Computational Neuroscience
IIT Madras

Assignment 2 : FitzHugh-Nagumo Neuron Model

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EE15B025

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1 Aim

Simulate and Understand FitzHugh-Nagumo neuron model

2 FitzHugh-Nagumo neuron model

Two variable FitzHugh-Nagumo neuron model can be simulated using the following equations :

$$\frac{d\nu}{dt} = f(\nu) - \omega + I_m \qquad \text{where } f(\nu) = \nu(a-\nu)(\nu-1)$$
$$\frac{d\omega}{dt} = b\nu - r\omega$$

To obtain the nullclines, set $\frac{d\nu}{dt} = 0$ and $\frac{d\omega}{st} = 0$, which gives us :

$$\nu - \text{nullcline} \longrightarrow \omega = \nu(a-\nu)(\nu-1) - I_m$$
$$\omega - \text{nullcline} \longrightarrow \omega = \frac{b\nu}{r}$$

The above equations have been used to generate the below plots using the following values of variables :

Variable	**Value**
a	0.5
b	0.1
r	0.1
I_m	0.1

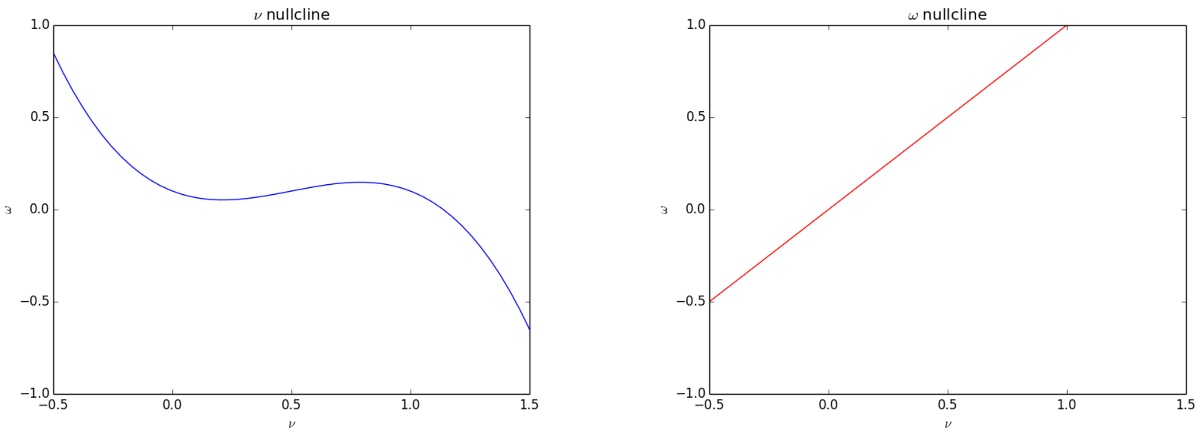


Figure 1: Nullclines for FitzHugh-Nagumo neuron model

The python file for creating the plots has been named *"null_cline.py"*

3 Model analysis for varying values of I_m

3.1 For $I_m = 0$

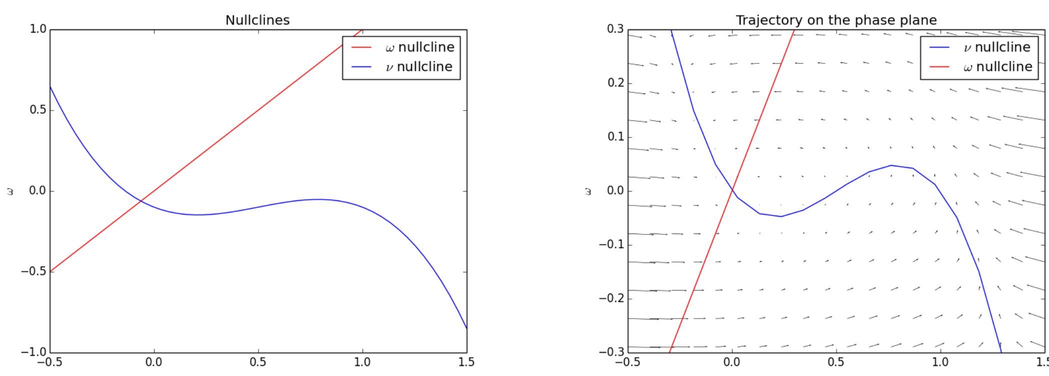
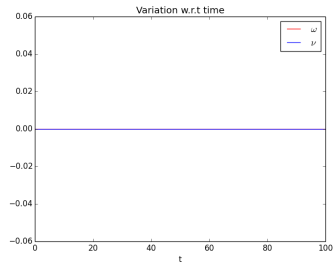
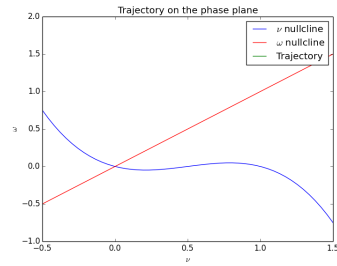
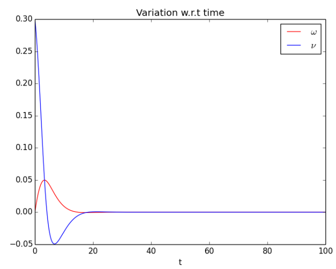
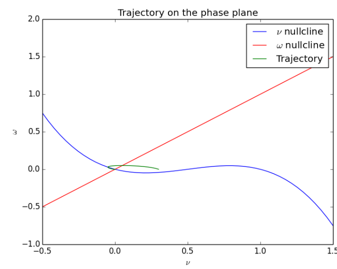
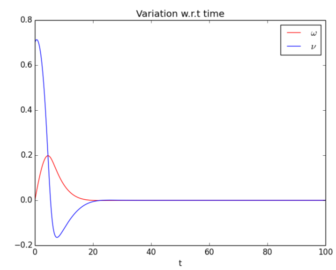
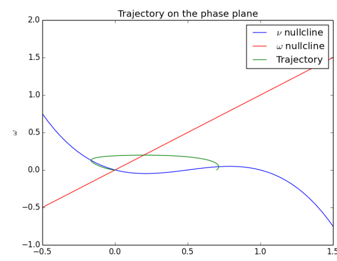
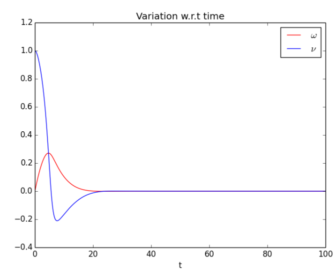
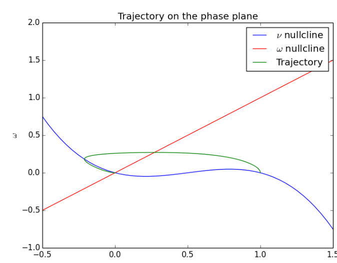


Figure 2: Phase Plot for $I_m = 0$

The table gives the values of variables used :

Variable	**Value**
a	0.5
b	0.1
r	0.1
I_m	0.0
$\omega(0)$	0.0

As we can see from the plots below, this is a stable fixed point i.e., for a small perturbation there is a return to the fixed point.

<table>
<tr><th>v(0)</th><th>Variation w.r.t time</th><th>Trajectory on Phase Plot</th></tr>
<tr><td>0.0 ($< a$)</td><td></td><td></td></tr>
<tr><td>0.3 ($< a$)</td><td></td><td></td></tr>
<tr><td>0.7 ($> a$)</td><td></td><td></td></tr>
<tr><td>1.0 ($> a$)</td><td></td><td></td></tr>
</table>

The python file for creating the plots has been named *"I_0.py"*

3.2 For $I_1 < I_m < I_2$

Oscillations are exhibited between I_1 and I_2. This happens when the ω-nullcline intersects the ν-nullcline at a point where the ν-nullcline has a positive slope. Hence I_1 and I_2 can be identified by finding the local minima and local peak in the ν-nullcline and finding the value of current for which the ω-nullcline intersects the ν-nullcline at those points.

For a better understanding, initially claiming $I_m = 0$, we get :

$$\nu - \text{nullcline} \longrightarrow \omega = \nu(a-\nu)(\nu-1) = f(v)$$
$$\omega - \text{nullcline} \longrightarrow \omega = \frac{b\nu}{r}$$

Find the values of ν for which ν-nullcline has a local minima and maxima using the python code attached. Now, increasing I_m just shifts the ν-nullcline up the y-axis. ω-nullcline is independent of the value of I_m. Hence, find the ω-nullcline values for the above specified minima and maxima and find the current value for which the ν-nullcline intersects the ω-nullcline at these values of ω. This gives us,

$$f(\nu) + I_m = \frac{b\nu}{r}$$
$$\implies I_m = \frac{b\nu}{r} - \nu(a-\nu)(\nu-1)$$

The computed values for I_1 and I_2 are :

$$I_1 = 0.19803611$$
$$I_2 = 0.75827886$$

The corresponding phase plots are :

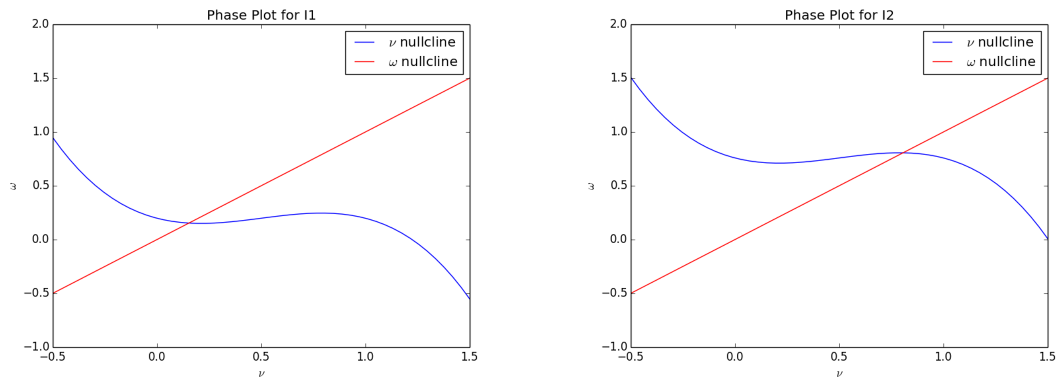


Figure 3: Phase Plot for I_1 and I_2

The code for finding the values of I_1 and I_2 and plotting the corresponding phase plots is named as *"find_I1I2.py"*

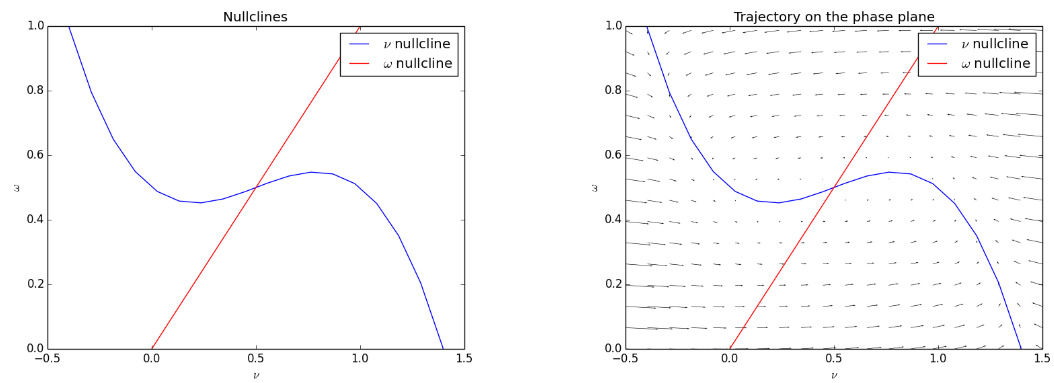
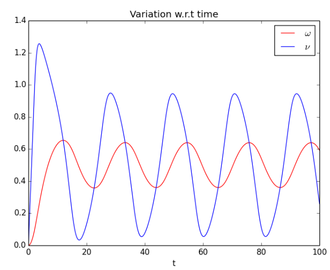
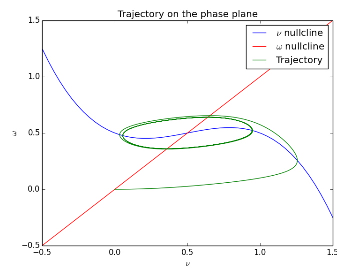
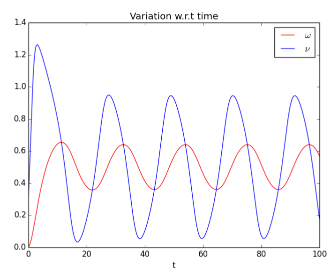
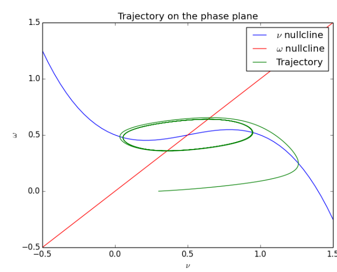
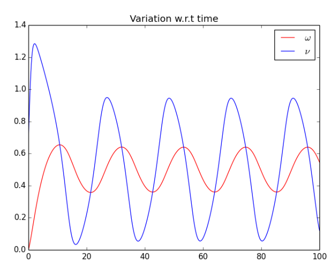
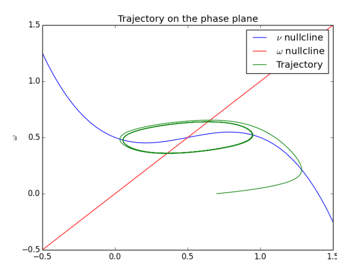
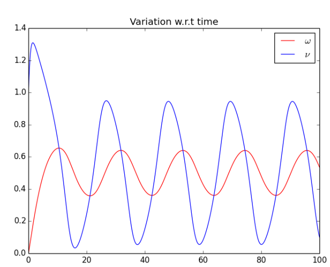
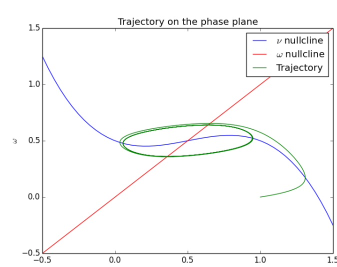


Figure 4: Phase Plot for $I_m = 0.5$

The table gives the values of variables used :

Variable	Value
a	0.5
b	0.1
r	0.1
I_m	0.5
$\omega(0)$	0.0

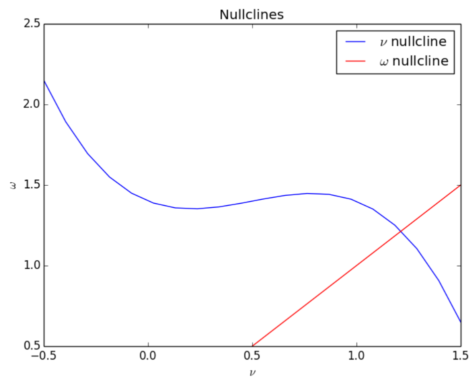
As we can see from the plots, the fixed point is unstable i.e., for a small perturbation there is a no return to the fixed point. The trajectory as well as the limit cycle can be seen on the phase plots in the below table.

v(0)	**Variation w.r.t time**	**Trajectory on Phase Plot**
0.0 ($< a$)		
0.3 ($< a$)		
0.7 ($> a$)		
1.0 ($> a$)		

The python file for creating the plots has been named *"I1_I2.py"*

3.3 For $I_m > I_2$

Since we know the value of I_2 to be 0.75827886, let's consider $I_m = 1.4$ for the purpose of demonstration.



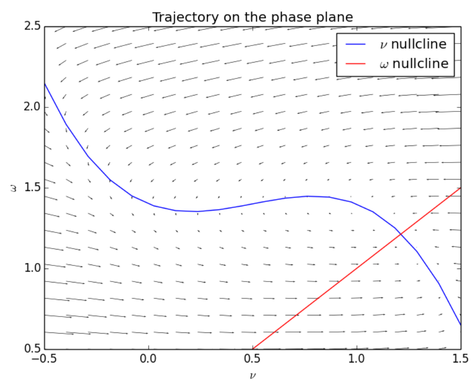
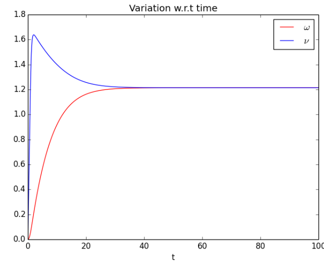
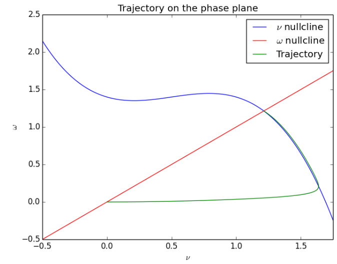
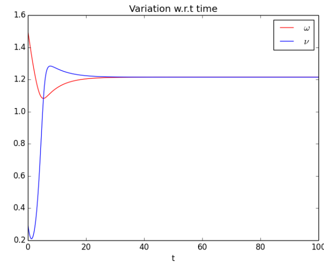
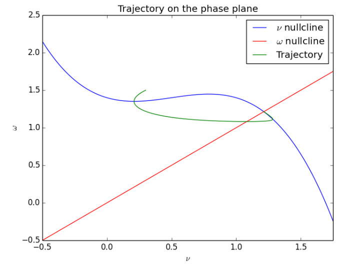
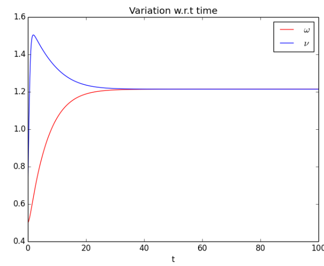
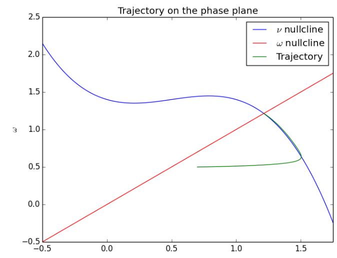
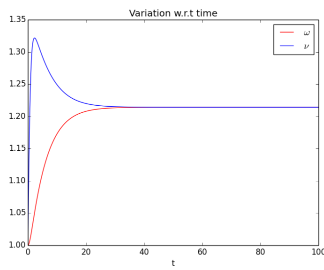
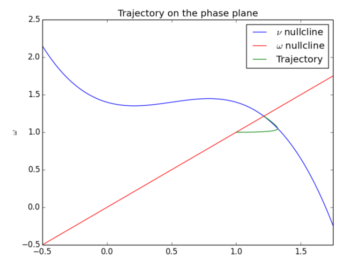


Figure 5: Phase Plot for $I_m = 1.4$

The table gives the values of variables used :

Variable	**Value**
a	0.5
b	0.1
r	0.1
I_m	1.4

As we can see from the plots below, this is a stable fixed point i.e., for a small perturbation there is a return to the fixed point.

$\nu(0)$	$\omega(0)$	Variation w.r.t time	Trajectory on Phase Plot
0.0 ($< a$)	0.0		
0.3 ($< a$)	1.5		
0.7 ($> a$)	0.5		
1.0 ($> a$)	1.0		

The python file for creating the plots has been named *"I_2.py"*

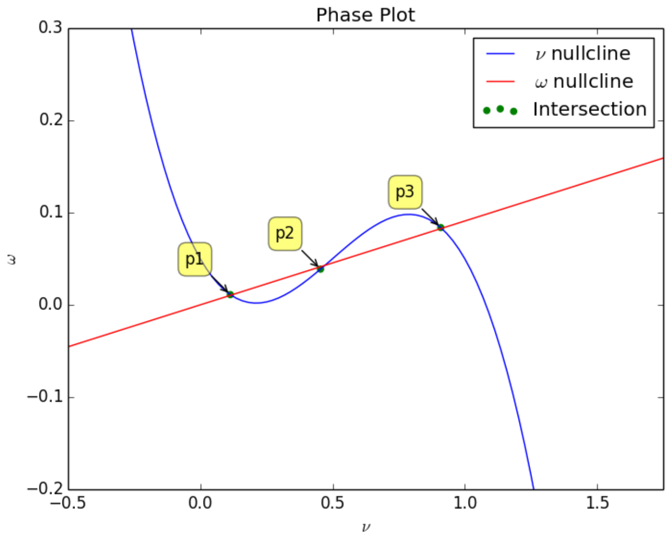
3.4 Bistability

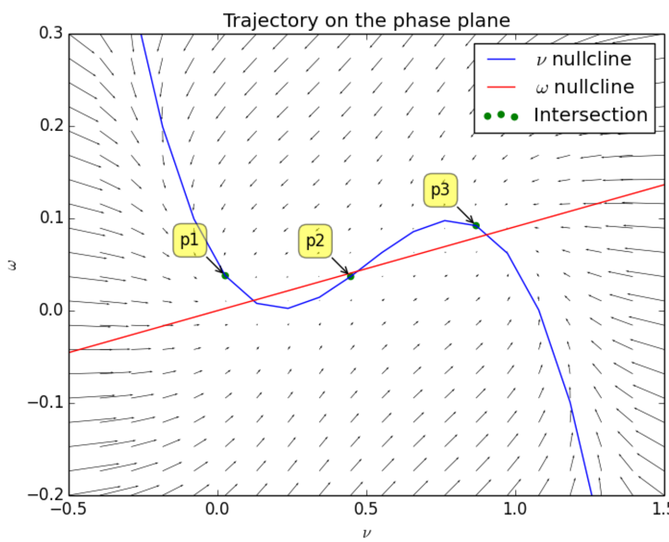
Some real neurons exhibit bistable behavior – their membrane voltage can remain at tonically high ("UP" state) or tonically low ("DOWN" state) values

The table gives the values of variables used :

Variable	Value
a	0.5
b	0.1
r	1.1
I_m	0.05

The phase plot is given by,

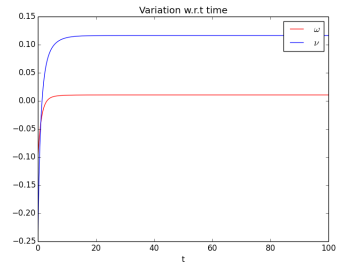
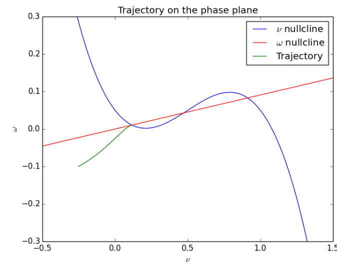
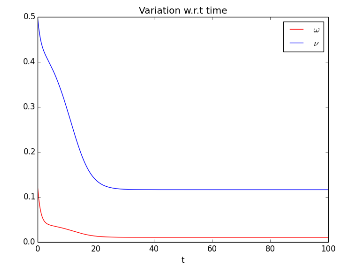
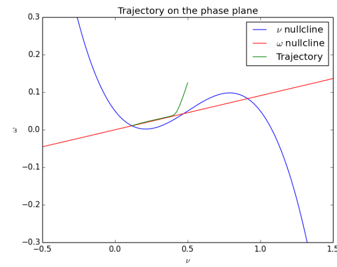
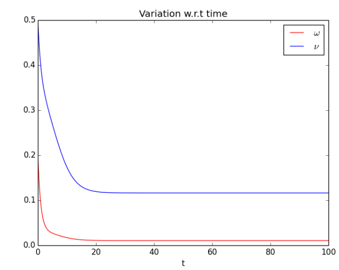
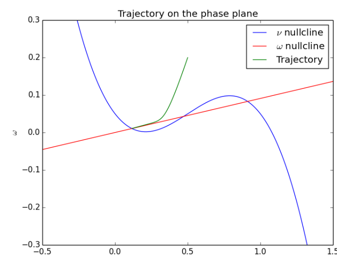
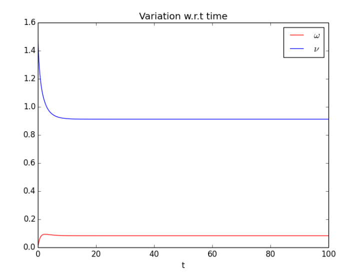
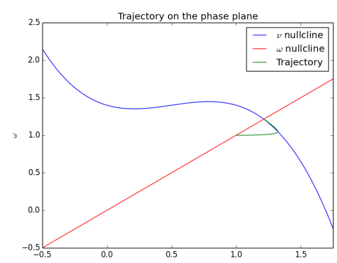




where the coordinates corresponding to the intersection are :

$$\begin{array}{llll} p1: & \nu = 0.113636363636, & \omega = 0.0110842411721 & ;\text{Stable} \\ p2: & \nu = 0.454545454545, & \omega = 0.0387302779865 & ;\text{Saddle} \\ p3: & \nu = 0.909090909091, & \omega = 0.0838091660406 & ;\text{stable} \end{array}$$

For different starting points, the trajectories have been depicted :

$\nu(0)$	$\omega(0)$	**Variation w.r.t time**	**Trajectory on Phase Plot**
-0.25	-0.1		
0.5	1.125		
0.5	0.2		
1.5	0.0		

The python file for creating the plots has been named *"bistable.py"*