EE2004: Digital Signal Processing

Tutorial 7

(March 9, 2017)

1. Using z-transform properties determine the transform of the following sequences; specify the RoC in each case: (a) $x_1(n) = 7 \cdot (3)^n u[n] - 4 \cdot (-1)^n u[n-1]$, (b) $x_2[n] = 6n\, u[n] + 2\, e^{-5n} u[n]$, (c) $x_3[n] = x_1[n-1]$, (d) $x_4(n) = x_2[n+2]$.

2. Let $x_1[n] = \alpha^n u[n]$ and $x_2[n] = \beta^n u[n]$, where $0 < \alpha, \beta < 1$ and $\alpha \neq \beta$. Find the $y[n] = x_1[n] * x_2[n]$ by computing the inverse z-transform of $X_1(z)\, X_2(z)$. To compute the inverse z-transform of $X_1(z)\, X_2(z)$, express it as $P_1(z)/Q_1(z) + P_2(z)/Q_2(z)$, and then find the inverse transform of each term, assuming the RoC to be outside of a certain region.

3. Consider the finite-duration sequence $h[n]$ defined in the range $0 \leq n \leq N-1$. Let $g[n] = h^*[N-1-n]$.
   (a) Express $G(z)$ in terms of $H(z)$.
   (b) Now let $g[n] = h[n]$, i.e., $h^*[N-1-n] = h[n]$. If z_0 is a non-trivial zero of H(z), can you find another zero related to z_0 ?
   (c) Suppose now $h[n] \in \mathbb{R}$ and z_0 is a non-trivial zero such that $|z_0| \neq 1$. How many other zeros of $H(z)$ can you determine from the knowledge of z_0 ? How will your answer change if $|z_0| = 1$?

4. The only non-trivial poles of $X(z)$ are at $1/3, 2/3$, and 2. What can you say about the RoC in each of the following cases?
   (a) $x[n]$ is right sided.
   (b) $X(e^{j\omega})$ exists.

5. Let $h[n]$ be the impulse response of a causal and stable system. The corresponding $H(z)$ is $\dfrac{1}{1 + \sum_{k=1}^{K} a_k z^{-k}}$, where $a_K \neq 0$.
   (a) What can you deduce about the zeros of $H(z)$?
   (b) Comment on the zeros and poles of the z-transforms corresponding to the sequences (a) $n\, h[n]$, (b) $h[-n]$, (c) $h[n+n_0]$, (d) $h[n-n_0]$.
   (c) Let $g[n]$ be such that $G(z) = \dfrac{1}{H(z)}$. Is the system causal? absolutely summable? Justify your answers. What can you say about the poles of $G(z)$ and hence about its RoC?

6. Let $x[n] = (0.3)^n \exp(j0.25\, \pi n)\, u[n]$. The *autocorrelation* sequence $r_{xx}[n]$ corresponding to $x[n]$ is defined as the convolution of $x[n]$ with $x^*[-n]$. Find (a) $X(z)$,

(b) the z-transform of $x^*[-n]$, (c) $R_{xx}(z)$ (by using properties), (d) $r_{xx}[n]$ from $R_{xx}(z)$.

7. A causal sequence $g[n]$ has the z-transform

$$G(z) = \left(1 + 3z^{-2} + 2z^{-4}\right) \cdot \sin(z^{-1})$$

Find $g[11]$.

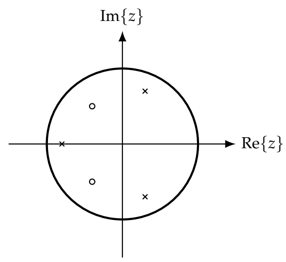
8. For each z-transform given below, find all possible inverse transforms: (a) $X_1(z) = \dfrac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$, and (b) $X_2(z) = \dfrac{1}{(1 - 0.5z^{-1})^2}$.

9. It is known that $x[n] \in \ell_1$ and has z-transform

$$X(z) = \frac{1 - 2z^{-1}}{1 - \frac{5}{2}z^{-1} + z^{-2}}$$

Determine $x[n]$ using partial fraction expansion.

10. The pole-zero plot shown on the right corresponds to that of a *causal* system. The corresponding *inverse system* $H_i(z)$ is defined to satisfy $H(z) \cdot H_i(z) = 1$. Can $H_i(z)$ be both causal and stable? Clearly justify your answer.



11. **Computer assignment** Consider

$$X(z) = \frac{1 - 0.9\,z^{-7}}{1 - 0.9\,z^{-1}}$$

Plot its poles and zeros. Lookup the following commands: `zplane`, `roots`, `impz`, `residuez`. Obtain the impulse response using `impz` and compare with the theoretical result. Repeat the experiment for the following two cases:

(a)

$$X(z) = \frac{1 - 0.9\,z^{-8}}{1 - 0.9\,z^{-1}}$$

(b)

$$X(z) = \frac{1}{1 - 0.9\,z^{-1}}$$