2 HOURS

## RENMIN UNIVERSITY OF CHINA

# Advanced Microeconomics I

Semester 1, Academic Year 2014-2015

5 January, 2015

Electronic calculators and dictionaries may be used, provided that they cannot store text or allow you to access to the internet.

P.T.O.

**Please answer Question 1. [50 marks]**

1. Consider the utility function



where the vector or parameters  satisfies  and .

(1) Use the Lagrange approach to solve for the Marshallian demand function. [10 marks]

(2) Determine the indirect utility function and verity Roy’s identity. [10 marks]

(3) Determine the expenditure function (without solving the expenditure minimisation problem) and use Shepard’s lemma to obtain the Hicksian demand function. [10 marks]

(4) Check whether your answer to (3) satisfies the relation . Write down the Slutsky matrix (including the decomposition of the elements in the matrix) and proof the Law of Demand. [20 marks]

（1）令收入为I，商品的价格为

构造拉格朗日方程有，依次对x和λ求偏导数有



将上面的n个式子变形为，再将这n个式子求和左边为1，右边等于λI，于是得到λ=1/I，于是将λ代入到前n个偏导数中可以得到马歇尔需求函数为

1. 由对偶性可知只需将上题中所得的马歇尔需求函数代入到效用函数的方程中去，则有间接效用函数

紧接着验证罗伊方程，则

1. 由（2）中的间接效用函数可知

于是可以得到，解出支出函数为

使用谢泼特引理可知希克斯需求函数为

1. 将（2）中的间接效用函数代入到希克斯需求函数中可得



**Please choose ONE from Questions 2-3 to answer. [50 marks]**

2. Suppose there are a group of migrants who have decided to move from rural to urban areas. An individual  may choose to live and work in one of the  cities. If living in city ,  has a Cobb-Douglas utility function and faces the following utility-maximisation problem:

 

where  is ’s consumption of a tradable composite good in city ; its price is the same everywhere and normalised to 1.  is ’s consumption of a nontradable composite good (e.g., housing) in city  and its price in city  is ;  and  represent the elasticities of  and , respectively;  measures the population size in city ;  is a vector including observed characteristics of city ;  represents a non-monetary cost of migration that is related to the distance from ’s home village to city ;  is ’s idiosyncratic component of utility which is assumed to be independent of migration distance and city characteristics (e.g., individual fixed effects attached to city );  is the income can earn in city ;  denotes the city-level unobserbables.

1. Please solve the individual ’s utility maximisation problem and find the optimal demand for tradable () and nontradable () composite goods in the city. [20 marks]
2. Plug the demand functions in (1) to the utility function and then, write down the indirect utility function  in log form. [5 marks]

(3) Because the price of nontradable goods, , is not directly observed, we further assume this price to be a linear function of observed city characteristics:



where  denotes the city-level unobservables. Substitute the above expression for  to the indirect utility function and work out ’s *marginal willingness to pay* which measures the amount of income  is willing to give up (i.e., ) in order to live in a larger city (i.e., a city with one more unit of population ). [15 marks]

(4) Comment on migrants’ preferences for larger cities based on your answer to (3). [10 marks]

（1）由题目中的代表性个体i的最大化问题（即效用最大化），这时可以构造拉格朗日函数得到



分别对C，H和λ求偏导可以得到，其中令





将（1）（2）移项作比可得进一步化简有于是得到对可贸易品与不可贸易品的消费分别为

（2）将（1）中所得到的马歇尔需求函数直接代入到效用函数方程中可到间接效用函数形



（3）将题目中所给的价格P的形式代入到（2）中所得到的间接效用函数可得



对S求偏导数可以得到



（4） 这里主要讨论偏导数的正负号问题，如何导数的结果为正，则会向大城市转移；如何导数为负，则不会向大城市转移；如果导数为0，则当前的状态即是最好的。

3. Suppose you are both a consumer and a producer, and have the following intertemporal utility maximisation problem:



where  denotes your consumption at every period ; there is a log instantaneous utility function; your capital stock  depreciates at the rate of  at every time period;  denotes your production function, which in turn determines your income.

1. Please solve the intertemporal maximisation problem and show the Euler equation(s). [20 marks]

(2) Draw a diagram of  against  and find the steady-state consumption and capital stock. [10 marks] Hint: At the steady state, the growth rate of a variable is zero.

(3) Use the diagram in (2) to discuss how consumption and capital stock evolve when they are off the steady state. [20 marks]