第三次上机作业(第4，5章)

1. 书(9版220页第 12题）

Use Romberg integration to compute approximations to

$$\int_0^{48} \sqrt{1+(\cos x)^2}\,dx$$

a. Determine $R1,1$, $R2,1$, $R3,1$, $R4,1$, and $R5,1$, and use these approximations to predict the value of the integral.

b. Determine $R2,2$, $R3,3$, $R4,4$, and $R5,5$, and modify your prediction.

c. Determine $R6,1$, $R6,2$, $R6,3$, $R6,4$, $R6,5$, and $R6,6$, and modify your prediction.

d. Determine $R7,7$, $R8,8$, $R9,9$, and $R10,10$, and make a final prediction.

e. Explain why this integral causes difficulty with Romberg integration and how it can be reformulated to more easily determine an accurate approximation.

$$
\begin{array}{lllllll}
R_{1,1} & & & & & & \\
R_{2,1} & R_{2,2} & & & & & \\
R_{3,1} & R_{3,2} & R_{3,3} & & & & \\
R_{4,1} & R_{4,2} & R_{4,3} & R_{4,4} & & & \\
\vdots & \vdots & \vdots & \vdots & \ddots & & \\
R_{n,1} & R_{n,2} & R_{n,3} & R_{n,4} & \cdots & & R_{n,n}
\end{array}
$$

$$
R_{k,2} = R_{k,1} + \frac{R_{k,1} - R_{k-1,1}}{3},
$$

$$
R_{k,j} = R_{k,j-1} + \frac{R_{k,j-1} - R_{k-1,j-1}}{4^{j-1} - 1}.
$$

2. 书(9版275页第 11题）

Given the initial-value problem

$$\begin{cases} y' = -y + t + 1, \ 0 \le t \le 5 \\ y(0) = 1 \end{cases}$$

with exact solution $y = e^{-t} + t$

Approximate *y*(5) with $h = 0.2$, $h = 0.1$, and $h = 0.05$.

a. **Euler's method;** **(P267)**
b. **Modified Euler's method;** **(P286)**
c. **Runge-Kutta Method order four.** **(P288)**

And analysis the error.