate p^{*k}.[19] In other words, the sample of data is not rich enough to permit estimation of every p^{*k} parameter: all candidate threshold parameters for the time period in question exceed every observed price, and thus the sample moment conditions fail to provide an estimate of the p^{*k}. For such time periods the model reduces to a simple first-order autoregression of the price (see eq. [26] or [27]), and only the regime for which positive inventories are demanded appears in the evidence. An advantage of the models with several threshold prices (PD_2 and PD_6 in the experiments

[19] In principle, such a failure could also occur if no observed price falls below p^{*k} so that the observed p_t equals a constant plus a random error for every corresponding period in the sample. This prospect seems remote in practice, at least for the commodities and time periods considered here.

TABLE 1

GMM Estimates for the i.i.d. Model, PD_1

	Coffee	Sugar	Wheat	Maize	Rice	Soybeans	Cotton
γ	1.0002	1.0280	.9992	.9988	.9997	1.0038	1.0002
	(.0038)	(.0173)	(.0022)	(.0024)	(.0034)	(.0033)	(.0023)
p^*	1.2611	.2148	1.2451	.8769	2.7717	1.6109	.9637
	(.0811)	(.0322)	(.0503)	(.0660)	(.2254)	(.0663)	(.0406)
Stock-out	1.2346	8.5938	3.3854	.5208	3.9063	6.5104	5.2083
H	9.2914	12.0086*	9.8896	10.8024	9.4332	10.7932	9.5802

* Significant at the 5 percent level.

here) is that they provide an element of flexibility that allows for circumstances in which prices never rise to their threshold levels in some of the sampled months. The row labeled "stock-out" in tables 1, 2, and 3 records the percentage of months for which there are estimated stock-outs, that is, for which the observed price is at least equal to the relevant p^{*k}.

Finally, each of the tables reports diagnostic statistics for the overidentifying restrictions.[20] In table 1 the number of degrees of freedom in the χ^2 test for H is equal to five for each commodity. Given that, in tables 2 and 3, several parameters are not estimable, the number of degrees of freedom differs from commodity to commodity, and these numbers are reported in the final row.

In the simplest (PD_1) model, in table 1, all the estimated γ coefficients are close to unity, although for three commodities they are fractionally below one. That they are found to be so close to unity is hardly surprising: with monthly time intervals, the implied values of δ and r should be very small. The estimated $\mathbf{p}^*$ coefficients all appear to be well determined and to imply values for stock-outs that are not unreasonable. With respect to the test of the overidentifying restrictions, H, the statistics reject the null hypothesis (that the restrictions are valid) for one commodity (sugar) at the 5 percent significance level.

In the PD_2 model, the results for which are reported in table 2, all the estimated γ coefficients exceed unity (by a small margin). The flexibility of the PD_2 model is apparent from the inability to estimate p^{*2} for one of the commodities (wheat). Otherwise, all the p^{*k} coefficients are well determined according to usual statistical criteria. The H statistic indicates that the overidentifying restrictions should be rejected for one commodity (sugar) at the 5 percent level.

[20] The test statistic, $H = T^{-1}\mathbf{u}(\hat{\boldsymbol{\beta}})'\mathbf{Z}\hat{\boldsymbol{\Phi}}^{-1}\mathbf{Z}'\mathbf{u}(\hat{\boldsymbol{\beta}})$, is that proposed by Hansen (1982), the heteroscedasticity-consistent estimate, (29), of the covariance matrix, $\boldsymbol{\Phi}$, being used in the computations.

TABLE 2

GMM Estimates for the Periodic Disturbances Model, PD_2

	Coffee	Sugar	Wheat	Maize	Rice	Soybeans	Cotton
γ	1.0010	1.0242	1.0007	1.0004	1.0042	1.0020	.9999
	(.0038)	(.0203)	(.0023)	(.0025)	(.0076)	(.0029)	(.0021)
p^{*1}	.9782	.3327	1.0697	.8711	2.9539	1.6003	1.0429
	(.0394)	(.1241)	(.0725)	(.1799)	(.5856)	(.0882)	(.1157)
p^{*2}	1.2727	.1800	[1.4582]	.7006	1.9903	2.0410	.9399
	(.0763)	(.0470)		(.0278)	(.4843)	(1.6053)	(.0481)
Stock-out	2.4691	7.2917	3.1250	2.8646	6.2500	3.6458	4.1667
H	4.8763	11.1167*	6.0254	8.3705	8.6441	6.0711	8.9033
Degrees of freedom	4	4	5	4	4	4	4

* Significant at the 5 percent level.

TABLE 3

GMM Estimates for the Periodic Disturbances Model, PD_6

	Coffee	Sugar	Wheat	Maize	Rice	Soybeans	Cotton
γ	1.0009	1.0500	1.0016	1.0060	1.0021	1.0017	1.0022
	(.0046)	(.0379)	(.0027)	(.0049)	(.0035)	(.0028)	(.0034)
p^{*1}	1.0160	.1932	[1.3558]	[.8711]	2.8723	1.7069	.7058
	(.0502)	(.1541)			(.3847)	(.4828)	(.0521)
p^{*2}	.9652	.1332	1.1625	[.8916]	2.5149	2.0114	.9058
	(.0601)	(.0531)	(.1787)		(.7808)	(.2452)	(.1648)
p^{*3}	[1.0986]	.1053	1.3898	[.8471]	2.5973	[1.8150]	[1.2806]
		(.0560)	(.0891)		(.6960)		
p^{*4}	.6128	.1993	[1.4518]	[.8769]	[3.7793]	1.7225	.9311
	(.1492)	(.0773)				(.0921)	(.1349)
p^{*5}	.7570	.3415	.9466	.5144	2.3571	1.5349	[1.1195]
	(.0783)	(.0669)	(.1293)	(.0396)	(1.2303)	(.0751)	
p^{*6}	1.2784	[.3361]	.7730	.6953	2.0107	[3.4382]	.8852
	(.1313)		(.1495)	(.0597)	(.6481)		(.0468)
Stock-out	5.2469	10.6771	4.9479	8.0729	4.1667	3.3854	8.5938
H	7.2481	8.2583	9.4468	7.2006	8.8613	13.3220	9.7571
Degrees of freedom	7	7	8	10	7	8	8

The results for the more general periodic disturbance model, PD_6, are reported in table 3, where, once again, it is seen that all the estimated γ coefficients exceed unity by a small margin. Once again, the flexibility of the model is apparent from the inability to estimate several of the p^{*k} coefficients in the sample. Those that are estimable appear to be well determined and, as for the simpler PD_2 model, tend to show significant differences among one another. The test for overidentifying restrictions is passed for all seven commodities in the sense that the null hypothesis cannot be rejected at the 5 percent significance level.

Although the estimates of γ tend to exceed unity, as predicted by the theory, their values are very close to one and, hence, imply very small values of the opportunity cost of storage (measured by $r + \delta$). A rough guide to the relevant magnitudes may be found by approximating *annual* values of $r + \delta$ by $\gamma^{12} - 1$. With $\gamma = 1.001$, $r + \delta \approx 1.2$ percent; $\gamma = 1.002$ implies $r + \delta \approx 2.4$ percent; $\gamma = 1.004$ implies $r + \delta \approx 4.9$ percent; $\gamma = 1.006$ implies $r + \delta \approx 7.4$ percent; and $\gamma = 1.008$ implies $r + \delta \approx 10.0$ percent. These calculations suggest that the estimates of γ are small for most commodities, in some cases implausibly so. It is interesting to note that Deaton and Laroque (1992, p. 17) obtained rather similar results, with the highest estimated $r + \delta$ being 9 percent for sugar and that for wheat being 0.5 percent. In the sample used here, sugar is also clearly an outlier when it is realized that $\gamma = 1.02$ implies $r + \delta \approx 26.8$ percent and $\gamma = 1.05$ implies $r + \delta \approx 79.6$ percent! As Deaton and Laroque (1992) assert, "discount rates are notoriously hard to estimate," a defense that must also apply for the sample of data considered here.

Formal tests designed to compare the three models are complicated by the inability to estimate several of the p^{*k} parameters from the available sample. For six of the seven commodities it is, however, possible to test whether $p^{*1} = p^{*2}$ using the information contained in tables 2 and 3. Under the null hypothesis of equality of the threshold prices, the difference between the H statistics for the PD_2 and PD_1 models is asymptotically distributed as χ^2_1. The differences between the sample values for H in the tables are as follows: coffee 5.0451; sugar 0.8919; maize 2.4319; rice 0.7891; soybeans 4.7221; and cotton 0.6769. At the 5 percent significance level, for two of the six commodities the evidence suggests that the threshold prices are unequal. Of course, it is possible that apparent equality could mask more complicated differences among threshold parameters.

Given that the sample renders inestimable at least one of the threshold parameters for each commodity in the PD_6 model, a test is constructed between each of the PD_1, PD_2, and PD_6 models in turn and a common alternative, namely, the first-order autoregression, AR_1, of prices. These pairwise comparisons are made using a "likelihood ratio" test methodology, the "restricted" estimates being those of the AR_1 model.[21] The relevant test statistics are reported in table 4.

In each case the test statistics are asymptotically distributed as χ^2 (with degrees of freedom in parentheses) under the null hypothesis that the AR_1 model is the "true" specification. The first row of table 4 shows that the AR_1 model is rejected in comparison with the PD_1 model for three of the commodities (wheat, rice, and cotton) at the 5 percent level of significance. With respect to the PD_2 model, the AR_1 model is rejected for the same three commodities. Finally, comparison between the AR_1 model and PD_6 shows that the AR_1 specification is rejected for coffee, sugar, maize, rice, and cotton.

Overall, the evidence in favor of the periodic disturbance models is encouraging. They compare very well with the autoregressive model with PD_2 or PD_6 serving to reject the AR_1 model for six of the seven commodities. The PD_1 model, with a single p^*, achieves this for only three of the seven commodities. For just one commodity, soybeans, the AR_1 model is not rejected against PD_2 and PD_6 models. It is interesting that in this case, AR_1 also survives against PD_1, the Deaton and Laroque model.

Although the results tend to be supportive of the models, it is important to note that the grouping of the months for the estimation of the p^{*k} parameters was arbitrary. For the PD_2 model, the two parameters were chosen for 6-month intervals beginning in July and January. For the PD_6 model, the six parameters were chosen for 2-month intervals beginning July through to the following May. Obviously, an allowance for more complicated patterns of disturbances could result in no more unfavorable results and, almost surely, significantly greater support for the models. A systematic investigation along these lines would, however, necessitate consideration of the underlying climatic and other disturbances that affect fluctuations in the supplies of, and demands for, the different commodities. Such analyses, while important in their own right, go far beyond what can be attempted here.

V. Summary and Conclusion

The previous sections have shown how far it is possible to obtain testable predictions, which require observations on prices alone, from

[21] When $\tilde{\mathbf{u}}$ is used to denote the AR_1 residual vector, the test statistic is given by $T^{-1}(\tilde{\mathbf{u}}'\mathbf{Z}\tilde{\mathbf{\Phi}}^{-1}\mathbf{Z}'\tilde{\mathbf{u}} - \hat{\mathbf{u}}'\mathbf{Z}\hat{\mathbf{\Phi}}^{-1}\mathbf{Z}'\hat{\mathbf{u}})$, which is asymptotically distributed as a χ^2_{m-1} variable under the AR_1 process. The $\tilde{\mathbf{\Phi}}$ and $\hat{\mathbf{\Phi}}$ matrices are computed to be robust to heteroscedasticity according to (29). Also it should be noted that, in estimation of the AR_1 model, the number of instruments is set equal to that used in the estimation of the alternative model in each case. Newey and West (1987) provide a detailed explanation of the method used for computing the test statistics.

TABLE 4

TESTS OF PD_1, PD_2, AND PD_6 AGAINST AR_1

	Coffee	Sugar	Wheat	Maize	Rice	Soybeans	Cotton
PD_1	1.1455	.3035	4.4543*	.0402	11.3200*	.6626	15.6610*
	(1)	(1)	(1)	(1)	(1)	(1)	(1)
PD_2	5.5606	1.1954	8.3185*	2.4721	12.1091*	5.3847	16.3379*
	(2)	(2)	(1)	(2)	(2)	(2)	(2)
PD_6	11.3811*	14.8503*	8.9378	8.1843*	15.6611*	3.0924	17.4016*
	(5)	(5)	(4)	(2)	(5)	(4)	(4)

NOTE.—Degrees of freedom are in parentheses.
* Significant at the 5 percent level.

a standard framework of short-term commodity price determination. The framework was outlined in Section II, which also pointed to the precise nature of the stochastic modeling that was required. Section III examined the properties of three models: (i) a model with independent and identically distributed disturbances ("harvests"), (ii) a model with temporally correlated disturbances, and (iii) a model with periodic disturbances. Temporal dependence of the disturbances was justified by reference to a persistence in shocks, commonly (but not exclusively) supposed to occur as a result of short-term weather patterns. Periodicity was also easy to justify and was interpreted as a restricted form of heterogeneous disturbances. For the case of periodic disturbances, time was divided into epochs (e.g., years) to represent finite sequences of primitive time periods (e.g., months). The probability distributions of the primitive time periods were then allowed to differ among one another within each epoch but were required to be identical among different epochs.

Rational expectations equilibrium price functions, of the form proposed by Deaton and Laroque (1992), were characterized in Section III, and the fundamental empirical implications of the models were derived. It was shown there how assumptions about the patterns of serial correlation and periodicity implied testable predictions about the sample paths of observed prices. While it would have been possible to construct models with both the temporal dependence and periodicity of disturbances, these models would have been unwieldy. Without the imposition of stringent restrictions, few if any testable predictions could be obtained. Even temporal dependence, alone, requires information on quantities (or some proximate determinant of the disturbances) as well as prices in order to be empirically implemented. For this reason it was decided to focus the empirical work on the case of periodic disturbances.

In Section IV an illustrative empirical exercise was conducted for monthly data on seven commodities that satisfied the criteria of the theory. With GMM techniques, qualified support was found for models of price determination with periodic disturbances. The evidence, although supportive, was not overwhelming, perhaps because we applied the same specifications to the different commodities rather than incorporate features (such as harvest patterns) specific to the markets in question. Further work is needed to identify features that are unique to the production and marketing of each commodity in order to take advantage of the restrictions that need to be placed on the probability distributions.

An outstanding issue meriting further research is the way in which temporal averaging of price data, resulting in different observation intervals, would affect the appropriateness of alternative assumptions about the disturbances. Casual observation suggests that fluctuations in the weather over very short intervals of time (days or weeks) tend to be serially correlated, and hence, this could be important for agricultural commodities. The presumption is that over longer intervals of time the hypothesis of serially correlated weather disturbances is less compelling, but the empirical implications of this remain to be investigated. In a similar way the heterogeneity of (serially independent) disturbances could be affected by time aggregation, and once again, its implications may well vary from commodity to commodity in ways that remain to be investigated.

Finally, although the statistical investigation of this paper is most suitable in circumstances for which observations on quantities of the commodities are not available, the methods could obviously have even greater value when such data are available. In particular, it would be possible to study the effects of autocorrelated disturbances that may well exist when the time interval is short.

Data Appendix

The data series used in the text are presented in table A1.

The coffee price data were provided by the International Coffee Organization (London), and the other series were provided by the World Bank (Washington). Neither institution is responsible for the way in which the data have been used. Each series was deflated by the U.S. index of all producer prices (1967 = 100). For each commodity the first 12 observations were reserved for the lags required by the instruments, thus leaving a sample size of T = 324 (27 years) for coffee and T = 384 (32 years) for the other commodities. Summary statistics for the data are provided in table A2.

The reported statistics for skewness and kurtosis have been adjusted so that, under the (separate) null hypotheses that neither of these phenomena is present, they would be drawn (asymptotically) from standard normal distributions.

TABLE A1

DATA SOURCES

Commodity	Source	Price	Time Period
Coffee	London (composite)	U.S. cents/lb.	July 1965–June 1993
Sugar	ISA (FOB Caribbean)	U.S. cents/kg.	July 1960–June 1993
Wheat	Canada (Northern Store)	U.S. dollars/ton	July 1960–June 1993
Rice	Thailand (FOB Bangkok)	U.S. dollars/ton	July 1960–June 1993
Maize	United States (Louisiana Gulf)	U.S. dollars/ton	July 1960–June 1993
Soybeans	United States (CIF Rotterdam)	U.S. dollars/ton	July 1960–June 1993
Cotton	United States (CIF Liverpool)	U.S. cents/kg.	July 1960–June 1993

NOTE.—ISA is the International Sugar Agreement (administered by the International Sugar Organization); CIF stands for cost, insurance, and freight (the price includes insurance and freight); the free-on-board (FOB) price does not.

TABLE A2

SUMMARY STATISTICS

	Coffee	Sugar	Wheat	Maize	Rice	Soybeans	Cotton
Means	.4494	.0999	.6597	.4738	1.3911	1.0748	.6401
Maxima:							
July	1.0723	.3412	1.1944	.8181	3.2096	2.4463	1.0862
August	1.0347	.4034	1.3558	.8711	3.1123	2.3196	1.1717
September	1.0025	.4503	1.4731	.8459	3.0876	1.8973	1.3632
October	.8787	.5122	1.4787	.8916	2.9559	1.9706	1.3801
November	.9678	.7200	1.4727	.8382	3.5920	1.8150	1.2592
December	1.0986	.5769	1.4483	.8471	3.6671	1.7913	1.2806
January	1.1569	.4916	1.4518	.7668	3.6630	1.7804	1.3565
February	1.2930	.4373	1.4490	.8769	3.7793	2.0410	1.2332
March	1.5892	.3415	1.4582	.8347	3.9366	1.9892	1.1195
April	1.6202	.3119	1.4383	.7477	4.1257	1.9893	1.0540
May	1.4212	.3361	1.2937	.7335	4.0323	2.6816	.9472
June	1.2503	.3329	1.1848	.7491	3.8295	3.4382	.9837
Minima	.1304	.0198	.3503	.2190	.6347	.6254	.2760
Standard deviations	.2205	.0859	.1991	.1366	.6049	.3361	.1733
Skewness	14.7348	24.0503	16.3639	3.4590	13.3920	15.1928	7.9346
Kurtosis	22.9380	49.7943	21.1944	.7869	17.1214	32.0262	11.4656

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