北京科技大学 200 ~200 学年度第一学期

Numerical Analysis Examination

试卷成绩（占课程考核成绩的 60%）								平时成绩	课程考核成绩
题号	一	二	三	四	五	六	小计		

学院：________
班级：________
学号：________
姓名：________

一、 **Answer question（40 points，4 points each）**

1. Let $x_1 = 0.5055\times 10^4, x_2 = x_3 = \cdots = x_{11} = 0.4500$, using four-digit rounding arithmetic to obtain the most accurate approximation to $S=\sum_{i=1}^{11} x_i$. Give your algorithm.

2. let $A=\begin{bmatrix}2&1&1\\1&3&2\\1&2&3\end{bmatrix}$, then $\rho(\boldsymbol{A})$=___, $\|A\|_1$=___, $\|A\|_\infty$=___, $\|A\|_F$ ___ $K_1(\boldsymbol{A})$ ___.

3. Steffensen's method is applied to a function g(x) using $p_0^{(0)}$=1 and $p_1^{(0)}=\sqrt{2}$ to obtain $p_0^{(1)} = 2.7802$. What is $p_2^{(0)}$?

4. A clamped cubic spline $S(x)$ for a function f is defined on $[1,3]$ by

$$S(x)=\begin{cases} S_0(x)=3(x-1)+2(x-1)^2-(x-1)^3, & \text{if} \quad 1\le x\le 2 \\ S_1(x)=a+b(x-2)+c(x-2)^2+d(x-2)^3, & \text{if} \quad 2\le x\le 3 \end{cases}$$

Given $f'(1)=f'(3)$, find a,b,c and d.

5. Let $f(x)\in C^n[a,b]$, if $f^{(n)}(x)=0, x\in[a,b]$, then divided difference $f[x_0,x_1,\cdots,x_n]$=______, for each $x_i\in[a,b], i=0,\cdots,n$. Let $f(x)=x^5\text{-}4x^3-8$, then the fifth divided difference f[-3,-2,-1,0,1,2]=_____

6. The explicit multistep method is given by

$\boldsymbol{w_{i+1}=\frac{4}{5}w_i+\frac{1}{5}w_{i-1}+3hf(t_i,w_i)}$, its characteristic equation is _______________,

Which has roots _______________ Does it satisfies root condition?_________(Yes/No)

Is it strongly stable or weakly stable or unstable?______________

7. Suppose C is positive constant , use Newton's method to equation $x^n - C = 0$, to find iteration sequence that approximate $\sqrt[n]{C}$, show that the sequence converge to $\sqrt[n]{C}$ of order 2.

8. Give some data of $f(x)$

x	1.8	1.9	2.0	2.1	2.2
$f(x)$	10.9	12.7	14.8	17.1	19.8

Use extrapolation formula

$$\begin{cases} N_1(h) = \dfrac{f(x_0+h) - 2f(x_0) + f(x_0-h)}{h^2} \\ N_j(h) = N_{j-1}(\dfrac{h}{2}) + \dfrac{N_{j-1}(\frac{h}{2}) - N_{j-1}(h)}{4^{j-1}-1} \end{cases}$$

to approximate $f''(2)$ as accurately as possible.

9. Integration $I_n = \int_0^1 x^n e^{x-1} dx, n = 0,1,2,\cdots$ satisfies the recursive equation $I_n = 1 - nI_{n-1}$. If $I_0 = 1 - e^{-1} \approx 0.632$ (3 significant digits), compute the error of I_5.Is this algorithm: $\begin{cases} I_n = 1 - nI_{n-1} \\ I_0 = 1 - e^{-1} \approx 0.632 \end{cases}$ stable ?

10. Let A be a $n \times n$ symmetric matrix, **x** be a n-dimensional vector .

Compute the number of operation for $x^t Ax$.

11. Complete the following **Matlab Progam** for Newton's method, which start with

```
function [x,k]=newton (F1,F2,x0,e) % Newton 法求 f(x)=0 的根
  % F1=f(x),F2=f'(x);
% x0:初值，e:误差限;
% x 近似解，k: 迭代步数;
```

12. According to the following data, Use Romberg integration to approximate $\int_{-2}^{2} f(x)dx$ as accurately as possible。

X	- 2	- 1	0	1	2
f (x)	2	1.5	1	-1	-2

13. let $\boldsymbol{A} = \begin{bmatrix} \mathbf{0} & \mathbf{2} & \mathbf{3} \\ \mathbf{1} & \mathbf{1} & \mathbf{-1} \\ \mathbf{0} & \mathbf{-1} & \mathbf{1} \end{bmatrix}$, find the permutation matrix $\boldsymbol{P}$ so that $\boldsymbol{PA=LU}$ (Doolittle form)

P= $\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$, L= $\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$, U= $\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$

二 、(**12 points**)Give some data of F(x) and F' (x):

X_i	-1	0	1
$F(x_i)$	1	2	1
$F'(x_i)$		3	1

Suppose $F(x) \in C^5[-2, 2]$, find osculating polynomial $P_4(x)$ of degree at most 4 such that

$$P_4(-1) = 1, \quad P_4(0) = 2, \quad P_4(1) = 1, \quad P_4'(0) = 3, \quad P_4'(1) = 1.$$

Find error: $F(x) - P_4(x)$。

三、(**10 points**) Give some data of $F(x) = -\int_x^{\infty} \frac{\sin t}{t} dt$ at x=0.32, 0.4, 0.55, 0.6, 0.7,

x_i	0.32	0.4	0.55	0.6	0.7
$F(x_i)$	0.2985	0.39646	0.49311	0.58813	0.68122

1) Find divided difference table for F(x) ;

2) Find third-order Newton forward divided difference polynomial, and find F(0.36).

四、 (**10 points**) Determine $A_1, A_2, x_1, and\ x_2$ so that quadrature formula

$$\int_{-1}^{1} f(x)dx \approx A_1 f(x_1) + A_2 f(x_2)$$

has the highest of precision。Find the highest of precision。

五、(**15 points**) Give the **Midpoint Method**: $y_{n+1} = y_{n-1} + 2hf(x_n, y_n)$

1) Find its local truncation error, what is its order?

2) Use the **Midpoint Method** to find the approximating solutions w_2, w_3 for the initial value problem $\begin{cases} y' = -y, & 0 \le x \le 2.25 \\ y(0) = 1 \end{cases}$, with $h = 0.25$;

and find actual errors。(小数点后保留 3 位,w_1 用 Euler 公式计算)。

3) Use the answers generated in part 2) and piecewise linear interpolation to approximate y(0.6).

六、((**13 points**)) The linear system ***Ax=b*** is given by

$$\begin{cases} x_1 - 2x_2 + 2x_3 = 1 \\ -x_1 + x_2 - x_3 = 1 \\ -2x_1 - 2x_2 + x_3 = 1 \end{cases}$$

(1) Find its **Jacobi** iterative method and **Gauss-Seidel** iterative method;

(2) Discussing the convergence of both iterations;

(3) Find the first two iterations of the convergent methods for above linear system, using $x^{(0)} = (0, 0, 0)^T$。