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Matrix Assignment

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1 Problem statement:

Construct a triangle PQR in which QR=6cm, $\angle Q = 60^\circ$ and PR - PQ = 2cm.

Law of Cosines

The law of Cosines relates the length of the triangle to the cosines of one of its angles. It states that, if the length of two sides and the angle between them is known for a triangle, then we can determine the length of the third side. It is given by:

$$q^2 = p^2 + r^2 - 2pr\cos\theta \quad (1)$$

SOLUTION:

Steps of Construction:

Step 1:

Let P,Q and R be the vertices of the triangle with coordinates.

Given QR length is a=6cm, So the coordinates of vertices Q,R and P are :

$$Q = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad R = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \quad P = r \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix}$$

Also given the angle is $Q = 60^\circ$, so by finding the coordinates of the other side we can form a required triangle.

For the input parameters in Table 1.

| Symbol | Value | Description |
|----------|--|--------------|
| Q | $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ | Q Point |
| R | $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$ | R Point |
| θ | 60° | $\angle PQR$ |
| r-q | 2 | PR-PQ |
| r | | PR |
| q | | PQ |
| p | 6 | QR |

Table 1

By using the Cosine formula in $\triangle PQR$

$$q^2 = p^2 + r^2 - 2pr\cos\theta \quad (2)$$

$$0 = p^2 + r^2 - q^2 - 2pr\cos\theta$$

$$0 = (r+q)(r-q) + 6^2 - 2 \times 6 \times .5r$$

After simplification

$$2r - q = 18 \quad (3)$$

Given that,

$$r - q = 2 \quad (4)$$

Step 2:

Using equation (3) and (4),

$$\begin{pmatrix} 2 & -1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} r \\ q \end{pmatrix} = \begin{pmatrix} 18 \\ 2 \end{pmatrix} \quad (5)$$

(1) The augmented matrix for the above matrix equation is

$$\left(\begin{array}{cc|c} 2 & -1 & 18 \\ 1 & -1 & 2 \end{array} \right) \quad (6)$$

$$\xleftrightarrow{R_1 \leftarrow \frac{R_1}{2}} \left(\begin{array}{cc|c} 1 & -\frac{1}{2} & 9 \\ 1 & -1 & 2 \end{array} \right)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - R_1} \left(\begin{array}{cc|c} 1 & -\frac{1}{2} & 9 \\ 0 & -\frac{1}{2} & 14 \end{array} \right)$$

$$\xleftrightarrow{R_1 \leftarrow \frac{R_2}{-\frac{1}{2}}} \left(\begin{array}{cc|c} 1 & 0 & 16 \\ 0 & 1 & 14 \end{array} \right)$$

$$\Rightarrow X = \begin{pmatrix} 16 \\ 14 \end{pmatrix} \quad (7)$$

Using equation (7) we get ,

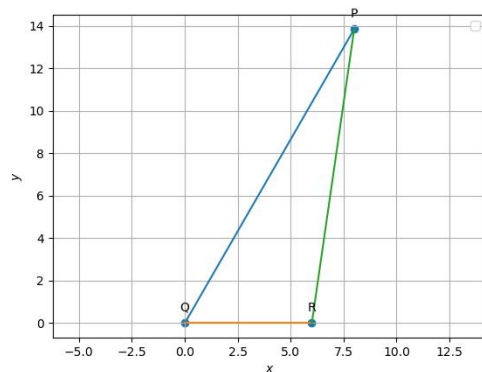
$$r = 16 \quad (8)$$

$$q = 14 \quad (9)$$

The vertices of $\triangle PQR$ are

$$p = 16 \begin{pmatrix} \cos 60^\circ \\ \sin 60^\circ \end{pmatrix}, Q = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, R = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \quad (10)$$

Result



Implementation

| Equation no | Role |
|-------------|--------------------------------|
| 1 | law of Cosines |
| 5 | Matrix form of Linear equation |
| 8 | Length of r |
| 9 | Length of q |

Construction

| vertex | coordinates |
|--------|---|
| P | $\begin{pmatrix} 8 \\ 14 \end{pmatrix}$ |
| Q | $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ |
| R | $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$ |

Download the code

<https://github.com/Gangagopinath/ASSIGNMENT/tree/main/assignment4>