Ganga Meghanath
gmeghana@purdue.edu

Project 3
CS 514 Numerical Analysis, Fall 2022

2022-11-28

1 Section 7.2 Qn.1

Write a program for computing $\int_0^x e^{-t^2} dt$ by summing an appropriate Taylor Series until individual terms fall below 10^{-8} in magnitude. Test your program by calculating the values of this integral for $x = 0.0, 0.1, 0.2, ..., 1.0$. Taylor expansion of

$$e^{-x^2} = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{k!}$$

Integrating this gives us,

$$\int_0^x e^{-t^2} dt = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1) \cdot k!}$$

x	$\int_0^x e^{-t^2} dt$	$\lvert 0.5\sqrt{\pi}\text{erf}(x) - \int_0^x e^{-t^2} dt \rvert$
0	0	0
0.1	0.099667667	2.38E-09
0.2	0.197365029	2.35E-09
0.3	0.291237884	1.33E-09
0.4	0.379652839	7.03E-10
0.5	0.461281007	3.93E-10
0.6	0.535153533	5.98E-09
0.7	0.600685665	3.24E-09
0.8	0.657669858	1.98E-09
0.9	0.706241514	1.34E-09
1	0.746824134	1.01E-09

2 Section 7.2 Qn.2

Section 7.2 Qn.2 : Write a computer program that estimates $\int_a^b f(x)dx$ by $\int_a^b S(x)dx$ where S is the natural cubic spline having knots $a + ih$ and interpolating f at these knots. Here $0 \le i \le n$ and $h = (b - a)/n$. First obtain a formula for

$$\int_{t_0}^{t_n} S(x)dx$$

starting with Equation (7) in section 6.4 (p.351). Then write the subprogram to compute this. Test your code on these well known integrals:

(a) $\frac{4}{\pi} \int_0^1 (1 + x^2)^{-1} dx$

(b) $\frac{1}{\log 3} \int_1^3 x^{-1} dx$

Suppose

$$\int_a^b f(x)dx \sim \int_a^b S(x)dx$$

We know,

$$S_i(x) = \frac{z_i}{6h_i}(t_{i+1} - x)^3 + \frac{z_{i+1}}{6h_i}(x - t_i)^3 + (\frac{y_{i+1}}{h_i} - \frac{z_{i+1}h_i}{6})(x - t_i) + (\frac{y_i}{h_i} - \frac{z_i h_i}{6})(t_{i+1} - x)$$

Integrating $S_i(x)$,

$$IS_i(x) = \int S_i(x)dx = -\frac{z_i}{6h_i}\frac{1}{4}(t_{i+1}-x)^4 + \frac{z_{i+1}}{6h_i}\frac{1}{4}(x-t_i)^4 + (\frac{y_{i+1}}{h_i} - \frac{z_{i+1}h_i}{6})\frac{1}{2}(x-t_i)^2 - (\frac{y_i}{h_i} - \frac{z_i h_i}{6})\frac{1}{2}(t_{i+1}-x)^2$$

Therefore,

$$\int_{t_0}^{t_n} S(x)dx = \sum_{i=0}^{n-1} \int_{t_i}^{t_{i+1}} S_i(x)dx$$

$$\int_{t_0}^{t_n} S(x)dx = \sum_{i=0}^{n-1} IS_i(t_{i+1}) - IS_i(t_i)$$

2.1 a. $\frac{4}{\pi}\int_0^1 (1+x^2)^{-1}dx$

n	$\int_0^1 S(x)dx$	$\lvert 1-\int_0^1 S(x)dx\rvert$
4	0.8182185716675513	0.1817814283324487
8	0.915112979528867	0.08488702047113295
16	0.9589259706642688	0.04107402933573123

2.2 b. $\frac{1}{\log 3}\int_1^3 x^{-1}dx$

n	$\int_1^3 S(x)dx$	$\lvert 1-\int_1^3 S(x)dx\rvert$
4	0.8384495257172797	0.1615504742827203
8	0.9214177502443295	0.07858224975567052
16	0.9613423101212144	0.038657689878785595